Beam Lifetime

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Topics

1) Machine Aperture
   - Physical
   - Dynamic
   - RF

2) Beam Gas Interactions
   - Elastic
   - Inelastic
   - Ions

3) Intra-beam Scattering
   - Large Angle Collisions - Touschek Lifetime
   - Small Angle Collisions

4) Quantum Lifetime

5) Increasing the Lifetime

The Machine Aperture

Machine Aperture

Machine Aperture determines beam lifetime

Two types of aperture: Physical and dynamic

Large apertures are required to:

- Account for particle-particle interactions within the vacuum chamber
- Efficiently accumulate a stored beam

During injection particles execute large betatron oscillations that sample strong non-linear fields.

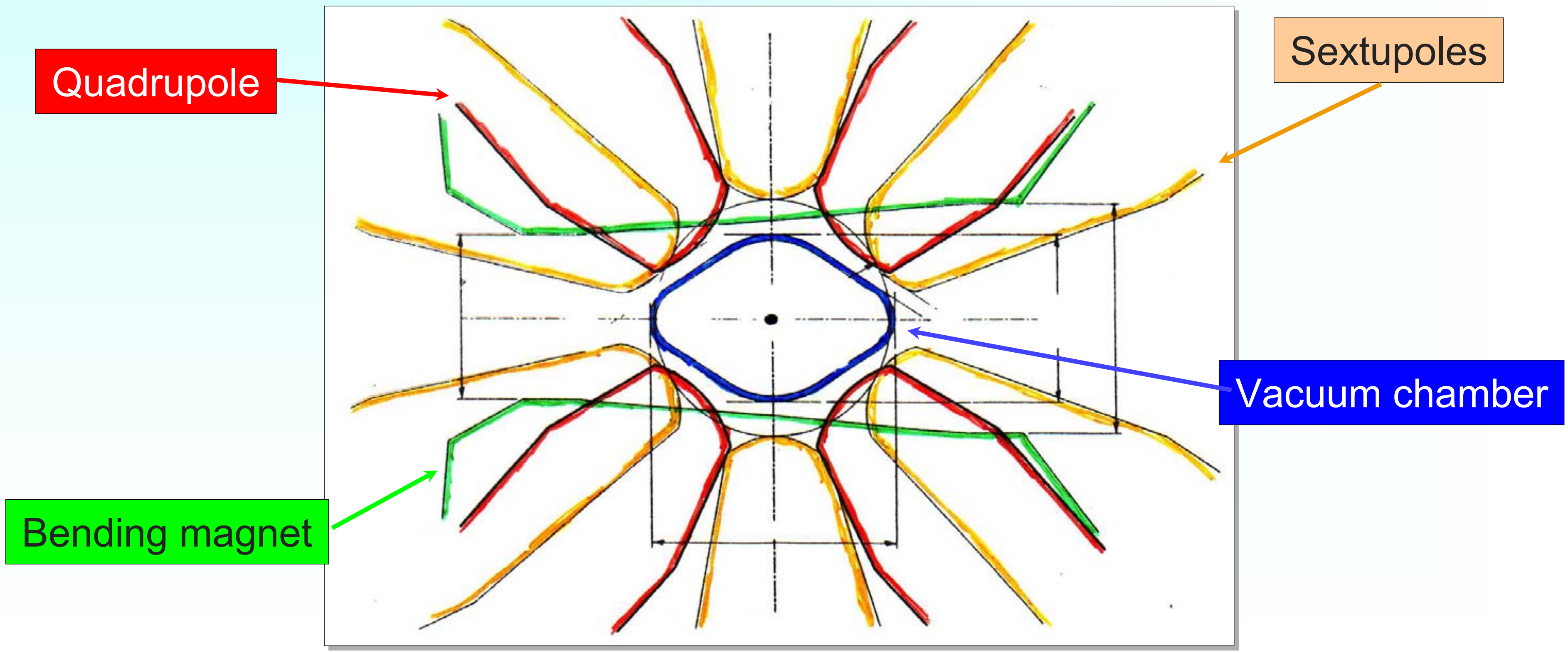
The aperture must be sufficiently large to accommodate these particles and others which undergo large betatron oscillations caused by collisions - **elastic gas scattering**.

An aperture must also exist for off-momentum particles, and this has to contain those particles that suffer energy changes because of interaction with other particles in the beam or with residual gas molecules - **the Touschek effect and inelastic gas collisions**.

The Physical Aperture

The Physical Aperture

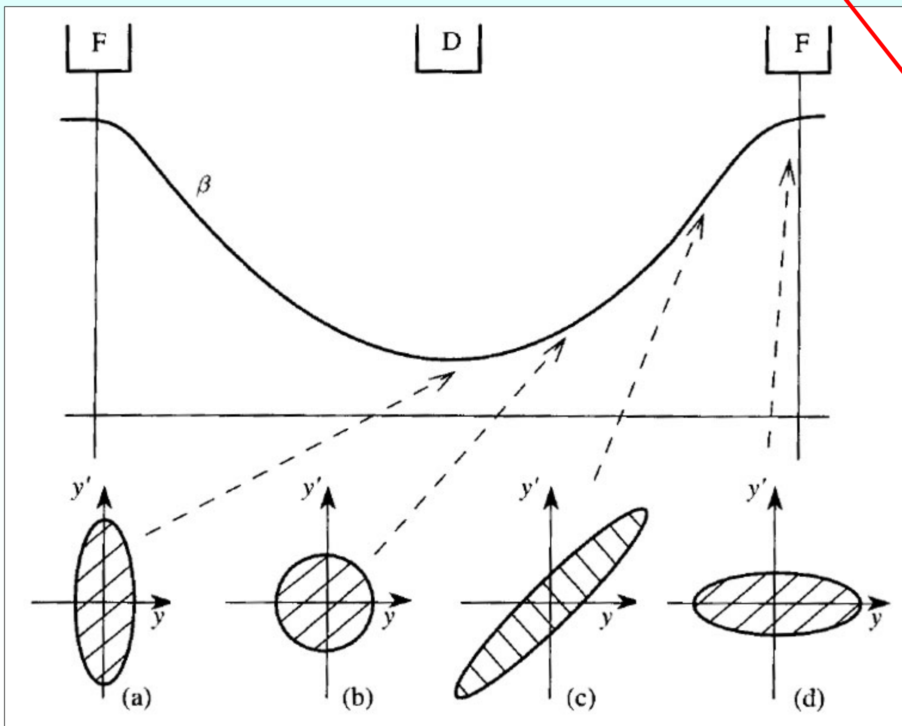
The vacuum chamber mostly stays within the magnet apertures.
It is seen by the beam as an impedance and can adversely effect beam stability.
For SR it contains not only the circulating beam but also the photon beam.
It’s design determines the vacuum pressure and therefore beam lifetime.
All particles are eventually lost at the chamber surface.

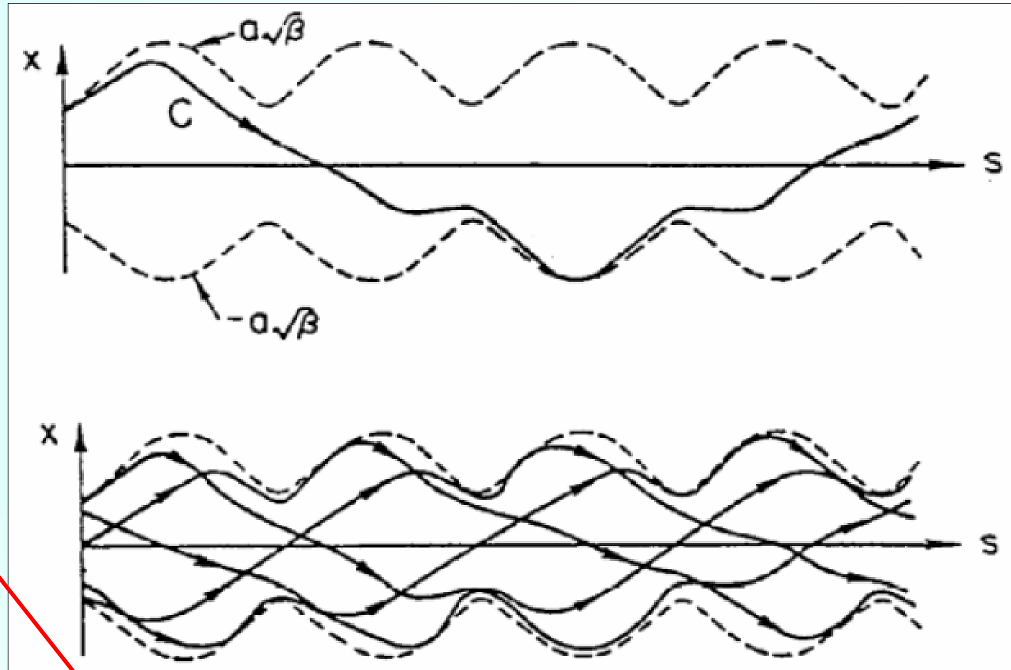


Invariants of Motion - reminder

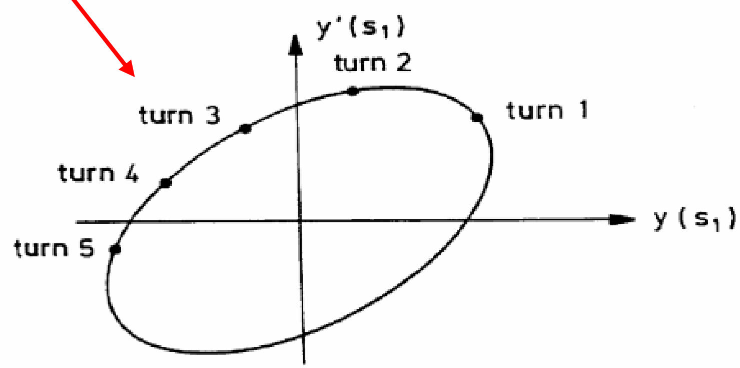
$$\sigma_x = \sqrt{\beta_x \varepsilon_x}, \quad \sigma_{x'} = \sqrt{\gamma_x \varepsilon_x}, \quad \gamma_x = \frac{1+\alpha_x^2}{\beta_x}$$

$$\boxed{\text{area} = \pi\varepsilon = \gamma^2 x^2 + 2\alpha x x' + \beta^2 x'^2}$$

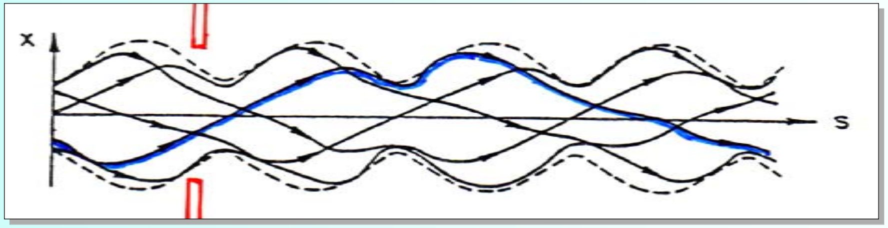




$$\boxed{x = \sqrt{a_x \beta_x(s)} \cdot \cos\varphi(s) + D(s) \cdot \delta_E}$$



Physical Acceptance



Off energy betatron trajectory: $x = \sqrt{a_x \beta_x(s)} \cdot \cos\varphi(s) + D(s) \cdot \delta_E$

The largest x amplitude at a point s is: $x_{\max} = \sqrt{a_{\max,x} \beta_x(s)} + D(s) \cdot \delta_E$

The particle is lost if x_{max} exceeds the physical aperture $x_{vac}(s)$

Physical **acceptance** is: $A_{phys,x}(\delta_E) = a_{\max,x} = \min\limits_{s \in [0,L]} \left(\frac{\left[x_{vac}(s) - \left| D(s) \cdot \delta_E \right| \right]^2}{\beta_x(s)} \right)$

Storage rings are flat and dispersion exists only in the horizontal plane.

$$A_{phys,y} = \min_{s \in [0,L]} \left(\frac{y_{vac}^2(s)}{\beta_y(s)} \right)$$

$A_{phys,x,y}$ defines the largest emittance that can be sustained.

The Dynamic Aperture

Dynamic Aperture

A lattice of only dipoles and quadrupoles is governed by linear dynamics.

Sextupoles for chromaticity correction, magnetic field errors, insertion devices will induce non-linear dynamics.

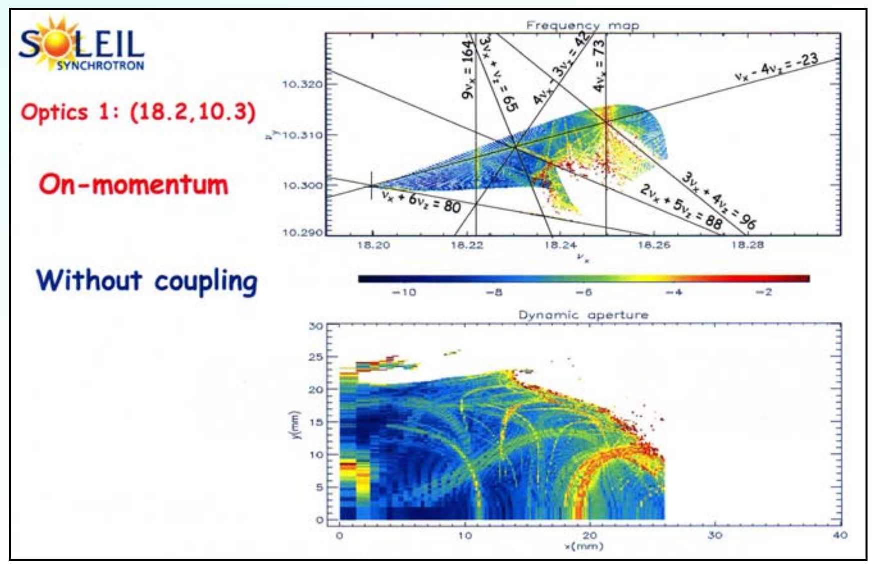
- Tune shift with amplitudes
- Tune shift with momentum

Particles diffuse and get lost as resonances are crossed.

Several families of sextupoles are used to compensate non-linear effects and maximise the dynamic aperture.

See S. Smith, "Optimisation of Modern Light Source Lattices", EPAC 2002, 44. (and slides)

and A. Streun, "Practical Guidelines for Lattice Design", CAS School, Benodet, 1999 (SLS-TME-TA-1999-0014, online at SLS).



Dynamic Aperture - Light Source

Greater Brilliance

Lower Emittance

Insertion Devices

Stronger Focusing

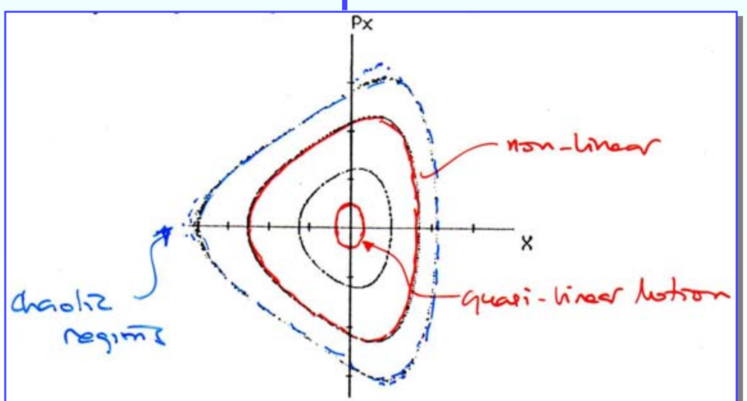
Errors

Non-linearities and Broken Symmetry

Stronger Sextupoles

Dynamic Aperture

$$A_{dynap,x,y}(\delta_E)$$



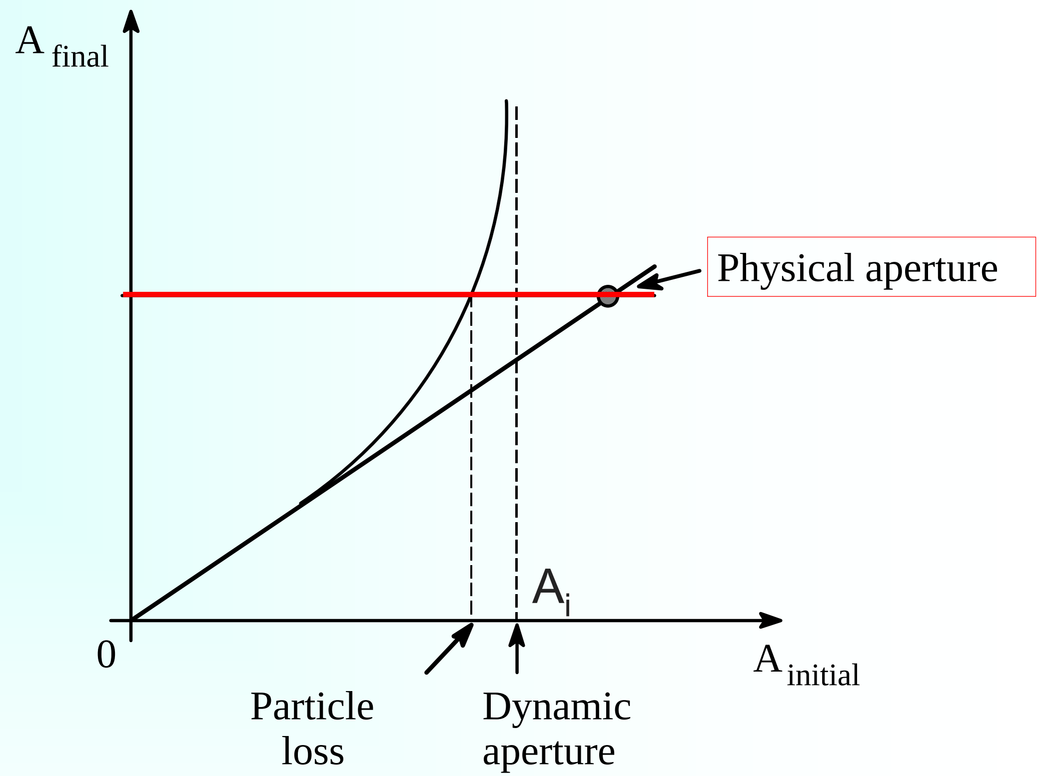
$$y'' = -\frac{1}{2\rho^2}\left\{\underbrace{\left(y - \frac{k^2}{6}y^3\right)}_{\textit{Linear optics}} + \underbrace{\frac{k\sin ks}{\rho}x'\left(y + \frac{k^2}{6}y^3\right) + \ldots}_{\textit{Dynamic aperture}}\right\}$$

Dynamic Aperture

Dynamic aperture = The maximum stable initial transverse amplitude in the presence of non-linearities.

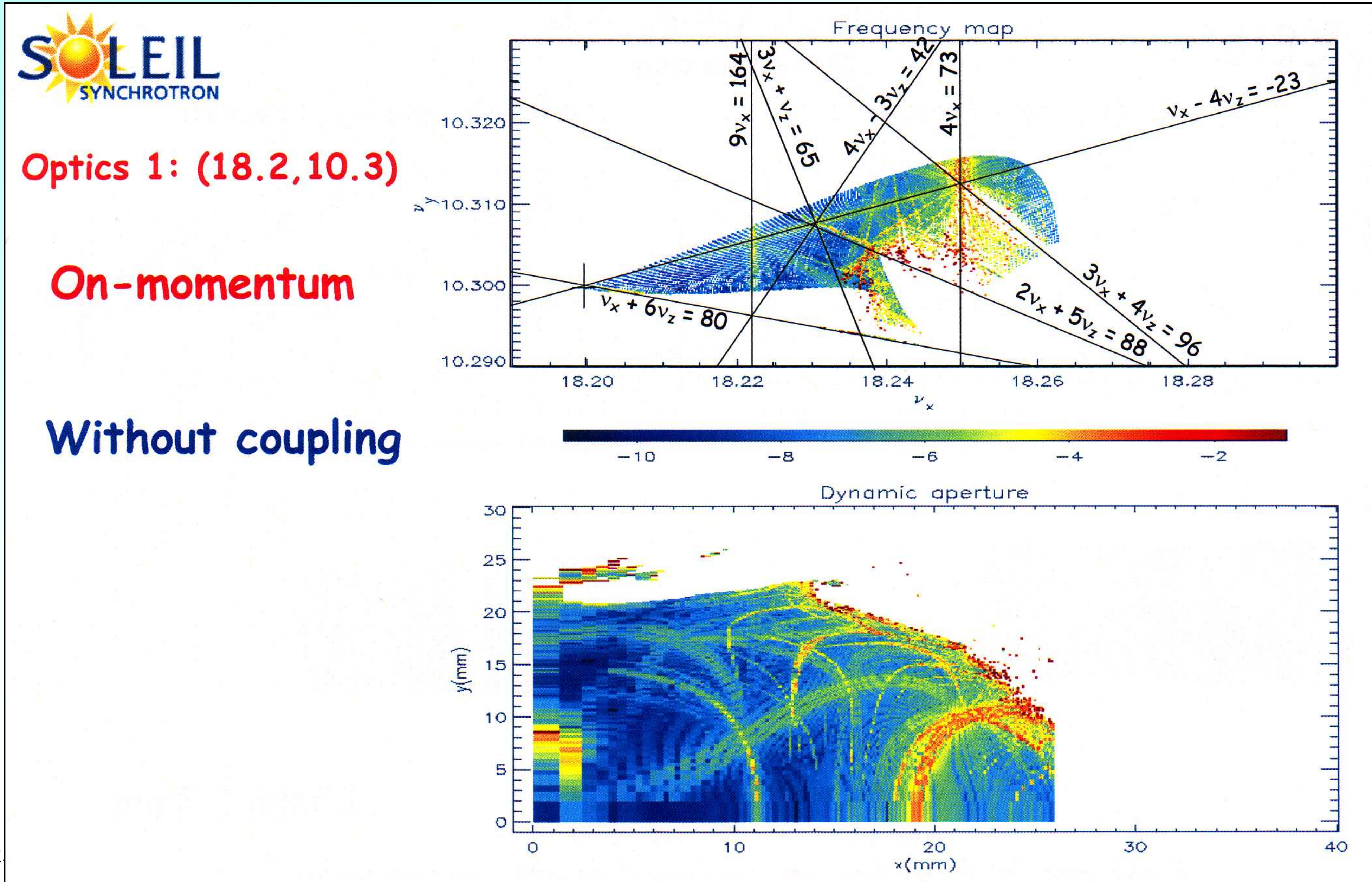
The physical aperture is represented by a horizontal line parallel to the initial co-ordinate axis.

The starting amplitude A_i at which a particle goes to infinity (i.e., the particle has unbounded motion) is usually referred to as the dynamic aperture.
This can either be inside or outside the physical aperture.



Note that although particles are always lost at the physical aperture, the dynamic aperture gives an indication of the reduction in phase space for stable motion.

Dynamic Aperture



Dynamic Aperture

DIAMOND, Conceptual Design

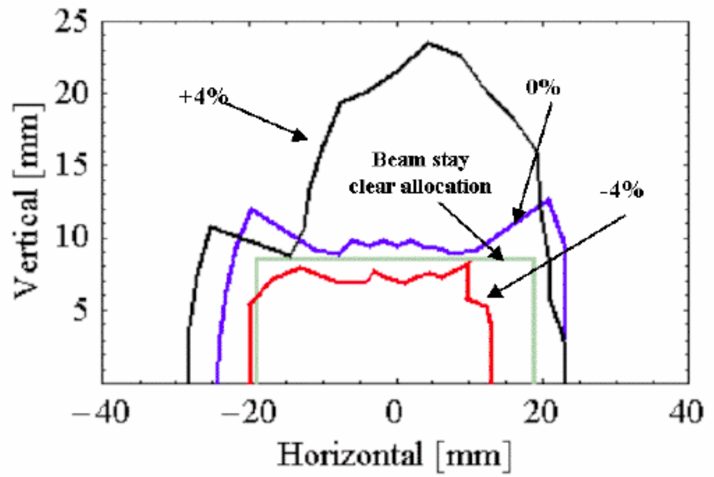


Figure 1.3-7 Dynamic aperture for the reference solution.

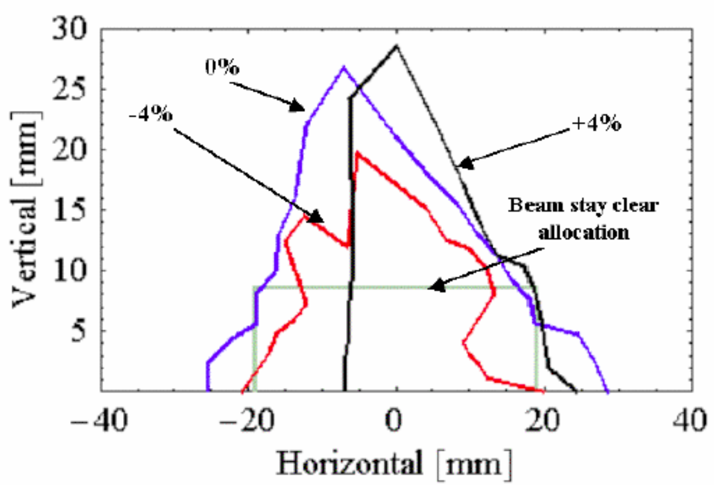
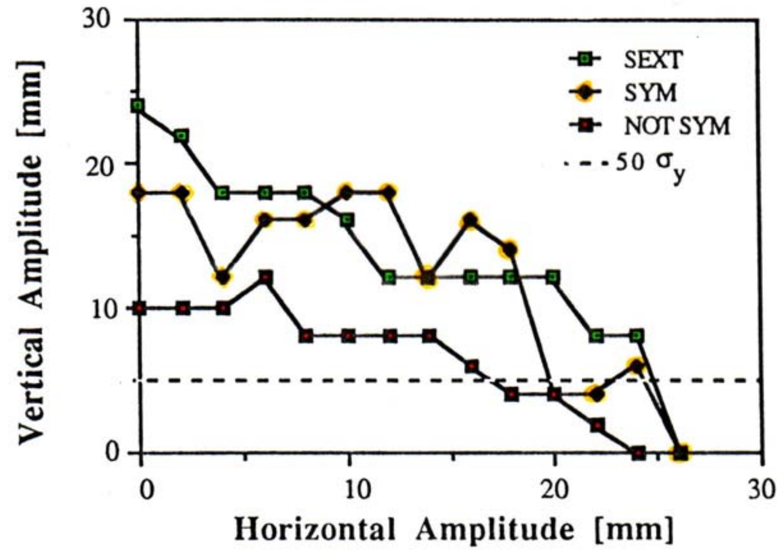


Figure 1.3-8 Dynamic aperture for the example lower chromaticity solution.

ELETTRA



6 Undulators arranged symmetrically and asymmetrically in the lattice

Transverse Momentum Acceptance

If a particle changes energy δ_E at position s_0 where the dispersion is nonzero it will circulate around a new off-momentum phase ellipse.

The coordinates with respect to the new origin:

$$\bar{x} = x + D_x \cdot \delta_E,\ \bar{x}' = x' + D'_x \cdot \delta_E$$

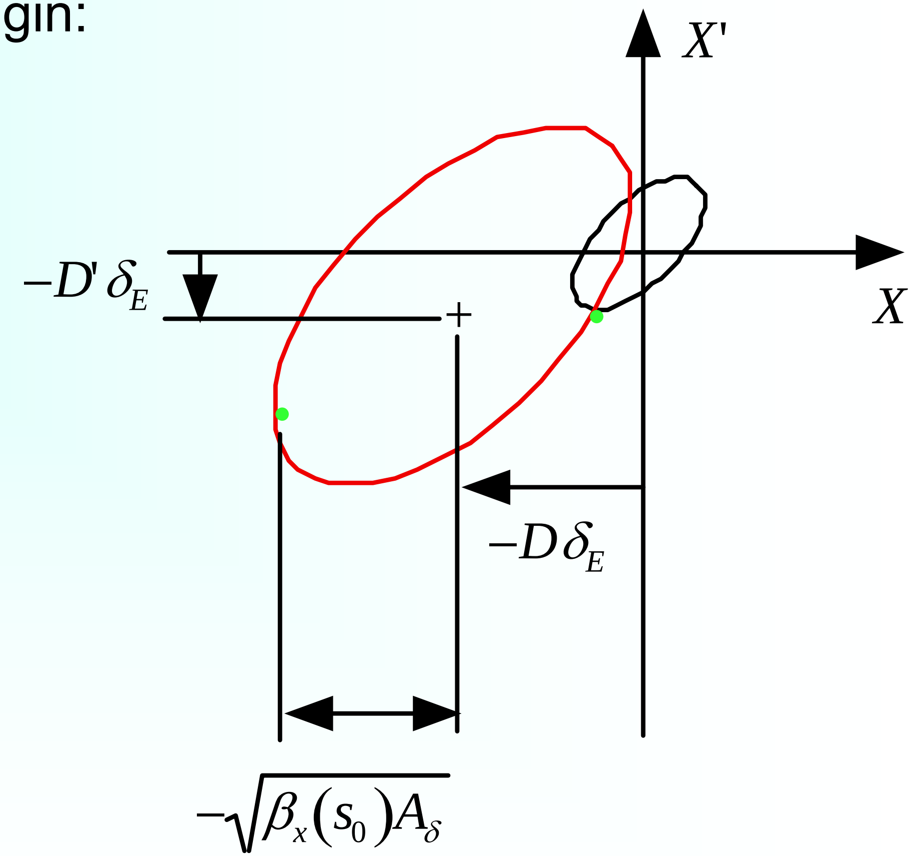
New ellipse has an invariant:

$$A_{x,\delta} = \gamma_x(s_0)\bar{x}^2 + 2\alpha_x(s_0)\bar{x}\,\bar{x}' + \beta_x(s_0)\bar{x}'^2$$

Eventual maximum displacement at s_0:

$$x_{\max}(s_0) = \left| D_x(s_0)\cdot\delta_E \right| + \sqrt{\beta_x(s_0)\,A_{x,\delta}}$$

The induced amplitudes can be very large.



Transverse Momentum Acceptance

Maximum displacement elsewhere at position s is simply:

$$x_{\max}(s) = \left| D_x(s) \cdot \delta_E \right| + \sqrt{\beta_x(s)\, A_{x,\delta}}$$

First term is the chromatic closed orbit, the second the induced betatron motion.

We simplify by considering only core particles. i.e., x=x'=0, then

$$\begin{aligned} A_{x,\delta} &= \gamma_x(s_0)\,\overline{x}^2 + 2\alpha_x(s_0)\,\overline{x}\,\overline{x}' + \beta_x(s_0)\,\overline{x}'^2 \\ &= \left[\gamma_x(s_0)\,D_x^2 + 2\alpha_x(s_0)\,D_x\,D_x' + \beta_x(s_0)\,D_x'^{\,2}\right]\delta_E^2 \\ &= \mathsf{H}_x(s_0)\,\delta_E^2 \end{aligned}$$

and $\quad x_{\max}(s) = \left[D_x(s) + \sqrt{\mathsf{H}_x(s_0)\beta_x(s)}\right] \cdot \delta_E$

If x_{max} > $A_{apr,x}$ (ring loss aperture,physical or dynamic) the particle is lost.

The maximum momentum acceptance $\delta_{E,\max}(s_0)$ at a point s_0 is:

$$\delta_{E,\max}(s_0) = \min_{s\in[0,L]} \left\lfloor \frac{A_{apr,x}(s)}{\left|D_x(s)\right| + \sqrt{\mathsf{H}_x(s_0)\beta_x(s)}} \right\rfloor$$

Longitudinal Momentum Acceptance

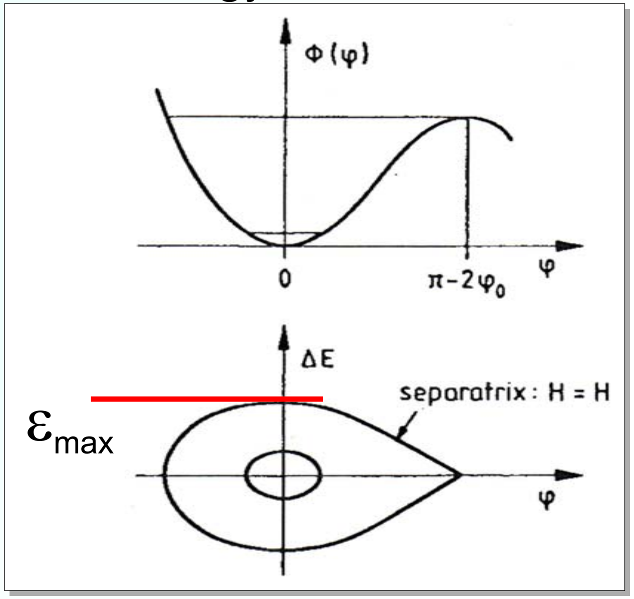
In the longitudinal plane the acceptance is defined by the rf system and is a function of the rf voltage and the magnet lattice. **The RF acceptance is a constant along the lattice.**

A large aperture in this plane is needed to contain particles which suffer energy losses - **inelastic gas collisions and the Touschek effect** and to guarantee sufficient **quantum lifetime**. The RF voltage and the lattice delimit a potential energy well in which the electrons are bound.

$$\varepsilon^2_{\max,RF} = \frac{eV\sin\psi_s}{\pi h\,\alpha_c\,E}\,2\left(\sqrt{q^2-1}-\cos^{-1}\left(\frac{1}{q}\right)\right)$$

$$q = \frac{eV}{U} = \frac{1}{\sin\psi_s}$$ = over-voltage factor, U is energy loss/turn

$$\alpha_c = \frac{\Delta L/L}{\Delta p/p}$$ = momentum compaction factor



The Separatrix is the limit of stable motion. From the equation describing this curve the maximum energy deviation can be found.

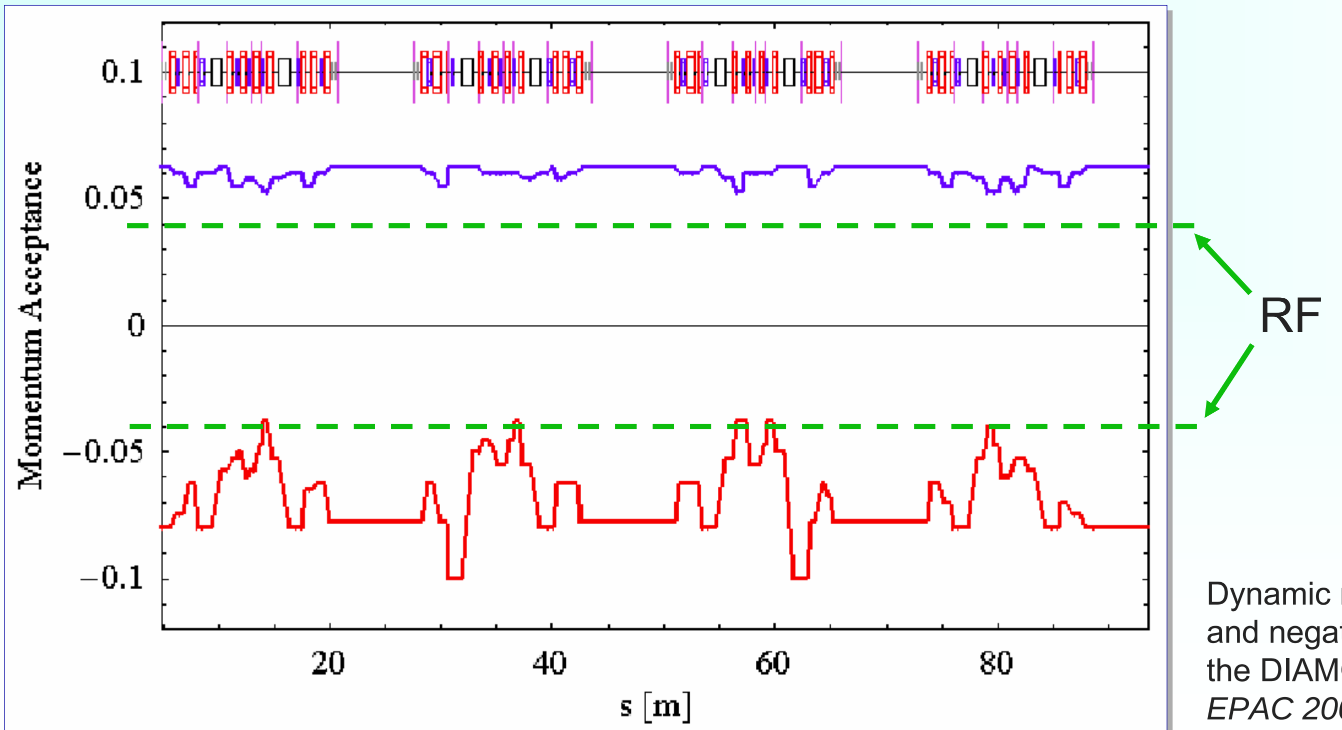
Typical values of the momentum acceptance: 2 to 4% - New machines ~4 to 5%

Momentum Acceptance - Summary

The overall momentum acceptance is the smallest of the RF acceptance, the physical acceptance and the dynamic acceptance, i.e.,

$$\delta_{E,\max}(s_0) = \min_{\delta\in[-\delta,+\delta]}\left[\varepsilon_{\max,RF},\delta_{x,y/g,phys},\delta_{x,y/g,dyn}\right]$$

Apart from the constant RF acceptance, the momentum acceptance is a function of location in the ring.



Dynamic momentum acceptance for positive and negative relative momentum particles for the DIAMOND lattice from *N. Wyles, et al., EPAC 2002, p 781*

Gas Scattering

Gas Scattering

The beam is scattered by the residual gas molecules in the vacuum chamber.

Particles are lost when their oscillation amplitudes exceed machine acceptances.

Minimized by:

Careful construction of vacuum chamber - pumping and minimization of photo-desorption

Sufficient pumping at right places (at absorbers, at high beta values, …)

There are two main single particle effects:

Elastic collisions - loss at physical or dynamic aperture

Circulating particles are deflected by gas nuclei resulting in an increase of the betatron amplitudes.

Inelastic collisions - loss at RF acceptance limit or off-momentum (physical or dynamic)

Circulating particles suffer energy loss either by

- Energy transfer to the residual gas molecule
- Photon emission - Bremsstrahlung effect

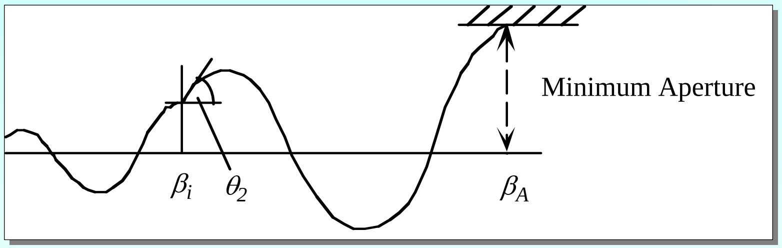
Gas Scattering

Elastic Collisions

Giving a circulating electron a kick θ_i results in an oscillation $u(s) = \theta_i \sqrt{\beta(s)\beta_i}\ \sin(\varphi(s) - \varphi_i)$

The maximum amplitude is $Max\left|u(s)\right| = A = \theta_i \sqrt{\beta_A \beta_i}$

If this exceeds the physical or dynamic aperture the particle is lost

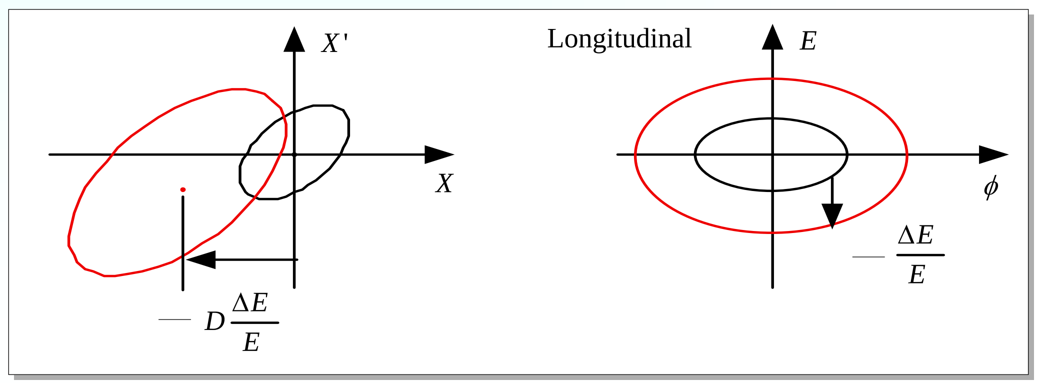


Machine acceptance H defined as $\left(A^2/\beta_A\right)_{\min}$

Inelastic Collisions: there are two effects

- Bremsstrahlung scattering: deflection by a nucleus and emission of a photon.
- Direct energy transfer from the circulating particle to the atom of the residual gas.

An electron that suffers an energy loss (ΔE/E) will be lost either at the transverse aperture limit (physical or dynamic) or at the RF limit.

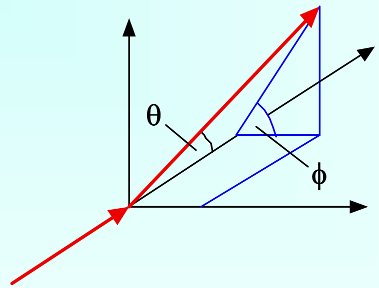


Gas Scattering - Elastic Collisions

The cross section: $$\sigma = \frac{\text{number of events per unit time}}{\text{number of incident particles per unit area per unit time}}$$

The probability of a collision resulting in a deflection between θ and $\theta+d\theta$ per unit time is proportional to the differential cross-section $d\sigma/d\Omega$ for the encounter and the number of scattering centres per unit volume. The differential **elastic** cross-section for scattering off nuclei is given by the classical Rutherford scattering formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z\ r_0}{2\gamma}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$



$$d\Omega = \sin\theta\, d\theta\, d\phi$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{m_0 c^2} = 2.82 \cdot 10^{-15} [\mathrm{m}]$$

Loss of electrons - Calculate collision cross-section that leads to a deflection angle greater than a maximum θ_{max} defined by the acceptance of the ring. Integrating $d\sigma$ from θ_{max} to π:

$$\sigma_{loss} = 2\pi \int_{\theta_{max}}^{\pi} \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{\pi}{2} \left(\frac{Z\, r_0}{\gamma}\right)^2 \cot^2\left(\frac{\theta_{max}}{2}\right)$$

$\theta_{max}=\sqrt{H/\beta_i}$ is a small angle, approximate tan θ_{max} as θ_{max} and average over the circumference:

$$\boxed{\sigma_{loss} = \frac{2\pi Z^2 r_o^2}{\gamma^2} \cdot \frac{1}{\theta_{max}^2} = \frac{2\pi Z^2 r_o^2}{\gamma^2} \frac{\beta_i}{H} = \frac{2\pi Z^2\, r_o^2}{\gamma^2} \cdot \frac{\langle\beta\rangle}{H}}$$

Gas Scattering - Inelastic Collisions

We consider only radiative losses - gas excitation cross-sections are small at relativistic energies.

The differential cross-section for energy loss from photon emission at the nucleus is (relativistic energies):

$$\left(\frac{d\sigma}{d\varepsilon}\right)_N = \alpha\,\frac{4Z^2 r_0^2}{\varepsilon}\left\{\left[\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\frac{\varepsilon^2}{E^2}\right]\left[183-\frac{1}{3}\,\ell n\,Z\right]+\left[\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right]\right\}$$

The differential cross-section for photon emission at a bound electron on a residual gas atom is (relativistic energies):

$$\left(\frac{d\sigma}{d\varepsilon}\right)_e = \alpha\,\frac{4Z r_0^2}{\varepsilon}\left\{\left[\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\frac{\varepsilon^2}{E^2}\right]\left[1194-\frac{2}{3}\,\ell n\,Z\right]+\left[\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right]\right\}$$

The total differential cross-section is the sum of the two. Integrating from ε_m, the lowest energy loss which results in particle loss, to *E* the highest energy loss gives the total cross-section.

$$\sigma(\varepsilon,E)=\int_{\varepsilon_m}^{E}\left(\frac{d\sigma}{d\varepsilon}\right)d\varepsilon=4\,\alpha\,r_0^2\left\{F(Z)\,\frac{4}{3}\left[\ell n\left(\frac{E}{\varepsilon_m}\right)-\frac{5}{8}\right]+\frac{Z(Z+1)}{9}\left[\ell n\left(\frac{E}{\varepsilon_m}\right)-1\right]\right\}$$

for $\varepsilon_m \ll E$, and $F(Z)=Z^2\,\ell n\left(\frac{183}{Z^{1/3}}\right)+Z\,\ell n\left(\frac{1194}{Z^{2/3}}\right)$

Note the strong dependence on the atomic number, but a weak dependence on the maximum energy acceptance that goes as -ln (ε_m/E).

Gas Scattering - Lifetime

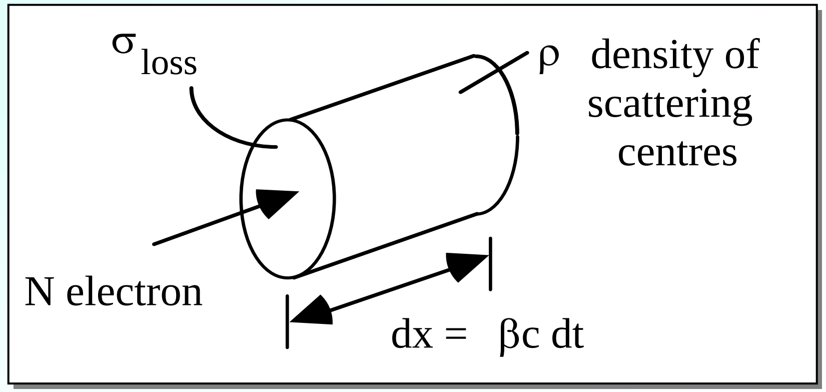
For independent events the total cross section is the sum of the cross-sections for the individual events,

$$\sigma_{loss} = \sigma_{elastic} + \sigma_{inelastic}$$

The number of particles lost dN per unit time is proportional to the cross-section, the number of scattering centres and the number of incident particles.

$$dN = -N\rho\sigma_{loss}dx$$

$$\frac{1}{N}\frac{dN}{dt} = -\rho\sigma_{loss}\beta c$$



leading to an exponential decay of the stored beam $\left(N = N_0\, e^{-t/\tau}\right)$

with a lifetime $\tau = \dfrac{1}{\beta c\ \sigma_{loss}\ \rho}$

$$\rho_i = \frac{p_i}{kT}$$

Partial Pressure

Gas Scattering - Lifetime

Desorption of gas molecules by synchrotron radiation is the main source of residual gas in light sources. The vacuum pressure depends on the circulating current and the desorption coefficient G, writing for ρ,

$$\rho = \rho_0 + GN \longrightarrow \frac{dN}{dt} = -\beta c\, \sigma_{loss} \left(\rho_0 + GN\right) N \longrightarrow \frac{N\left(\rho_0 + GN_0\right)}{N_0\left(\rho_0 + GN\right)} = e^{-\rho_0 \beta c\, \sigma_{loss} t}$$

with N_0 the starting number of electrons. Define the lifetime as the time it takes for the starting number of electrons to be reduced by 1/e, we get:

$$\tau_e = \frac{1}{\left(\beta c\, \sigma_{loss} \rho_0\right)} \ell n \left\{ \frac{e\rho_0 + GN_0}{\rho_0 + GN_0} \right\}$$

For no desorption (G=0) we have the previous result.

For situations where desorption effects dominate $\rho_0 \to 0$, and

$$\tau_e = \frac{e - 1}{\beta c\, \sigma_{loss}\, G\, N_0}$$

Gas Scattering - Lifetime

The lifetime is inversely proportional to the initial current and accounts for the increasing lifetime as the current decays.

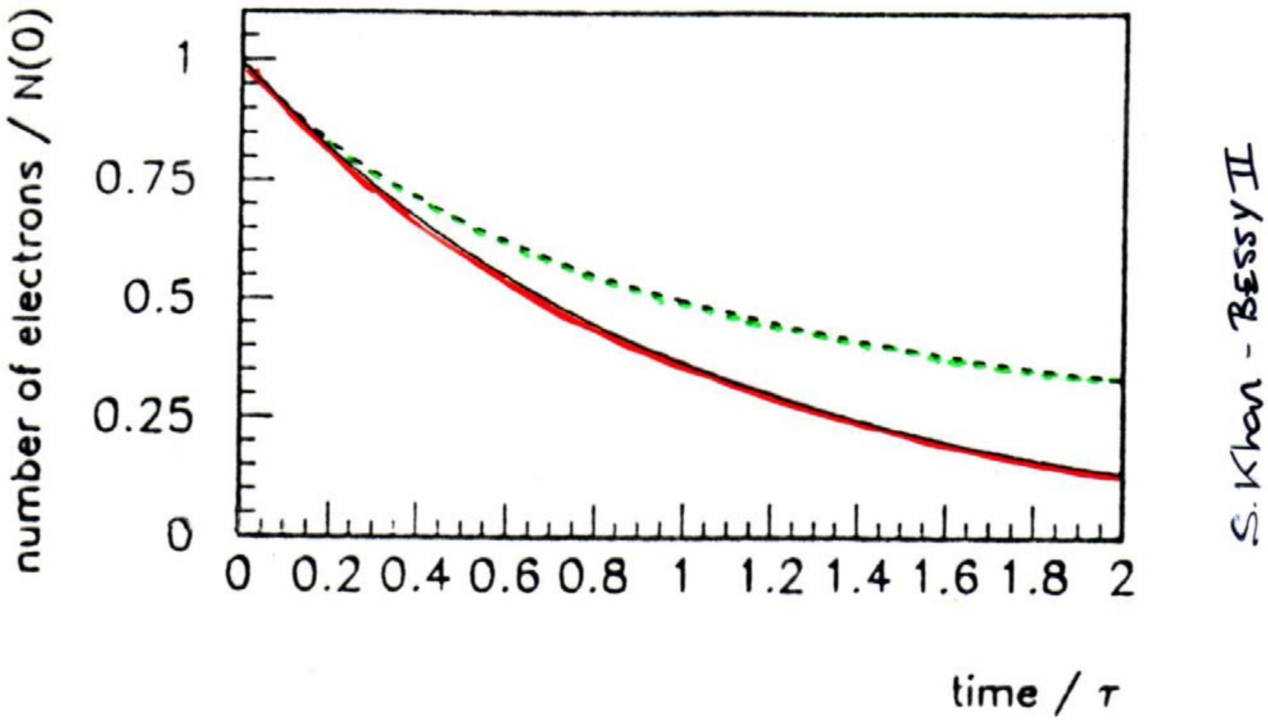
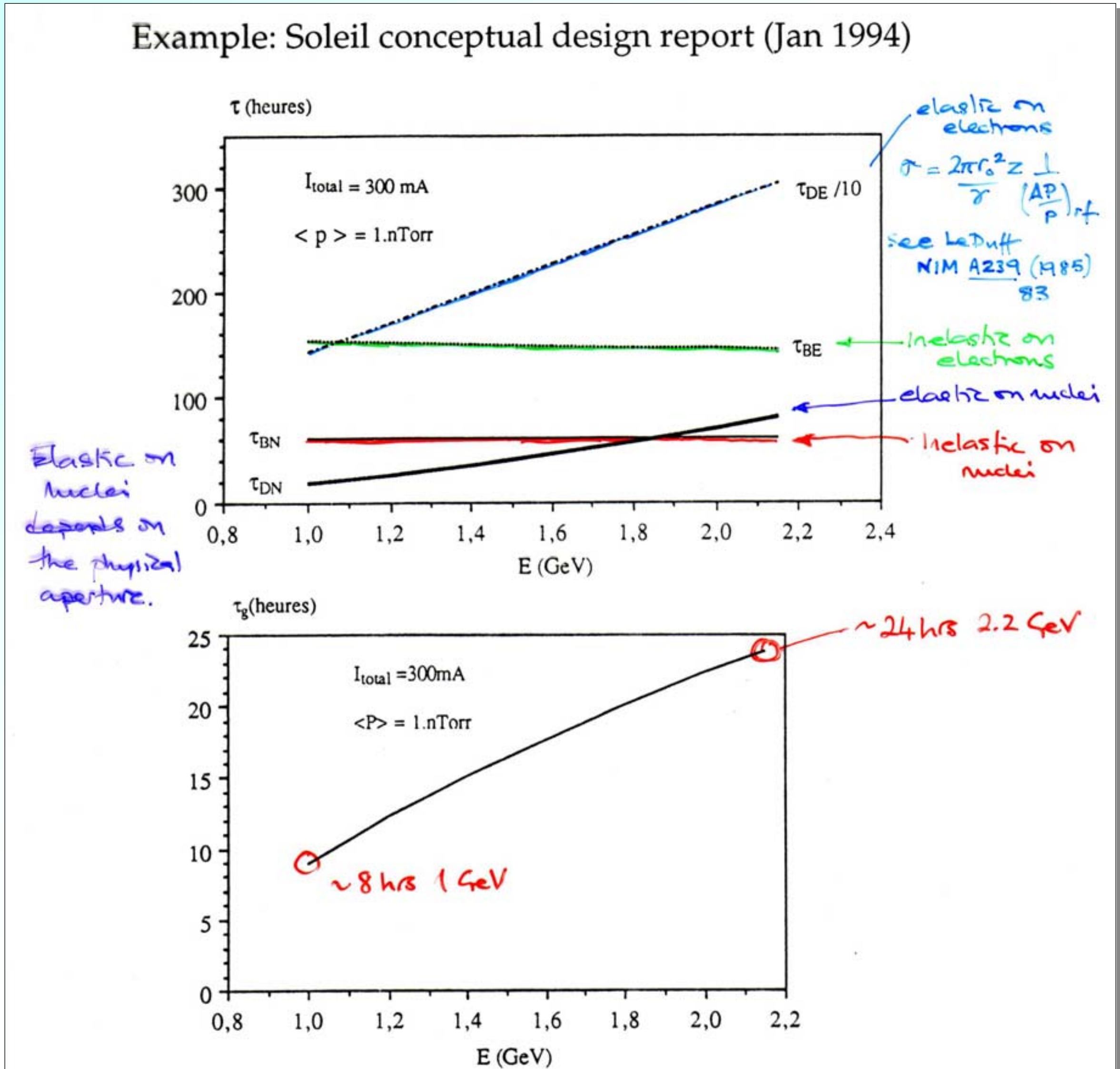


Figure 2: Number of electrons $N(t)$ in units of $N(0)$ versus time in units of $\tau = -N(0)/\dot{N}(0)$ for $\dot{N} \sim -N$ (solid line) and $\dot{N} \sim -N^2$ (dashed line).

Gas Scattering - Lifetime

The total lifetime is given by the sum of the individual reciprocal lifetimes

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_{elastic}} + \frac{1}{\tau_{inelastic}}$$



Ion Trapping

Ion Trapping

A circulating bunch of negatively charged particles colliding with the residual gas can knock out electrons and form positive ions.

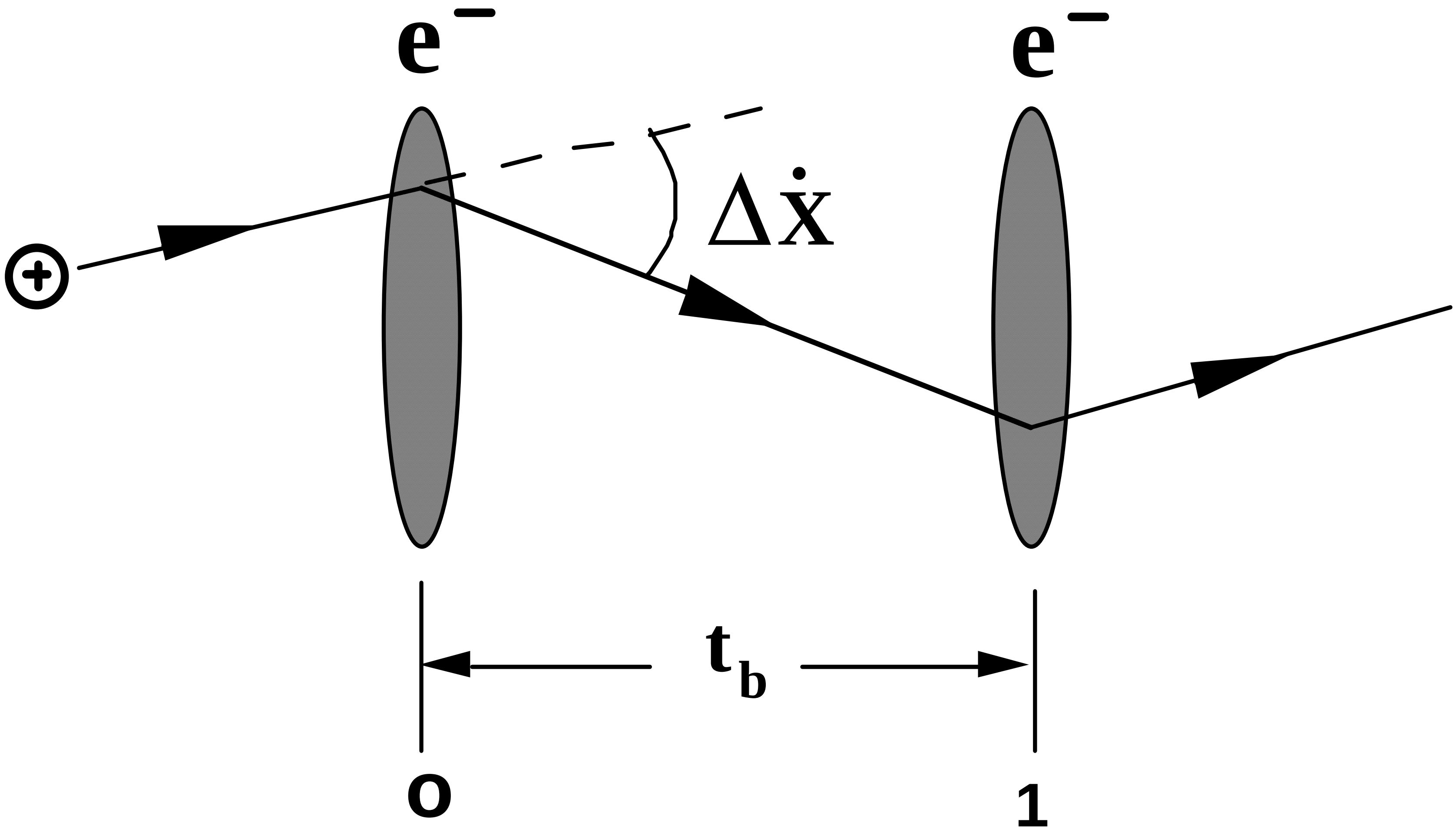
The ions can then be trapped by the circulating electrons. When this happens we find:

A local increase in gas pressure
  reduction in lifetime
  emittance blow-up
Tune shifts and spreads
Transverse beam coupling
Beam instabilities (ion-cloud to electron bunch)
Increased γ Bremsstrahlung

Possible gas species to found

H	H_2	C	N	O/CH_4	CO/N_2	O_2	Ar	CO_2
1	2	12	14	16	28	32	40	44

Ion Trapping



Ion Trapping

An ion is kicked towards the electron bunches and then drifts freely for a time t_b between bunches.

$$M_i \Delta\dot{X} = \int_0^{\Delta T} eE_\perp \, dt$$

The kicks are non-linear

r_p: classical proton radius

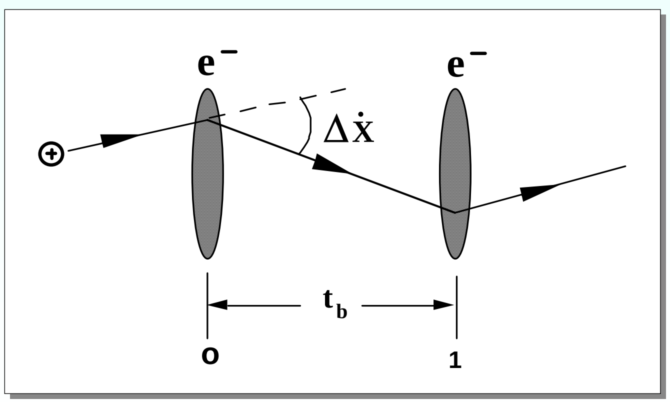
$$\begin{pmatrix} \Delta\dot{x} \\ \Delta\dot{y} \end{pmatrix} = \frac{Nr_p c}{An}\sqrt{\frac{2\pi}{\left(\sigma_x^2-\sigma_y^2\right)}}\begin{pmatrix} \mathrm{Im} \\ \mathrm{Re} \end{pmatrix}\left(\mathrm{W}\left[\frac{x+\mathrm{i}y}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right] - \exp\left[-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right]\mathrm{W}\left[\frac{\mathrm{x}\frac{\sigma_y}{\sigma_x}+\mathrm{iy}\frac{\sigma_x}{\sigma_y}}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right]\right)$$

Close to the bunch centre we linearise

$$\mathrm{W}[x+iy] \sim 1 + \frac{2}{\sqrt{\pi}}(x-iy)$$

Change in ion coordinate from kick + drift

$$\begin{pmatrix} x \\ \dot{x} \end{pmatrix}_j = \begin{pmatrix} 1 & t_b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix}_i \qquad a = \frac{2Nr_p c}{An\sigma_x\left(\sigma_x+\sigma_y\right)}$$



$$\mathbf{X}_j = \mathbf{M}\mathbf{X}_i$$

Motion is stable if |Trace ($\mathbf{M}_x$) | < 2

Solving we find the smallest stable ion mass

$$A_{x,c} = \frac{NCr_p}{2n^2\sigma_x\left(\sigma_x+\sigma_y\right)}$$ is the critical mass parameter.

Ion Trapping

Asymmetric bunch filling

$$\mathbf{M} = \left[\begin{pmatrix} 1 & t_b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -a & 1 \end{pmatrix}\right]^n \begin{pmatrix} 1 & t_b \\ 0 & 1 \end{pmatrix}^{(h-n)}$$

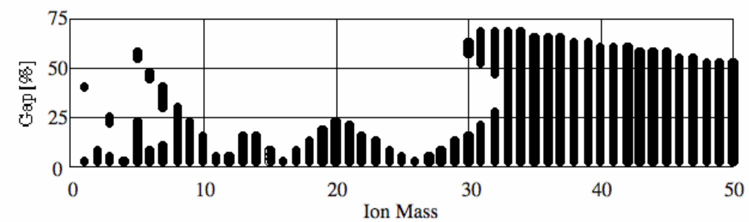
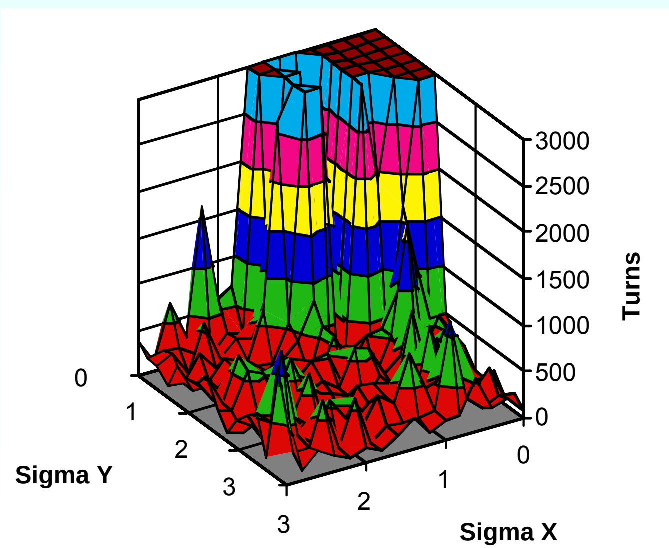


Figure 15 Prediction of linear theory for the trapping of ions in the ANKA [90] storage ring (400 mA, 2.5 GeV, 3% coupling at the position for maximum trapping).

Solve for the trace numerically. The gap represents an error in the focusing and drives resonances. The ion ladder is disrupted.

Non-linear

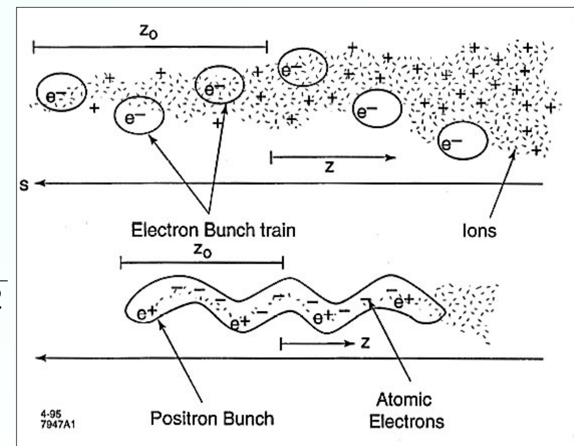


Fast Ion Instability - single pass effect

$$y_b \sim \frac{y_0 \exp\sqrt{t/\tau_c}}{(t/\tau_c)^{1/4}}$$

$$\frac{1}{\tau_c} \approx \frac{n_{gas}\sigma_i r_p^{1/2}}{A^{1/2}} \frac{N_b^{3/2} n_b^2 r_e L_{sep}^{1/2} c}{\gamma\sigma_y^{3/2}(\sigma_x+\sigma_y)^{3/2}}$$

Micro-sec growth rates



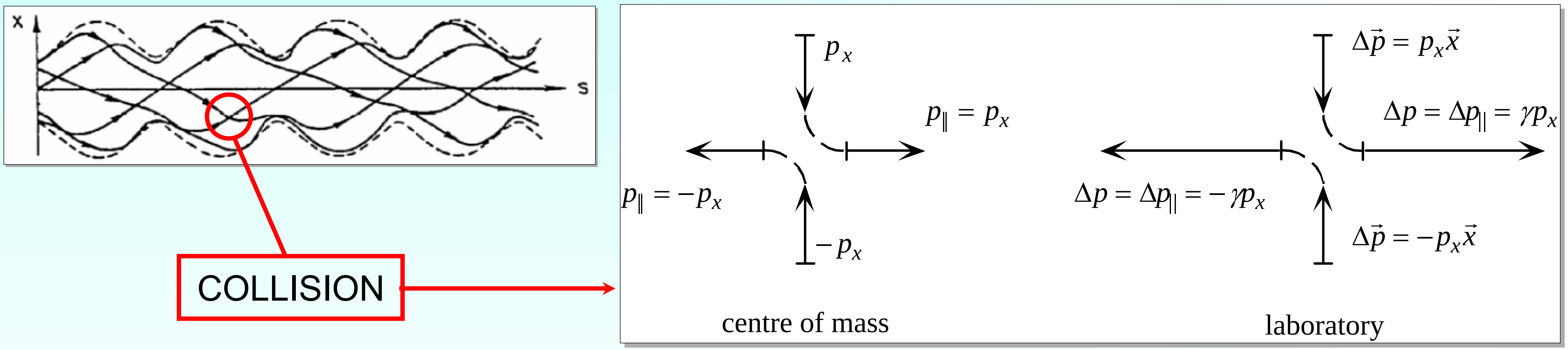
The Touschek Effect

Large Angle intra-Beam Scattering

Touschek Effect

The Touschek effect is a loss mechanism driven by large angle Coulomb collisions in the electron bunch that lead to momentum transfers into the longitudinal plane.

The change in the longitudinal momentum can lead to particle loss if the momentum exceeds the rf acceptance or the transverse (physical or dynamic) momentum acceptance.

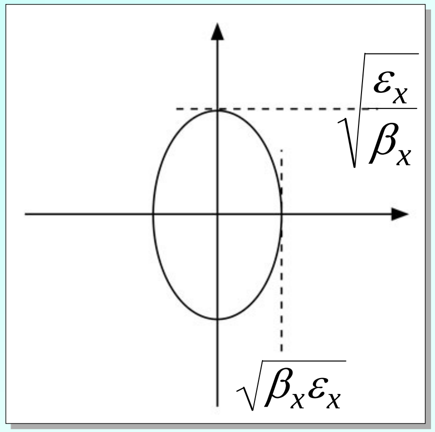


The Touschek effect is one of the limiting mechanism for low emittance machines.
For light sources the requirement of high brilliance results in an enhanced particle loss via the Touschek effect.

High Brilliance → Low Emittance → High Bunch Densities → High Collision Probabilities

Touschek Effect - Orders of Magnitude

At a position where the electron's amplitude is σ_x which has a maximum betatron value of β_x, the maximum divergence is:



$$\sigma'_x = \sqrt{\frac{\varepsilon_x}{\beta_x}}$$

and $\sigma_x = \sqrt{\beta_x \varepsilon_x}$

$$\Rightarrow \sigma'_x = \frac{\sigma_x}{\beta} = \frac{p_x}{p}$$

and $p_x = p\sigma_{x'}$

See Le Duff, CERN 89-01, pp.114

If the transverse momentum p_x is **all** transferred to the longitudinal plane it is boosted by γ:

$$\Rightarrow \Delta p = \gamma\, p_x = \gamma\, \frac{p\sigma_x}{\beta_x}$$

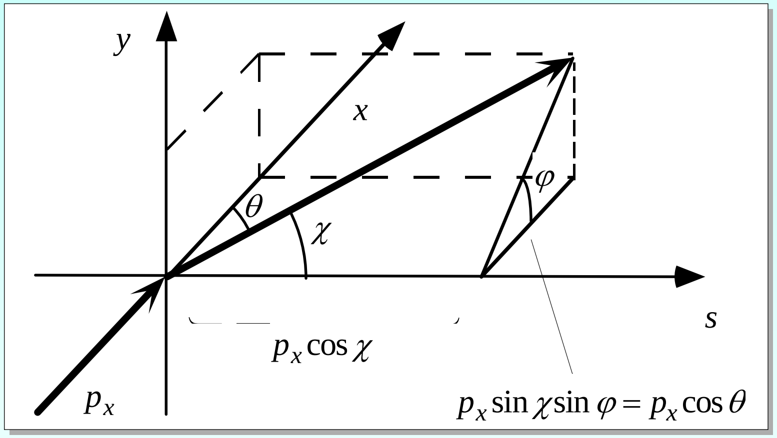
for a displacement of 100 μm at a point with β_x = 10 m and beam energy 2.0 GeV (γ = 3914), Δp = 78 MeV/c, which is 3.9 % energy deviation. This value is the same order of magnitude as the energy acceptance.

Since the electron distribution is Gaussian, higher energy transfers can occur leading to enhanced probability for particle loss.

In the vertical plane the beam is usually very small because of the small coupling, and results in roughly an order of magnitude less effect for a 1% coupled beam.

Touschek Effect - Cross-Section

We consider the collision in the centre of mass system. The transverse momenta of the two colliding particles are equal and opposite, and we assume that they are non-relativistic in this frame. The collision geometry is shown below:



The momentum transferred into the longitudinal laboratory direction is $\gamma\, p_x \cos\chi$.

The particle is lost if this exceeds the longitudinal acceptance. Define,

$$|\cos\chi| \geq \frac{\Delta p_{rf}}{\gamma p_x} = \mu$$

Momentum Acceptance

Boosted transferred momentum

The differential cross-section in the centre of mass frame is given by the Möller cross-section (non-relativistic):

$$\frac{d\sigma}{d\Omega} = \frac{4 r_0^2}{(v/c)^4}\left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta}\right]$$

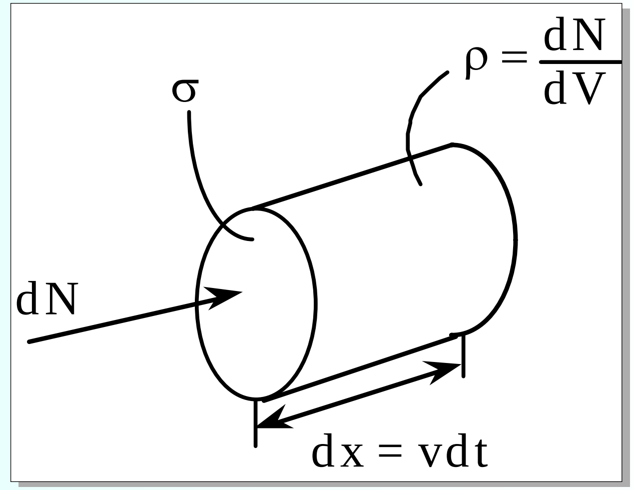
where v is the relative velocity of the two colliding particles, r_0 the classical electron radius and θ the deflection angle. The total cross-section leading to particle loss is given by:

$$\sigma_T = \int_{|\cos\chi|>\mu} \left(\frac{d\sigma}{d\Omega}\right) \cdot d\Omega = \frac{8\pi r_0^2}{(v/c)^4}\left[\frac{1}{\mu^2} - 1 + \ln\mu\right]$$

In centre of mass coordinates

Touschek Effect - Lifetime

The fraction dN_i/N_i per unit time of incoming particles with momentum p_i which suffer a collision is equal to the cross-section, the density ρ_j of the scattering centres with momentum p_j and the distance traversed:



$$dN_i = N_i\ \sigma\ \rho_j\ v\ dt \qquad N_i = \rho_i\ dV$$

Collision rate: $\Rightarrow \dfrac{dN_i}{dt} = N_i\ \sigma\ v\ \rho_j = (\sigma v)\ \rho_i\ \rho_j\ dV$

The total loss rate is now determined by fully integrating the above equation over all space and momentum coordinates which lead to particle loss:

$$\left(\frac{dN}{dt}\right)_{loss} = 2\left(\frac{dN}{dt}\right)_{collisions} = -\frac{2}{\gamma^2}\int(\sigma v)\rho_i\ \rho_j\ dV$$

Touschek Effect - Lifetime

The factor γ^2 comes from transforming the integral from the cm system to the laboratory.
For a flat beam, laminar in the vertical plane, the phase space distribution in the laboratory is:

$$\rho_i = N\ \rho_x(x_i\ x_i')\ \rho_y(y_i)\ \rho_z(z_i)$$

where

$$\rho_x(x_i, x_i') = \frac{\beta_x}{2\pi\sigma_x^2} \exp\left\{-\frac{x_i^2 + (\beta_x x_i' + \alpha_x x_i)^2}{2\sigma_x^2}\right\}, \quad \rho_z(z_i) = \frac{1}{\sqrt{2\pi}\ \sigma_z} \exp\left\{-\frac{z_i^2}{2\sigma_z^2}\right\}, \quad \rho_y(y_i) = \frac{1}{\sqrt{2\pi}\ \sigma_y} \exp\left\{-\frac{y_i^2}{2\sigma_y^2}\right\}$$

we have:

$$\frac{dN}{dt} = -\frac{\sqrt{\pi}\, c\, r_o^2\, N^2}{\gamma^3 V_b\, \sigma_x' \left(\frac{\Delta p}{p}\right)_{rf}^2}\ \varepsilon \int_\varepsilon^\infty \frac{1}{u^2}\left[\left(\frac{u}{\varepsilon}\right) - \ln\left(\frac{u}{\varepsilon}\right)^{1/2} - 1\right] e^{-u}\, du = -\frac{\sqrt{\pi}\, c\, r_o^2\, N_b^2}{\gamma^3 V_b\, \sigma_x' \left(\frac{\Delta p}{p}\right)_{rf}^2}\ C(\varepsilon)$$

bunch volume $V_b = 8\pi^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z$ and $\varepsilon = \frac{1}{(\gamma\sigma_x')^2}\left(\frac{\Delta p}{p}\right)_{rf}^2, \quad \gamma >> 1$

Relative momentum acceptance

The function c (ε) for $\varepsilon < 1$ can be approximated by: $C(\varepsilon) \sim -\left[\ell n(1.732\ \varepsilon) + 3/2\right]$

Touschek Effect - Lifetime

$$\frac{dN}{dt} = \dot{N} = -a\,N^2 \;\Rightarrow\; N(t) = \frac{N_0}{1 + N_0\,at}$$

and the half-life is now given by:

$$\tau_{1/2} = \frac{1}{aN_0} = \frac{N_0}{\dot{N}_0} = \frac{\gamma^3 V_b\,\sigma'_x \left(\frac{\Delta p}{p}\right)^2_{rf}}{\sqrt{\pi}\;c\;r_0^2\;N_b} \cdot \frac{1}{C(\varepsilon)}$$

and is related to 1/e lifetime by: $\tau_{1/2} = \ln 2 \cdot \tau_{1/e}$

The derivation applies to one point in the ring, the overall lifetime is obtained by taking the average over the whole circumference: $\tau = \langle\tau(s)\rangle$

More accurate calculations take into account vertical motion, relativistic effects and dispersion.

Typical lifetime values are a few to ten’s of hours.

Touschek Effect - Parameters

The energy dependence of the Touschek lifetime is very strong and several factors affect the lifetime: the emittance, coupling, bunch length, energy spread, beam divergence, relativistic effects and small angle intra-beam scattering.
The lifetime scales as E^3 to E^4. The Touschek effect is more severe for low energy machines and for high energy machines with few bunches.

The Touschek lifetime depends on

The bunch volume $V_b = 8\pi^{3/2}\sigma_x\sigma_y\sigma_z$
- Transverse emittance
- Bunch length
  - Machine impedance
  - The RF voltage
  - The RF frequency
- Betatron coupling

The momentum acceptance
- RF
- Transverse (dynamic, physical)

$$\tau_{1/2} = \frac{\gamma^3 V_b \sigma'_x \left(\frac{\Delta p}{p}\right)^2_{rf}}{\sqrt{\pi}\, c\, r_0^2\, N_b} \cdot \frac{1}{C(\varepsilon)}$$

Beam Energy

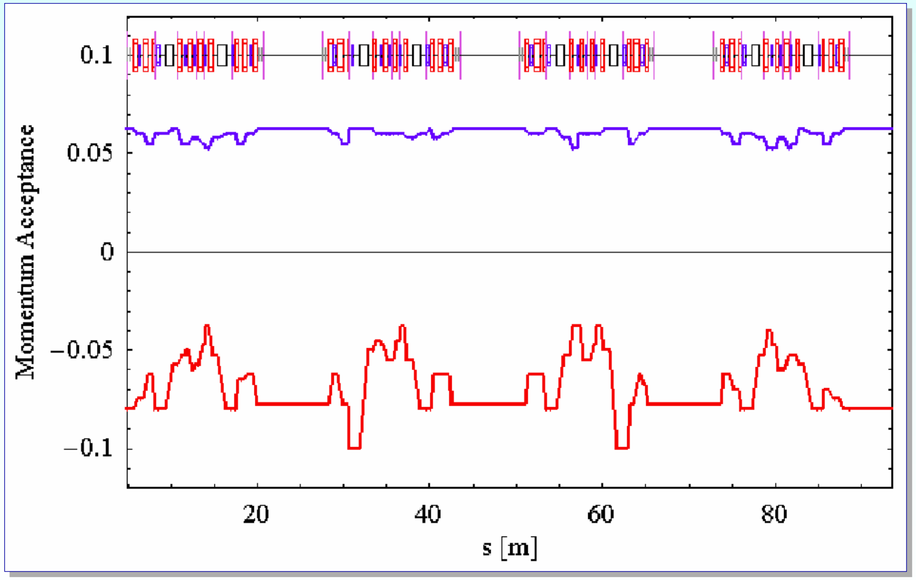
Current

Improve Touschek lifetime by acting on these parameters

Touschek Effect - Particle Loss

The above treatment explicitly referred to the RF momentum acceptance. In general the scattering rate is calculated for each location in the ring based on local particle density and momentum acceptance (whether RF, dynamic or physical). This is then averaged over the ring.

We note that for most third generation storage rings the limiting acceptance (if not optimised) is mostly in the transverse plane. For the latest rings (DIAMOND & SOLEIL) much effort has been put into finding optics giving large transverse momentum apertures.



DIAMOND - Transverse

Ways of improving the lifetime

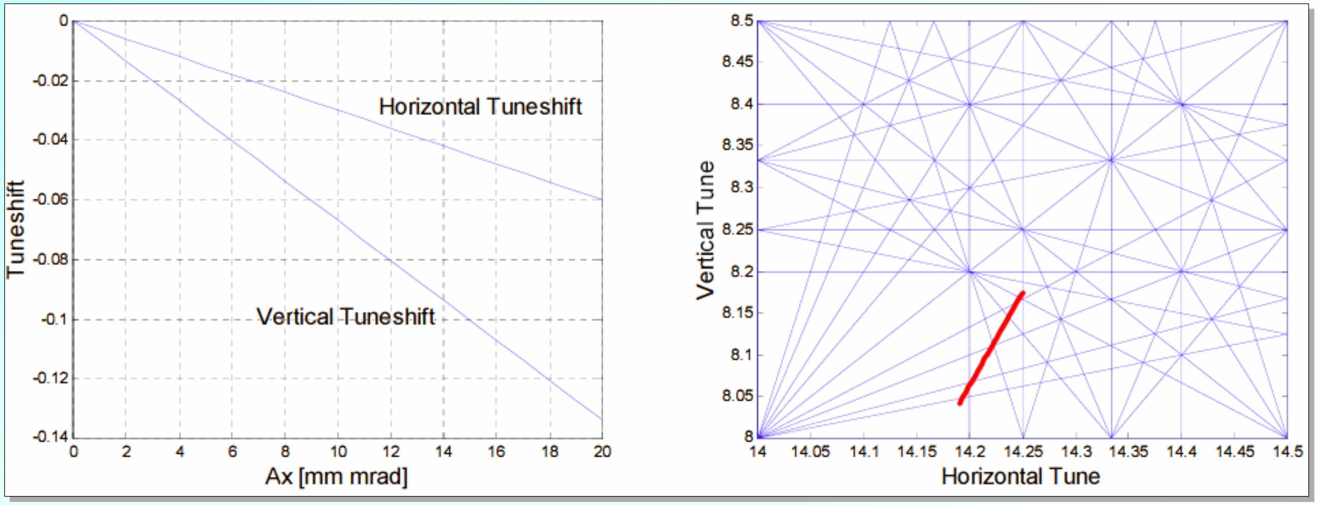
- Increase the coupling
- Superconducting RF system for large V_{RF}
- Phase modulation of the RF signal - quadrupolar excitation of the bunch

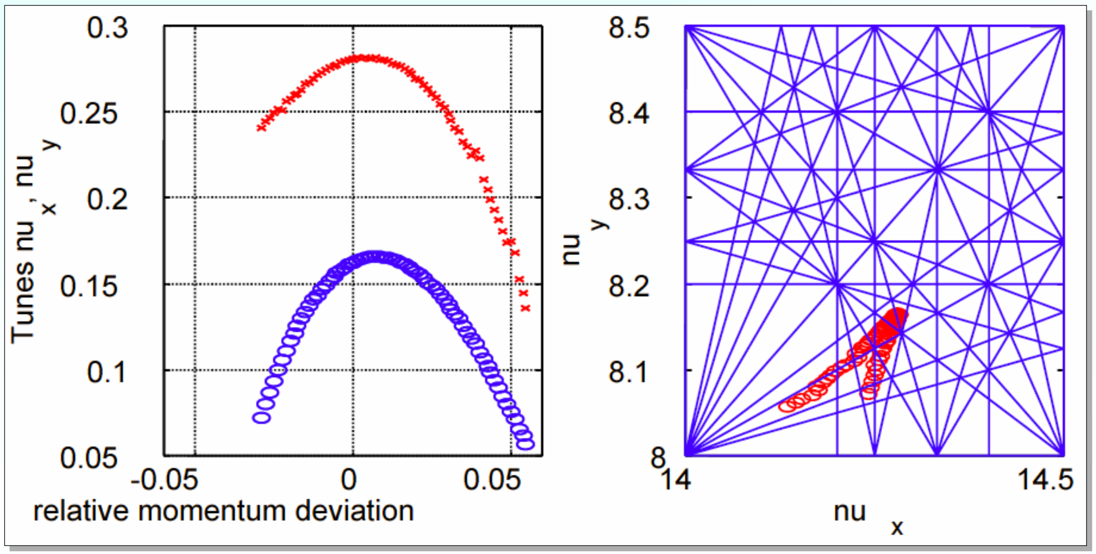
Reduce the bunch density without compromising the brightness nor energy spread. i.e., Lengthen the bunch through the use of a higher harmonic cavity.

Ultimate lifetime improvement is given by top-up mode of operation.

Touschek Effect - ALS Example

Tune shifts with amplitude and energy

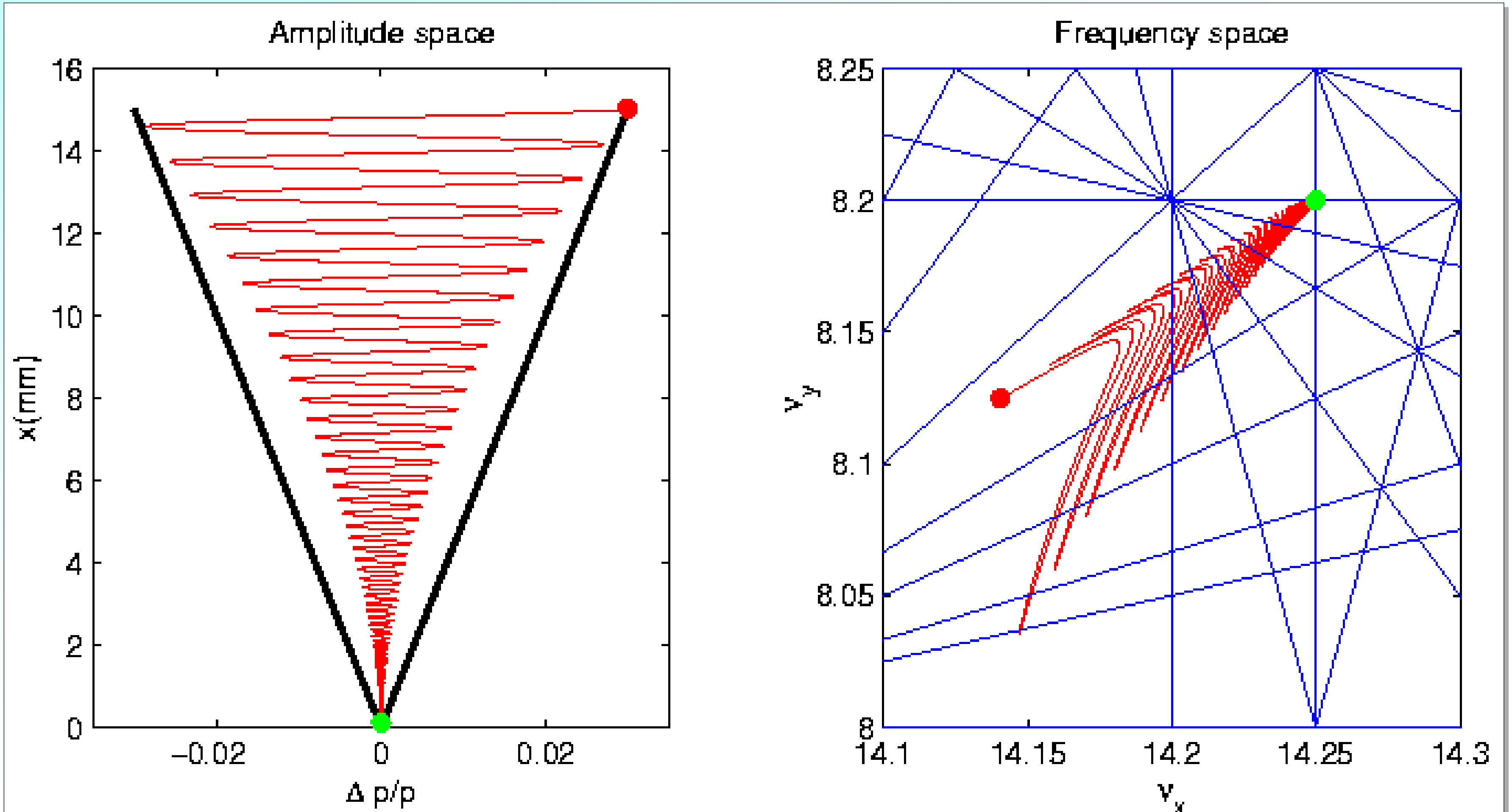




Courtesy of D. Robin, ALS

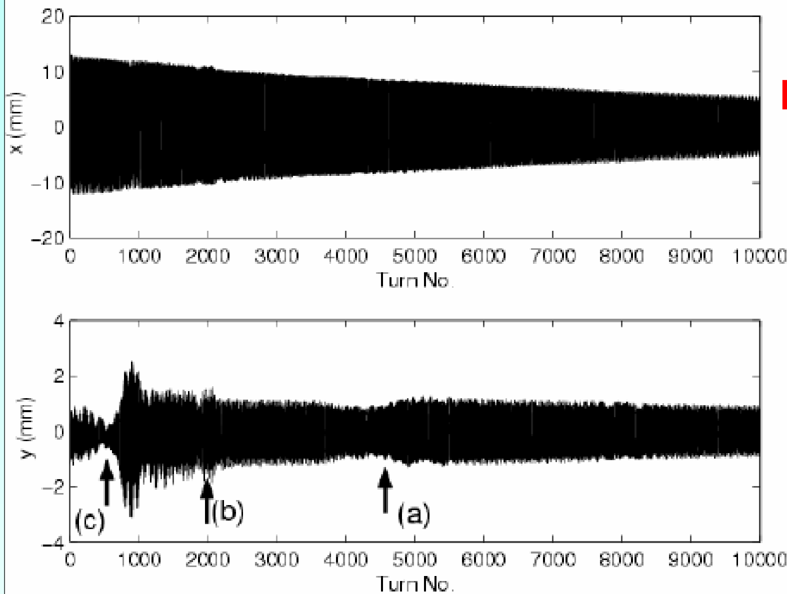
Touschek Effect - ALS Example

A particle suffers a sudden energy change. After which it slowly radiation damps back to being on-energy and all the time traversing the tune diagram

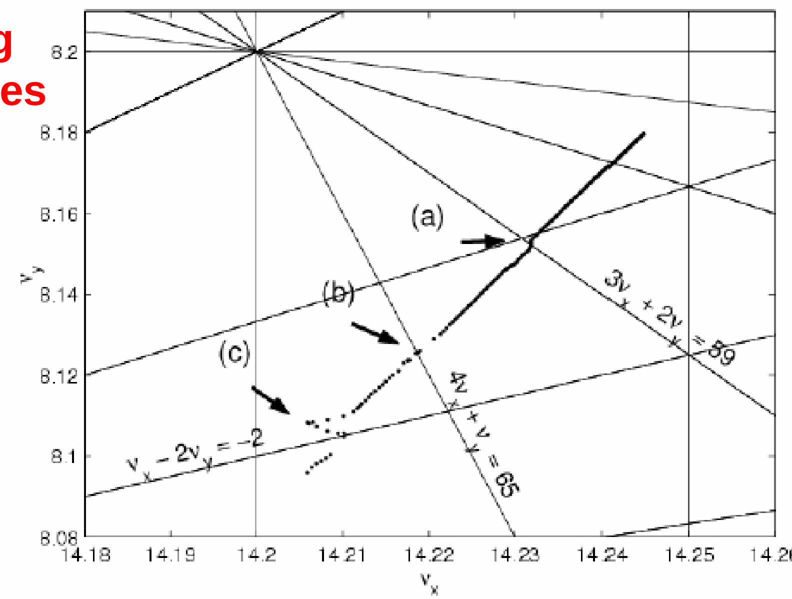


Courtesy of D. Robin, ALS

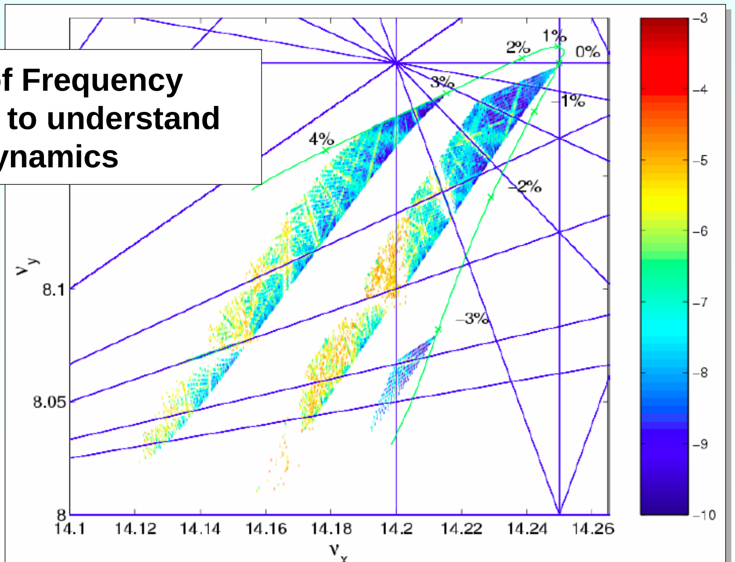
Touschek Effect - ALS Example

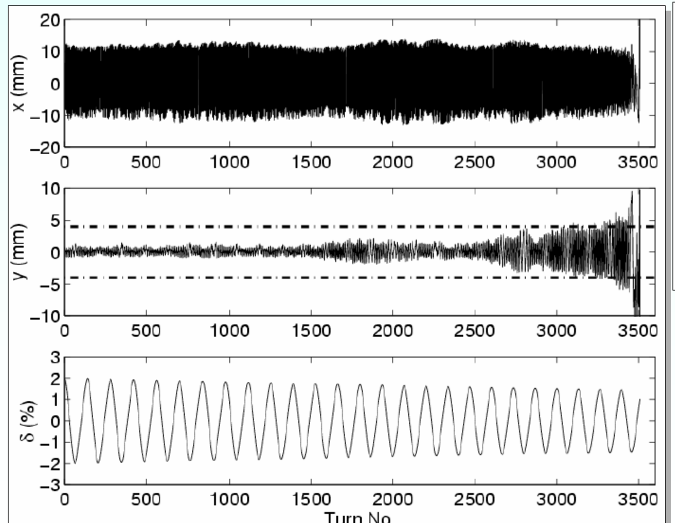


Crossing Resonances



Use of Frequency maps to understand the dynamics





Particle lost in the vertical plane.
Highest level of radiation at the end of the narrow gap insertion device chamber.

Courtesy of D. Robin, ALS

Intra Beam Scattering

Intra Beam Scattering

Intra-beam scattering deals with all those collisions which were ignored in deriving the Touschek effect. **The many small angle collisions lead to a re-distribution of the six dimension phase space distribution**. In the centre of mass frame the transverse momentum spreads are larger than the longitudinal.

$$\left.\begin{array}{l} \sigma'_x p \\ \sigma'_y p \\ \sigma'_y p/\gamma \end{array}\right\} cms$$

The multiple collisions between particles will predominantly transfer momentum from the transverse plane to the longitudinal. We must, however, consider synchrotron oscillations.

The Touschek effect showed that the transverse collision in the bunch frame resulted in a re-distribution of momenta. The transferred transverse momentum p_x is boosted to γp_x. The longitudinal blow-up is compensated by the transverse collapse.

The Touschek event neglected transfer of momentum from the longitudinal plane to the transverse because of the larger transverse acceptance.

Intra Beam Scattering

Following Sorensen (CERN 87-10, pp 135) and examining the emittance (at a symmetry point) prior to the collision of two electrons we have:

$x_\beta = x$ and $x_\beta' = +p_x/p$ for one particle
and $x_\beta = x$ and $x_\beta' = -p_x/p$ for the other

after the collision with full transfer of momentum to the longitudinal plane, we excite betatron motion at positions of non-zero dispersion.

$x_\beta = x + D\gamma p_x/p$ and $x_\beta' = 0$ for one particle
and $x_\beta = x - D\gamma p_x/p$ and $x_\beta' = 0$ for the other

If the original emittance of the particles was: $\varepsilon_x = \dfrac{1}{\beta_x}\left(x_\beta^2 + \beta_x^2 x_\beta'^2\right)$

the change in the sum of the emittances for the two particles after the collision is:

$$\Delta(\varepsilon_{x1} + \varepsilon_{x2}) = \frac{2}{\beta_x}\left(\frac{p_x}{p}\right)^2 \left[(D\gamma)^2 - \beta_x^2\right]$$

For $(D\gamma/\beta_x)^2 > 1$

there is simultaneous longitudinal and transverse growth since the beam can absorb any amount of energy from the rf system.

Intra Beam Scattering

The full treatment is mathematically rather complicated since it involves integration over 12 dimensions for two particles. Just the recipe is given and results quoted.

Computation of the growth rate of the intra-beam effect given by the recipe of Piwinski, is as follows:

1) Transformation of the momenta of the two colliding particles to the centre of mass frame.
2) Calculation of the change in momenta due to an elastic collision.
3) Transformation of the momenta back to the laboratory frame.
4) Relate the changes in momenta to changes in emittances.
5) Average over the distribution of scattering angles using the small angle Moller cross-section
6) Average over the distribution of the particles within a bunch to get the growth time.

(Piwinski, Proc. 9th Int. Conf. On High Energy Accel. 105, 1974)
See also the CERN Yellow Books.

Intra Beam Scattering

The derivation for a weak focusing machine yields the following condition:

$$\langle H \rangle \left(\frac{1}{\gamma^2} - \alpha_c \right) + \frac{\langle \varepsilon_x \rangle}{\beta_x} + \frac{\langle \varepsilon_y \rangle}{\beta_y} = 0$$

<> denotes the average around the ring circumference ε_x and ε_y the transverse emittances and H is the longitudinal emittance. Below transition $\gamma^{-2} < \alpha_c$ the sum of the three positive invariants is limited, they only exchange energy.

Above transition there is no equilibrium distribution and the invariants can grow without bounds.

In the presence of radiation damping an equilibrium will exist:

since

- **radiation damping becomes stronger with increasing amplitudes**
- **intra-beam scattering will becomes weaker with increasing amplitudes**

and

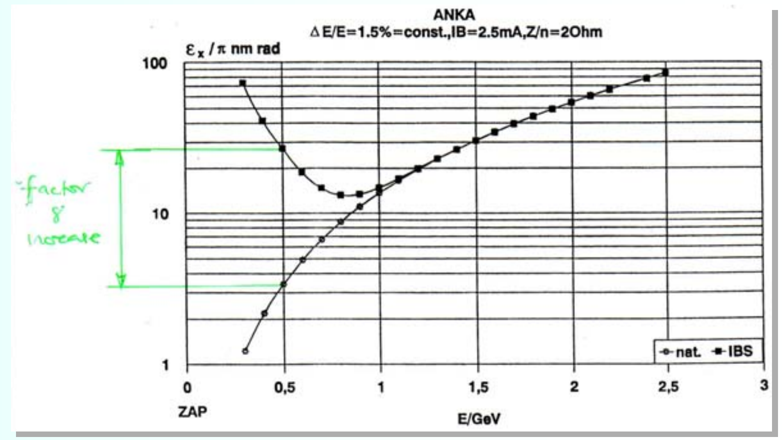
- **quantum fluctuations are independent of amplitudes.**

Distribution growth rates go as the 4th power of the beam energy and Intra-beam scattering is (usually) only of importance at low energies where the emittance is low and high particle densities can arise. For light sources above 1.0 GeV its effect rapidly diminishes with increasing energy.

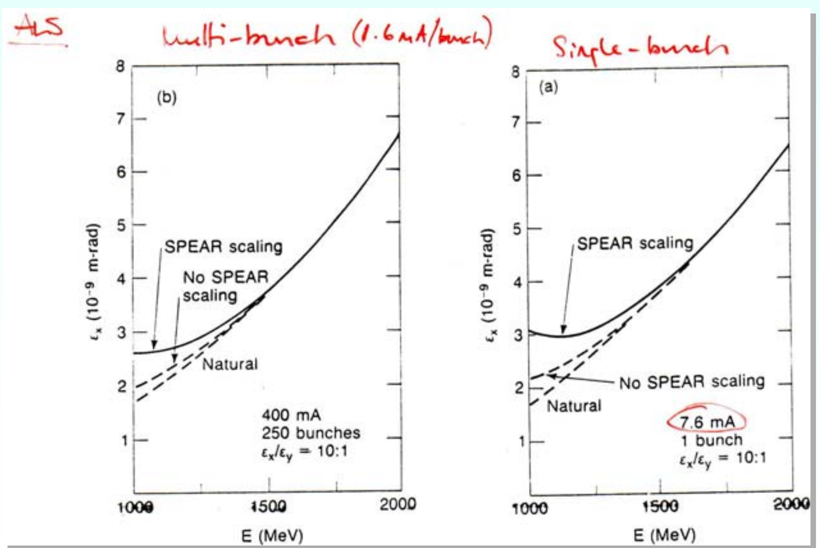
Intra Beam Scattering

IBS may become evident when bunch currents are increased in exotic, few or single bunch operation. However, bunch lengthening from potential well distortion and turbulent bunch lengthening will decrease the bunch density and therefore lessen the effect of IBS.

ANKA - Conceptual Design



ALS - Conceptual Design



Soleil - Conceptual Design

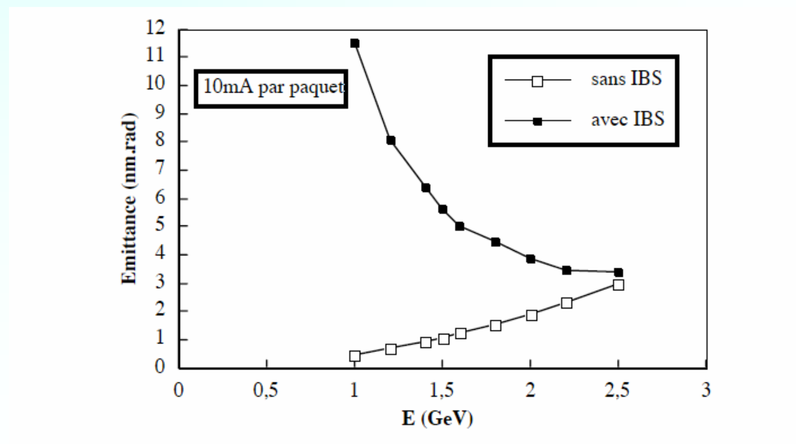


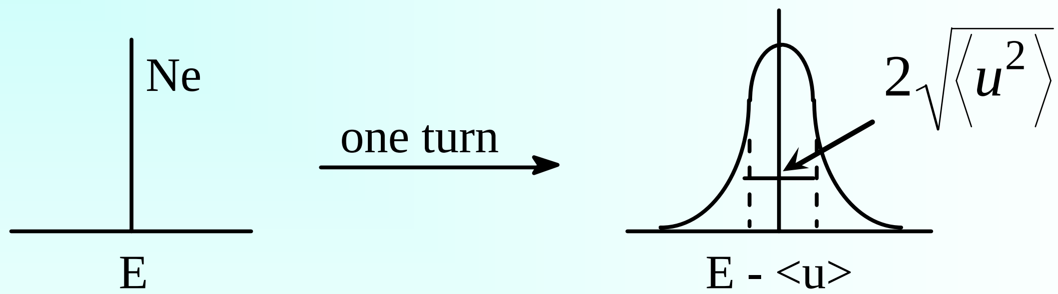
Figure 3.2.3-1. Effet IBS en fonction de l'énergie de la machine pour un courant par paquet de 10 mA.

Quantum Lifetime

Quantum Lifetime - Fluctuations

The number of photons emitted per electron per turn is a rather small number. For a single electron the number of emissions per meter is about 6.2 B[T].

This small number will give rise to noticeable statistical effects.



The broadening of the distribution is due to quantum fluctuations which are proportional to $< u^2 >$, with u the photon energy. The mean value of the energy loss <u> is associated with radiation damping.

The quantum effect is similar to a diffusion process.

The equilibrium between radiation damping and quantum excitation determines the distribution of the electrons.
The Central Limit theorem tells us that the distribution will be Gaussian independent of the distribution function for u.

Quantum Lifetime - Transverse

The equilibrium established between quantum fluctuations and radiation damping results in a Gaussian phase space distribution,

$$\frac{1}{2\pi\varepsilon_x}\exp\left\{-\frac{1}{2\varepsilon_x}\left(\gamma x^2+2\alpha x x'+\beta x'^2\right)\right\}dx\,dx'$$

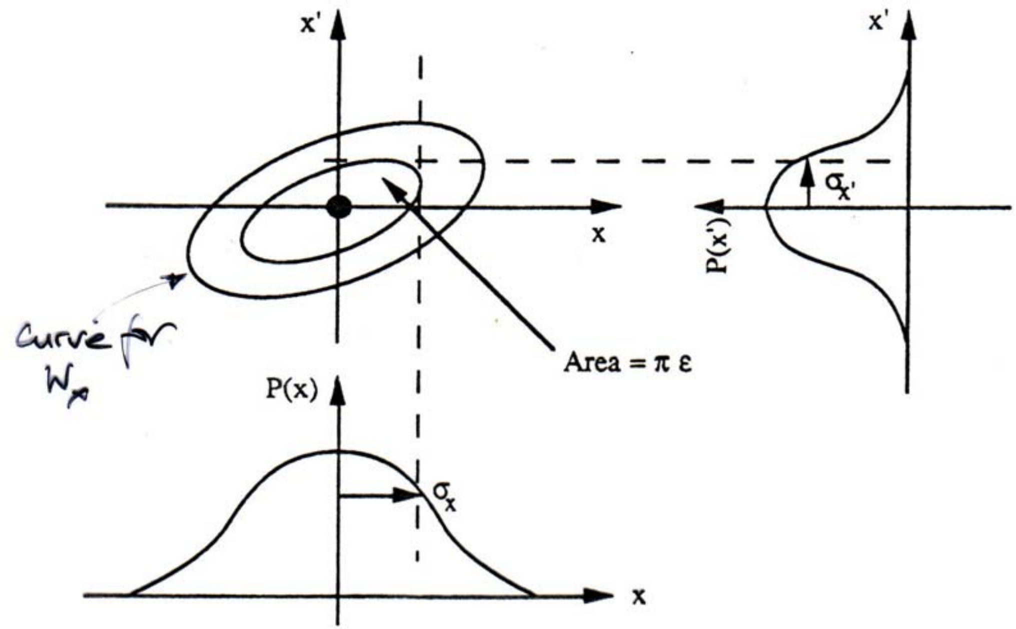


Fig. 1 Gaussian distribution of electron density in the (x,x') plane

The Courant-Snyder invariant for a particular trajectory x, x' is

$$W_x=\gamma_x x^2+2\alpha_x x x'^2+\beta_x x'^2$$

with the coordinates described as

$$x=(W_x\beta_x)^{1/2}\cos\phi,\quad x'=-(W_x/\beta_x)^{1/2}(\sin\phi+\alpha_x\cos\phi)$$

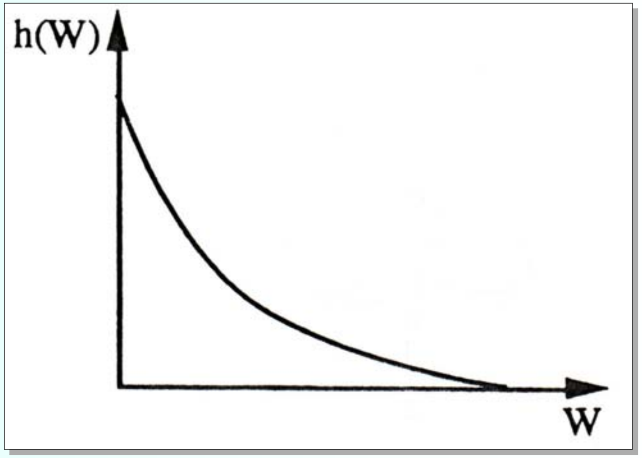
where ϕ is an arbitrary phase.

Use these coordinates to describe the distribution in terms of W and ϕ.

Quantum Lifetime - Transverse

The phase space distribution can be written in terms of the invariant as:

$$h(W_x)dW_x d\phi = \frac{1}{2\pi\langle W_x\rangle}\exp\left(\frac{-W_x}{\langle W_x\rangle}\right)dW_x d\phi$$



And has the property: $\int h(W_x)dW_x = 1$

The average $\langle W_x\rangle = 2\varepsilon_x$.

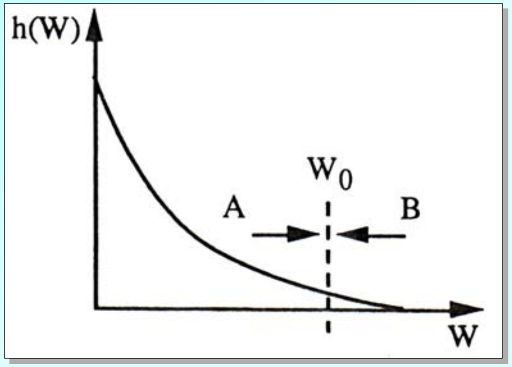
In a stored beam of N electrons the number having values of W between W_x and W_x+dW_x is:

$$dN = Nh(W_x)dW_x = \frac{N}{\langle W_x\rangle}\exp\left(\frac{-W_x}{\langle W_x\rangle}\right)dW_x$$

The distribution being Gaussian, in principle extends to infinity. The vacuum chamber, however, will truncate the distribution and lead to a constant particle loss at the tails.

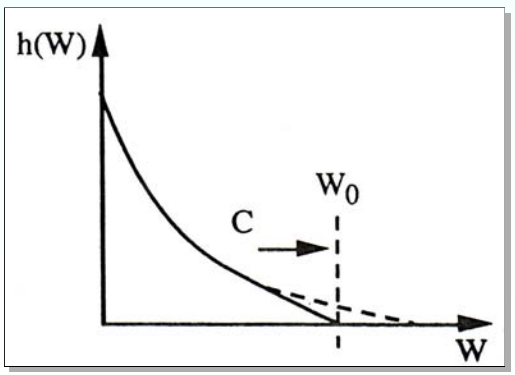
Quantum Lifetime - Transverse

Without the restriction we have a stationary situation where,



the number of particles crossing an arbitrary point W_0 due to quantum excitation (process A) equals the number entering due to radiation damping (process B).

With a restriction in amplitudes at W_0 (due the chamber wall), if we are very far away from the core as will be the case, the number of electrons crossing W_0 and hence lost will be very nearly the same as if there were no aperture.



i.e. $C \cong A = B$

Quantum Lifetime - Transverse

The loss rate can be estimated as that due to process B, due to radiation damping. Then,

$$\left(\frac{dN}{dt}\right)_{W_0} = \left(\frac{dN}{dW}\cdot\frac{dW}{dt}\right)_{W_0}$$ (i.e. evaluated at the restriction)

$$dN = Nh(W)dW \quad \Rightarrow \quad \frac{dN}{dW} = Nh(W)$$

For a given electron the rate of change of W due to radiation damping is:

$$\frac{dW_x}{dt} = -\frac{2W_x}{\tau_x}$$

radial damping time

and

$$\frac{dN}{dt} = -N\frac{2}{\tau_x}\frac{W_0}{\langle W\rangle}\exp\left(\frac{-W_0}{\langle W\rangle}\right) \quad \Rightarrow \quad N = N_0 \exp\left(\frac{-t}{\tau_q}\right)$$

quantum lifetime

where τ_q is the quantum lifetime defined by:

$$\tau_q = \frac{\tau_x}{2}\frac{\langle W\rangle}{W_0}\exp\left(\frac{W_0}{\langle W\rangle}\right) = \frac{\tau_x}{2}\frac{\exp(\xi^2)}{\xi^2}$$

where $\xi^2 = \frac{W_0}{\langle W\rangle}$

Quantum Lifetime - Transverse

Assuming the limiting aperture occurs at a point with β-function, β_{max} then (α=0),

$$\xi^2 = \frac{x_{\max}^2 / \beta_{\max}}{2\sigma_x^2 / \beta_{\max}} = \left(\frac{x_{\max}}{\sqrt{2}\,\sigma_x} \right)^2$$

Because of the exponential factor the quantum lifetime increases rapidly with x_{max}/σ_x.
Due to the approximations made the expression is valid for $x_{max} >> \sigma_x$

For $x_{max} > \sim 5\ \sigma_x$ accurate values of τ_q can be computed:

x_{max}/σ_x	5	5.5	6.0	6.5	7.0
τ_q	1.8 min	20.4 min	5.1 hrs	98.3 hrs	103 days

"Golden Rule" for τ_q ~ 100 hrs means:

$$\boxed{\frac{x_{\max}}{\sigma_x} \geq 6.5}$$

Quantum Lifetime - Longitudinal

In the longitudinal plane a Gaussian distribution also exists for the energy and phase deviations. Here the aperture restriction is the finite potential rf well. A result similar to the transverse case is found:

longitudinal damping time

$$\tau_q = \frac{\tau_\varepsilon}{2} \frac{\exp \xi^2}{\xi^2}$$

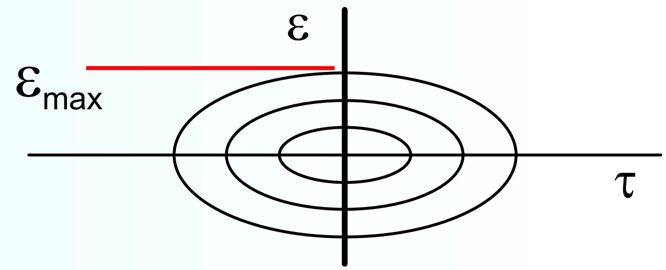
$$W = \varepsilon^2 + \left(\frac{E_0 \Omega}{\alpha}\right) \tau^2$$

where:

maximum energy deviation

$$\xi^2 = \frac{W_0}{\langle W \rangle} = \frac{\varepsilon_{\max}^2}{2\,\sigma_\varepsilon^2}$$

standard deviation of the energy spread



Again as for the transverse case the rf acceptance is determined for the Touschek lifetime ~2-3%. With relative energy spreads of the order a few 10^{-4} the longitudinal quantum lifetime is practically infinite.

Increasing the Lifetime

Gas Desorption and NEG

To minimise gas scattering effects the vacuum pressure has to be maintained at low values.

Important in ID vacuum chambers

- Conventional pumping requires expensive solutions for low gaps
- Gas Bremsstrahlung radiation is a hazard for users

Recently use is made of sputtered NEG coated chambers

- Composed of a few micron thick layer of Titanium, Vanadium and Zirconium
- Acts as a distributed pump
- Reduces the emission of adsorbed gas molecules produced by photo-electron desorption

The NEG coating is activated by heating. Subsequent activations require higher temperatures.

It can be deposited on Stainless Steel and Aluminium.

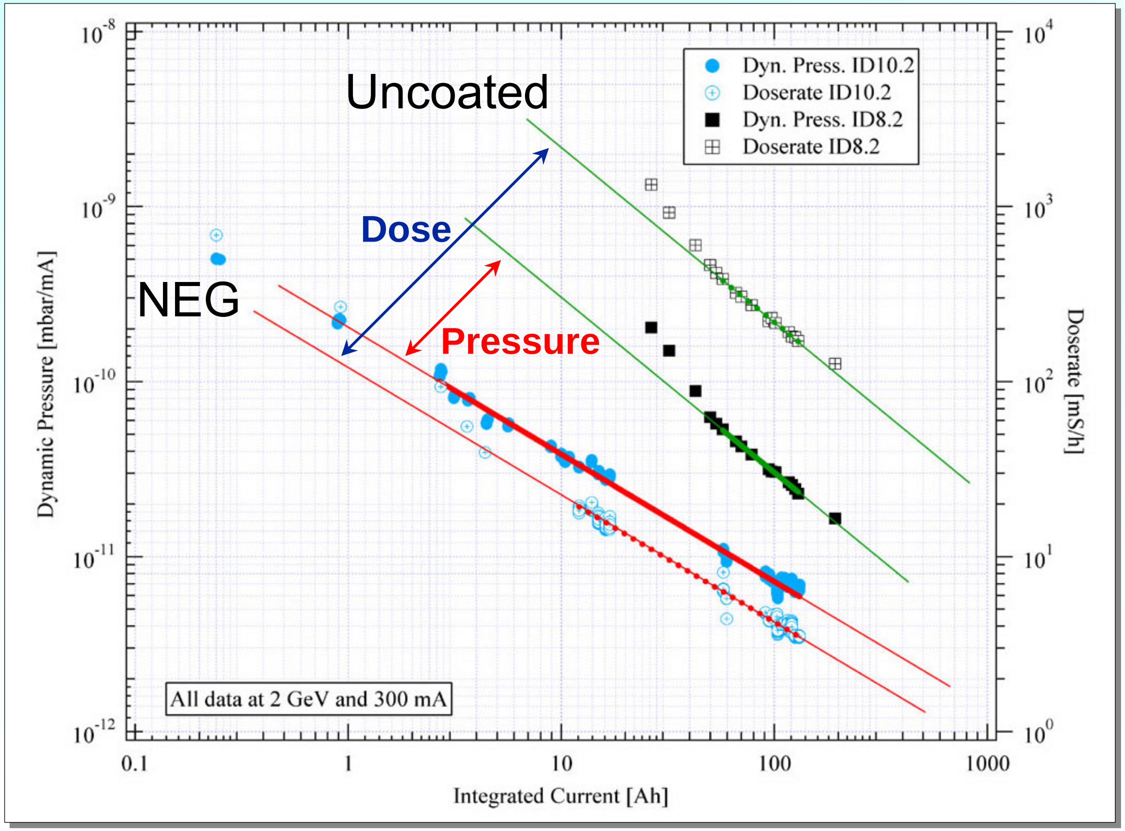
NEG Coated Chambers

NEG Coated Chambers at ELETTRA

Comparison of Normal pump free Al chamber and NEG coated Al chamber.

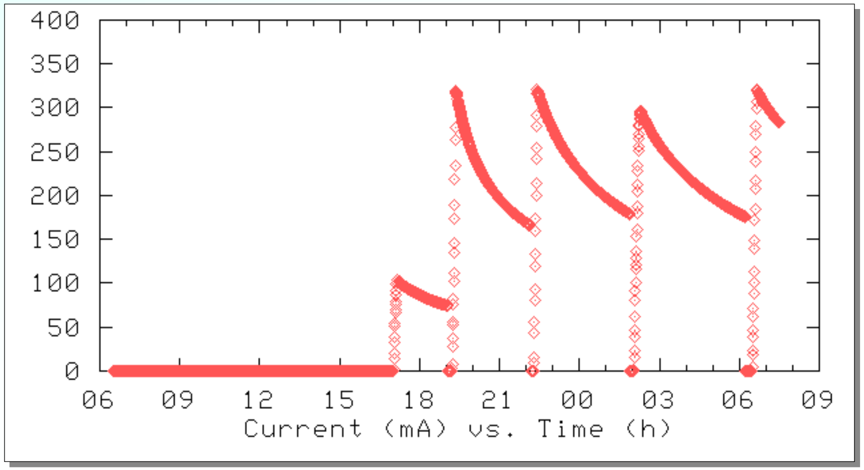
NEG coating gives excellent conditioning and operational results

Dynamic Pressure and Doserate



F. Mazzolini et al., "Performance of Insertion Device Vacuum Chambers at ELETTRA", Proc. EPAC 2002, Paris

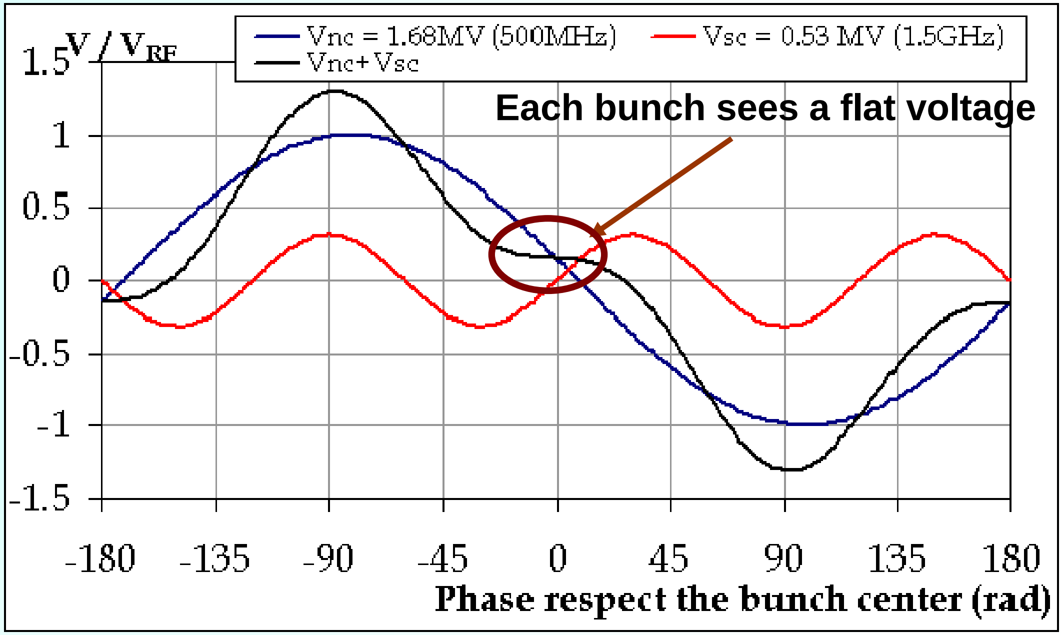
Little Beam Conditioning Needed



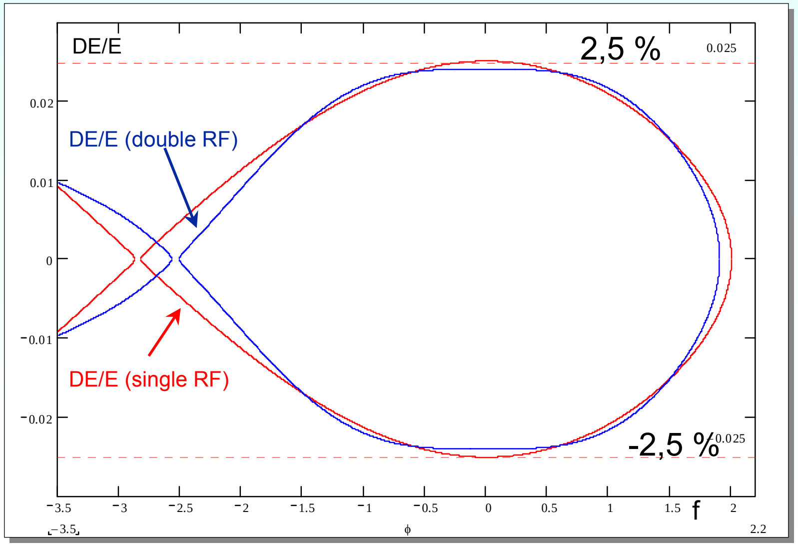
Increasing the Bunch Length - 3rd HC

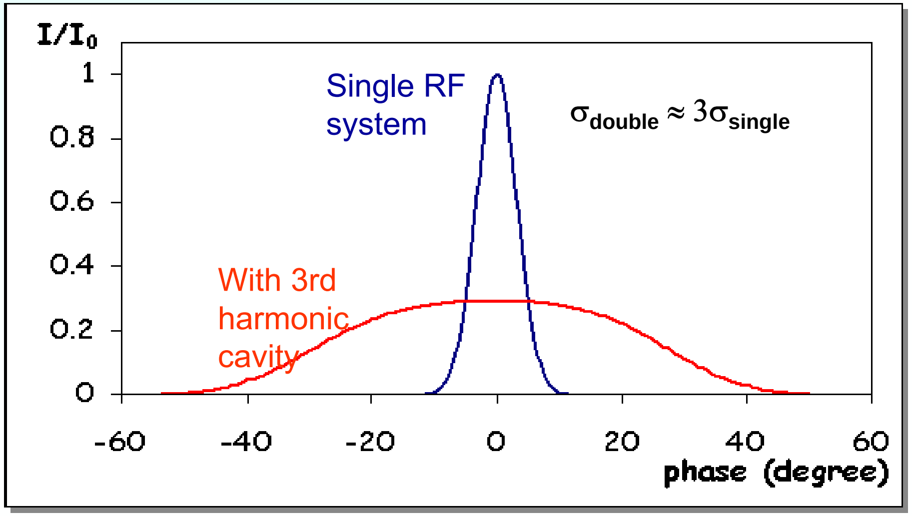
The most typical choice is a 3rd harmonic system

A flattened potential well is obtained if the voltage slope at the bunch center is zero



The RF acceptance is not significantly modified by adding an harmonic cavity

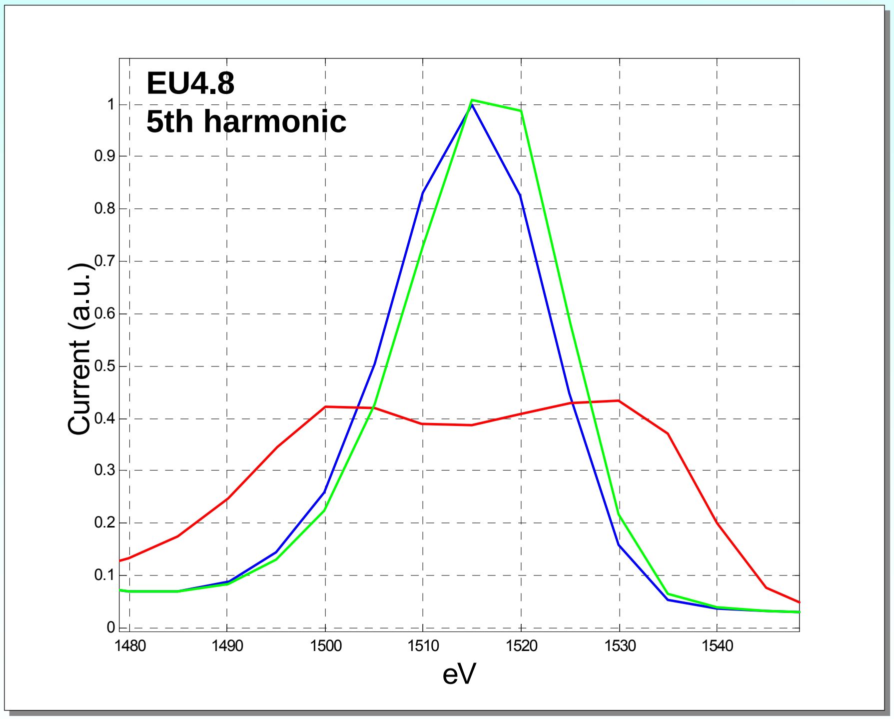




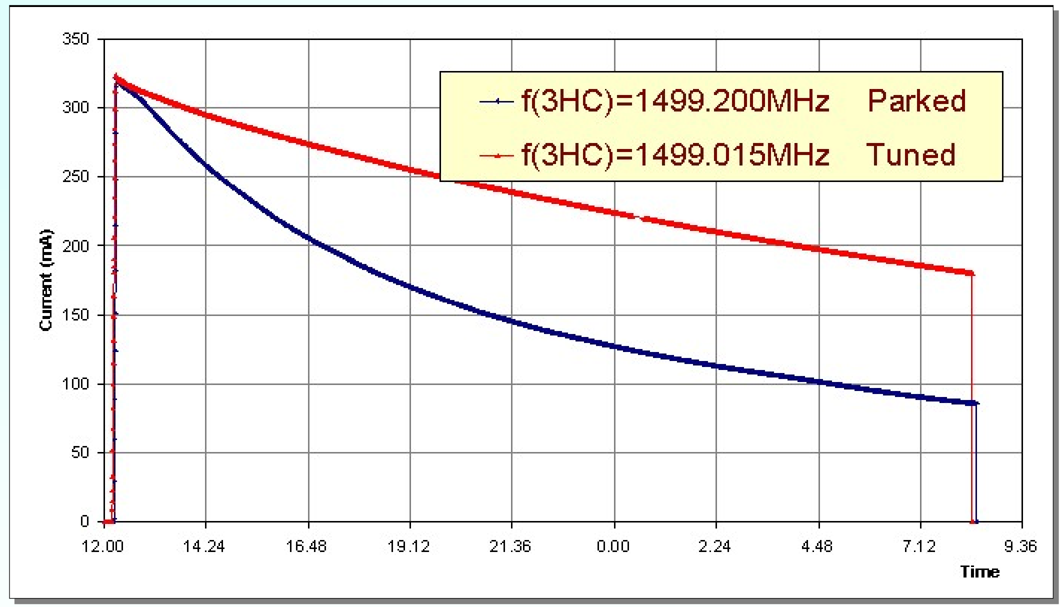
The Twofold Effect of 3HC on the Beam

Elettra as an example

Landau damping of longitudinal multibunch instabilities increases the beam brightness.



Lengthening of the bunch increases the lifetime.

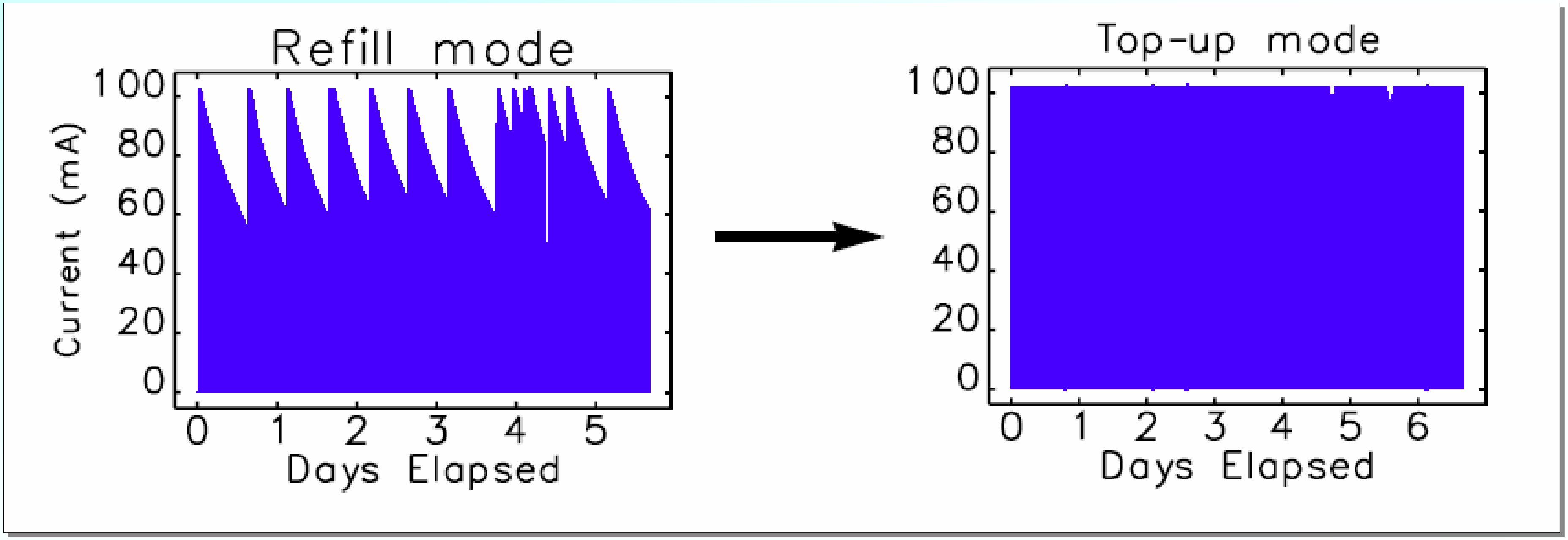


Top Up - APS

Top-up for light sources was pioneered at the APS

Injection at the APS every two minutes. Users want constancy in injection.

Routine Operation in top-up mode



Injection is performed in single bunch mode.
The algorithm computes which bucket needs to be refilled.
The constancy of bunch charge is determined by the injector parameters (charge/pulse, repetition rate and injection efficiency).