

# Diagnostics I

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DESY

Introduction  
Beam Charge / Intensity  
Beam Position  
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} Diagnostics I

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Summary

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Introduction  
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Summary

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# Introduction

Accelerator performance depends critically on the ability to carefully measure and control the properties of the accelerated particle beams

In fact, it is not uncommon, that beam diagnostics are modified or added after an accelerator has been commissioned

This reflects in part the increasingly difficult demands for high beam currents, smaller beam emittances, and the tighter tolerances placed on these parameters (e.g. position stability) in modern accelerators

A good understanding of diagnostics (in present and future accelerators) is therefore essential for achieving the required performance

A beam diagnostic consists of

the measurement device  
associated electronics and  
processing hardware  
high-level applications



focus of this lecture

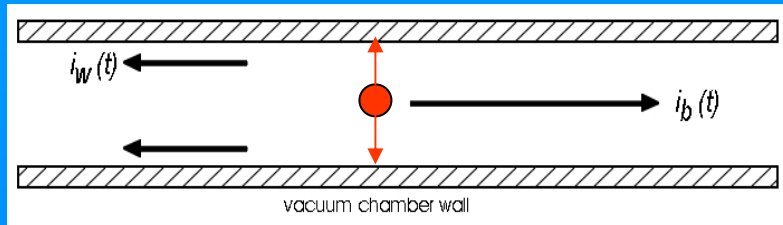


subject of many recent publications  
and internal reports (often application  
specific)



reference: "Beam Diagnostics and Applications",  
A. Hofmann (BIW 98)

# Fields of a relativistic particle

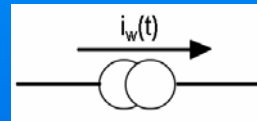


induced wall current  $i_w(t)$   
has opposite sign of beam  
current  $i_b(t)$ :  $i_b(t) = -i_w(t)$

Lorentz-contracted "pancake"

## Detection of charged particle beams - beam detectors:

$i_w$  is a current source



with infinite output impedance,  $i_w$  will flow through any  
impedance placed in its path

many "classical" beam detectors consist of a modification of the  
walls through which the currents will flow

## Sensitivity of beam detectors:

beam  
charge:

$$S(\omega) = \frac{V(\omega)}{I_w(\omega)}$$

(in  $\Omega$ )

= ratio of signal size developed  $V(\omega)$  to  
the wall current  $I_w(\omega)$

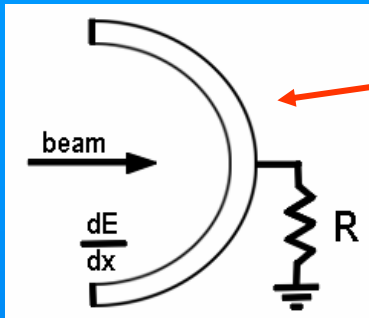
beam  
position:

$$S(\omega) = \frac{V(\omega)}{D(\omega)}$$

(in  $\Omega/m$ )

= ratio of signal size developed /dipole mode  
of the distribution, given by  $D(\omega) = I_w(\omega) z$ ,  
where  $z = x$  (horizontal) or  $z = y$  (vertical)

# Beam Charge - the Faraday Cup



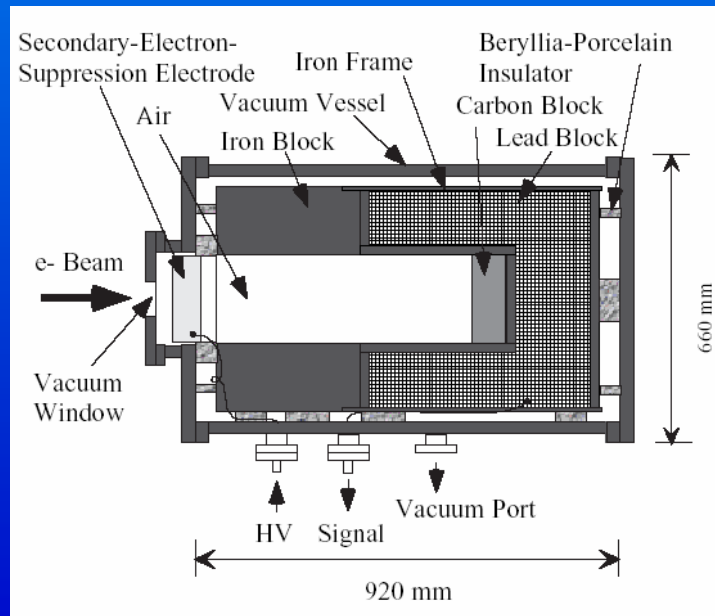
thick (e.g. ~0.4 m copper for 1 GeV electrons) or series of thick (e.g. for cooling) charge collecting receptacles

Principle: beam deposits (usually) all energy into the cup (invasive)  
charge converted to a corresponding current  
voltage across resistor proportional to instantaneous current absorbed

In practice:

termination usually into 50  $\Omega$ ; positive bias to cup to retain e<sup>-</sup> produced by secondary emission; bandwidth-limited (~1 GHz) due to capacitance to ground

cross-sectional view of the FC of the KEKB injector linac (courtesy T. Suwada, 2003)



cylindrically symmetric blocks of lead (~35 rad lengths) carbon and iron (for suppression of em showers generated by the lead)  
bias voltage (~many 100 Volts) for suppression of secondary electrons

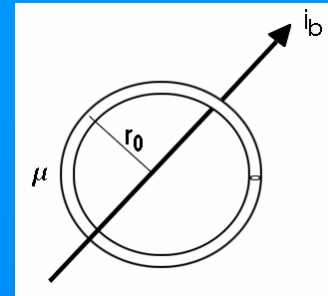
# Beam Intensity - Toroids (1)

Consider a magnetic ring surrounding the beam, from Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu I$$

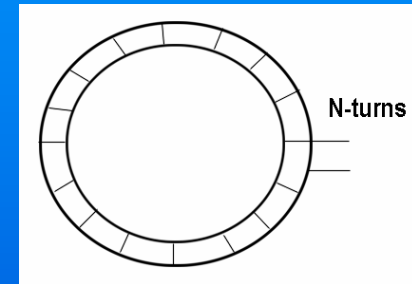
if  $r_0$  (ring radius)  $\gg$  thickness of the toroid,

$$B = \frac{\mu i_b}{2\pi r_0}$$

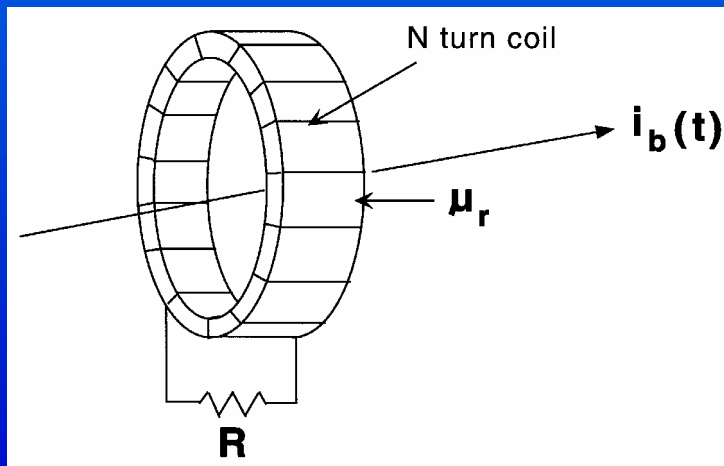


Add an N-turn coil - an emf is induced which acts to oppose B:

$$\begin{aligned} \epsilon &= \frac{d\phi}{dt} \text{ where } \phi = \int \vec{B} \cdot d\vec{a} \\ &= \frac{\mu A}{2\pi r_0} \frac{di_b}{dt} \end{aligned}$$

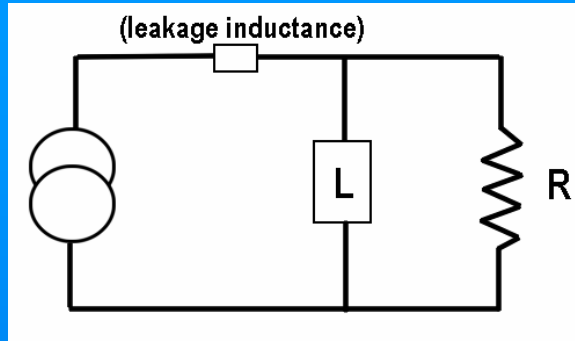


Load the circuit with an impedance; from Lenz's law,  $i_R = i_b/N$ :



Principle: the combination of core, coil, and  $R$  produce a current transformer such that  $i_R$  (the current through the resistor) is a scaled replica of  $i_b$ . This can be viewed across  $R$  as a voltage.

# Beam Intensity - Toroids (2)



$$L = \frac{N^2}{R_h}$$

with  $R_h$  = reluctance of magnetic path

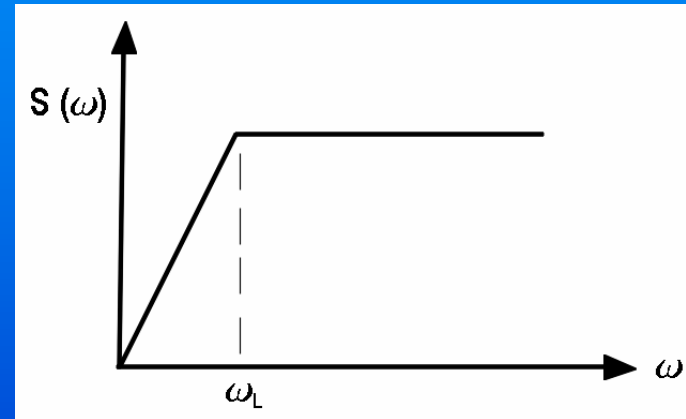
$$R_h = \frac{l}{\mu A} [\text{H}^{-1}]$$

$$L = \frac{N^2 \mu_r \mu_0 A}{l}$$

sensitivity:

$$S = \frac{R}{\sqrt{1 + \left(\frac{\omega_l}{\omega}\right)^2}}$$

$$\omega_l = \frac{R}{L}$$



cutoff frequency,  $\omega_l$ , is small if  $L \sim N^2$  is large

detected voltage:

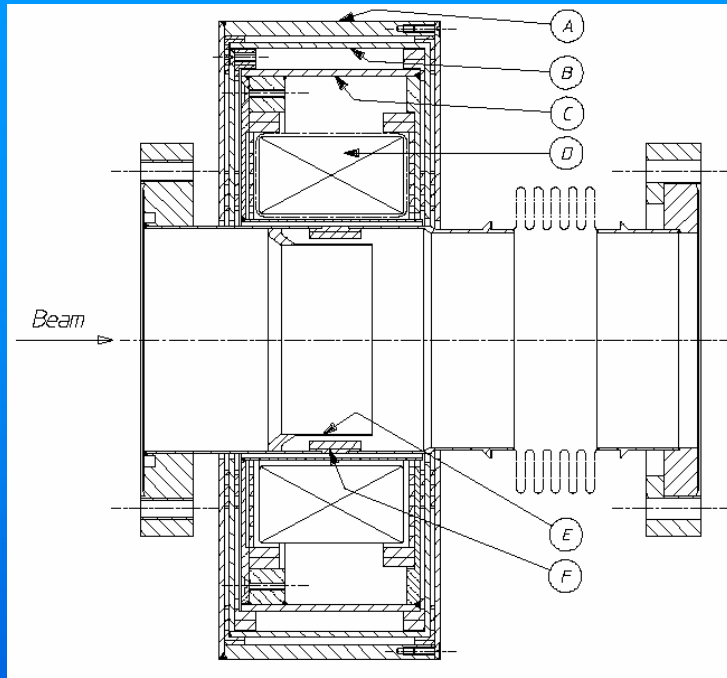
$$V(t) = \frac{i_b R}{N} e^{-(\frac{R}{L})t}$$

if  $N$  is large, the voltage detected is small

trade-off  
between  
bandwidth  
and signal  
amplitude

# Beam Intensity - Toroids (3)

schematic  
of the toroidal  
transformer for the TESLA  
Test facility  
(courtesy,  
M. Jablonka,  
2003)



A iron  
B Mu-metal  
C copper  
D "Supermalloy" (distributed  
by BF1 Electronique,  
France) with  $\mu \sim 8 \times 10^4$   
E electron shield  
F ceramic gap

(one of many)  
current trans-  
formers available  
from Bergoz  
Precision Instru-  
ments (courtesy  
J. Bergoz, 2003)



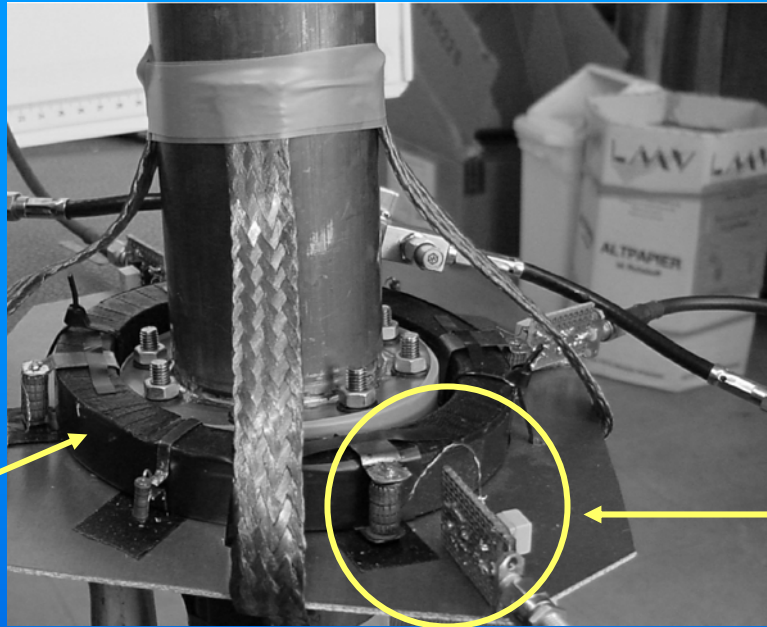
(based on design of K. Unser  
for the LEP bunch-by-bunch  
monitor at CERN)

linacs: resolution of  $3 \times 10^6$   
storage rings: resolution  
of 10 nA rms

details: [www.bergoz.com](http://www.bergoz.com)

# Beam Intensity - Toroids (4)

recent developments of toroids for TTF II (DESY)



2 iron halves

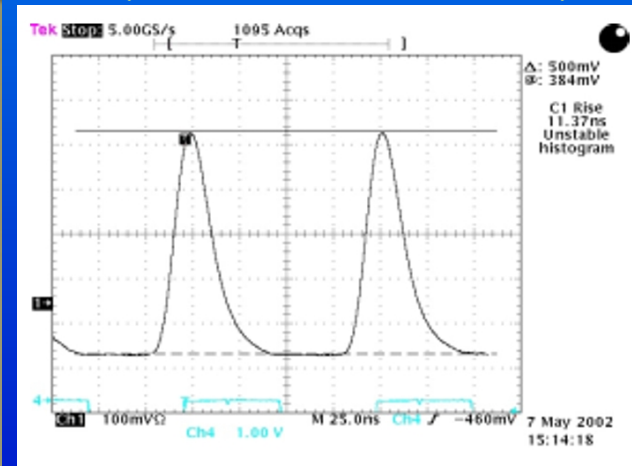
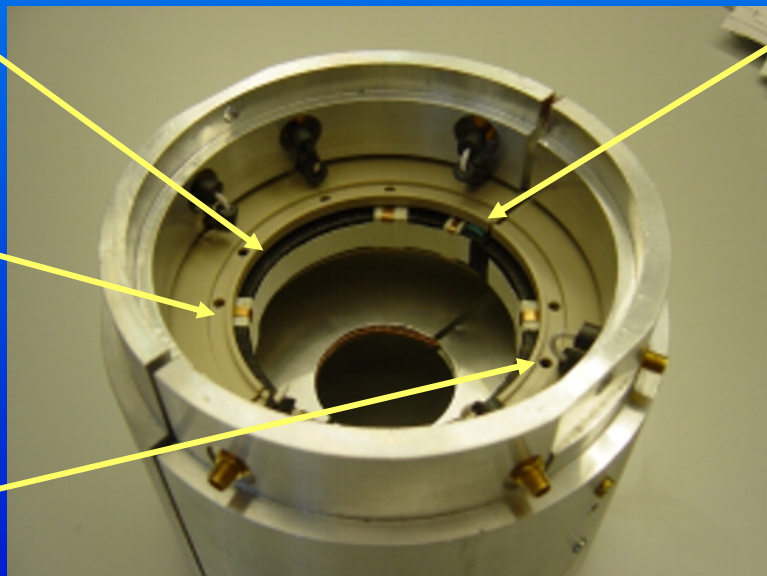
ferrite ring

50  $\Omega$  output impedance

calibration windings  
(25 ns , 100 mV / divsn)

bronze pick-ups

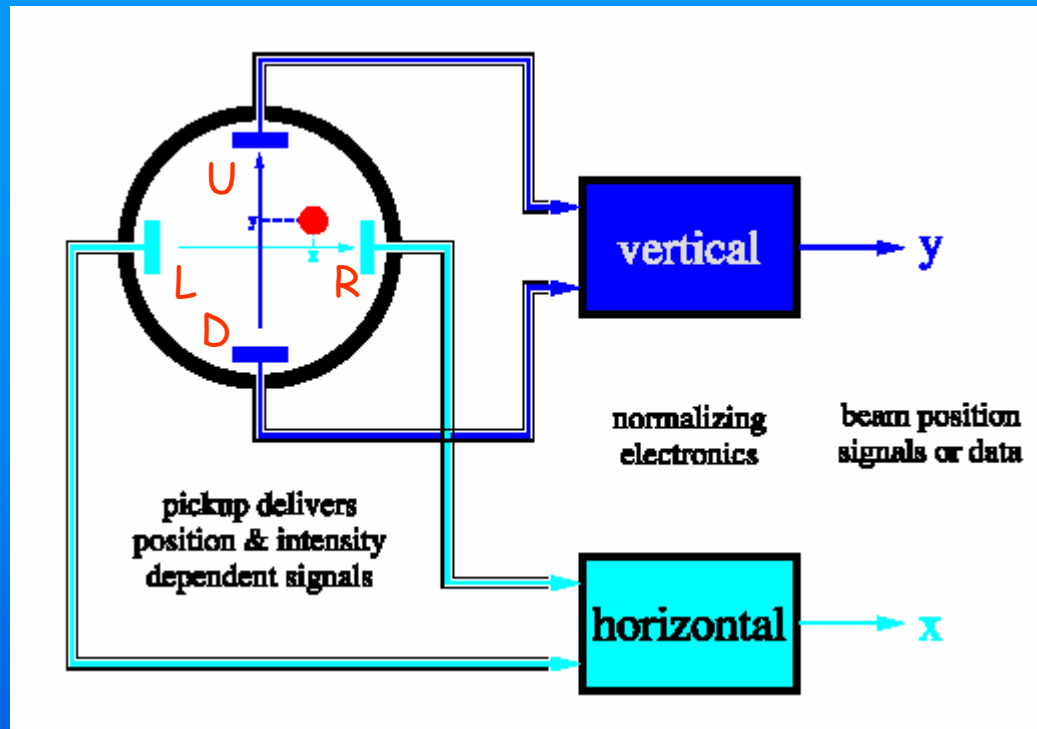
ferrite rings  
(for suppression of high frequency resonance)



(courtesy D. Noelle, L. Schreiter, and M. Wendt, 2003)



# Beam Intensity - BPM Sum signals



U ~ up  
D ~ down  
L ~ left  
R ~ right

(figure, courtesy M. Wendt, 2003)

beam "position"  $V_R - V_L$  (horizontal)  
 $V_U - V_D$  (vertical)

beam intensity  $V_R + V_L, V_U + V_D, V_R + V_L + V_U + V_D$

normalized (intensity-independent) beam position =  $\frac{\text{"position"}}{\text{intensity}}$

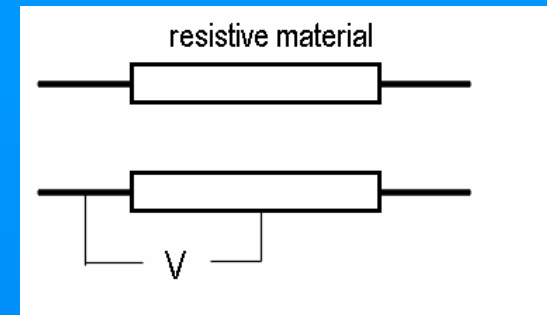
- Remarks:
- 1) as we will see, higher-order nonlinearities must occasionally be taken into account
  - 2) in circular  $e^{\pm}$  accelerators, assembly is often tilted by 45 degrees

# Beam Position - Wall Gap Monitor (1)

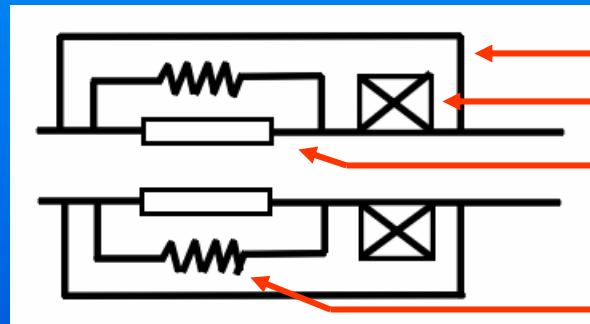
principle:

remove a portion of the vacuum chamber and replace it with some resistive material of impedance  $Z$

detection of voltage across the impedance gives a direct measurement of beam current since  $V = i_w(t) Z = -i_b(t) Z$



(susceptible to em pickup and to ground loops)



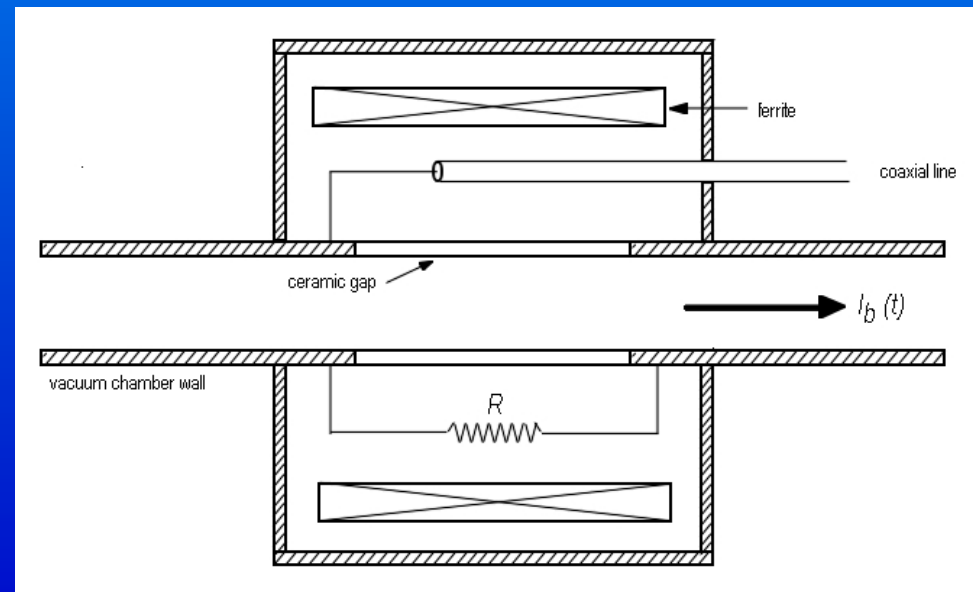
add high-inductance metal shield

add ferrite to increase  $L$

add ceramic breaks

add resistors (across which  $V$  is to be measured)

alternate topology - one of the resistors has been replaced by the inner conductor of a coaxial line



# Beam Position - WGM (2)

sensitivity:

circuit model using parallel  
RLC circuit:

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

high frequency response is determined by C:

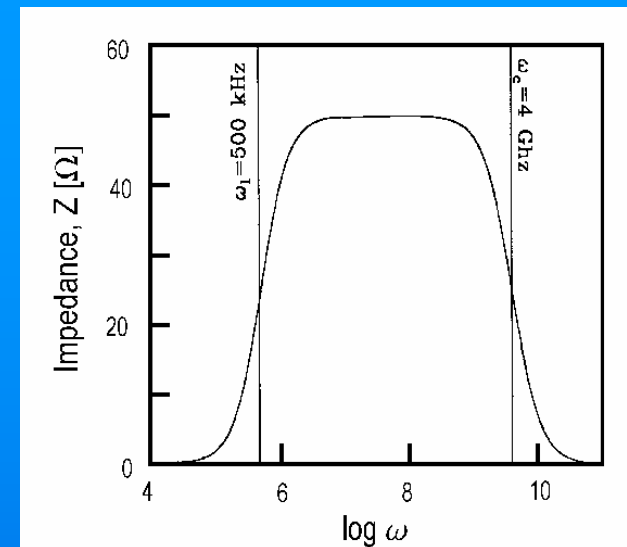
$$|Z(\omega \rightarrow \infty)| = \frac{R}{\sqrt{1 + \left(\frac{\omega}{\omega_C}\right)^2}} \quad (\omega_C = 1/RC)$$

low frequency response determined by L:

$$|Z(\omega \rightarrow 0)| = \frac{R}{\sqrt{1 + \left(\frac{\omega_L}{\omega}\right)^2}} \quad (\omega_L = R/L)$$

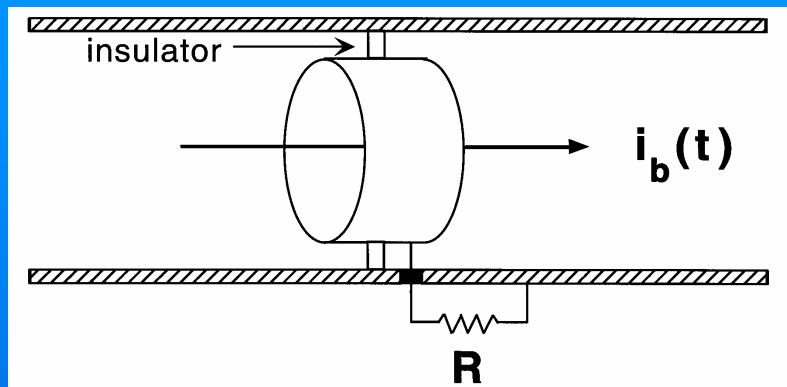
intermediate regime:  $R/L < \omega < 1/RC$  - for high bandwidth, L should be large and C should be small

remark: this simplified model does not take into account the fact that the shield may act as a resonant cavity



# Beam Position - Capacitive Monitors (1)

(capacitive monitors offer better noise immunity since not only the wall current, but also PS and/or vacuum pump return and leakage currents, for example, may flow directly through the resistance of the WGM)



principle: vacuum chamber and electrode act as a capacitor of capacitance,  $C_e$ , so the voltage generated on the electrode is  $V=Q/C_e$  with  $Q = i_w t = i_w L/c$  where  $L$  is the electrode length and  $c = 3 \times 10^8$  m/s

## long versus short bunches:

since the capacitance  $C_e$  scales with electrode length  $L$ , for a fixed  $L$ , the output signal is determined by the input impedance  $R$  and the bunch length  $\sigma$

for  $\omega \ll \omega_c$

(bunch long compared to electrode length  $\sigma > L$ )

the electrode becomes fully charged during bunch passage  
signal output is differentiated

signal usually coupled out using coax attached to electrode

for  $\omega \gg \omega_c$

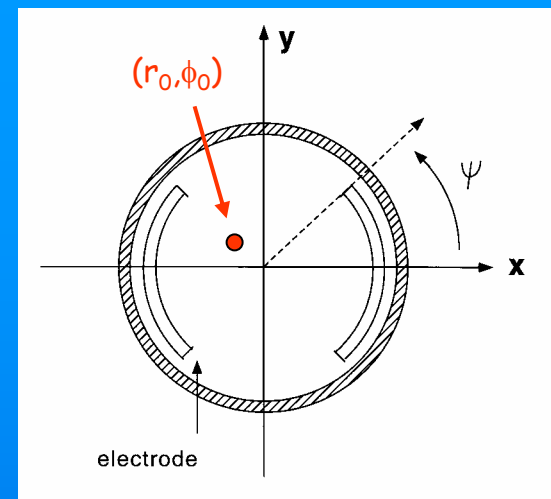
output voltage rises rapidly and is followed by extended negative tail (since dc component of signal is zero)

induced voltage usually detected directly through a high impedance amplifier

# Beam Position - Capacitive Monitors (2)

position information:

replace cylinder by curved electrodes (usually 2 or 4) symmetrically placed with azimuth  $\pm \psi$  (usually small to avoid reflections between the edges and the output coupling)



example - capacitive split plate:

$$\sigma = \frac{1}{2\pi a} \left[ 1 + \sum_{n=1}^{\infty} \left(\frac{r_0}{r}\right)^2 \cos n(\phi - \phi_0) \right]$$

surface charge density  $\sigma$  due to a unit line charge collinear to electrodes at  $(r_0, \phi_0)$

$$\begin{aligned} I_R &= \int_{-\psi}^{+\psi} \sigma(r d\phi) \\ &= \frac{i_w}{2\pi} \left[ 2\psi + 2\frac{x_0}{a} \sin \psi + \frac{x_0^2 - y_0^2}{a^2} \sin 2\psi + \dots \right] \end{aligned}$$

$$\begin{aligned} I_L &= \int_{\pi-\psi}^{\pi+\psi} \sigma(r d\phi) \\ &= \frac{i_w}{2\pi} \left[ 2\psi - 2\frac{x_0}{a} \sin \psi + \frac{x_0^2 - y_0^2}{a^2} \sin 2\psi + \dots \right] \end{aligned}$$

integrate over area of electrode

the voltage on a single electrode depends on the detector geometry via the radius  $a$  and the angle subtended by the electrode; e.g. if the signal from a *single* electrode is input into a frequency analyzer, higher harmonics arise due to these nonlinearities

voltage across impedance  $R$

$$\begin{aligned} V &= (I_R - I_L)R \\ &= \frac{2i_w R}{\pi a} (\sin \psi) x_0 + \dots \end{aligned}$$

sensitivity

$$S = \frac{V}{i_w x_0} = \frac{2R}{\pi a} \sin \psi + \dots$$

the voltage and sensitivity are large if the azimuthal coverage is large or the radius  $a$  is small; e.g.  $\psi = 30 \text{ deg}$ ,  $R = 50 \Omega$ ,  $a = 2.5 \text{ cm} \rightarrow S = 2 \Omega/\text{mm}$

# Beam Position - Capacitive Monitors (3)

example - capacitive split cylinder:

charge in each detector half is found by integrating the surface charge density:

$$Q_i = \frac{\lambda}{2} \left[ L \pm \frac{r_0}{2\pi} \sin \phi_0 \tan \theta \right]$$

$$C_e = \frac{C}{2}$$

$$C = \frac{L}{Z_0 c}$$

(can be shown)

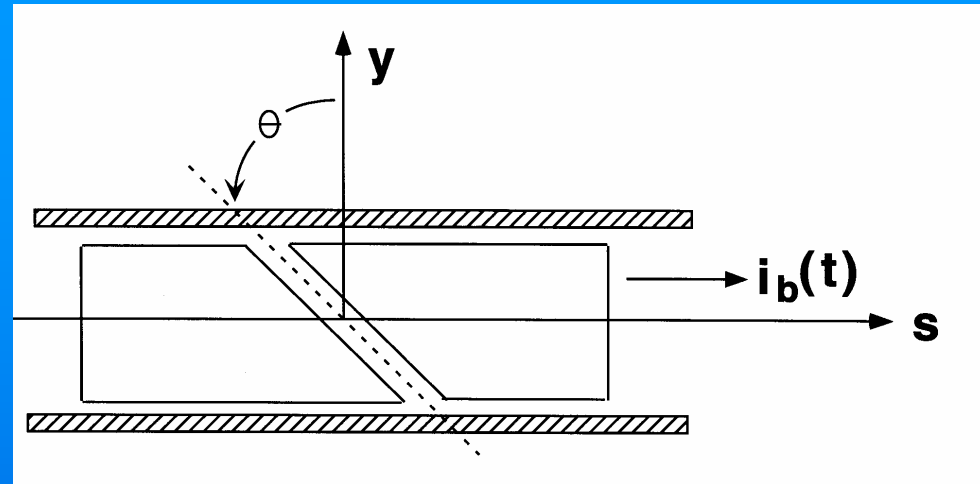
$$(\Delta x = r_0 \cos \phi_0)$$

detected voltage

$$V = \frac{Q_l - Q_r}{C_e} = \frac{Z_0 \tan \theta}{2\pi L} (-i_w) \Delta x$$

sensitivity

$$S = \frac{Z_0 \tan \theta}{2\pi L}$$



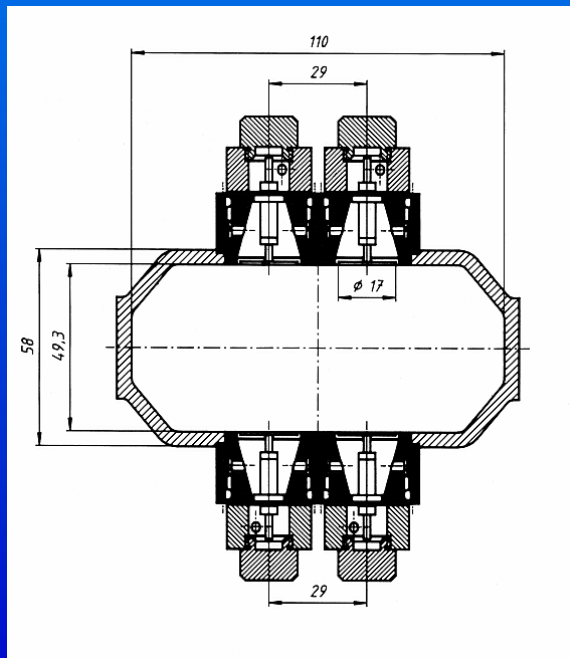
the capacitive split cylinder is a linear detector; there are no geometry -dependent higher order contributions to the position sensitivity.

# Beam Position - Button Monitors

Buttons are used frequently in synchrotron light sources are a variant of the capacitive monitor (2), however terminated into a characteristic impedance (usually by a coax cable with impedance  $50 \Omega$ ). The response obtained must take into account the signal propagation (like for transmission line detectors, next slide)



button electrode for use between the undulators of the TTF II SASE FEL (courtesy D. Noelle and M. Wendt, 2003)



cross-sectional view of the button BPM assembly used in the DORIS synchrotron light facility

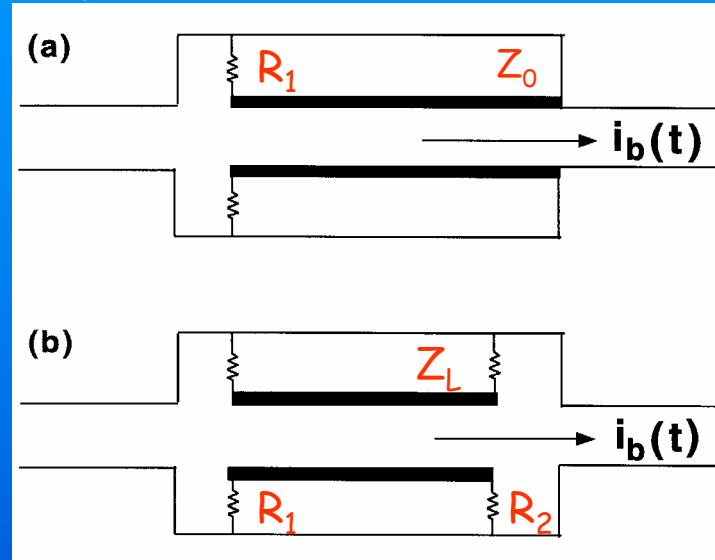
design reflects geometrical constraints imposed by vacuum chamber geometry

note: monitor has inherent nonlinearities (courtesy F. Peters, 2003)

# Beam Position - Stripline / Transmission Line Detectors (1)

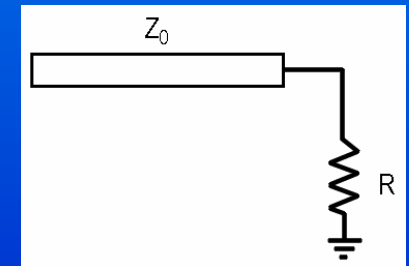
principle: electrode (spanning some azimuth  $\psi$ ) acts as an inner conductor of a coaxial line; shield acts as the grounded outer conductor  $\rightarrow$  signal propagation must be carefully considered

unterminated transmission line



transmission line terminated (rhs) to a matched impedance

reminder: characteristic impedance  $Z_0$  terminated in a resistor  $R$



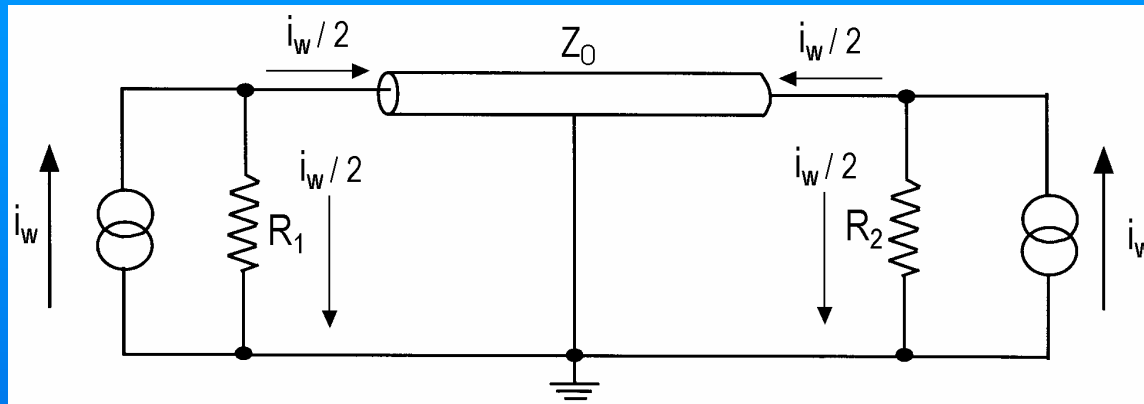
$$\rho = \text{reflection coefficient} = \frac{R - Z_0}{R + Z_0} = \begin{cases} 0 & \text{if } R = Z_0 \\ -1 & \text{if } R = 0 \\ > 0 & \text{if } R > Z_0 \\ < 0 & \text{if } R < Z_0 \end{cases}$$

$\Gamma = \sqrt{1 - \rho^2}$  = transmission coefficient



# Beam Position - Stripline / Transmission Line Detectors (2)

equivalent circuit (approximation: velocity of  $i_w$  = velocity of  $i_b$ , approximately true in absence of dielectric and/or magnetic materials)



the voltage appearing across each resistor is evaluated by analyzing the current flow in each gap:

voltage at  $R_1$ :

$$V_{R_1, g_1} = \frac{i_w}{2} \left[ 1 + \left( \frac{R_2 - Z_0}{R_2 + Z_0} \right) e^{-2j\omega\Delta t} \right] R_1$$

initial reflection

$$V_{R_1, g_2} = -\frac{i_w}{2} e^{-j\omega\Delta t} \left[ 1 - \left( \frac{R_1 - Z_0}{R_1 + Z_0} \right) \right] e^{-j\omega\Delta t} R_1$$

beam delay transmission

# Beam Position - Stripline / Transmission Line Detectors (3)

similarly, voltage at  $R_2$ :

$$V_{R_2, g_1} = \frac{i_w}{2} e^{-j\omega\Delta t} \left[ 1 - \left( \frac{R_2 - Z_0}{R_2 + Z_0} \right) \right] R_2$$

signal delay      transmission

$$V_{R_2, g_2} = -\frac{i_w}{2} e^{-j\omega\Delta t} \left[ 1 + \left( \frac{R_1 - Z_0}{R_1 + Z_0} \right) e^{-2j\omega\Delta t} \right] R_2$$

voltage on each resistor:

$$\begin{aligned} V_{R_1} &= V_{R_1, g_1} + V_{R_1, g_2} \\ V_{R_2} &= V_{R_2, g_1} + V_{R_2, g_2} \end{aligned}$$

$$\Delta t = \frac{L}{c}$$

beam delay      initial      reflection

special cases:

(i)  $R_1 = Z_0$ ,  $R_2 = 0$  (terminated to ground)

$$V_{R_1} = \frac{i_w}{2} \left( 1 - e^{-\frac{2j\omega L}{c}} \right) R_1$$

(no signal generated at  $g_2$ )

$$V_{R_2} = 0$$

(ii)  $R_1 = R_2 = Z_L$  (matched line)

$$V_{R_1} = \frac{i_w}{2} \left( 1 - e^{-\frac{2j\omega L}{c}} \right) Z_L$$

$$V_{R_2} = 0$$

(iii)  $R_1 = R_2 \neq Z_L$  then solution as in (ii) to second order in  $\rho$

# Beam Position - Stripline Monitors (4)

again, 
$$V_{R_1} = \frac{i_w}{2} \left( 1 - e^{-\frac{2j\omega L}{c}} \right) R_1$$

sensitivity

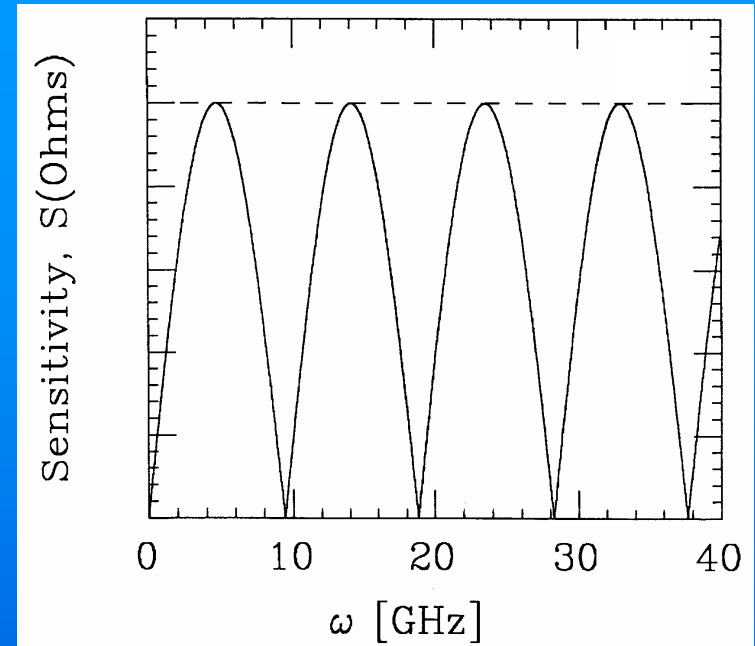
$$|S| = \left| \frac{V}{i_w} \right| = R_1 |\sin^2 \omega \Delta t|$$

signal peaks at

$$\omega \Delta t = \frac{2\pi L}{\lambda} = \frac{\pi}{2} \longrightarrow L = \frac{\lambda}{4}$$

spacing between zeros

$$\omega \Delta t = 0 \longrightarrow L = \frac{\lambda}{2}$$



sensitivity of a matched transmission line detector of length  $L=10$  cm

the LEUTL at Argonne shorted S-band quarter-wave four-plate stripline BPM (courtesy R.M. Lill, 2003)

specially designed to enhance port isolation (using a short tantalum ribbon to connect the stripline to the molybdenum feedthrough connector) and to reduce reflections

$L=28$  mm (electrical length  $\sim 7\%$  longer than theoretical quarter-wavelength),  $Z_0=50 \Omega$



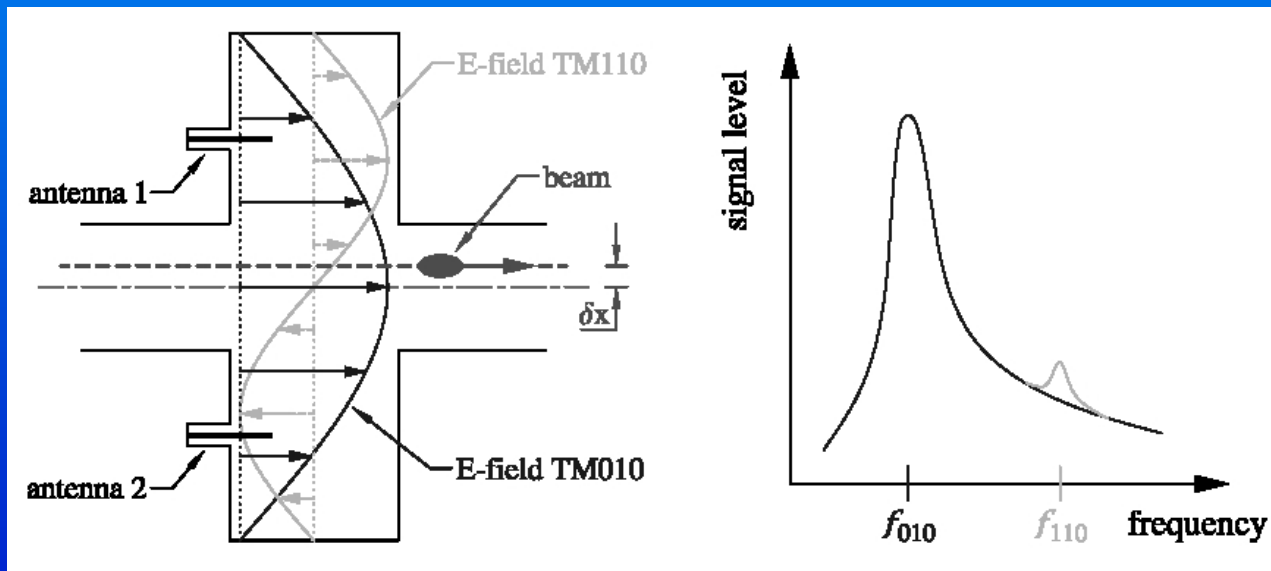
# Beam Position - Cavity BPMs (1)

principle: excitation of discrete modes (depending on bunch charge, position, and spectrum) in a resonant structure; detection of dipole mode signal proportional to bunch charge,  $q \times$  transverse displacement,  $\delta x$

theoretical treatment: based on solving Maxwell's equations for a cylindrical waveguide with perpendicular plates on two ends

motivation: high sensitivity (signal amplitude /  $\mu\text{m}$  displacement)  
accuracy of absolute position, LCLS design report

dipole mode cavity BPM consists of (usually) a cylindrically symmetric cavity, which is excited by an off-axis beam:



reference:  
"Cavity BPMs", R. Lorentz  
(BIW, Stanford, 1998)

$TM_{010}$ , "common mode" ( $\propto I$ )

$TM_{110}$ , dipole mode of interest

} amplitude detected at position of antenna contains contributions from both modes  $\rightarrow$  signal processing

# Beam Position - Cavity BPMs (2)

$$V_{110}^{out}(\delta x) = V_{110}^{in}(\delta x) \left(\frac{R}{Q}\right)_{110}^{-1/2} \sqrt{\frac{50\Omega}{Q_L}} \sqrt{1 - \frac{Q_L}{Q_0}}$$

$$V_{110}^{in} \approx \delta x \cdot q \frac{l T_{tr}^2}{r^3} \cdot 0.2474$$

$$T_{tr} = \frac{\sin \eta}{\eta} \quad \text{with} \quad \eta = \frac{\pi l}{\lambda_{mn0}}$$

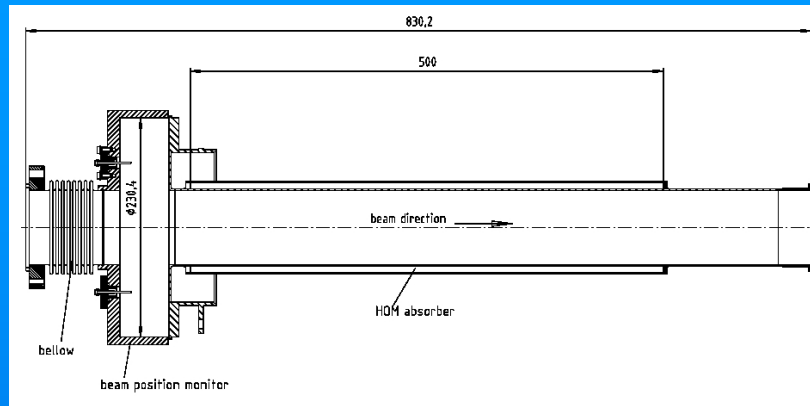
$T_{tr}$  transit time factor  
 $(R/Q)$  geometrical property of cavity  
 $Q_0, Q_L$  unloaded and loaded Q-factors  
 $L$  cavity length  
 $r$  cavity radius  
 $\lambda_{mn0}$  wavelength of mode of interest  
 $\delta x$  transverse displacement

for the TTF cavity BPM:

$$r = 115.2 \text{ mm}$$

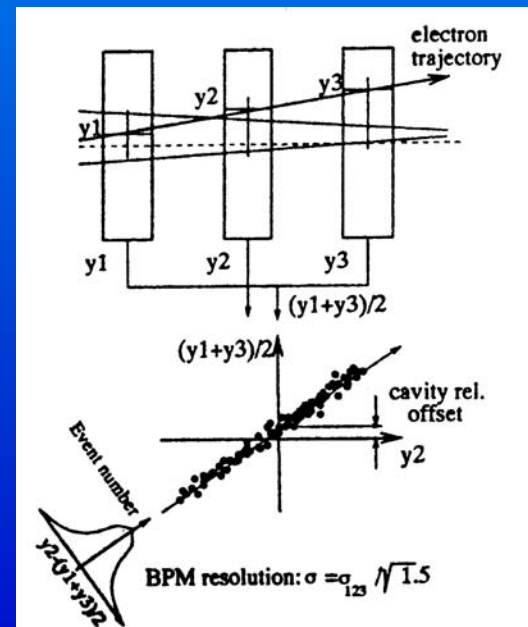
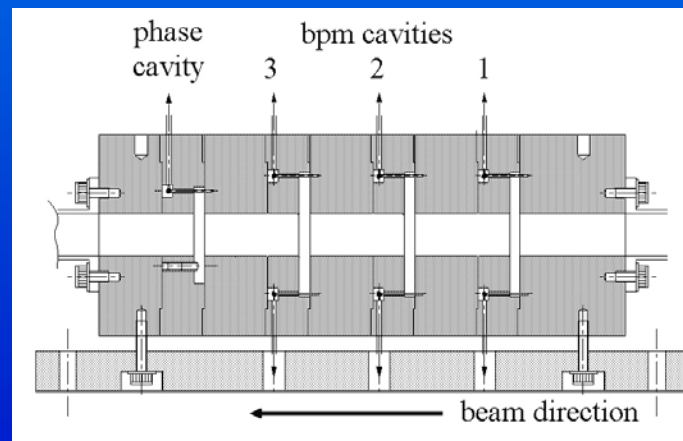
$$L = 52 \text{ mm}$$

$$\rightarrow V_{110}^{out} \sim 115 \text{ mV/mm for } 1 \text{ nC}$$



schematic of a "cold" cavity BPM tested at TTF I (Lorenz)

pioneering experiments: 3 C-band cavity "RF" BPMs in series at the FFTB (SLAC)  $\rightarrow$  25 nm position resolution at 1 nC bunch charge



(courtesy, T. Shintake, 2003)

# Beam Position - Reentrant Cavity BPMs

principle: detection of the evanescent field of the cavity fundamental mode (those waves with exponential attenuation below the cut-off frequency):

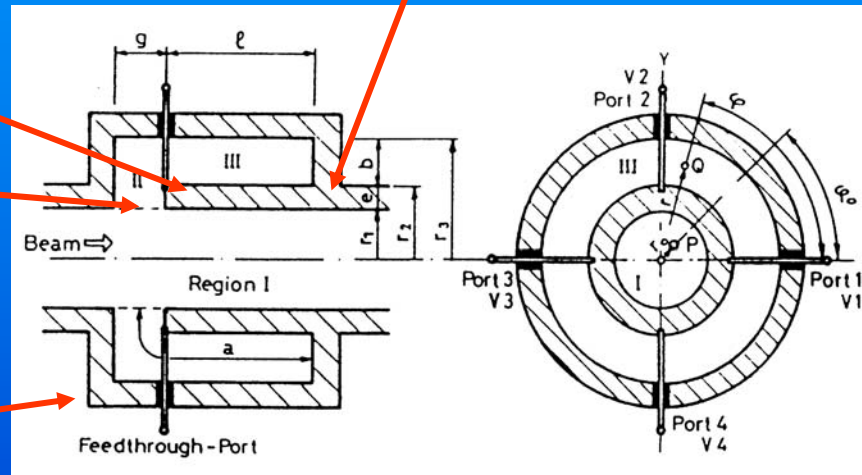
excite cavity at frequency  $f_0$  with respect to cavity resonant frequency  $f_r$  while Q-factor decreases by  $\sqrt{f_0/f_r}$ , the attenuation constant of evanescent fields below  $\sim 1/2$  the cut-off frequency is practically constant  $\rightarrow$  maintain high signal amplitude

(short to ground)

vacuum chamber

gap

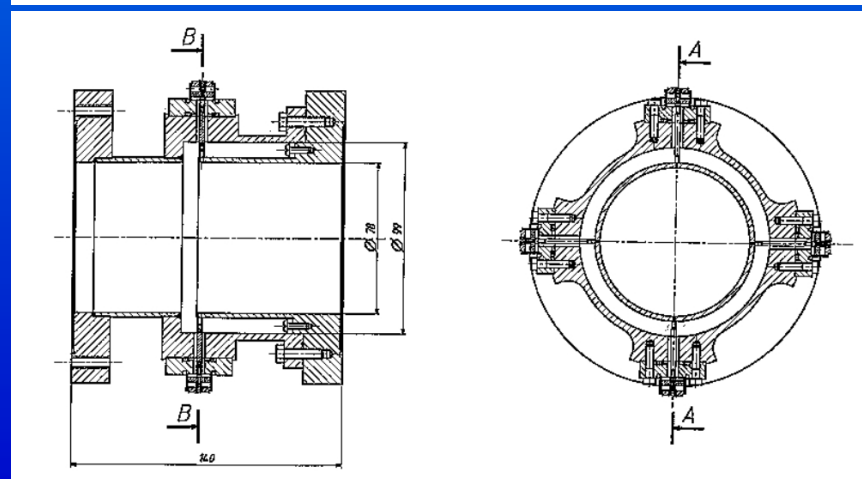
coaxial cylinder



from R. Bossart,  
"High Precision BPM  
Using a Re-Entrant  
Coaxial Cavity",  
LINAC94

using URMEL, the  
equivalent circuit for  
impedance model was  
developed

schematic of the  
reentrant cavity  
BPM used success-  
fully at TTF I and  
planned for use  
at TTF II (courtesy  
C. Magne, 2003)



# Summary

Detection of the wall current  $I_w$  allows for measurements of the beam intensity and position

The detector sensitivities are given by

$$S(\omega) = \frac{V(\omega)}{I_w(\omega)} \quad \text{for the beam charge and intensity}$$

$$S(\omega) = \frac{V(\omega)}{D(\omega)} \quad \text{with} \quad \begin{array}{l} D(\omega) = I_w(\omega)x \quad \text{for the horizontal position} \\ D(\omega) = I_w(\omega)y \quad \text{for the vertical position} \end{array}$$

We reviewed basic beam diagnostics for measuring:

the beam charge - using Faraday cups

the beam intensity - using toroidal transformers and BPM sum signals

the beam position

- using wall gap monitors
- using capacitive monitors (including buttons)
- using stripline / transmission line detectors
- using resonant cavities and re-entrant cavities

We note that the equivalent circuit models presented were often simplistic.

In practice these may be tailored given direct measurement or using computer

models. Impedances in the electronics used to process the signals must also

be taken into account as they often limit the bandwidth of the measurement.

Nonetheless, the fundamental design features of the detectors presented were

discussed (including variations in the designs) highlighting the importance of detector geometries and impedance matching as required for high sensitivity.