



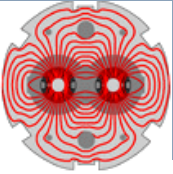
Linear Imperfections

AS@ESI, June 2018

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RN Beams Department

eration group – LHC section



Introduction

Imperfection - sources

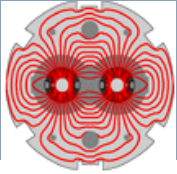
Orbit perturbations

Optics perturbations

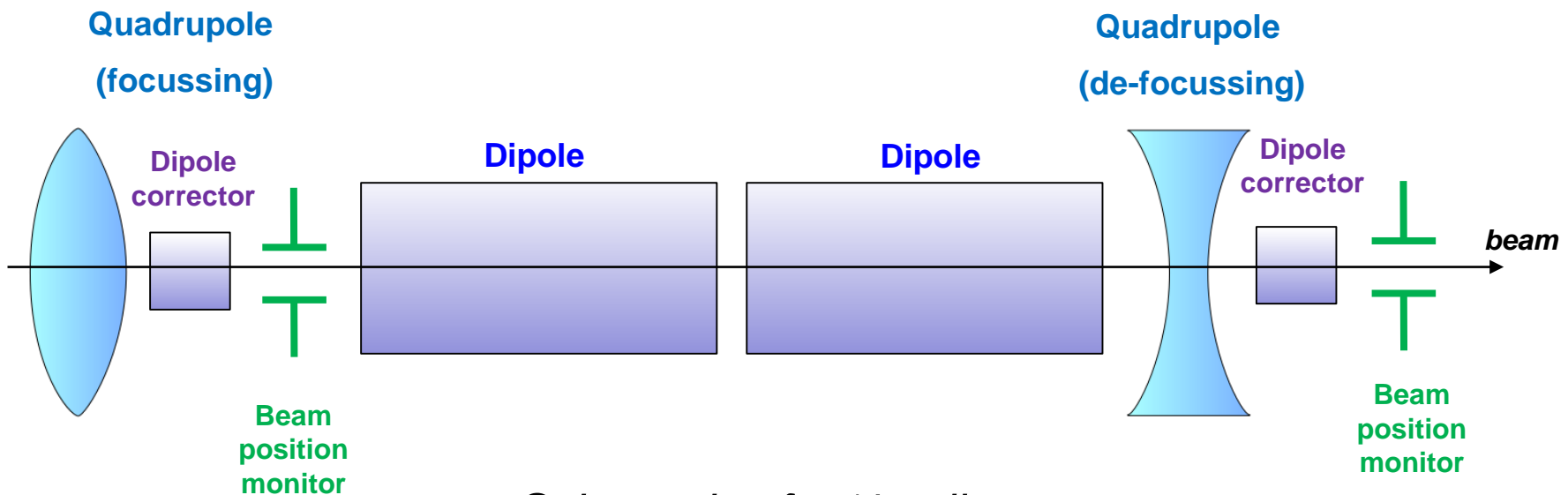
Coupling between planes

Linear imperfections and geology

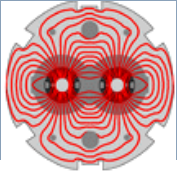
Summary



- ❑ An accelerator is usually build using a number of basic ‘cells’.
- ❑ The cell layouts of an accelerator come in many subtle variants.
- ❑ For today we consider a simple FODO cell containing:
 - **Dipole magnets** to bend the beams,
 - **Quadrupole magnets** to focus the beams,
 - **Beam position monitors** (BPM) to measure the beam position,
 - **Small dipole corrector magnets** for beam steering.



Schematic of a 1/2 cell

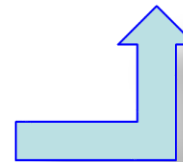
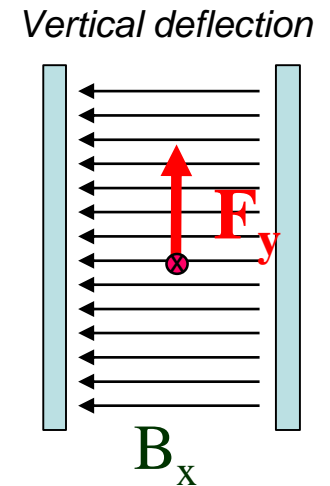
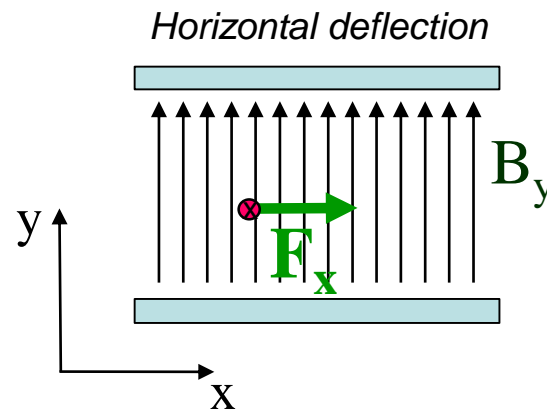


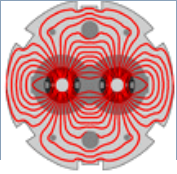
- The dipole has two magnetic poles and generates a **homogeneous field** providing a constant force on all beam particles – used to **deflect** the beam.
 - A dipole corrector is just a small version of such a magnet, dedicated to steer the beam as we will see later.

Lorentz force:

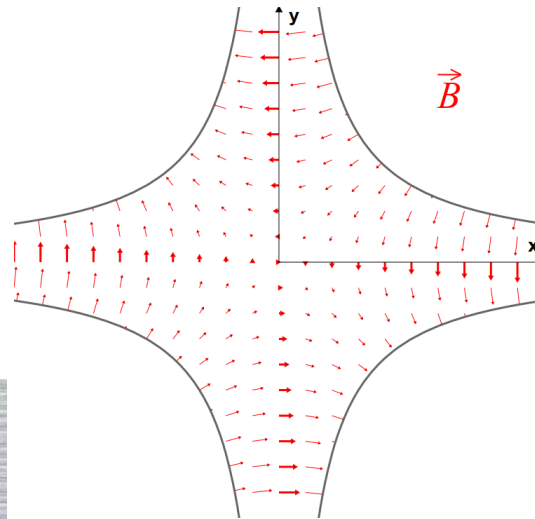
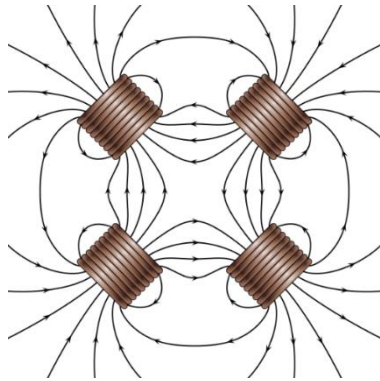
$$F = q \vec{v} \times \vec{B}$$

orthogonal to the speed and magnetic field directions

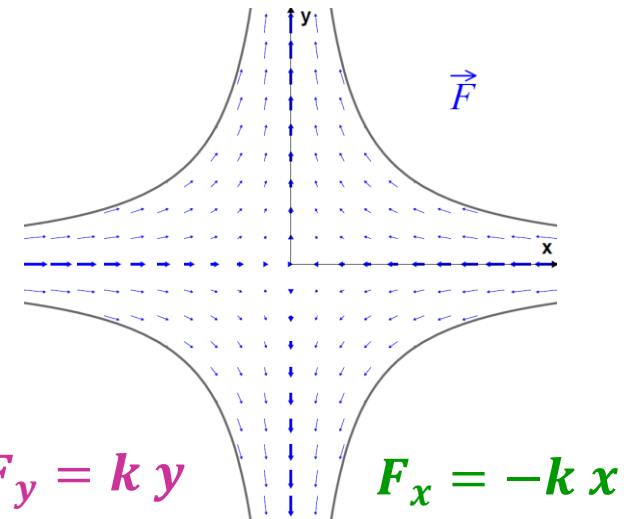




- A quadrupole has 4 magnetic poles.
- A quadrupole provides a field (force) that **increases linearly** with the distance to the quadrupole center – provides **focussing** of the beam.
 - Similar to an optical lens, except that a quadrupole is focussing in one plane, defocussing in the other plane.



Force pushes the particle away from the center →
defocussing

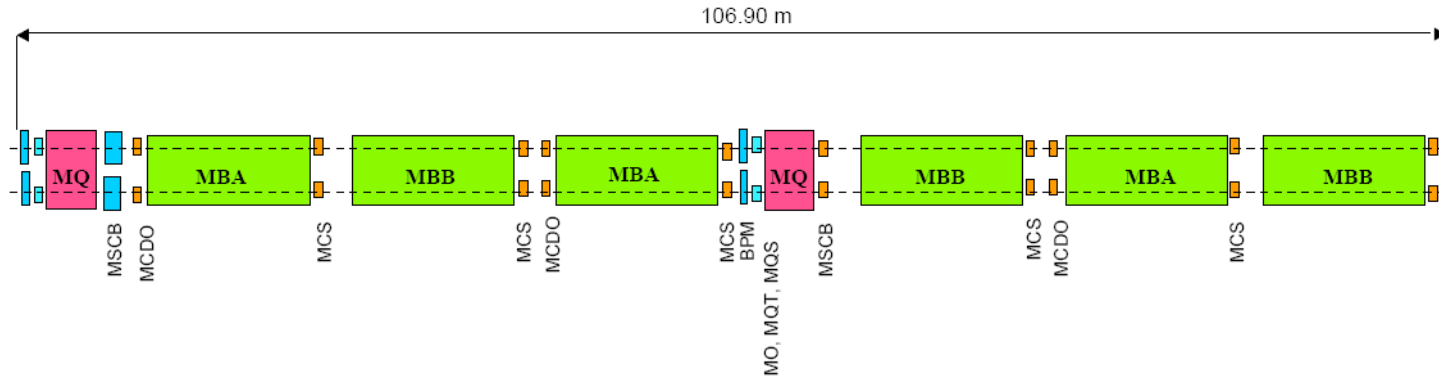


Force pushes the particle towards the center →
focussing





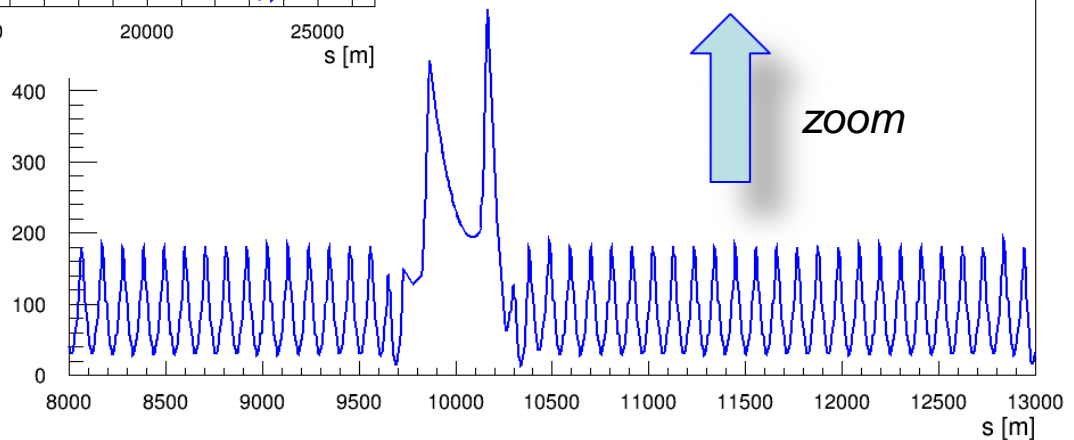
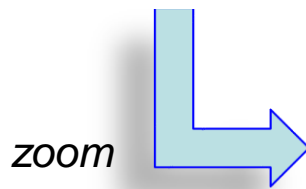
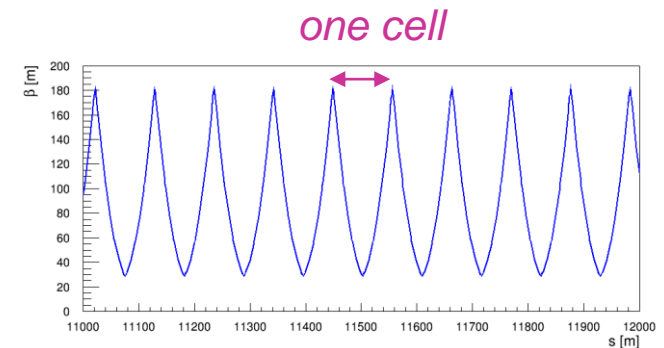
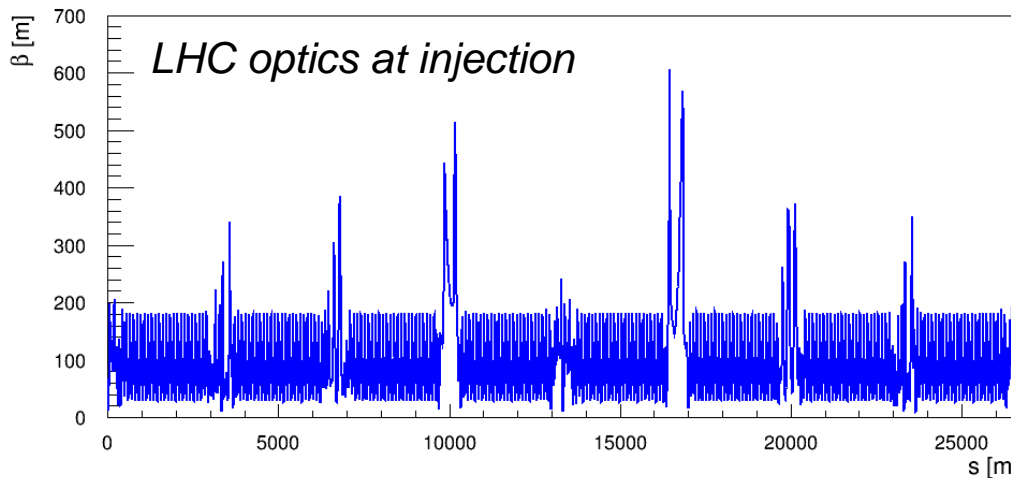
- The LHC arc section are equipped with 107 m long F0D0 cells. Besides our 3 main elements the LHC cell is equipped with other correction (trim) magnets.

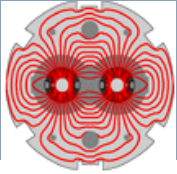


- **MB:** main dipole
- **MQ:** main quadrupole
- MQT: trim quadrupole
- MQS: skew trim quadrupole
- MO: lattice octupole (Landau damping)
- **MSCB:** sextupole + orbit corrector dipole
- MCS: Spool piece sextupole
- MCDO: Spool piece 8 / 10 pole
- **BPM:** Beam position monitor

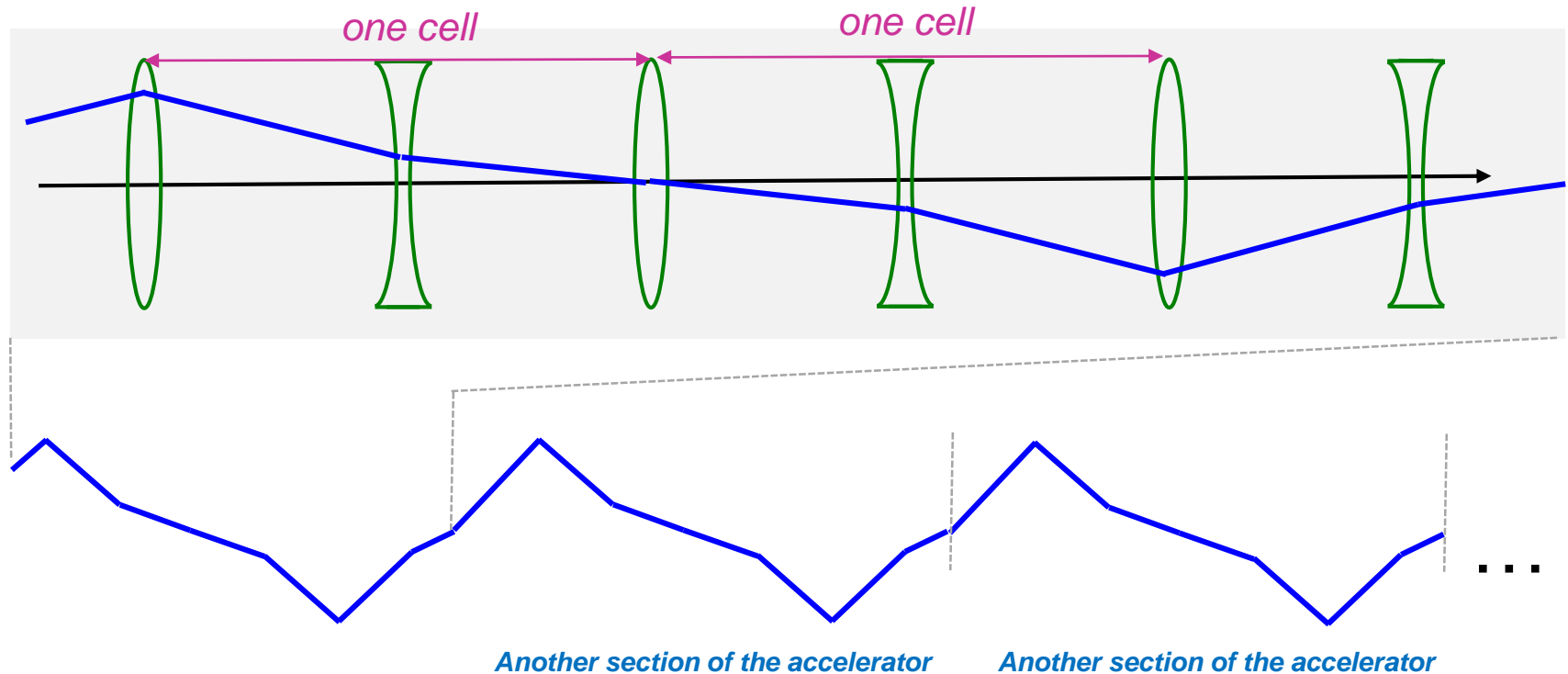


- There are a few quantities related to a beam optics in a circular accelerator that we will need for the lecture:
 - The **betatron function** (β) that defines the beam envelope,
 - **Beam size / envelope is proportional to $\sqrt{\beta}$**
 - The **betatron phase advance** (μ) that defines the phase of an oscillation.





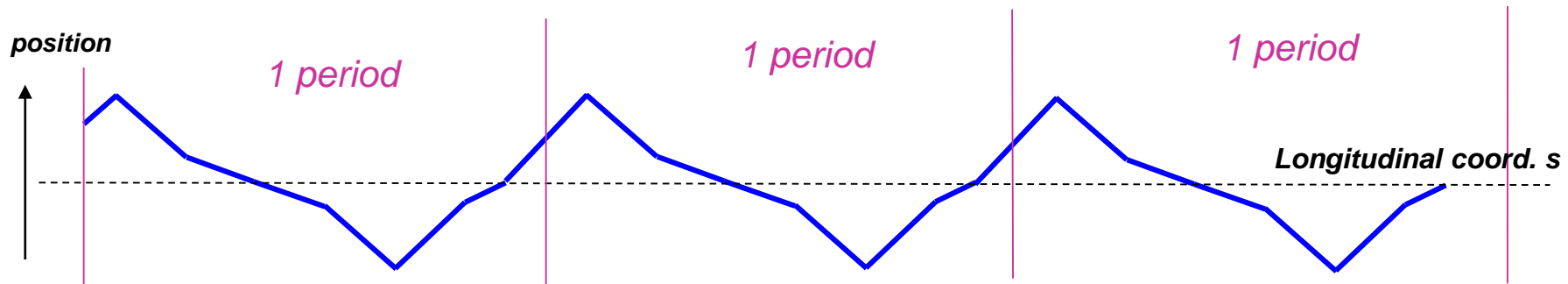
- Consider a particle moving in a section of the accelerator lattice. The focussing elements make it bounce back and forth.



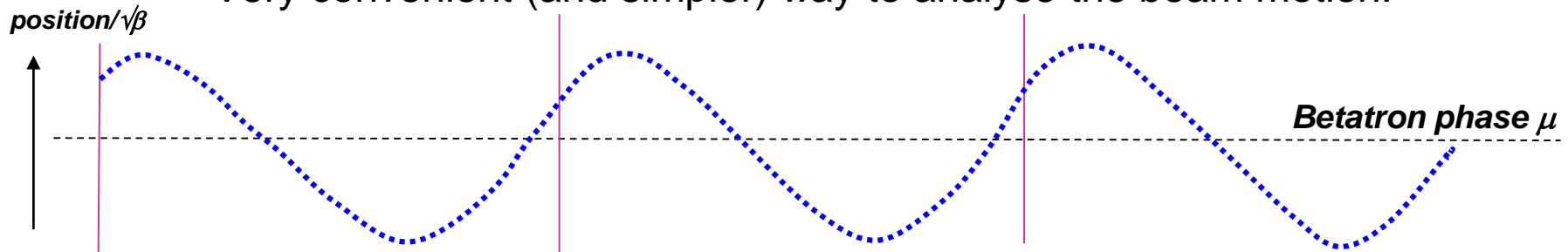
- Does this not look a bit like a periodic oscillation? This is called a **betatron oscillation**.

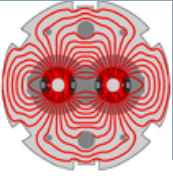


- The number of oscillation periods for one turn of the machine is called the machine **tune (Q)** or **betatron tune**.
 - In this **example Q** is around 2.75 – 2 periods and $\frac{3}{4}$ of a period.



- It is possible to change the **coordinates** (from the longitudinal position in meters to the betatron phase advance in degrees) and transform this ‘rocky’ oscillation into a pure sinusoidal oscillation.
 - Very convenient (and simpler) way to analyse the beam motion.





Introduction

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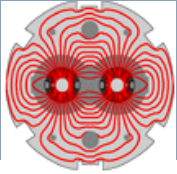
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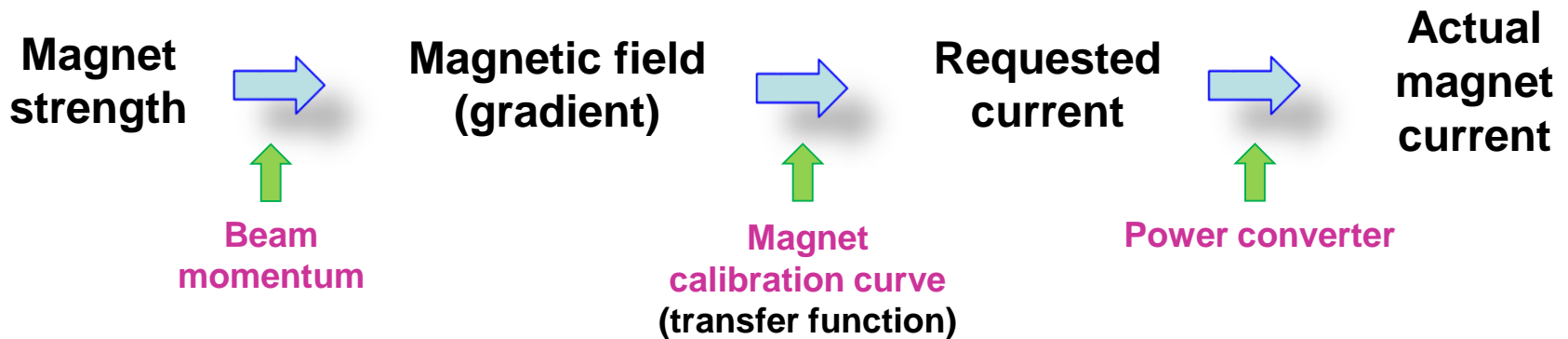
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Linear imperfections and geology

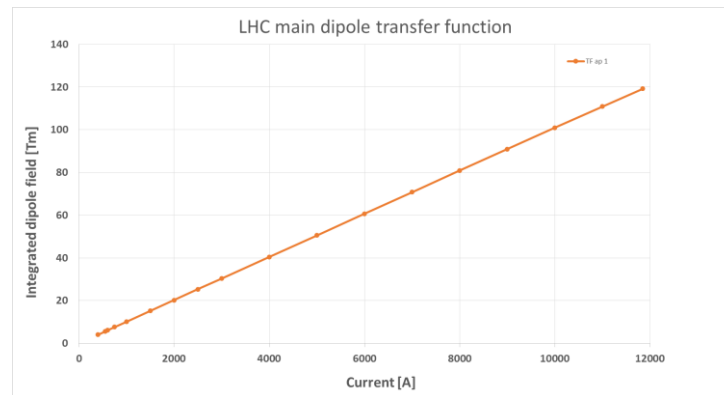
Summary



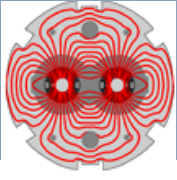
- The **physical units** of the machine model defined by the accelerator physicist must be converted into **magnetic fields** and eventually into **currents** for the power converters that feed the magnet circuits.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by **uncertainties** or **errors** on:
 - Beam momentum, magnet calibrations and power converter regulation.



Example of the LHC main dipole calibration curve

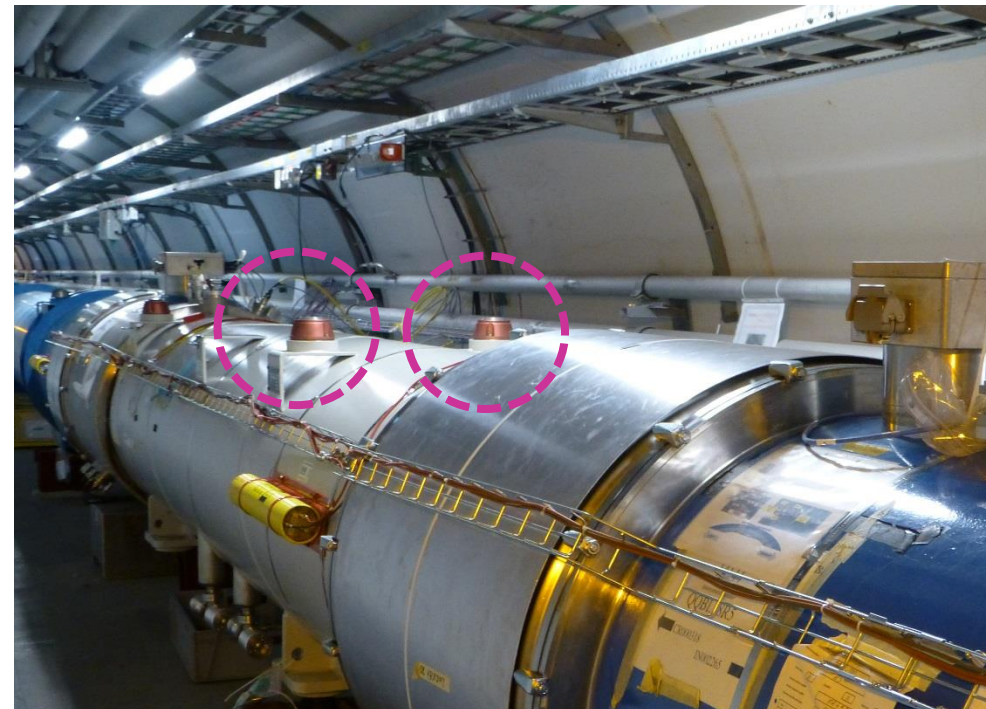
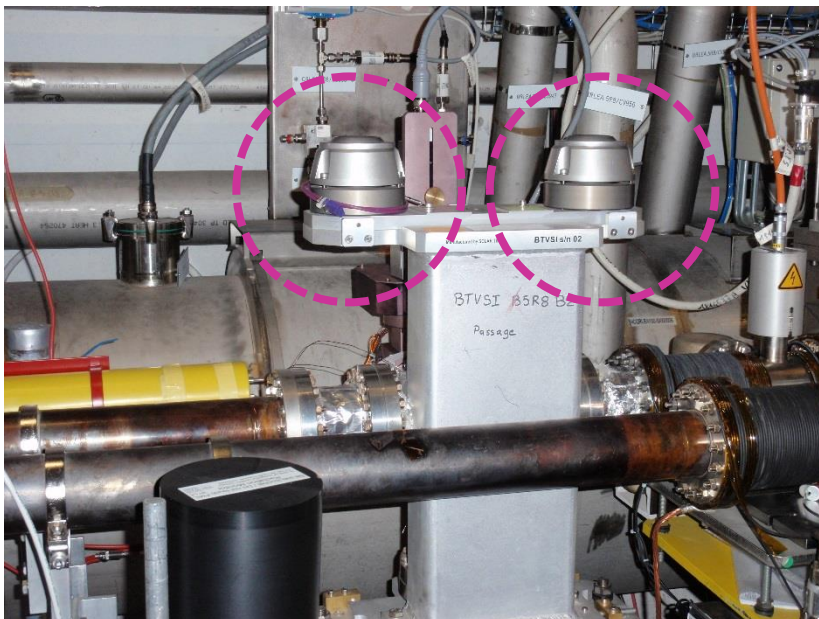


From the lab to the tunnel

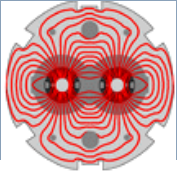




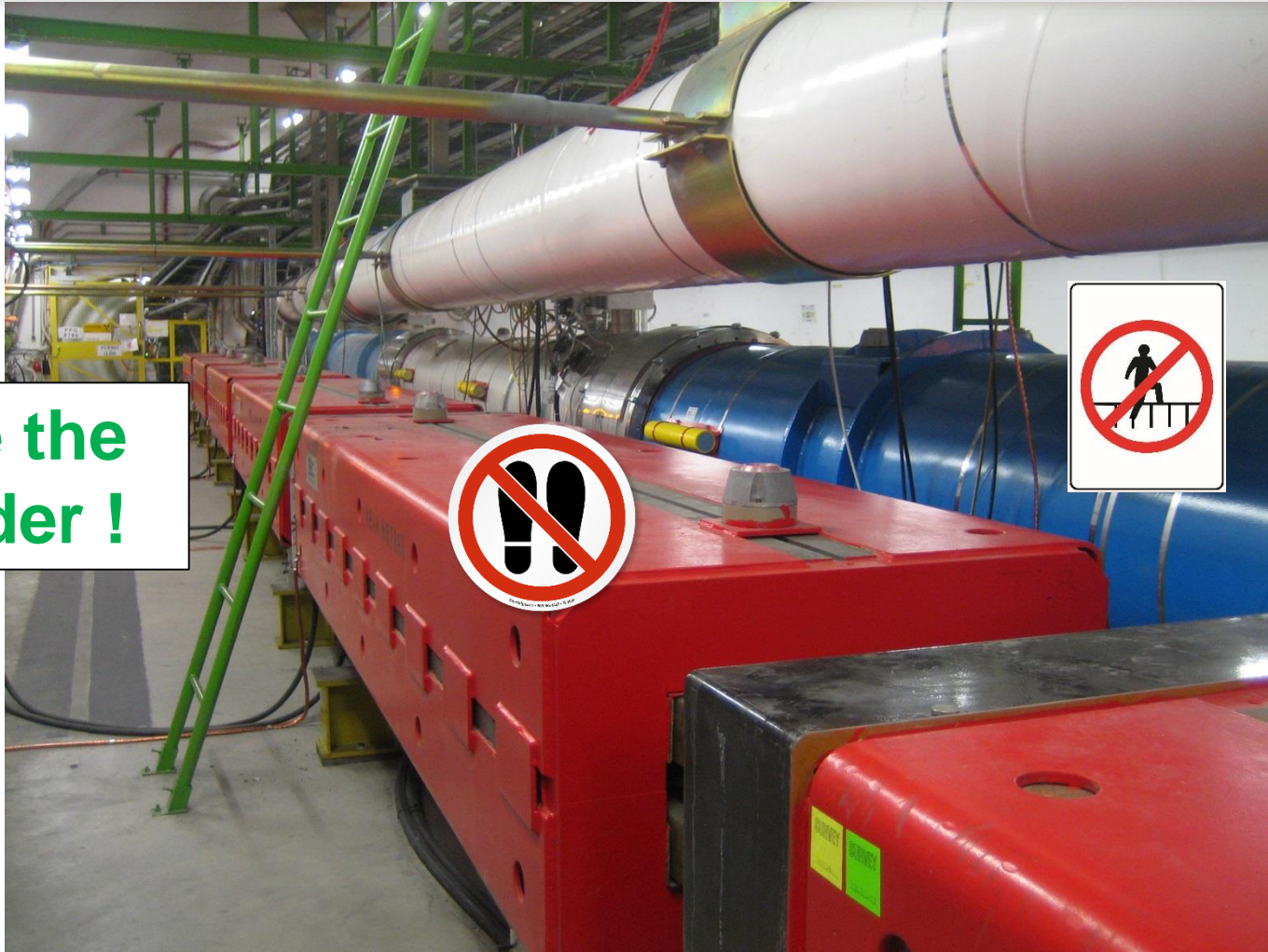
- ❑ To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of micrometres for CLIC !
 - At the CERN hadron machines we aim for accuracies of around **0.1 mm**.
- ❑ The alignment process implies:
 - Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
 - Precise in-situ alignment (position and angle) of the element in the tunnel.
- ❑ **Alignment errors** are a common source of imperfections.



A good attitude in the tunnel

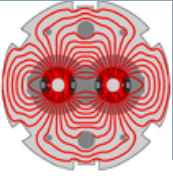


Please remember that accelerator components in the CERN tunnels are carefully aligned – please treat with respect !



Use the ladder !





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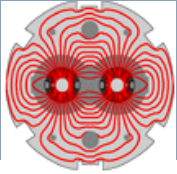
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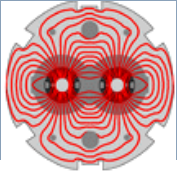
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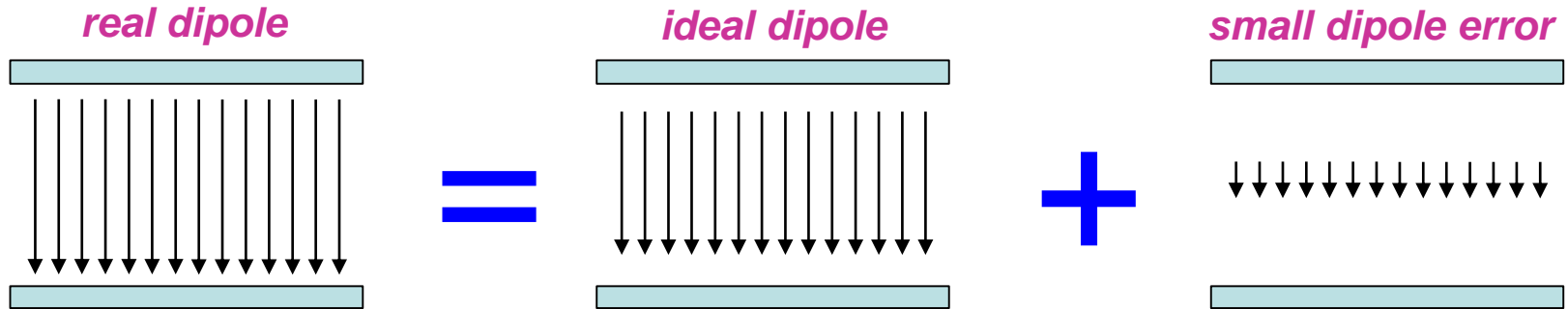
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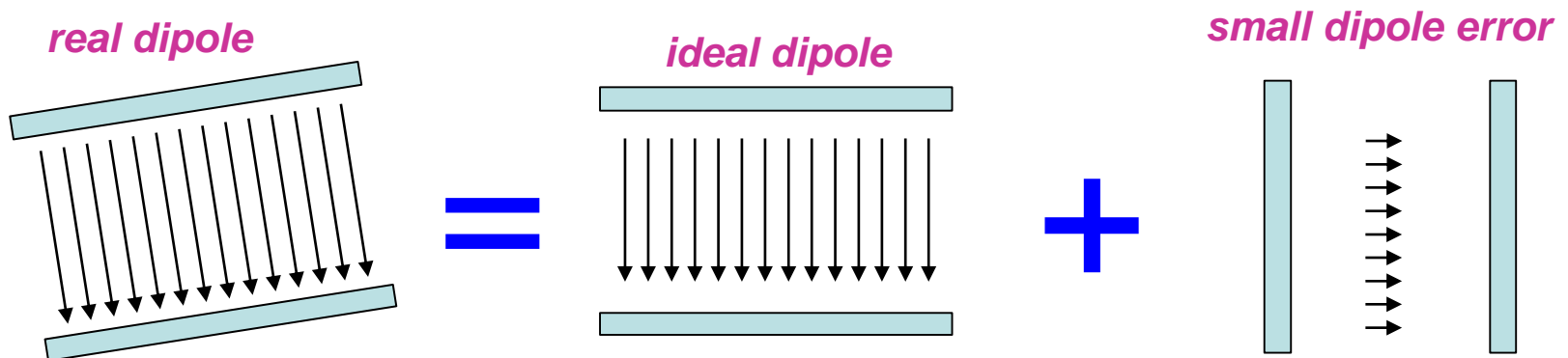
- ❑ The presence of an **unintended deflection** along the path of the beam is a first category of imperfections.
- ❑ This case is also in general the first one that is encountered when beam is first injected...
- ❑ The **dipole orbit corrector** is added to the cell to **compensate** the effect of **unintended deflections**.
 - With the orbit corrector we can generate a deflection of opposite sign and amplitude that compensates locally the imperfection.
- ❑ How can an **unintended deflection** appear?

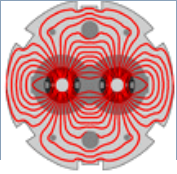


- The first source is a **field error** (deflection error) of a dipole magnet.
- This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc).
 - The imperfect dipole can be expressed as a perfect one + a small error.



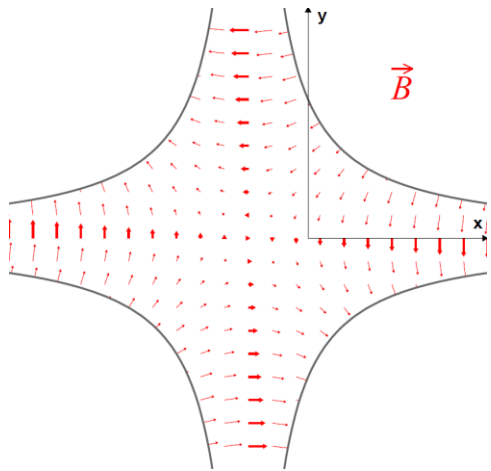
- A small **rotation (misalignment)** of a dipole magnet has the same effect, but in the other plane.





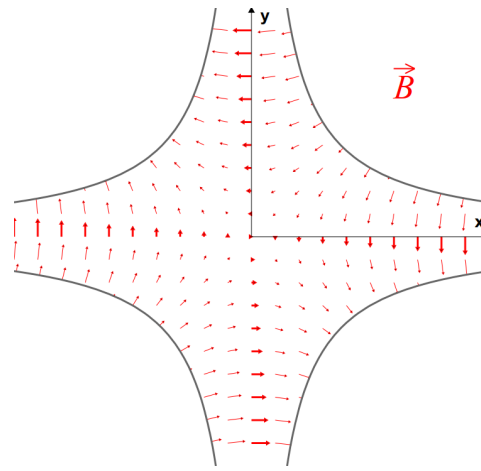
- The second source is a **misalignment** of a quadropole magnet.
 - The misaligned quadropole can be represented as a perfectly aligned quadropole plus a small deflection.

real quadropole



*Non-zero magnetic field
on the beam axis !*

ideal quadropole

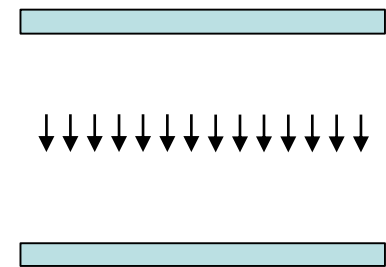


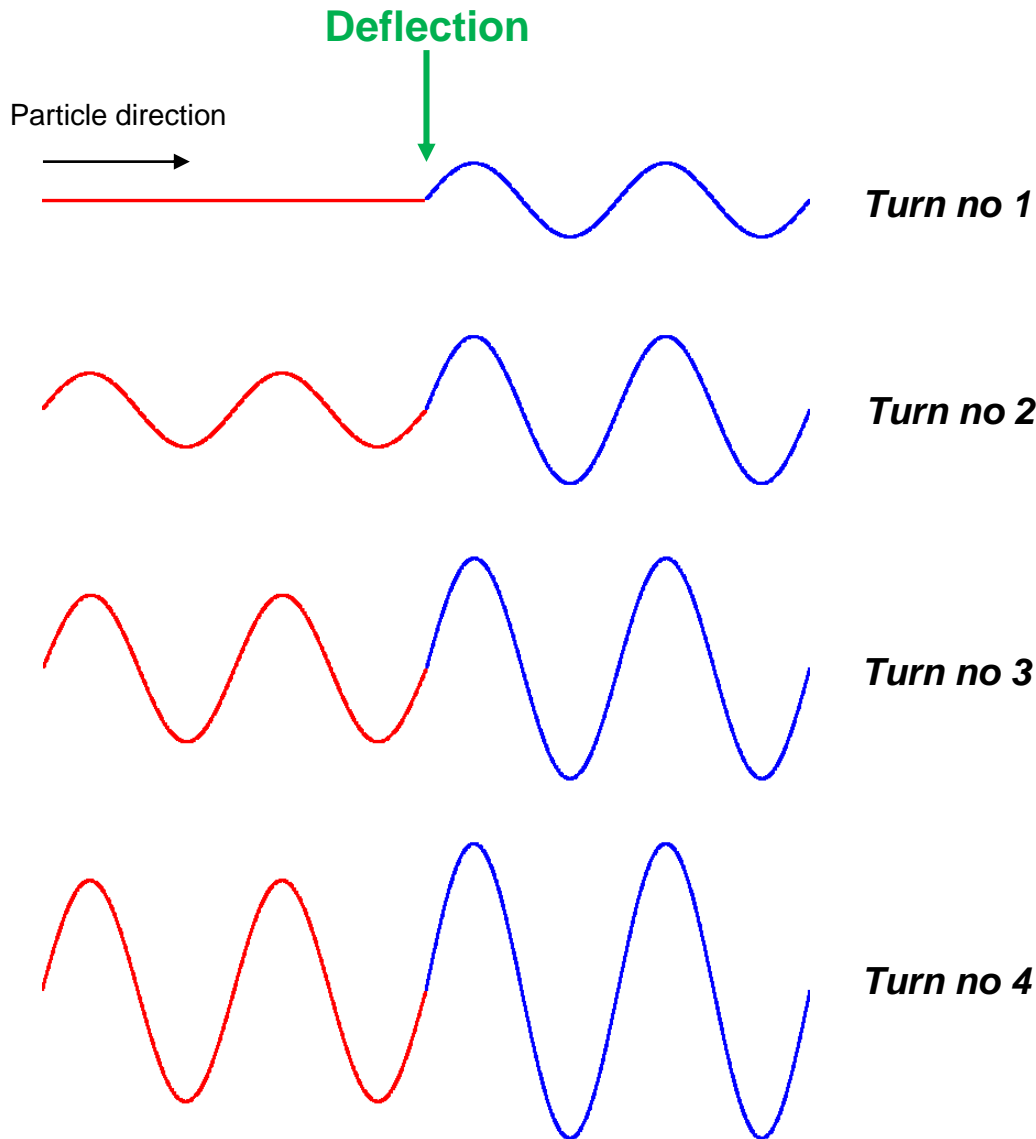
*No magnetic field on
the beam axis*

=

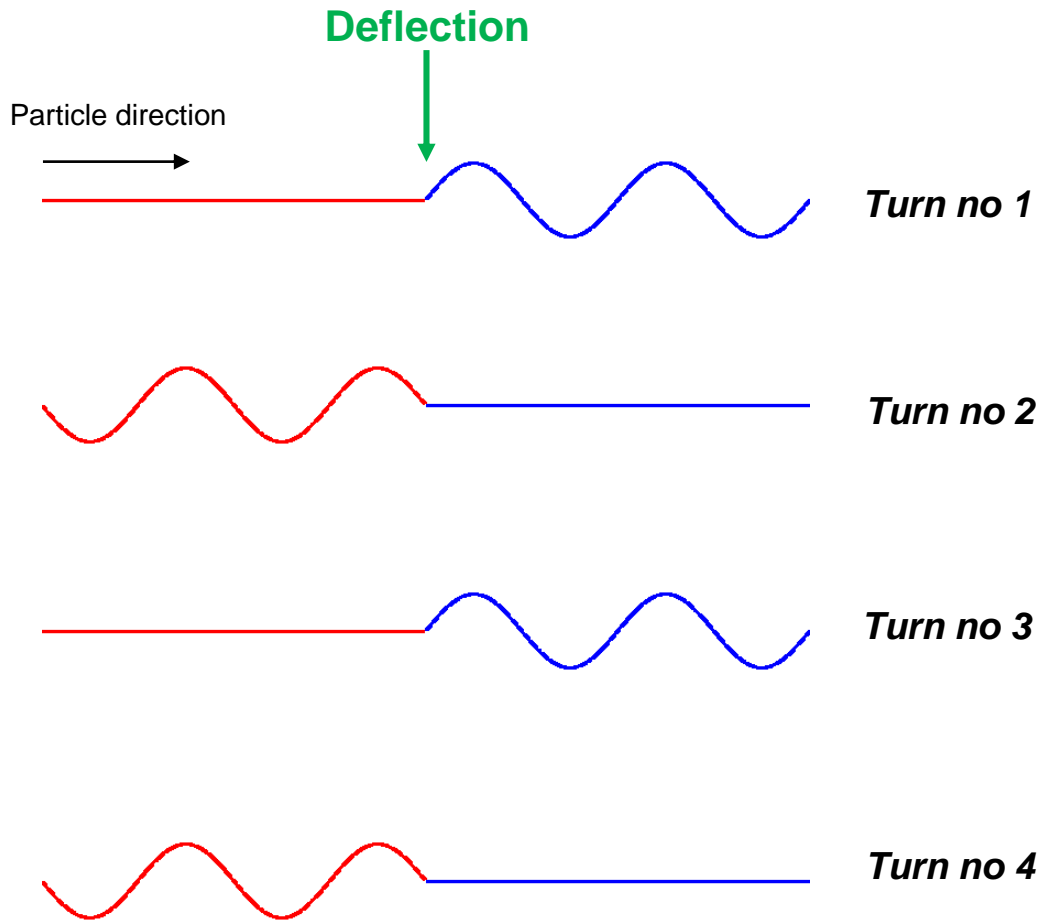
+

small dipole error

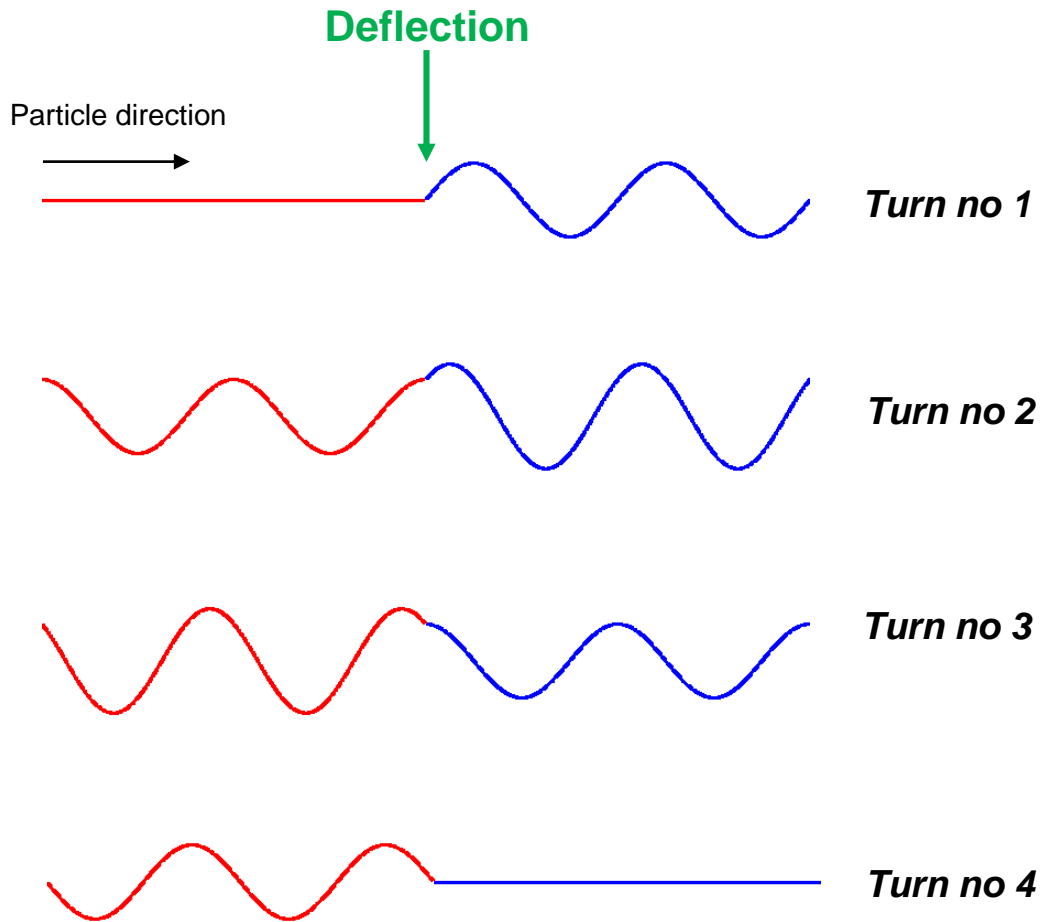
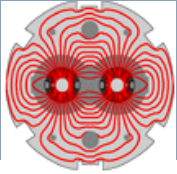




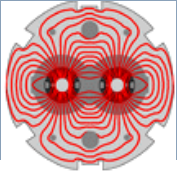
- We set the machine tune to an integer value:
 - $Q = n \in \mathbb{N}$
- When the tune is an integer number, **the deflections add up on every turn !**
 - The amplitudes diverge, the particles do not stay within the accelerator vacuum chamber.
- We just encountered our first **resonance** – the integer resonance that occurs when $Q = n \in \mathbb{N}$



- We set the machine tune to a half integer value:
 - $Q = n+0.5, n \in \mathbb{N}$
- For half integer tune values, **the deflections compensate on every other turn !**
 - The amplitudes are stable.
- This looks like a much better working point for Q !

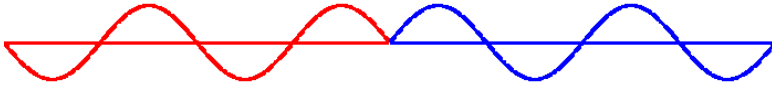


- We set the machine tune to a quarter integer value:
 - $Q = n + 0.25, n \in \mathbb{N}$
- For quarter tune values, **the deflections compensate every four turns!**
 - The amplitudes are stable.
- Also a reasonable working point for $Q!$



- Let's plot the 50 first turns on top of each other and change Q .
 - All plots are on the same scale

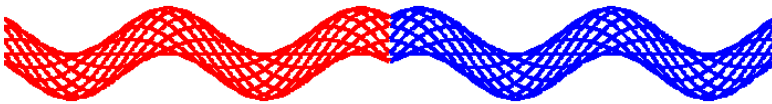
$Q = n + 0.5$



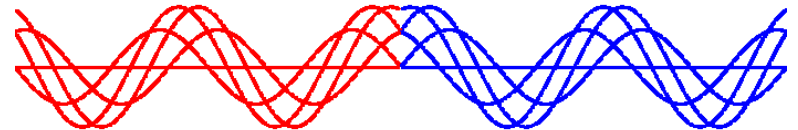
$Q = n + 0.4$



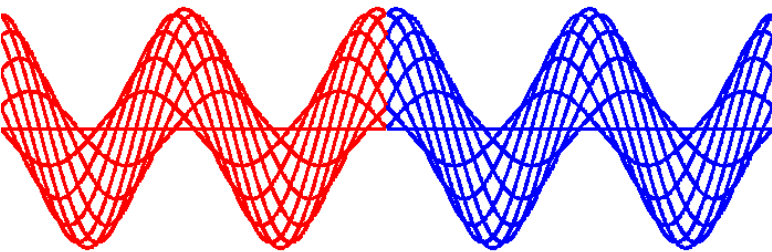
$Q = n + 0.3$



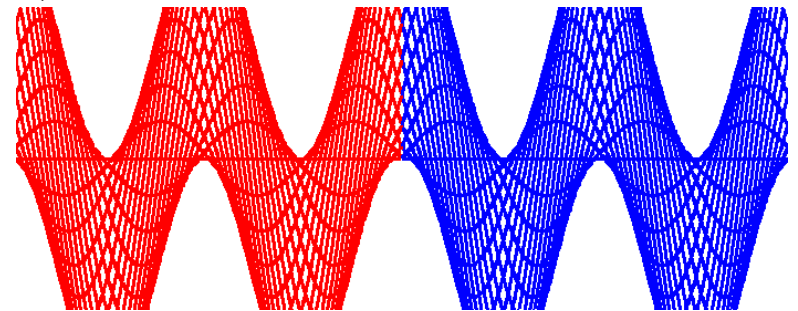
$Q = n + 0.2$



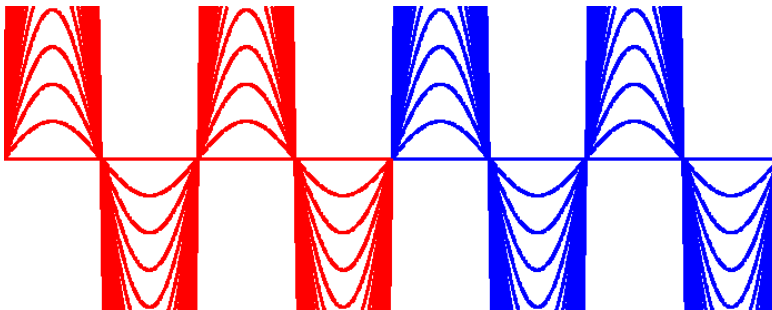
$Q = n + 0.1$



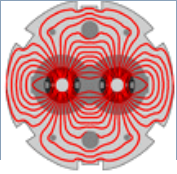
$Q = n + 0.05$



$Q = n$



- The particles **oscillate around a stable mean value** ($Q \neq n$)!
- The amplitude diverges as we approach $Q = n \rightarrow$ integer resonance



- The stable mean value around which the particles oscillate is called the **closed orbit**.
 - Every particle in the beam oscillates around the closed orbit.
 - As we have seen the closed orbit ‘does not exist’ when the tune is an integer value.
- The general expression of the **closed orbit $x(s)$** in the presence of a **deflection θ** is:

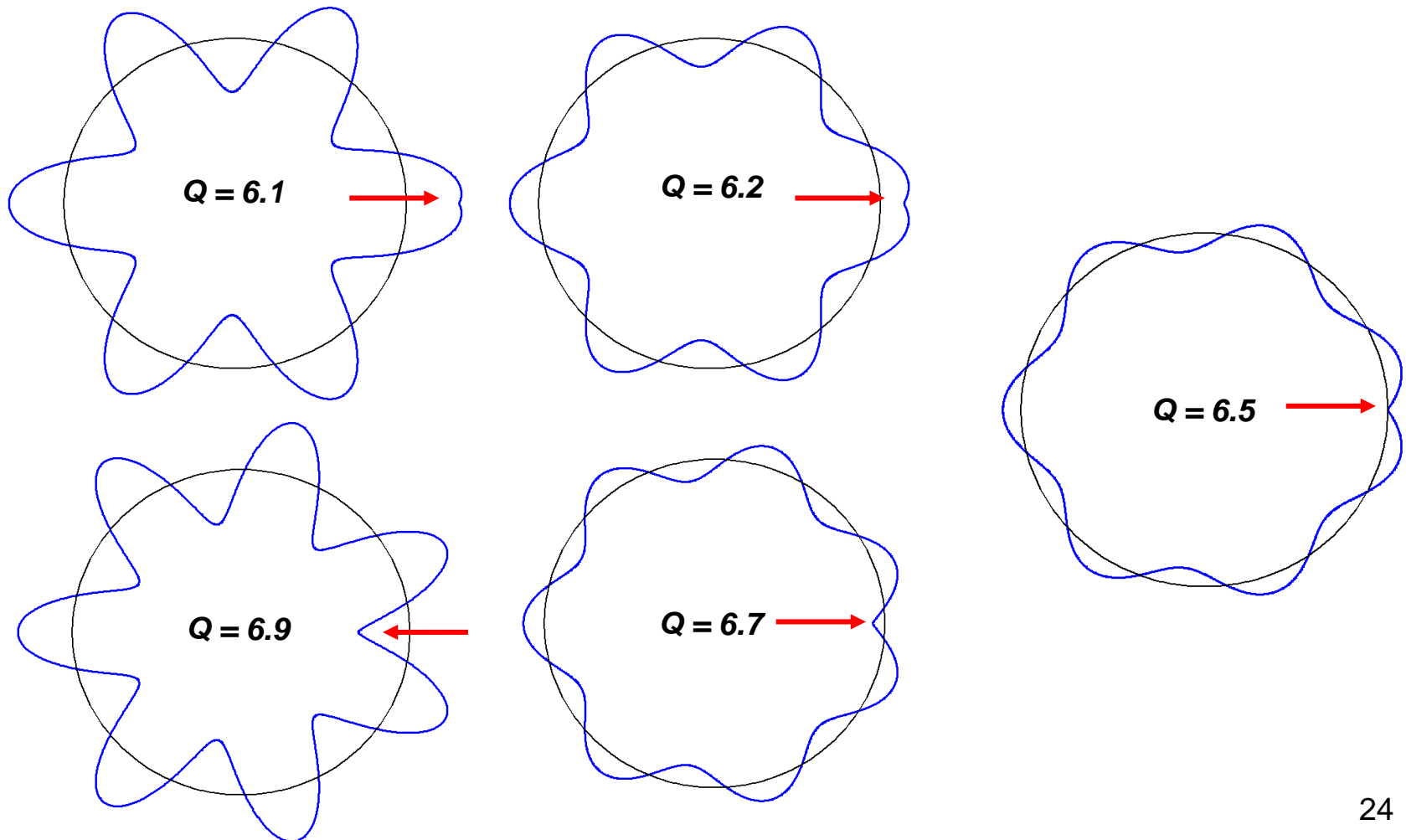
$$x(s) = \frac{\sqrt{\beta(s)\beta_\theta} \cos(|\mu(s) - \mu_\theta| - \pi Q)}{2 \sin(\pi Q)} \theta$$

amplitude modulated by the envelope β → $\sqrt{\beta(s)\beta_\theta}$
oscillating term → $\cos(|\mu(s) - \mu_\theta| - \pi Q)$
kink at the location of the deflection → θ
divergence for $Q = n$ → $2 \sin(\pi Q)$

Closed orbit example

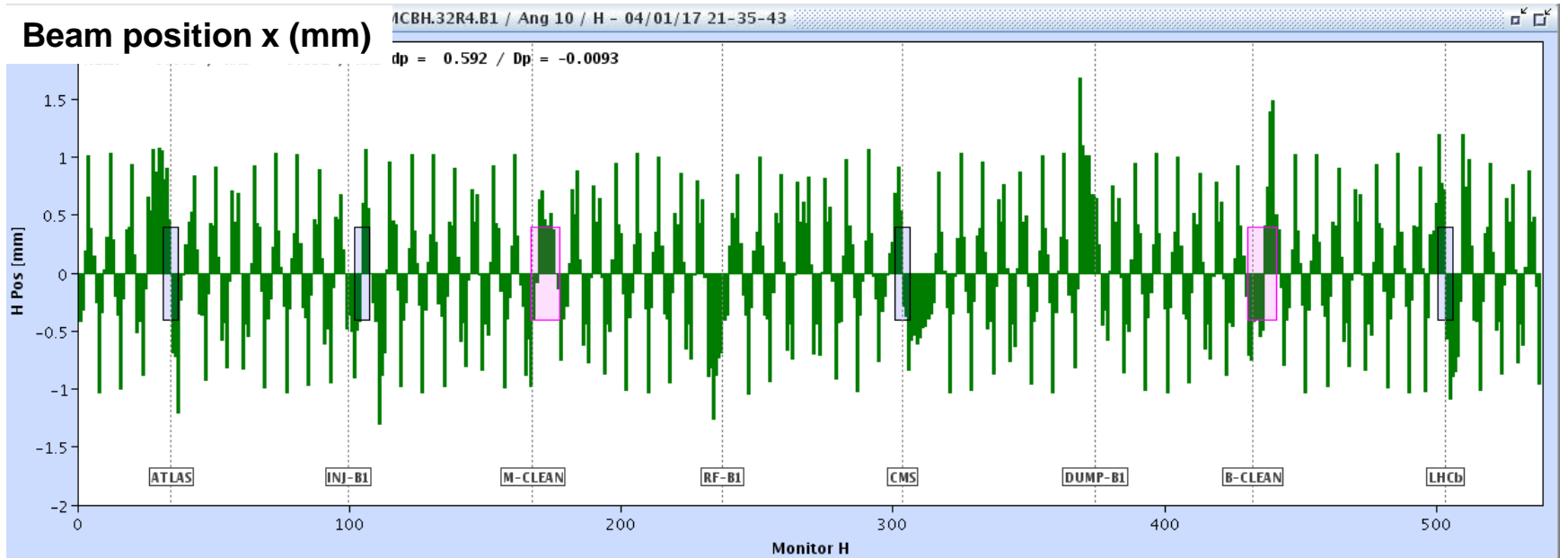


- Example of the horizontal closed orbit for a machine with tune $Q = 6 + q$.
- The **kink at the location of the deflection** (\rightarrow) can be used to localize the deflection (if it is not known) \rightarrow can be used **for orbit correction**.



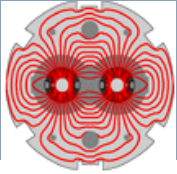


- In the example below for the 26.7km long LHC, there is **one undesired deflection**, leading to a perturbed closed orbit.



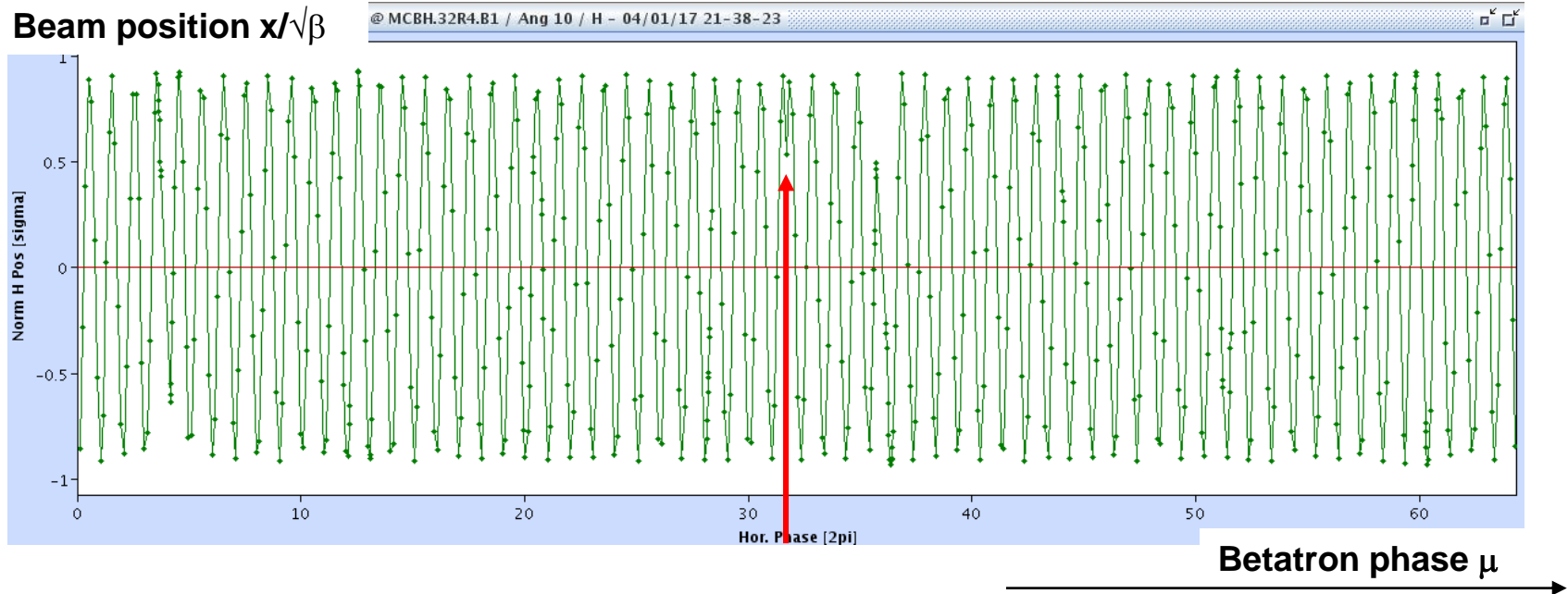
BPM index along the LHC circumference

Where is the location of the deflection?

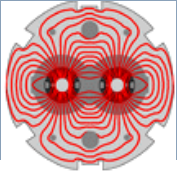


- To make our life easier we divide the position by $\sqrt{\beta(s)}$ and replace the BPM index by its phase $\mu(s)$.

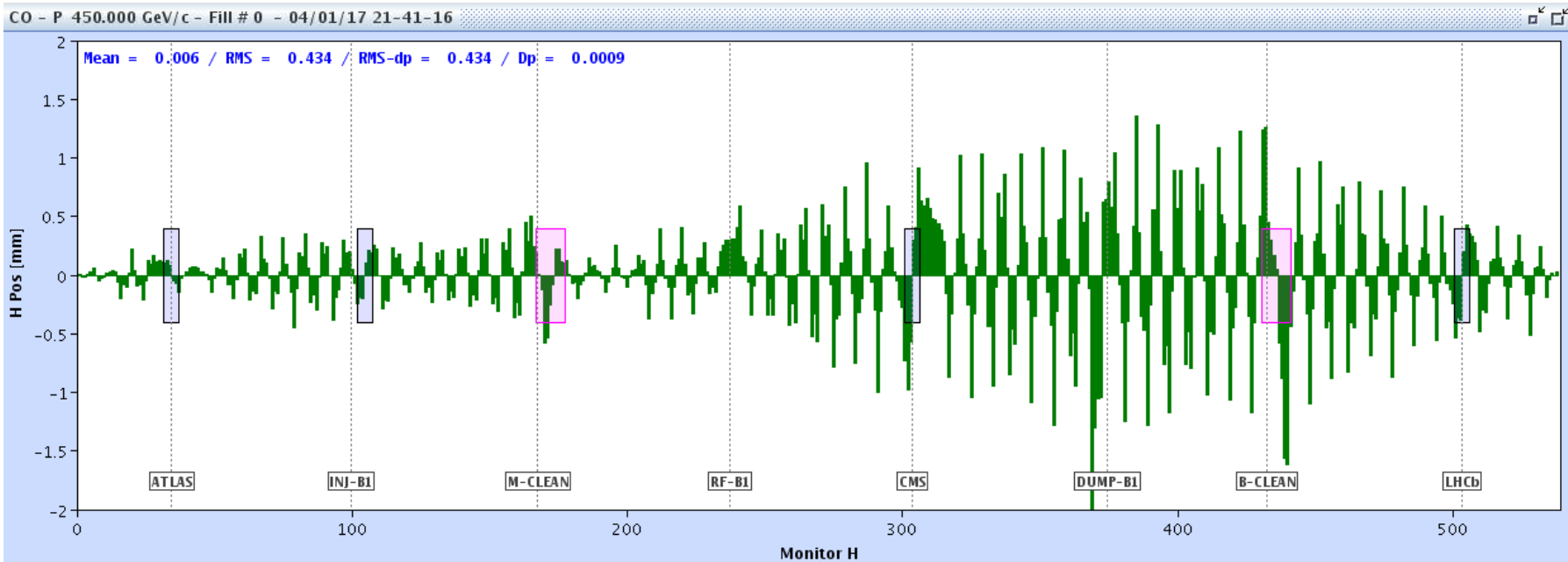
$$\frac{x(s)}{\sqrt{\beta(s)}} = \frac{\sqrt{\beta_\theta} \cos(|\mu(s) - \mu_\theta| - \pi Q)}{2 \sin(\pi Q)} \theta \propto \cos(|\mu(s) - \mu_\theta| - \pi Q)$$



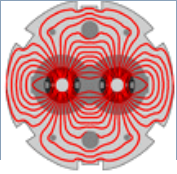
Can you localize the deflection now?



- Now a more realistic orbit with 100's of deflections.



How do we proceed to correct?



- The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

- B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines:

CLOSED ORBIT CORRECTION OF A.G. MACHINES
USING A SMALL NUMBER OF MAGNETS

by

B. Autin & Y. Marti

- **MICADO***

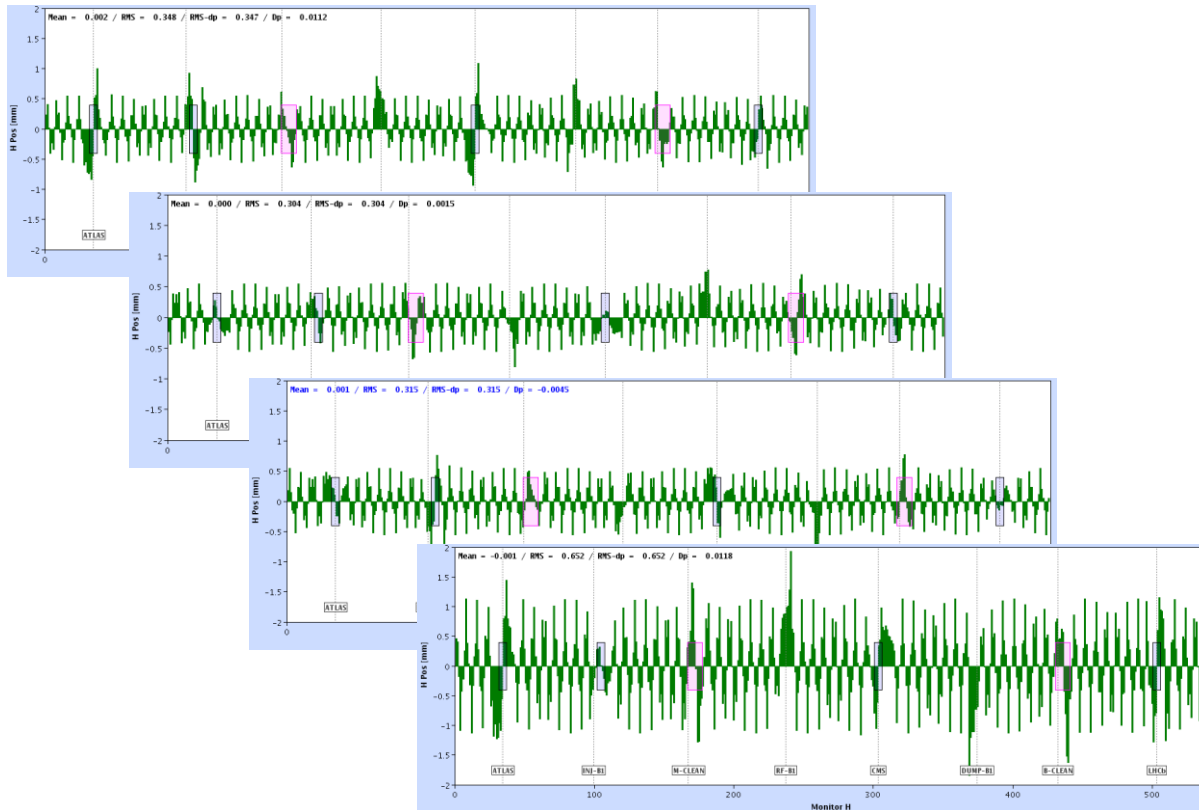
```
CALL MICADO (A, B, NDIM, M, N, AP, XA, NA, NB, NC, EPS, ITER, DP, X, NX, R, RHO).
```

* `MINimisation des CArrés des Distortions d'Orbite.`

(Minimization of the quadratic orbit distortions)



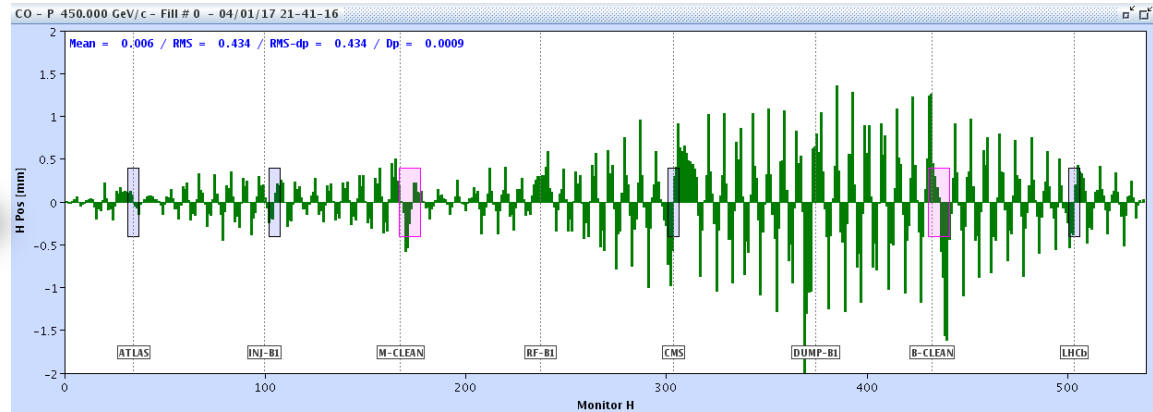
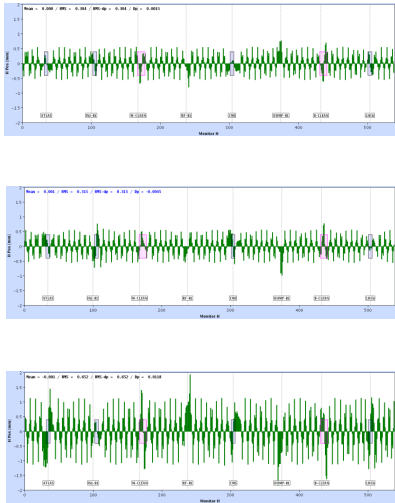
- The intuitive principle of MICADO is rather simple.
- Preparation:
 - You need a model of your machine,
 - You compute for each orbit corrector what the effect (response) is expected to be on the orbit.



■ ■ ■



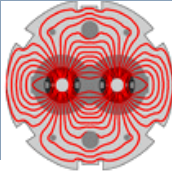
- MICADO compares the response of every corrector with the raw orbit.



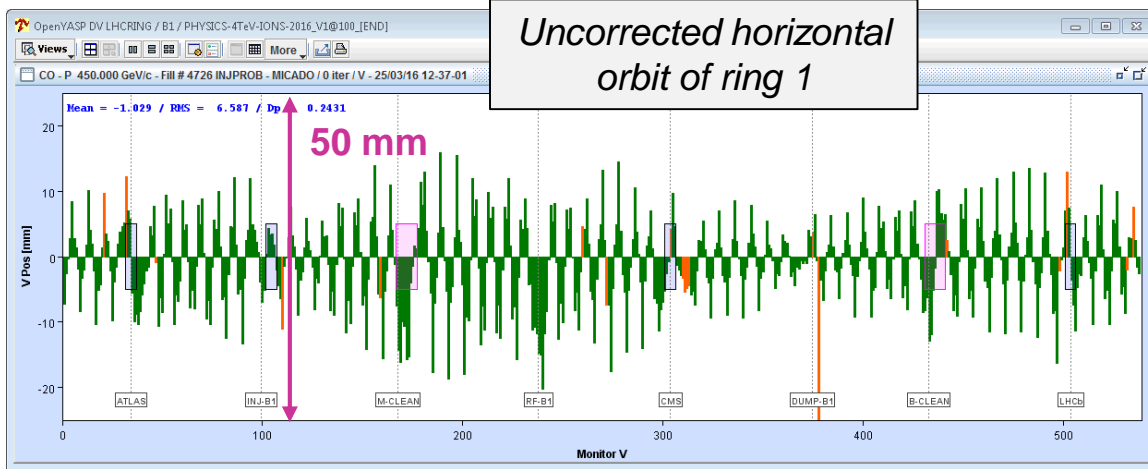
...

- MICADO then picks out the corrector that has the **best match** with the orbit, and that will give the largest improvement to the orbit deviation rms.
- The procedure can be **iterated** until the orbit is good enough (or as good as it can be).

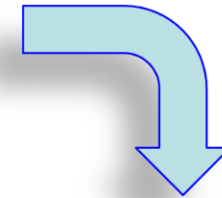
LHC orbit correction example



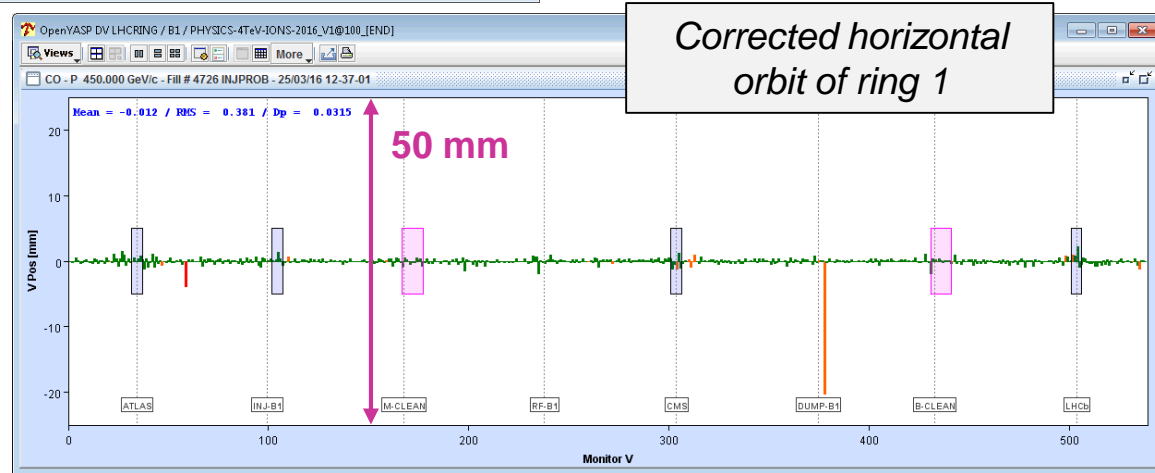
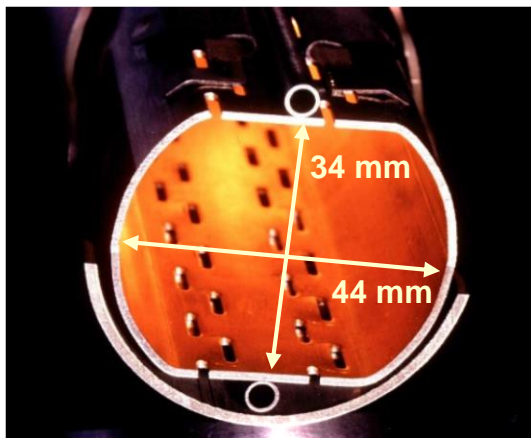
- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by more than a factor 20.



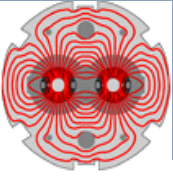
MICADO & Co



LHC vacuum chamber



At the LHC a good orbit correction is vital !



Introduction

Imperfection - sources

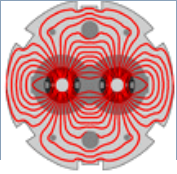
Orbit perturbations

Optics perturbations

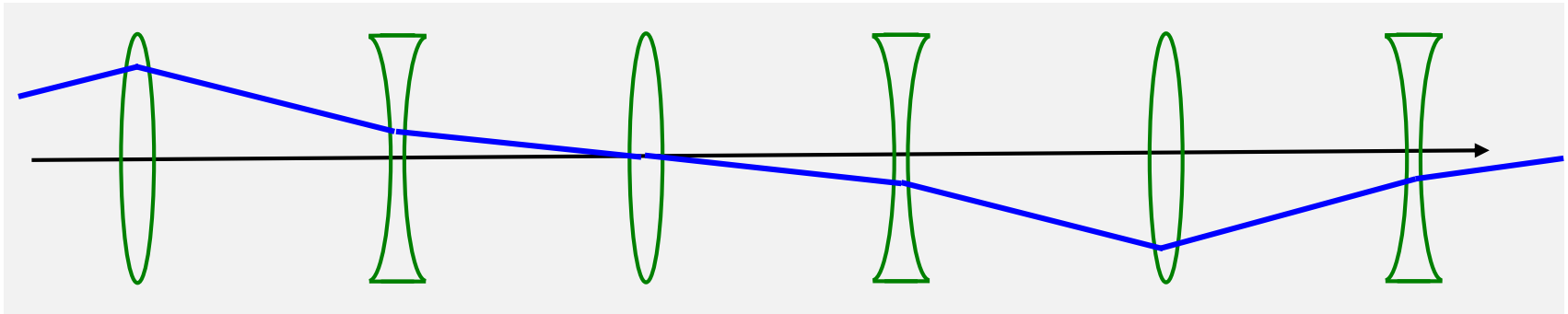
Coupling between planes

Linear imperfections and geology

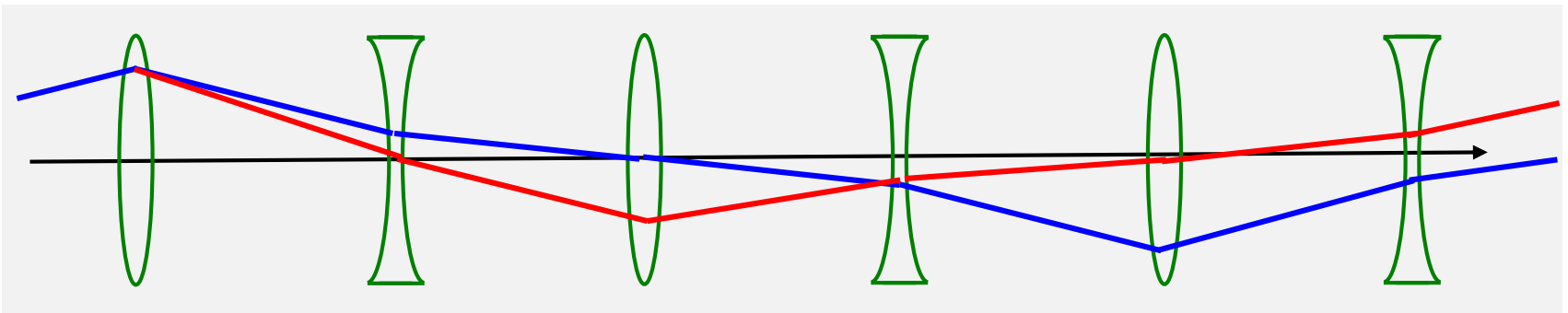
Summary



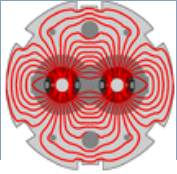
- What is the impact of a **quadrupole gradient error**?
 - Let us consider a particle oscillating in the lattice.



Too strong gradient / lens

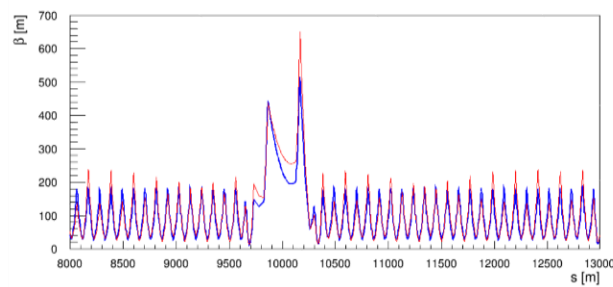
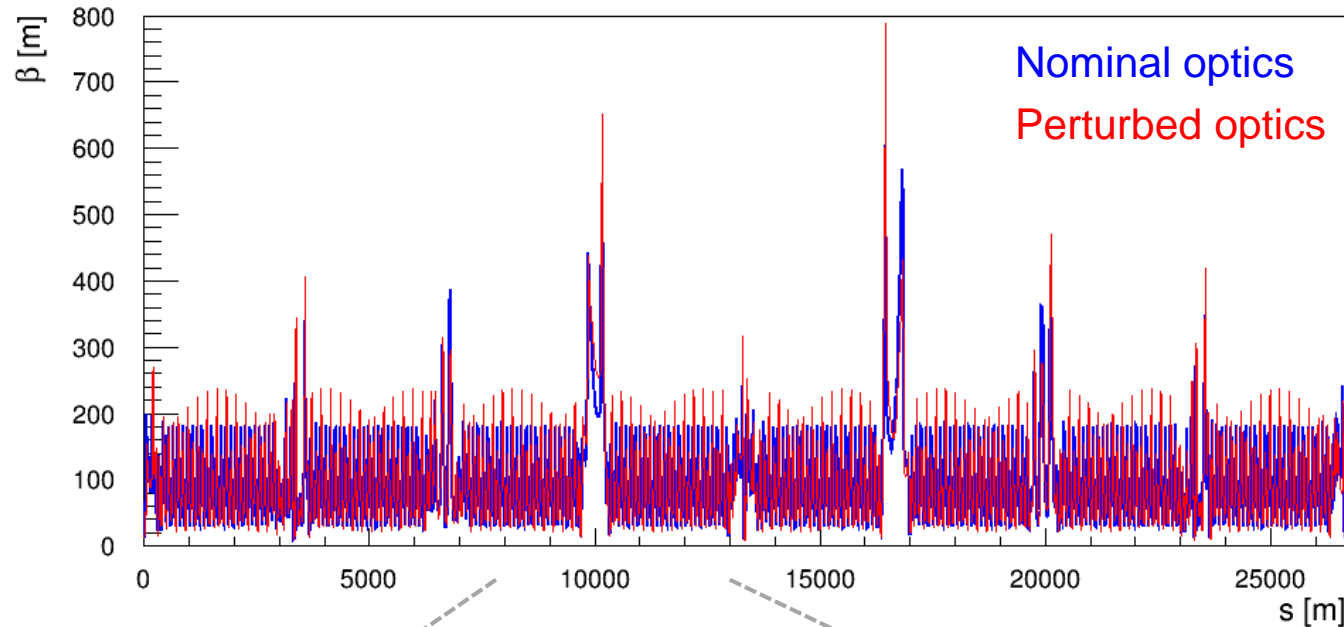


*The oscillation period is affected → **change of tune**, here Q increases !*

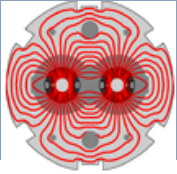


- In a ring a focussing error affects the beam optics and envelope (size) over the entire ring ! It also changes the tune.

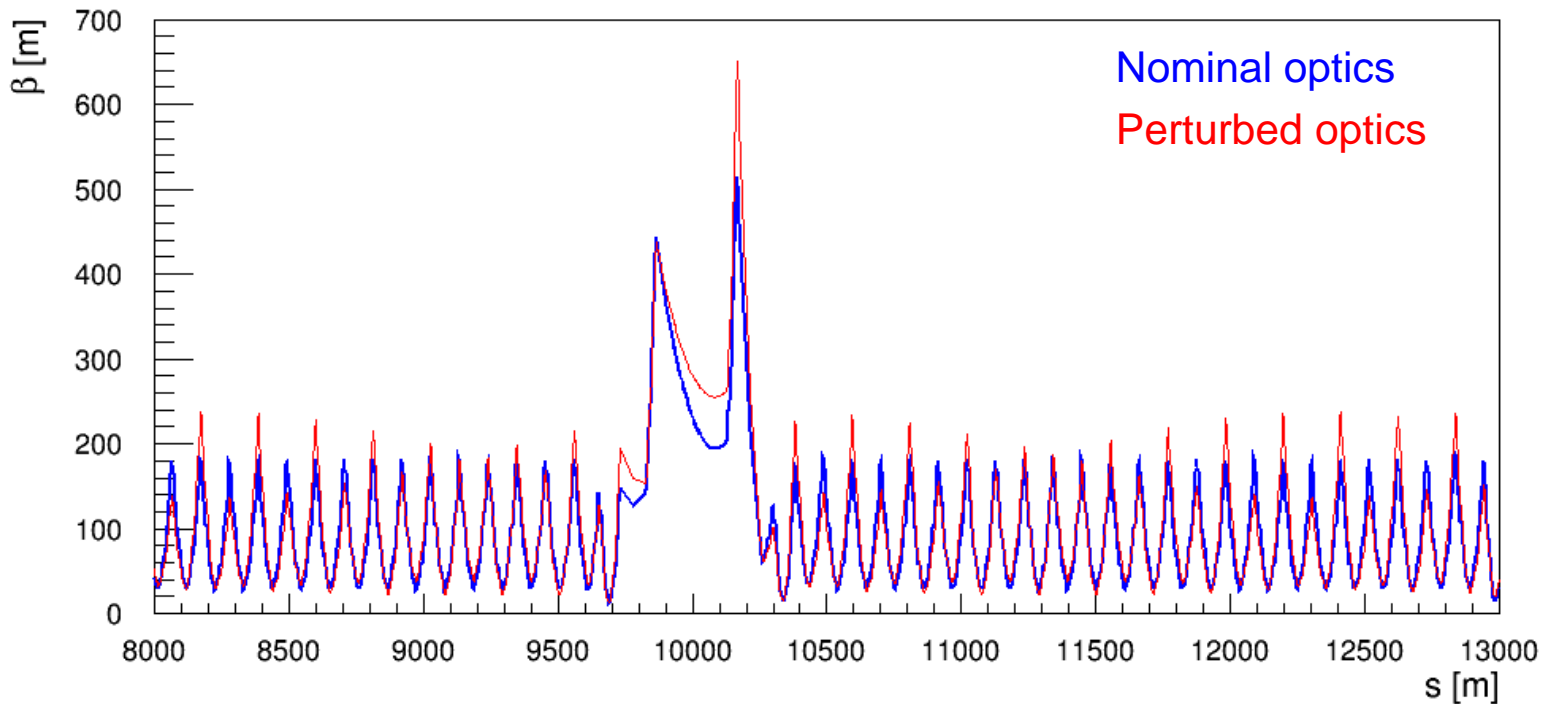
Example for LHC: one quadrupole gradient is incorrect



Zoom into a subsection

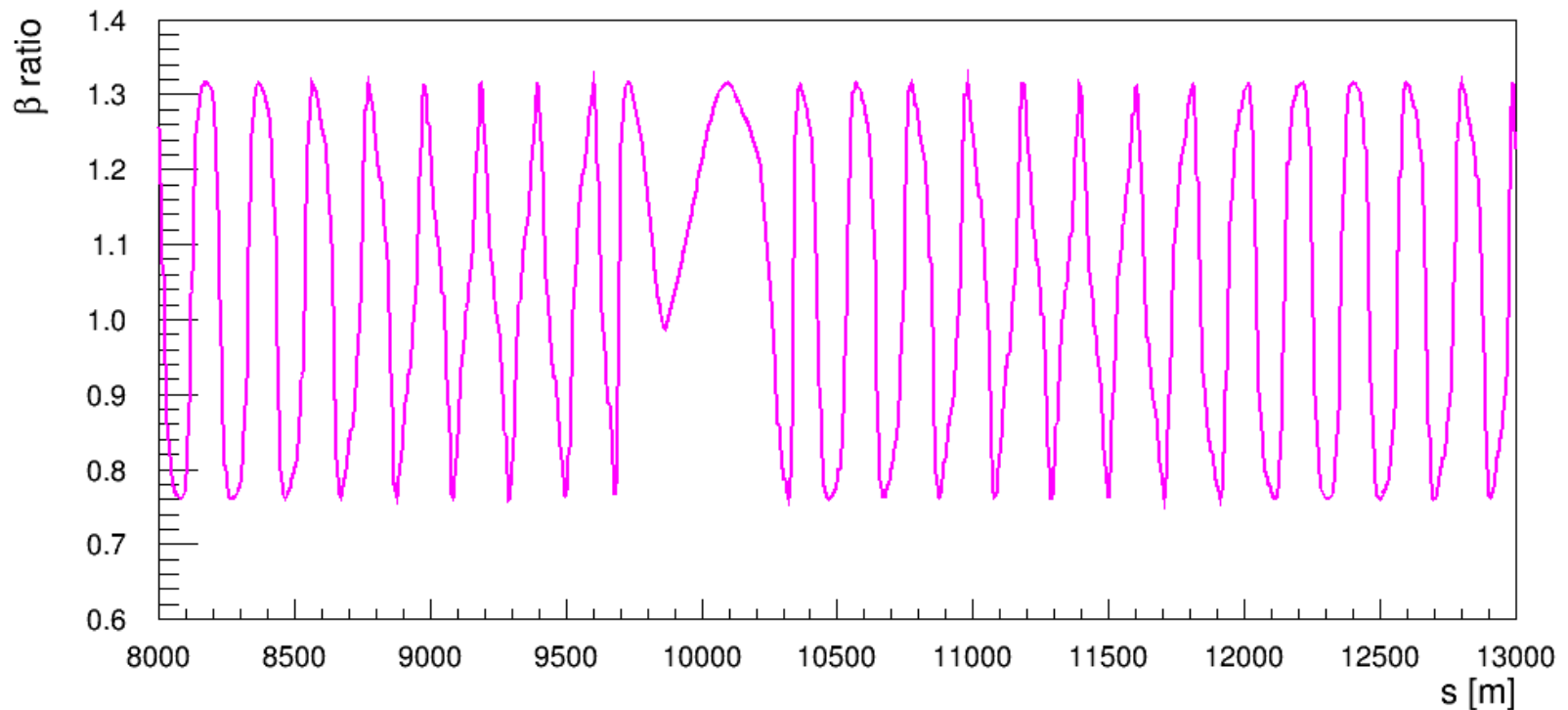


- The local beam optics perturbation... note the oscillating pattern of the error.



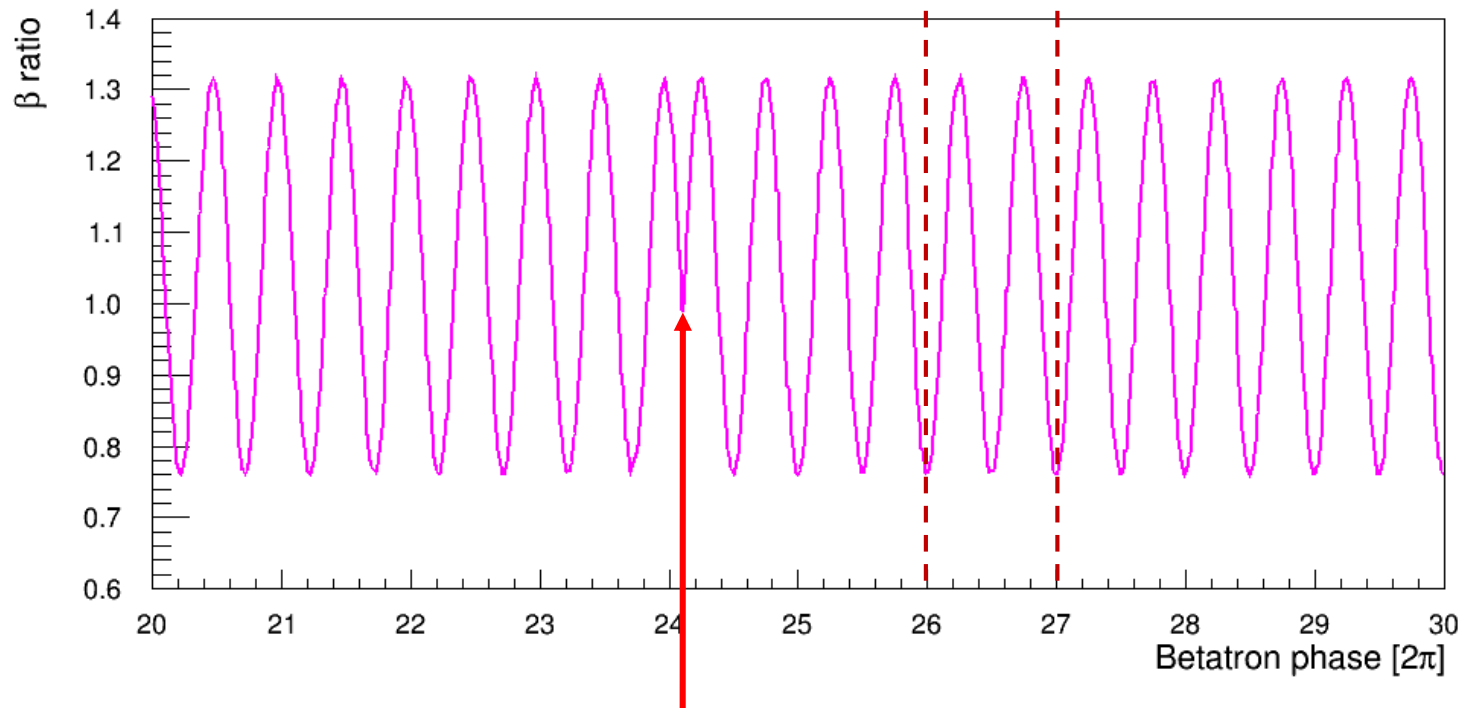


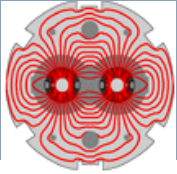
- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** ('beta-beating'). The amplitude of the perturbation is the same all over the ring !



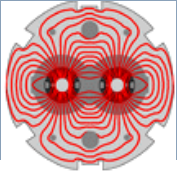


- The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance.
- The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink !
 - If you watch closely you will observe that there are **two oscillation periods per 2π (360 deg) phase**. The beta-beating frequency is **twice** the frequency of the orbit !

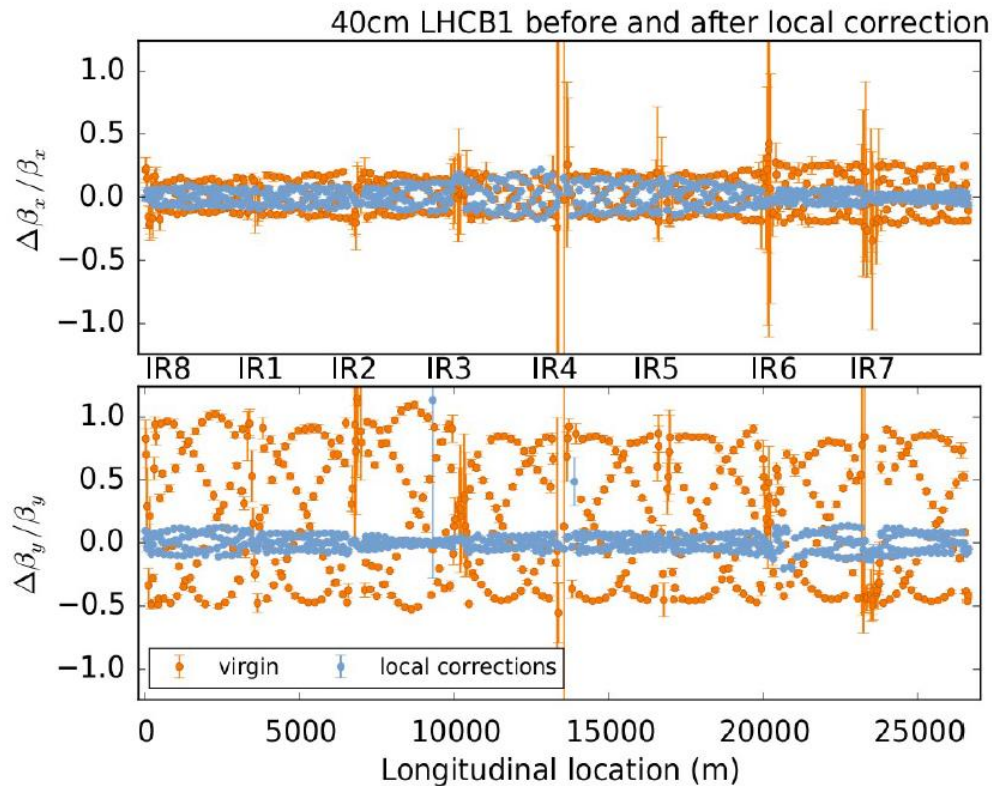


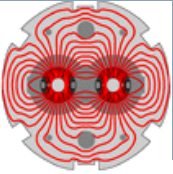


- ❑ How can one correct such beta-beating?
- ❑ The correction strategy with MICADO can be applied !
 - You can build the response of any gradient change on the optics (β).
 - You can use MICADO to look for the best possible solution.
 - The correcting elements are the quadrupole themselves (adjust their current).
- ❑ For optics corrections more sophisticated and powerful algorithm provide however better correction strategies.



- In collision at top energy of 6.5 TeV, the optics is wrong by 100% before correction.
 - Can be corrected to a few % residual error with modern correction algorithms.





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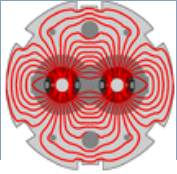
Orbit perturbations

Optics perturbations

Coupling between planes

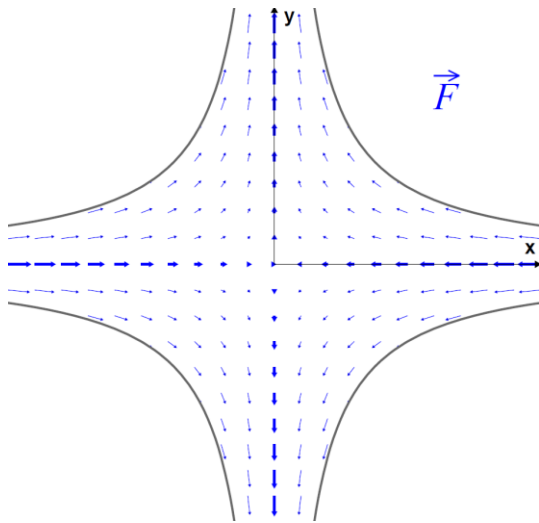
Linear imperfections and geology

Summary



- If a quadrupole is rotated by 45° ('skew quadrupole') one obtains an element where the force (deflection) in x depends on y and vice-versa: **the horizontal and vertical planes are coupled**.

normal quadrupole

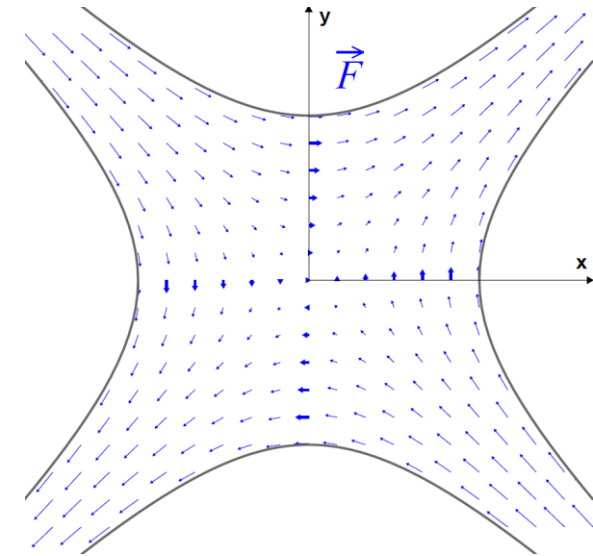
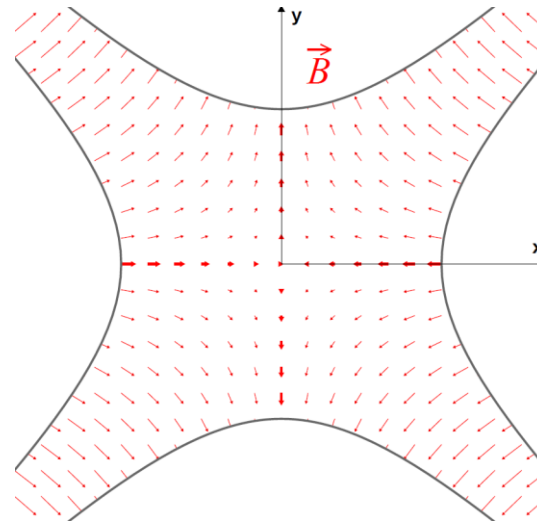


$$F_x = -k x$$

$$F_y = k y$$

No mixing of planes

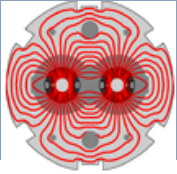
skew quadrupole



Full mixing of planes

$$F_x = k y$$

$$F_y = -k x$$

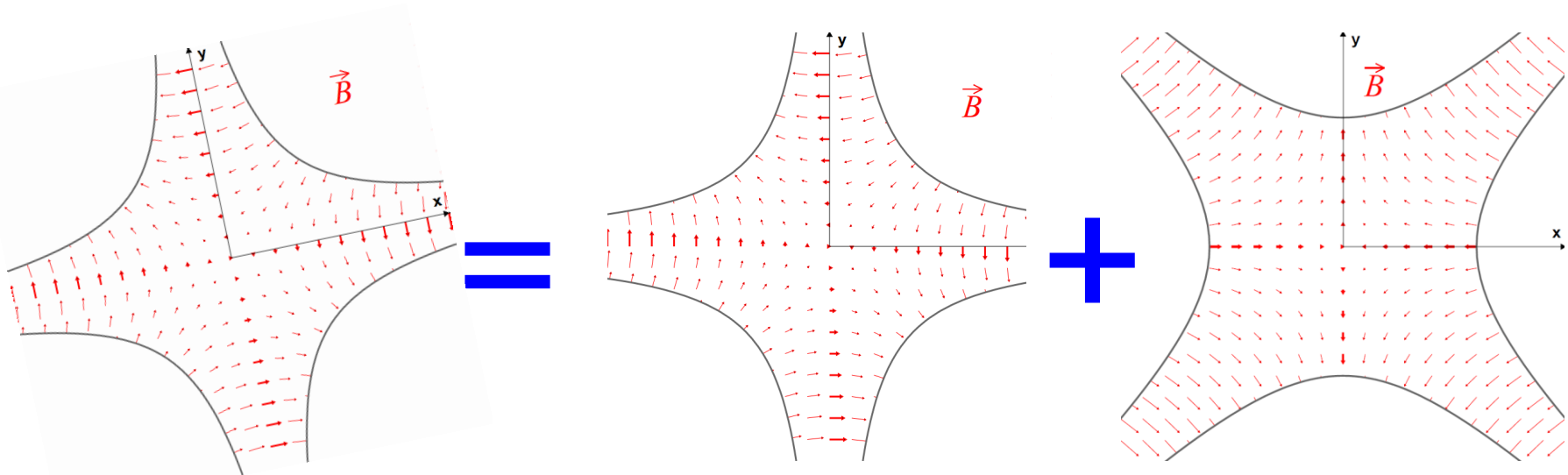


- Small quadrupole tilts lead to coupling of the x and y planes.
- The coupling can be corrected by installing dedicated skew quadrupoles to compensate for alignment errors.

tilted quadrupole

ideal quadrupole

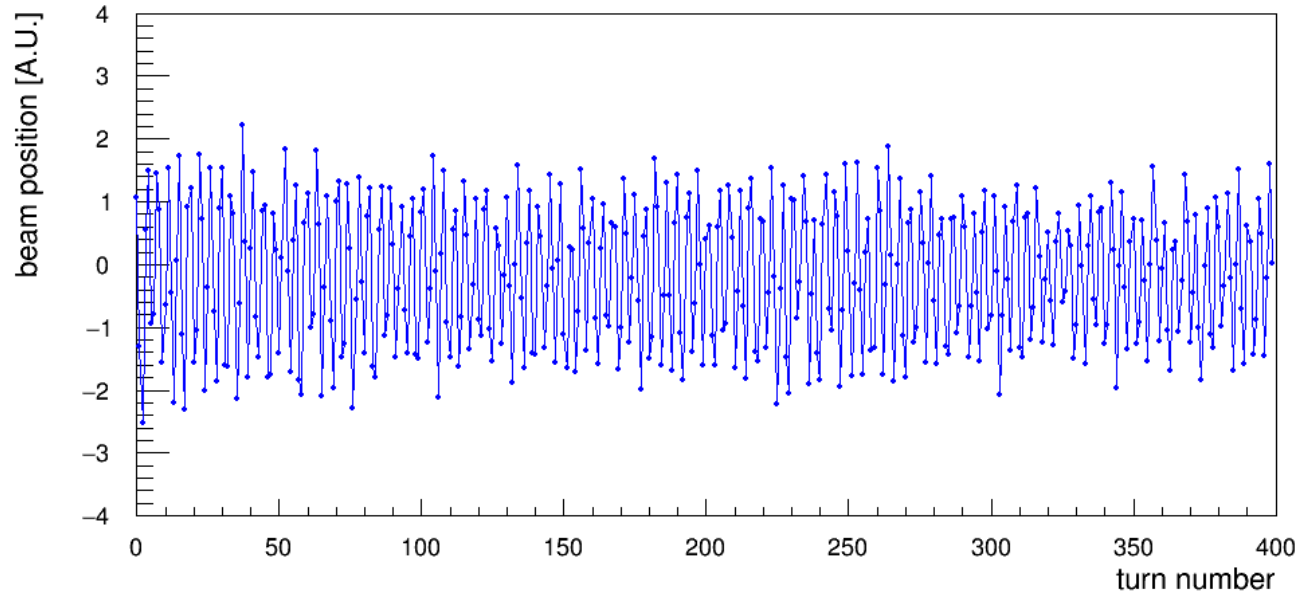
skew quadrupole





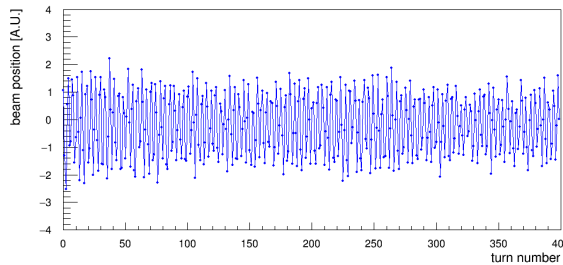
- The simplest thing to determine if there is coupling is to kick the beam in one plane to generate an oscillation, and then observe the oscillations or the frequency content.
 - Or just use the natural beam oscillations if they exist.
- If coupling is present, then for a horizontal kick there will be a small vertical oscillation (and vice-versa).

Turn by turn recording of the beam position at one BPM



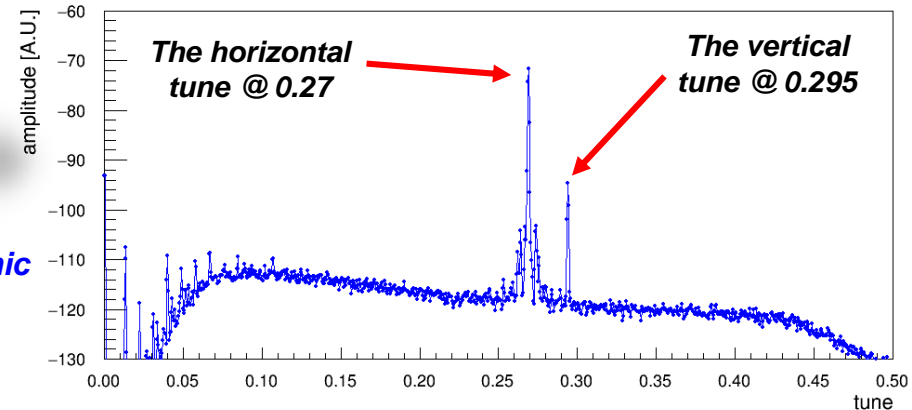
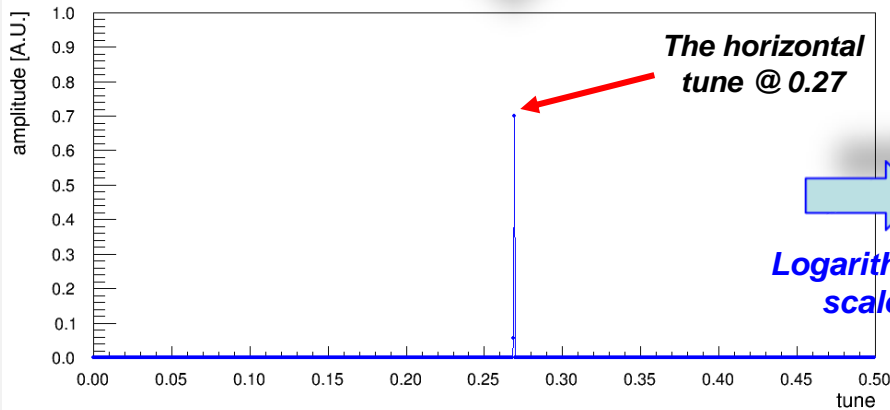


- We apply a **Fourier analysis** to the position data to extract the beam oscillation frequencies.

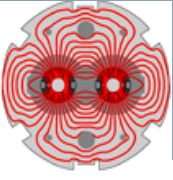


Example : horizontal beam position at a BPM observed turn by turn

Fourier analysis



- The **ratio of the vertical to horizontal amplitude** measures the amount of coupling → now one can tune the skew quadrupoles until the vertical tune peak disappears.



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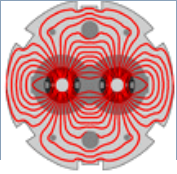
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Linear imperfections, geology and celestial bodies



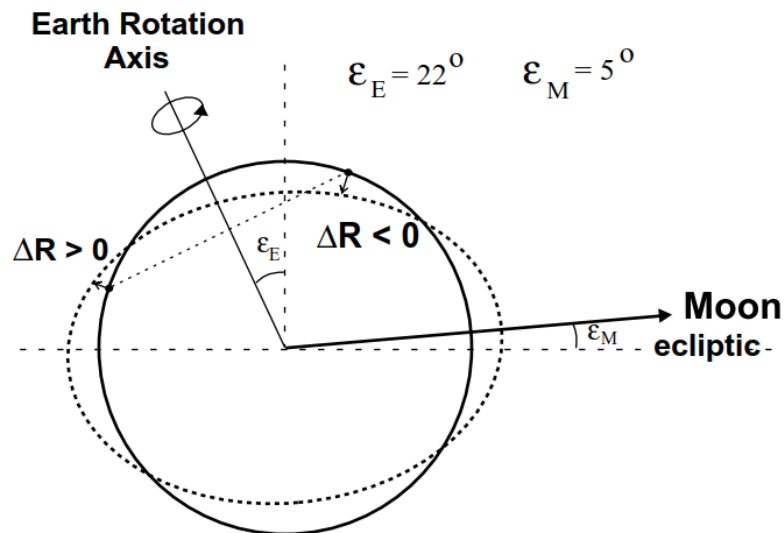


Tide bulge of a celestial body
of mass M at a distance d

$$\Delta R \sim \frac{M}{2d^3} (3\cos^2\theta - 1) \quad \theta = \text{angle}(\text{vertical, the celestial body})$$

induces surface deformations and affects the water levels of the oceans.

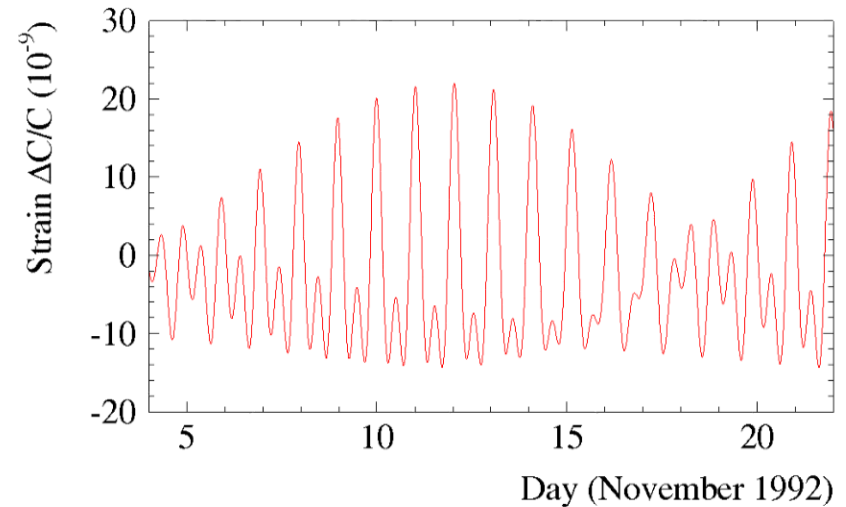
→ impacts the alignment of the entire accelerator !



Such **Earth tides** alter the accelerator circumference:

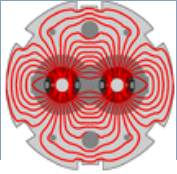
- The Moon contributes 2/3, the Sun 1/3.
- Not resonance-driven (unlike Sea tides !).
- Accurate predictions possible (~%).

LEP tide predictions for Nov. 1992



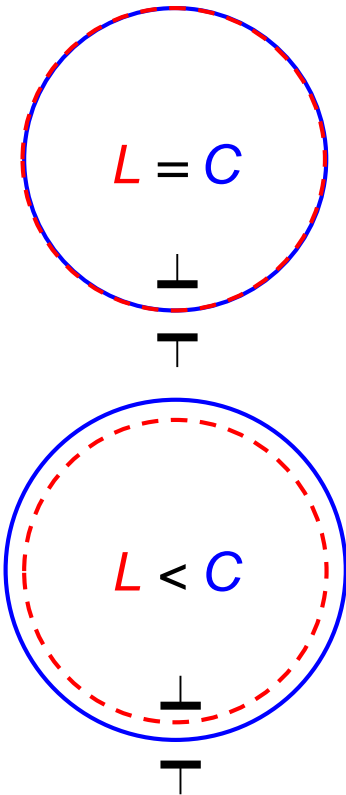
The relative circumference change amounts to $\sim 10^{-8} \sim 1 \text{ mm}$.

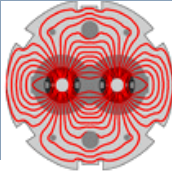
Gravitational wave detectors achieve sensitivities of $\sim 10^{-21}$!



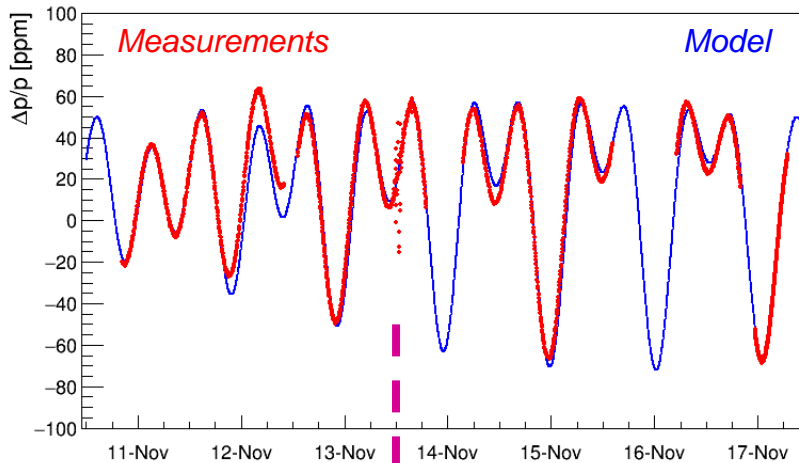
- ❑ At the LHC the beams are ‘captured’ by the RF system which forces the beams to remain synchronous with the RF frequency.
 - Because at LHC the speed $\cong c = \text{constant}$, this fixes the length of the orbit.

- ❑ When the frequency is well adjusted, the length of the orbit L matches the circumference C .
- ❑ If for any reason C varies, the beam has to move radially if L is kept constant.
- ❑ A mismatch between C and L can be observed on the mean radial orbit using the BPMs that move with the ring.
 - As a side effect it also changes very slightly the beam energy (level of 0.01%).





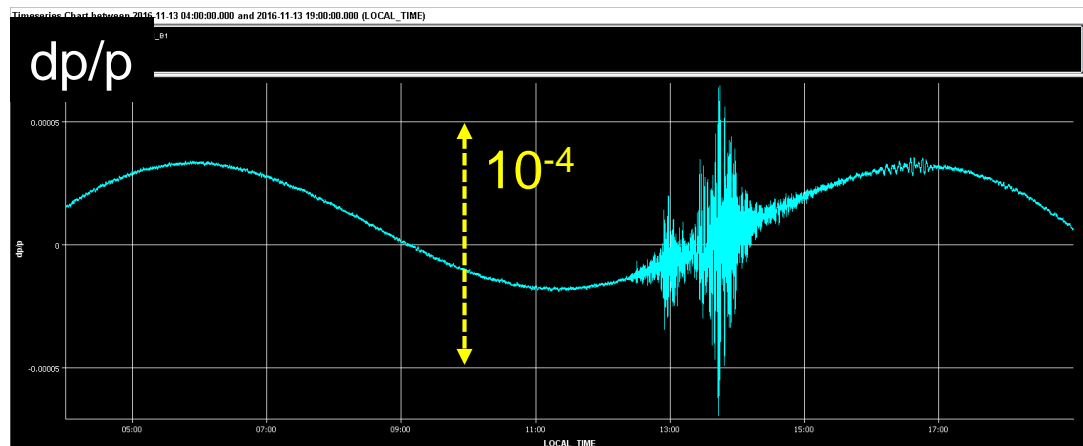
- Tides are observed very clearly on the LHC circumference by measuring the mean radial (=horizontal) beam position.



Tide observations (from orbit changes) over one week at 4 TeV in 2016 (expressed in energy change $\Delta p/p$)

Earthquake in New Zealand

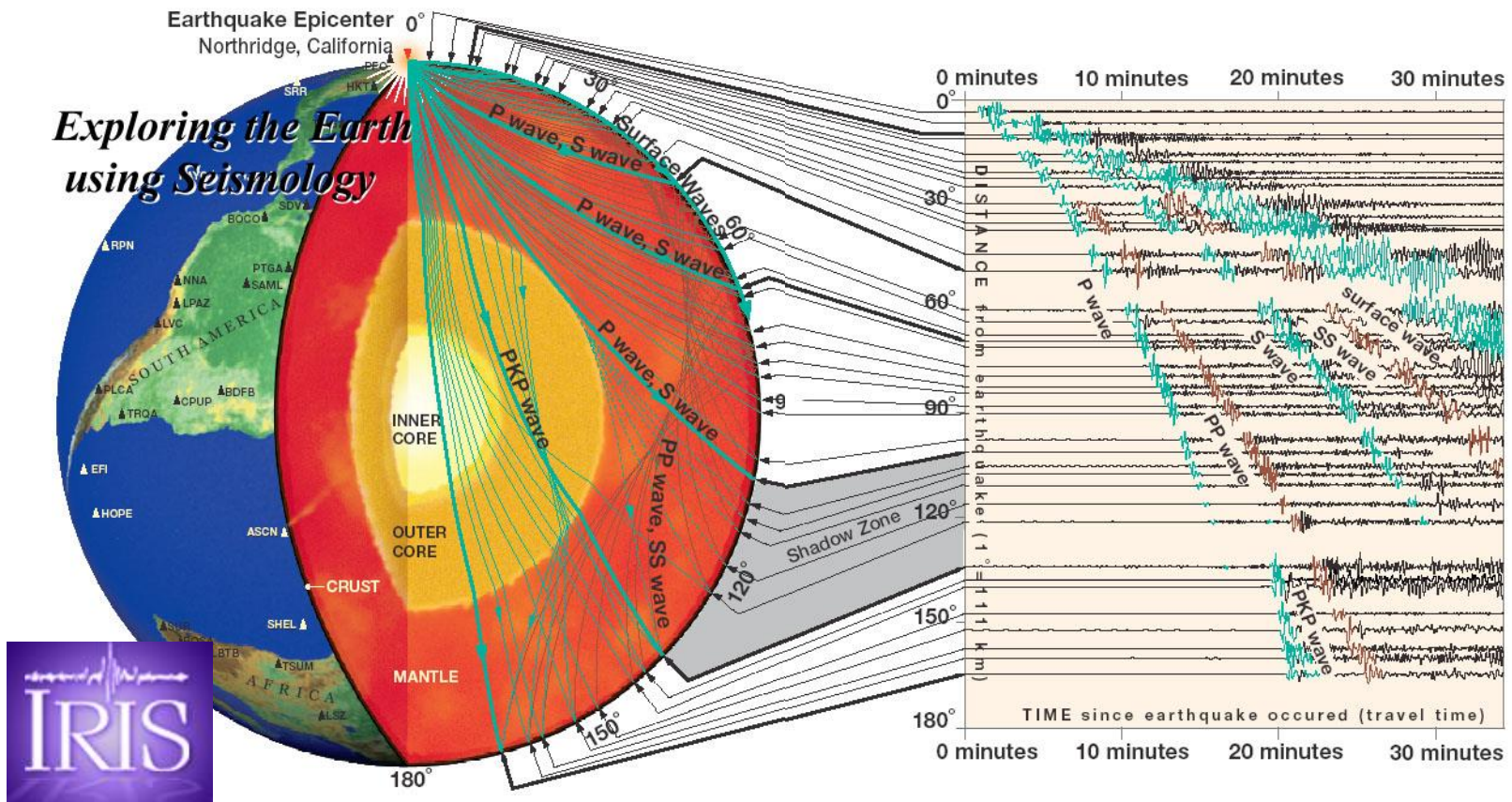
The pressure waves induce a modulation of the circumference

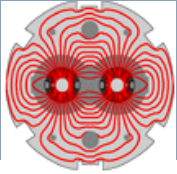


Waves from earthquakes

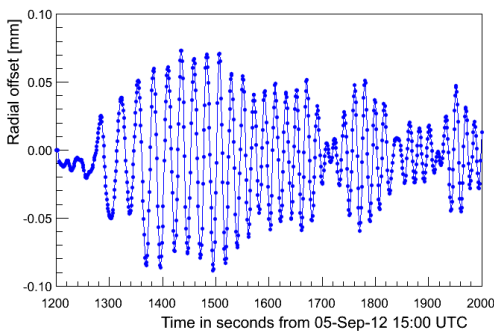
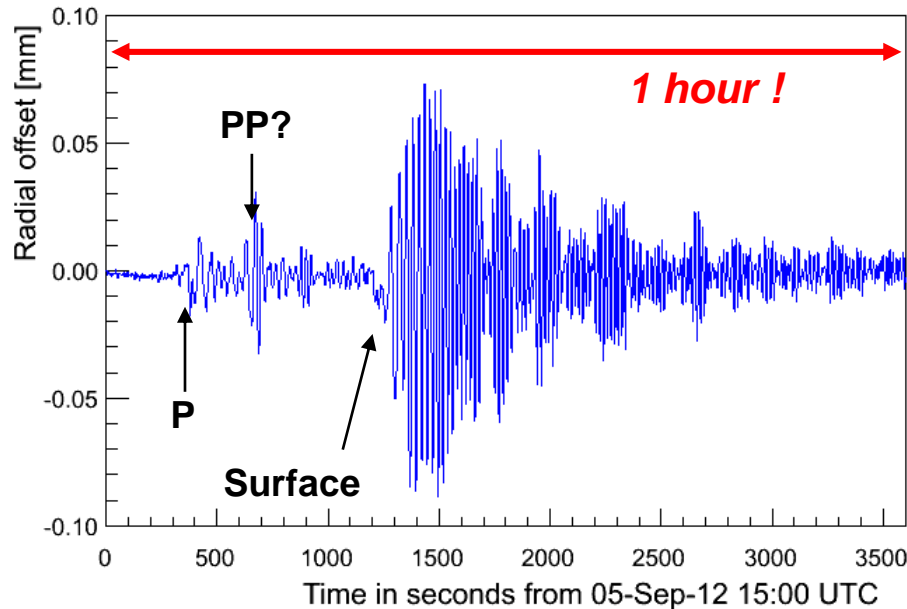


Different types of body (**P**ressure, **S**hear) and surface waves (**R**aleigh, **L**ove), multiple paths and reflections produce a complex signature of earthquakes at seismic measurement stations – also at the LHC.

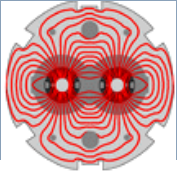




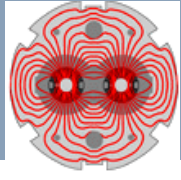
- A **magnitude 7.6** earthquake in Costa Rica (05/09/2012 @ 14:42:10 UTC) 'struck' the LHC in **fill 3032** during stable colliding beams.
 - Arrival of the first waves at CERN ~15:06 UTC.



- The arrival of the different waves can be observed on the radial beam position – equivalent to largest tides.



- ❑ We have seen that magnetic field errors and misalignments of accelerator components induce:
 - Errors on the beam orbit,
 - Errors on the optics and the tune,
 - Coupling between the horizontal and vertical planes.
- ❑ The errors are often sufficiently large (for sure at LHC) that the machine operates poorly or not at all.
- ❑ Since the 1970's ever improving tools and algorithms have been developed to correct for such errors.
- ❑ However to minimize the imperfections from the start we need:
 - well measured calibration curves of magnets,
 - precise power converters,
 - the best possible machine alignment.





- ▣ Various collider tune working points.

