Conventional Magnets for Accelerators

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Contents

The presentation deals with d.c. magnets only.

i) No current or steel:

- Laplace's equation with scalar potential;
- Cylindrical harmonic solutions in two dimensions;

ii) Introduce steel yoke:

- Ideal pole shapes for dipole, quad and sextupole;
- Field harmonics-symmetry restraints and significance;

iii) Introduce current:

- Ampere-turns in dipole, quad and sextupole;

Contents (cont.)

iv) Magnet geometry:

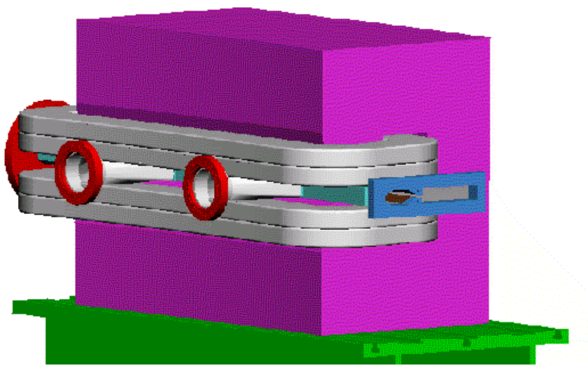
- Backleg and coil geometry- 'C', 'H' and 'window frame' designs;
- Coil economic optimisation-capital/running costs;

v) Field computation and pole optimisation:

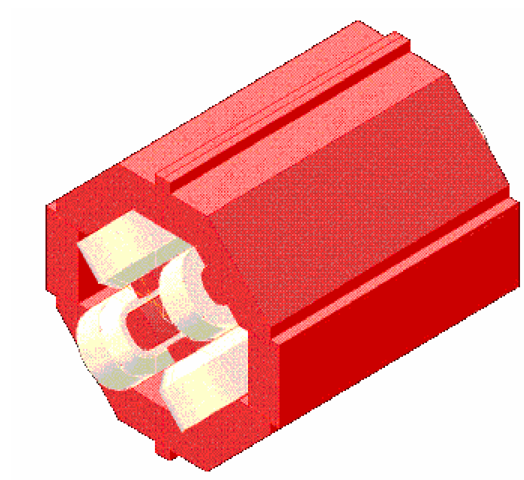
- Field computation software-OPERA, TOSCA;
- Design of pole geometry for dipole, quad and sextupole;
- Magnet ends-computation and design;

Magnets we know about:

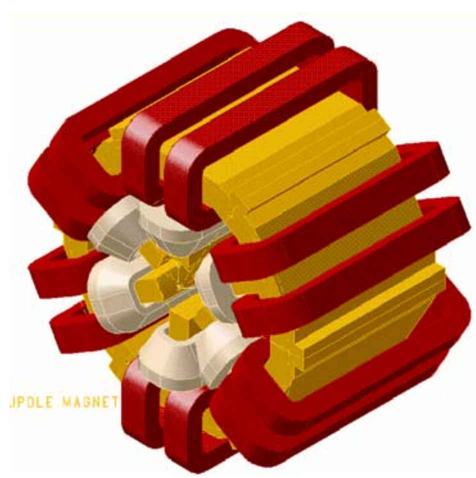
Dipoles to bend the beam:



Quadrupoles to focus it:



Sextupoles to correct chromaticity:



We need to establish a formal approach to describing these magnets.

But first – nomenclature!

Magnetic Field: (the magneto-motive force produced by electric currents)

symbol is **H** (as a vector);

units are Amps/metre in S.I units (Oersteds in cgs);

Magnetic Induction or **Flux Density:** (the density of magnetic flux driven through a medium by the magnetic field)

symbol is **B** (as a vector);

units are Tesla (Webers/m^2 in mks, Gauss in cgs);

Note: induction is frequently referred to as "Magnetic Field".

Permeability of free space:

symbol is μ_0 ;

units are Henries/metre;

Permeability (abbreviation of **relative permeability**):

symbol is μ;

the quantity is dimensionless;

i) No Currents - Maxwell's equations:

$$\underline{\nabla}.\underline{\mathbf{B}} = 0 \;;$$

$$\underline{\nabla}\ \underline{\mathbf{H}} = \underline{\mathbf{j}} \;;$$

In the absence of currents: $\mathbf{j} = 0.$

Then we can put: $\underline{\mathbf{B}} = -\,\underline{\nabla}\phi$

So that: $\underline{\nabla}^2\phi = 0$ (Laplace's equation).

Taking the two dimensional case (ie constant in the z direction) and solving for cylindrical coordinates (r,θ):

$$\phi = (E+F\,\theta)(G+H\ln r) + \Sigma_{n=1}^{\infty}\,(J_n\, r^{\,n}\cos n\theta + K_n\, r^{\,n}\sin n\theta$$
$$+ L_n\, r^{\,-n}\cos n\,\theta + M_n\, r^{\,-n}\sin n\,\theta\,)$$

In practical situations:

The scalar potential simplifies to:

$$\phi = \Sigma_n (J_n r^{n} \cos n\theta + K_n r^{n} \sin n\theta),$$

with n integral and J_n,K_n a function of geometry.

Giving components of flux density:

$$B_r = - \Sigma_n (n J_n r^{n-1} \cos n\theta + nK_n r^{n-1} \sin n\theta)$$
$$B_\theta = - \Sigma_n (-n J_n r^{n-1} \sin n\theta + nK_n r^{n-1} \cos n\theta)$$

Significance

This is an infinite series of cylindrical harmonics; they define the allowed distributions of **B** in 2 dimensions in the absence of currents within the domain of (r,θ).

Distributions not given by above are not physically realisable.

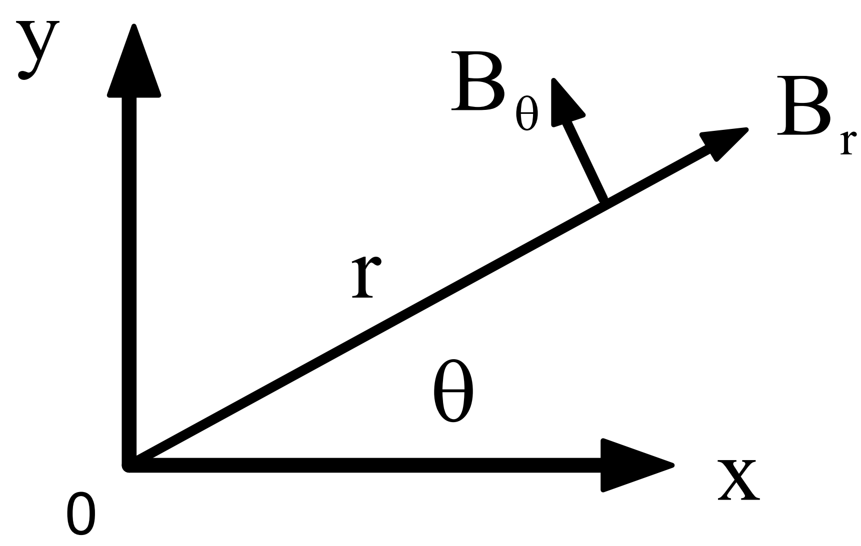
Coefficients J_n, K_n are determined by geometry (iron boundaries or remote current sources).

Cartesian Coordinates

In Cartesian coordinates, the components are given by:

$$B_x = B_r \cos\theta - B_\theta \sin\theta,$$

$$B_y = B_r \sin\theta + B_\theta \cos\theta,$$



Dipole field: n = 1

Cylindrical:

$B_r = J_1 \cos\theta + K_1 \sin\theta;$

$B_\theta = -J_1 \sin\theta + K_1 \cos\theta;$

$\phi = J_1 r \cos\theta + K_1 r \sin\theta.$

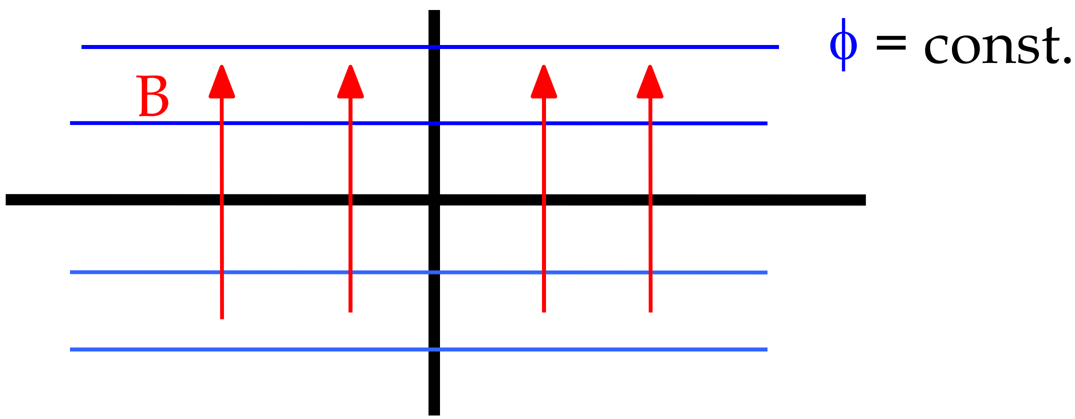
Cartesian:

$B_x = J_1$

$B_y = K_1$

$\phi = J_1 x + K_1 y$

So, $J_1 = 0$ gives vertical dipole field:



$K_1 = 0$ gives horizontal dipole field.

Quadrupole field: n = 2

Cylindrical:

$B_r = 2\, J_2\, r\ \cos 2\theta + 2K_2\, r\ \sin 2\theta;$

$B_\theta = -2J_2\, r\ \sin 2\theta + 2K_2\, r\ \cos 2\theta;$

$\phi\ =\ J_2\, r\, 2 \cos 2\theta + K_2\, r\, 2 \sin 2\theta;$

Cartesian:

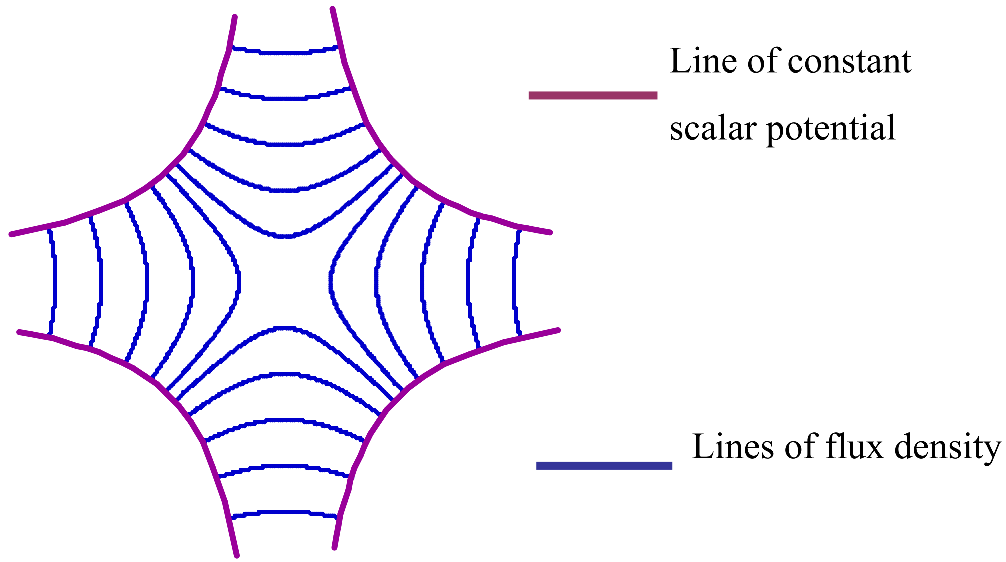
$Bx = 2\,(J_2\, x + K_2\, y)$

$B_y = 2\,(-J_2\, y + K_2\, x)$

$\phi = J_2\,(x^2 - y^2) + 2K_2\, xy$

$J_2 = 0$ gives 'normal' or 'right' quadrupole field.

$K_2 = 0$ gives 'skew' quad fields (above rotated by $\pi/4$).



Sextupole field: n = 3

Cylindrical;

$B_r = 3\, J_3 r^2 \cos 3\theta + 3K_3 r^2 \sin 3\theta;$

$B_\theta = -3J_3\, r^2 \sin 3\theta + 3K_3\, r^2 \cos 3\theta;$

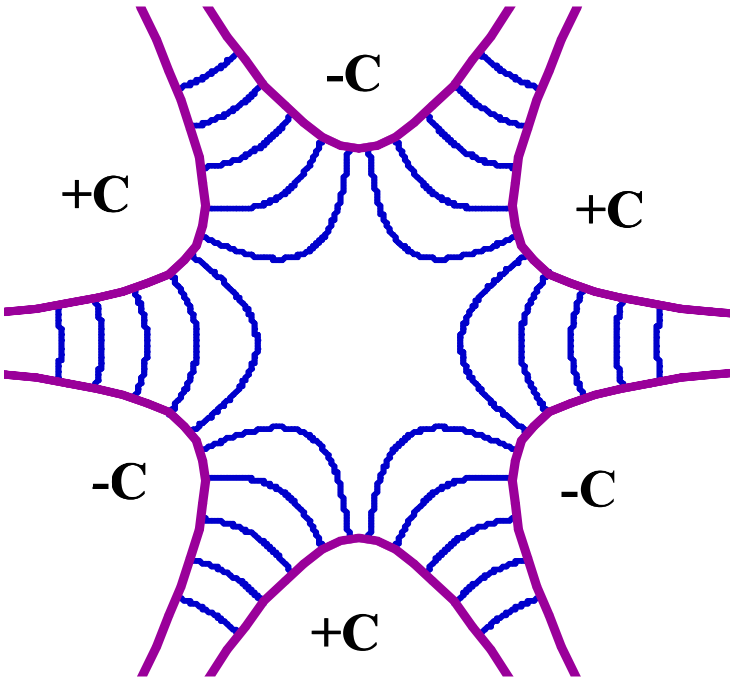
$\phi = J_3\, r^3 \cos 3\theta + K_3\, r^3 \sin 3\theta;$

Cartesian:

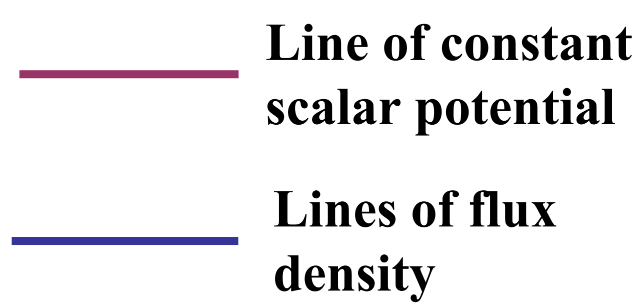
$B_x = 3\{J_3\,(x^2 - y^2) + 2K_3 yx\}$

$B_y = 3\{-2\, J_3\, xy + K_3(x^2 - y^2)\}$

$\phi = J_3\,(x^3 - 3y^2x) + K_3(3yx^2 - y^3)$

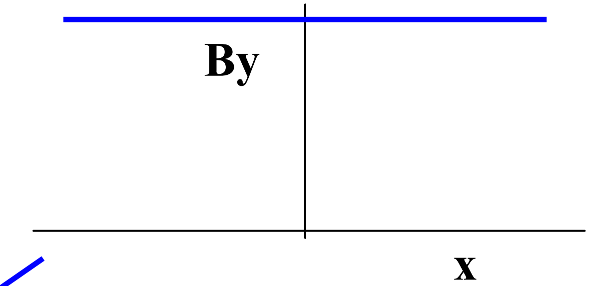


$J_3 = 0$ giving 'normal' or 'right' sextupole field.

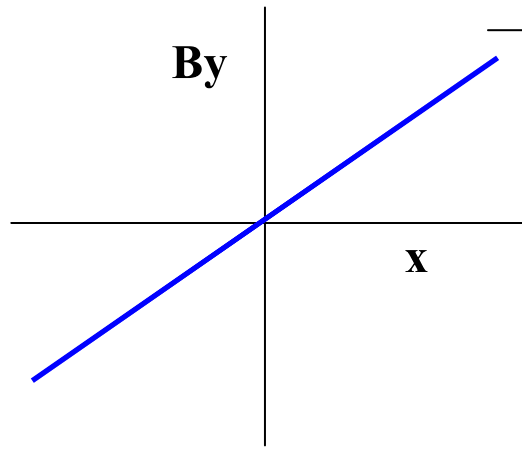


Summary; variation of B_y on x axis

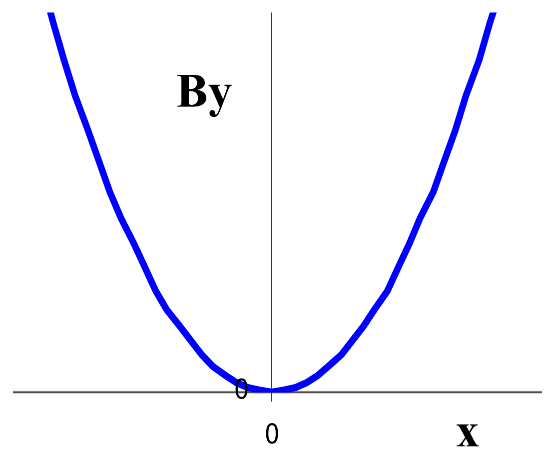
Dipole; constant field:



Quad; linear variation:



Sext.: quadratic variation:



Alternative notification (USA)

$$B(x) = B\rho \sum_{n=0}^{\infty} \frac{k_n\ x^n}{n!}$$

magnet strengths are specified by the value of k_n;
(normalised to the beam rigidity);

order n of k is different to the 'standard' notation:

dipole is	$n = 0$;
quad is	$n = 1$; etc.

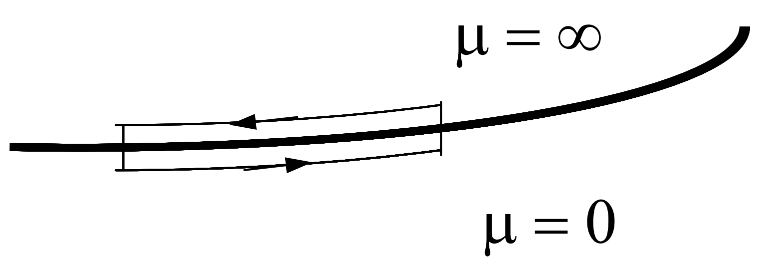
k has units:

k_0 (dipole)	m^{-1};
k_1 (quadrupole)	m^{-2}; etc.

ii) Introducing Iron Yokes

What is the ideal pole shape?

•Flux is normal to a ferromagnetic surface with infinite μ:



curl H = 0

therefore $\int$ H.ds = 0;

in steel H = 0;

therefore parallel H air = 0

therefore B is normal to surface.

•Flux is normal to lines of scalar potential, ($\underline{\mathbf{B}} = - \underline{\nabla}\phi$);

•So the lines of scalar potential are the ideal pole shapes!

(but these are infinitely long!)

Equations for the ideal pole

Equations for Ideal (infinite) poles;

($J_n = 0$) for **normal** (ie not skew) fields:

Dipole:

$$y = \pm g/2;$$

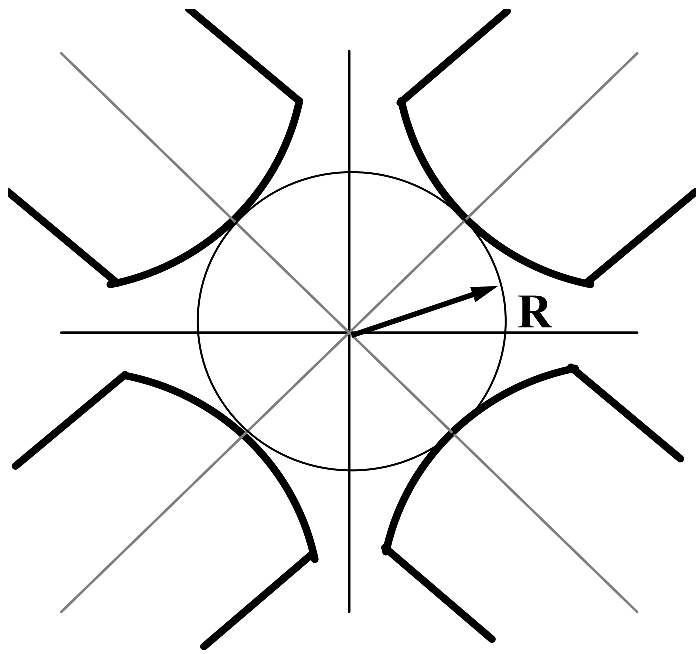
(g is interpole gap).

Quadrupole:

$$xy = \pm R^2/2;$$

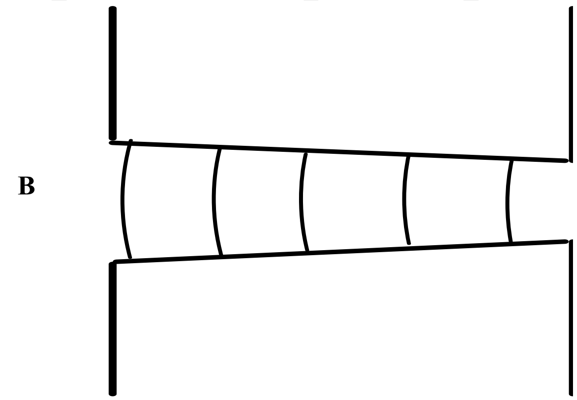
Sextupole:

$$3x^2y - y^3 = \pm R^3;$$



Combined function magnets

'Combined Function Magnets' - usually dipole and quadrupole field combined:



A quadrupole magnet with physical centre shifted from magnetic centre.

Characterised by 'field index' n, +ve or -ve depending on direction of gradient; do not confuse with harmonic n!

$$n = -\left(\frac{\rho}{B_0}\right)\left(\frac{\partial B}{\partial x}\right),$$

ρ is radius of curvature of the beam;

B_o is central dipole field

Pole equations for c.f. magnet

If physical and magnetic centres are separated by X_0

Then $$B_0 = \left(\frac{\partial B}{\partial x}\right) X_0;$$

therefore $$X_0 = -\rho / n;$$

in a quadrupole $$x' y = \pm R^2 / 2$$

where x' is measured from the true quad centre;

Put $$x' = x + X_0$$

So pole equation is $$y = \pm \frac{R^2}{2} \frac{n}{\rho} \left(1 - \frac{nx}{\rho}\right)^{-1}$$

or $$\boxed{y = \pm g \left(1 - \frac{nx}{\rho}\right)^{-1}}$$

where g is the half gap at the physical centre of the magnet

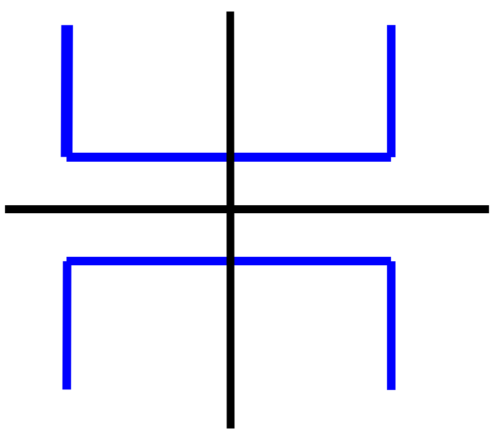
The practical Pole

Practically, poles are finite, **introducing errors**;

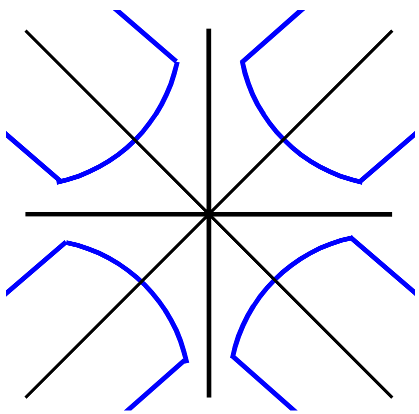
these appear as higher harmonics which degrade the field distribution.

However, the iron geometries have certain symmetries that **restrict** the nature of these errors.

Dipole:



Quadrupole:



Possible symmetries:

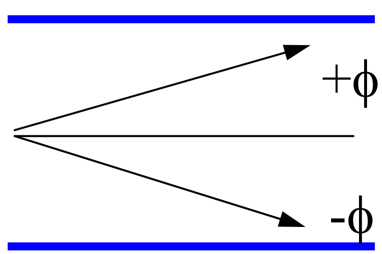
Lines of symmetry:

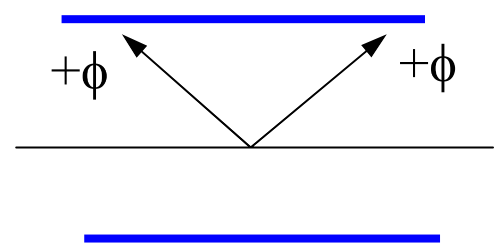
	Dipole:	Quad
Pole orientation determines whether pole is normal or skew.	$y = 0$;	$x = 0$; $y = 0$
Additional symmetry imposed by pole edges.	$x = 0$;	$y = \pm x$

The additional constraints imposed by the symmetrical pole edges limits the values of n that have non zero coefficients

Dipole symmetries

Type	**Symmetry**	**Constraint**
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	all $J_n = 0$;
Pole edges	$\phi(\theta) = \phi(\pi - \theta)$	K_n non-zero only for: $n = 1, 3, 5$, etc;





So, for a fully symmetric dipole, only 6, 10, 14 etc pole errors can be present.

Quadrupole symmetries

Type	**Symmetry**	**Constraint**
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	All $J_n = 0$;
	$\phi(\pi) = -\phi(\pi - \theta)$	$K_n = 0$ all odd n;
Pole edges	$\phi(\theta) = \phi(\pi/2 - \theta)$	K_n non-zero only for: $n = 2, 6, 10$, etc;

So, a fully symmetric quadrupole, only 12, 20, 28 etc pole errors can be present.

Sextupole symmetries

Type	**Symmetry**	**Constraint**
Pole orientation	$\phi(\theta) = -\phi(-\theta)$ $\phi(\theta) = -\phi(2\pi/3 - \theta)$ $\phi(\theta) = -\phi(4\pi/3 - \theta)$	All $J_n = 0$; $K_n = 0$ for all n not multiples of 3;
Pole edges	$\phi(\theta) = \phi(\pi/3 - \theta)$	K_n non-zero only for: n = 3, 9, 15, etc.

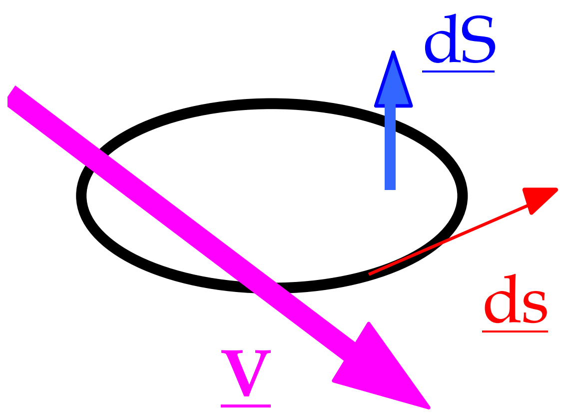
So, a fully symmetric sextupole, only 18, 30, 42 etc pole errors can be present.

iii) Introduction of currents

Now for $\underline{j} \neq 0$ $\qquad \underline{\nabla}\ \underline{\mathbf{H}} = \underline{\mathbf{j}}$;

To expand, use Stoke's Theorum:
for any vector $\underline{\mathbf{V}}$ and a closed curve s :

$$\int \underline{\mathbf{V}}.\underline{\mathbf{ds}} = \iint \mathbf{curl}\ \underline{\mathbf{V}}.\underline{\mathbf{dS}}$$



Apply this to: $\quad \mathbf{curl\ H} = \underline{\mathbf{j}}$;

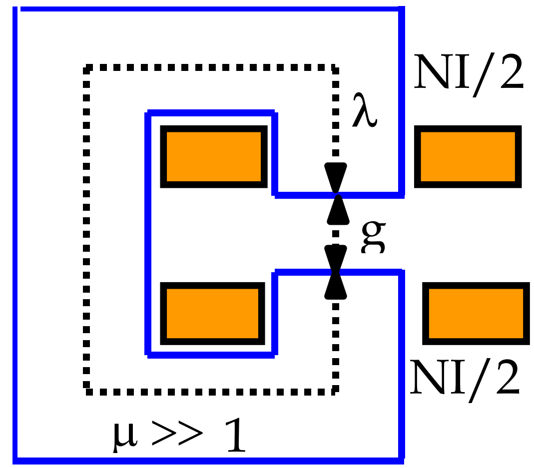
then in a magnetic circuit:

$$\boxed{\int \underline{\mathbf{H}}.\underline{\mathbf{ds}} = \mathrm{N\ I};}$$

N I (Ampere-turns) is total current cutting $\underline{\mathbf{S}}$

Excitation current in a dipole

B is approx constant round the loop made up of λ and g, (but see below);



But in iron, $\mu >> 1$,
and $H_{iron} = H_{air} / \mu$;

So

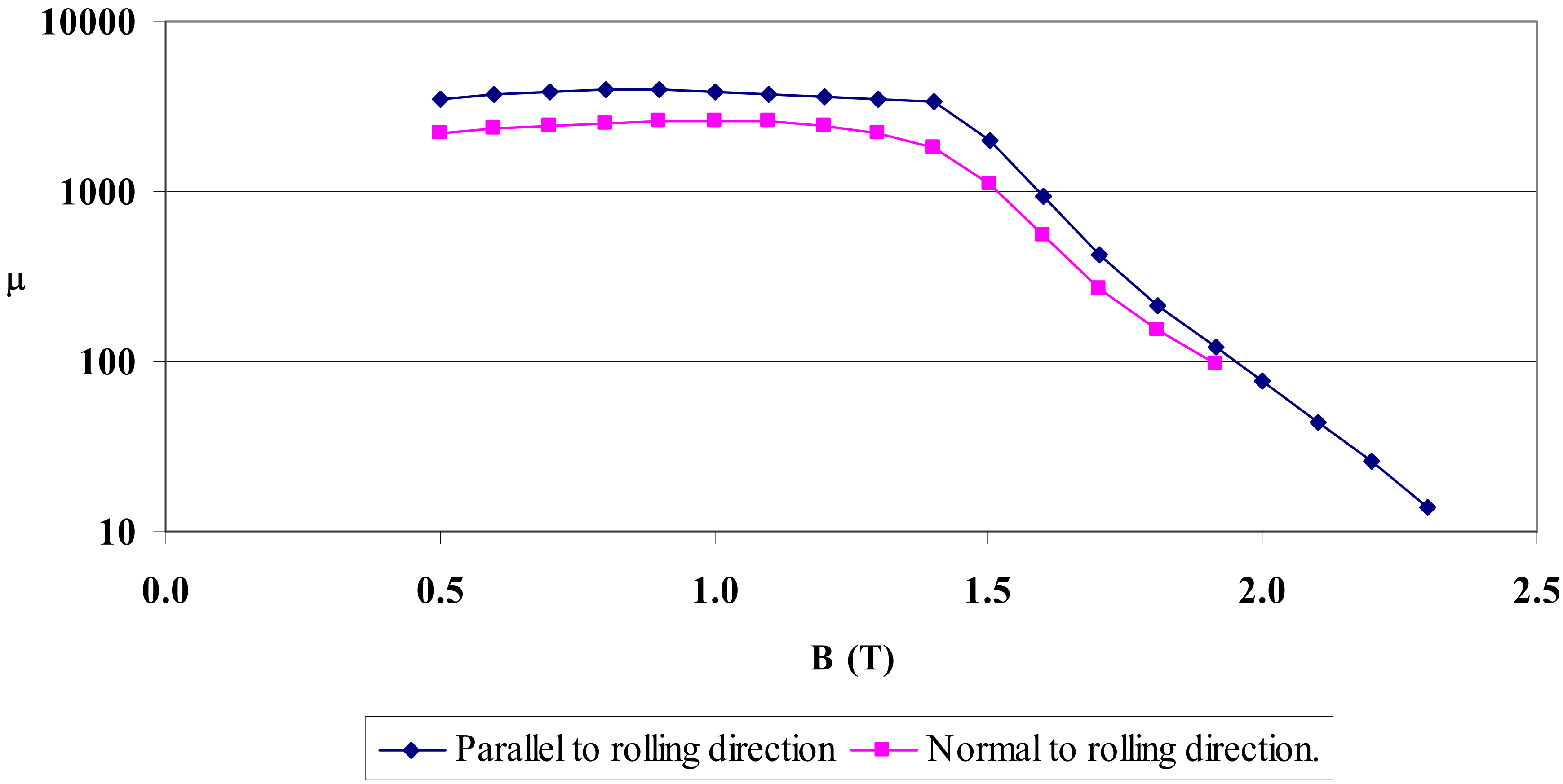
$$\boxed{B_{air} = \mu_0 NI / (g + \lambda/\mu);}$$

g, and λ/μ are the 'reluctance' of the gap and iron.

Approximation ignoring iron reluctance ($\lambda/\mu << g$):

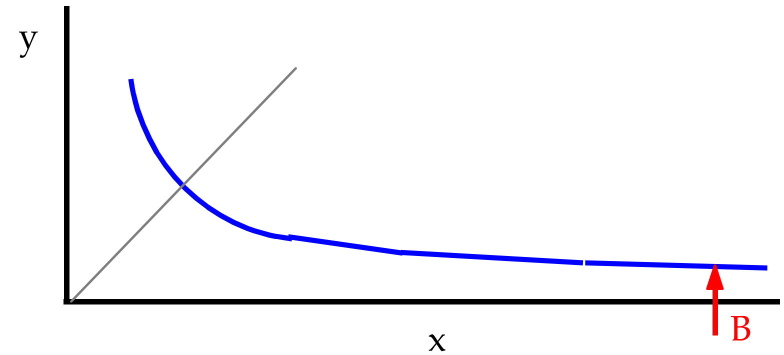
$$\boxed{NI = B g / \mu_0}$$

Relative permeability of low silicon steel



Excitation current in quad & sextupole

For quadrupoles and sextupoles, the required excitation can be calculated by considering fields and gap at large x. For example: **Quadrupole:**



Pole equation: $xy = R^2 /2$
On x axes $B_Y = gx$;
where g is gradient (T/m).

At large x (to give vertical lines of B):

$$N\,I = (gx)\,(R^2 /2x)/\mu_0$$

ie

$$\boxed{N\,I = g\,R^2 /2\,\mu_0 \text{ (per pole).}}$$

The same method for a **Sextupole**, (coefficient g_S,), gives:

$$\boxed{N\,I = g_S\,R^3/3\,\mu_0 \text{ (per pole)}}$$

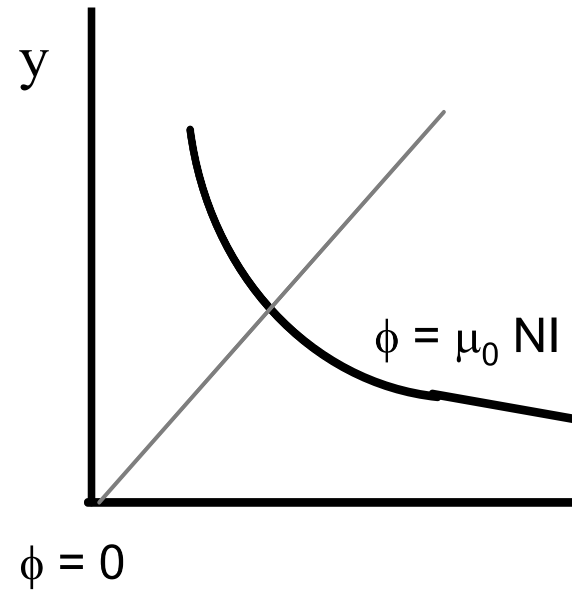
General solution for magnets order n

In air (remote currents!), $\mathbf{B} = \mu 0\, \mathbf{H}$

$\underline{\mathbf{B}} = - \underline{\nabla}\phi$

Integrating over a limited path
(not circular) in air: $N\,I = (\phi_1 - \phi_2)/\mu_o$
ϕ_1, ϕ_2 are the scalar potentials at two points in air.
Define $\phi = 0$ at magnet centre;
then potential at the pole is:

$\mu_o\, NI$



Apply the general equations for magnetic field harmonic order n for non-skew magnets (all Jn = 0) giving:

$$N\,I = (1/n)\,(1/\mu_0)\,\{B_r/R^{(n-1)}\}\,R^{n}$$

Where:

NI is excitation per pole;
R is the inscribed radius (or half gap in a dipole);
term in brackets {} is magnet strength in T/m $^{(n-1)}$.

iv) Magnet geometry

Dipoles can be ‘C core’ ‘H core’ or ‘Window frame’

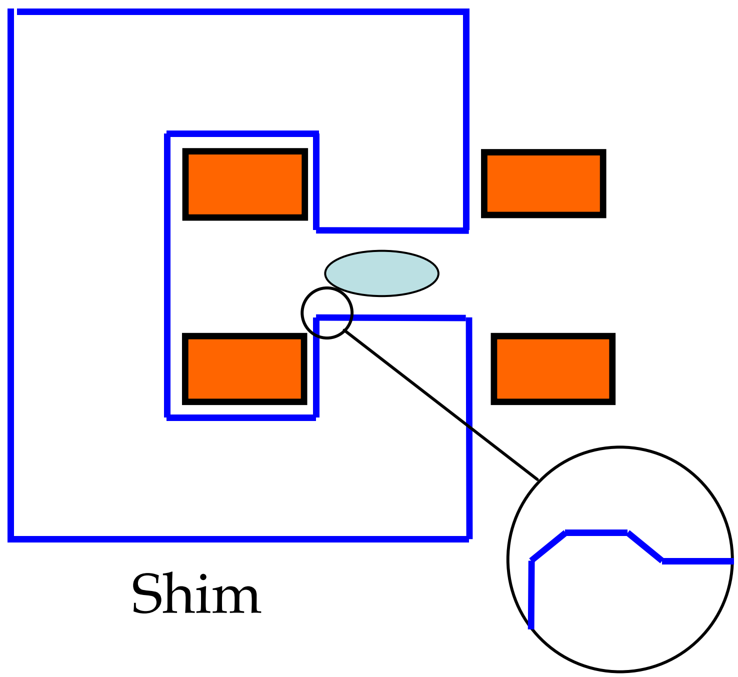
"C' Core:

Advantages:

- Easy access;
- Classic design;

Disadvantages:

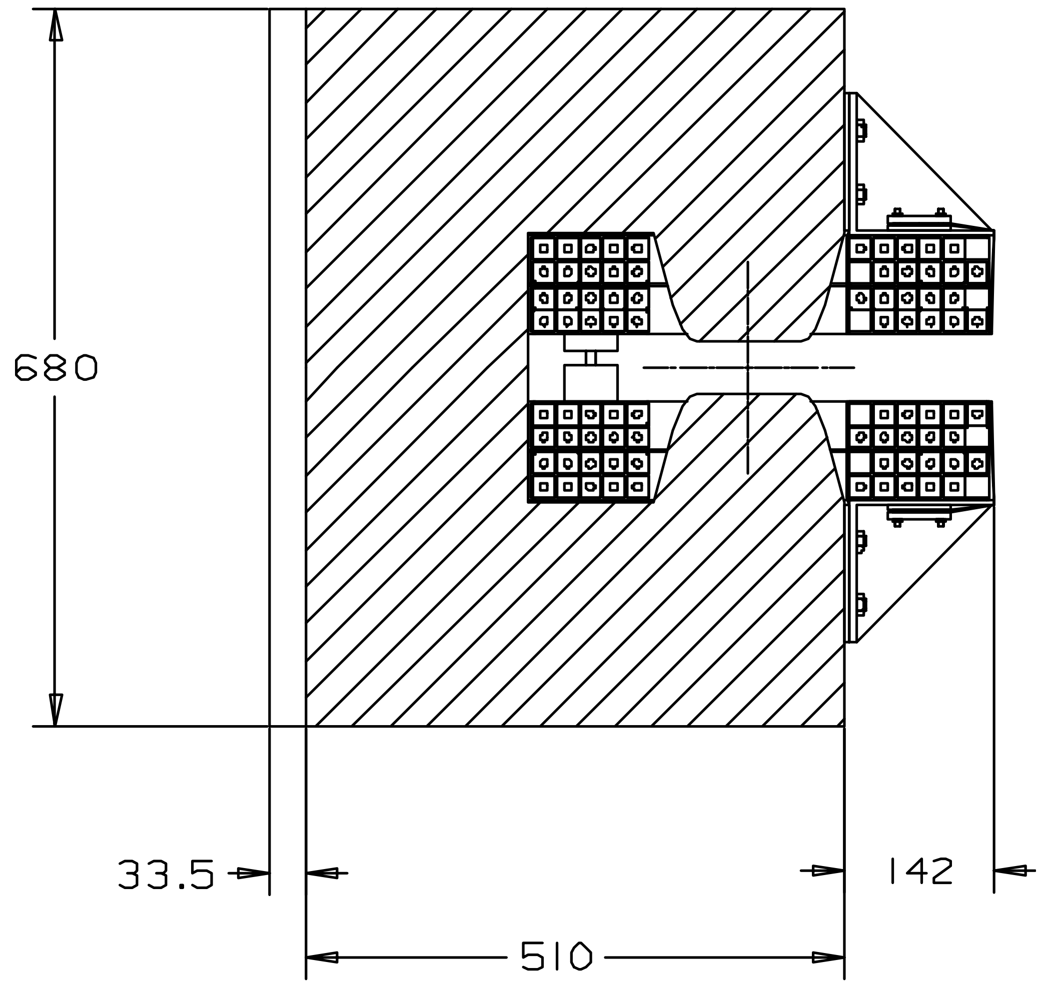
- Pole shims needed;
- Asymmetric (small);
- Less rigid;



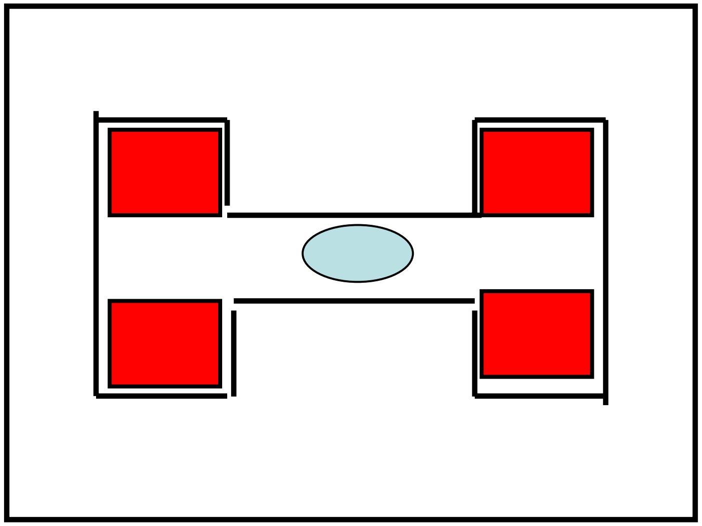
The ‘shim’ is a small, additional piece of ferro-magnetic material added on each side of the two poles – it compensates for the finite cut-off of the pole, and is optimised to reduce the 6, 10, 14...... pole error harmonics.

A typical 'C' cored Dipole

Cross section of the Diamond storage ring dipole.



H core and window-frame magnets



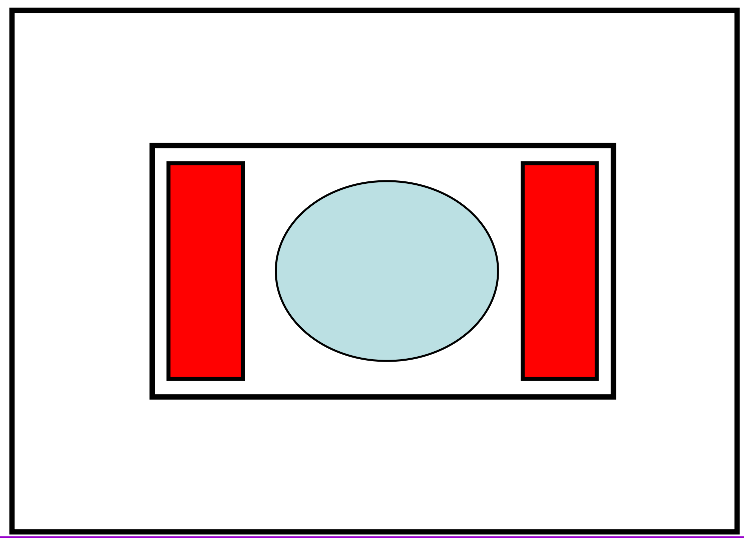
"Window Frame"

Advantages:

- High quality field;
- No pole shim;
- Symmetric & rigid;

Disadvantages:

- Major access problems;
- Insulation thickness



'H core':

Advantages:

- Symmetric;
- More rigid;

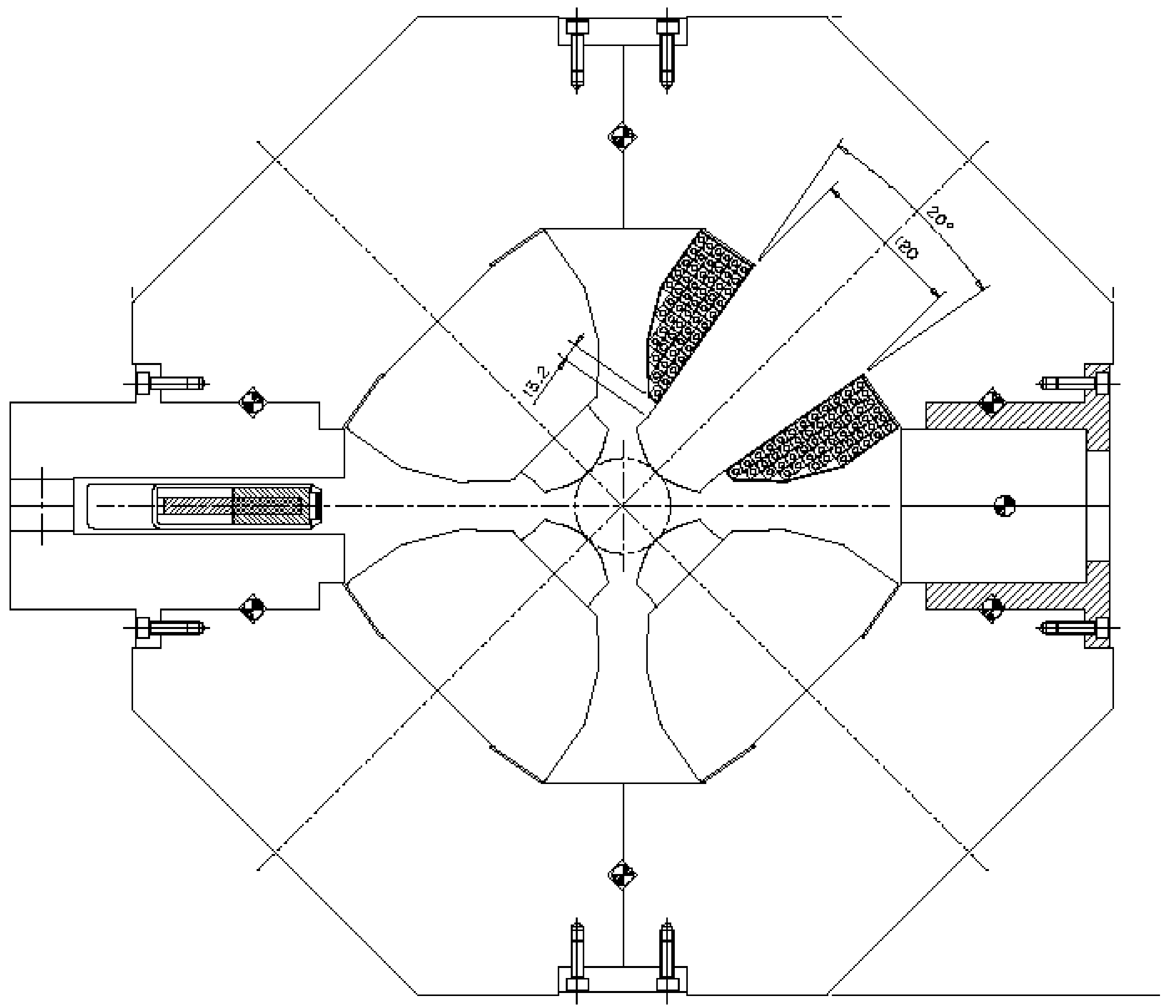
Disadvantages:

- Still needs shims;
- Access problems.

An open-sided Quadrupole.

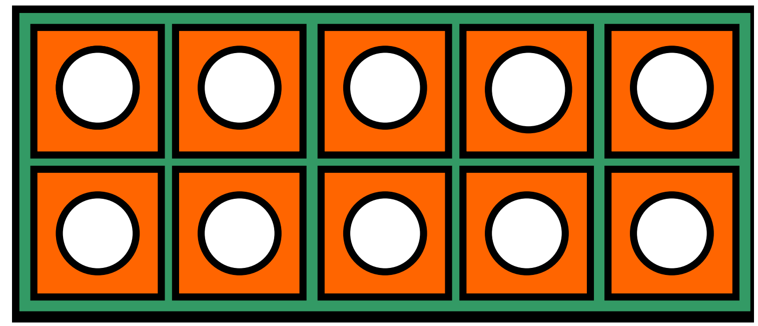
'Diamond' storage ring quadrupole cross section.

The yoke support pieces in the horizontal plane need to provide space for beam-lines and are not ferro-magnetic



Coil geometry

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.



Amp-turns (NI) are determined, but total copper area (A_{copper}) and number of turns (N) are two degrees of freedom and need to be decided.

Current density:
$j = NI/A_{copper}$
Optimum j determined from **economic** criteria.

Current density - optimisation

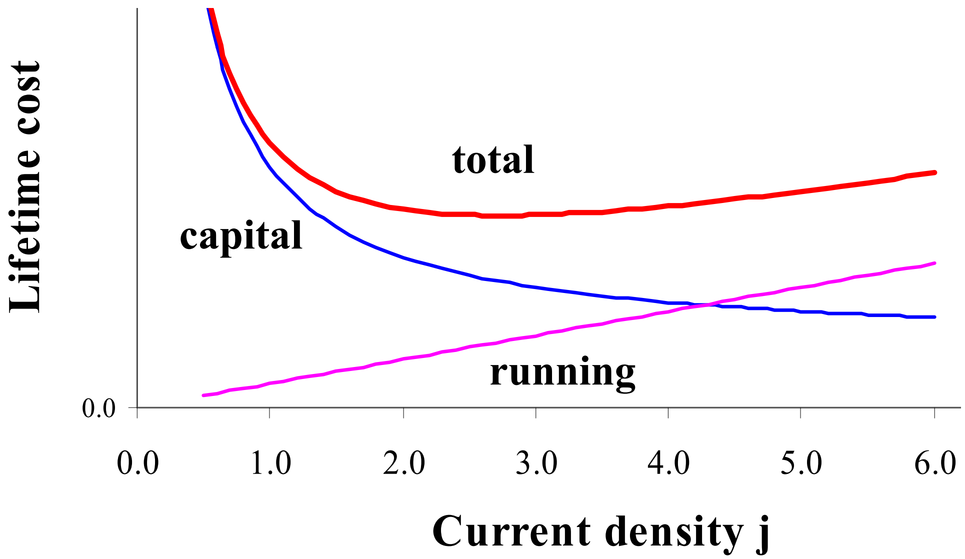
Advantages of low j:

- **lower power loss** – power bill is decreased;
- **lower power loss** – power converter size is decreased;
- **less heat** dissipated into magnet tunnel.

Advantages of high j:

- **smaller coils;**
- **lower capital cost;**
- **smaller magnets.**

Chosen value of j is an optimisation of magnet capital against power costs.



Number of turns, N

The value of number of turns (N) is chosen to match power supply and interconnection impedances.

Factors determining choice of N:

Large N (low current)	Small N (high current)
Small, neat terminals.	Large, bulky terminals
Thin interconnections-hence low cost and flexible.	Thick, expensive connections.
More insulation layers in coil, hence larger coil volume and increased assembly costs.	High percentage of copper in coil volume. More efficient use of space available
High voltage power supply -safety problems.	High current power supply. -greater losses.

Examples of typical turns/current

From the Diamond 3 GeV synchrotron source:

Dipole:

N (per magnet):	40;	
I max	1500	A;
Volts (circuit):	500	V.

Quadrupole:

N (per pole)	54;	
I max	200	A;
Volts (per magnet):	25	V.

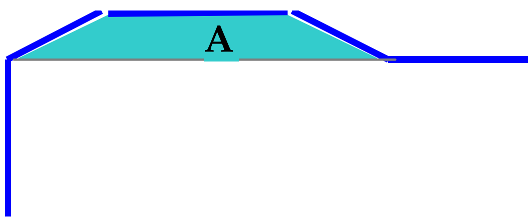
Sextupole:

N (per pole)	48;	
I max	100	A;
Volts (per magnet)	25	V.

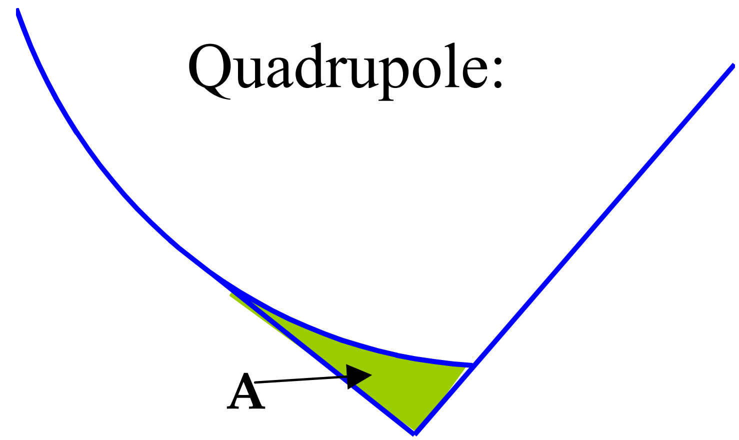
v) Pole design

To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Dipole:



Quadrupole:

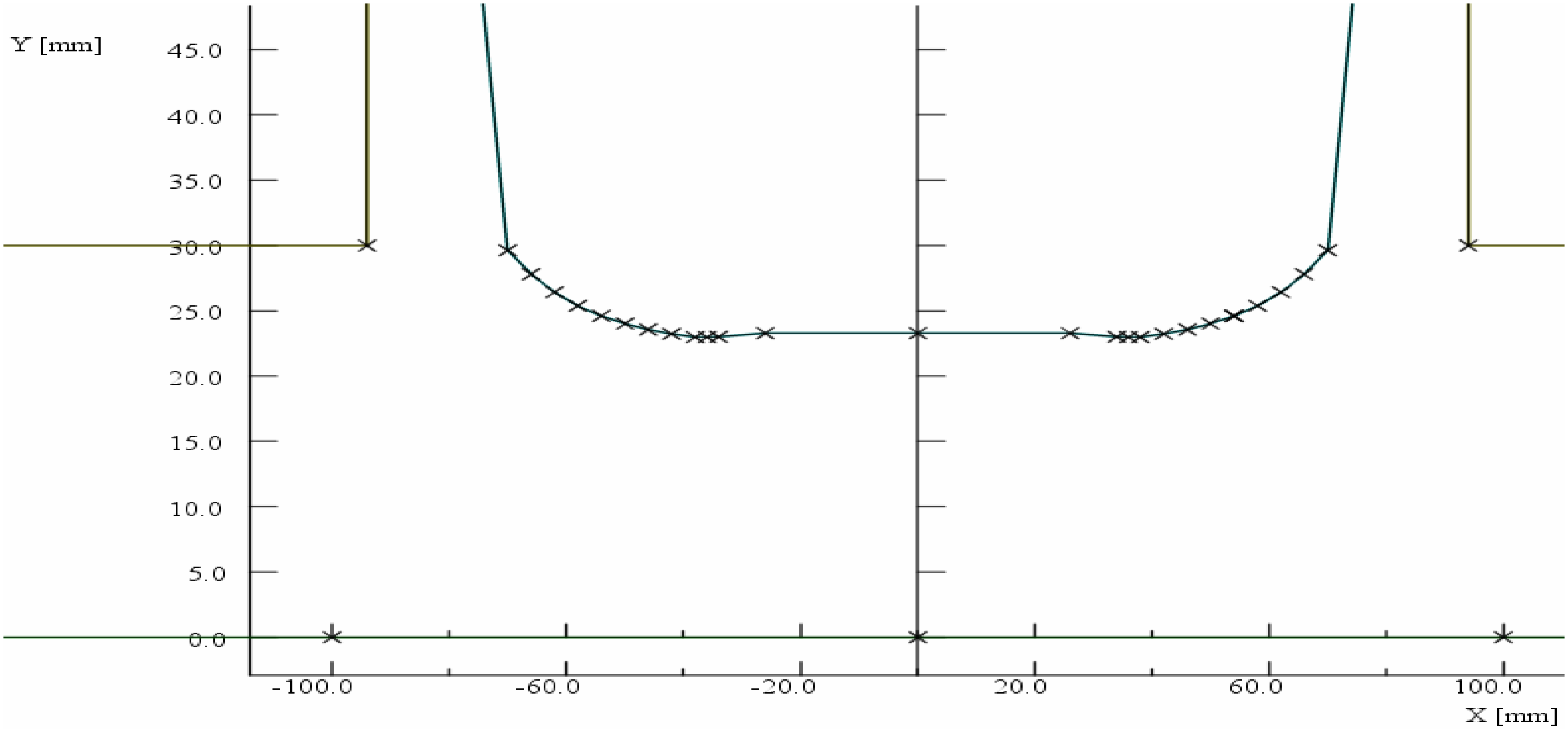


The designer optimises the pole by 'predicting' the field resulting from a given pole geometry and then adjusting it to give the required quality.

When high fields are present, chamfer angles must be small, and tapering of poles may be necessary

Diamond s.r dipole

Pole profile, showing shim and Rogowski roll-off for Diamond 1.4 T dipole.:



Computing magnetic fields

Advanced 2 D and 3 D 'finite element' codes predicting field distributions between poles and at the magnet ends give accurate predictions of the strength of the magnet:

$\int B.\,dl$	for dipole;
$\int g.\,dl$	for quadrupole.

For a dipole, very small variations must be examined;
For a quadrupole or sextupole the field variation is an inadequate criterion; the differentials must be examined.

Judgement of field quality, plot:

Dipole:	$(B_y\,(x) - B_y\,(0))/B_Y\,(0)$
Quad:	$dB_y\,(x)/dx$ (first differences);
Sextupoles:	$d^2B_y(x)/dx^2$ (second differences)

Computational methods

Pre computers, numerical methods and other maths methods were used to predict field distributions.

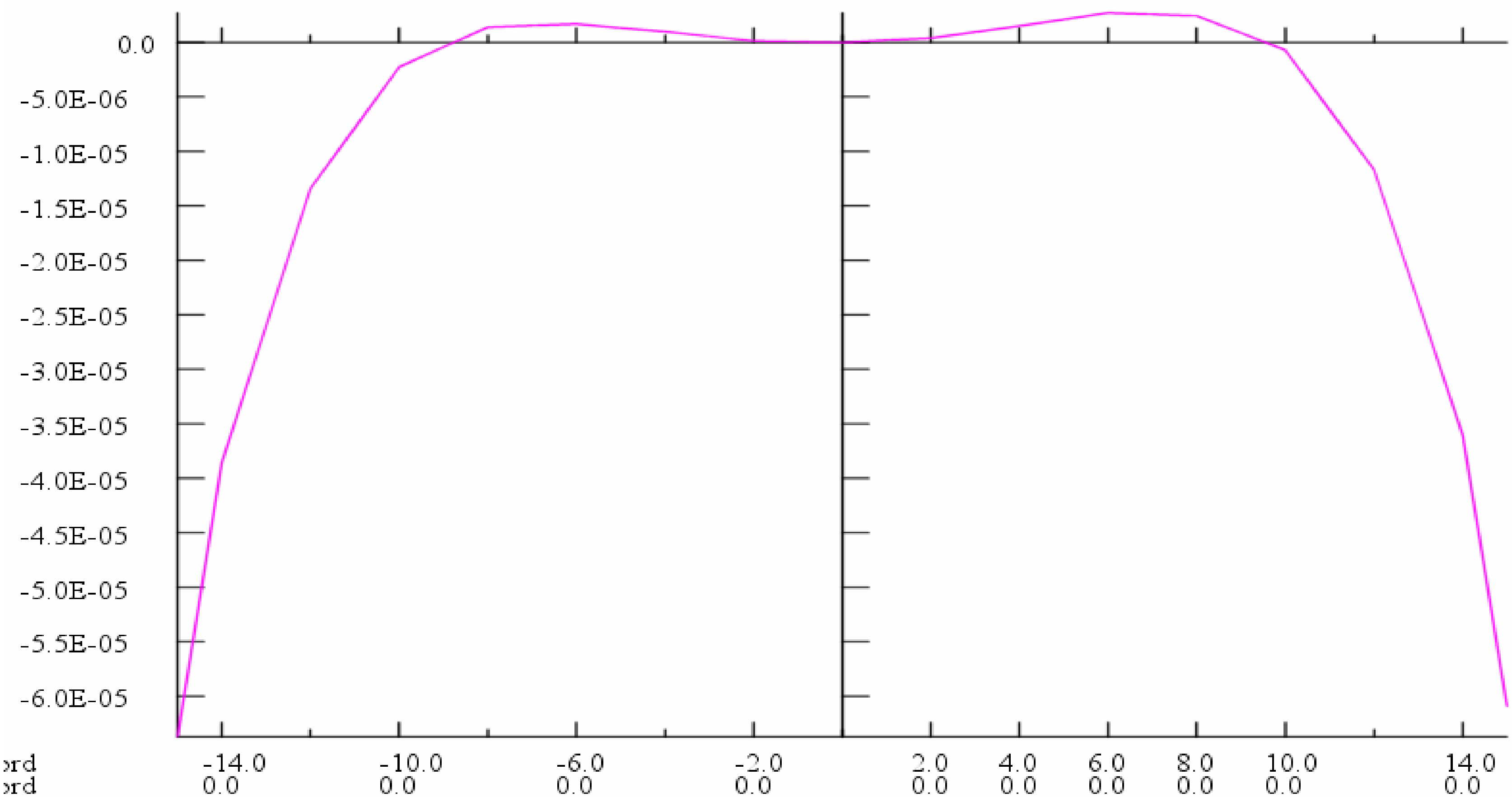
Still used - ‘conformal transformations’; mapping between complex planes representing the magnet geometry and a configuration that is analytic. The study of this is beyond the scope of the present course.

Computer codes are now used; eg the Vector Fields codes - ‘OPERA 2D’ and ‘TOSCA’ (3D) – as presented.

These have:

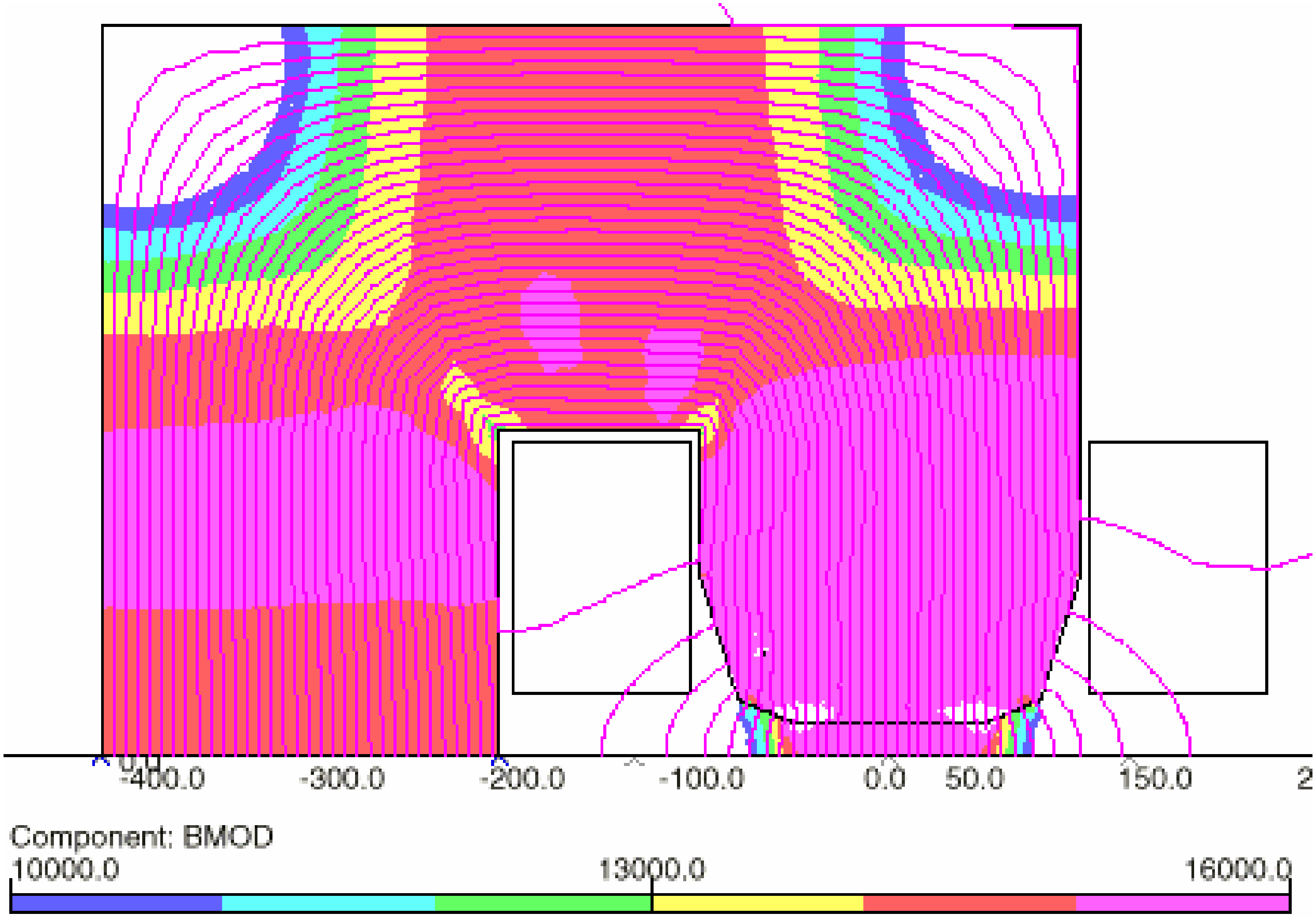
- finite elements with variable triangular mesh;
- multiple iterations to simulate steel non-linearity;
- extensive pre and post processors;
- compatibility with many platforms and P.C. o.s.

2 D Dipole field homogeneity on x axis



Diamond s.r. dipole: $\Delta B/B = \{By(x)- B(0,0)\}/B(0,0)$;
typically $\pm$ 1:10^4 within the 'good field region' of $-12mm \leq x \leq +12$ mm..

2 D Flux density distribution in a dipole.

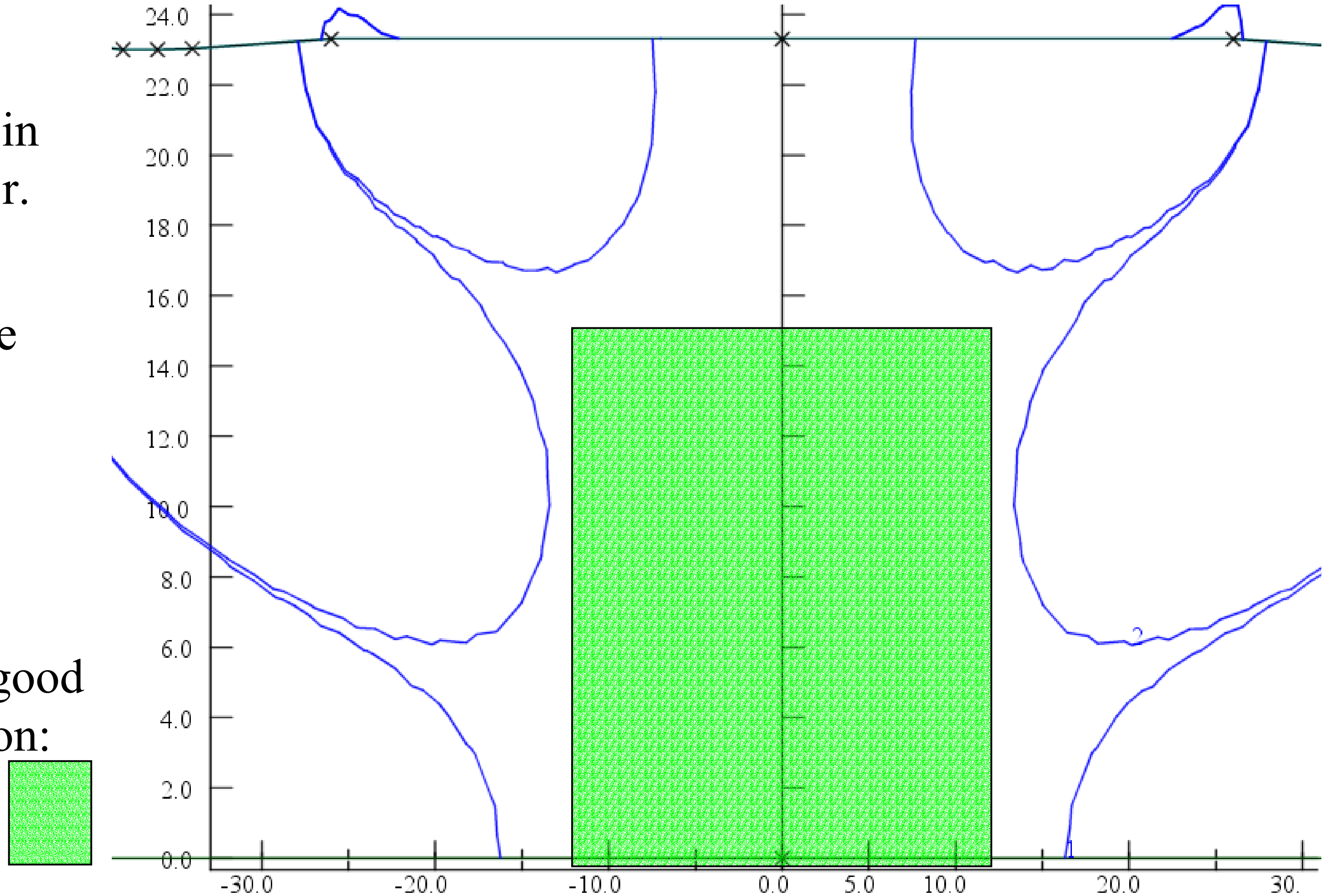


2 D Dipole field homogeneity in gap

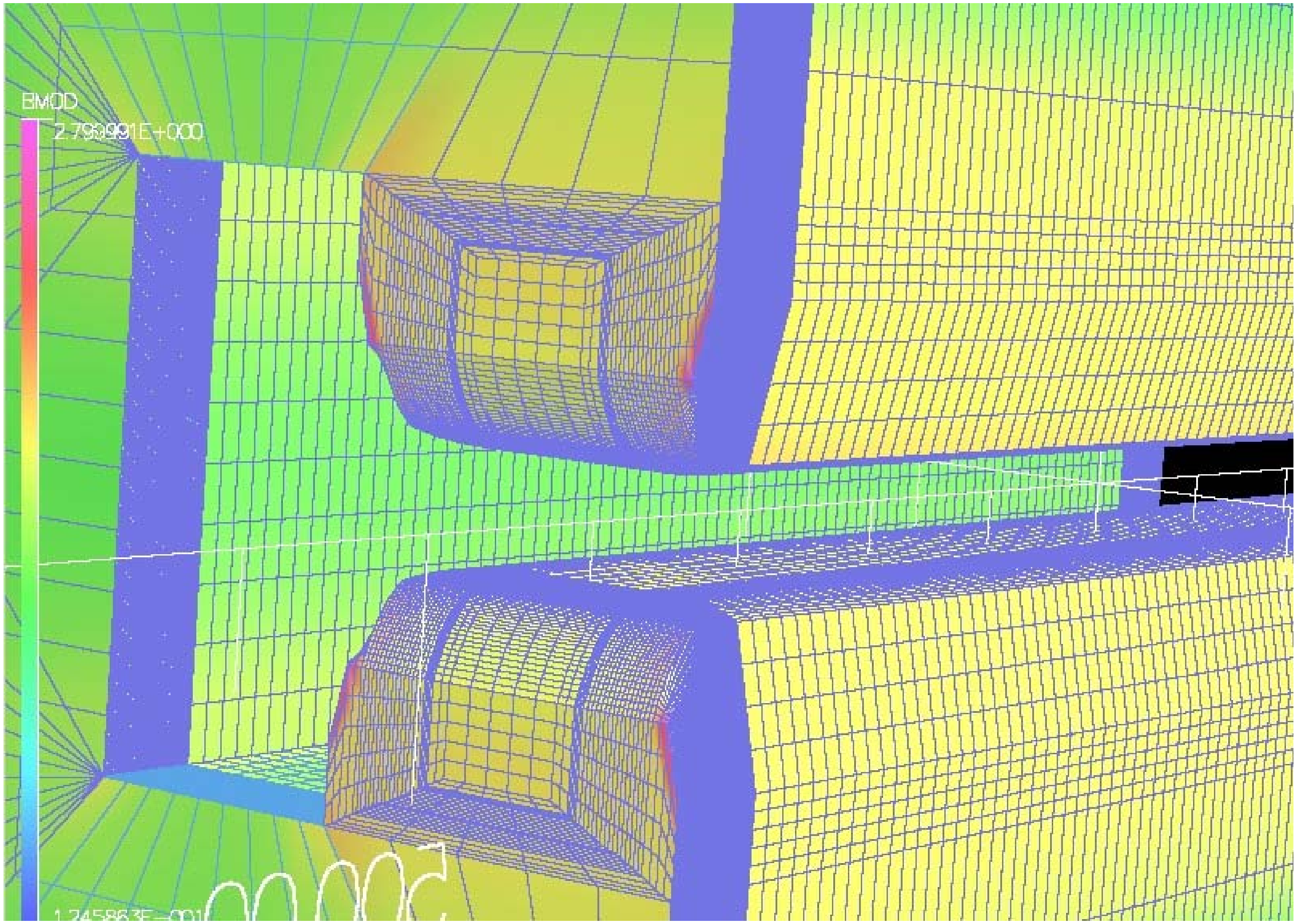
Transverse (x,y) plane in Diamond s.r. dipole;

contours are ±0.01%

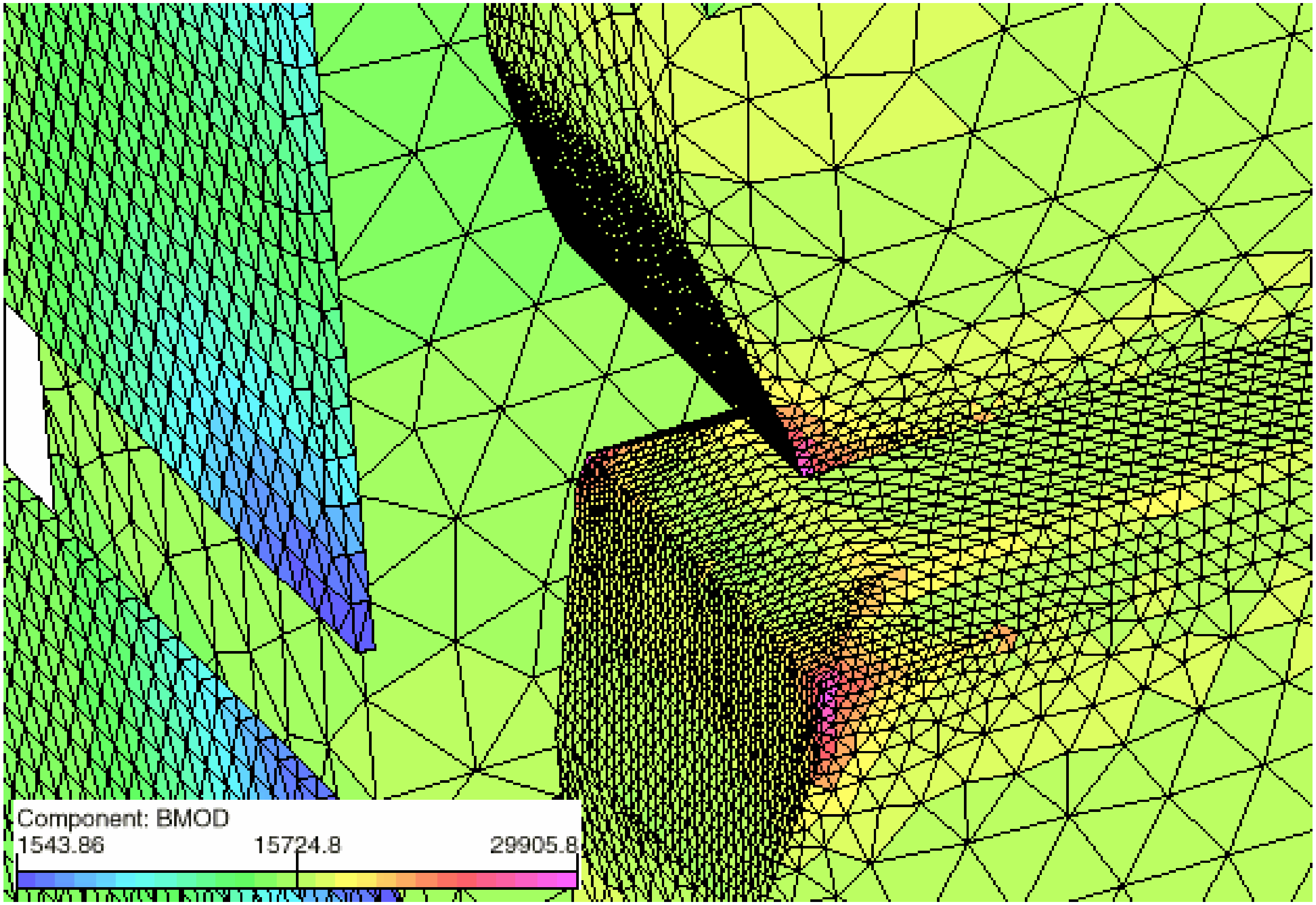
required good field region:



3D model of Diamond dipole.



‘Soleil’ dipole end.

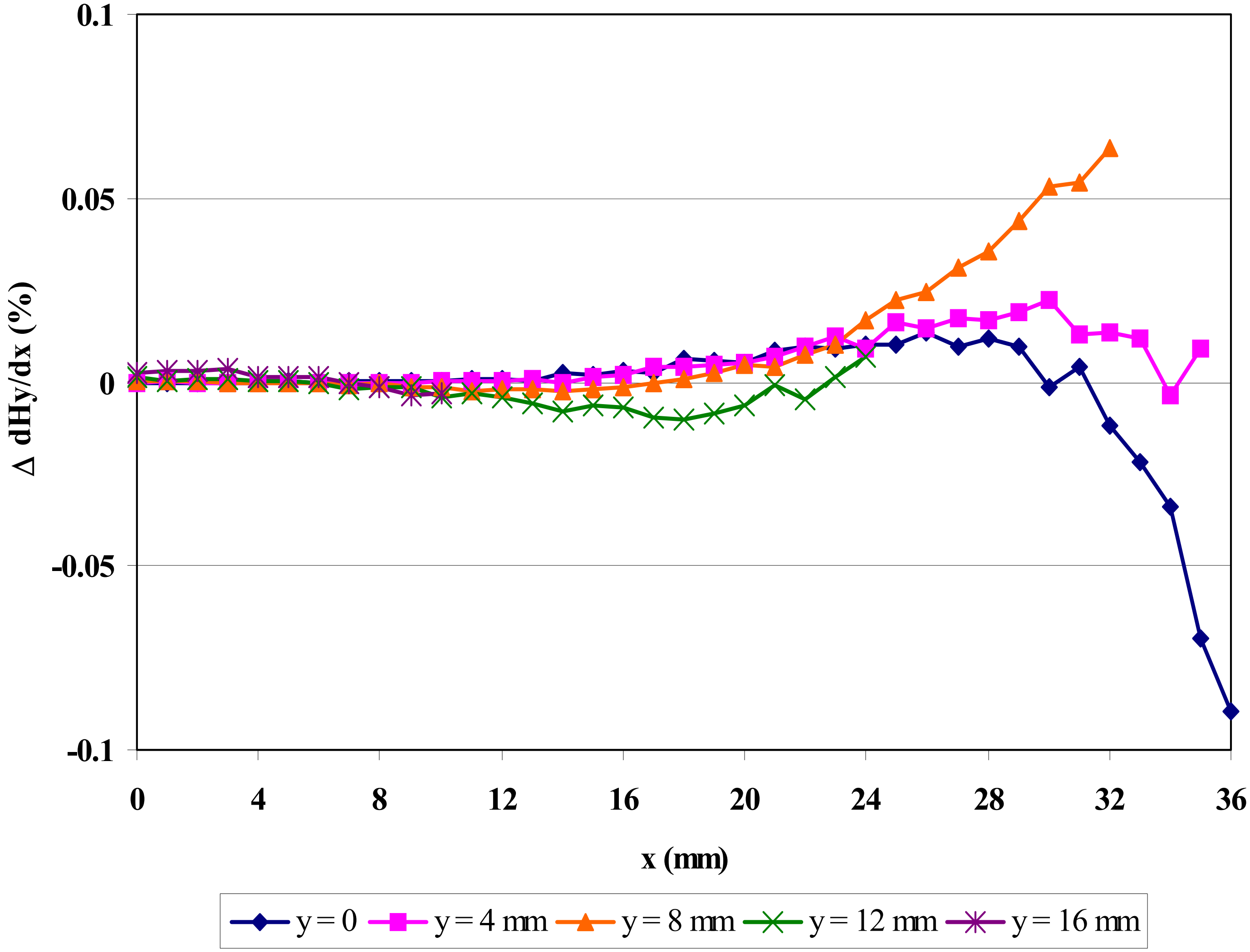


2 D Assessment of quadrupole gradient quality

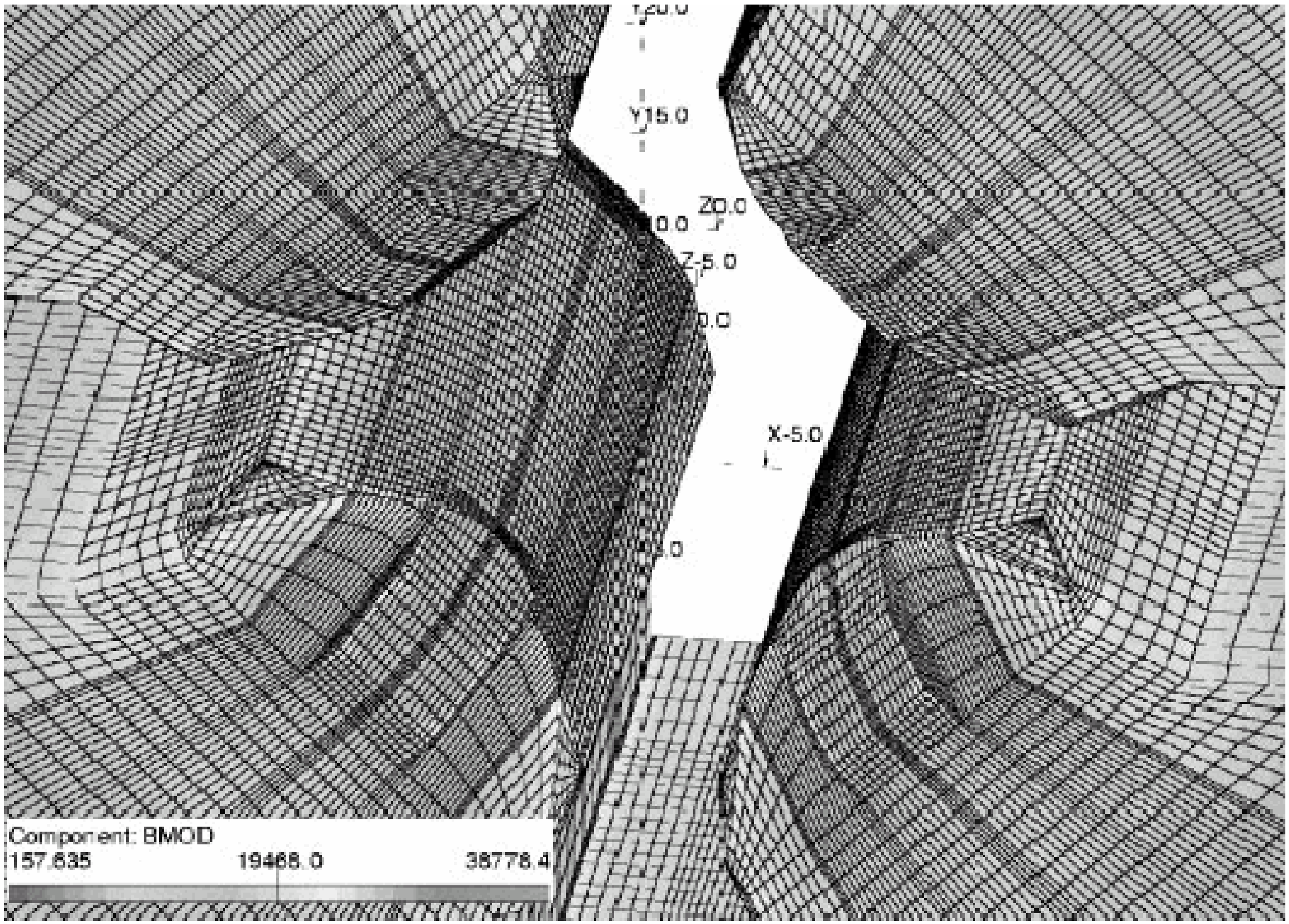
Diamond WM quadrupole:

graph is <u>percentage</u> variation in dBy/dx vs x at different values of y.

Gradient quality is to be ± 0.1 % or better to x = 36 mm.

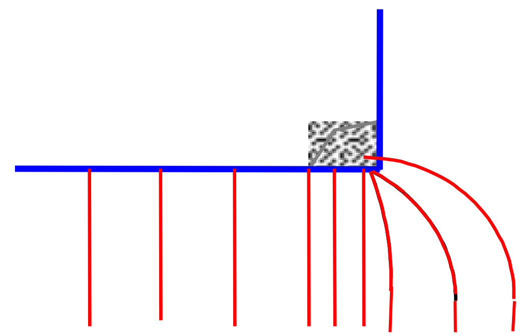


3D finite element model of Soleil quadrupole.

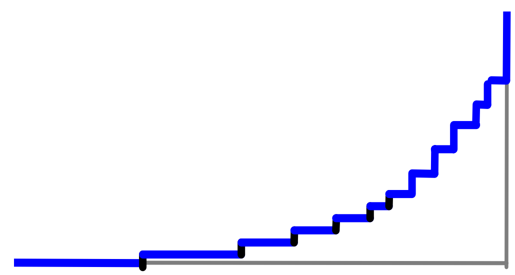


Magnet Ends

It is necessary to terminate the magnet in a controlled way:
•to define the length (strength);
•to prevent saturation in a sharp corner (see diagram);
•to maintain length constant with x, y;
•to prevent flux entering normal
to lamination (ac).



The end of the magnet is therefore 'chamfered' to give increasing gap (or inscribed radius) and lower fields as the end is approached



Classical end solution

The 'Rogowski' roll-off:

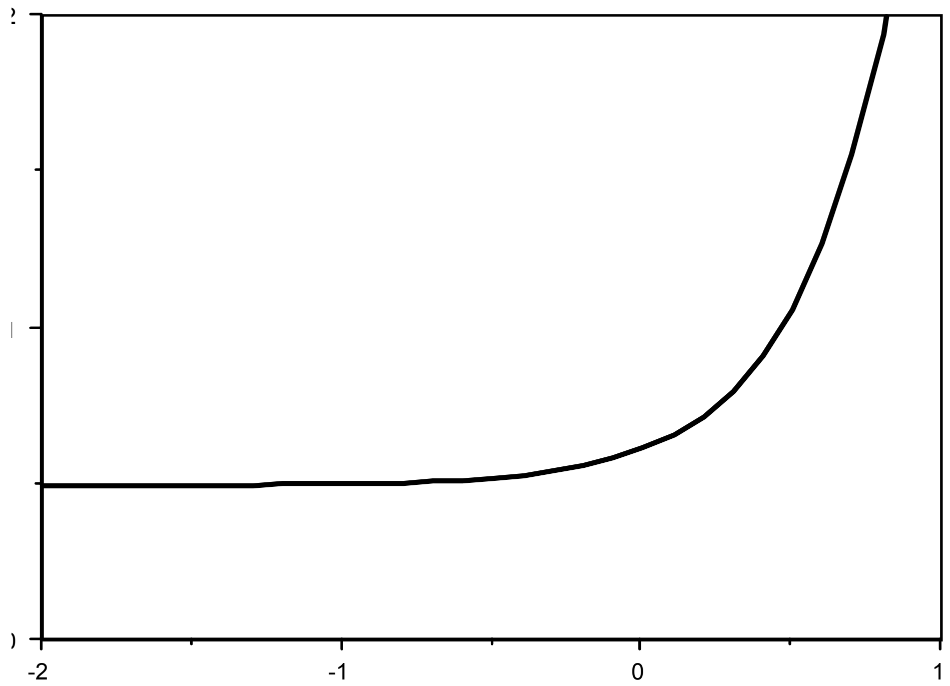
Equation:

$$y = g/2 + (g/\pi) \exp((\pi x/g) - 1);$$

g/2 is dipole half gap;

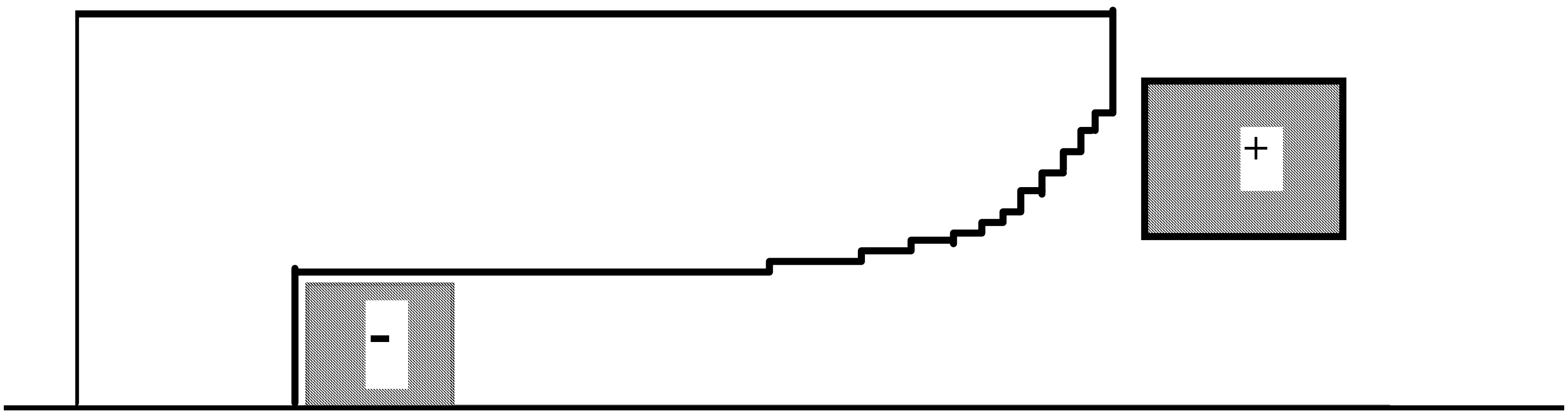
y = 0 is centre line of gap.

This profile provides the maximum rate of **increase** in gap with a monotonic **decrease** in flux density at the surface ie no saturation



Calculation of end field distribution

Calculation of end effects in longitudinal plane using 2D codes, with correct end geometry (including coil), but 'idealised' return yoke:



This provides a reasonable estimate of the distribution in the third plane using a 2D code. BUT it is not as accurate as using a full 3D code.