Luminosity and Beam-Beam Effects in the Large Hadron Collider (LHC)

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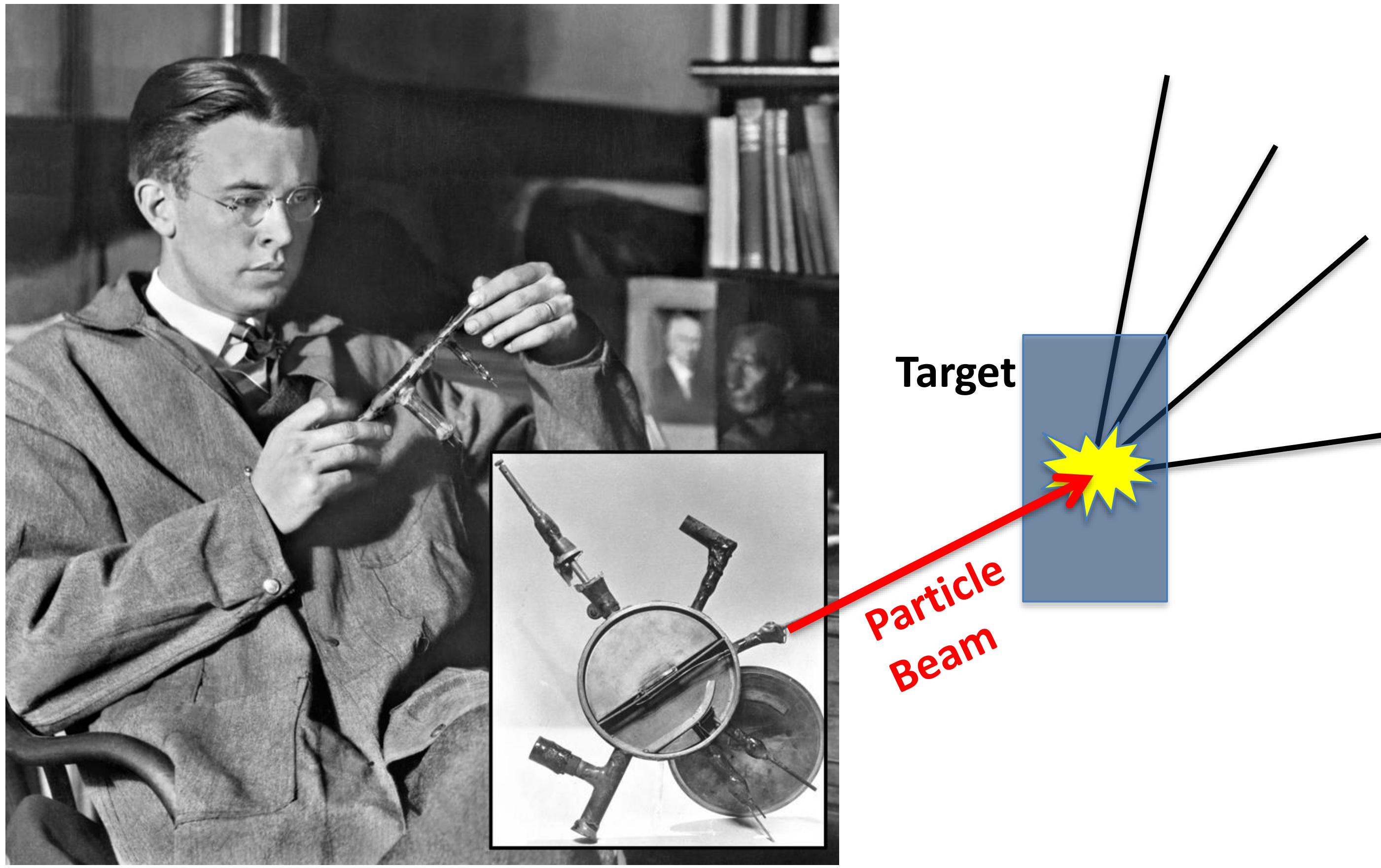
Circular Accelerators: acceleration occurs at every turn!



E. Lawrence 1930

Circular Accelerators: acceleration occurs at every turn!

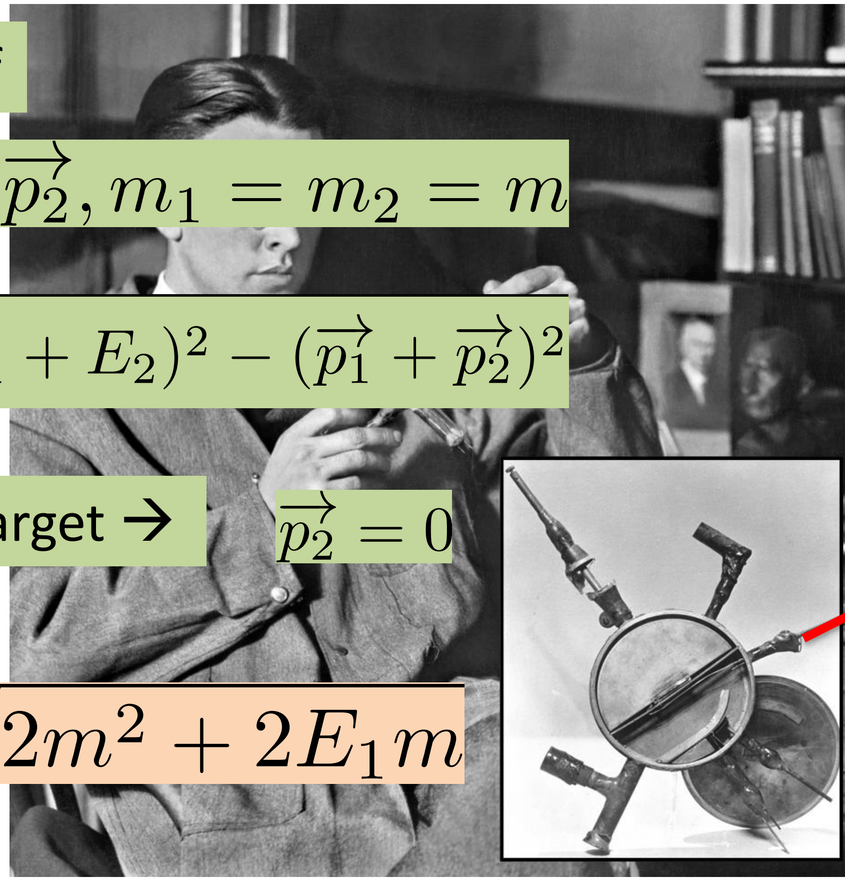
Two Beams of

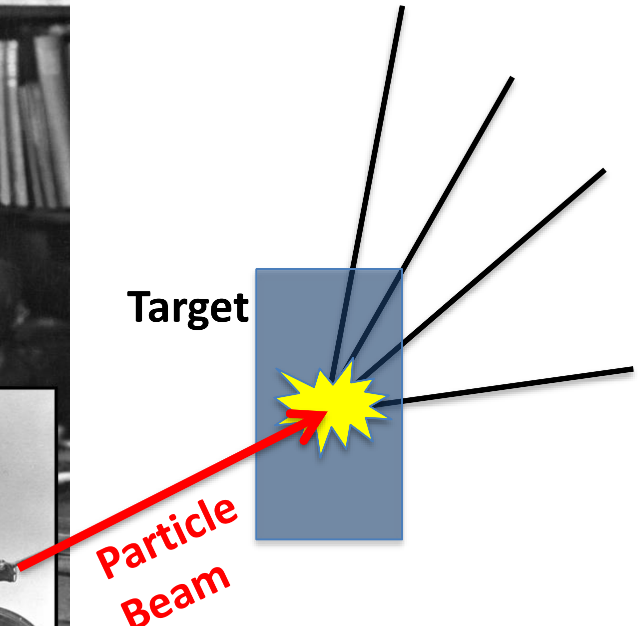
$$E_1, \overrightarrow{p_1}, E_2, \overrightarrow{p_2}, m_1 = m_2 = m$$

$$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\overrightarrow{p_1} + \overrightarrow{p_2})^2}$$

Beam 2 is a Target → $\overrightarrow{p_2} = 0$

$$E_{cm} = \sqrt{2m^2 + 2E_1 m}$$

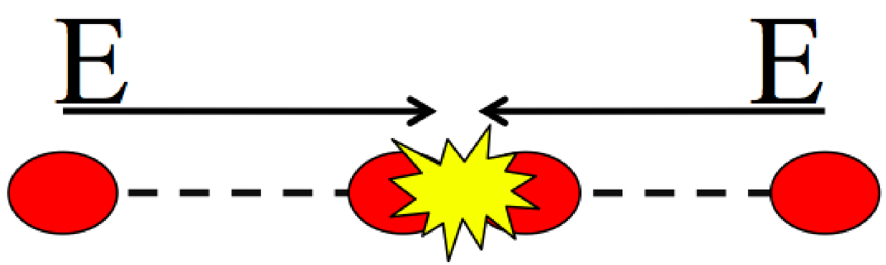




7 TeV proton beam against fix target →115 GeV

Colliders: higher energy

Anello di Accumulazione AdA B. Touschek 1960



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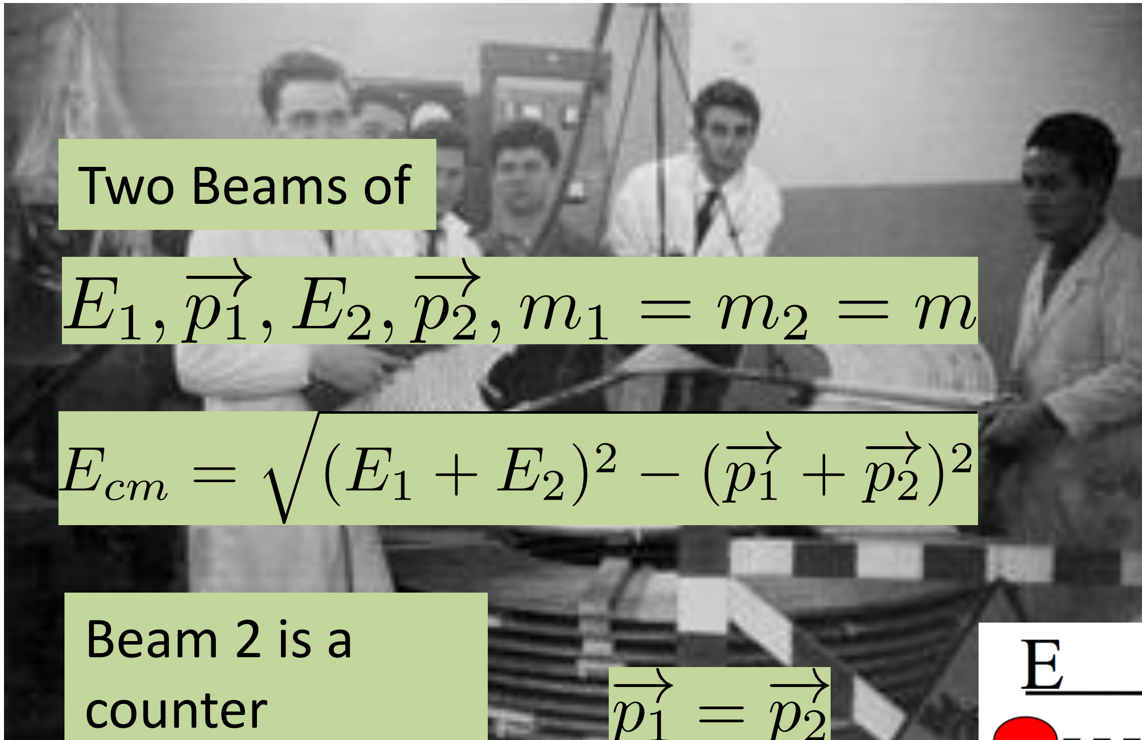

Two Beams of

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$$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\overrightarrow{p_1} + \overrightarrow{p_2})^2}$$

Beam 2 is a counter rotating beam

$$\overrightarrow{p_1} = \overrightarrow{p_2}$$

Anello di Accumulazione AdA
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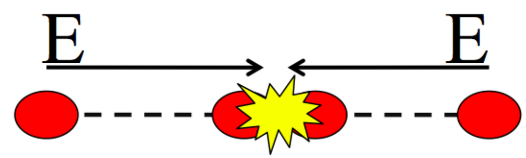
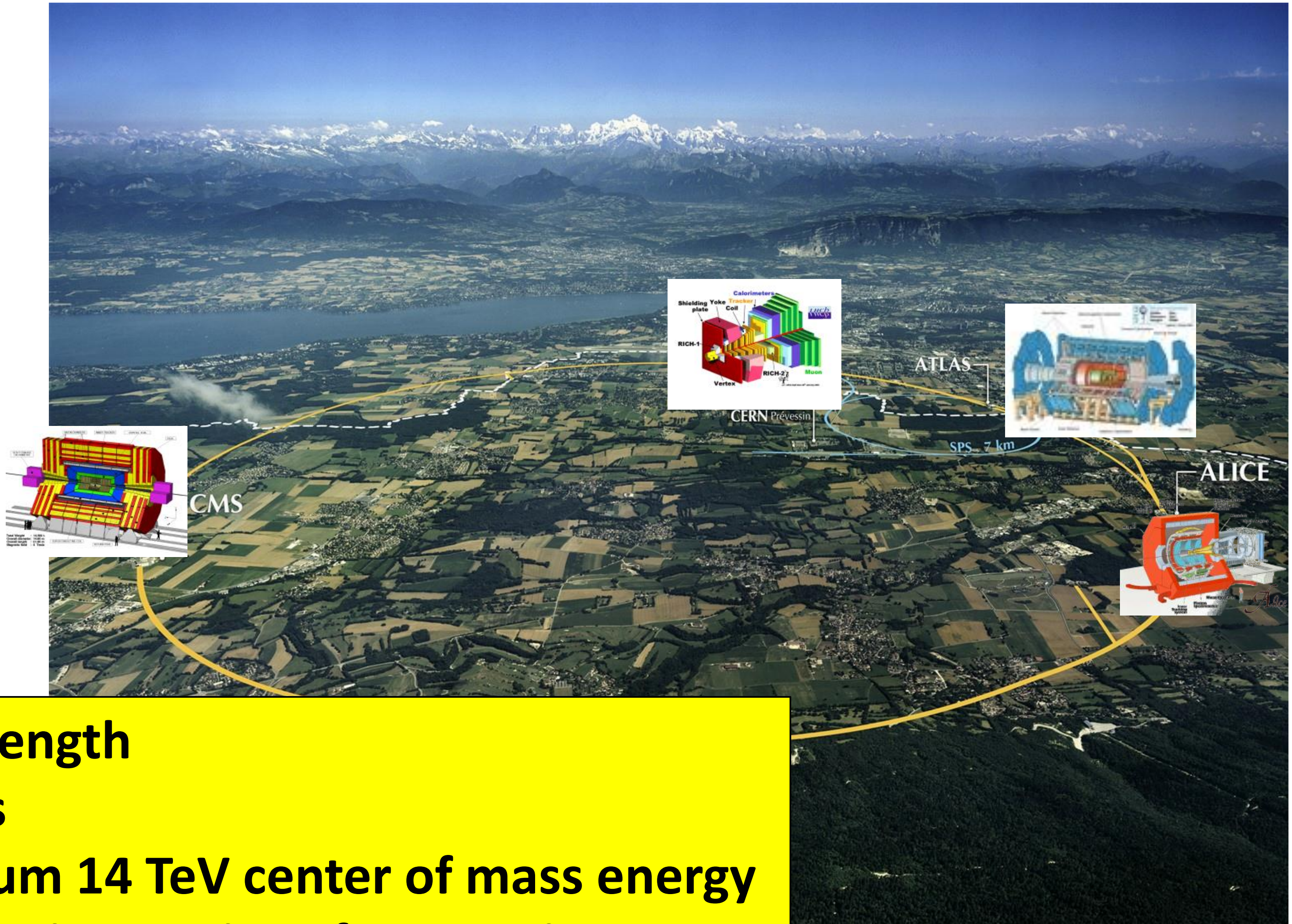


$$E_{cm} = E_1 + E_2$$

7 TeV proton beam colliding → 14 TeV

The Large Hadron Collider



27 Km length
Protons
Maximum 14 TeV center of mass energy
4 Interaction Regions for Experiments

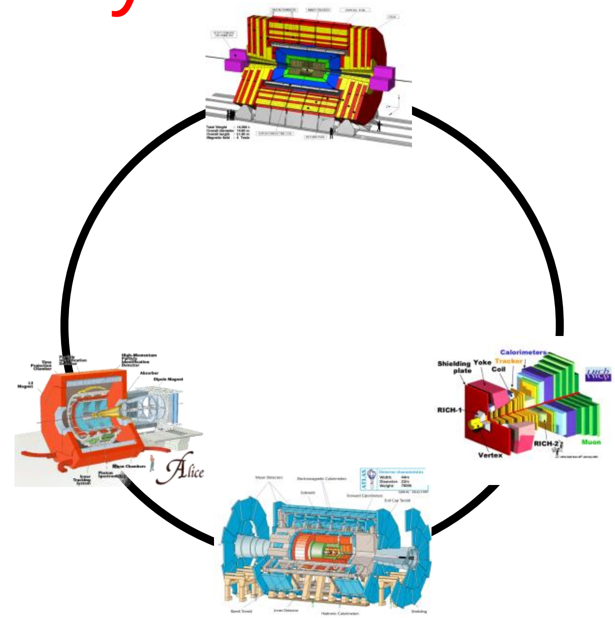
Circular colliders: Luminosity

Collider Luminosity $\mathcal{L}$

is the proportionality factor between

the cross section σ_{event}

and the number of events per second $\frac{d\mathcal{R}}{dt}$



$$\frac{d\mathcal{R}}{dt} = \mathcal{L} \times \sigma_{event} \qquad units : cm^{-2}s^{-1}$$

$$\int \mathcal{L}(t)dt$$

$fb^{-1} = 10^{39}\ cm^{-2}$

RUN1 1400 Higgs events with 30 fb^{-1}
RUN2 we are now around 120 fb^{-1}

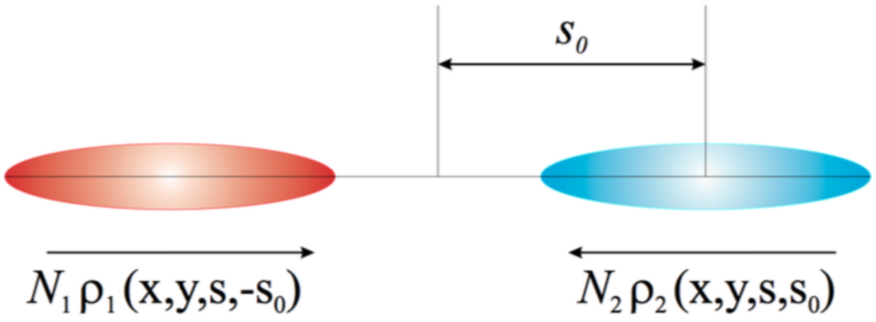
Luminosity is a machine parameter

→Independent of the physical reaction

→Reliable procedure to compute and measure

Luminosity calculation

The overlap integral of two bunches crossing each other head-on is proportional to the luminosity and it is given by:



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,+s_0)\,dx\,dy\,ds\,ds_0$$

$$s_0 = c \cdot t$$

Time variable

$$K = \sqrt{(\overrightarrow{v_1} - \overrightarrow{v_2})^2 - (\overrightarrow{v_1} \times \overrightarrow{v_2})^2/c^2}$$

Kinematic Factor

Luminosity formula

$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,+s_0)dxdydsds_0$$

Uncorrelated densities in all planes

→Factorize the distribution density as:

$$\rho_1(x,y,s,-s_0) = \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0)$$

For head-on collisions where

→ "Kinematic Factor" ***K = 2***

To have the luminosity per second

→Needs to multiple by revolution frequency ***f***

In the presence of many bunches $\boldsymbol{n_b}$

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot$$

$$\int\int\int\int_{-\infty}^{+\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0)dxdydsds_0$$

Closed solution for Gaussian distributions

Simplest case assumptions:

- Gaussian distributions
- No dispersion at the collision point
- Head-on collision

$$\rho_{i,z}(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\exp(\frac{z^2}{2\sigma_z^2})$$

$$\sigma_{x,y} = \sqrt{\epsilon \cdot \beta^*_{x,y}}$$

K = 2

$$\mathcal{L} = \frac{2N_1N_2fn_b}{(\sqrt{2\pi})^6\sigma_s^2\sigma_x^2\sigma_y^2)}\int\int\int\int e^{-\frac{x^2}{\sigma_x^2}}e^{-\frac{y^2}{\sigma_y^2}}e^{-\frac{s^2}{\sigma_s^2}}e^{-\frac{s_0^2}{\sigma_s^2}}dxdydsds_0$$

Equal Transverse beams “Round” beams

$$\sigma_{1x} = \sigma_{2x}$$
$$\sigma_{1y} = \sigma_{2y}$$

$$\mathcal{L} = \frac{N_1N_2fn_b}{4\pi\sigma_x\sigma_y}$$

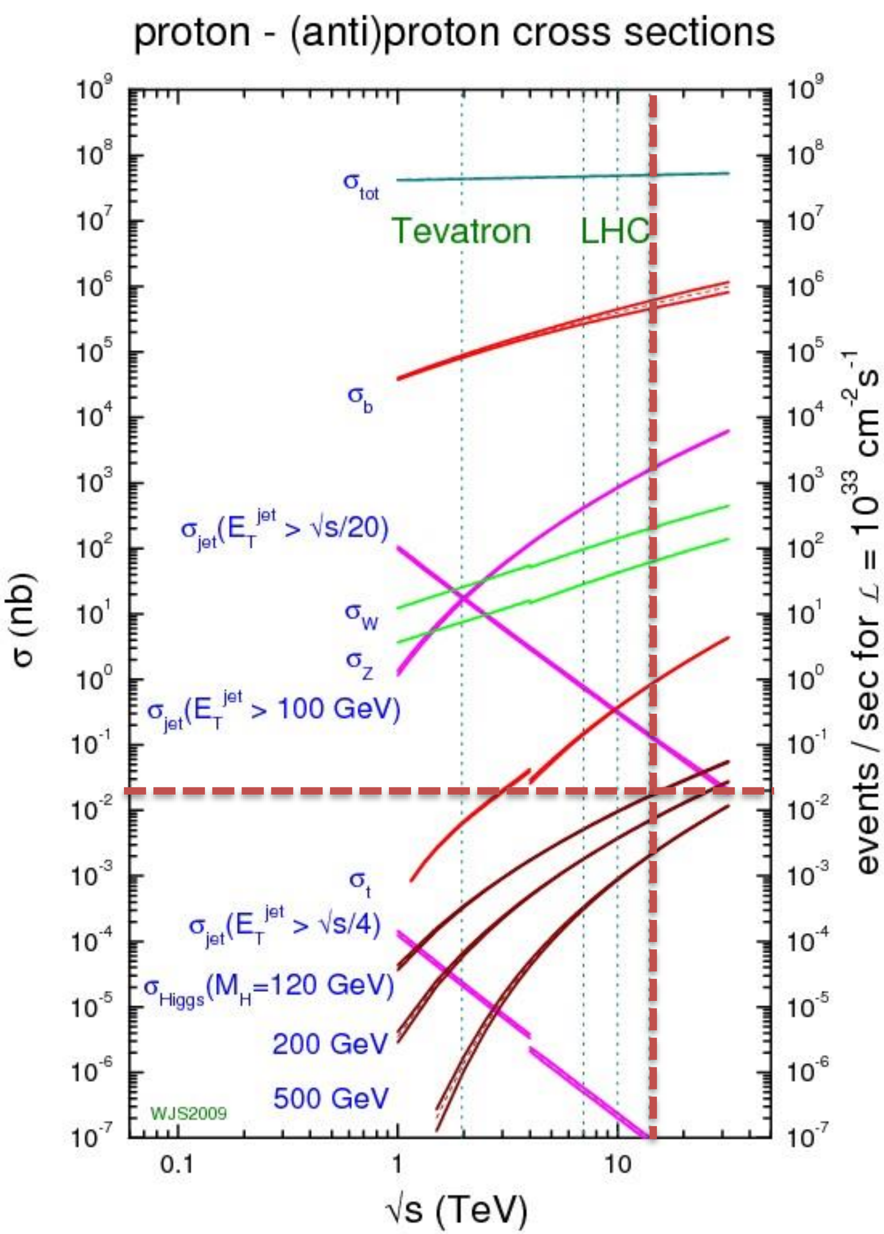
Un-Equal Transverse beams “Flat” beams or optics

$$\sigma_{1x} \neq \sigma_{2x}$$
$$\sigma_{1y} \neq \sigma_{2y}$$

$$\mathcal{L} = \frac{N_1N_2fn_b}{2\pi\sqrt{\sigma_{1x}^2+\sigma_{2x}^2}\sqrt{\sigma_{1y}^2+\sigma_{2y}^2}}$$

The LHC design parameters

$$\mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4\pi \epsilon_{x,y} \beta^*}$$



LHC Design

$N_1 = N_2 = 1.15\ 10^{11}$ protons per bunch

$\sigma_x = \sigma_y = 16.6\ \mu m$

$\beta^* = 55$ cm

→ $L=10^{34}\ cm^{-2}s^{-1}$

LHC Record

$N_1 = N_2 = 1.15\ 10^{11}$ protons per bunch

$\sigma_x = \sigma_y = 9.5\ \mu m$

$\beta^* = 30$ cm

→ $L=2 \times 10^{34}\ cm^{-2}s^{-1}$

High Luminosity Upgrade of LHC

$N_1 = N_2 = 2.2\ 10^{11}$ protons per bunch

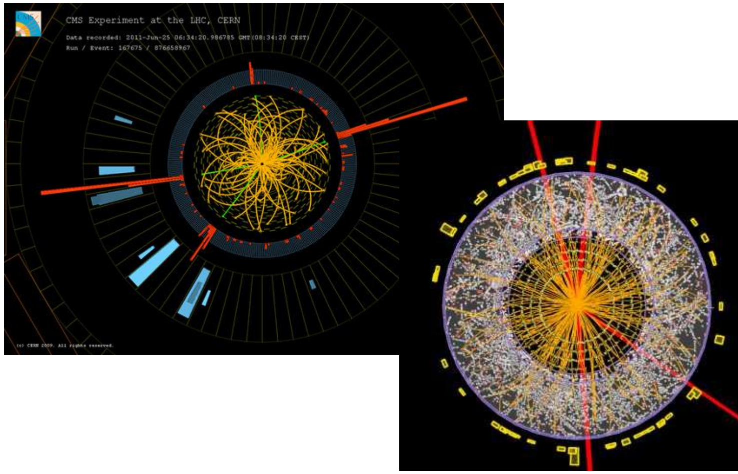
$\sigma_x = \sigma_y = 7.0\ \mu m$

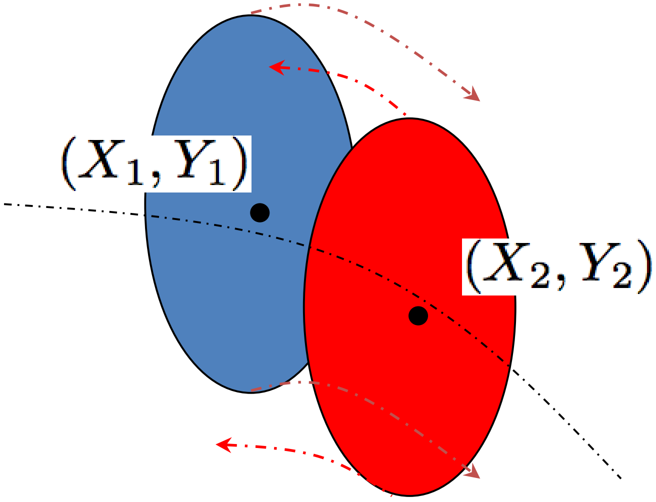
$\beta^* = 64 \rightarrow 15$ cm

→ $L=(10\text{-}20) \times 10^{34}\ cm^{-2}s^{-1}$

Different types of collisions

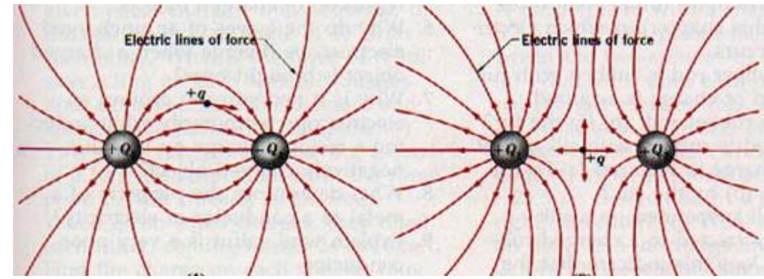
- They occur when two beams get closer and collide
- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)
- Unfortunately: usually both go together…
- 0.001% (or less) of particles collide
- 99.999% (or more) of particles are distorted



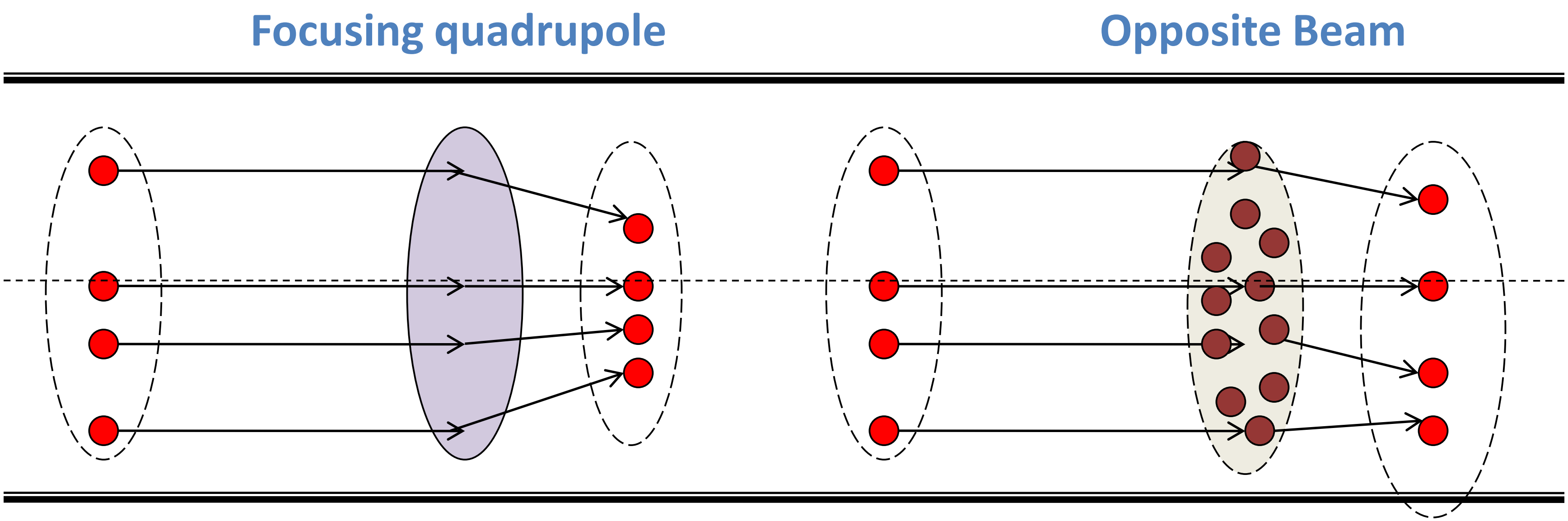


Proton Beams → Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges



Force on itself (space charge) and opposing beam (beam-beam effects)



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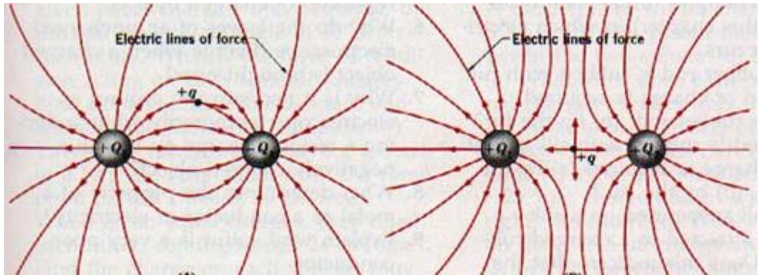
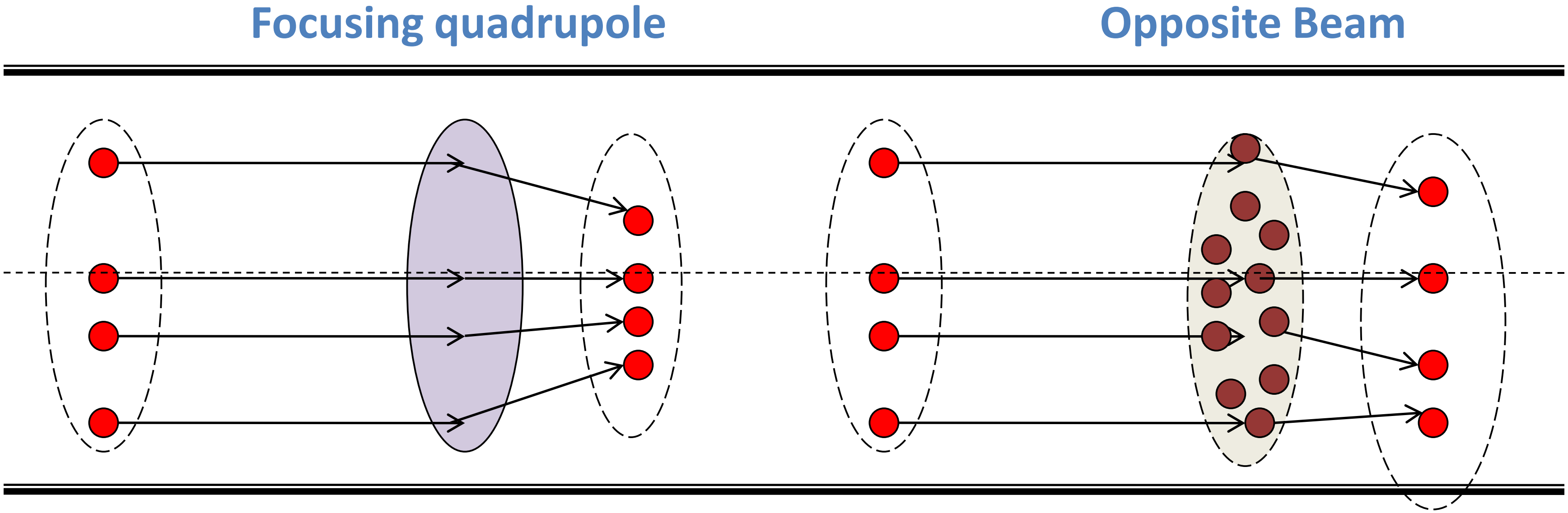


Force on itself: space charge effects goes with $1/\gamma^2$ factor for high energy colliders this contribution is negligible
(i.e. force scales LHC $1/\gamma^2 = 1.8\ 10^{-8}$)



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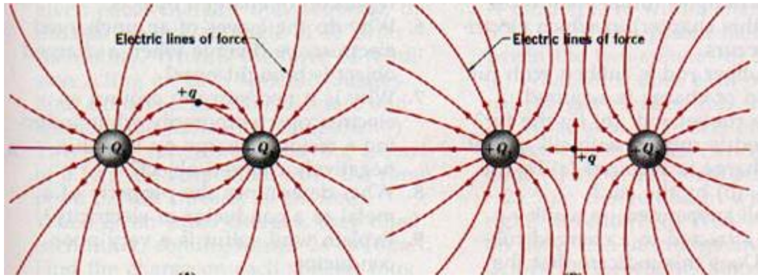
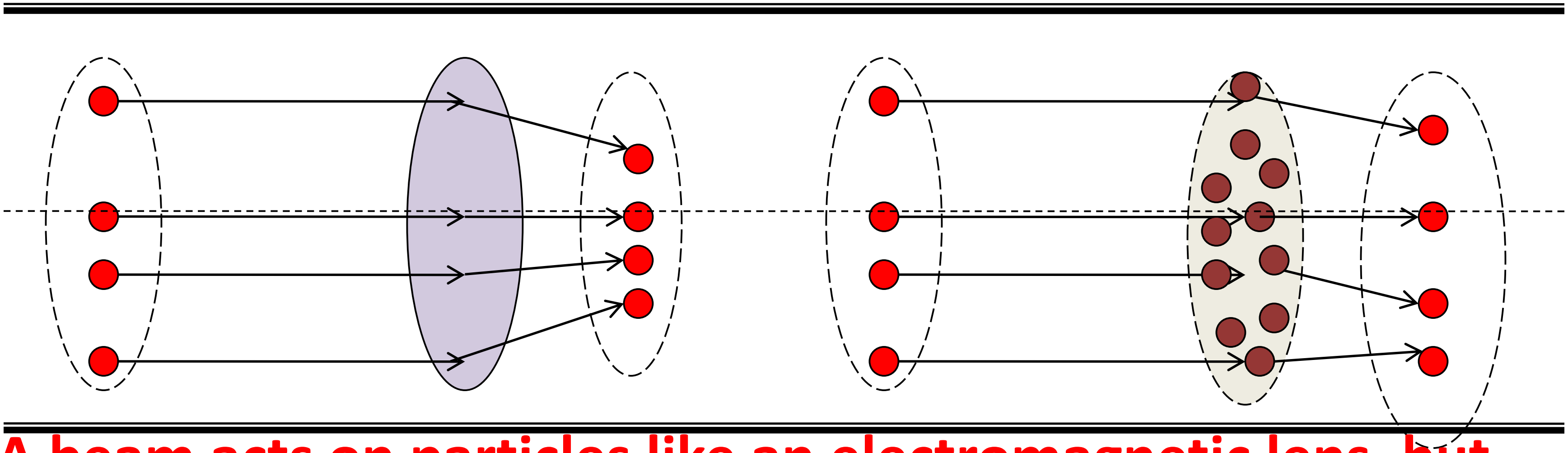


Electromagnetic force from opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted



A beam acts on particles like an electromagnetic lens, but…

Beam-beam Force derivation

General approach in electromagnetic problems **Reference[5]** already applied to beam-beam interactions in **Reference[1,3, 4]**

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int \frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}$$

$$\overrightarrow{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force…

Beam-Beam Force for Round Gaussian distributions

Gaussian distribution for charges
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

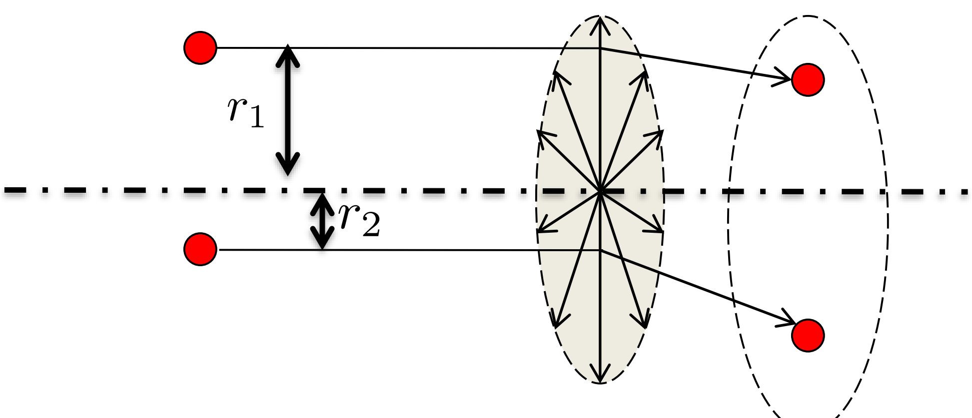
Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r,s,t)\ dt$$

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2}[1 - e^{-\frac{r^2}{2\sigma^2}}]$$

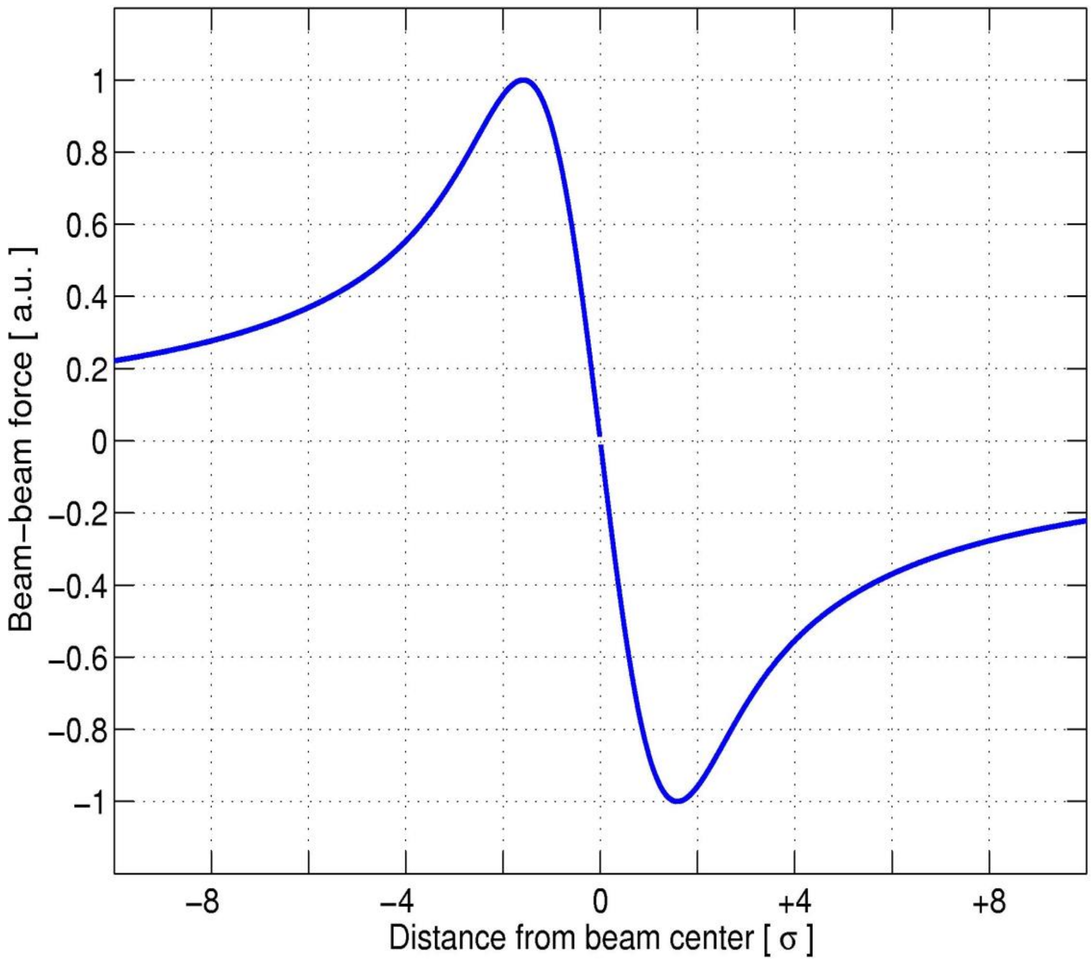
Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

Only radial component in relativistic case



How does this force looks like?

Beam-beam Force

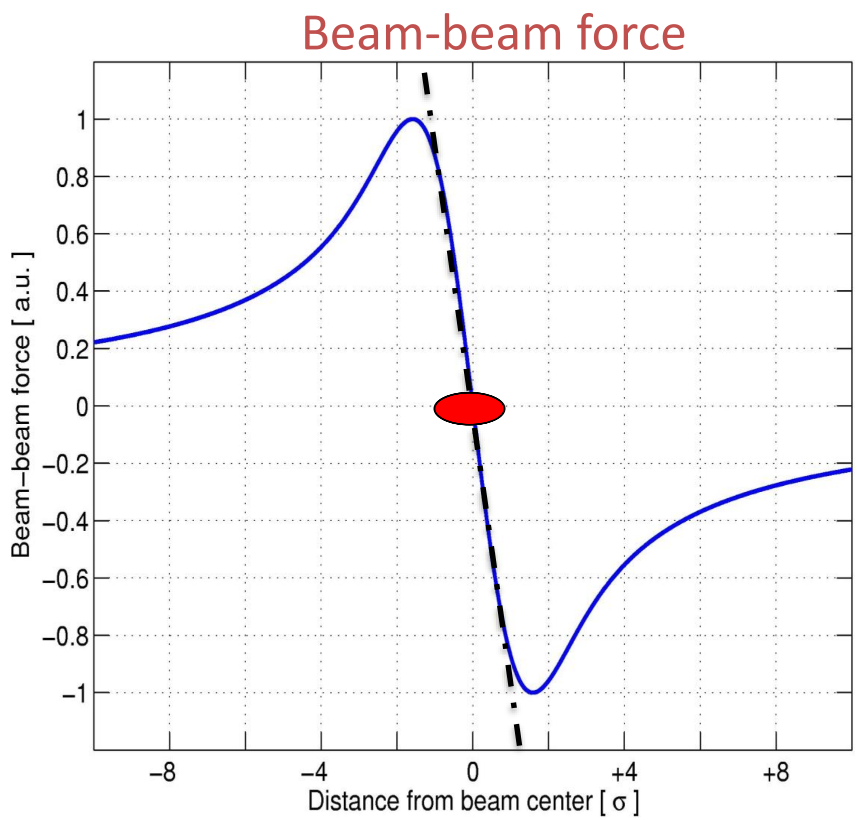


$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

Can we quantify the beam-beam strenght?

For small amplitudes: linear force (quadrupole) $F \propto -\xi \cdot r$

The slope of the force gives you the beam-beam parameter ξ



ξ Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force

Beam-Beam Parameter

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r,s,t)\ dt$$

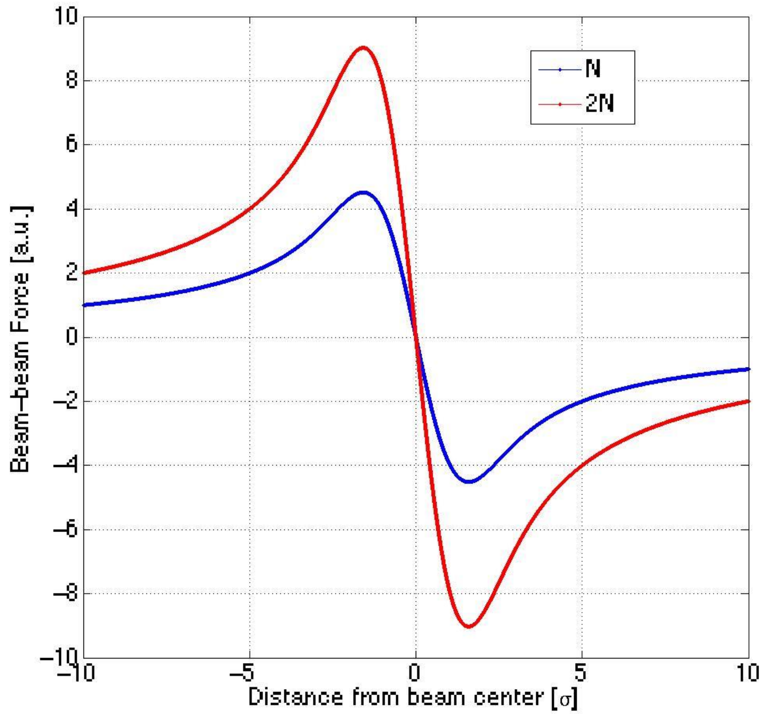
For small amplitudes: linear force

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \ldots\right)\right]$$

$$\Delta r'|_{r\to 0} = \frac{N r_0 r}{\gamma\sigma^2} = +f \cdot r$$

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$



For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Beam-Beam Parameter

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For non-round beams:

Parameters	LHC TDR	LHC 2012
Intensity $N_{p,e}$/bunch	$1.15\ 10^{11}$	$1.8\ 10^{11}$
Energy GeV	7000	4000
Beam size H	16.6 μm	16.6 μm
Beam size V	16.6 μm	16.6 μm
$\beta_{x,y}{}^*$ m	0.55	0.60
Crossing angle μrad	285	290
ξ_{bb}	**0.0037**	**0.007**

HL-LHC
$2.2\ 10^{11}$
7000
14 μm
14 μm
0.64-0.15
0
0.01

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Strongest non-linearity in a collider YOU CANNOT AVOID!

Strong non-linear electromagnetic distortion

→ impact on beam quality (particle losses and emittance blow-up)

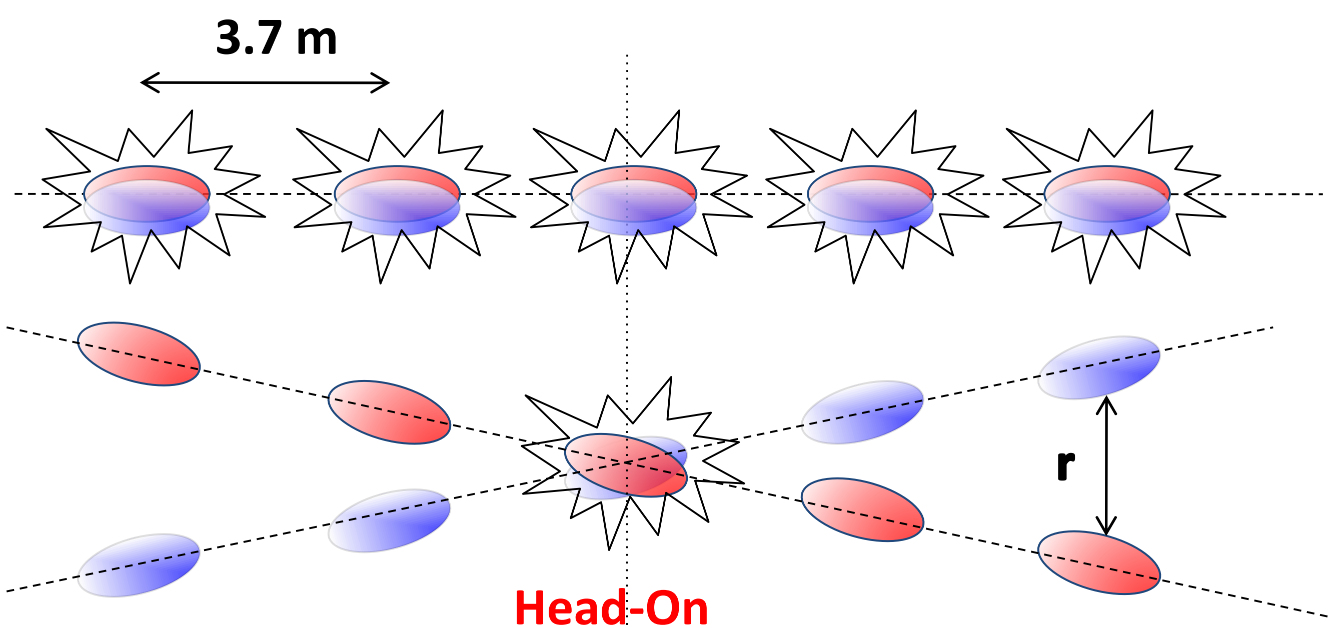
→ luminosity reduction

Crossing angle operation

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

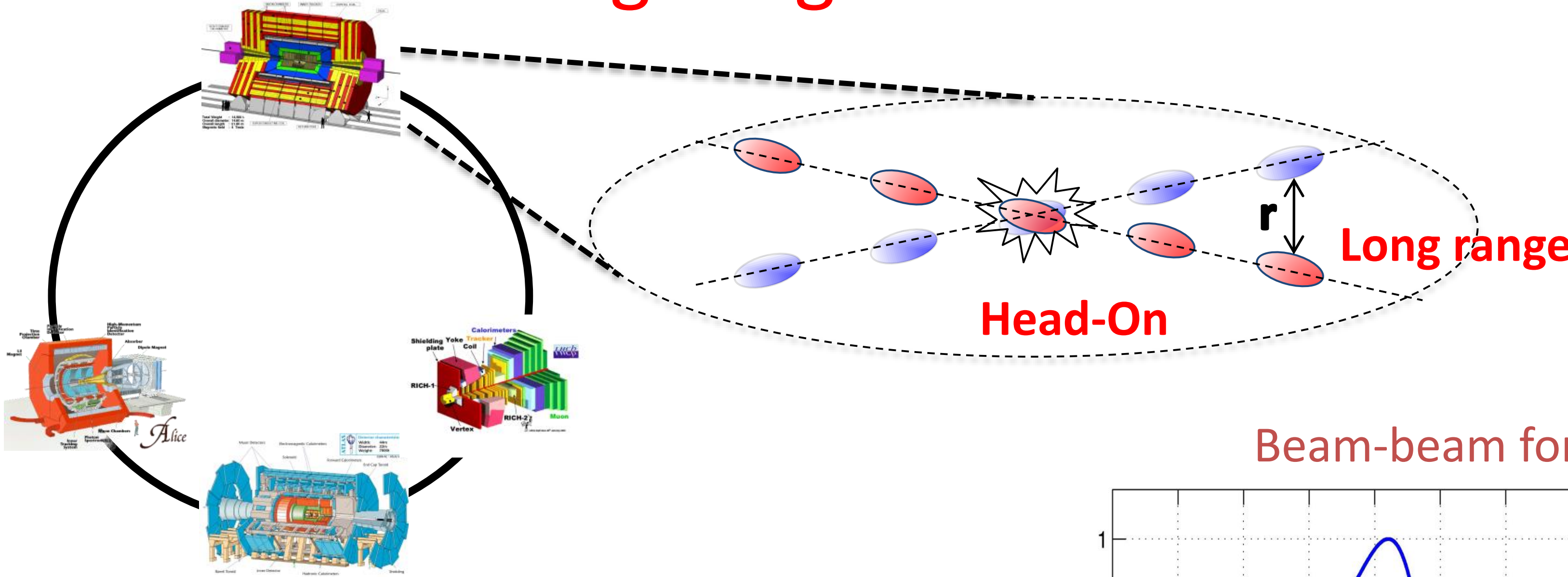
Num. of maximum bunches $n_b = 2808$

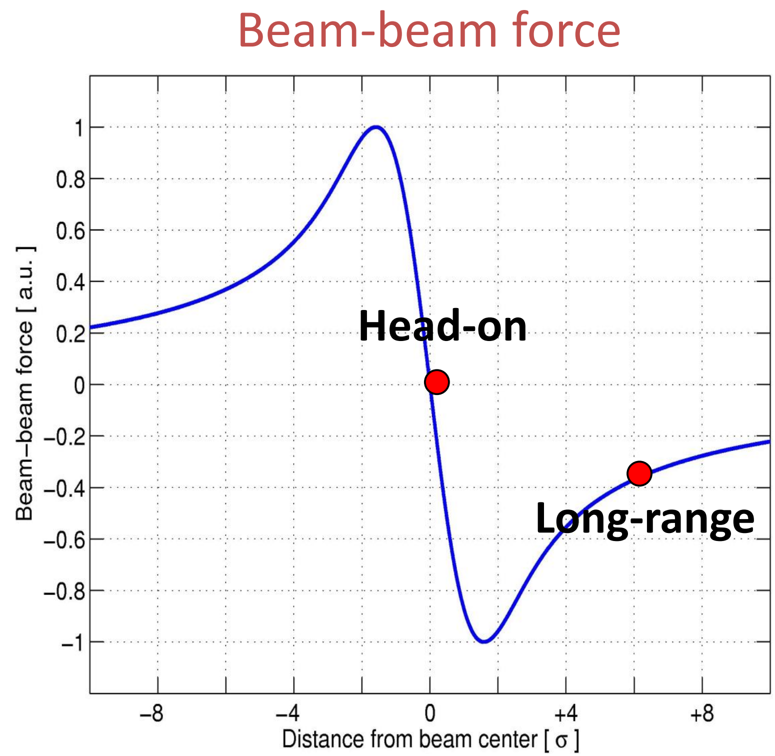
Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments



A finite crossing angle needed to avoid multiple collision points

Head-on and Long-range beam-beam interactions





Two type of interactions:

Other beam passing in the center force

→ HEAD-ON beam-beam interaction

→ LHC has 4 experiments:

- → ATLAS and CMS colliding head-on
- → ALICE and LHCB with transverse offset

Other beam passing at an offset r

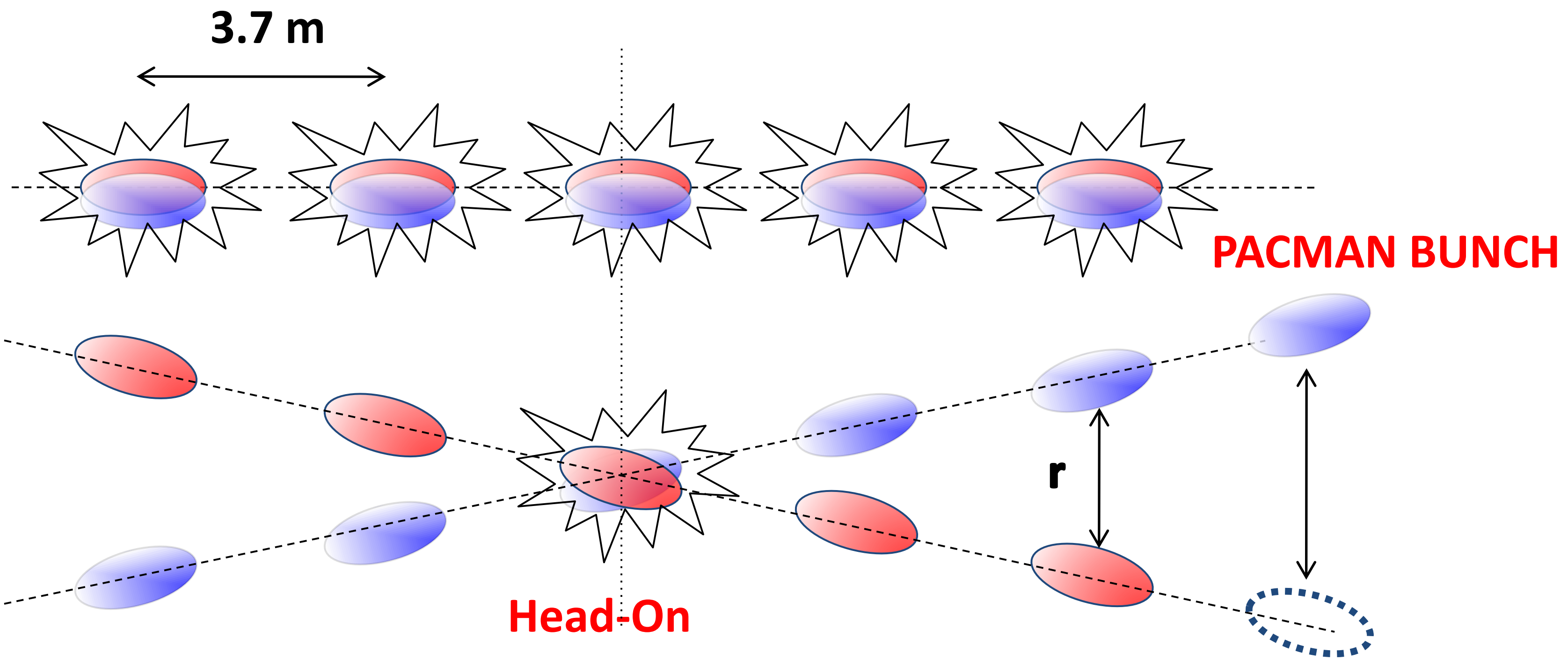
→ LONG-RANGE beam-beam interaction

→ **LHC has up to 120 LR interactions**

Multiple bunch Complications

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

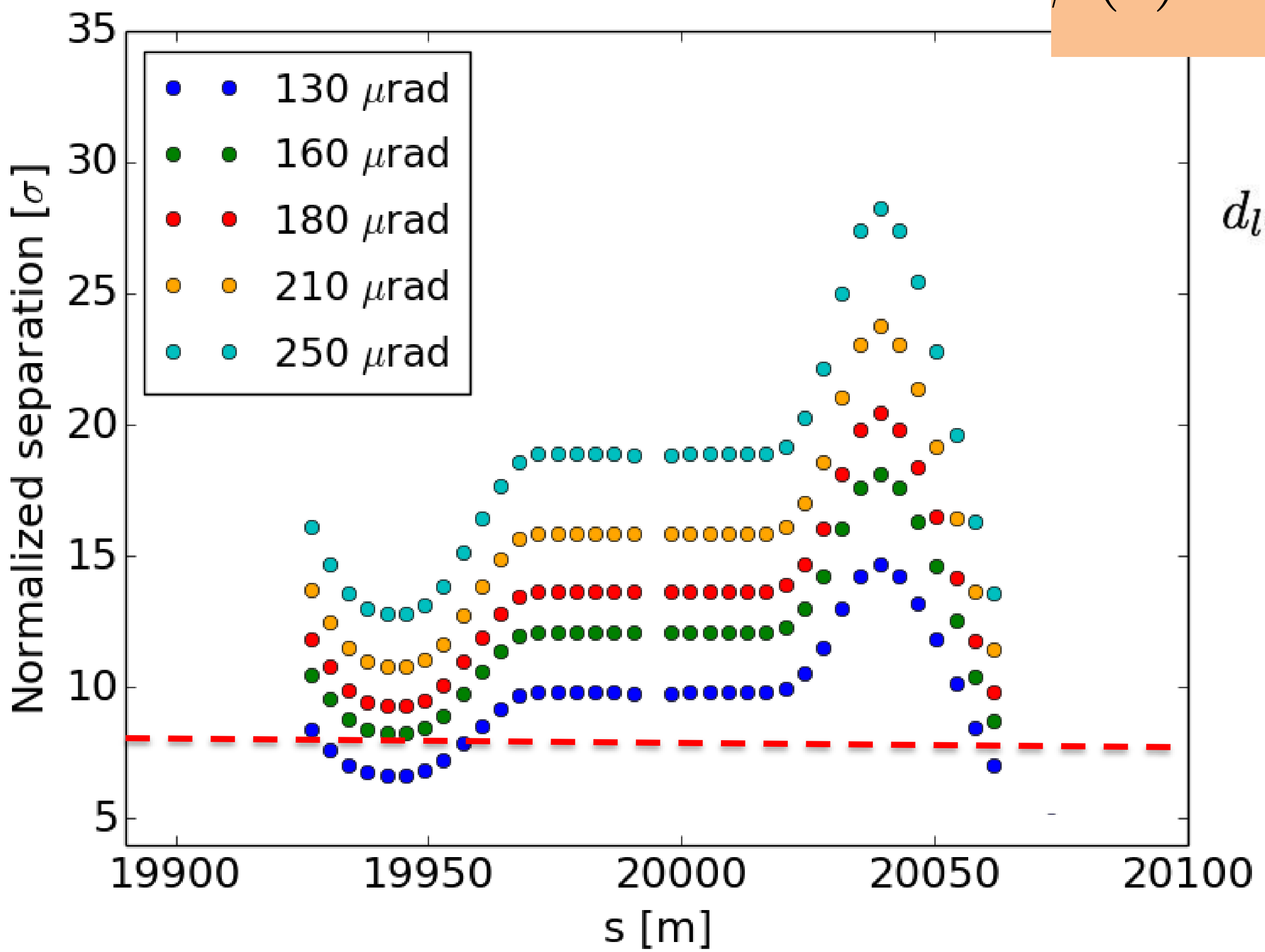
Num. of bunches : $n_b = 2808$



Due to the train structure of the beams → different bunches will experience a different number of interactions!

Long-Range separations

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$



$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments

Luminosity Geometric reduction factor

Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot \mathcal{S}$$

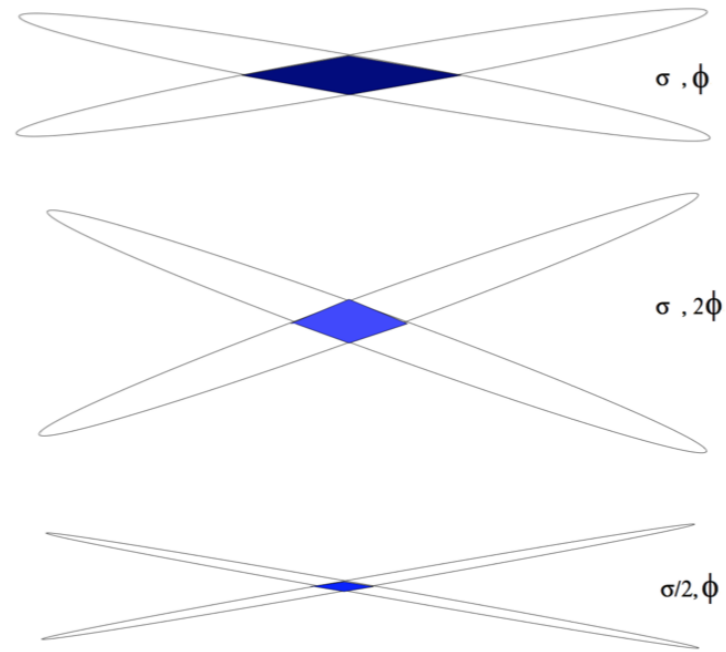
S is the geometric reduction factor

$$\mathcal{S} = \frac{1}{\sqrt{1 + (\frac{\sigma_x}{\sigma_s} \tan \frac{\phi}{2})^2}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2})^2}}$$

$$\sigma_s >> \sigma_{x,y}$$

Always valid for LHC and HL-LHC
σ_x = 17-7 μm, σ_s = 7.5 cm

$$S \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \frac{\phi}{2})^2}}$$



LHC design: ϕ = 285 μrad, σ_x = 17 μm, σ_s = 7.5 cm, S=0.84
LHC 2018: ϕ = 320 μrad, σ_x = 9.3 μm, σ_s = 7.5 cm, S=0.61

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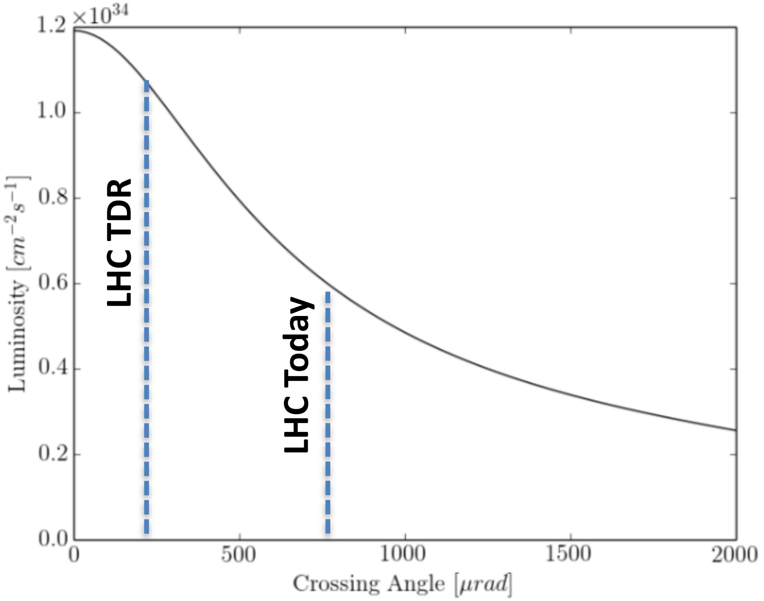

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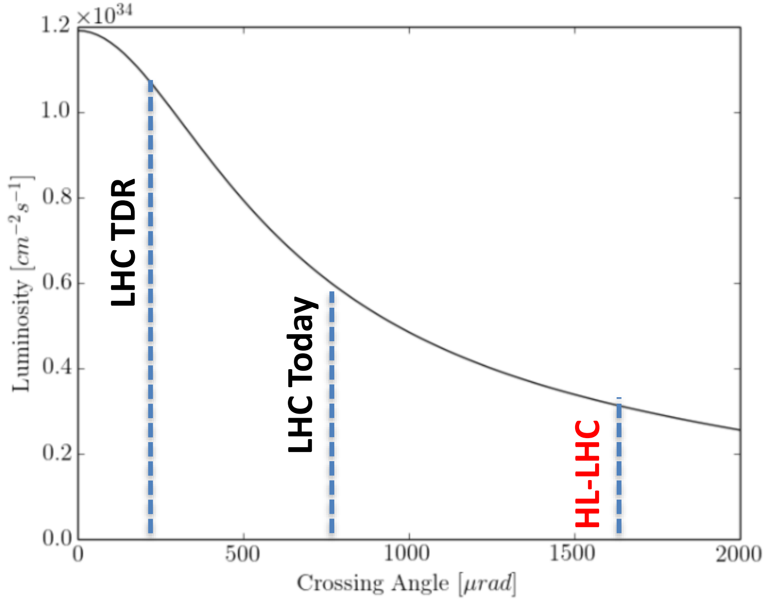

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LHC operates at finite crossing angle

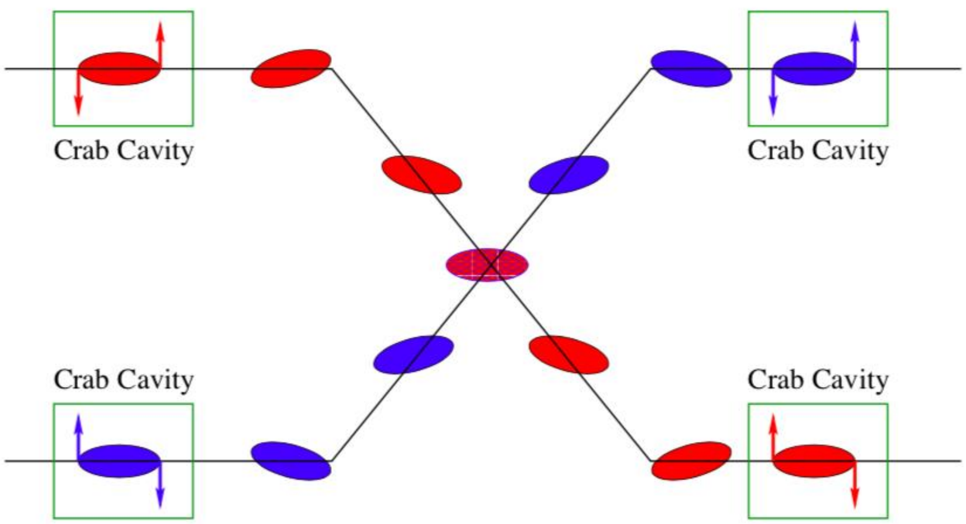
HL-LHC will have bunches of 2.2 10^{11} protons per bunch

ϕ = 590 µrad, σ_x = 9.3 µm, σ_s = 7.5 cm, S=0.26 → 73% of luminosity lost!

Courtesy of R. Calaga

$$\sigma_s >> \sigma_{x,y}$$

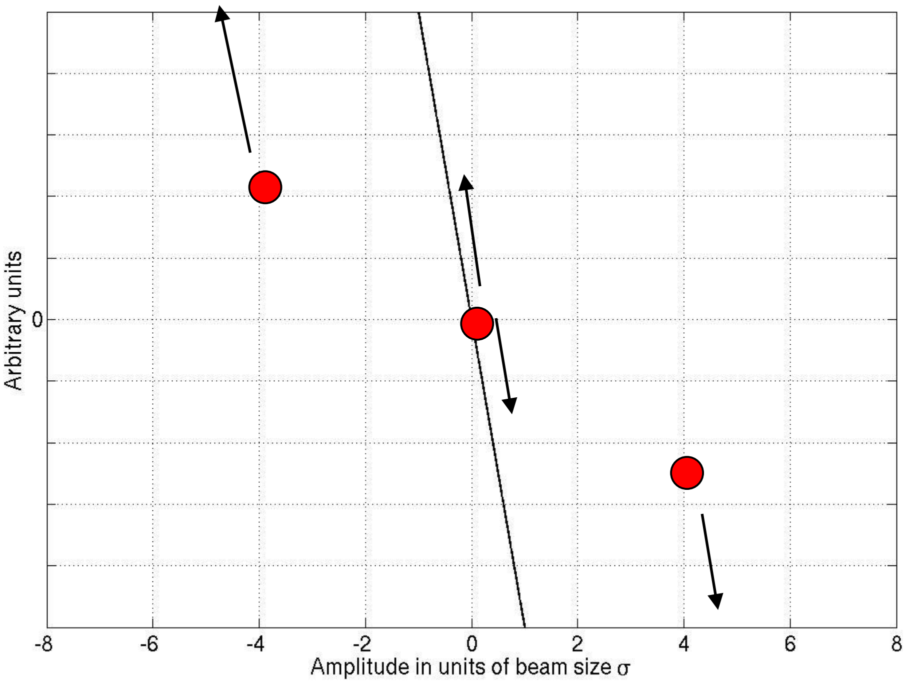
$$S \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$



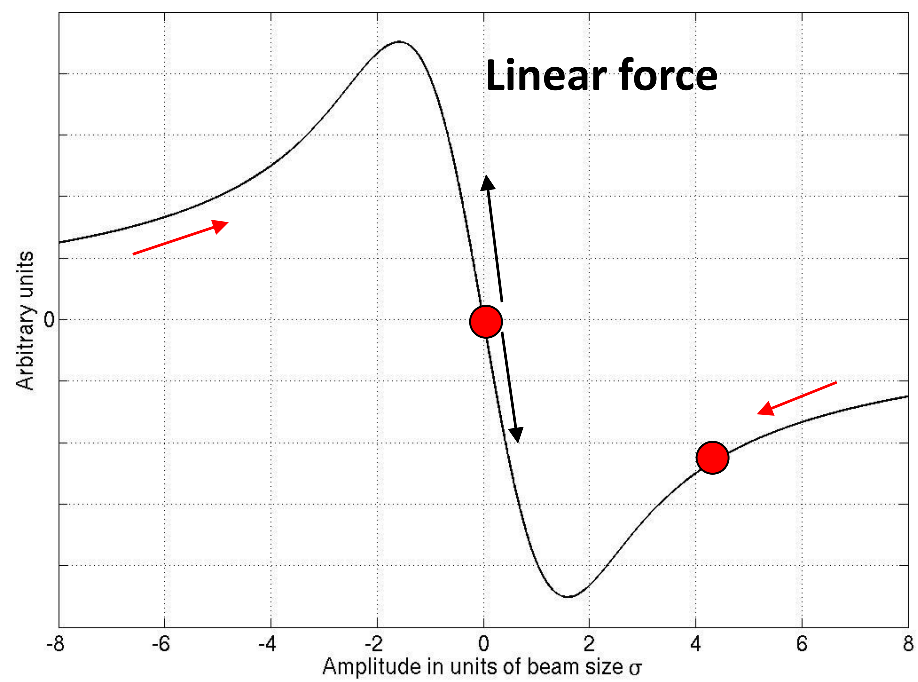
Crab Cavities used to tilt the bunches longitudinally and compensate for the crossing angle at the collision point!
Testing of crab cavities on-going in SPS!

Beam-Beam Force: single particle head-on collision

Lattice defocusing quadrupole



Beam-beam force



$$F = -k \cdot r$$

$$\mathrm{F} \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

Linear Tune shift due to head-on collision

For small amplitude particles beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

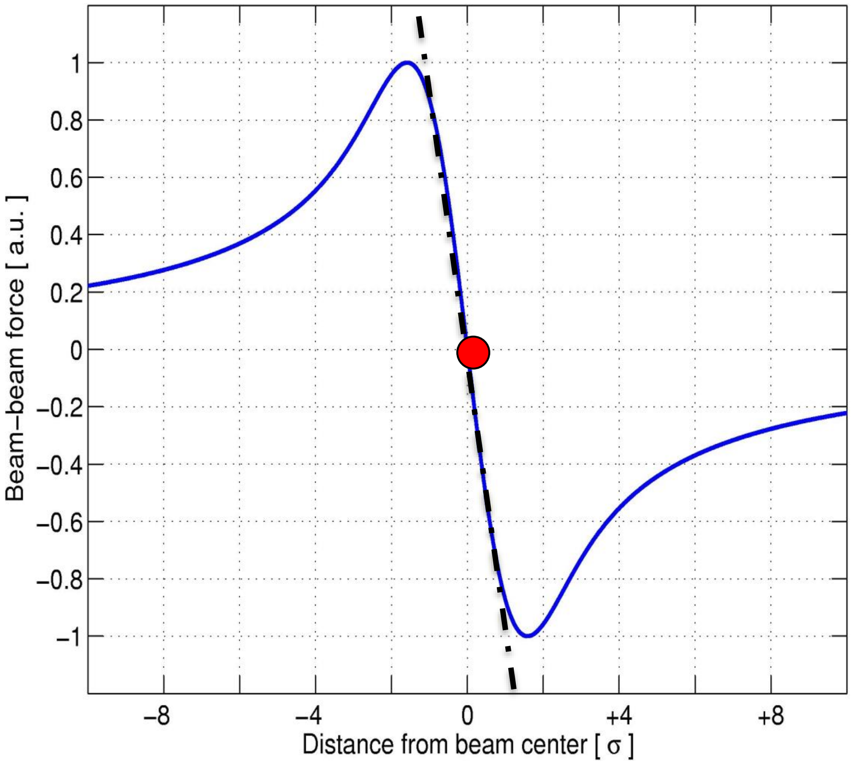
Focal length is given by the beam-beam parameter:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

Beam-beam force



Equivalent to tune shift

Perturbed one turn matrix

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β^*:

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Tune shift and dynamic beta

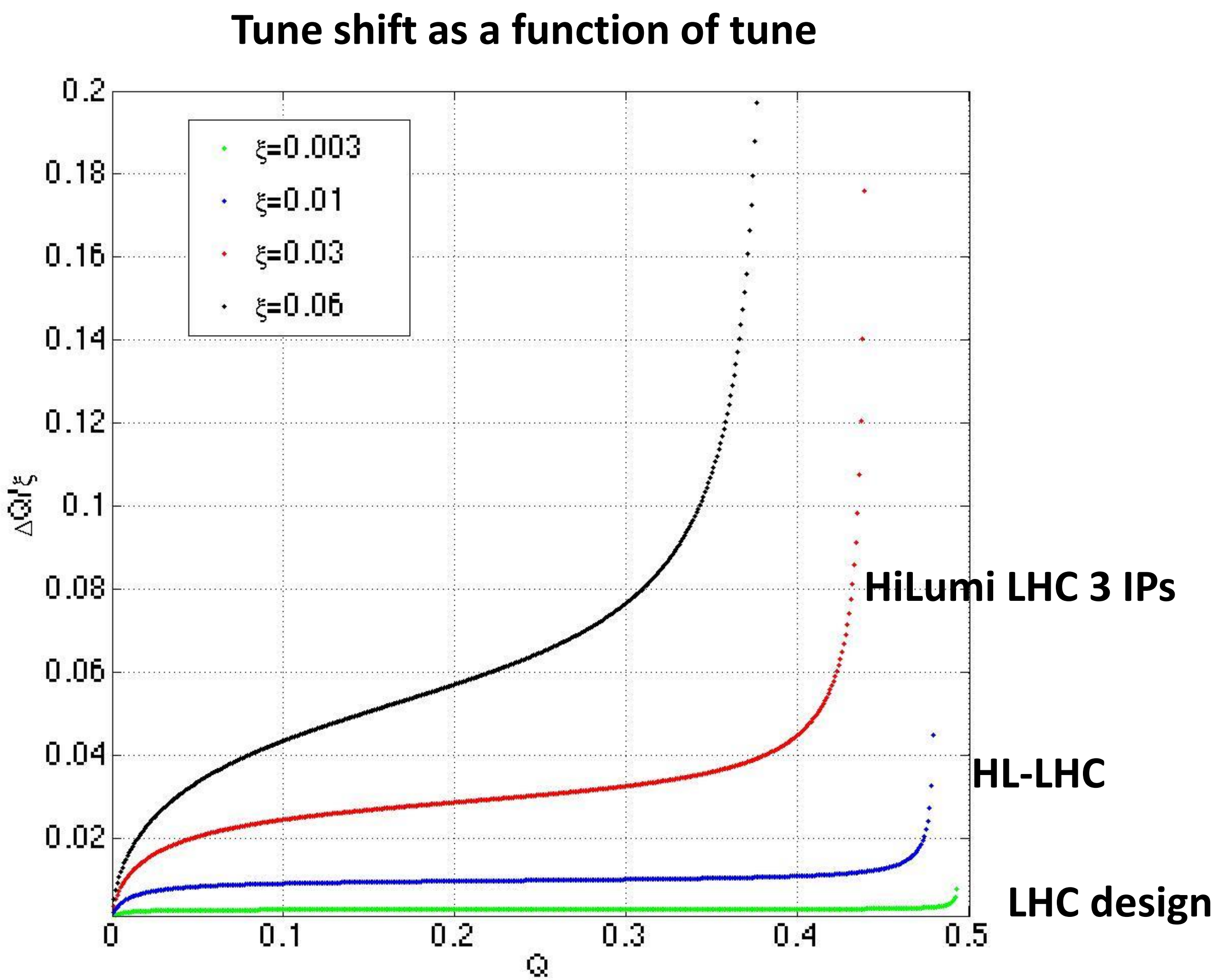
Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β^*:

Tune is changed

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} sin(2\pi Q)$$

...how does the tune changes?

Tune shift due to beam-beam interactions

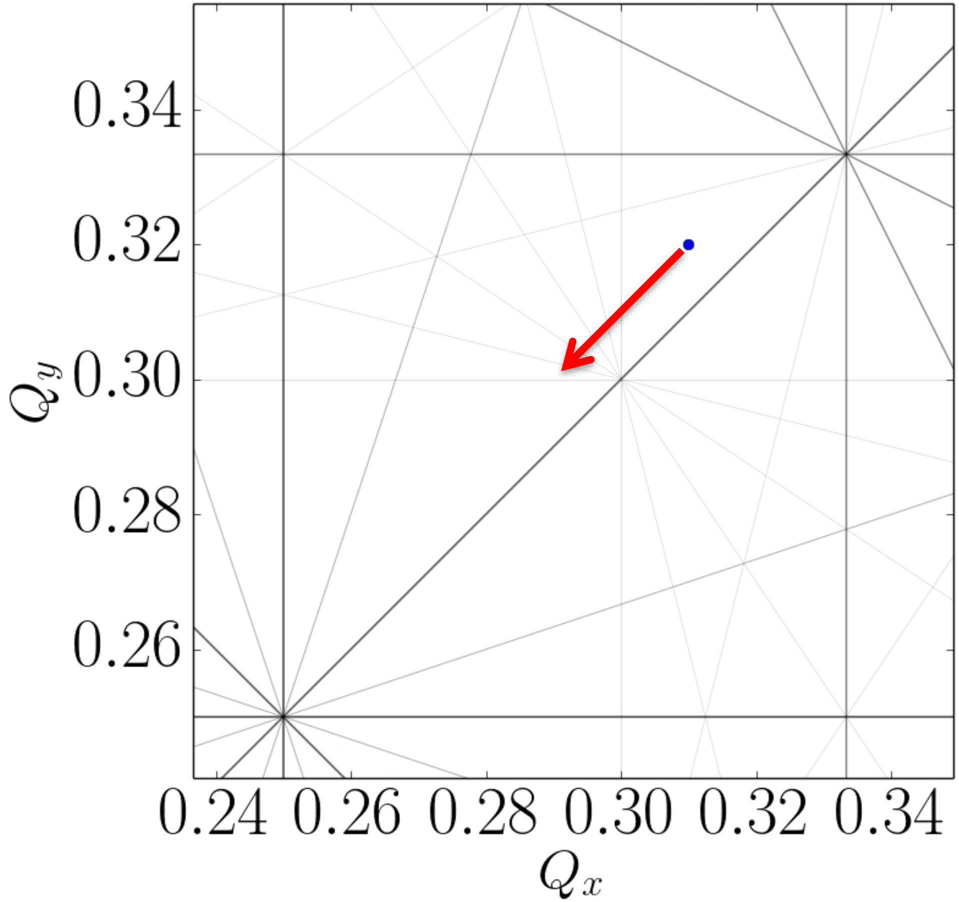


Larger ξ → Strongest variation with Q

Effects of multiple Interaction Points does not add linearly (phase advance between IP..)

Linear head-on Tune shift

Tune shift in 2 dimensional case equally charged beams and tunes far from integer and half

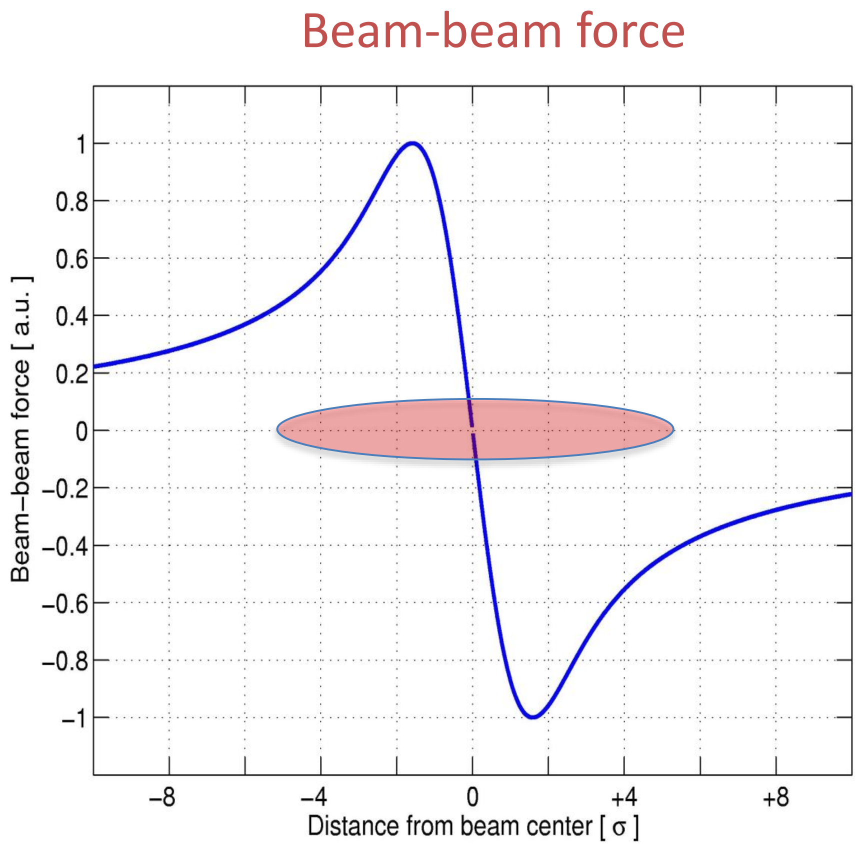


$$\xi_{bb} = 0.02$$

Zero amplitude particle will fill an extra defocusing term

$$\Delta Q \approx \xi_{bb}$$

A beam is a collection of particles

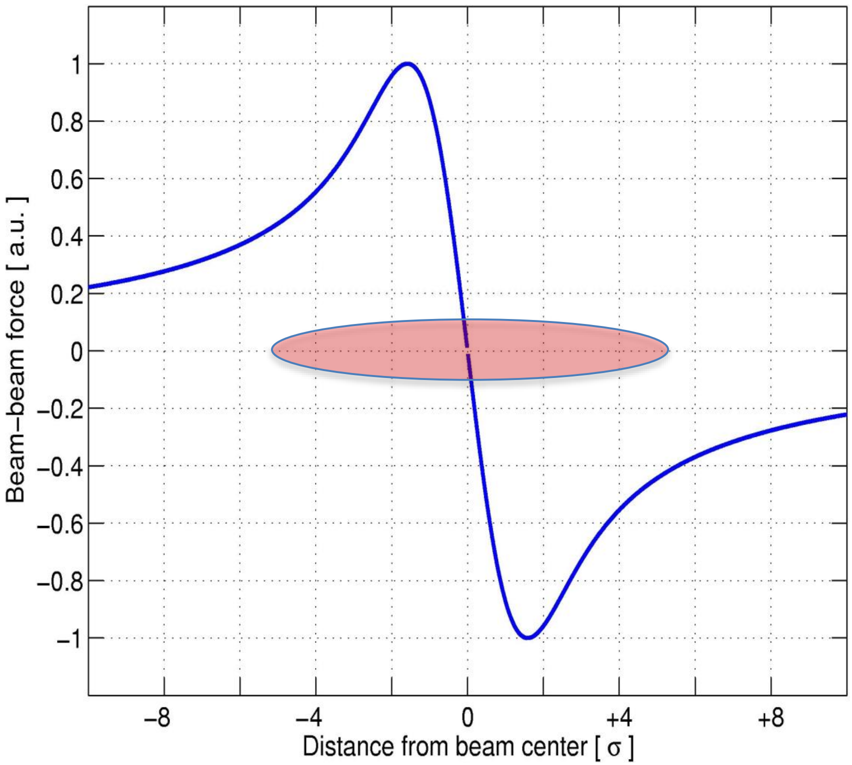


Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)

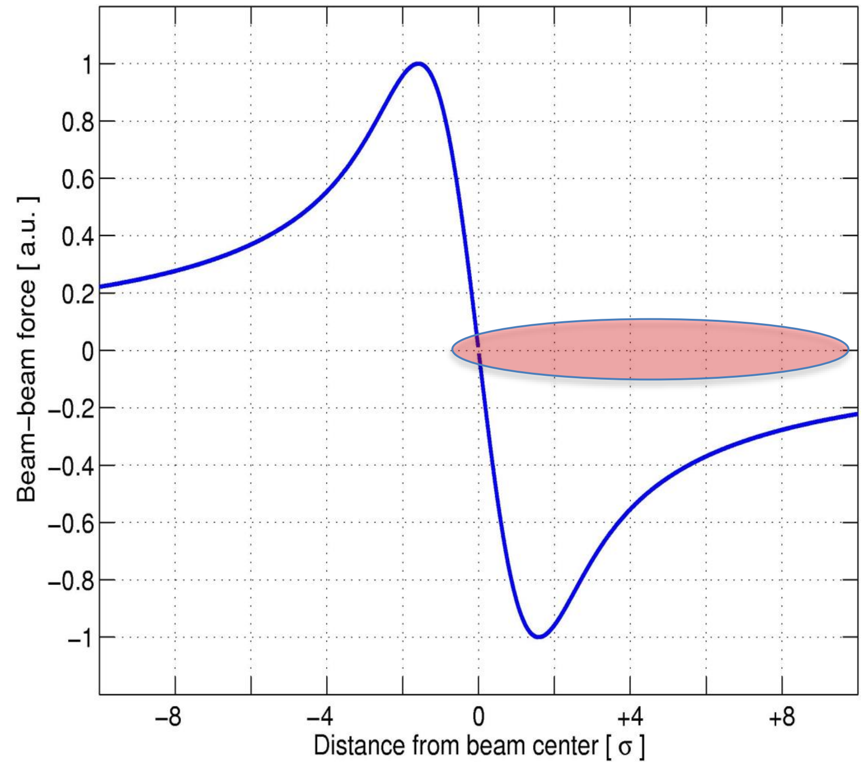
A beam will experience all the force range

Beam-beam force



Second beam passing in the center
HEAD-ON beam-beam interaction

Beam-beam force



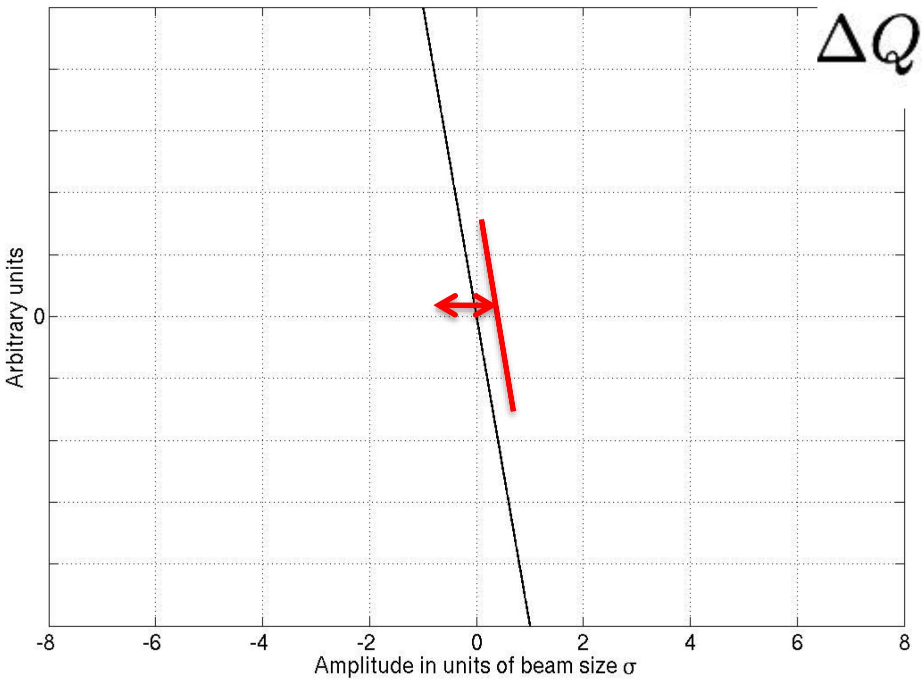
Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force

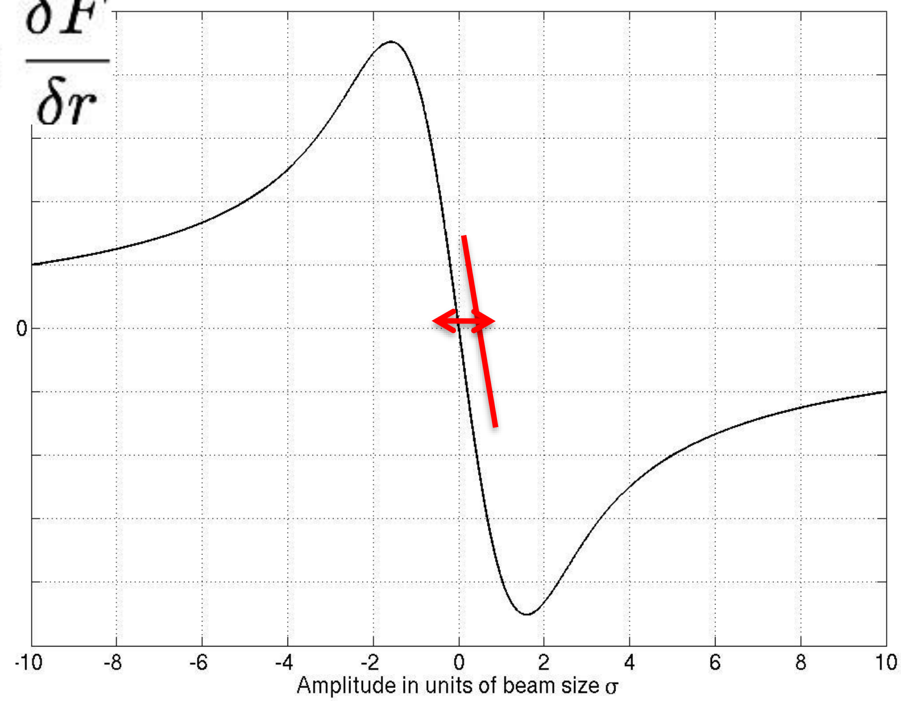
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude

$$\Delta Q \propto \frac{\delta F}{\delta r}$$



$$\Delta Q_{quad} = const$$



$$\Delta Q_{bb} \approx const$$

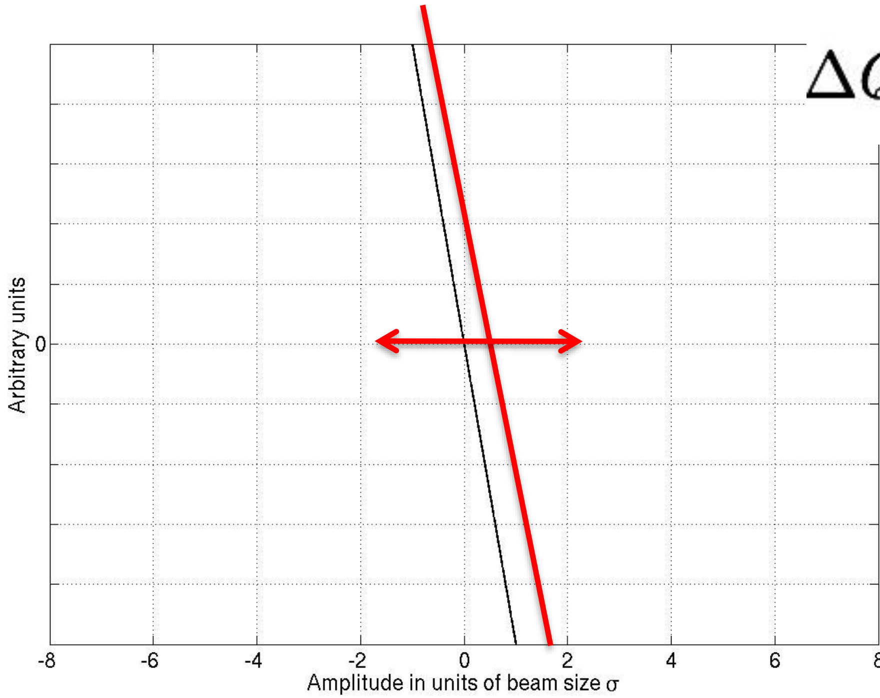
For small amplitude test particle linear tune shift

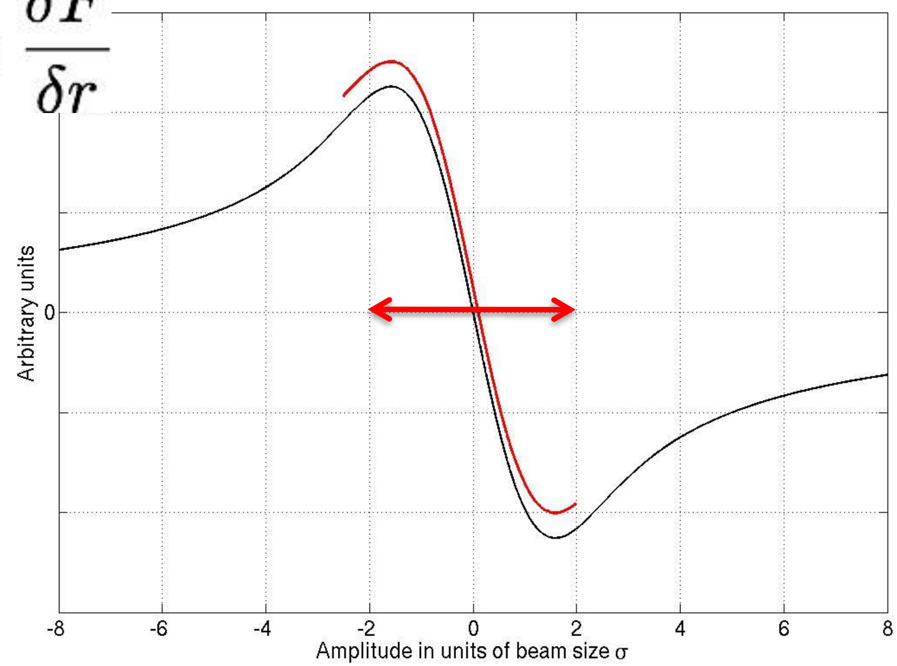
$$\lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread

$$\Delta Q \propto \frac{\delta F}{\delta r}$$





$$\Delta Q_{quad} = const$$

$$\Delta Q_{bb} \neq const$$

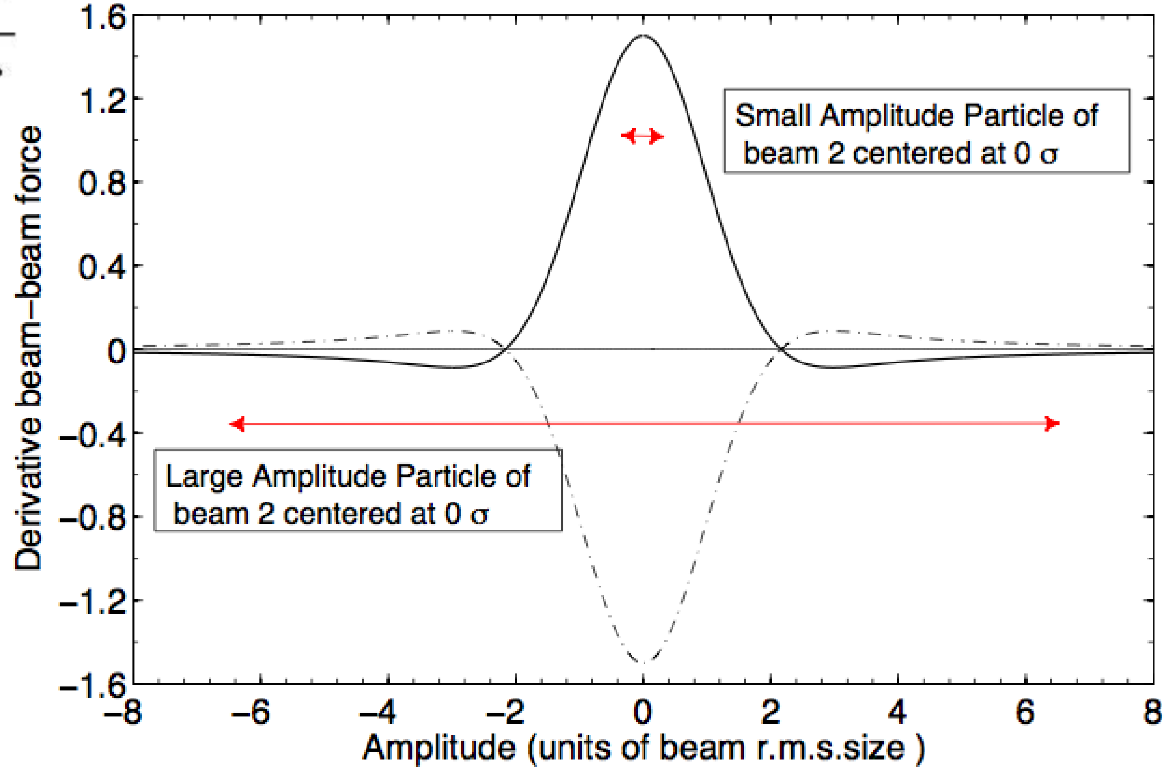
$$\Delta Q(x) = \frac{N r_0 \beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0(\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

Head-on detuning with amplitude

1-D plot of detuning with amplitude for opposite and equally charged beams

$$\Delta Q \propto \frac{\delta F}{\delta r}$$

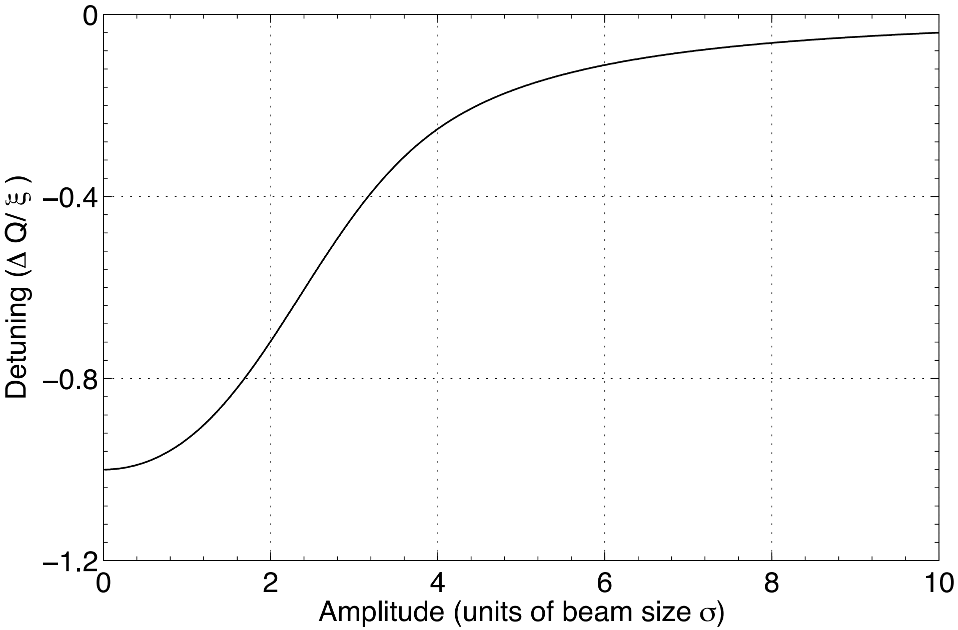


Maximum tune shift for small amplitude particles
Zero tune shift for very large amplitude particles

And in the other plane? THE SAME DERIVATION

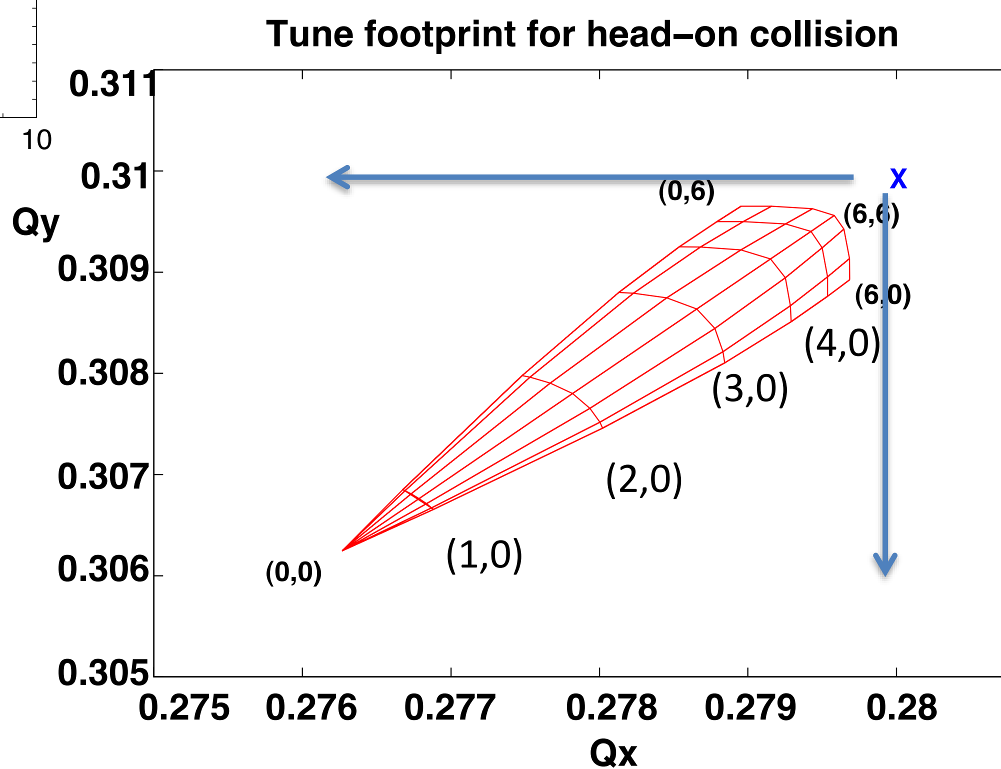
Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude



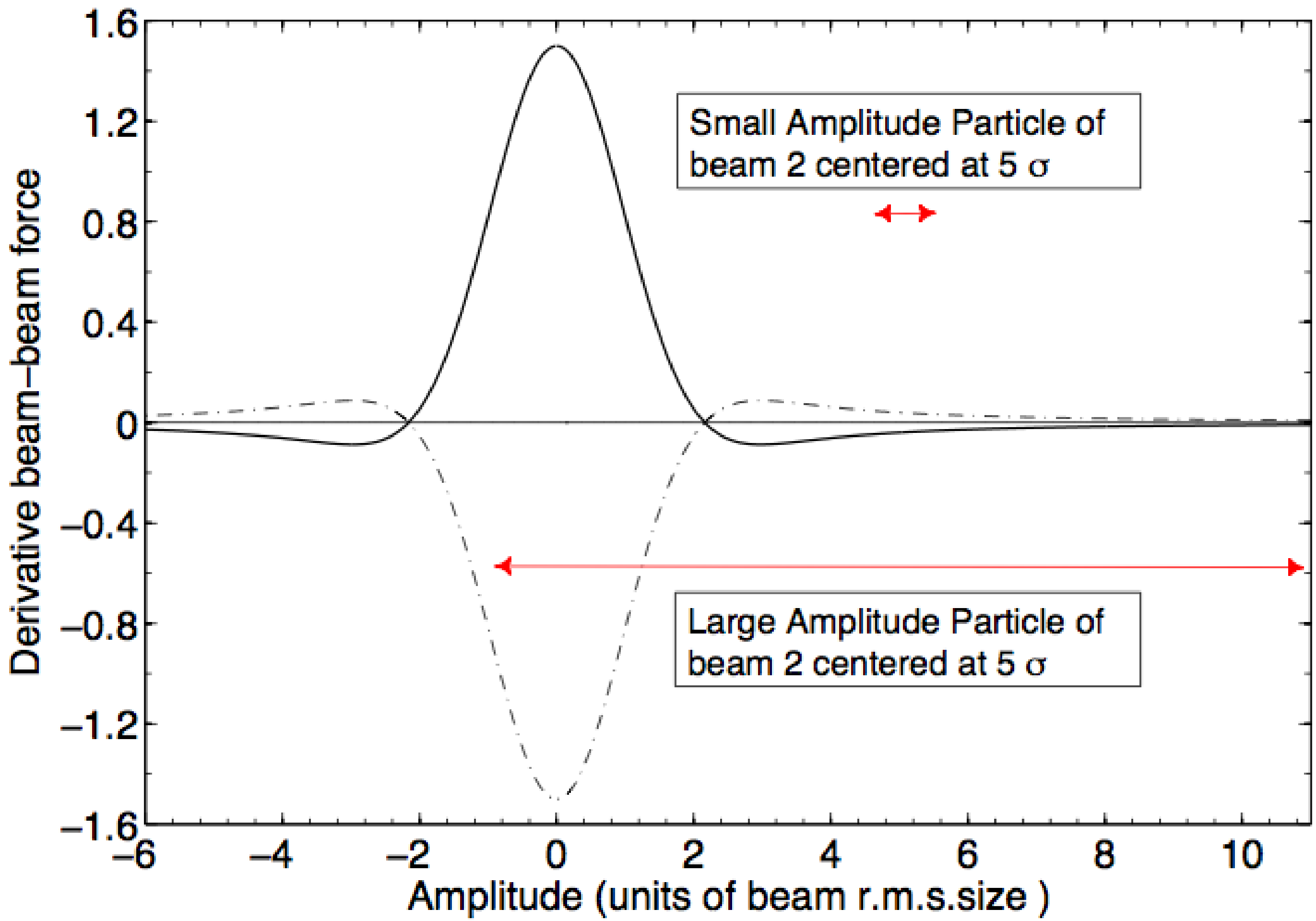
FOOTPRINT

2-D mapping of the detuning with amplitude of particles



Long Range detuning with amplitude

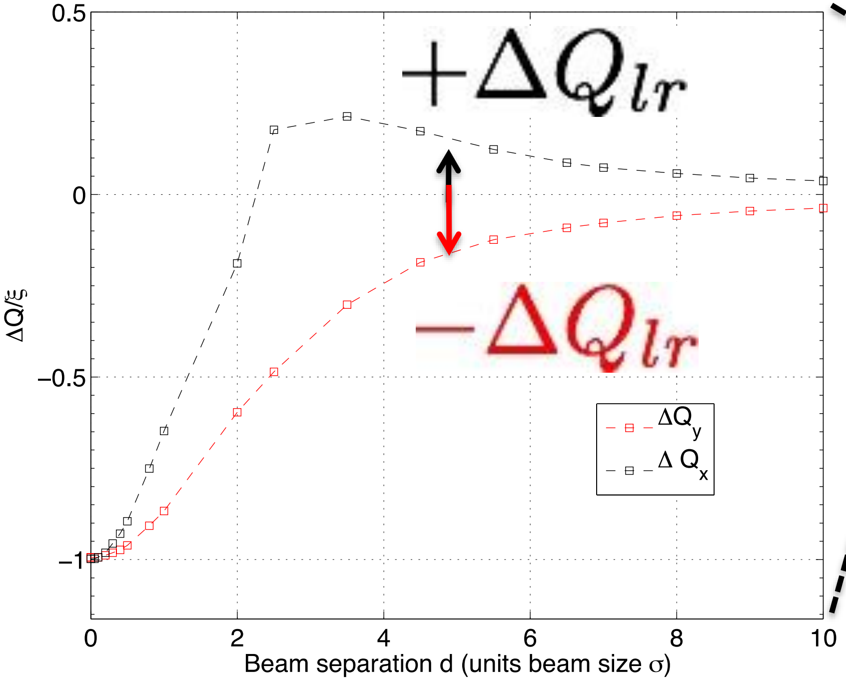
1-D plot of detuning with amplitude for opposite and equally charged beams



Maximum tune shift for large amplitude particles

Smaller tune shift detuning for zero amplitude particles and opposite sign

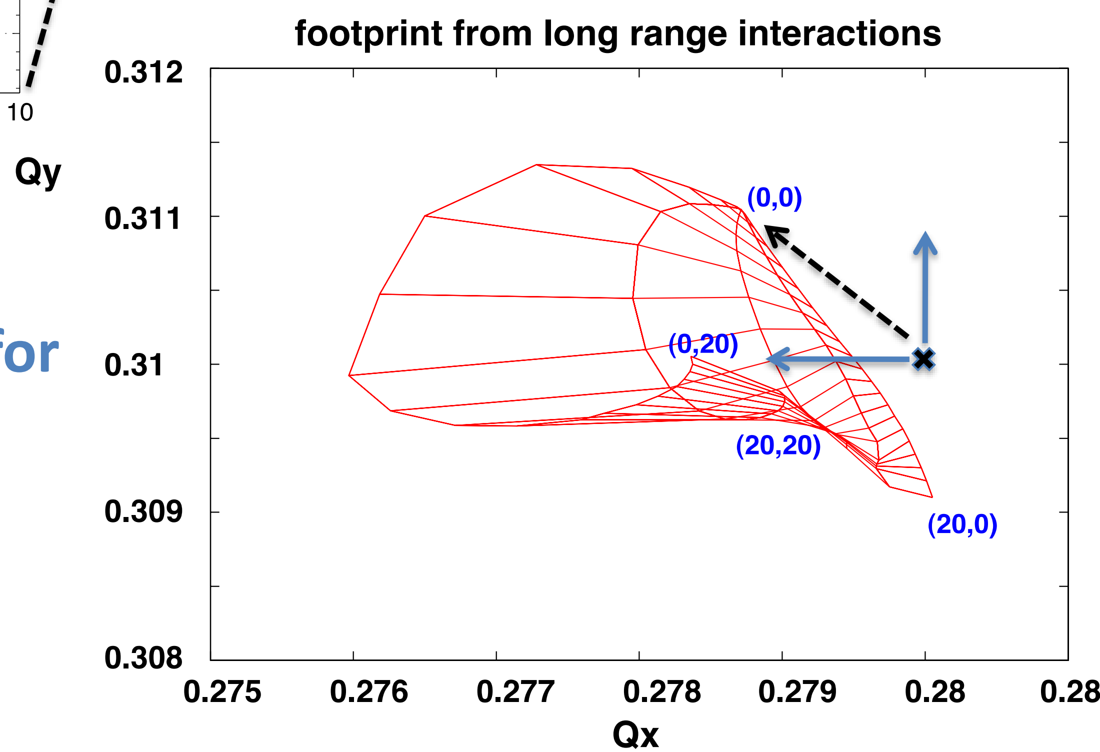
2-D Long Range detuning with amplitude



Tune shift as a function of separation in horizontal plane

In the **horizontal** plane **long range tune shift**

In the **vertical plane opposite sign!**

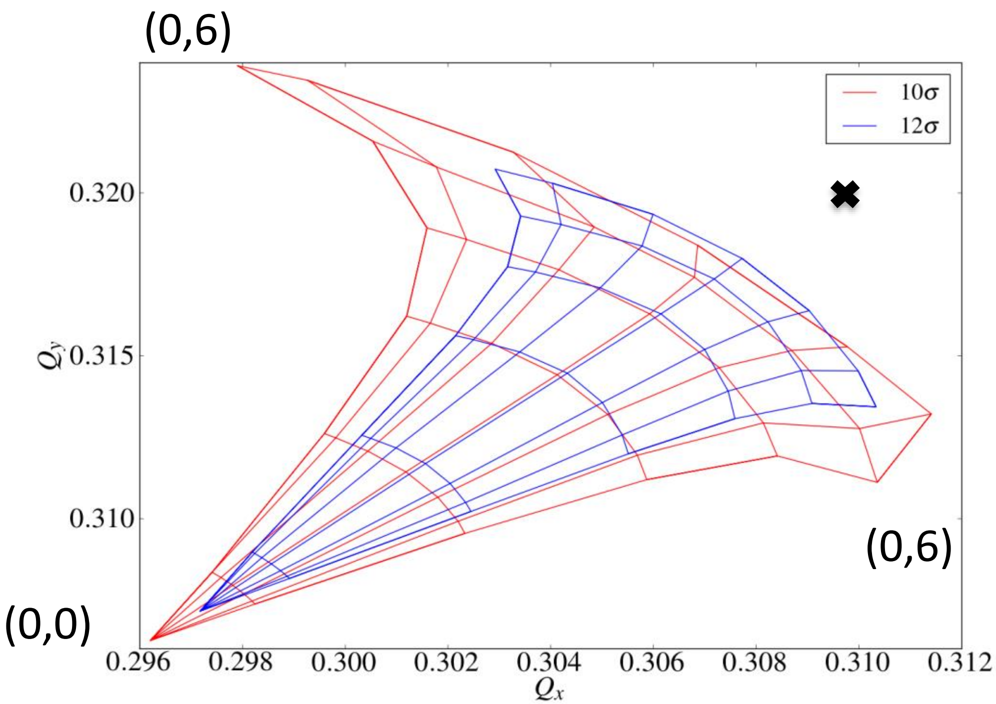


Long range tune shift scaling for distances $d > 6\sigma$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

Beam-beam tune shift and tune spread

Head-on and Long range interactions detuning with amplitude



Footprints depend on:

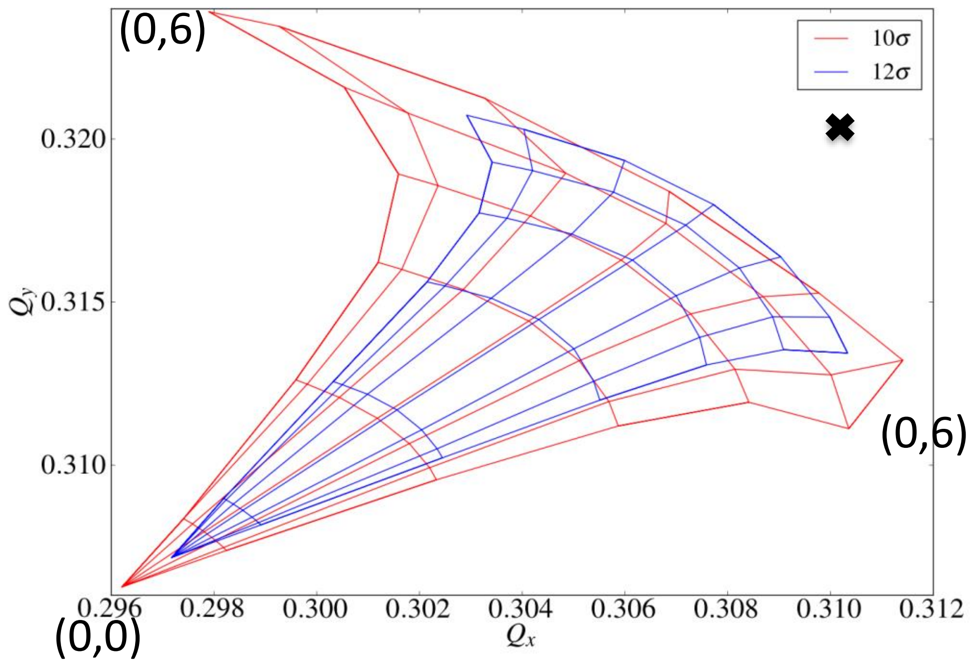
- **number of interactions (124 per turn)**
- **Type (Head-on and long-range)**
- **Separation**
- **Plane of interaction**

Very complicated depending on collision scheme

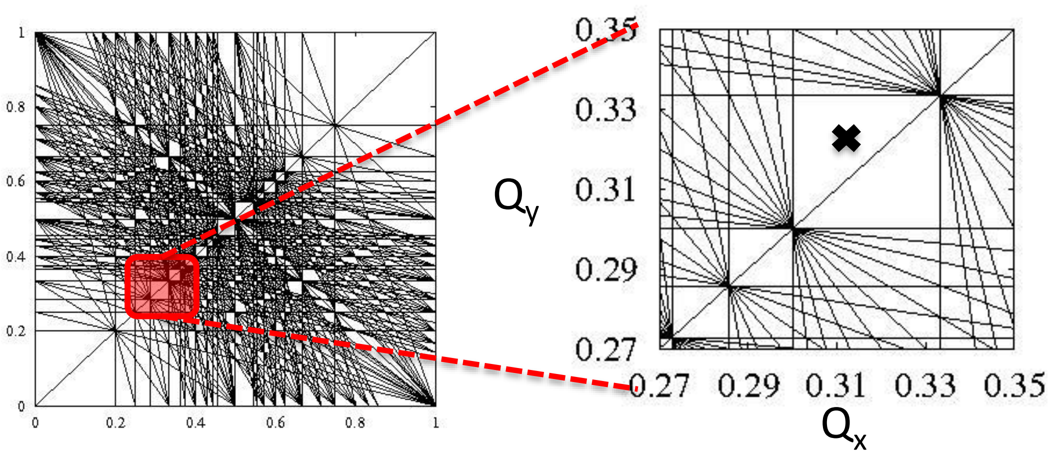
Pushing luminosity increases this area while we need to keep it small to avoid resonances and preserve the stability of particles

Strongest non-linearity in a collider

Beam-beam tune shift and spread



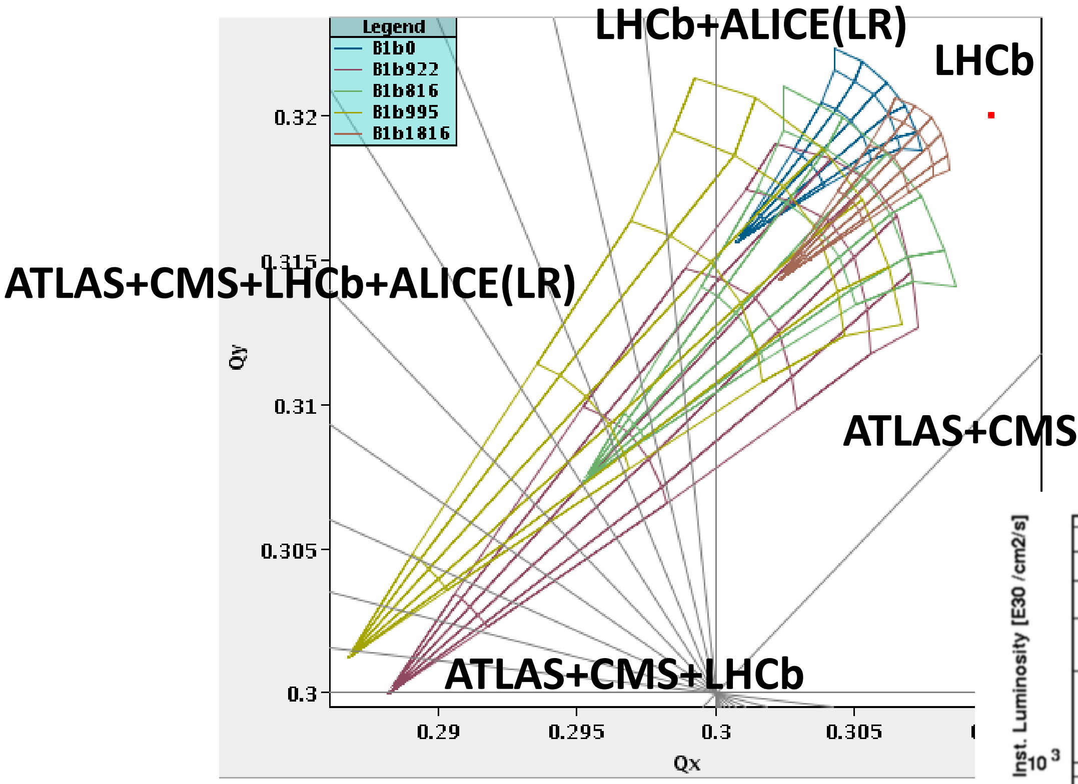
Higher Luminosity → increases this area
We need to keep it small to avoid resonances and preserve the long term stability of particles



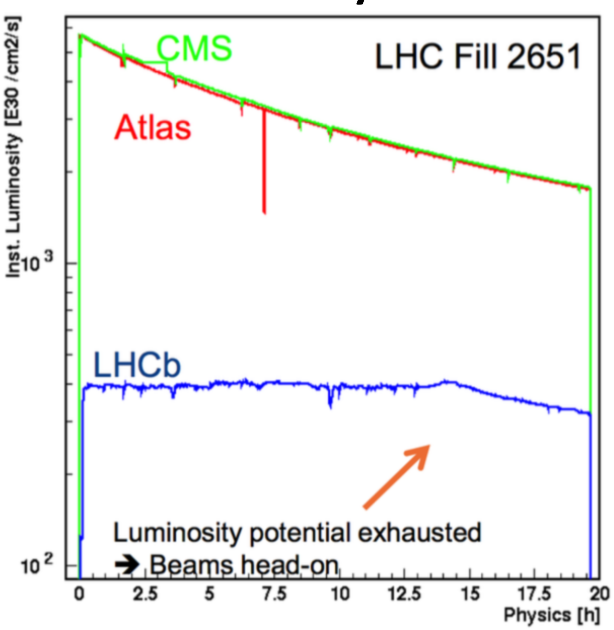
The footprint from beam-beam sits in the tune diagram

LHC Footprints and multiple experiments

LHC 2012 example



Luminosity Evolution



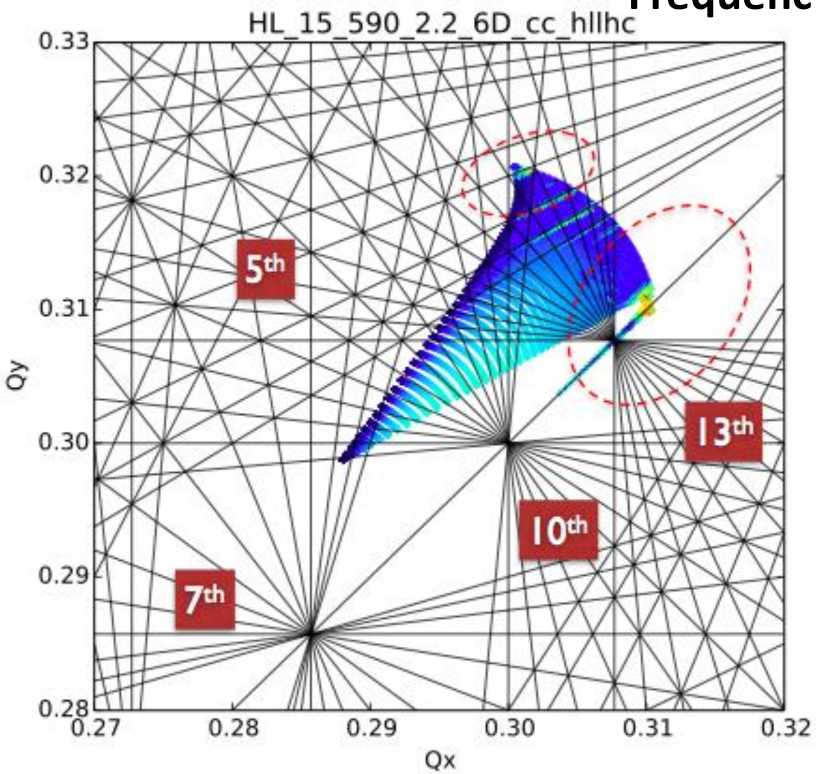
...operationally it is even more complicated!
...different intensities, emittances...

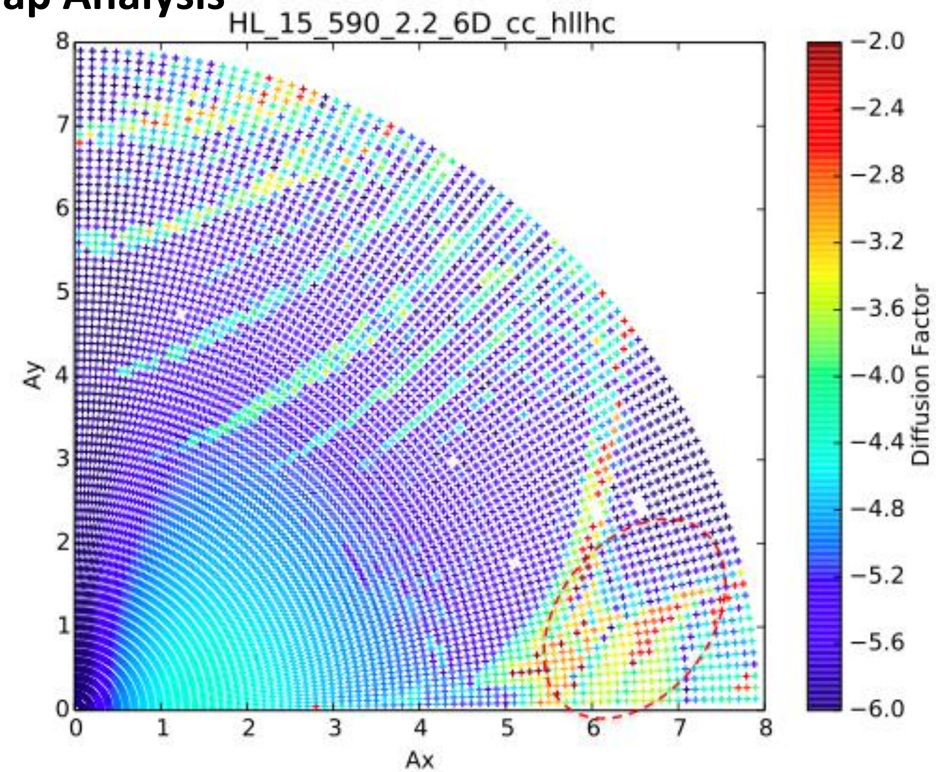
Dynamical Aperture and Particle Losses

Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle

Frequency Map Analysis

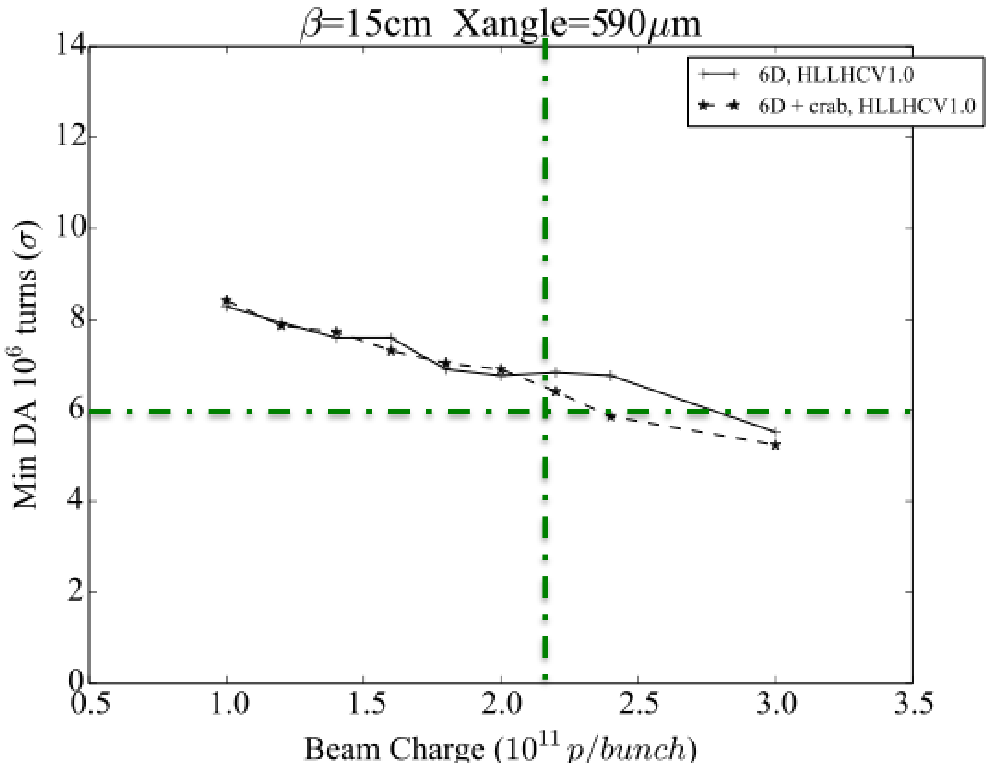




Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β^*, intensity) is the result of careful study of different effects!

Dynamical Aperture and Particle Losses

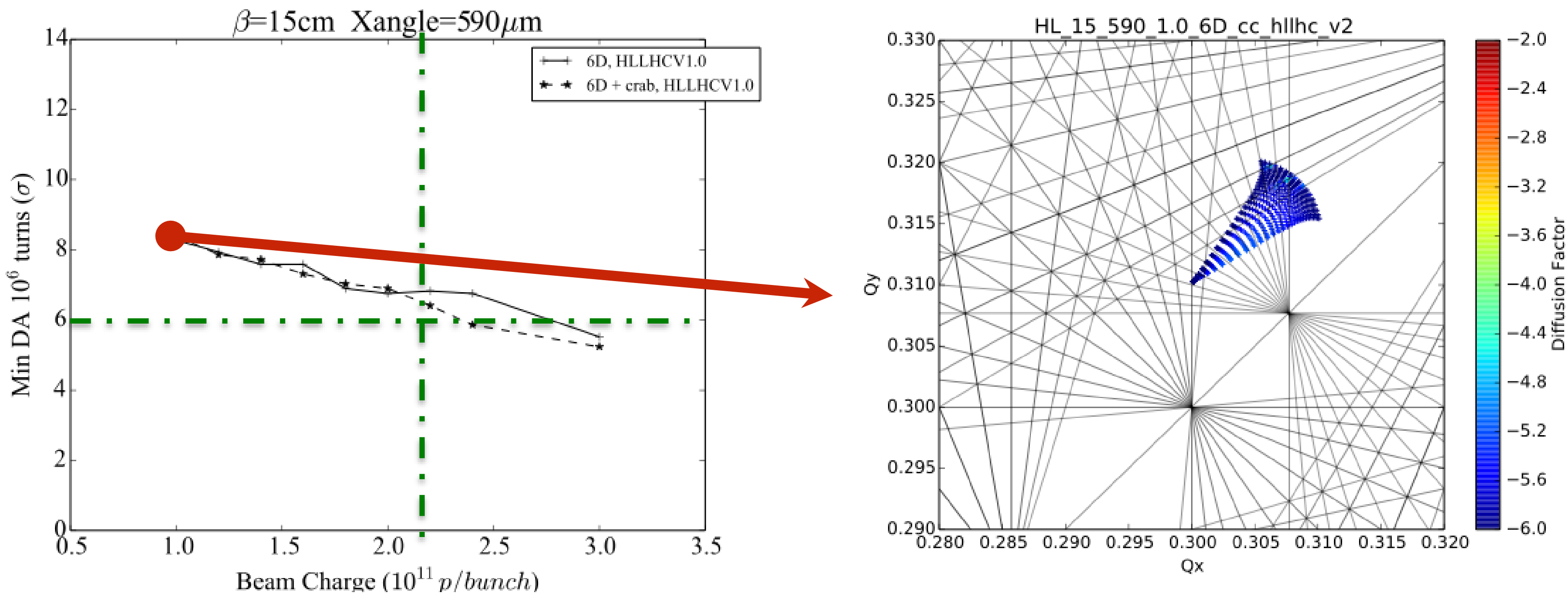
Beam-beam linear dependency with Intensity



Our goal: keep dynamical aperture as large as possible → all particles not lost over long tracking time (10^6 turns in simulation) equivalent to 1 minute of collider

Example collider collision time : 24 hours

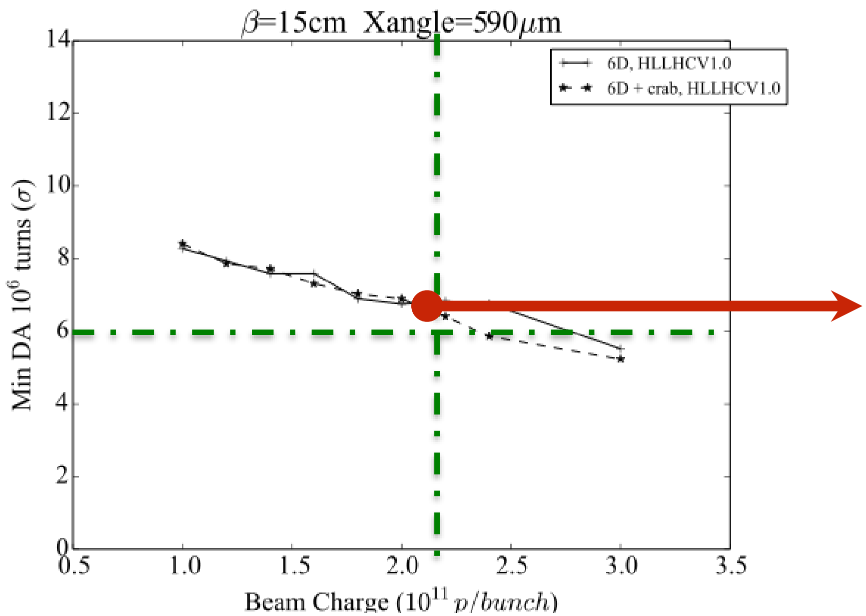
Round optics15 cm, 590μrad: intensity scan

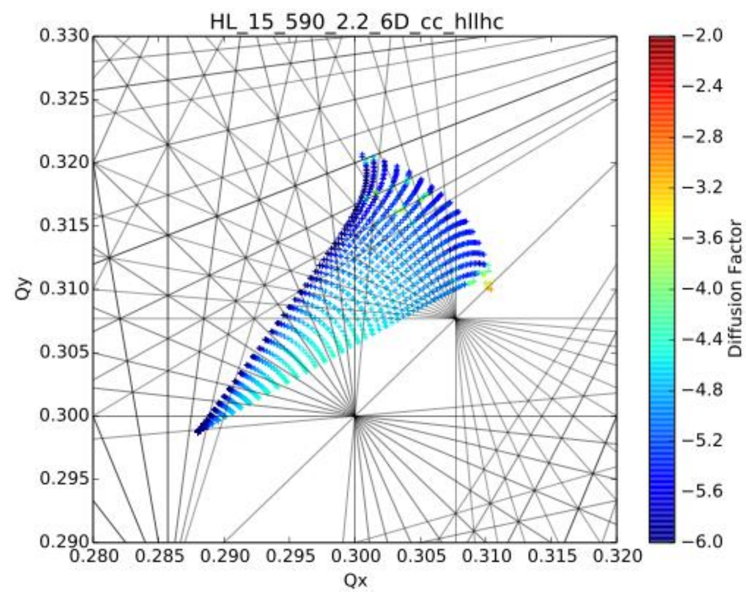


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

$$\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

Round optics15 cm, 590μrad: intensity scan

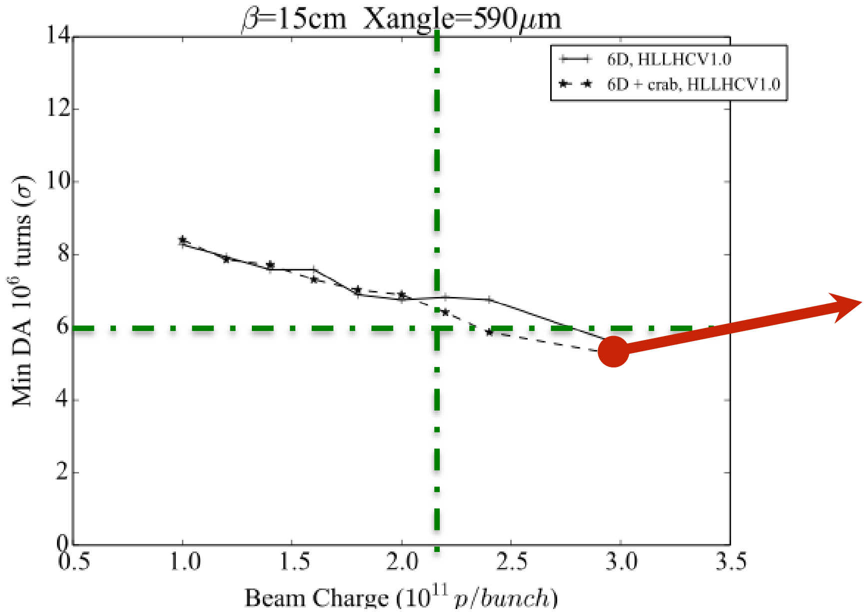


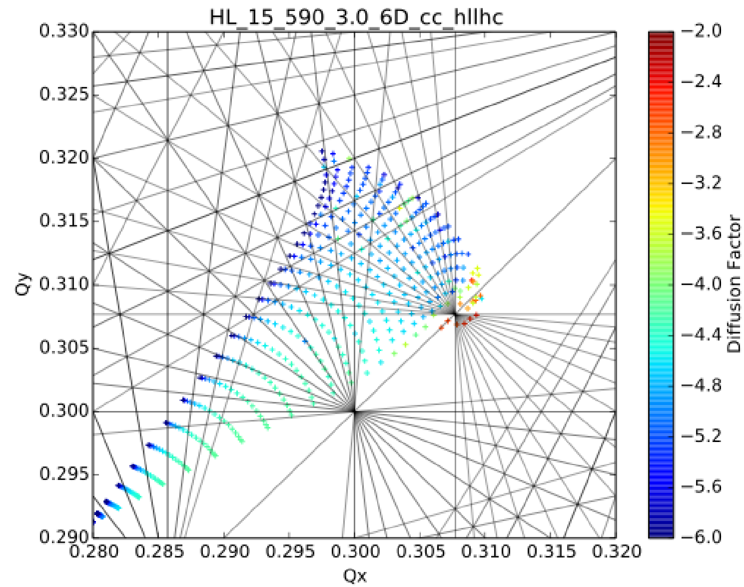


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \nearrow$$

$$\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} \nearrow$$

Round optics15 cm, 590µrad: intensity scan

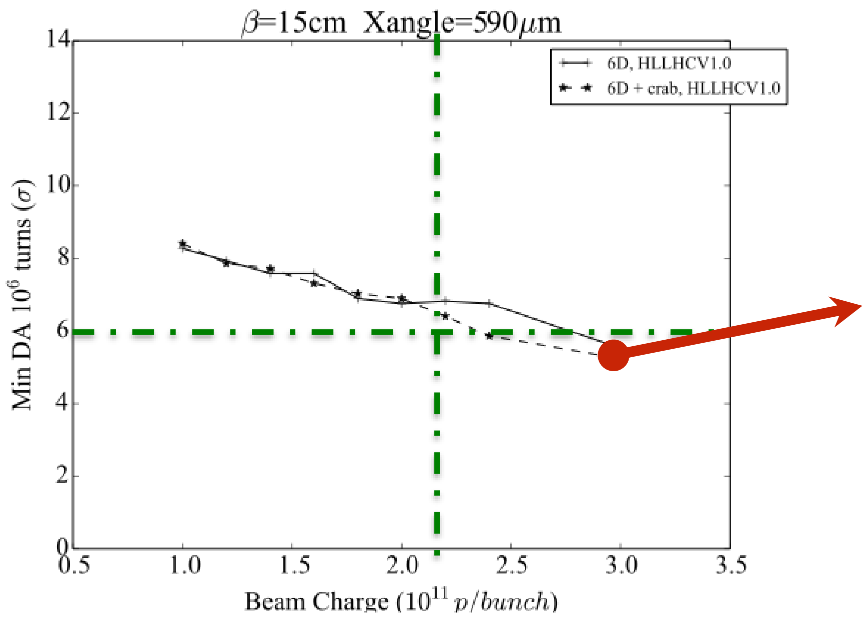


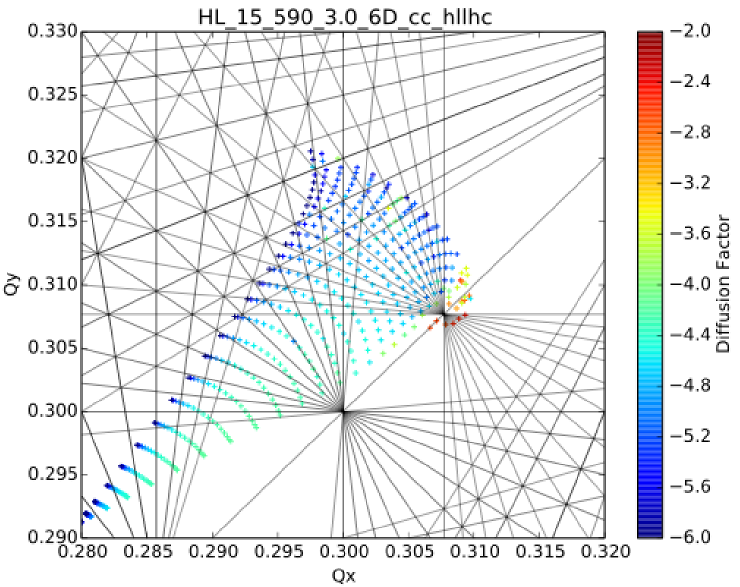


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \searrow$$

$$\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} \nearrow$$

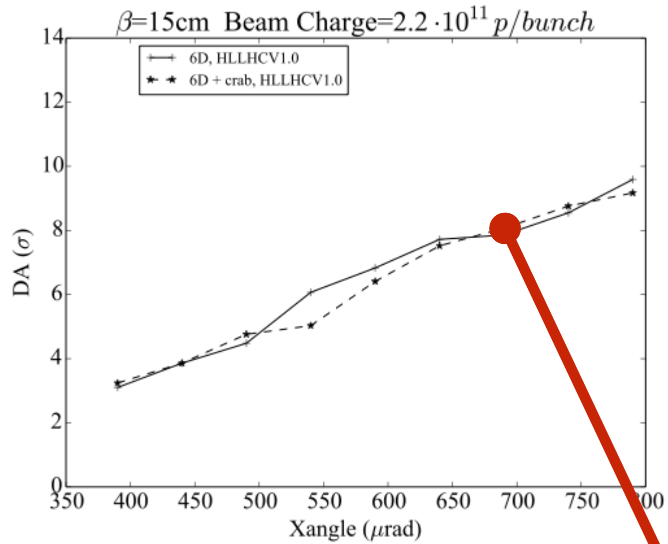
Round optics15 cm, 590µrad: intensity scan





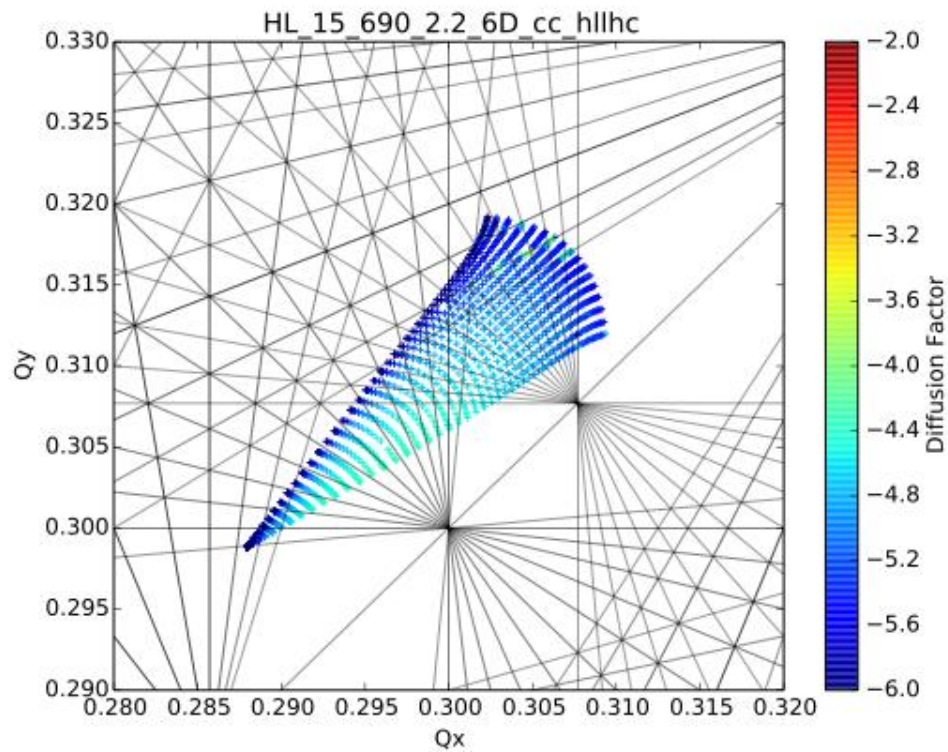
AT high intensity the beam-beam force gets too strong and makes particles unstable and eventually are lost

Round 15cm, 2.2E11, 690μrad



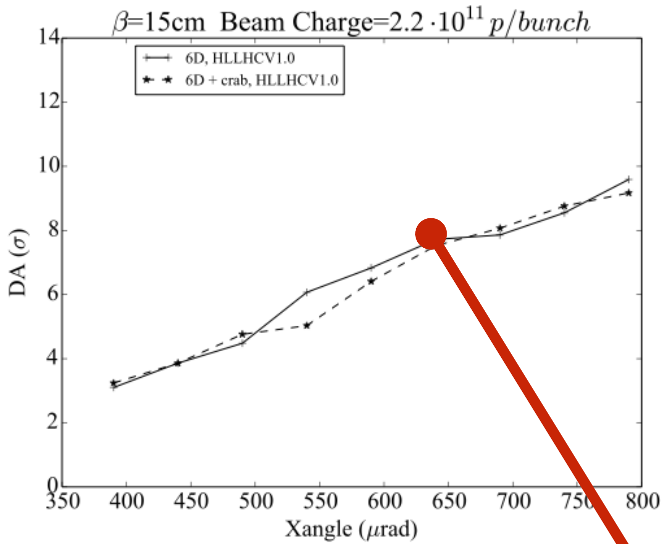
$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$$

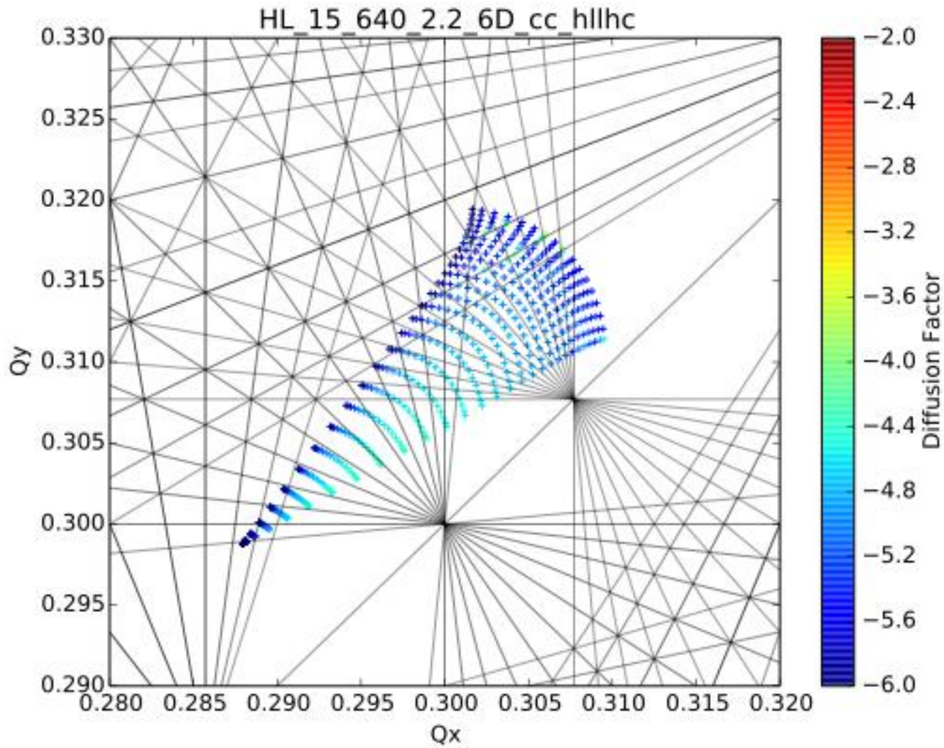


Smaller beam-beam separation at parasitic long-range encounters stronger non linearities → smaller stable area → losses

Round 15cm, 2.2E11, 650μrad



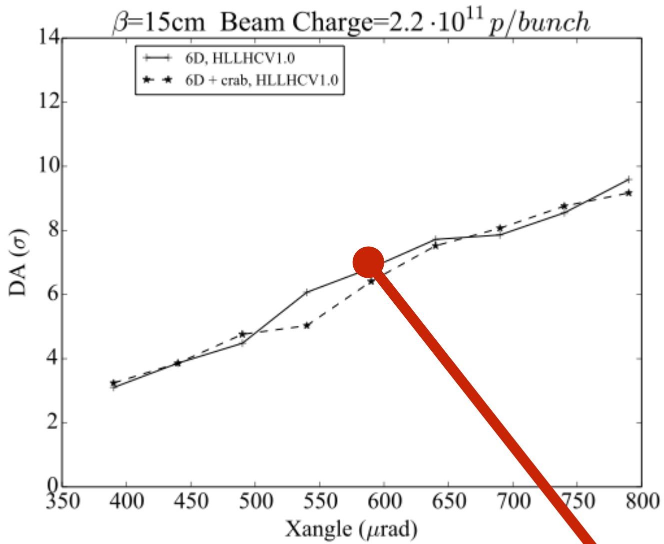
$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$



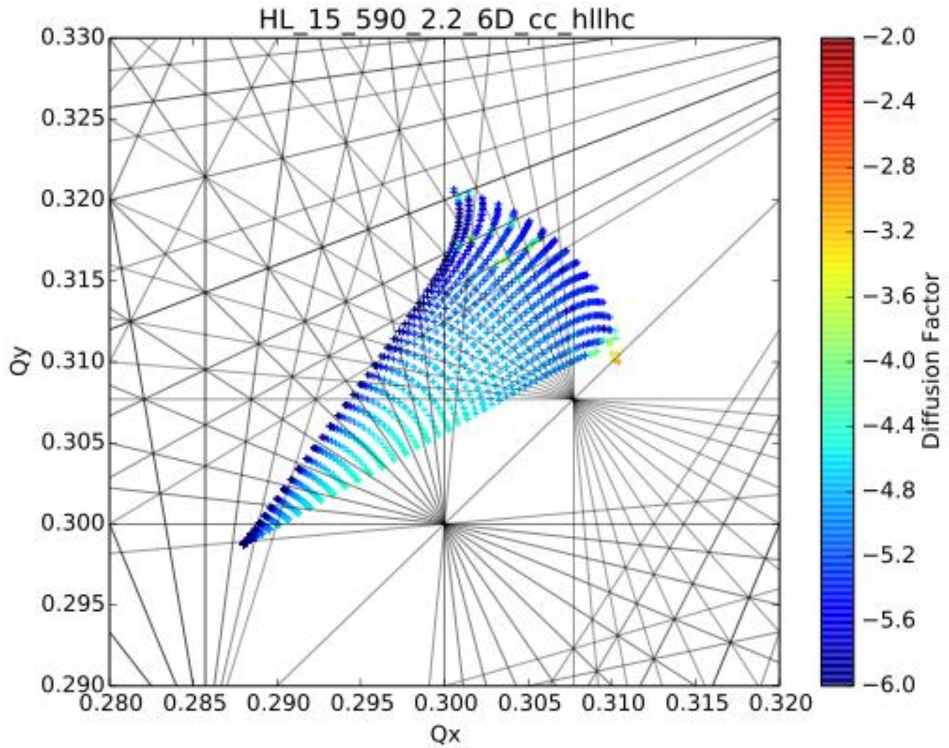
$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

5

Round 15cm, 2.2E11, 590μrad

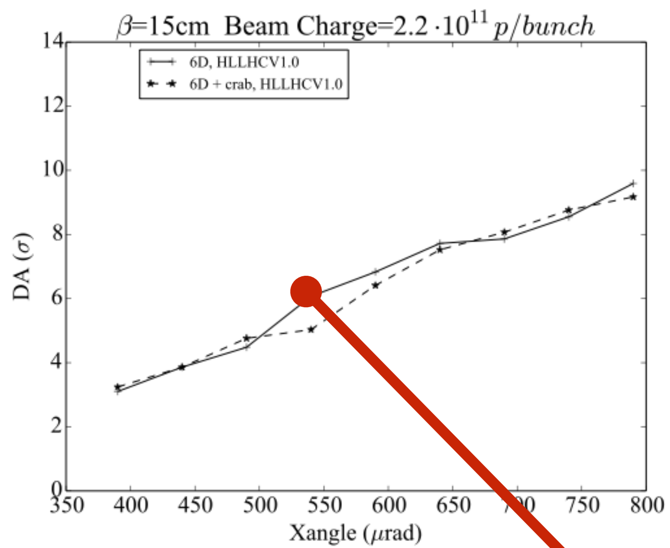


$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$



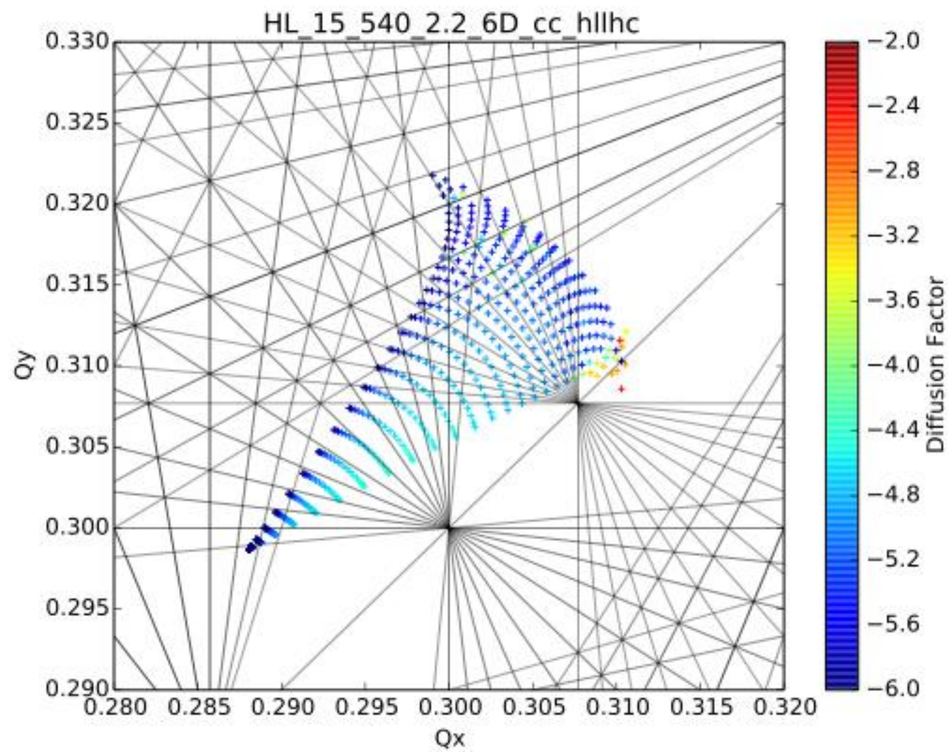
$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$$

Round 15cm, 2.2E11, 540μrad

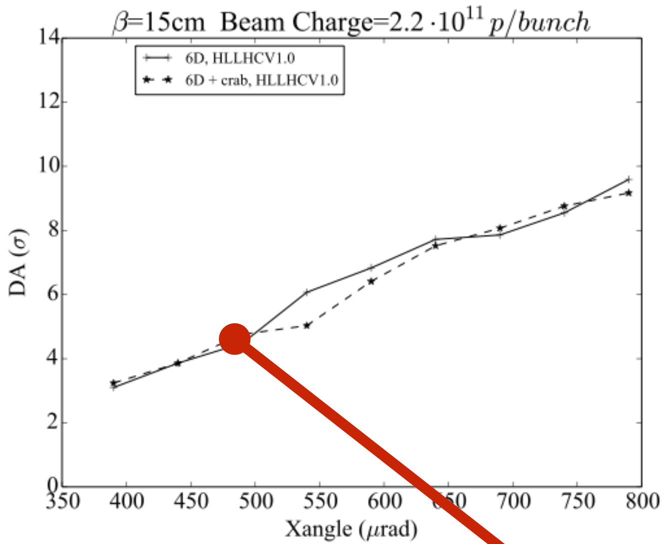


$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

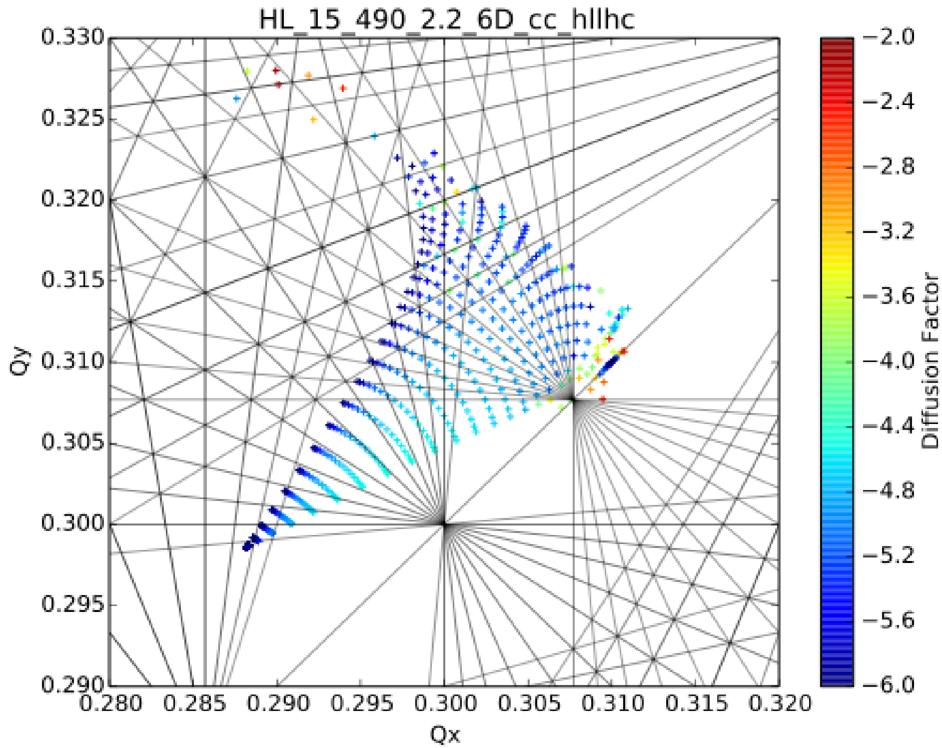
$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$$



Round 15cm, 2.2E11, 490μrad

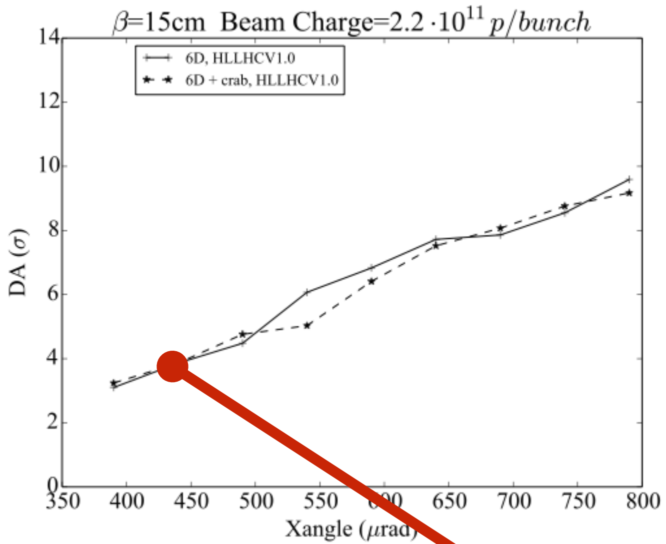


$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

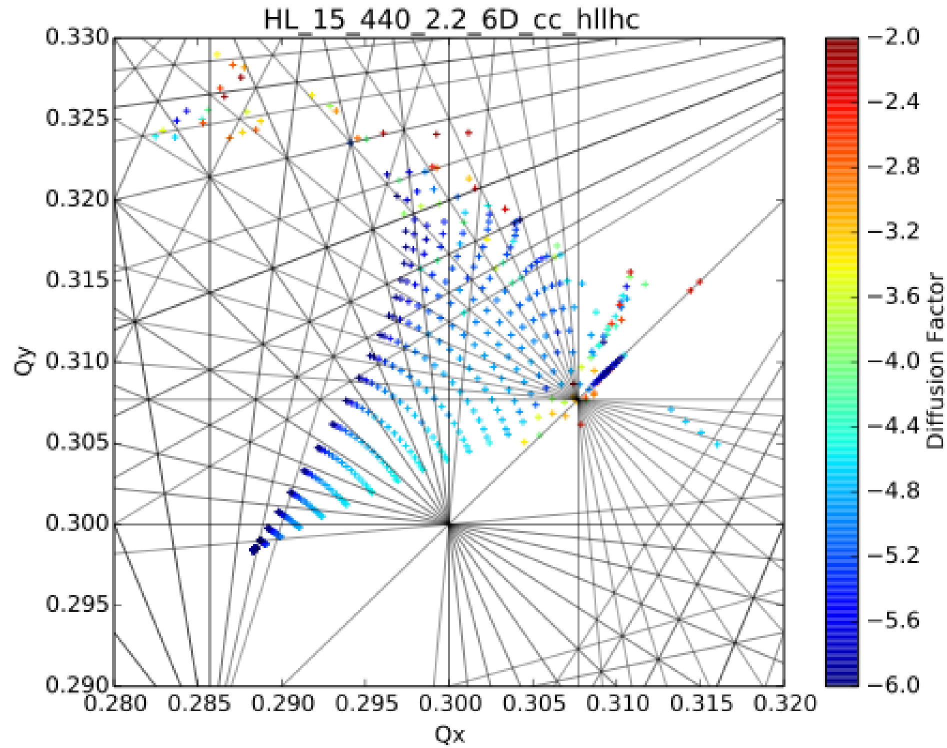


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

Round 15cm, 2.2E11, 440μrad

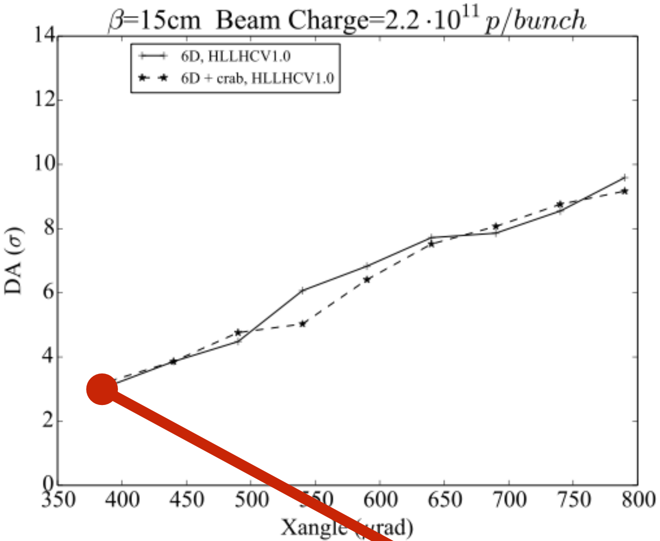


$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

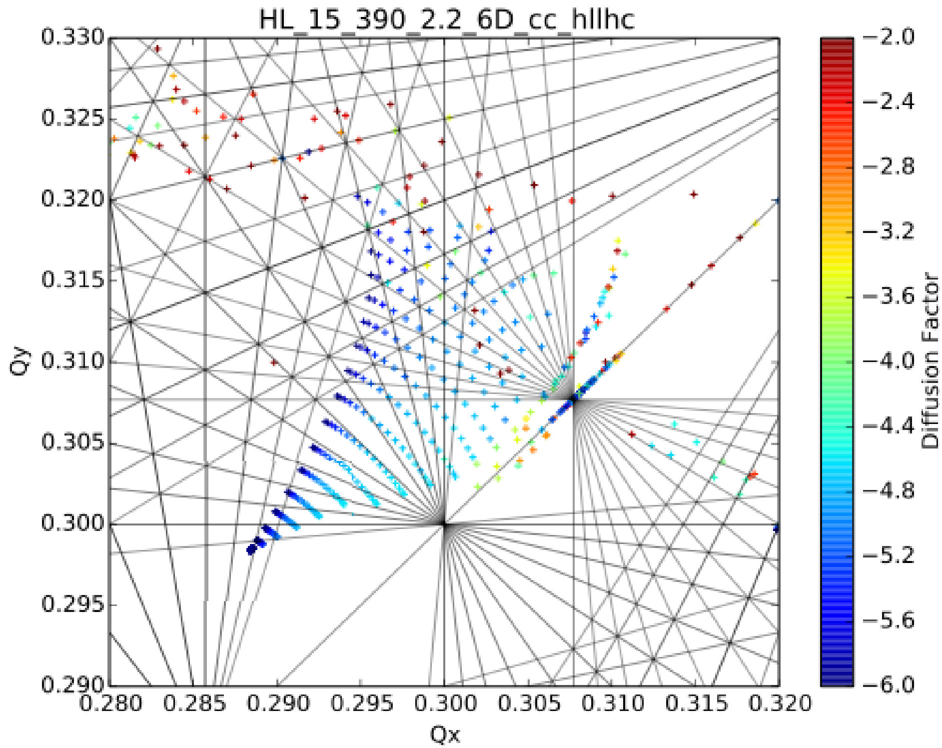


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

Round 15cm, 2.2E11, 390µrad

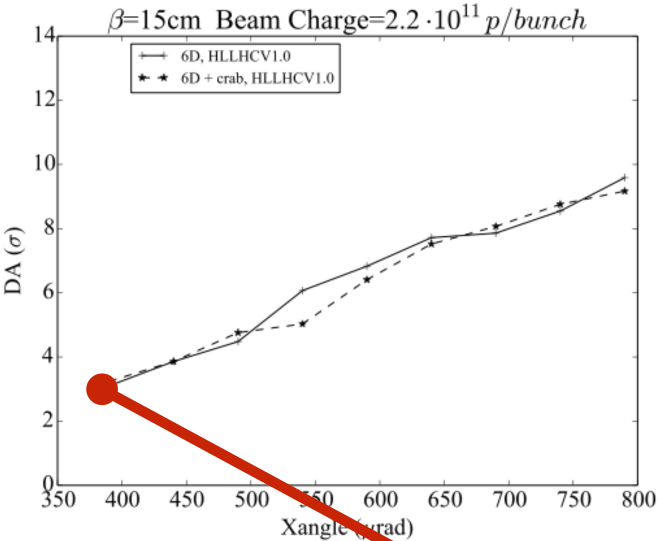


$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

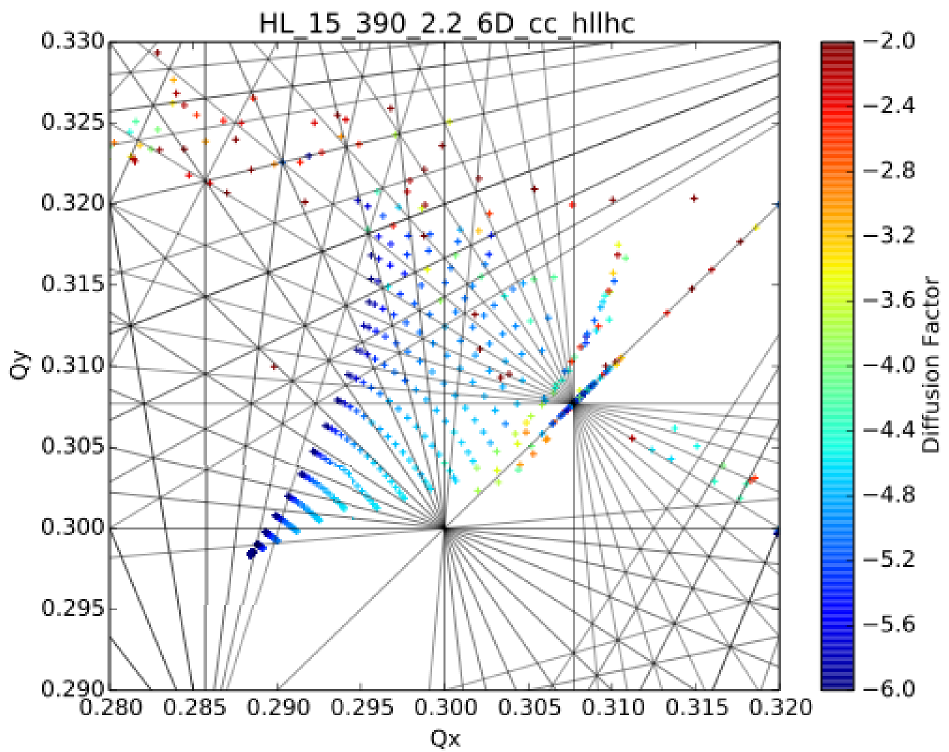


$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

Round 15cm, 2.2E11, 390μrad



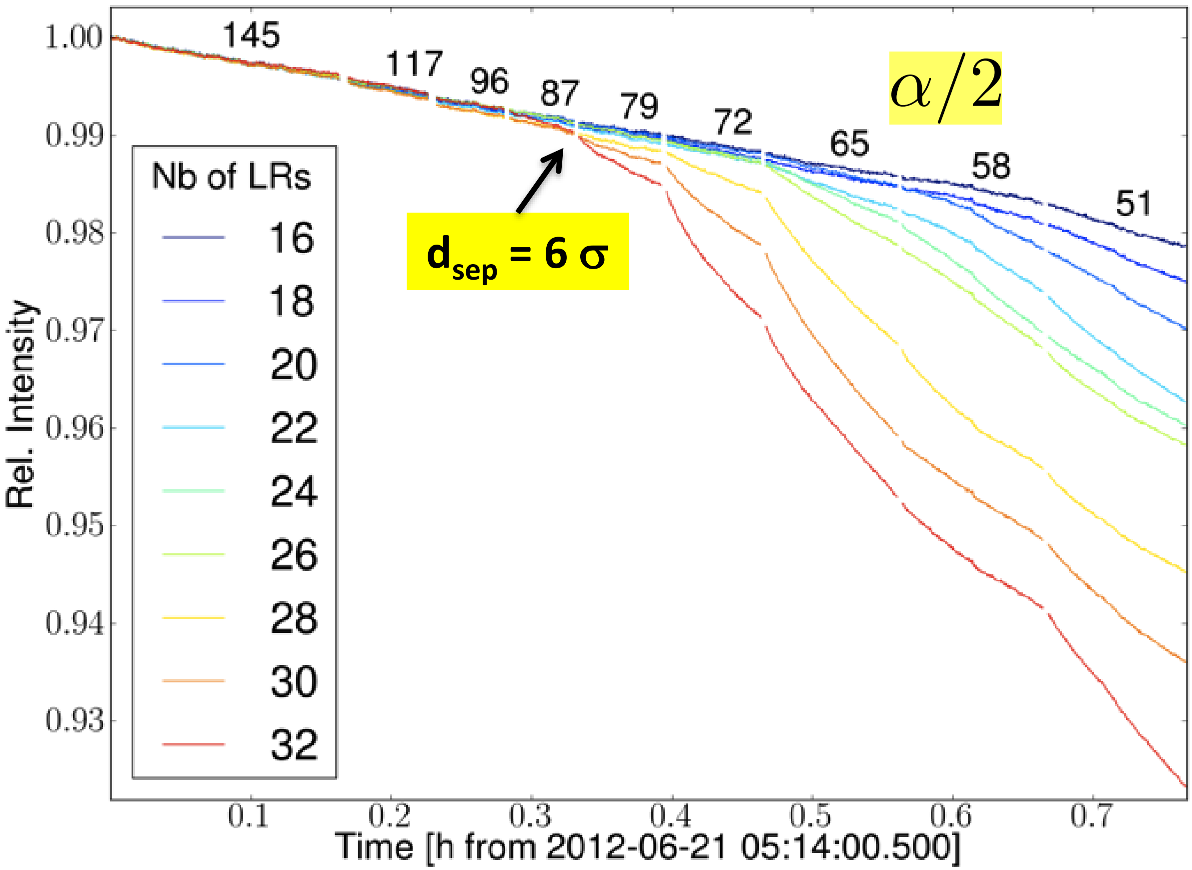
Crossing angle changes the separation and the strength of BB-LR that strongly affect the dynamics of particles tails first then if too strong core



At small separation particles gets unstable and eventually lost

How does it look like in the LHC?

Relative intensity decay 2012 experiment



Beam-Beam separation at first LR

$$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$$

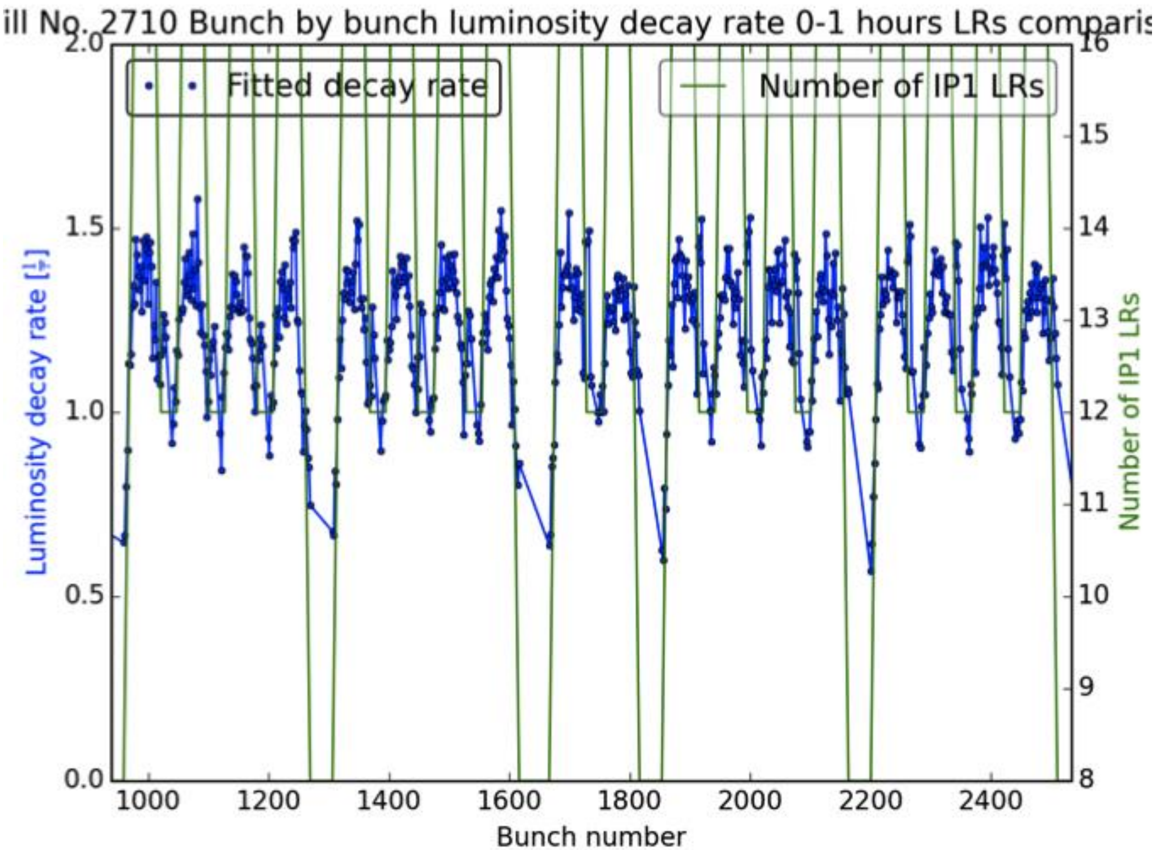
Small crossing angle = small separation

If separation of long range too small particles become unstable and are lost proportionally to the number of long range encounters

Particle losses follow number of Long range interactions

Do we see the particle losses?

Regular Physics Fill of 2012 RUN LHC



Beam-Beam separation at first LR

$$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$$

Small crossing angle = small separation

Luminosity decays following the long range numbers… higher number of long range interactions larger losses

Particle losses follow number of Long range interactions
Machine protection implication and beam lifetimes gets worse…

Best peformance of collider always a trade off between beam-beam and luminosity

Long-range Beam-Beam effects: orbit

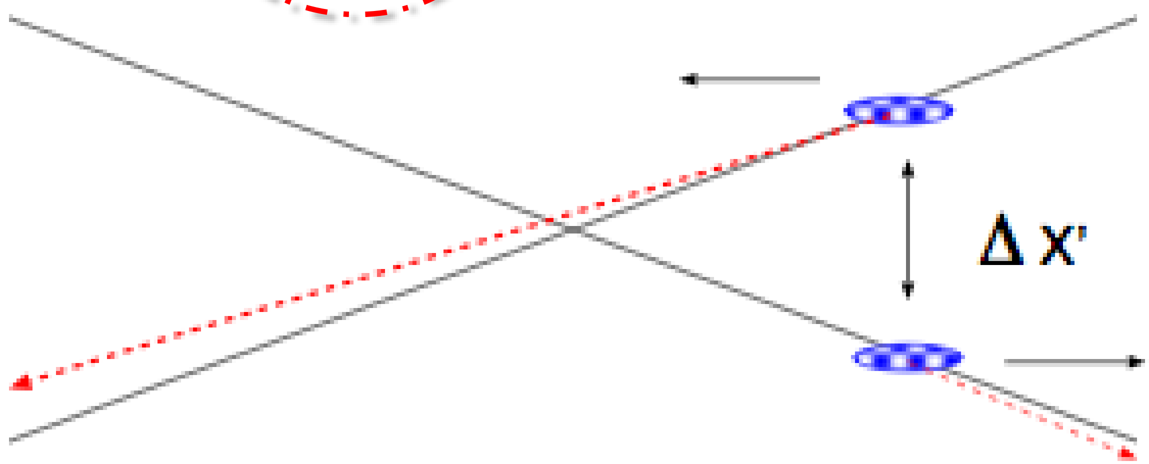
Long Range Beam-beam interactions lead to several effects…

Long range angular kick $$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}[1-\exp(-\frac{r^2}{2\sigma^2})$$

For well separated beams $$d \gg \sigma$$

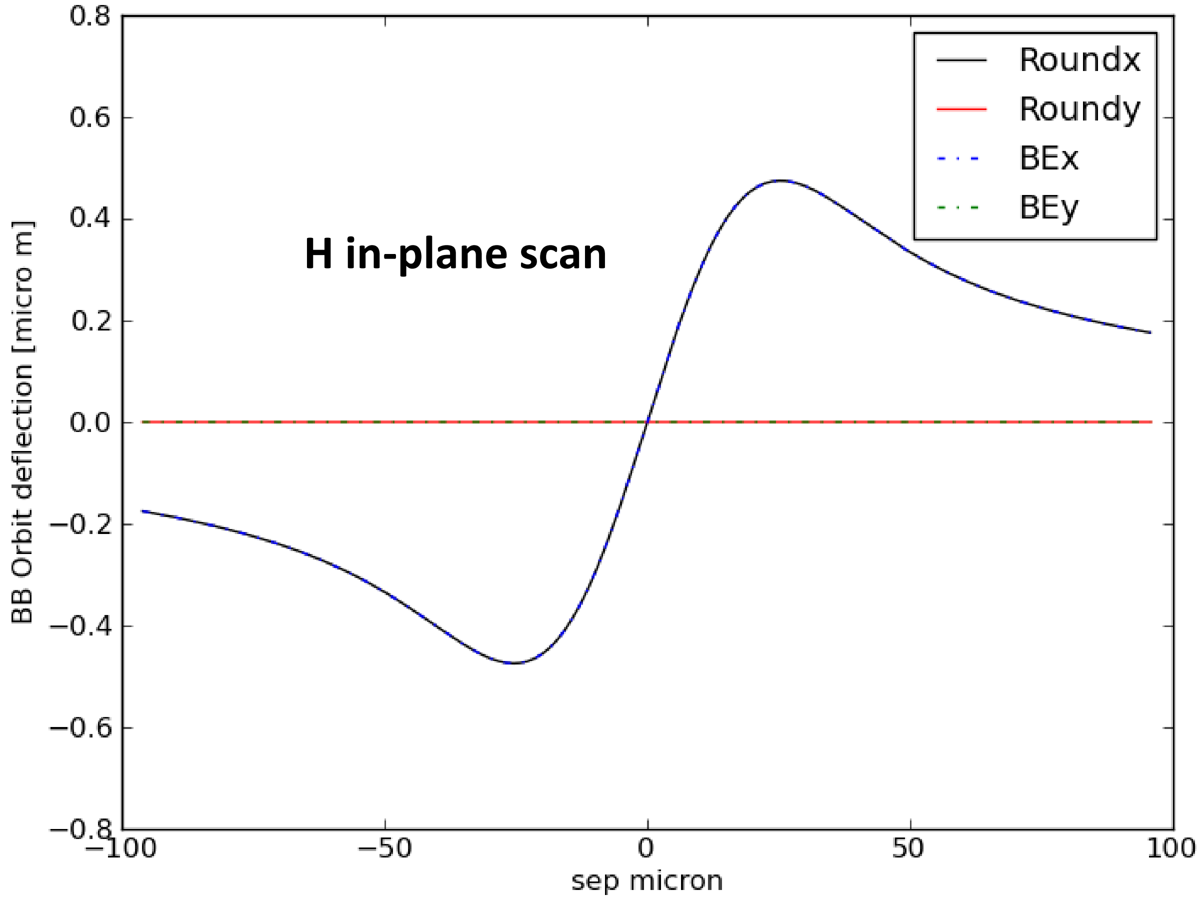
The force has several components at first order we have an amplitude independent contribution: ORBIT KICK

$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \ldots]$$



In simple case (1 interaction) one can compute it analytically

Orbit effect as a function of separation



Angular Deflections: $$\theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma)$$

Closed Orbit effect: $$Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})}$$

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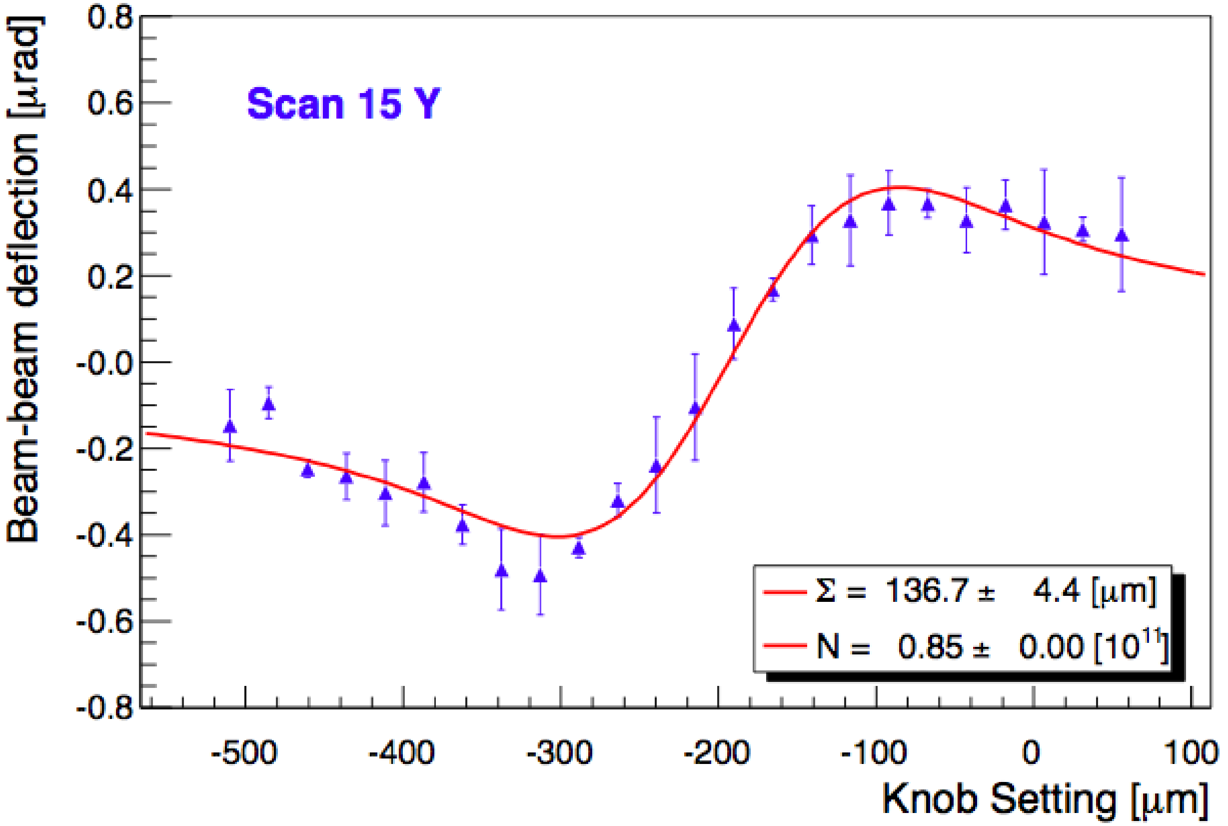

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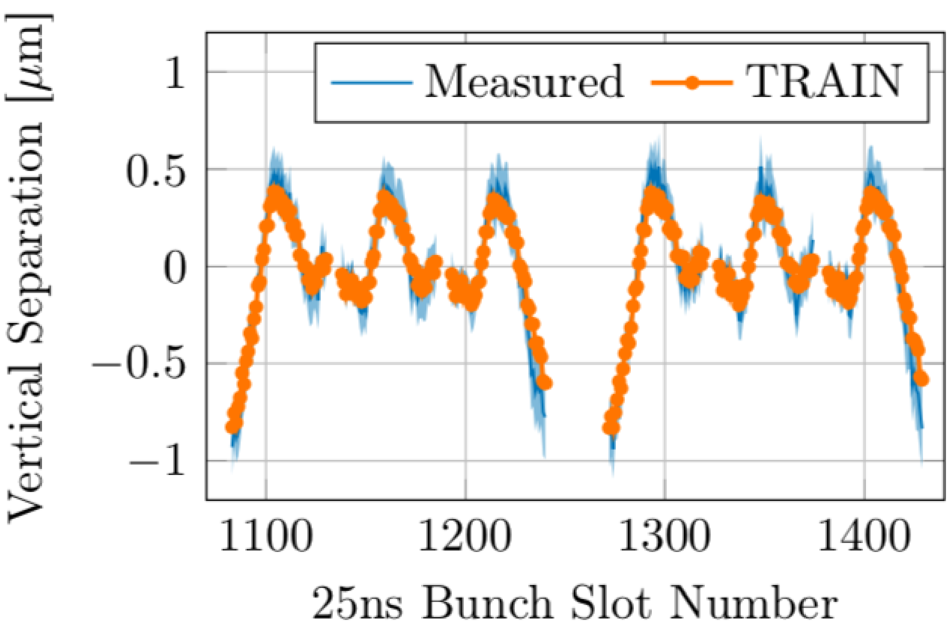
Orbit can be corrected but we should remember PACMAN effects

LHC orbit effects

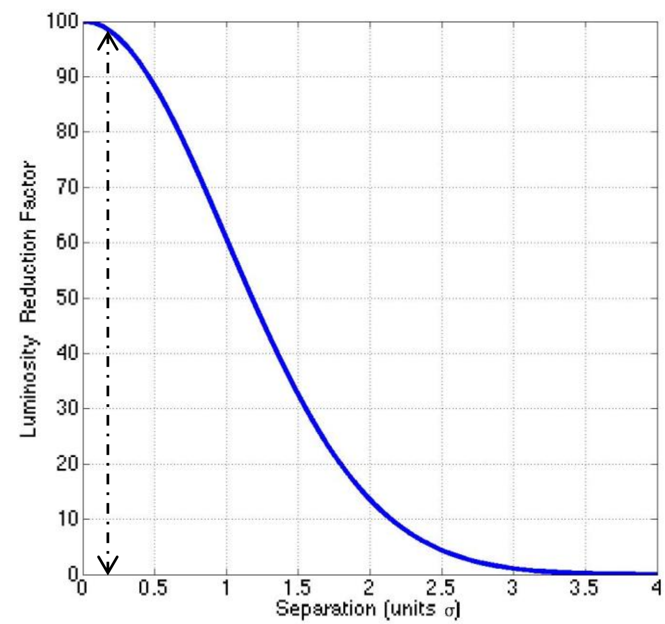
Many long range interactions could become important effect!
Holes in bunch structure leads to PACMAN effects this cannot be corrected!

$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

Self consistent evaluation



d = 0 – 0.2 units of beam size



1-2% Luminosity loss due to beam-beam orbit effects

Summary

Head-on Interactions

Long-Range Effects

Beam-Beam parameter

Dynamic Beta

Tune spread

$$\mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4\pi \epsilon_{x,y} \beta^*} \cdot F$$

Orbit effects

Particle Losses

Emittance Increase

Crossing angles

L measurements uncertainties

…not covered here…

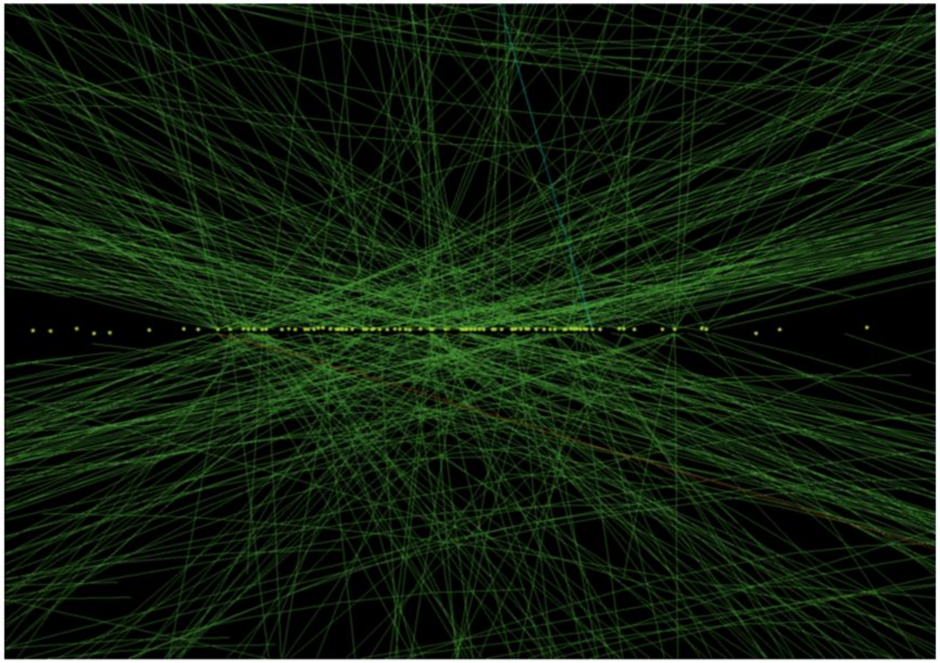
- *Beam-Beam compensation schemes*
- *Landau damping and beam-beam*
- *Beam-Beam coherent effects*
- *Asymmetric beams effects*
- *Noise on colliding beams*
- *Luminosity hourglass effect*
- *Measuring Luminosity: Van der Meer scans*
- *Pile-up and leveling luminosity*
- *….*

Pile-up and Luminosity leveling

Experiments might need luminosity control

- if too high can cause high voltage trips then impact efficiency of the detectors
- might have event size or bandwidth limitations in read-out
- too many simultaneous event cause loss of resolution

...experiments also care about the average number of inelastic interactions per bunch crossing



78 Event Vertices from CMS High Pile-up test

$$\mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4\pi \epsilon_{x,y} \beta^*} \cdot F$$

- β^* leveling
- Offset leveling
- Crossing angle leveling

ALICE and LHCb level with transverse offset

$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

Pile-up and Luminosity leveling

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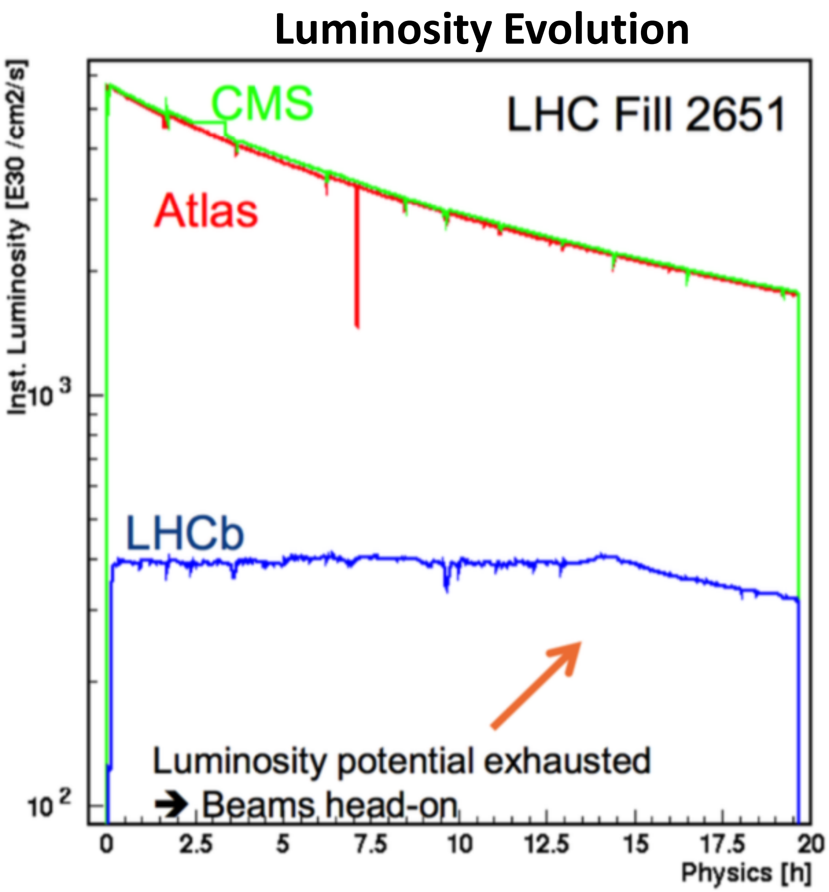

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- Crossing angle leveling

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$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

Thank you!

Questions?

References:

[*] *http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf*

[*] *W. Herr and B. Muratori "Concept of Luminosity", CAS 2006.*

[*] *W. Herr and T. Pieloni, "Beam-beam Effects" CAS Trondheim, 2016.*

[*] *V. Shiltsev et al, "Beam beam effects in the Tevatron", Phys. Rev. ST Accel. Beams 8, 101001 (2005)*

[*] *Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)*

[*] *Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)*

[*] *J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.*

[*] *A. Hofmann,"Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.*

[*] *R. Assmann et al., "*Results of long-range beam-beam studies - scaling with beam separation and intensity "

…much more on the LHC Beam-beam webpage:

http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/

Luminosity Basics

$$N_{events} = L \times \sigma_{event}$$

$\mu_{vis} = \varepsilon * \mu$ = **Mean number of interactions per Bunch crossing seen by detector**

Mean number of inelastic interactions per Bunch crossing

$$\mathcal{L} = \frac{\mu \, n_b \, f_r}{\sigma_{inel}} = \frac{\mu_{vis} \, n_b \, f_r}{\sigma_{vis}}$$

Inelastic cross section (unknown)

Cross section seen by detector

- σ_{vis} **is determined in dedicated fills based on beam parameters**

W. Kozanecki

Ref. S. Van der Meer, "Calibration of the Effective Beam Height in the ISR" *CERN-ISR-PO-68-31*, 1968.

Van der Meer Scans

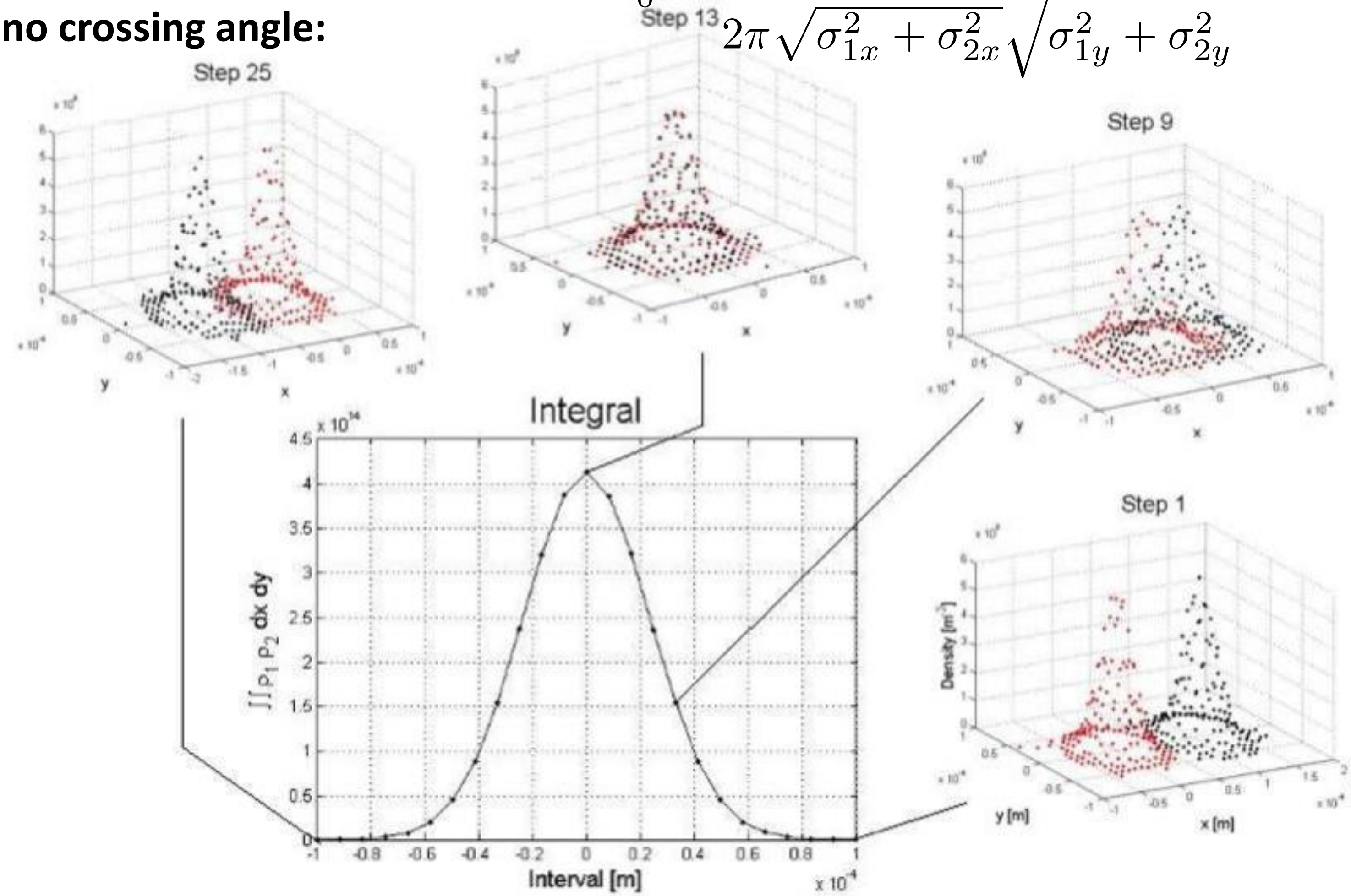
- Luminosity in terms of beam densities ρ_1 and ρ_2 in machine:

Luminosity in general

$$\mathcal{L} = n_b\, f_r\, n_1 n_2 \int \rho_1(x,y)\rho_2(x,y)\,dx\,dy$$

Gaussian beams and uncorrelated x & y components no crossing angle:

$$L_0 = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2+\sigma_{2x}^2}\sqrt{\sigma_{1y}^2+\sigma_{2y}^2}}$$



Calibrating σ_{vis} during van der Meer Scans

Gaussian fit of Lumi scans to extrapolate μ_{vis}^{Max} and Σ_x

$$\mathcal{L} = \frac{\mu\, n_b\, f_r}{\sigma_{inel}} = \frac{\mu_{vis}\, n_b\, f_r}{\sigma_{vis}}$$

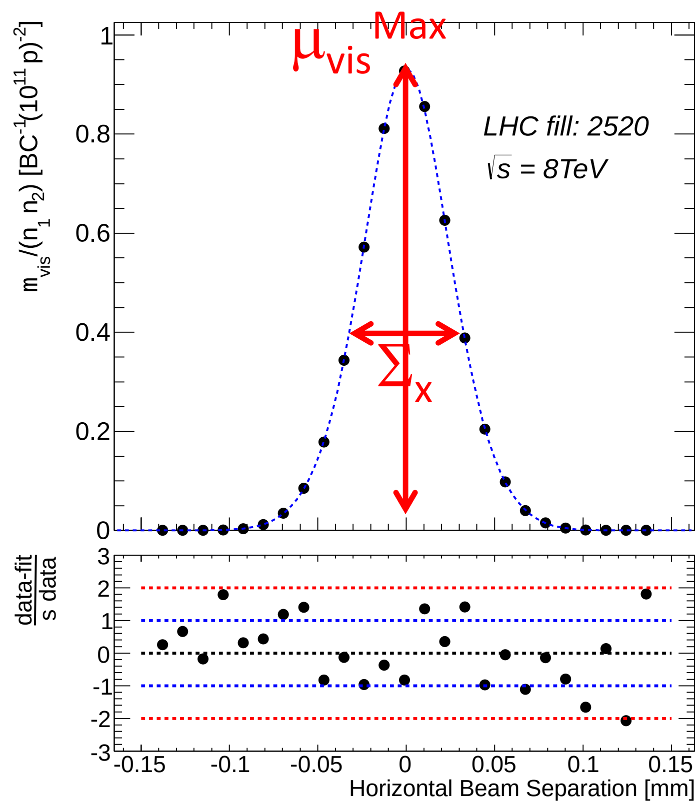
Measured in VdM scan

Detector independent

$$\sigma_{vis} = \boxed{\mu_{vis}^{Max}} \frac{2\pi \boxed{\Sigma_x \Sigma_y}}{\boxed{n_1 n_2}}$$

Detector dependent

Measured by beam instrumentation

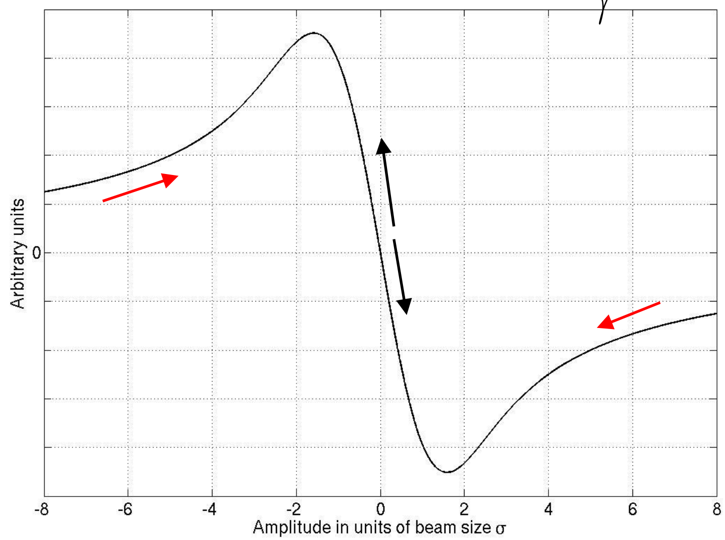


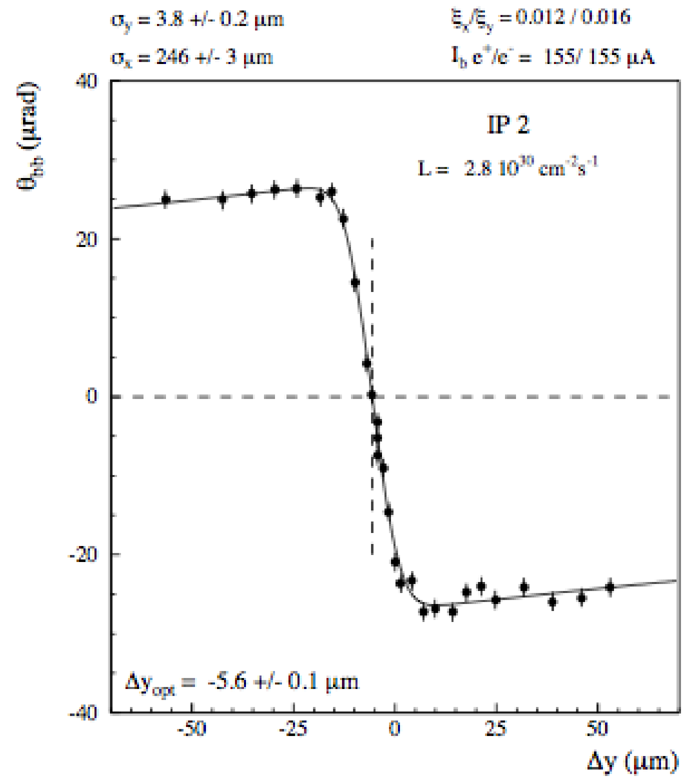
W. Kozanecki

Van der Meer scans and Beam-beam

Beam-Beam force

$$\theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma)$$



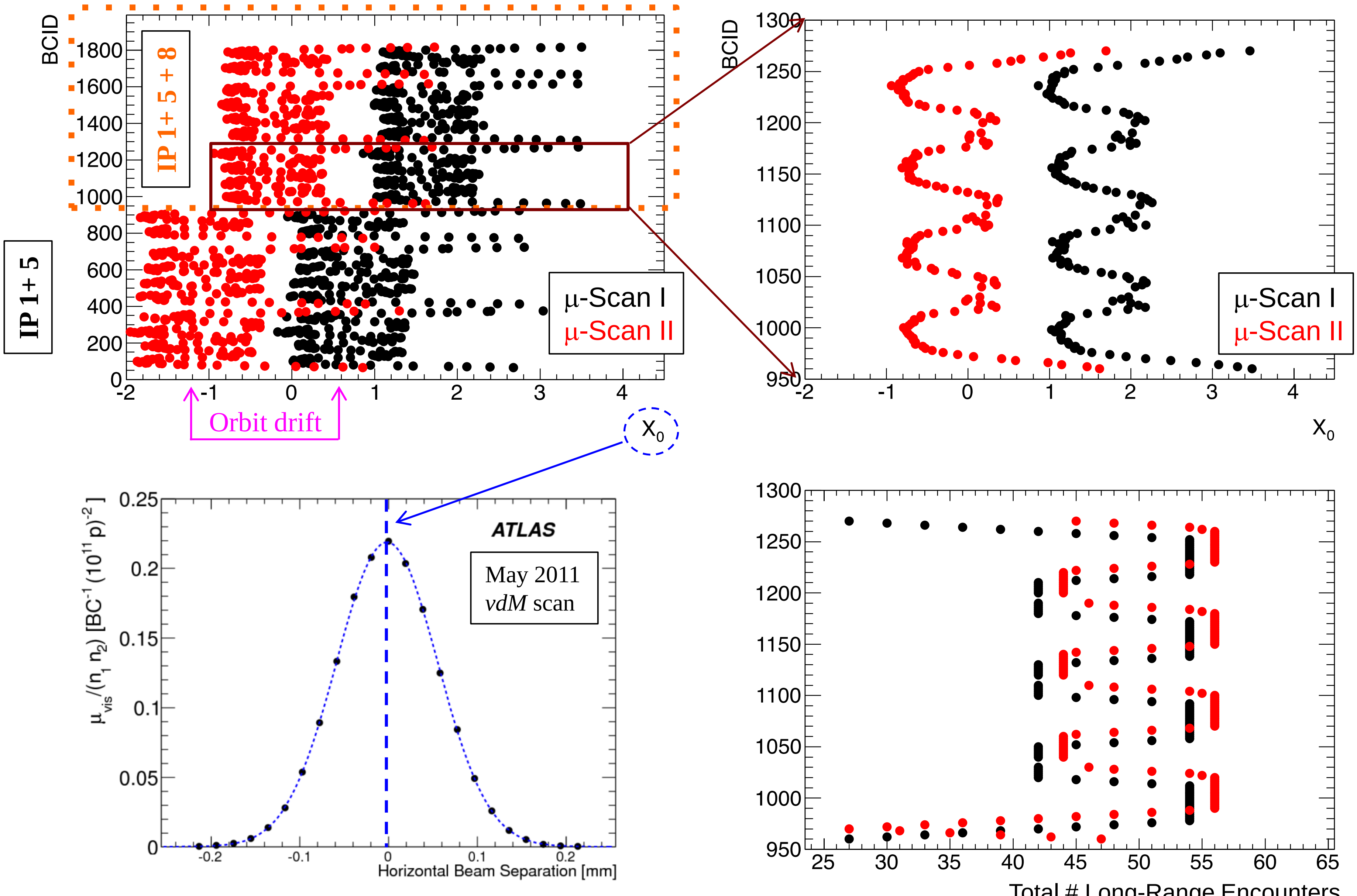


Beam-beam angular kick produces orbit change

Dynamic beta effects: beam sizes affected by beam-beam

Uncertainties corrected for during Van der Meer calibration scans

Impact of long-range encounters on $\mathcal{L}$ scans: data



W. Kozanecki