Diagnostics III

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Introduction

<u>circular accelerators:</u>

single-particle equation of motion

E=beam energy
T=revolution period

derivatives wrt time, t

$e^{+/-}$: α=momentum compaction factor
p: $\alpha \leftarrow \alpha - 1/\gamma_t^2$

$2\pi f_{rf}$

$$\dot{\phi} = \alpha\omega\delta$$
$$\dot{\delta} = -\frac{e\dot{V}}{\omega E T}\phi$$

energy deviation

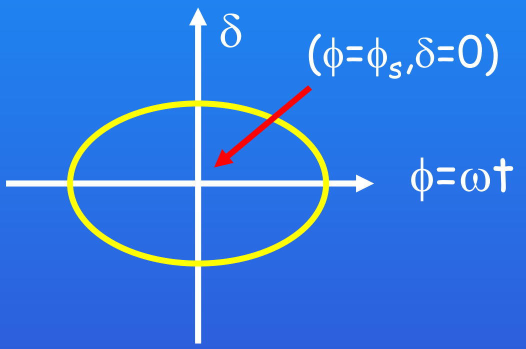
$\left.\frac{dV}{dt}\right|_{\phi=\phi_s}$

phase deviation

harmonic oscillator equations

$$\ddot{\phi} = \alpha\omega\dot{\delta} = -\frac{\alpha e\dot{V}}{ET}\phi$$
$$\ddot{\delta} = -\frac{e\dot{V}}{\omega ET}\dot{\phi} = -\frac{\alpha e\dot{V}}{ET}\delta$$

$$\ddot{\phi} + {\omega_s}^2\phi = 0$$
$$\ddot{\delta} + {\omega_s}^2\delta = 0$$



ellipses in phase space (for linear rf; i.e. small amplitude particle motion)

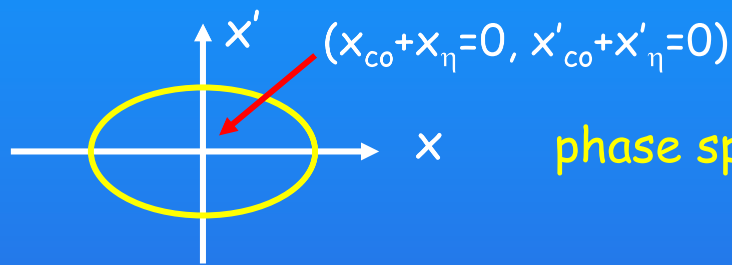
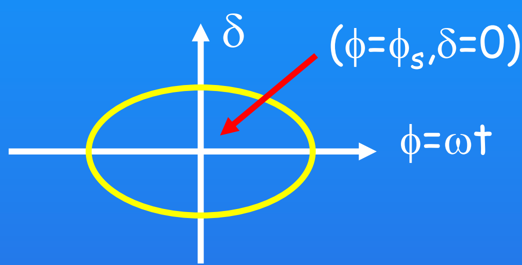
synchrotron frequency

$$\omega_s = 2\pi f_s = \left[\frac{\alpha e\dot{V}}{ET}\right]^{\frac{1}{2}}$$

synchrotron tune

$$\nu_s = \frac{f_s}{f_{rev}}$$

Analogy between transverse and longitudinal motion

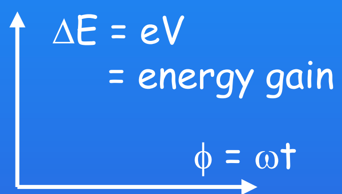
	transverse		longitudinal
equations of motion	$\begin{aligned}\ddot{x} + kx &= 0 \\ \ddot{p}_x + kp_x &= 0\end{aligned}$ $k \approx (\frac{\omega_\beta}{c})^2$		$\begin{aligned}\ddot{\phi} &= \alpha\omega\dot{\delta} = -\frac{\alpha e\dot{V}}{ET}\phi \\ \ddot{\delta} &= -\frac{e\dot{V}}{\omega ET}\dot{\phi} = -\frac{\alpha e\dot{V}}{ET}\delta\end{aligned}$ $\begin{aligned}\ddot{\phi} + {\omega_s}^2\phi &= 0 \\ \ddot{\delta} + {\omega_s}^2\delta &= 0\end{aligned}$
phase space			
second moments	$\sigma_x = \langle x^2\rangle^{\frac{1}{2}}$ $\sigma_{\dot{x}} = \langle \acute{x}^2\rangle^{\frac{1}{2}}$		$\sigma_\phi =< \phi^2 >^{\frac{1}{2}}$ $\sigma_\delta =< \delta^2 >^{\frac{1}{2}}$ $\sigma_z = \sigma_\phi \frac{c}{\omega}$
beam matrix	$\sigma_T = \begin{pmatrix} < x^2 > & < xx' > \\ < xx' > & < x'^2 > \end{pmatrix}$ (taking $<x>=0$, $<x'>=0$)		$\sigma_L = \begin{pmatrix} < \phi^2 > & < \phi\delta > \\ < \phi\delta > & < \delta^2 > \end{pmatrix} \frac{E}{\omega}$ (taking $<\phi>=0$, $<\delta>=0$)
emittance	$\begin{aligned}\epsilon_T &= (\det\,\sigma_T)^{\frac{1}{2}} \\ &= \sqrt{< x^2 >< x'^2 > - < xx' >^2}\end{aligned}$		$\begin{aligned}\epsilon_L &= (\det\,\sigma_L)^{\frac{1}{2}} \\ &= \sqrt{< \phi^2 >< \delta^2 > - < \phi\delta >^2}\,(\frac{E}{\omega})\end{aligned}$

linear accelerators:

Longitudinal emittance defined in the same way. Again:

$$
\begin{aligned}
\epsilon_L &= (\det\,\sigma_L)^{\frac{1}{2}} \\
&= \sqrt{<\phi^2><\delta^2>-<\phi\delta>^2}\,\left(\frac{E}{\omega}\right)
\end{aligned}
$$

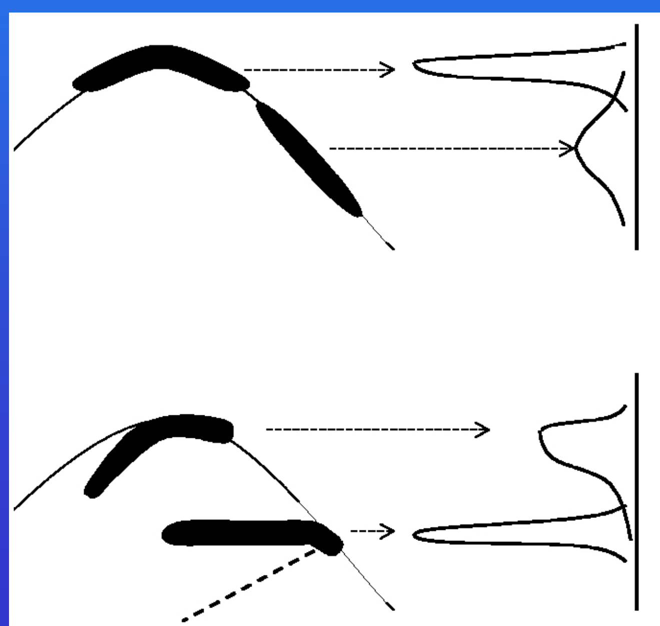
However, there is no analogous equation of motion in the longitudinal plane; the variation in energy of the single particles within the bunch is determined to a large extent by the length of the bunch and its overlap with the sinusoidal accelerating voltage



projection onto the
vertical (energy) axis → energy spread
horizontal (time) axis → bunch length

low bunch current

high bunch current

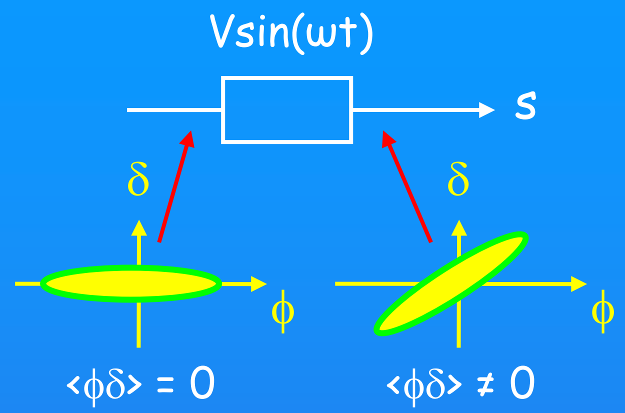


energy spread for low current bunches minimized by placing beam "on-crest"

energy spread for high currents minimized using "off-crest" acceleration (the accelerating and beam-induced voltages cancel)

Again,

$$\begin{aligned}\epsilon_L &= (\det \sigma_L)^{\frac{1}{2}} \\ &= \sqrt{<\phi^2><\delta^2> - <\phi\delta>^2}\ (\frac{E}{\omega})\end{aligned}$$



If the particle coordinates are uncorrelated (~~$\langle\phi\text{-}\delta\rangle=0$~~) $\langle\phi\delta\rangle=0$,

$$\begin{aligned}\epsilon_L &= <\phi^2>^{\frac{1}{2}}<\delta^2>^{\frac{1}{2}}\ (\frac{E}{\omega}) \\ &= \sigma_\phi \sigma_\delta\ (\frac{E}{\omega})\end{aligned}$$

where the bunch length (expressed in different ways) is

σ_z [m]

σ_ϕ [rad] = $\sigma_z \times (\omega / c)$

σ_t [s] = σ_z / c

and the energy spread is σ_δ

<u>linear accelerators (all particle types):</u>

the particle coordinates are generally correlated ($\langle\phi\delta\rangle \neq 0$)

the quantities measured and referred to are usually σ_z and σ_δ

<u>circular accelerators (e+/-)</u> with $\langle\phi\delta\rangle=0$ a reasonable approximation:

the energy spread σ_δ is small (~10^{-4}) and usually only σ_z is of interest

<u>circular accelerators (p)</u> with $\langle\phi\delta\rangle=0$ a reasonable approximation:

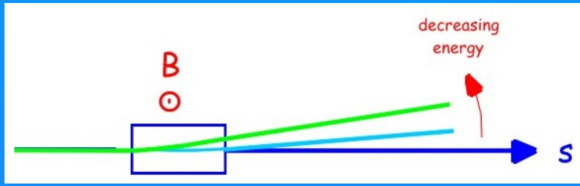
both σ_z and σ_δ are of interest so the longitudinal emittance may be computed

Beam Energy Spread (1)

principle: the beam size as measured, e.g. with a screen or wire, is the convolution of the natural beam size σ_β, and the contribution from the energy spread $\delta = \Delta E/E$:

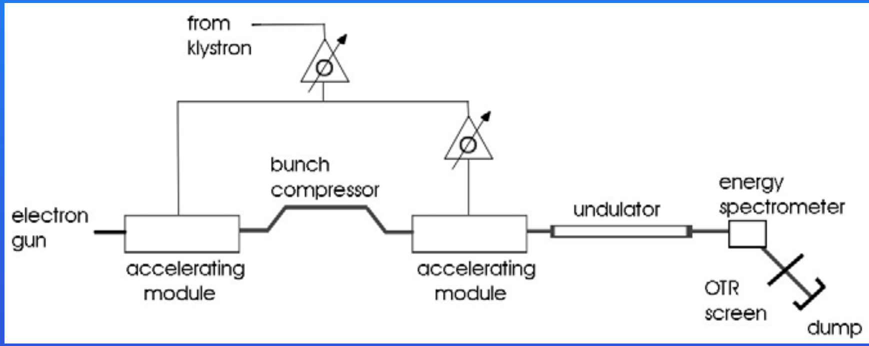
dispersion function (property of lattice)

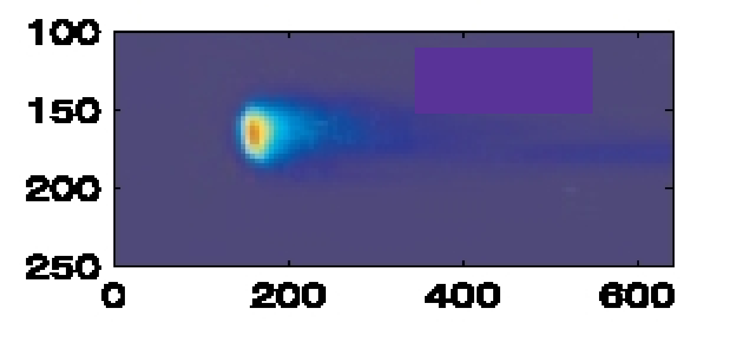
$$\sigma = \text{sqrt}\ [\sigma_\beta^2 + (\eta\delta)^2]$$



By proper selection of location (for a screen /wire), where η is large, the beam energy spread δ can be directly measured

example: TTF, single-bunch OTR images following spectrometer (courtesy F. Stulle, 2003)





energy spread measurement (as for screens):

digitized and fitted image

calibration: with etched lines of known spacing

issues (same as those for normal screen measurements):

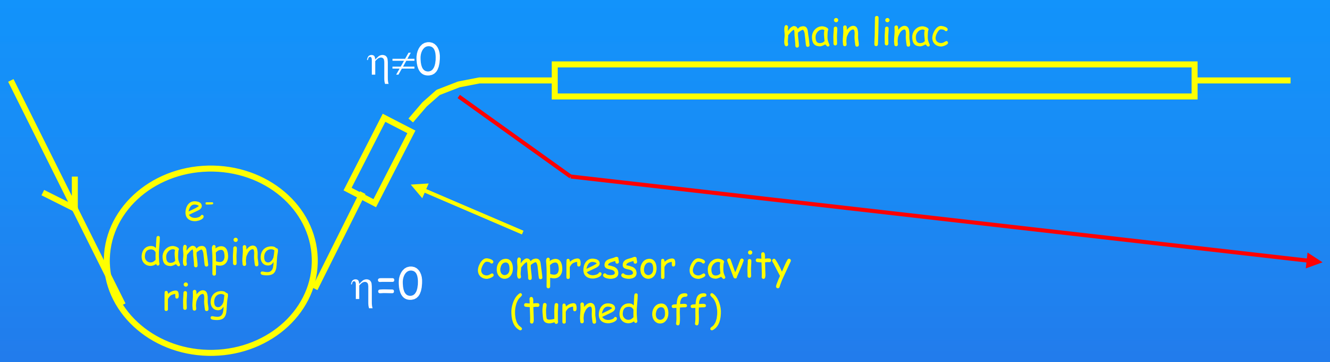
spatial and temporal resolution

radiation hardness of optics and camera

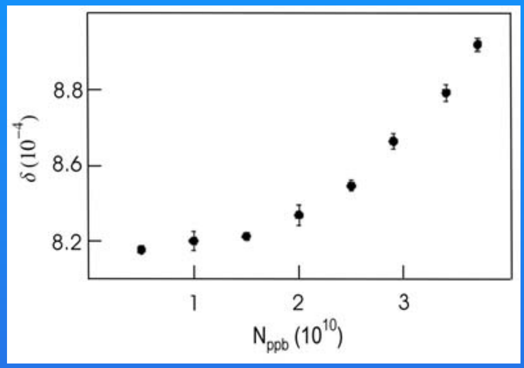
dynamic range (saturation of screen)

Beam Energy Spread (2) $\sigma = \mathrm{sqrt}\ [\sigma_\beta^2 + (\eta\delta)^2]$

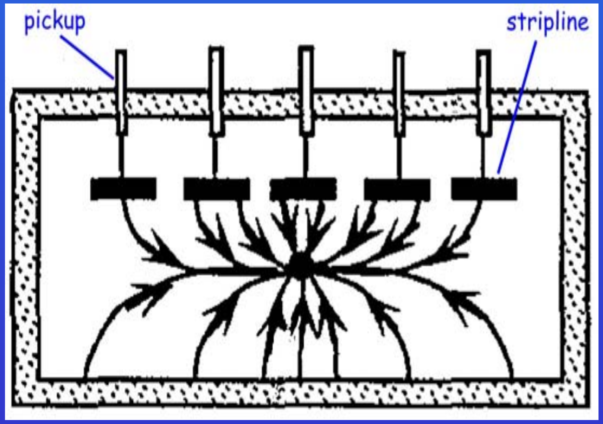
example: energy spread in the SLC electron damping ring (after extraction)



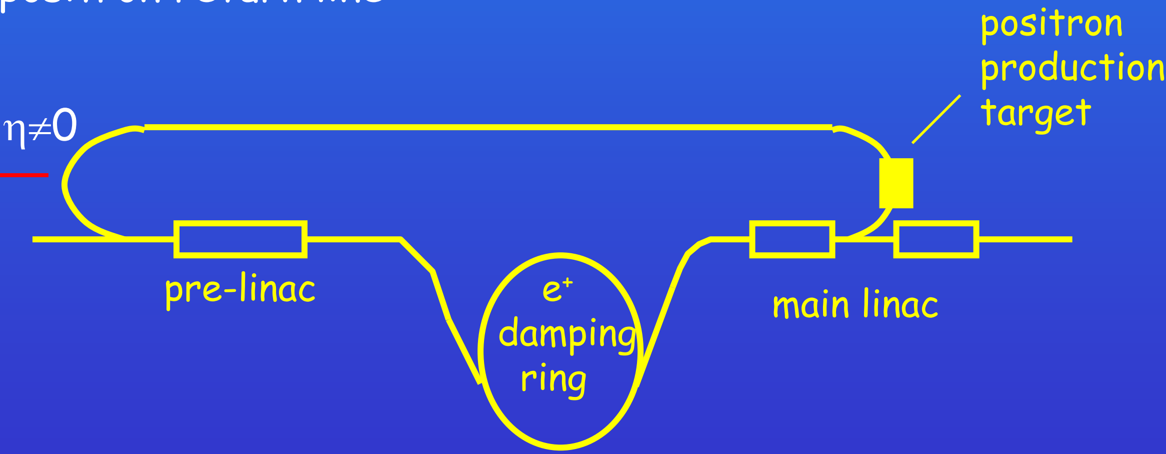
(wire scanner data)



example: energy spread in the SLC "positron return line"



BESM layout (courtesy T. Suwada, 2004)



Beam Energy Spread (3)

<u>principle:</u> in a storage ring, particles within the bunch perform phase and energy oscillations (out of phase by 90 deg) about a common reference point defined by the rf; measurement of the longitudinal distribution may therefore be used to determine the energy distribution of the bunch

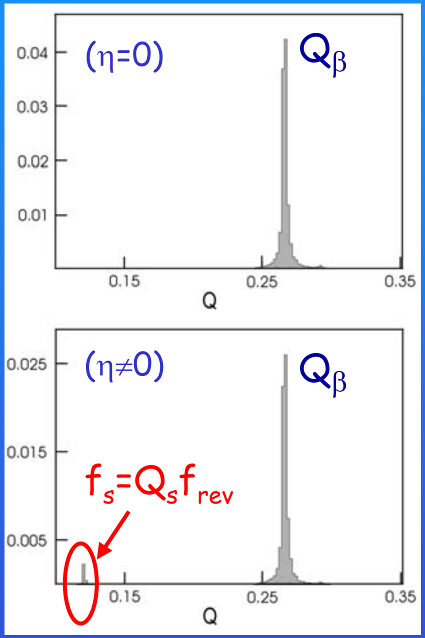
synchrotron frequency

$$\sigma_t = \frac{\alpha}{2\pi f_s}\left(\frac{\sigma_\delta}{E}\right) \rightarrow \frac{\sigma_\delta}{E} = \frac{2\pi f_s}{\alpha}\sigma_t$$

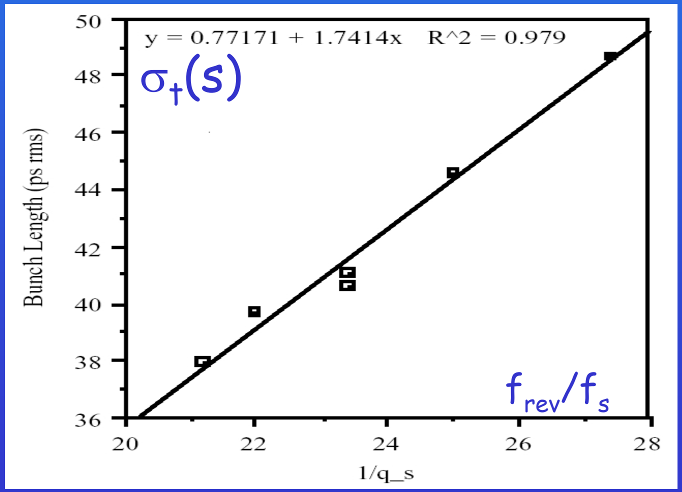
bunch length

momentum compaction

(ref: H. Wiedemann, Eq. 8.70)



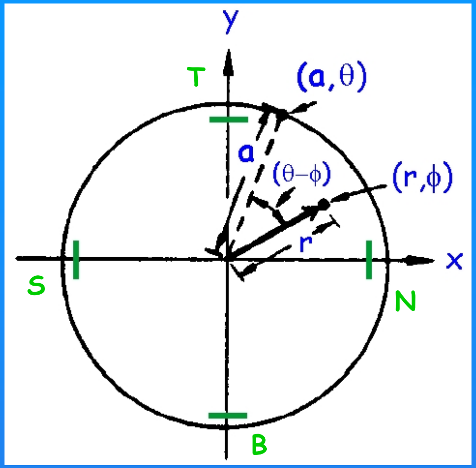
synchrotron frequency measured using FFT of turn by turn data in LEP (courtesy A. Müller, 2001)



example: energy spread in PEP-II B-Factory streak camera data (courtesy A. Fisher, 2004)

Beam Energy Spread (4), "Nonintercepting emittance monitor" R.H. Miller et al SLAC-PUB-3186

principle: measure the multipole moments induced by the passage of a charged particle beam



$$J_{image}(r,\phi,a,\theta) = \frac{I(r,\phi)}{2\pi a}\left[1+\sum_{k=1}^{\infty}\left(\frac{r}{a}\right)^{k}\cos k(\theta-\phi)\right]$$

integral of surface line density over area of electrode (see first lecture)

$$= \frac{I(r,\phi)}{2\pi a}\Bigg\{1+2\left[\frac{x}{a}\cos\theta+\frac{y}{a}\sin\theta\right]$$
$$+2\left[\frac{x^2-y^2}{a^2}\cos 2\theta+2\frac{xy}{a^2}\sin 2\theta\right]$$
$$+2\left[\left(\frac{x^3}{a^3}-3\frac{xy^2}{a^3}\right)\cos 3\theta-\left(\frac{y^3}{a^3}-3\frac{yx^2}{a^3}\right)\sin 3\theta\right]$$
$$+\text{higher order terms}\Bigg\}$$

(expansion of integral)

$$J_{image}(a,\theta)=\frac{I_{beam}}{4\pi^2 a\sigma_x\sigma_y}\int\int_{\substack{area\\ of\ pipe}}$$
$$\times\left[1+2\sum_{k=1}^{\infty}\left(\frac{r}{a}\right)^{k}\cos k(\theta-\phi)\right]$$
$$\times\exp\left[\frac{(x-\bar{x})^2}{2\sigma_y^2}\right]\exp\left[\frac{(y-\bar{y})^2}{2\sigma_y^2}\right]da$$

integrate over assumed Gaussian beam distributions in both transverse planes

Result:

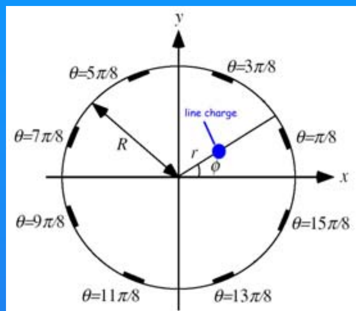
$$J_{image}(a,\theta)\simeq\frac{I_{beam}}{2\pi a}\Bigg\{1+2\left[\frac{\bar{x}}{a}\cos\theta+\frac{\bar{y}}{a}\sin\theta\right]$$
$$+2\left[\left(\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right)\cos 2\theta+2\frac{\bar{x}\bar{y}}{a^2}\sin 2\theta\right]$$
$$+2\left[3\left(\frac{\sigma_x^2-\sigma_y^2}{a^2}\right)+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right]\left(\frac{\bar{x}}{a}\cos 3\theta+\frac{\bar{y}}{a}\sin 3\theta\right)$$
$$+\text{higher order terms}\Bigg\}$$

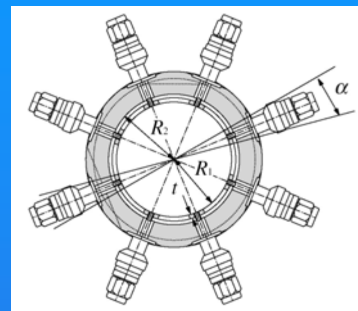
monopole terms
~ beam intensity
dipole terms
~ beam position
quadrupole terms
~ beam size
sextupole terms

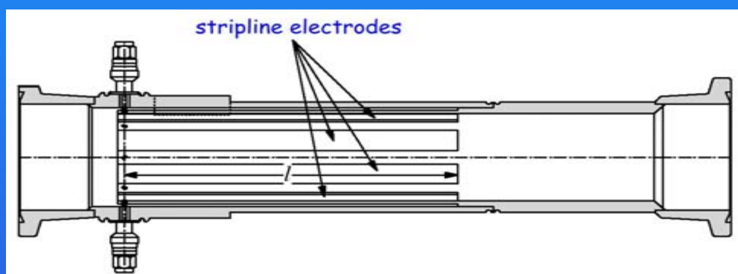
$$q\equiv\frac{(N+S)-(T+B)}{N+S+T+B}$$
$$=2\left[\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\quad+\text{h.o.t.}\right]$$

Beam Energy Spread (5), T. Suwada, M. Satoh, K. Furukawa "Nondestructive beam energy-spread monitor using multi-strip-line electrodes" PRSTAB 6, 032801 (2003)

principle: measure the energy spread derived from the quadrupole moments of the beam in regions of nonzero dispersion

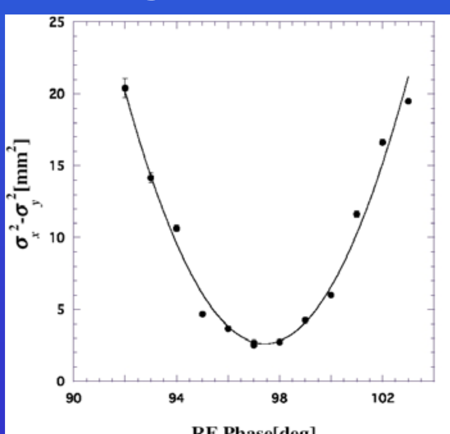




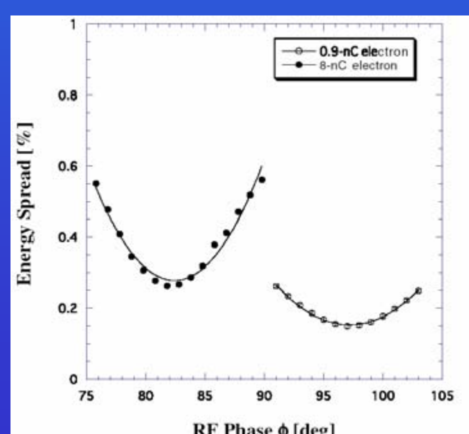


cylindrically symmetric with 8 electrodes
length (as long as possible) constrained by geometrical considerations (space)
50 Ohm output impedance
rotation to avoid synchrotron radiation

cross-calibration using OTR screen



energy spread measurements



energy spread found by solving:

measured with BESM

(correction for gain imbalance and geometrical errors)

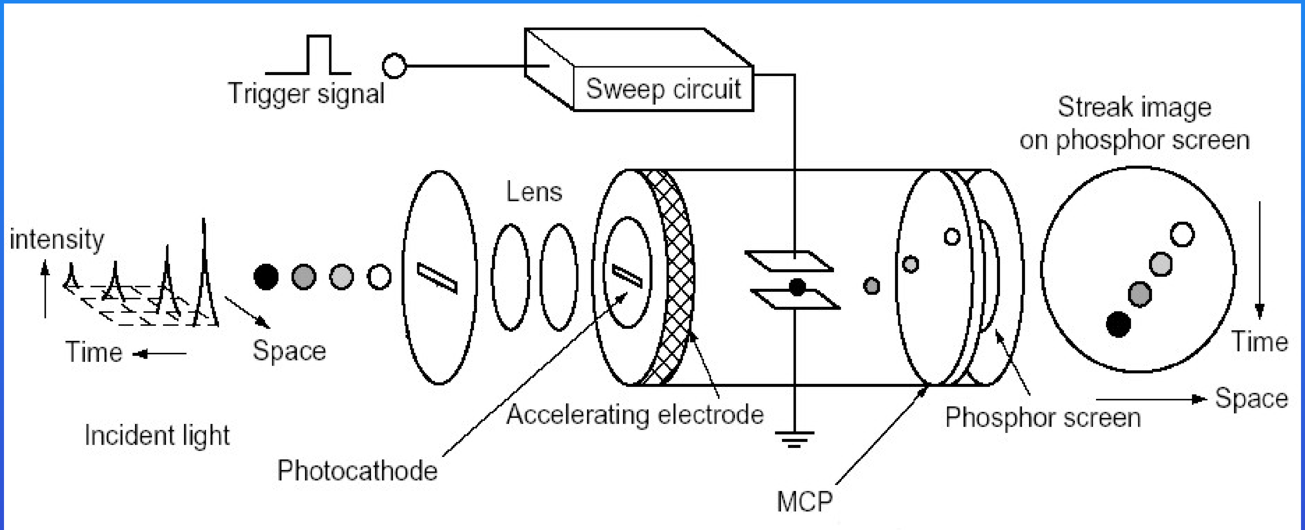
$$\overbrace{\langle x^2 \rangle - \langle y^2 \rangle}^{\text{measured with BESM}} \approx \beta_x \epsilon_x + (\eta_x \Delta E/E)^2 - \beta_y \epsilon_y + g$$

derived from independent measurement

(all figures courtesy T. Suwada, 2004)

Bunch Length (1) – conventional methods: streak camera

<u>principle:</u> photons (generated e.g. by SR, OTR, or from an FEL) are converted to e- (photocathode) , which are accelerated and deflected using a time-synchronized, ramped HV electric field; e- signal is amplified (MCP), converted to γ's (phosphor screen) and detected using an imager (e.g. CCD array), which converts the light into a voltage

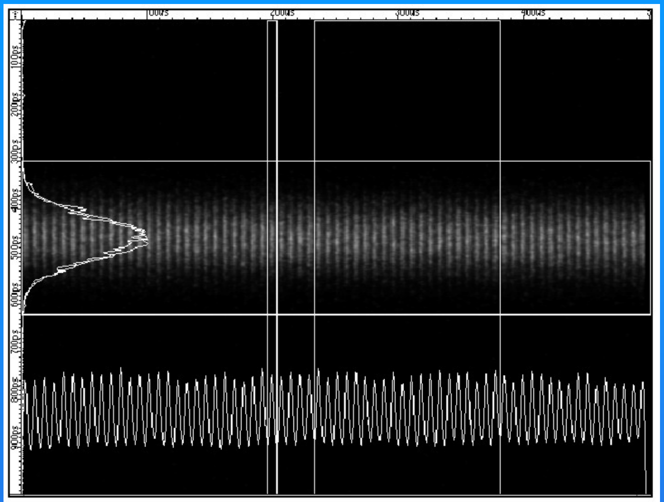


from Hamamatsu home page, C5680 series (www.hama-comp.com)

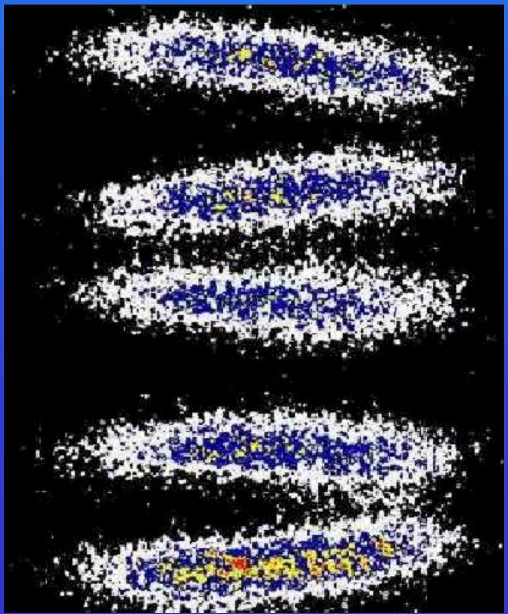
<u>issues:</u> energy spread of e- from the photocathode (time dispersion)
space charge effects following the photocathode
chromatic effects ($dt/dE_{\gamma}(\lambda)$) in windows
trigger jitter

lower limit on resolution ~ 0.5 ps

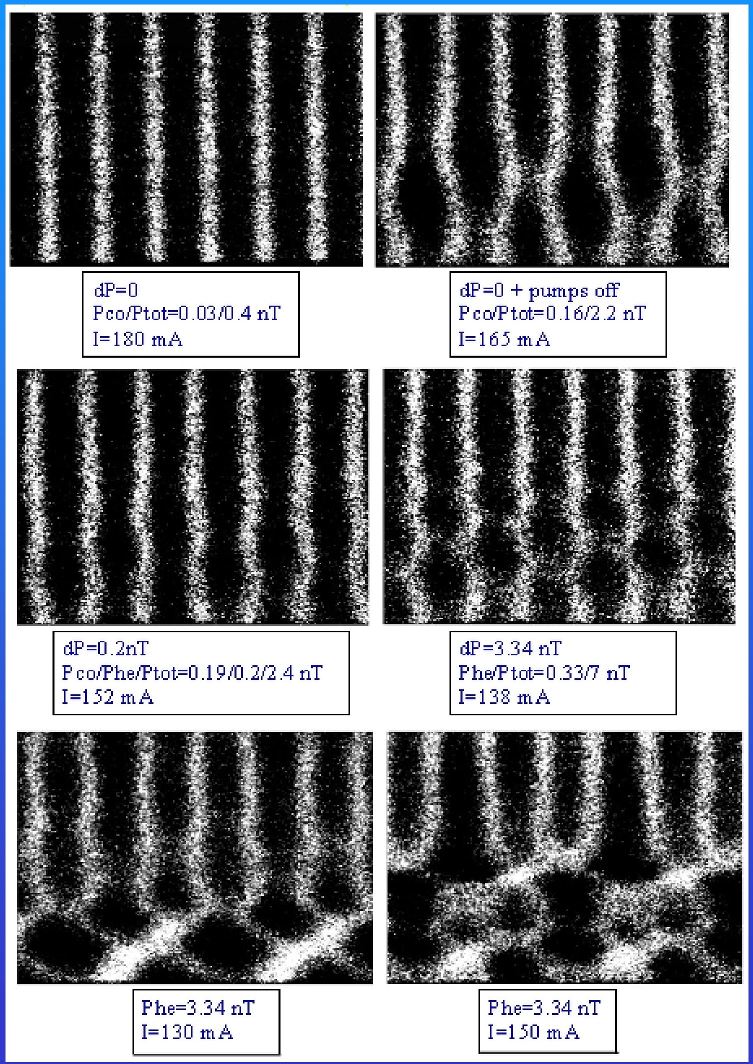
example: bunch length in PEP-II B-Factory (courtesy A. Fisher, 2004)



example: bunch length in ESRF over 5 turns (courtesy K. Scheidt, 2004)



example: streak camera images from the Pohang light source evidencing beam oscillations arising from the fast-ion instability (courtesy M. Kwon, 2000)

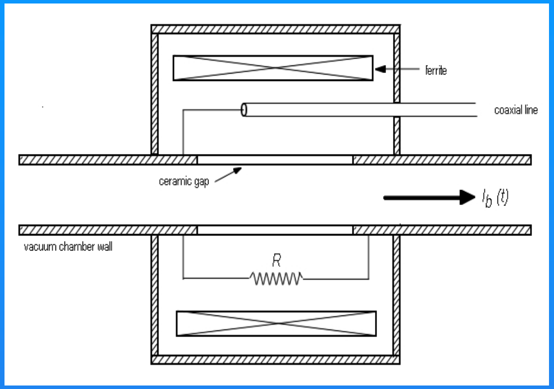


25 μs FS

500 ns (~1/2 turn) FS

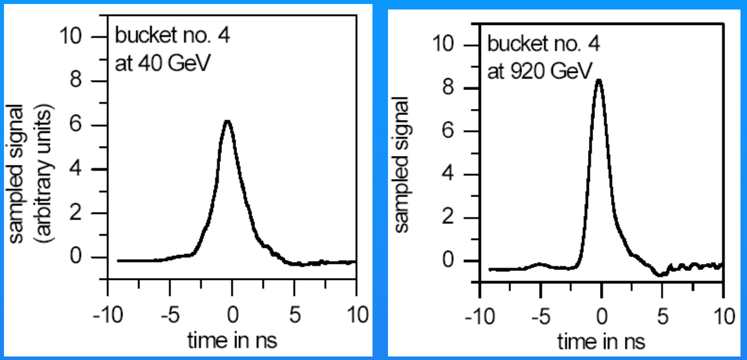
Bunch Length (2) – conventional methods: wall gap monitor

recall: resistive wall gap monitor:

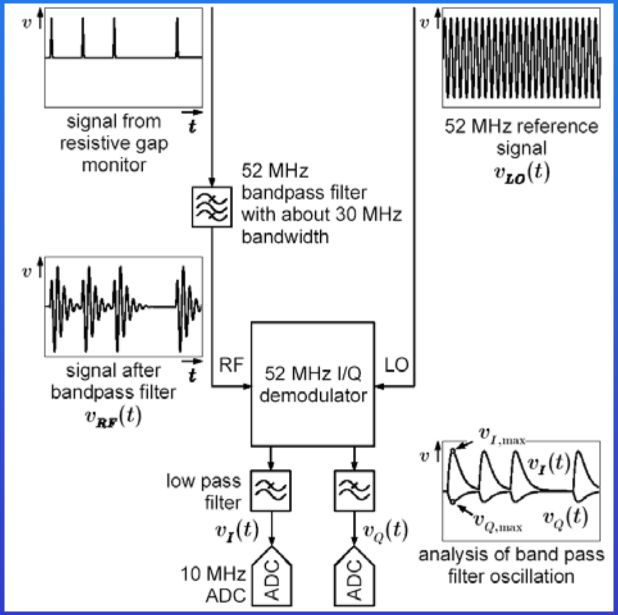


the wall current, after conversion to voltage, is measured with a high bandwidth (~GHz) scope

example: R-WGM bunch length measurements at HERA (courtesy E. Vogel, 2004)



with only a single monitor, not all bunches could be simultaneously measured (here ~s, possible: ~ms)
→ processing of rf signals for real-time measurements:



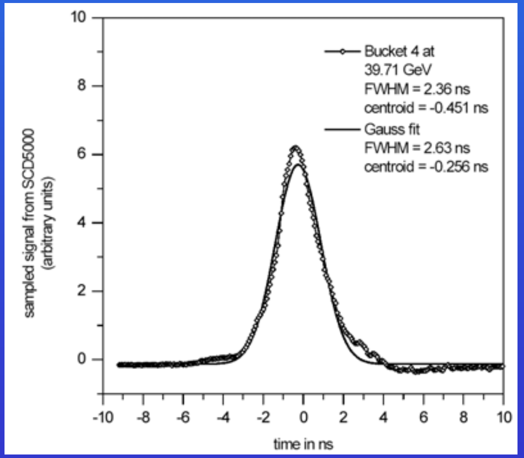
$$A(t) = \sqrt{(v_{I,max})^2 + (v_{Q,max})^2}$$

measurement of in-phase (I) and out-of-phase (Q) component of bunch spectrum at some frequency yields signal amplitude, A(t):

$$A(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma}}$$

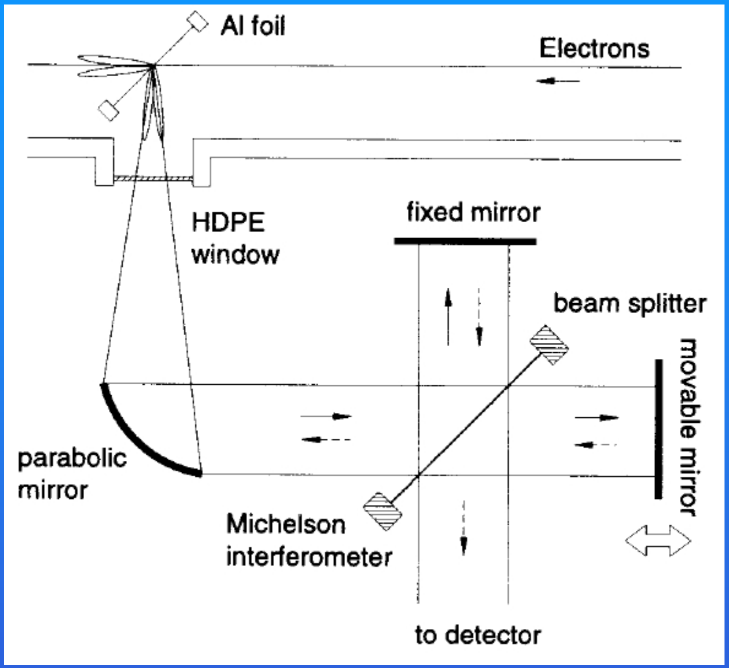
ratio of signals at two different frequencies
→ simple solution for the bunch length σ

example: bunch length measurements at HERA from frequency mixing (courtesy E. Vogel 2004)



Bunch Length (3) – newer methods: the Michelson interferometer

principle: use the interferometer to probe the bunch-length dependent frequency spectrum of the bunch



spectrum of a monochromatic wave:

$$\frac{2\pi}{\lambda}$$

$$S(\Delta x) = \underbrace{I(k)}(1 + \cos k\Delta x)$$

intensity of incoming radiation

path length difference between two arms of the spectrometer

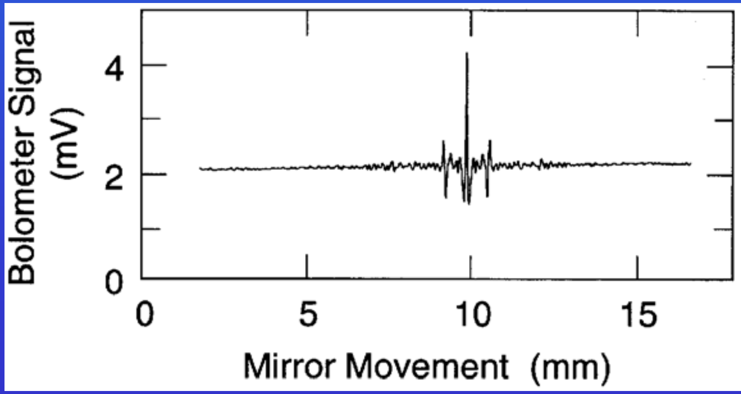
finite spectrum:

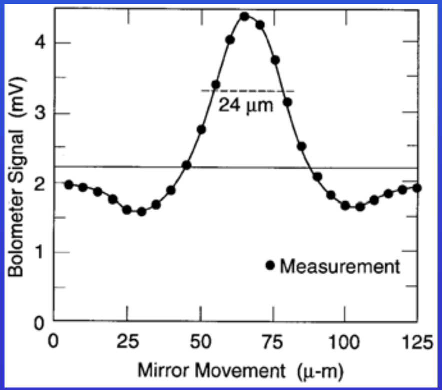
$$\begin{aligned} S(\Delta x) &= \int_0^\infty I(k)(1 + \cos k\Delta x)\, dk \\ &= S_0 + \int_0^\infty I(k) \cos k\Delta x \, dk \end{aligned}$$

signal with large Δx

interferogram: S(Δx)-S_0

examples: interferograms measured at the Stanford "sunshine facility" (courtesy H. Wiedemann, 2004)





signal FWHM in interferogram (assuming Gaussian bunches):

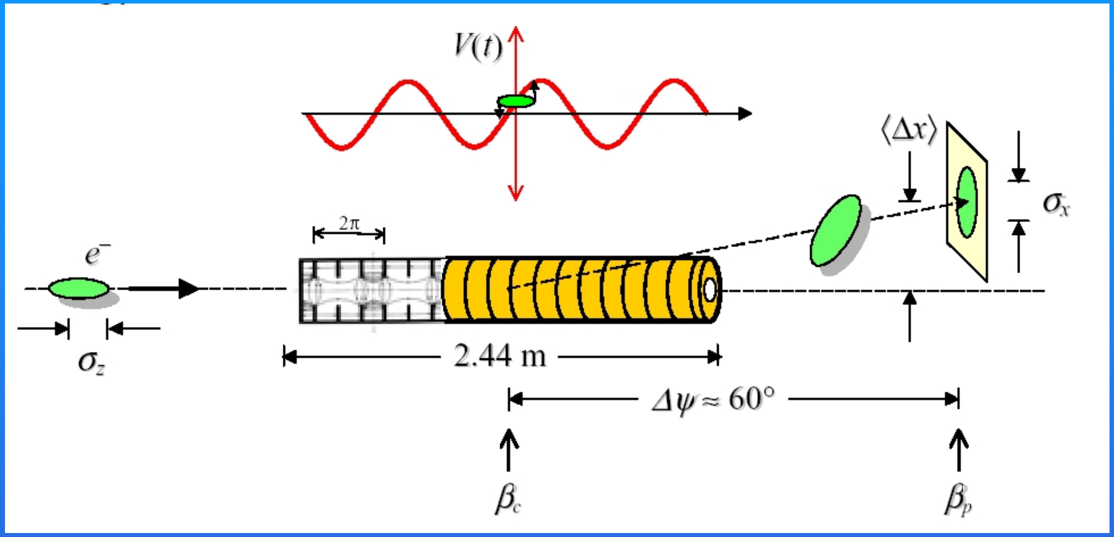
$$\left(\frac{2.35}{\sqrt{2}}\right)\sigma_z$$

issues:

long time averaging
water absorption lines

Bunch Length (4) – newer methods: transverse-mode cavity

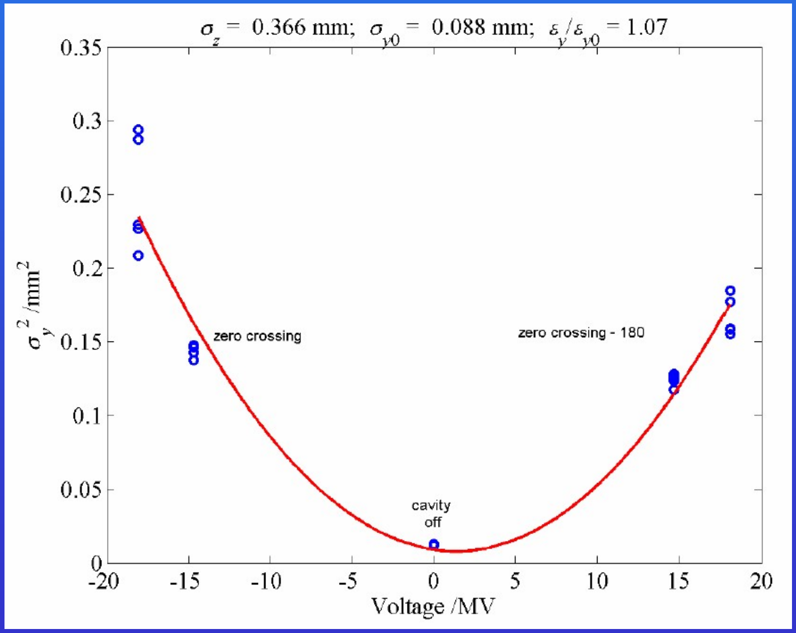
Principle: use transverse mode deflecting cavity to "sweep"/kick the beam, which is then detected using standard profile monitors



transverse mode deflecting cavity used at SLAC

introduce x-z correlation

oriented to displace beam vertically while a horizontal bending magnet deflects the beam onto the screen



$\sigma_y^2 = A(V_{rf} - V_{rf,min})^2 + \sigma_{y0}^2$

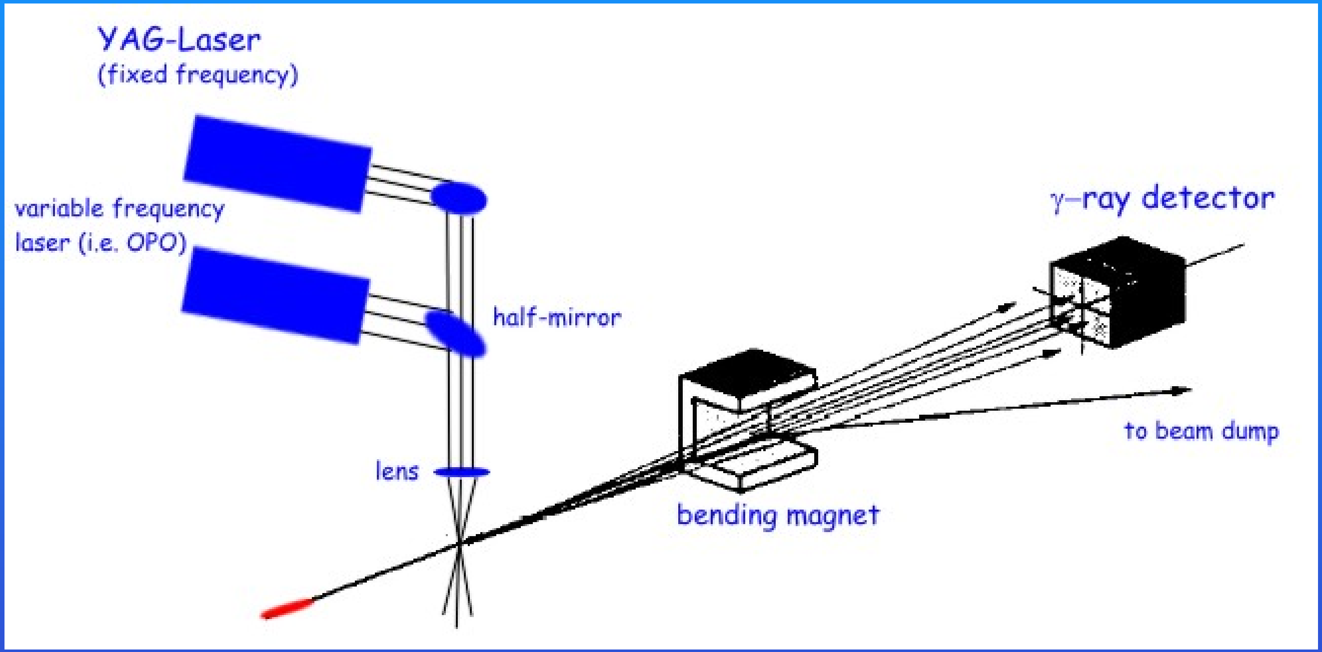
$\sigma_z = A^{1/2} E_0 \lambda_{rf} / 2\pi R_{34}$ where R_{34} is transfer function between structure and downstream BPM used for calibration of voltage

➡ σ_z expressed in terms of fit parameter, A

(figures courtesy R. Akre, 2003)

Bunch Length (5) – proposed methods: "beat wave" interferometer (T. Shintake)

principle: the light of a tunable laser is combined with a fixed wavelength laser providing a "beat-wave" (intensity modulation with frequency equal to the frequency difference of the two laser beams) with which the bunch interacts; signal amplitude measured as a function of variable frequency laser (or bunch arrival time)



bunch length measurements: scattered Compton γ's as a function of frequency

issues:

- laser power imbalance (mirror "asymmetries") and laser alignment
- electron beam crossing angle (electron beam not parallel to the plane of the fringes)
- longitudinal extent of the interference pattern (compare "hour-glass" effect)
- spatial and temporal coherence of the laser (alignment distortions)
- laser jitter

Summary

We motivated the concept of longitudinal emittance by analogy of the transverse and longitudinal equations of motion in circular accelerators.

In linear accelerators, equivalent equations of motion do not exist.

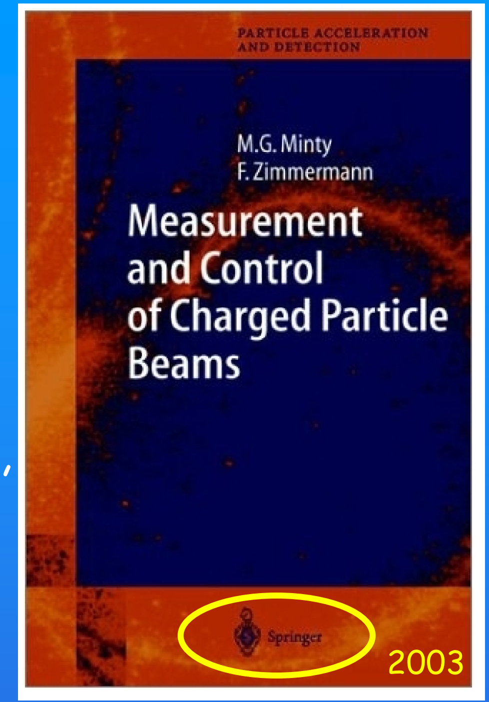
The longitudinal emittance ε_L was defined. Most relevant observables are:

linear accelerator:	σ_z , σ_δ	($\langle \phi_z \phi_\delta \rangle \neq 0$)
circular e+/- accelerator:	σ_z , (σ_δ)	($\langle \phi_z \phi_\delta \rangle = 0$)
circular p accelerator:	σ_z , σ_δ , ε_L	($\langle \phi_z \phi_\delta \rangle = 0$)

Measurements of the bunch energy spread are, basically, motivated by demand. "Spectrometer-based" examples were shown from a transport line and after extraction from a damping ring) using the same hardware (screens and wire scanners) as for transverse beam size measurements applied in locations with nonzero dispersion: σ = sqrt ($\sigma_\beta^2 + [\eta\delta]^2$). In a storage ring, measurement of the bunch length and assumption of α gives also the beam energy spread. The nonintercepting emittance monitor applied to energy spread measurements was also reviewed.

Selected techniques for measurement of the bunch length were described including
- conventional methods: streak cameras, resistive wall gap monitors
- newer methods : the Michelson interferometer and transverse-mode cavity
- proposed methods: laser-based "beat-wave" monitor

Book describes various <u>high-level applications</u> for measuring properties of beams, diagnosing errors, correcting errors, and manipulating / controlling beam parameters



Text aims to:

- organize in coherent way multitude of measurement and control techniques presented in conference proceedings, lab internal notes, some journal articles, and obtained from experience
- "bridge the gap" between theory and experimental results

  example - beam steering:
  - single element alignment
  - alignment with multi-magnet families
  - higher-level-applications including steering algorithms (one-to-one, SVD, dispersion-free steering, beam-based alignment, model-independent analysis,...)

- provide (mostly easy-to-access) references for each topic
- provide wherever appropriate real data from existing accelerators to support each topic of discussion
- include exercises (mostly from experiences) and their solutions