Accelerator Physics

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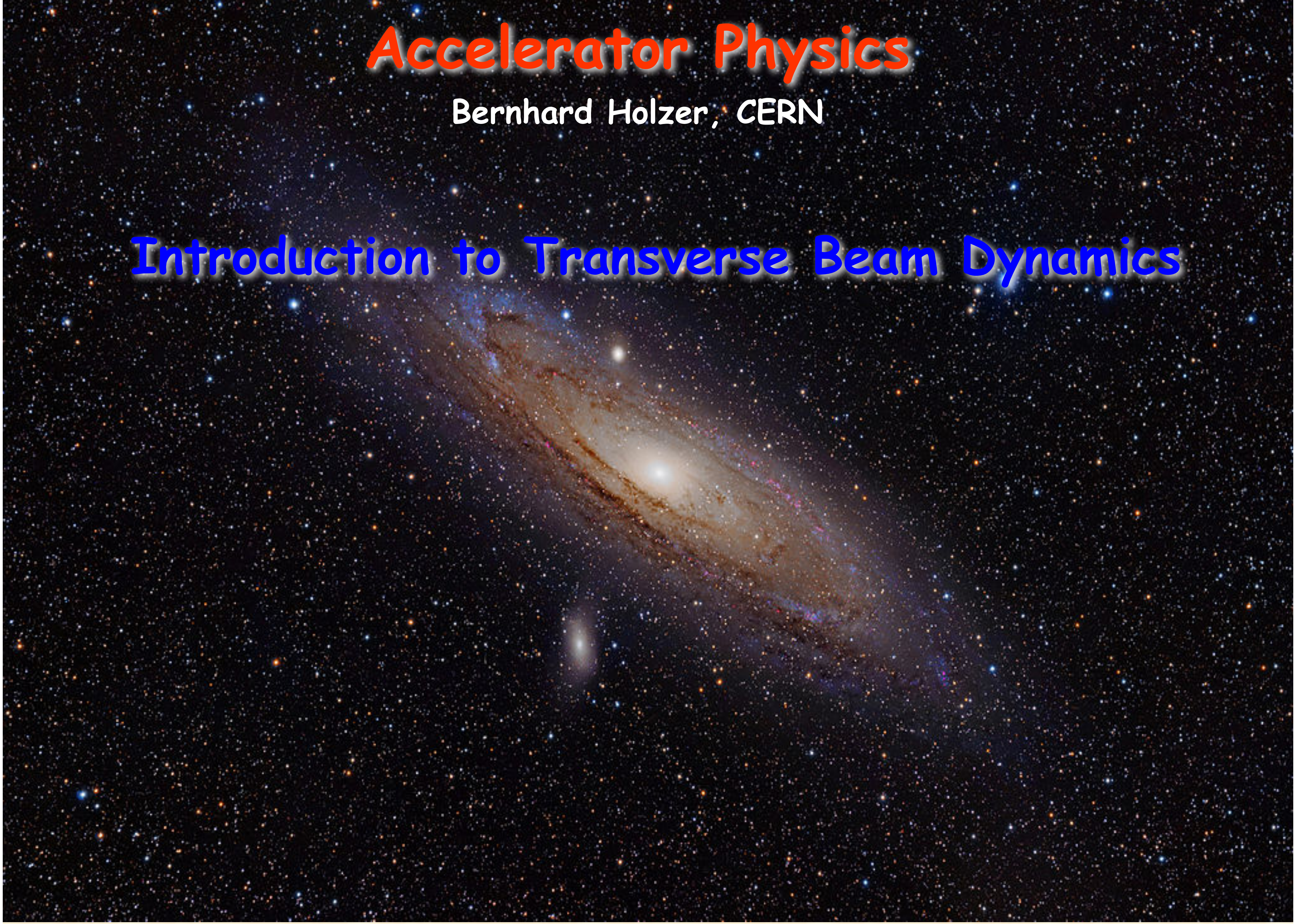
Introduction to Transverse Beam Dynamics



Transverse Beam Dynamics III

I) Linear Beam Optics

- ***Single Particle Trajectories***
- ***Magnets and Focusing Fields***
- ***Tune & Orbit***

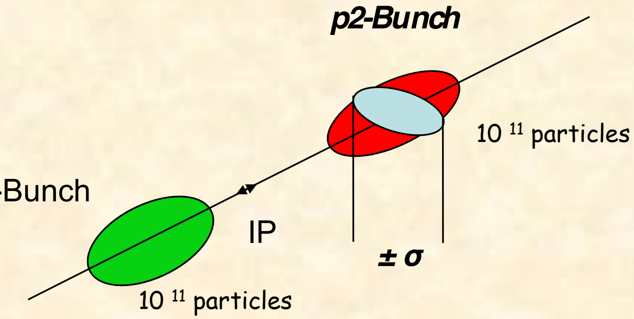
II) The State of the Art in High Energy Machines:

- ***The Beam as Particle Ensemble***
- ***Emittance and Beta-Function***
- ***Colliding Beams & Luminosity***

III) Errors in Field and Gradient:

- ***Liouville during Acceleration***
- ***The $\Delta p/p \neq 0$ problem***
- ***Dispersion***
- ***Chromaticity***

Luminosity



Example: Luminosity at LHC

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$\sigma = \sqrt{\varepsilon\beta}$$

$$\beta^*_{x,y} = 0.55\ m$$

$$f_0 = 11.245\ kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$$

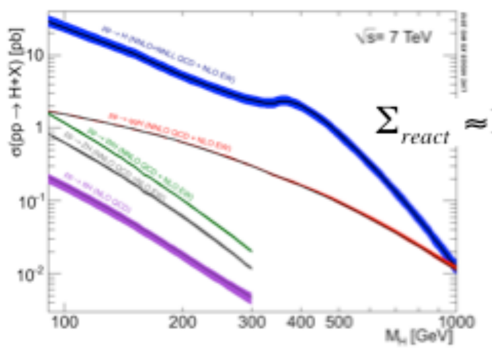
$$n_b = 2808$$

$$\sigma_{x,y} = 17\ \mu m$$

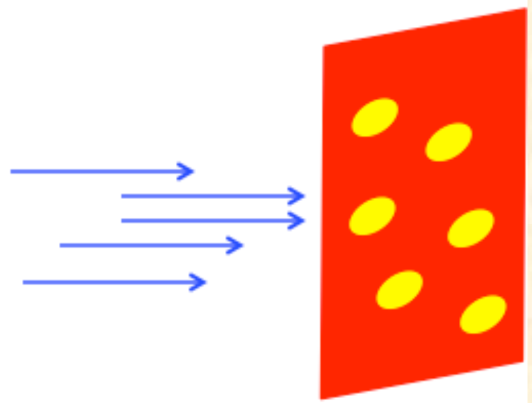
$$I_p = 584\ mA$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\ pb$$

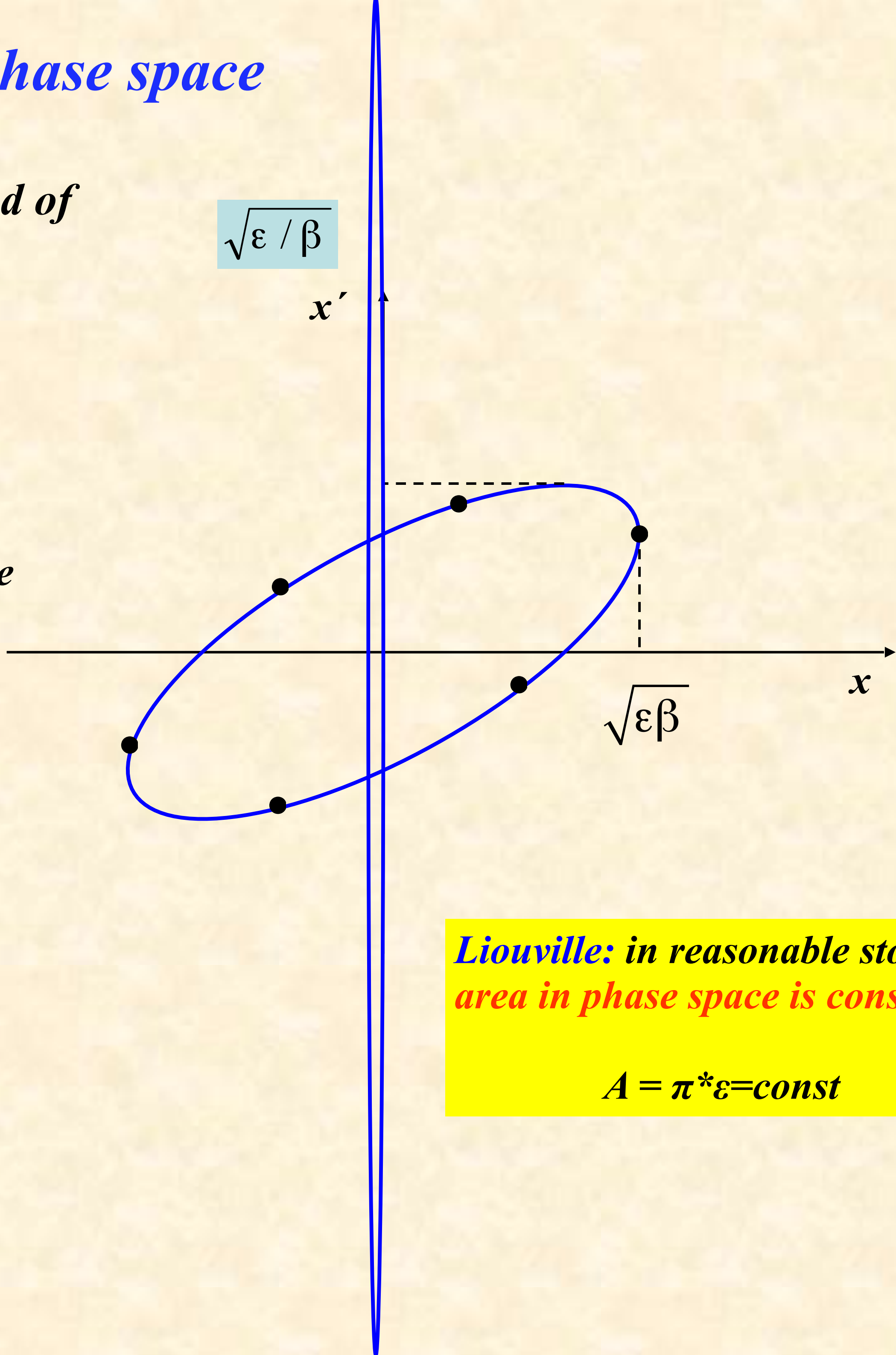


Make β as small as possible !!!*

Mini-Beta-Insertions in phase space

A mini-β insertion is always a kind of
special symmetric drift space.
→ greetings from Liouville

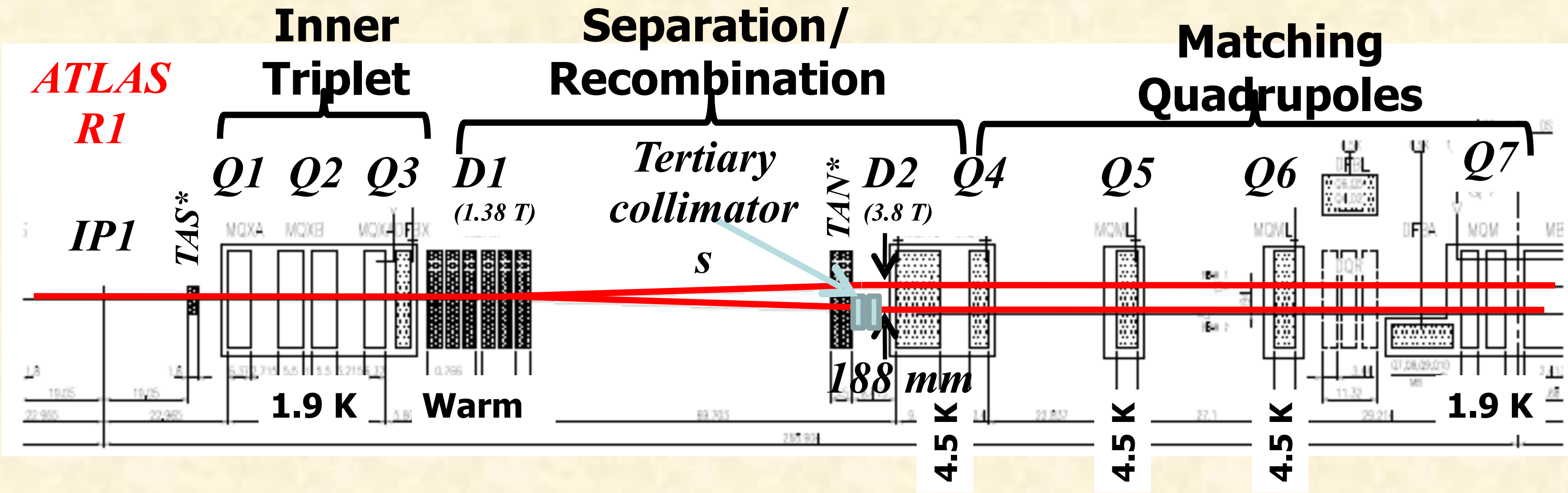
the smaller the beam size
the larger the beam divergence

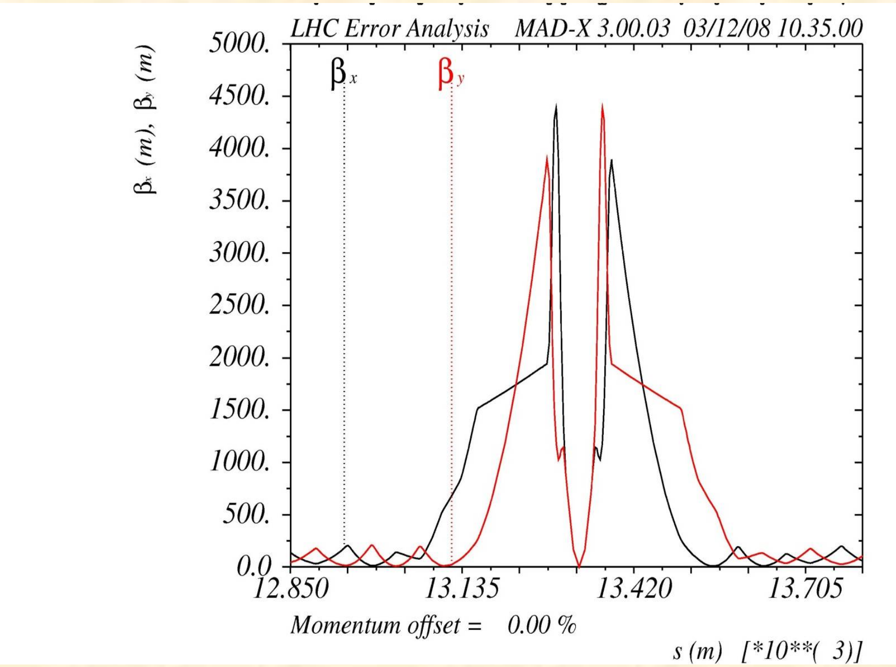


***Liouville:* in reasonable storage rings**
area in phase space is constant.

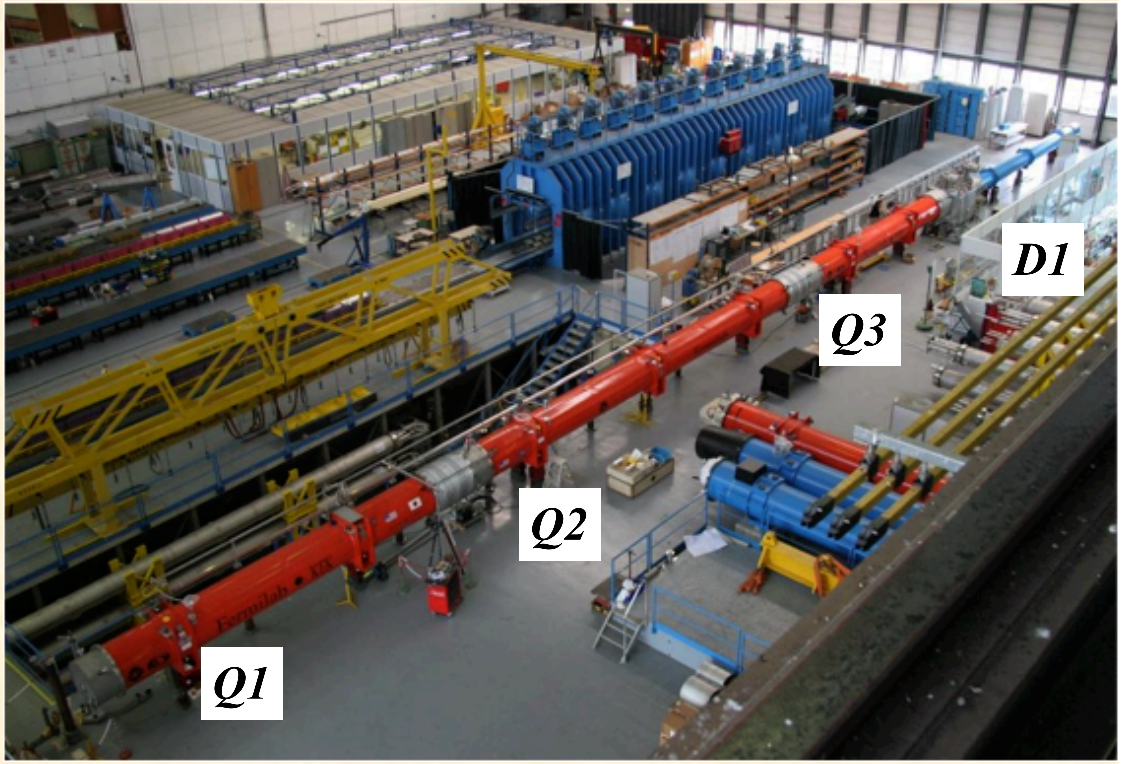
$$A = \pi * \varepsilon = const$$

The LHC Insertions

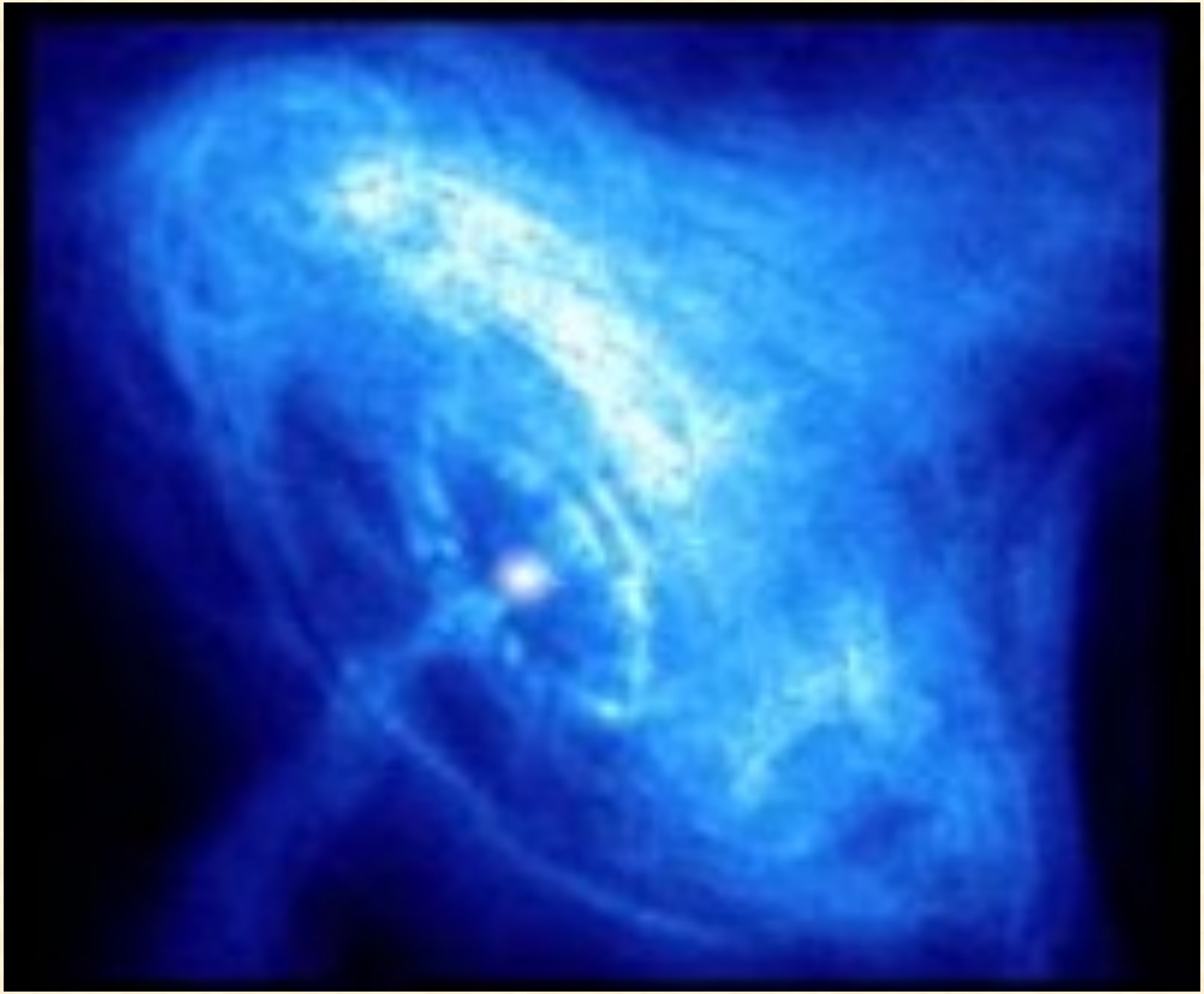




mini β optics



… finally … let´s talk about acceleration

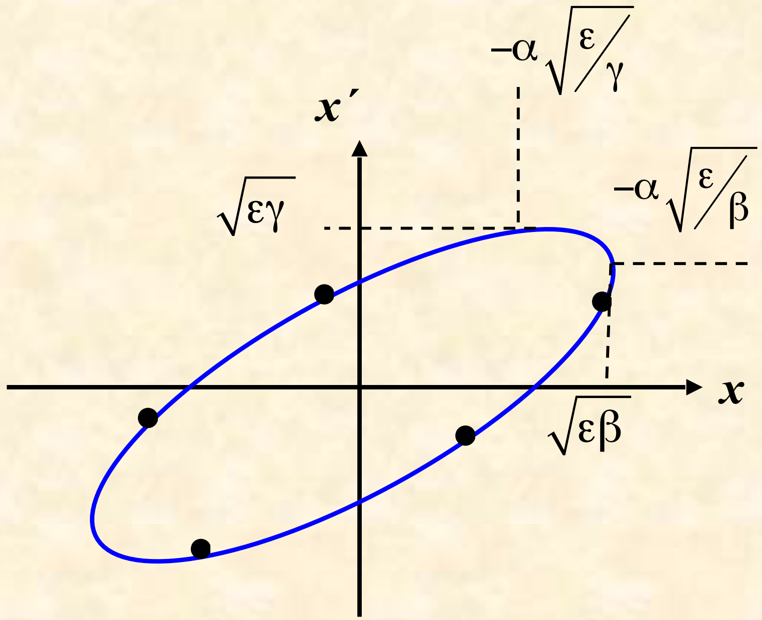


crab nebula,

burst of charged particles $E = 10^{20}$ eV

14.) Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the*
x, x´ Phase Space Ellipse

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq$ const !

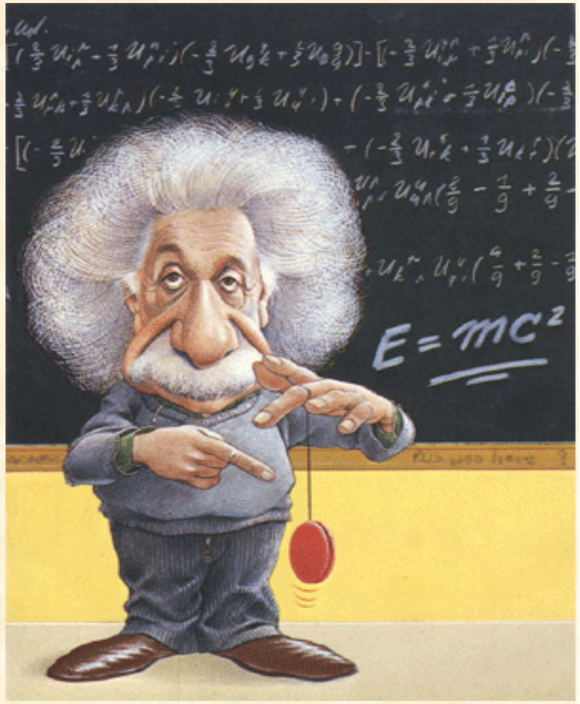
Classical Mechanics:

***phase space** = diagram of the two canonical variables*

position** & **momentum

x p_x

According to Hamiltonian mechanics: phase space diagram relates the variables q and p

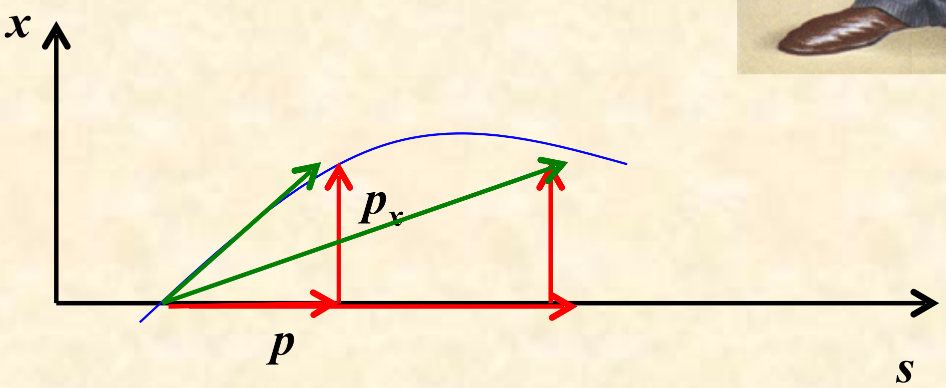


Liouvilles Theorem: $\int p\,dq = const$

… referring to the hor. plane $\int p_x dx = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\,\frac{dt}{ds} = \frac{\beta_x}{\beta} = \frac{p_x}{p}$$



$$\underbrace{\int x' dx}_{\varepsilon} = \frac{\int p_x dx}{p} \propto \frac{const}{m_0 c\ \gamma\beta}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

$$\beta_x = \frac{v_x}{c}$$

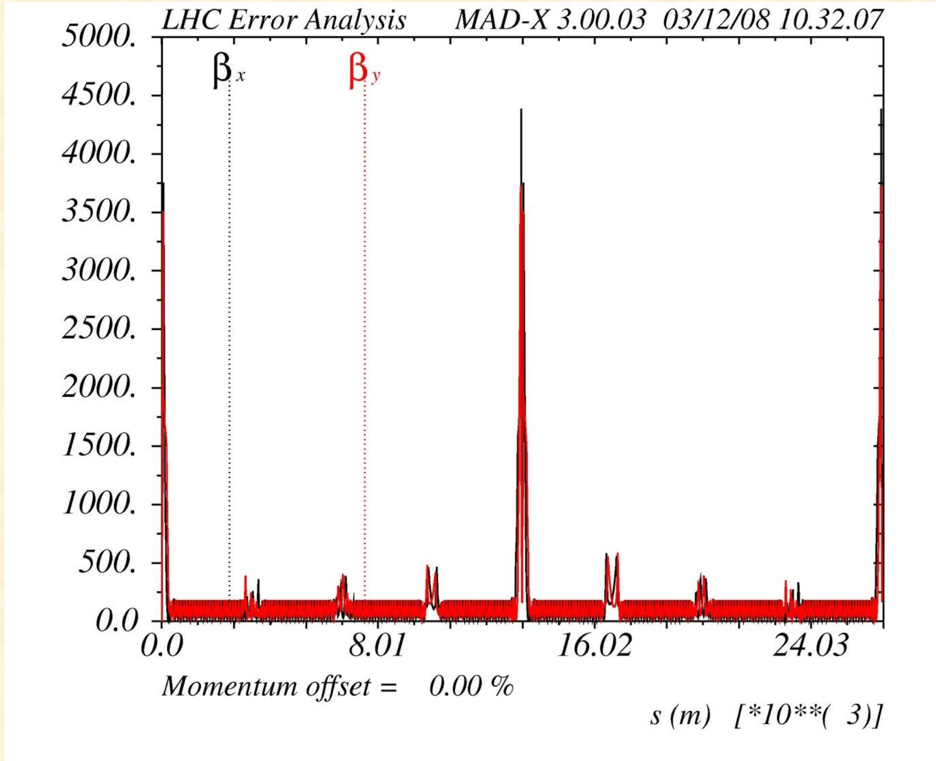
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

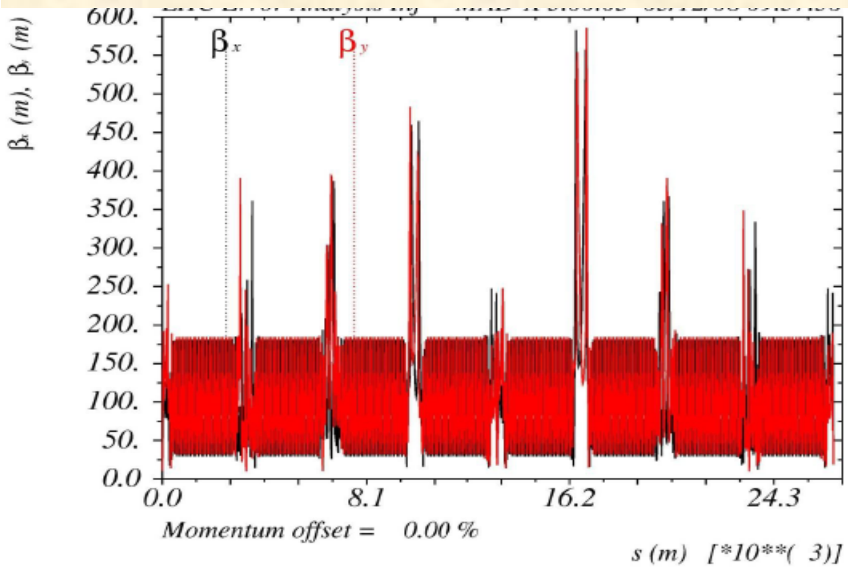
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

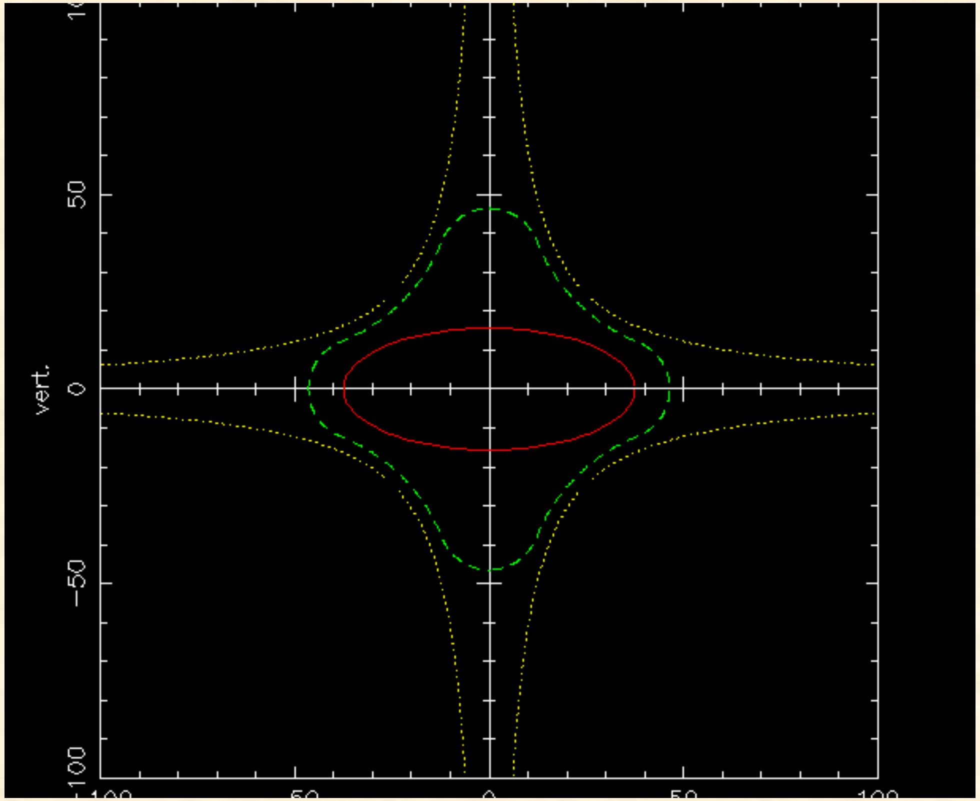


LHC injection optics at 450 GeV

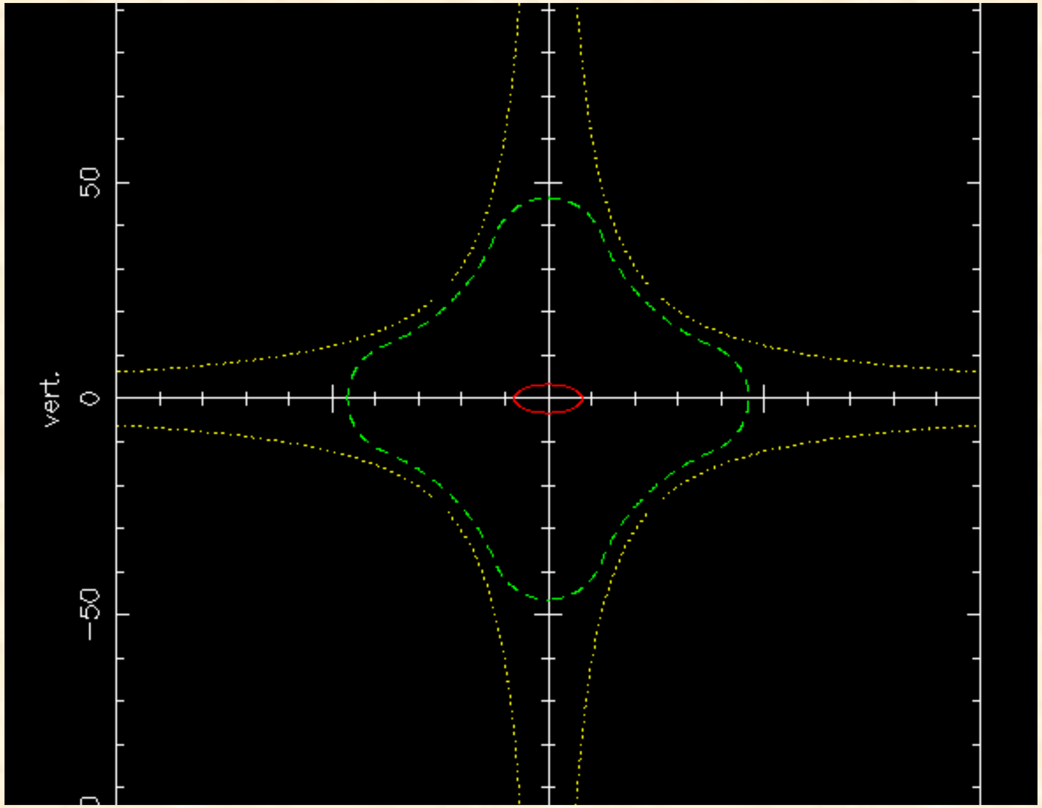
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon\,(40GeV) = 1.2 * 10^{-7}$
$\varepsilon\,(920GeV) = 5.1 * 10^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

15.) The „ $\Delta p / p \neq 0$“ Problem

ideal accelerator: *all particles will see the same accelerating voltage.*

→ $\Delta p / p = 0$

„nearly ideal“ accelerator: *Cockroft Walton or van de Graaf*

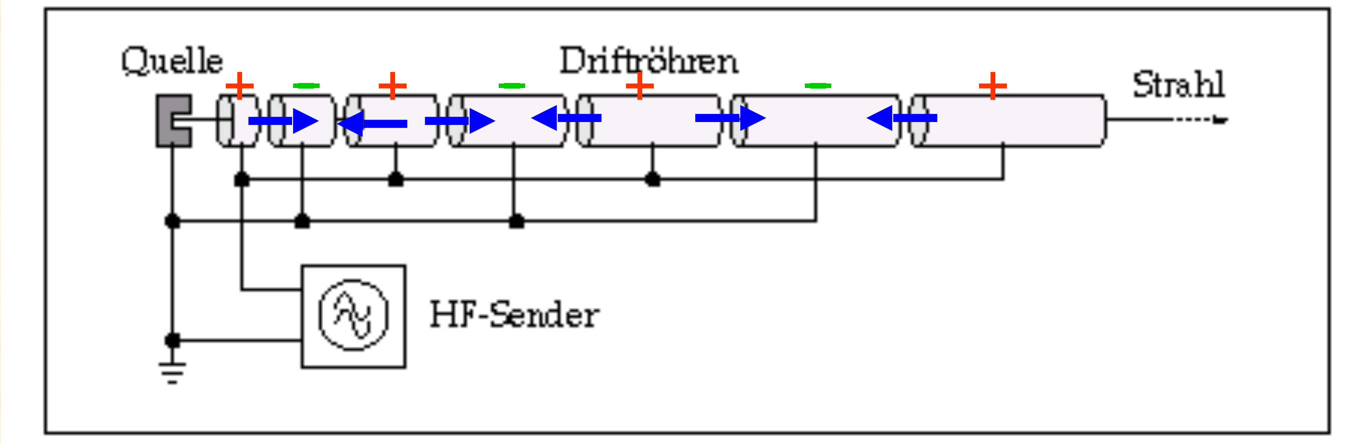
$\Delta p / p \approx 10^{-5}$

Vivitron, Straßbourg, inner structure of the acc. section

MP Tandem van de Graaf Accelerator at MPI for Nucl. Phys. Heidelberg

RF Acceleration

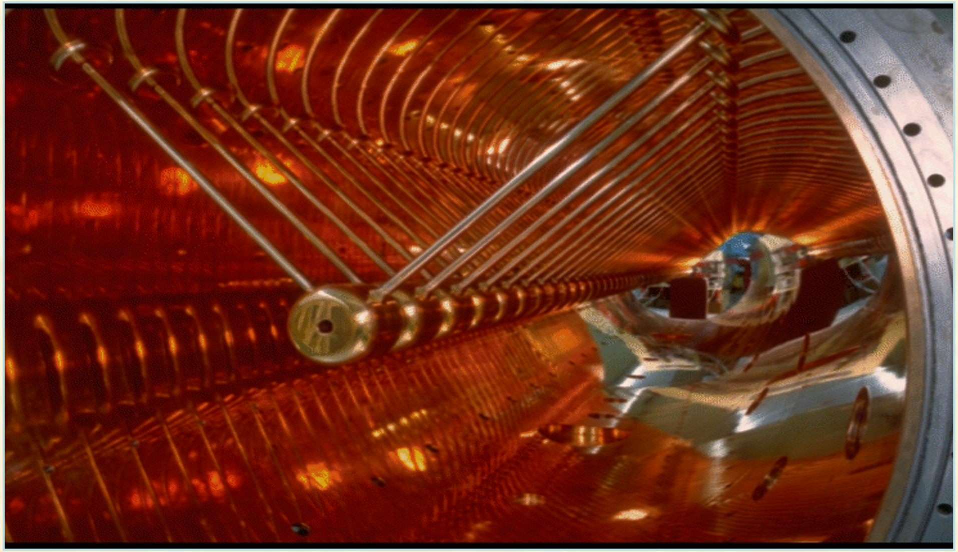
1928, Wideroe



Energy Gain per „Gap":

$$W = n * q U_0 \sin\omega_{RF} t$$

drift tube structure at a proton linac (GSI Unilac)



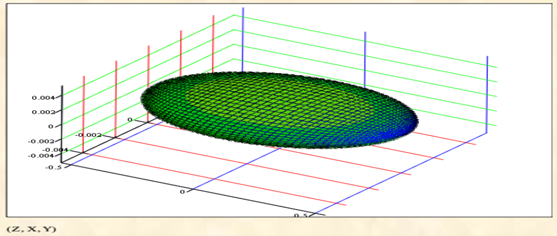
n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

500 MHz cavities in an electron storage ring

* **RF Acceleration:** multiple application of the same acceleration voltage; brillant idea to gain higher energies

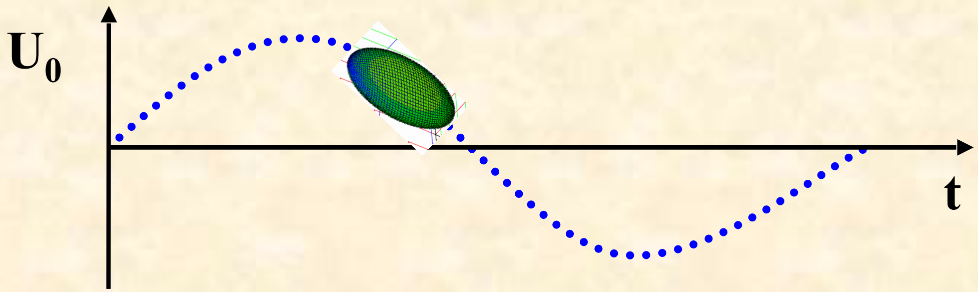
RF Acceleration-Problem:
panta rhei !!!

(Heraklit: 540-480 v. Chr.)



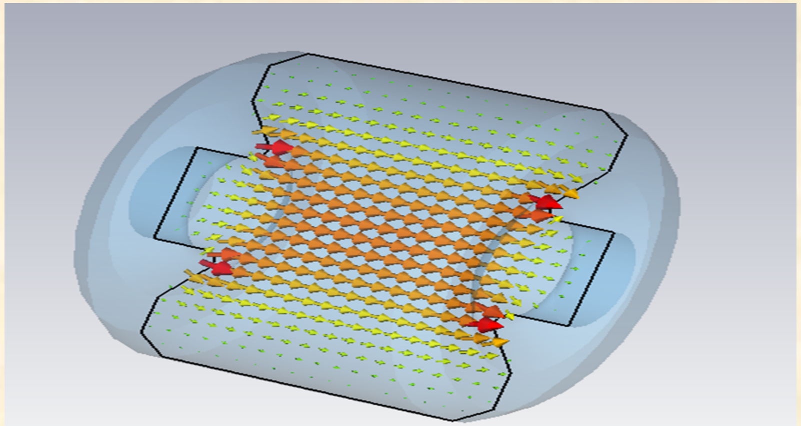
Bunch length of Electrons ≈ 1cm

just a stupid (and a little bit wrong) example)



$$\nu = 400\,MHz, \quad c = \lambda \nu \quad \Rightarrow \quad \lambda = 75\,cm$$

$\lambda = 75\,cm$



$$\sin(90^o) = 1$$

$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

Sure there are !!!

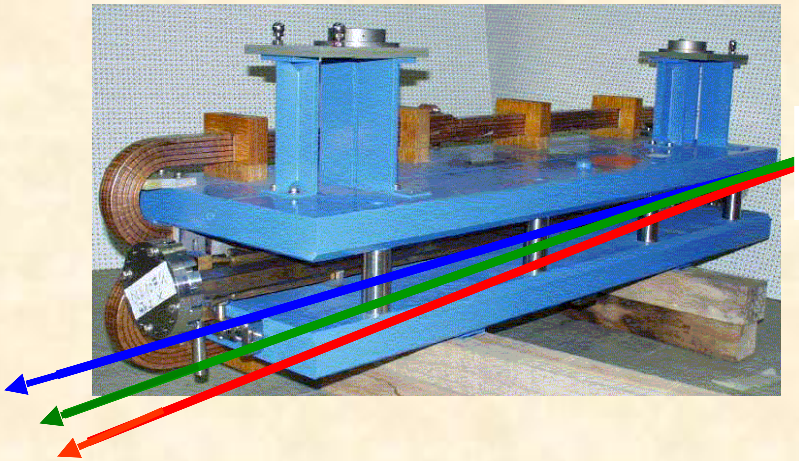
font colors due to pedagogical reasons

16.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: ***prop. to magn. field & prop. zu 1/p***

dipole magnet

$$\alpha = \frac{\int B\,dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

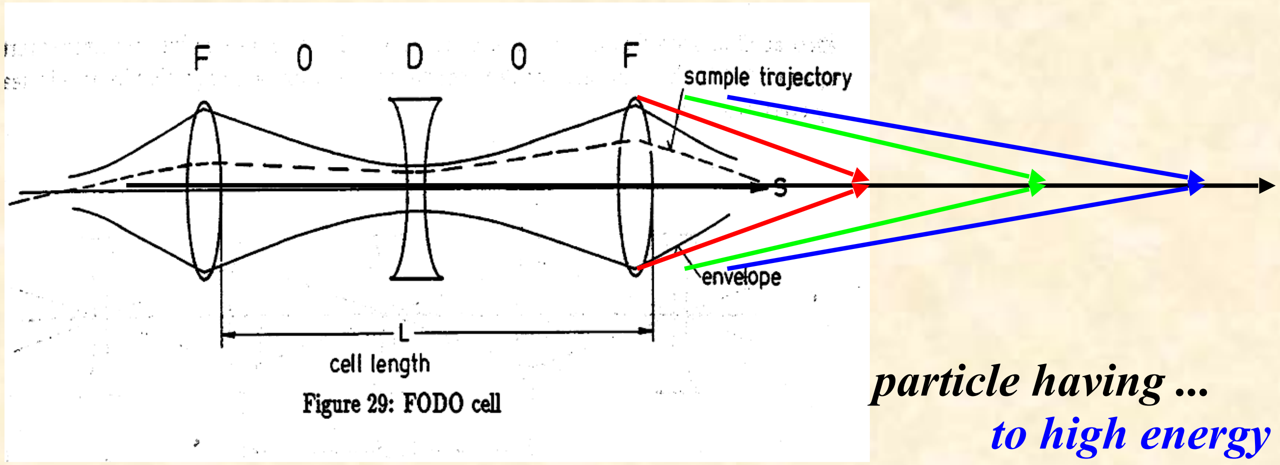


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

Dispersion

the typical Formula 1 effect:

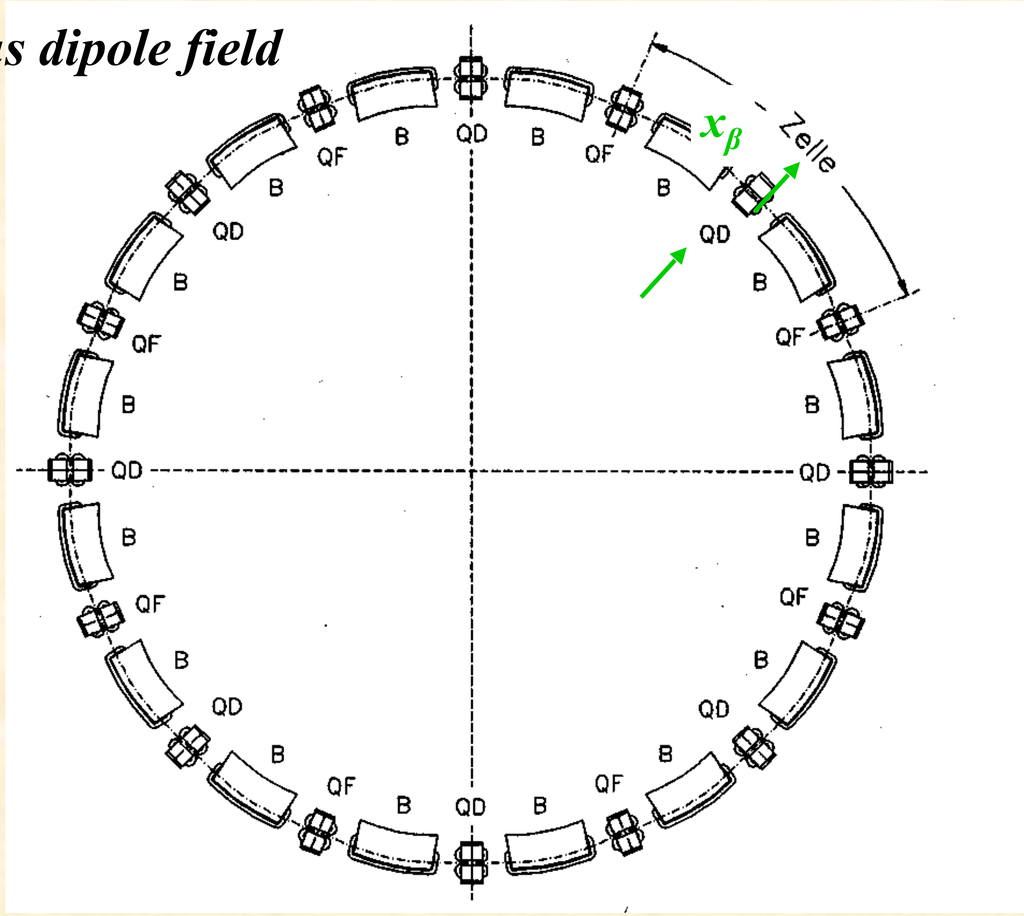
Those who are faster (have higher momentum) …
… are running on a larger circle.

BUT

they are focused nevertheless.

Dispersion

Example: homogeneous dipole field



it for Δp/p > 0

$$D(s)\cdot\frac{\Delta p}{p}$$

Matrix formalism:

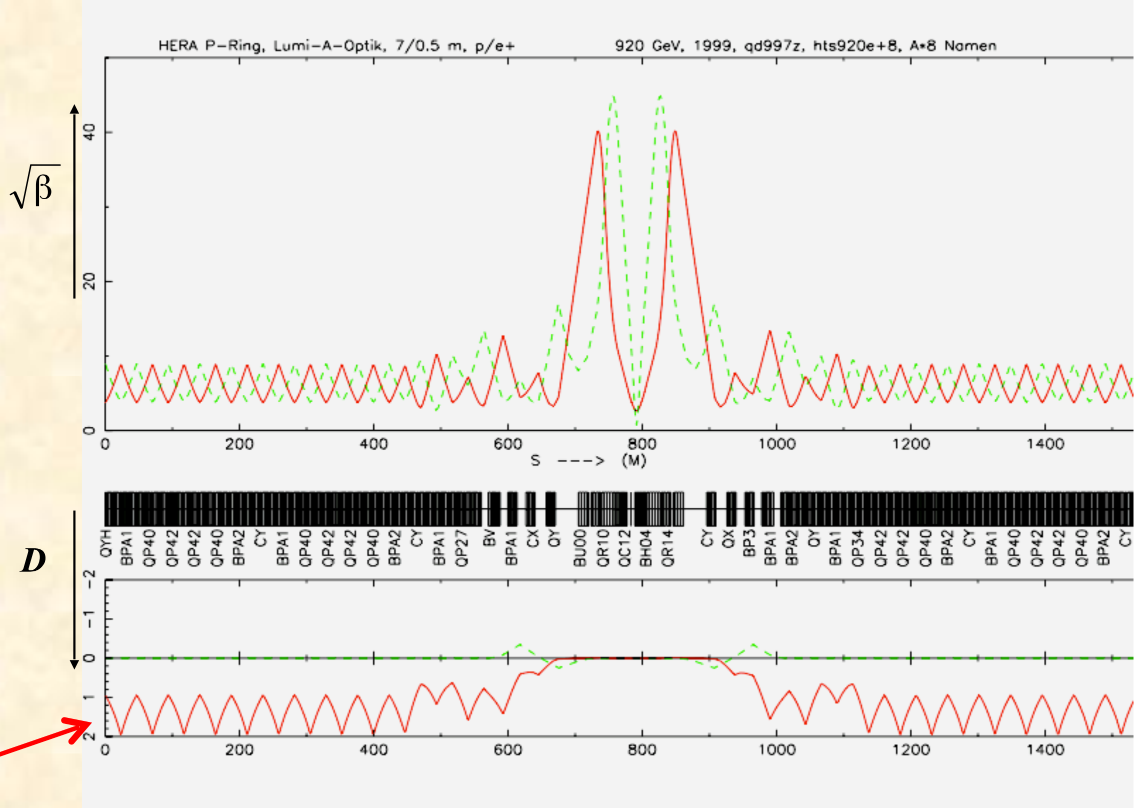
$$x(s) = x_\beta(s) + D(s)\cdot\frac{\Delta p}{p}$$

$$x(s) = C(s)\cdot x_0 + S(s)\cdot x_0' + D(s)\cdot\frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p}\begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \ldots 2\, mm$$
$$D(s) \approx 1 \ldots 2\, m$$
$$\Delta p/p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation contribution due to Dispersion ≈ beam size

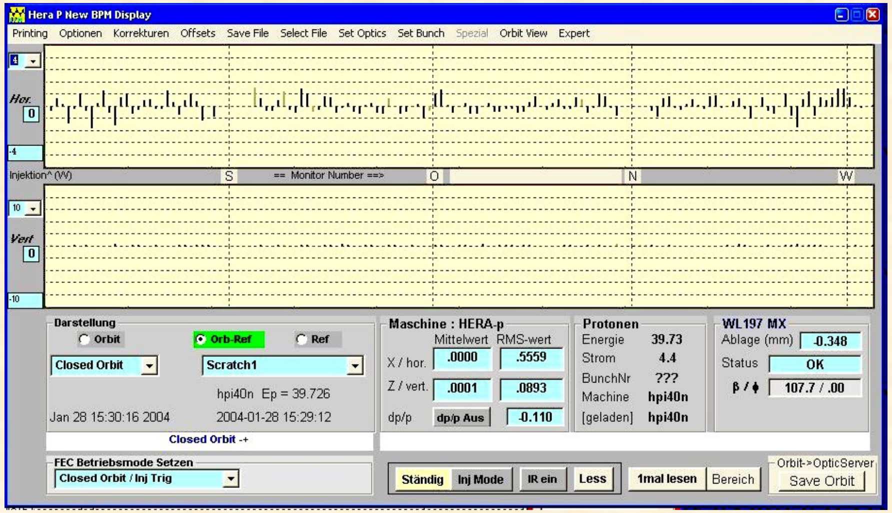
→ ***Dispersion must vanish at the collision point*** !

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

$$D(s) = S(s) \int_{s_0}^{s_1} \frac{1}{\rho} C(\tilde{s})\, d\tilde{s} - C(s) \int_{s_0}^{s_1} \frac{1}{\rho} S(\tilde{s})\, d\tilde{s}$$

(proof see CAS proc.)

Dispersion is visible



HERA Standard Orbit

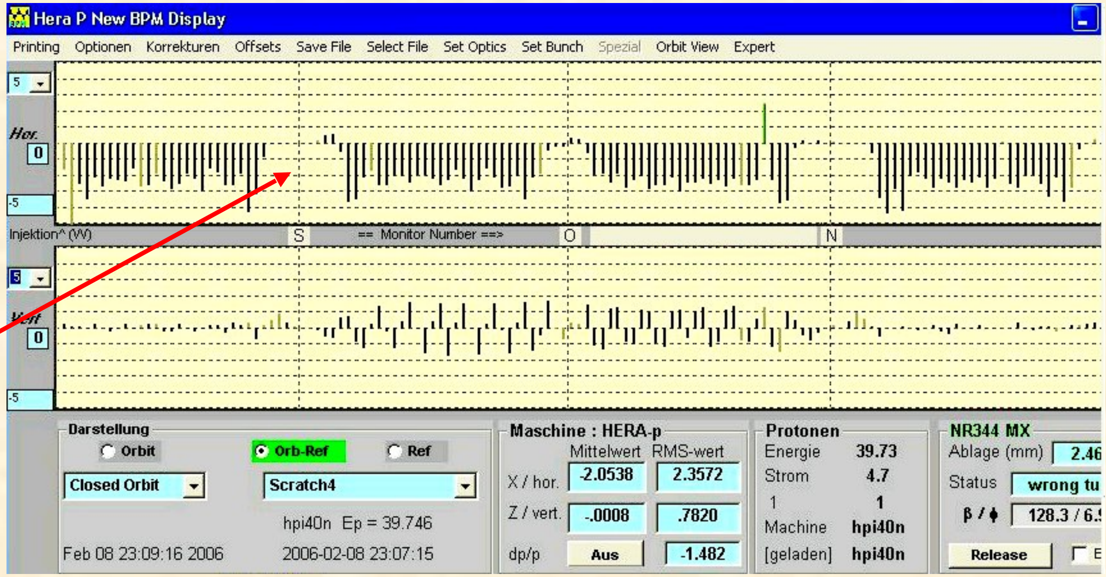
HERA Dispersion Orbit

dedicated energy change of the stored beam

→ closed orbit is moved to a dispersions trajectory

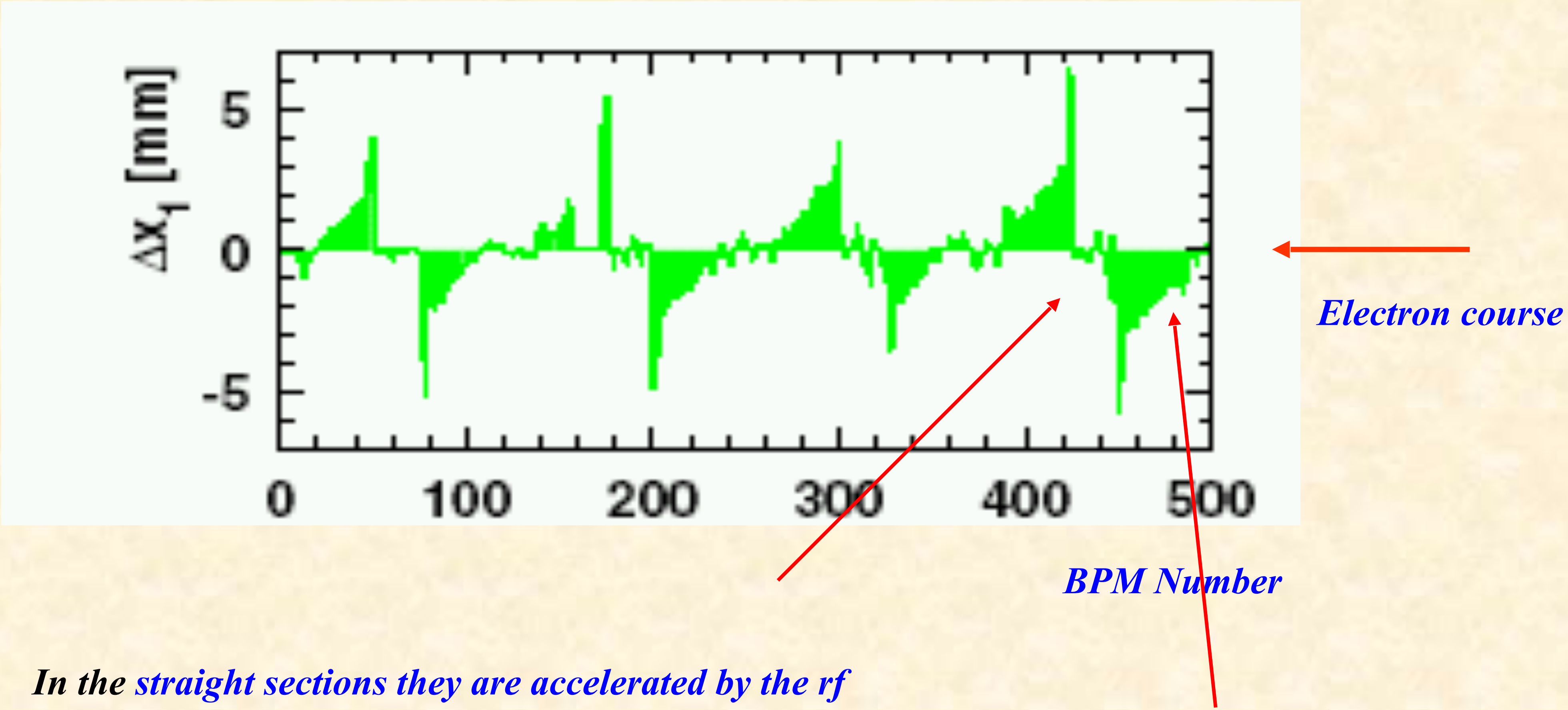
$$x_D = D(s) * \frac{\Delta p}{p}$$

Attention: at the Interaction Points we require D=D′= 0



Periodic Dispersion:

„Sawtooth Effect" at LEP (CERN)



In the straight sections they are accelerated by the rf cavities so much that they „overshoot" and reach nearly the outer side of the vacuum chamber.

In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.

17.) Chromaticity:
A Quadrupole Error for $\Delta p/p \neq 0$

***Influence of external fields on the beam:** prop. to magn. field & prop. zu 1/p*

Remember the normalisation of the external fields:

focusing lens $$k = \frac{g}{p/e}$$

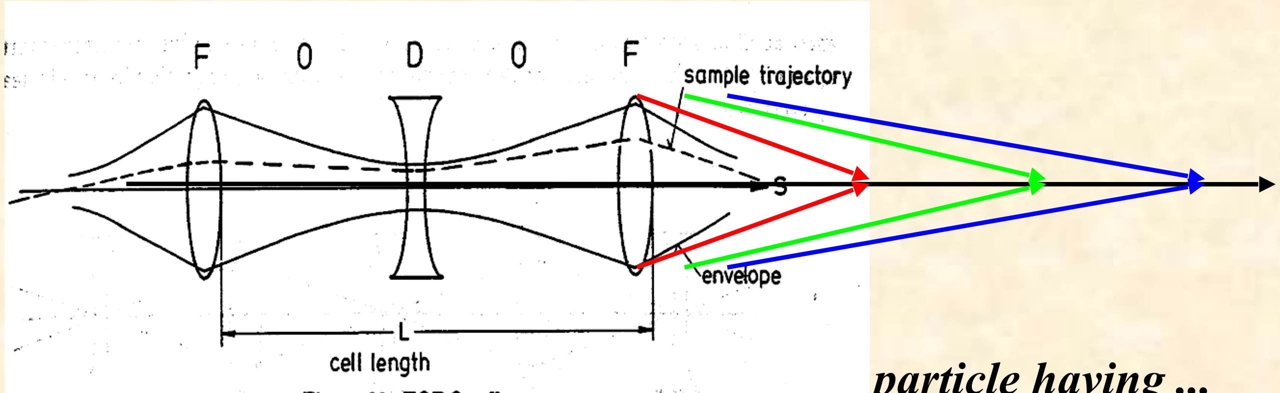


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

a particle that has a higher momentum feels a weaker quadrupole gradient and has a lower tune.

definition of chromaticity: $$\Delta Q = Q' \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Every individual particle has an individual momentum and thus an individual tune.

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ *it is determined by the focusing strength k of all quadrupoles*

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds \qquad Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

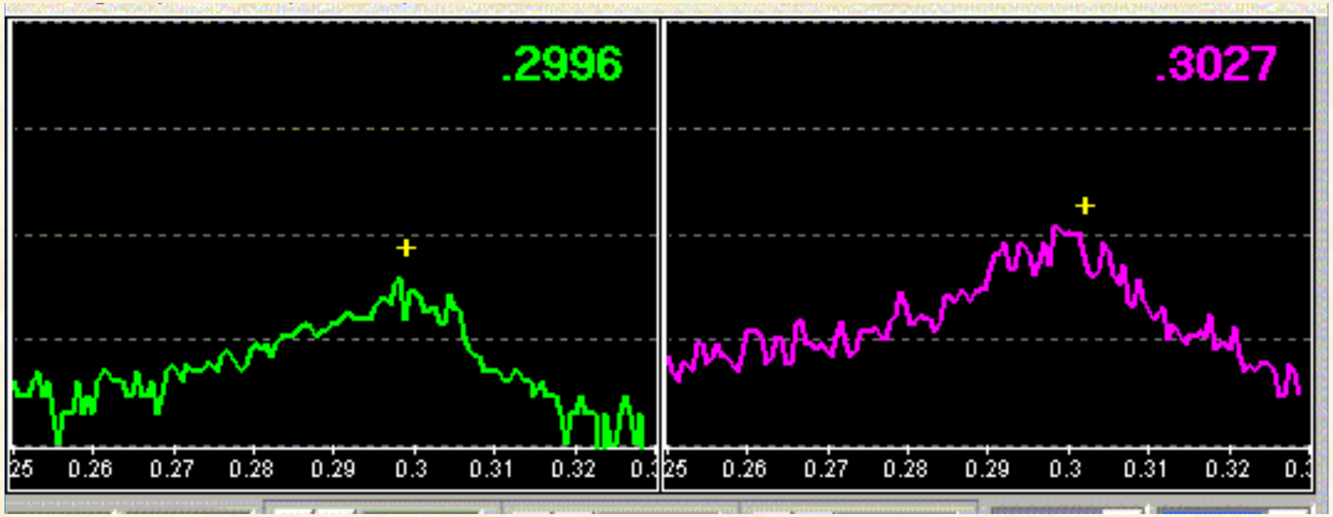
k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields

Example: LHC

Q' = 250
*Δ p/p = +/- 0.2 *10^-3^*
Δ Q = 0.256 … 0.36

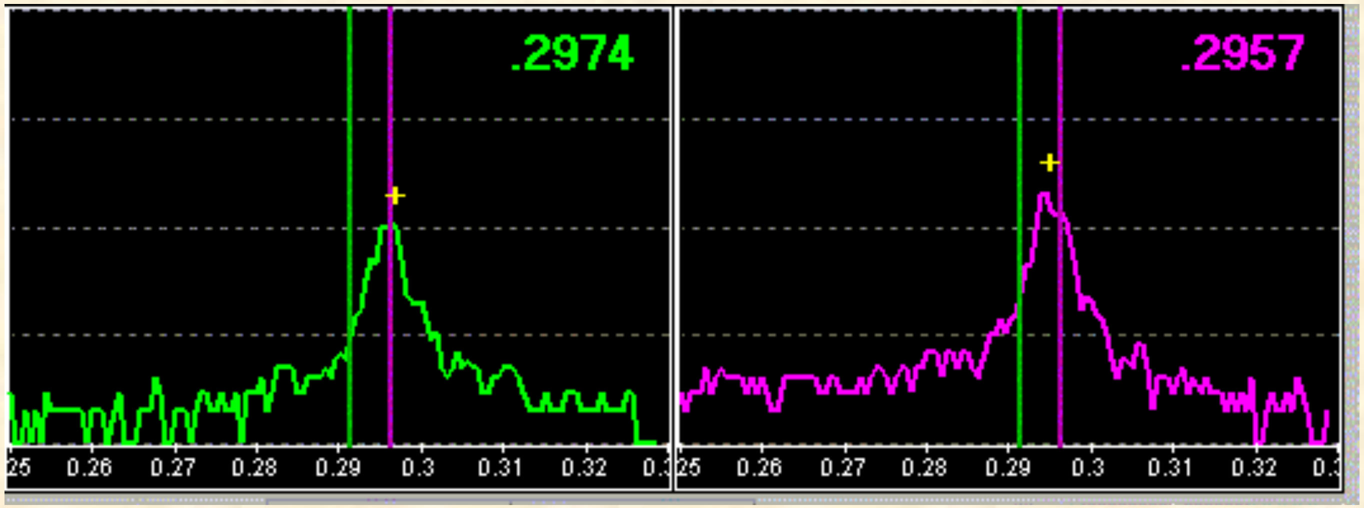
→ *Some particles get very close to resonances and are lost*

in other words: the tune is not a point it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

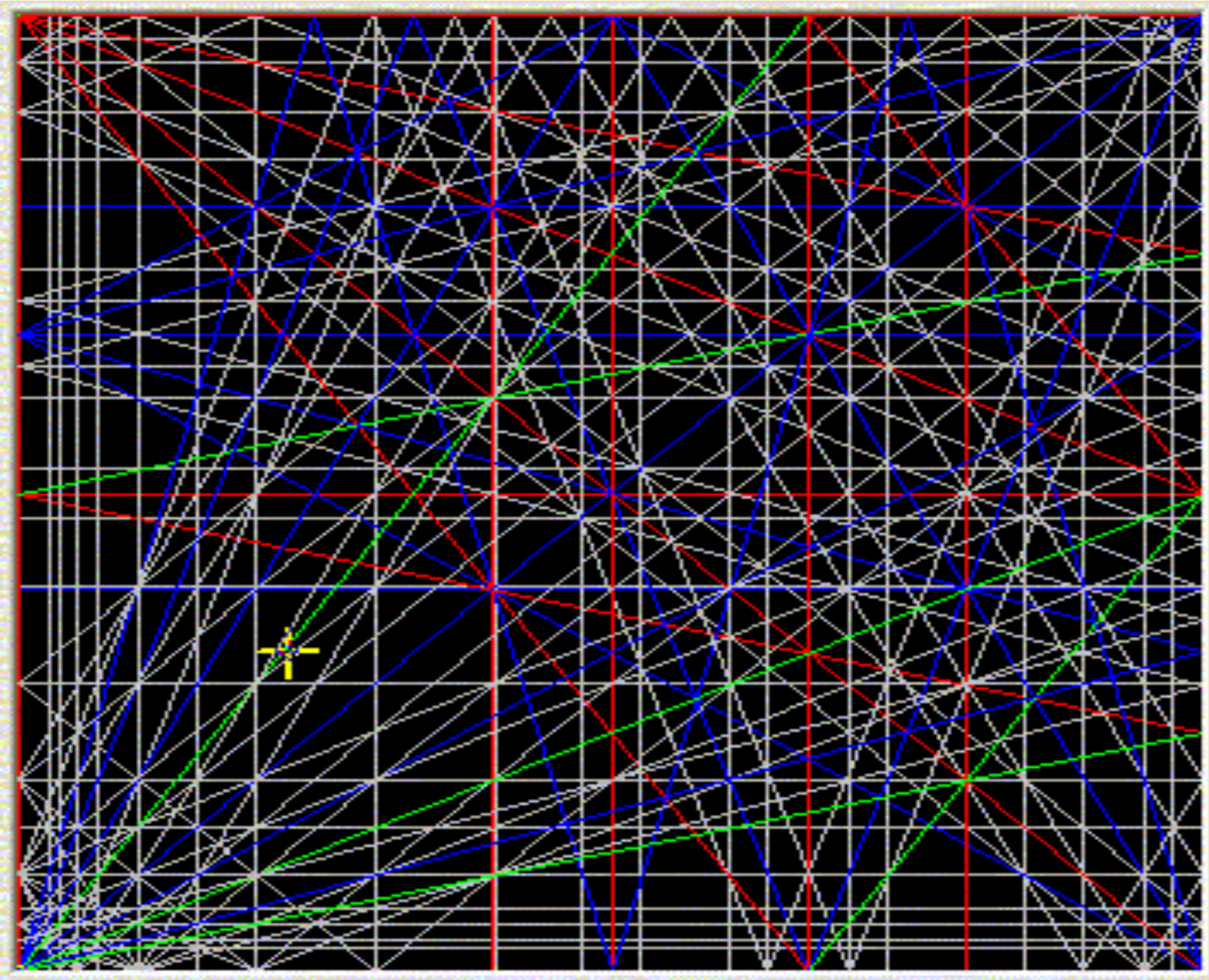
Ideal situation: cromaticity well corrected, (Q' ≈ 1)



Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

Tune diagram up to 3rd order



… and up to 7th order

Homework for the operateurs:
find a nice place for the tune
where against all probability
the beam will survive

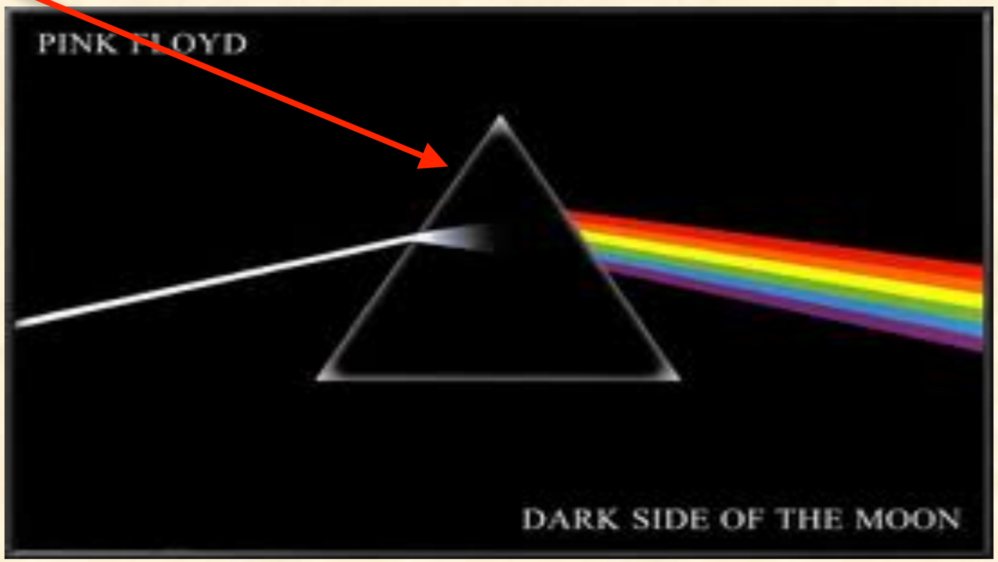
Chromaticity Correction:

We need a magnetic field that focuses stronger those individual particles that have larger momentum and focuses weaker those with lower momentum.

... but that does not exist.

The way the trick goes:

1.) sort the particle trajectories according to their energy
we use the dispersion to do the job



2.) introduce magnetic fields that increase stronger than linear with the distance Δx to the centre

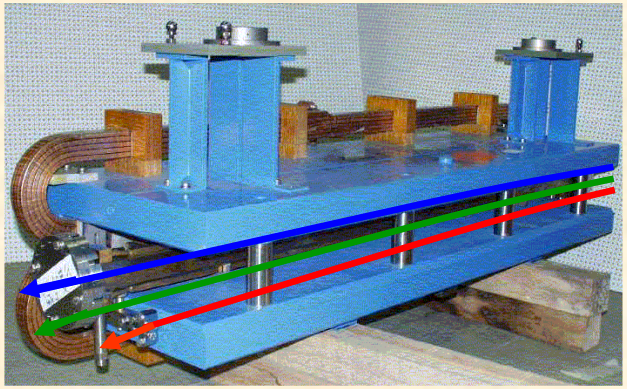
3.) calculate these fields (sextupoles) in a way that the lack of focusing strength is exactly compensated.

Correction of Q':

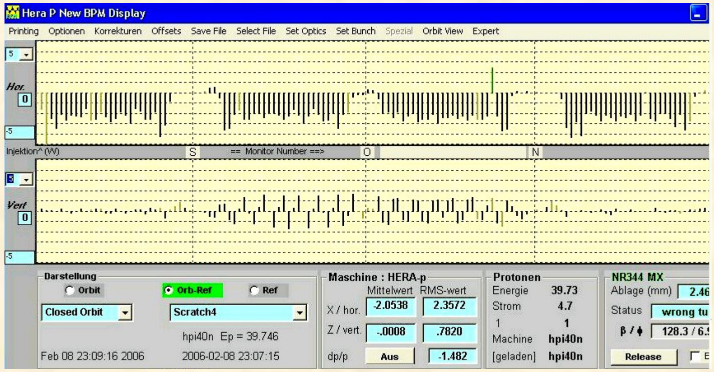
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



***2.) apply a** magnetic field that rises quadratically **with x** (sextupole field)*

$$\left.\begin{aligned} B_x &= \tilde{g}xy \\ B_y &= \frac{1}{2}\tilde{g}(x^2 - y^2) \end{aligned}\right\} \qquad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x$$

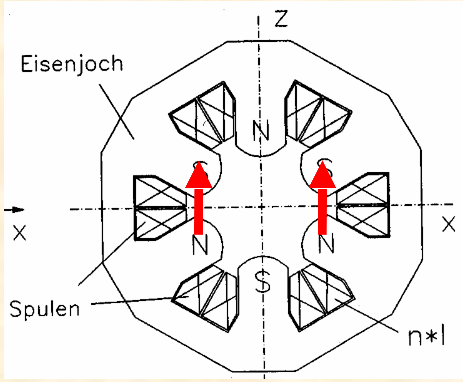
—> amplitude dependent gradient

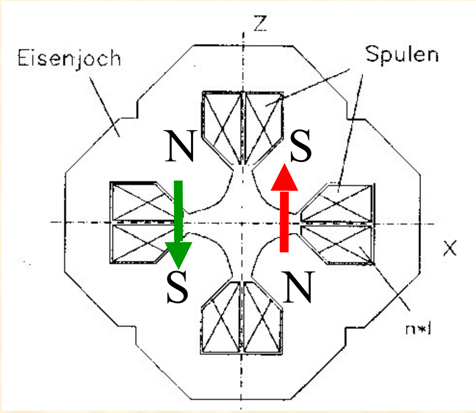
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

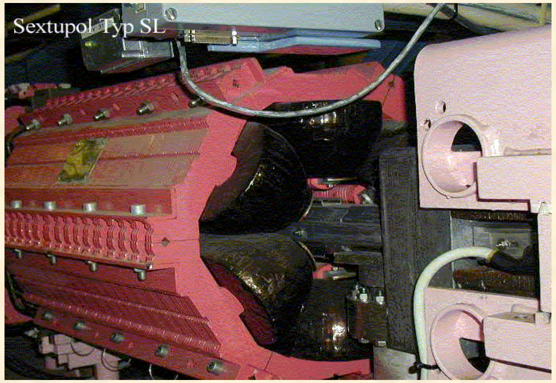
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}x}{p/e} = k_2 * x$$

$$= k_2 * D\frac{\Delta p}{p}$$



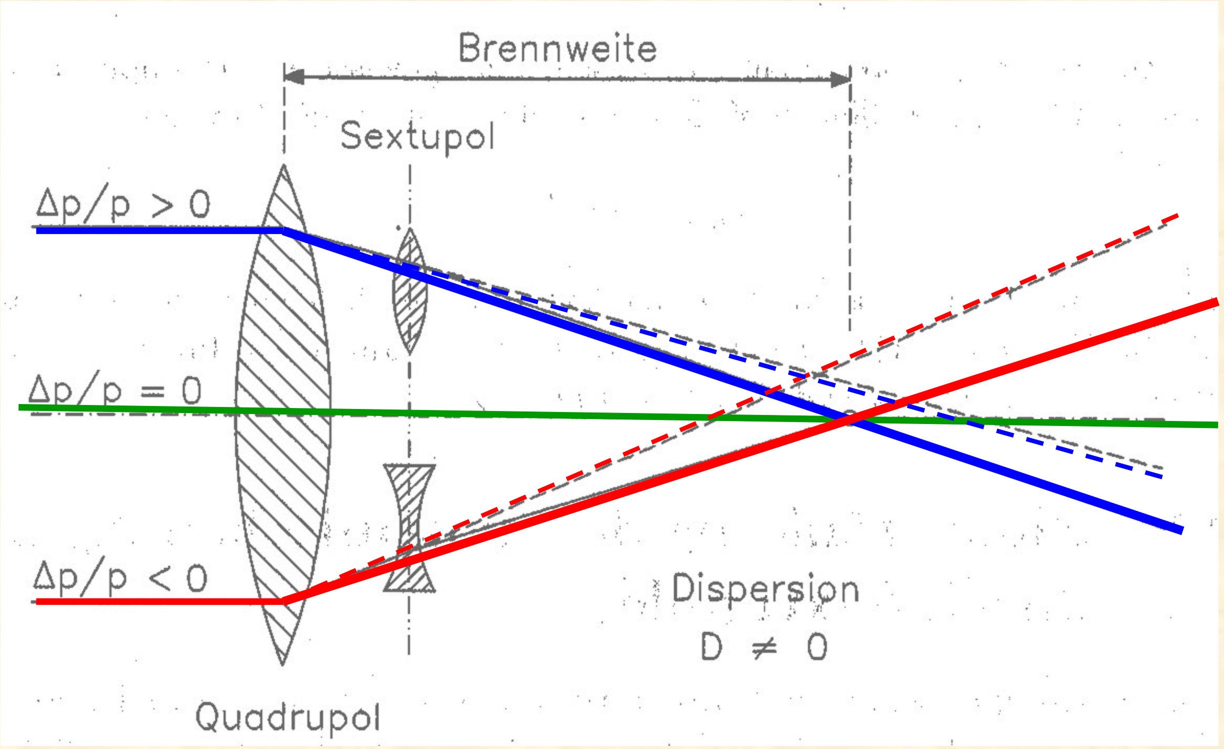
Combined effect of „natural chromaticity“ and Sextupole Magnets:

$$Q' = -\frac{1}{4\pi}\{\int k_1(s)\beta(s)\ ds + \int k_2(s)D(s)\beta(s)\ ds\}$$

You only should not forget to correct Q‘ in both planes ...
and take into account the contribution from quadrupoles of both polarities.

Chromaticity Correction:

schematical view



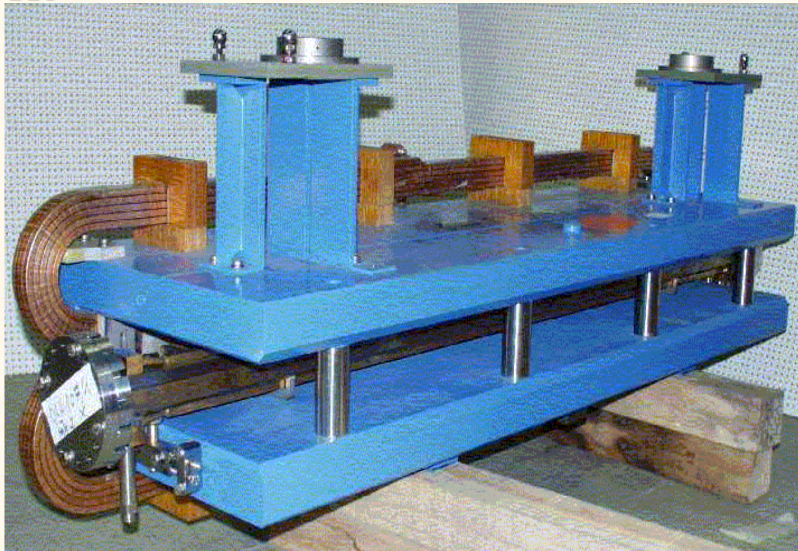
A word of caution: keep non-linear terms in your storage ring low.

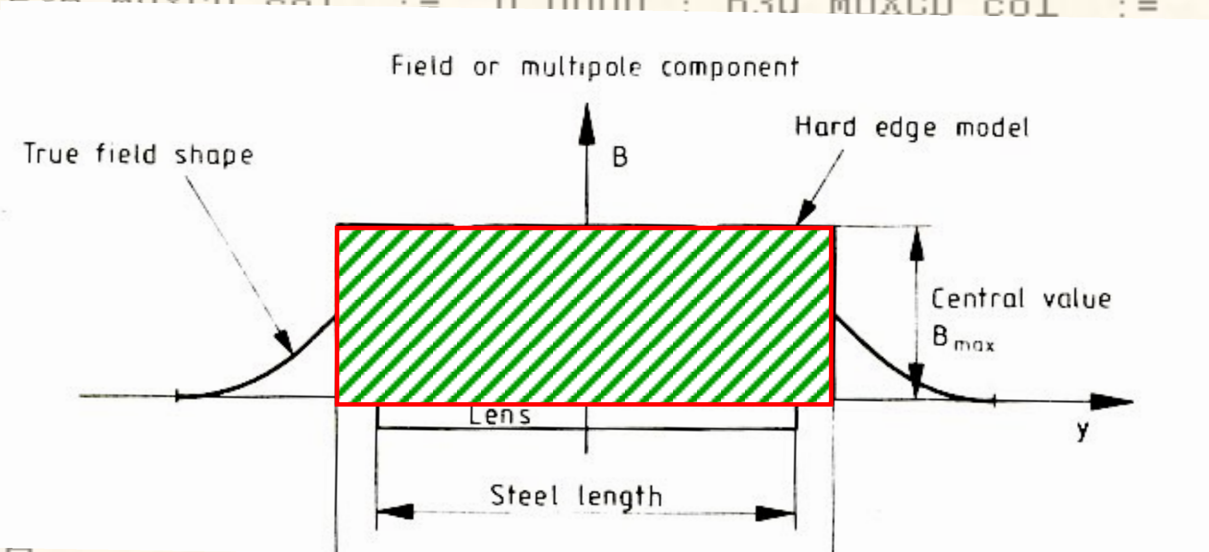
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$

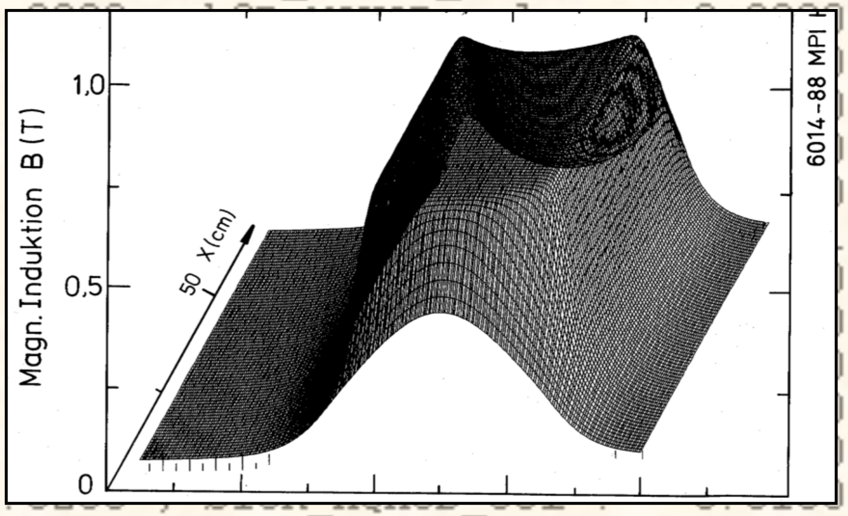
"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B ds$$

```
!
!                           bn at injection
b1M_MQXCD_inj  :=  0.0000 ; b1U_MQXCD_inj  :=
b2M_MQXCD_inj  :=  0.0000 ; b2U_MQXCD_inj  :=
b9M_MQXCD_inj  :=  0.0000 ; b9U_MQXCD_inj  :=
b10M_MQXCD_inj :=  0.5000 ; b10U_MQXCD_inj :=
b11M_MQXCD_inj :=  0.0000 ; b11U_MQXCD_inj :=
                            b12U_MQXCD_inj :=
                            b13U_MQXCD_inj :=
b14M_MQXCD_inj := -0.2700 ; b14U_MQXCD_inj :=  0.0300 ; b14R_MQXCD_inj :=  0.0100
b15M_MQXCD_                 b15U_MQXCD_inj :=  0.0000 ; b15R_MQXCD_inj :=  0.0000
!
!                           bn in collision
b1M_MQXCD_c                 b1U_MQXCD_col  :=  0.0000 ; b1R_MQXCD_col  :=  0.0000
b2M_MQXCD_col  :=  0.0000 ; b2U_MQXCD_col  :=
                                               0.0400 ; b14R_MQXCD_col :=  0.0100
                                               0.0000 ; b15R_MQXCD_col :=  0.0000
```







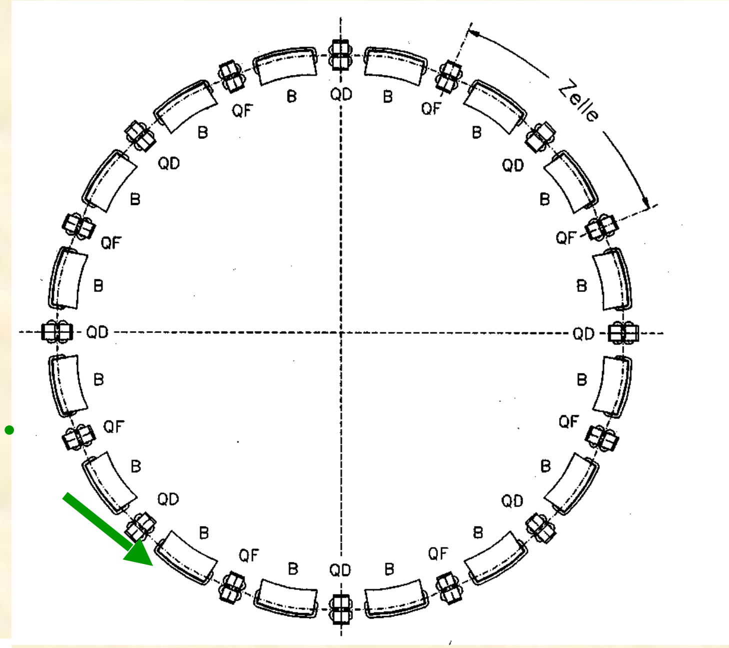
Clearly there is another problem ...

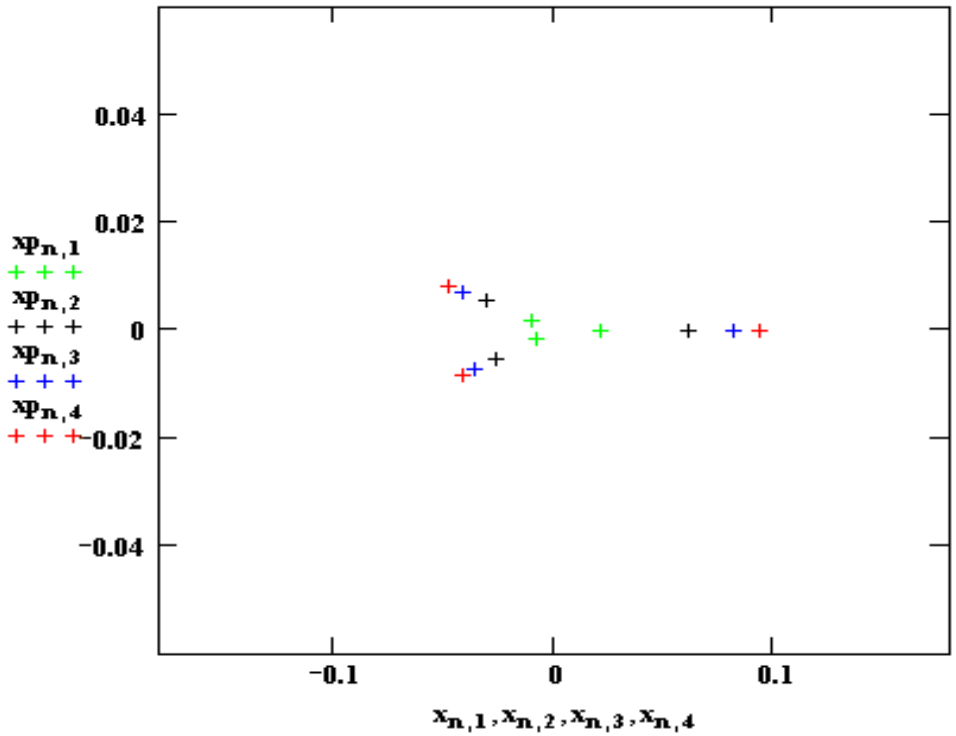
... if it were easy everybody could do it

Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$





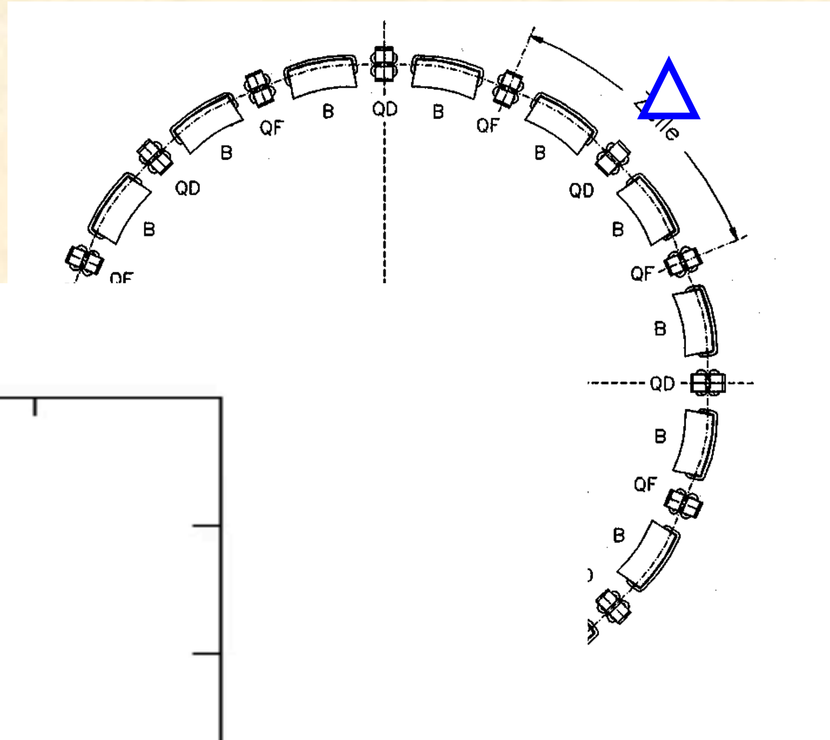
A beam of 4 particles

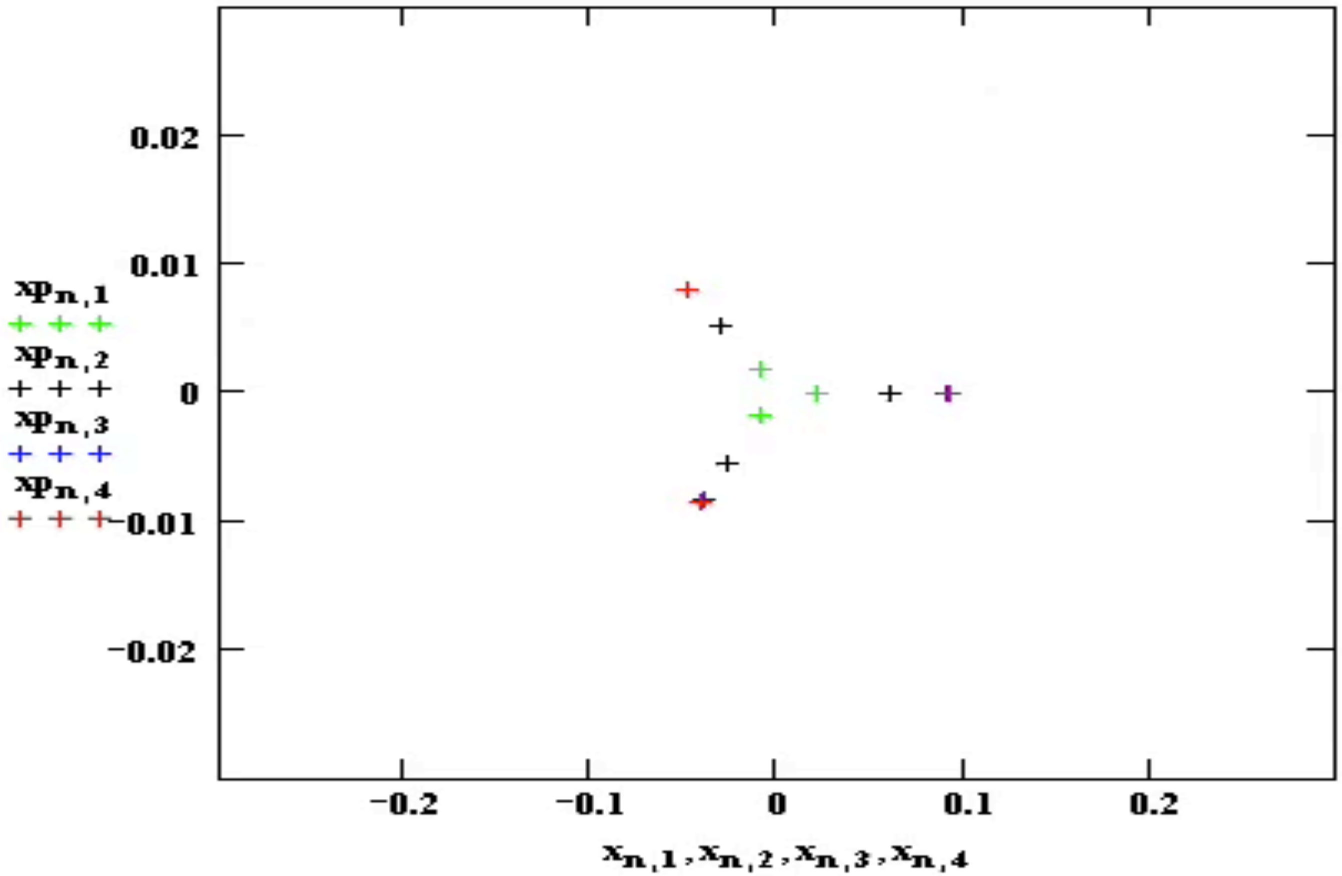
– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

The good news: sextupole fields in accelerators cannot be treated analytically anymore.

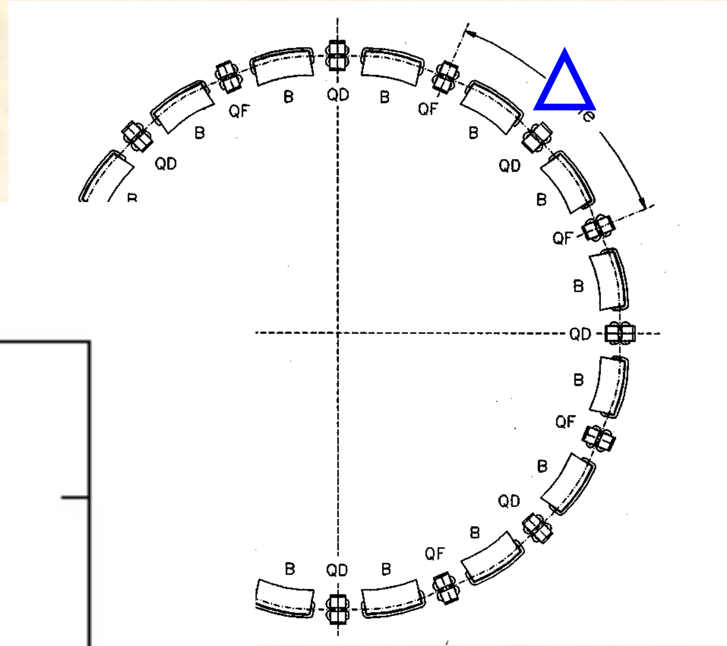
→ no equatiuons; instead: Computer simulation „ particle tracking "

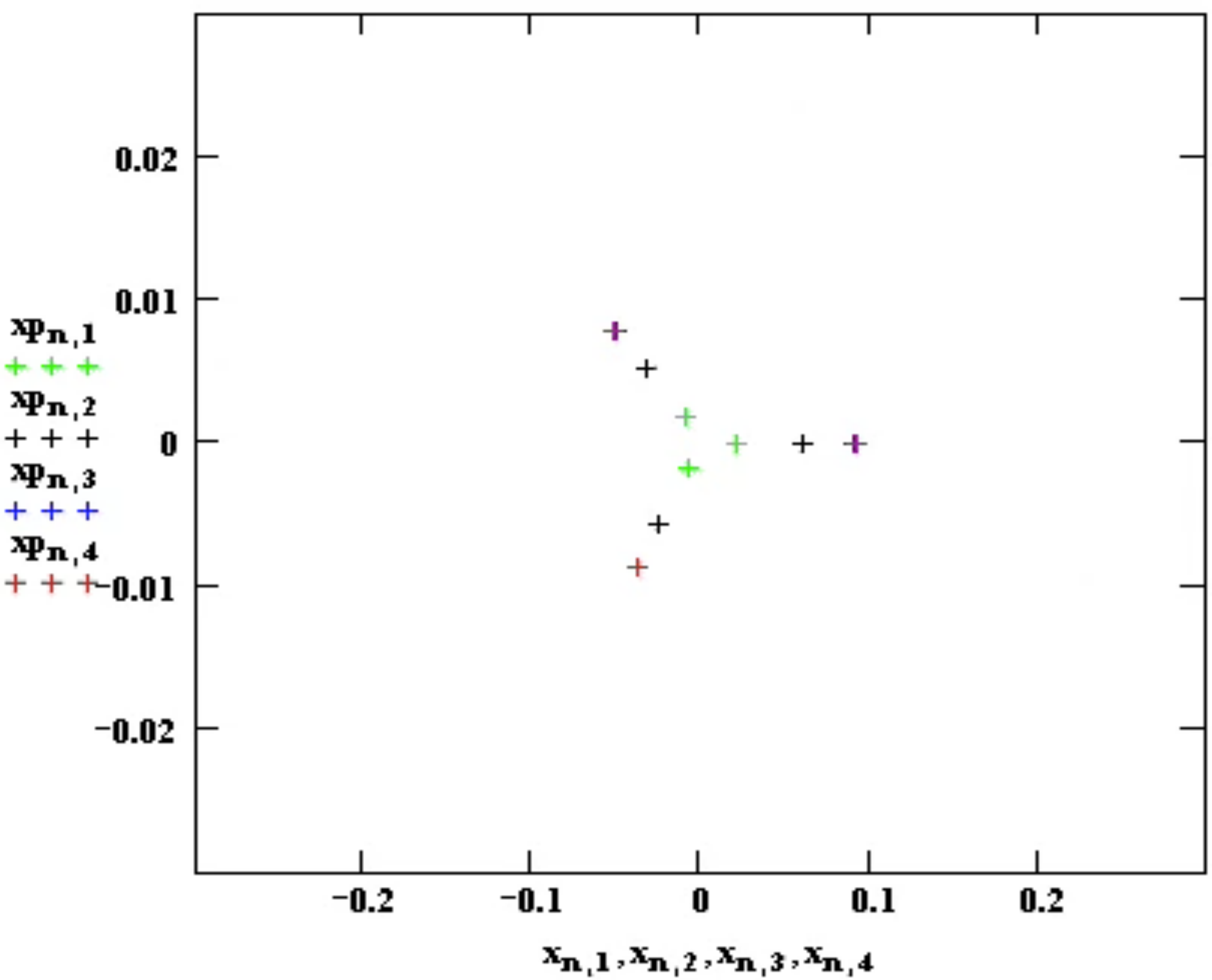




Effect of a strong (!!!) Sextupole …

→ *Catastrophy !*





„dynamic aperture“

The Mini-Beta scheme ...

... focusses strongly the beams to achieve smallest possible beam sizes at the IP. The obtained small beta function at the IP is called β^*.

Don't forget the cat.

Beam dimension during acceleration: A proton beam shrinks during acceleration in both ytransverse dimensions. We call it unfortunately „adiabatic shrinking".

Nota bene: An electron beam in a ring is *growing* with energy !!

Dispersion ...

... is the particle orbit for a *given* momentum difference.

Chromaticity ...

... is a focusing problem. Different momenta lead to different tunes

→ attention ... resonances !!

Sextupoles ...

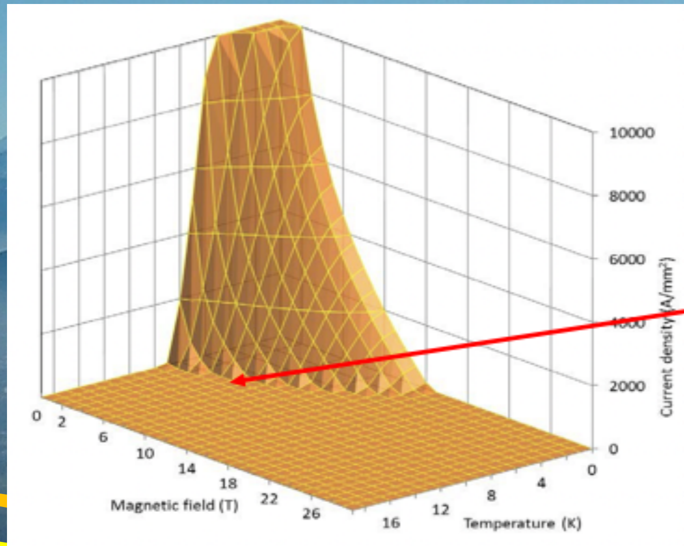
have non-linear fields and are used to compensate chromaticity. However we have to be careful: Strong non-linear fields can lead to particle losses (dynamic aperture)

2.) Where do we go ?

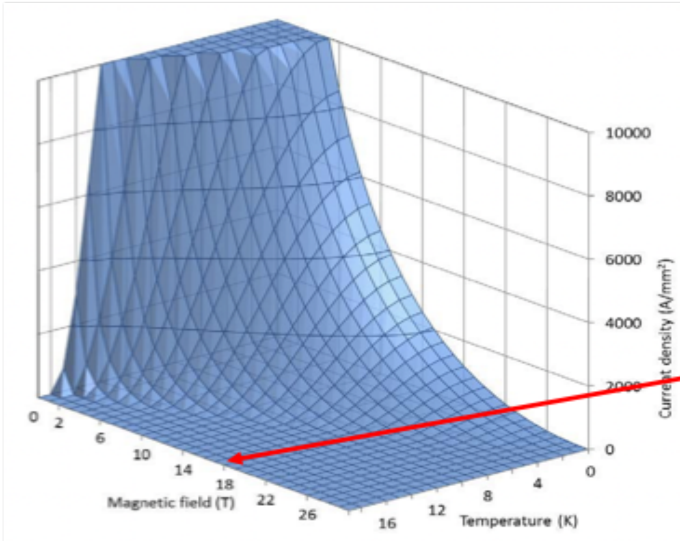
** Physics beyond the Standard Model*
** Dark Matter / Dark Energy*

FCC-pp - Collider

The Next Generation Ring Collider



NbTi:
B = 8.3 T

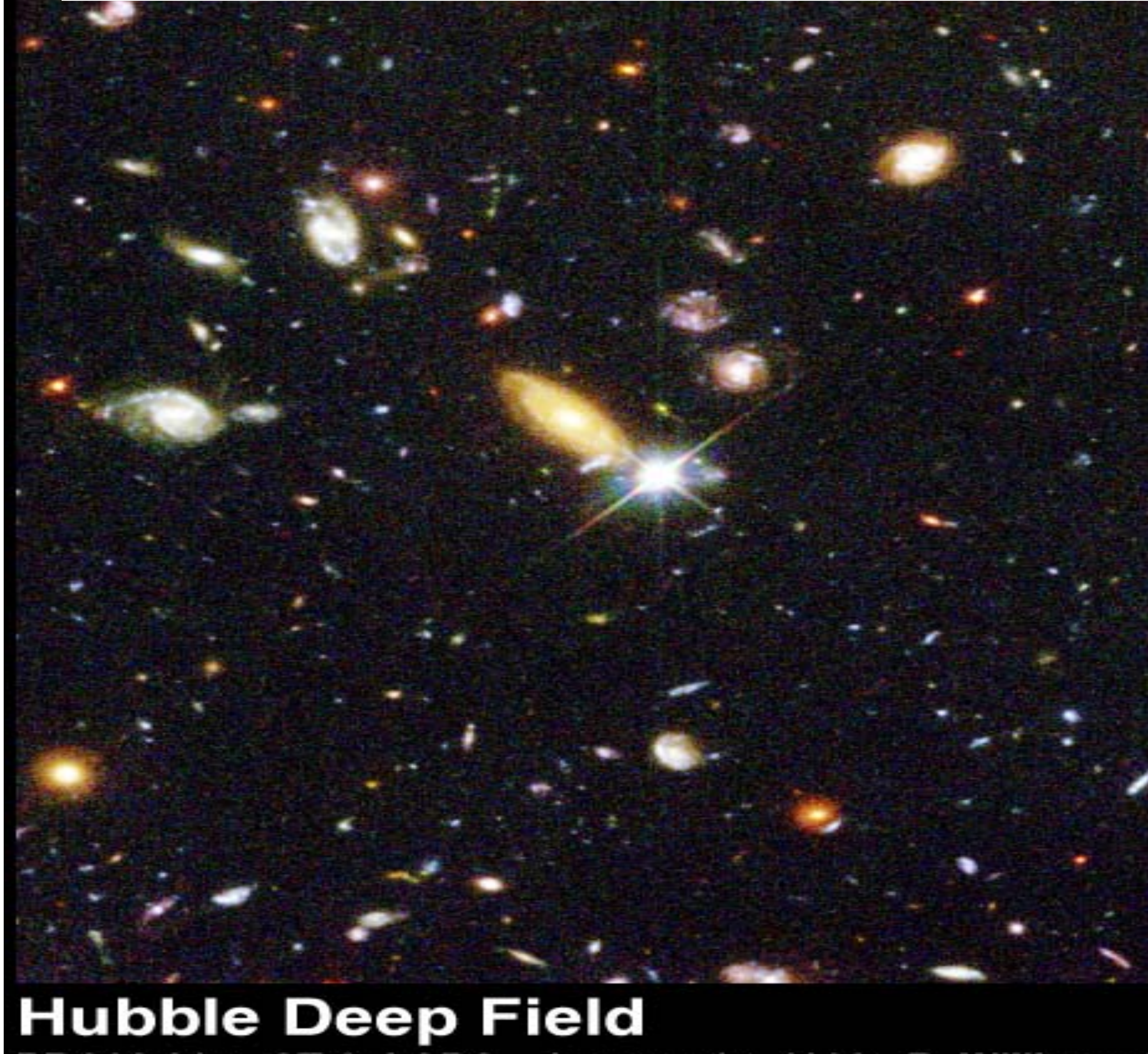


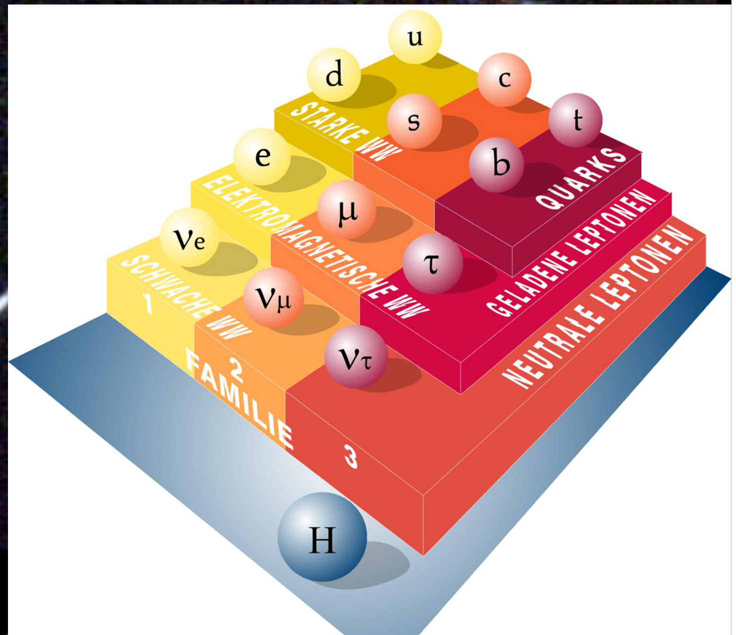
Nb_3Sn
B =16 T

What's next ???

Dark Matter & Dark Energy

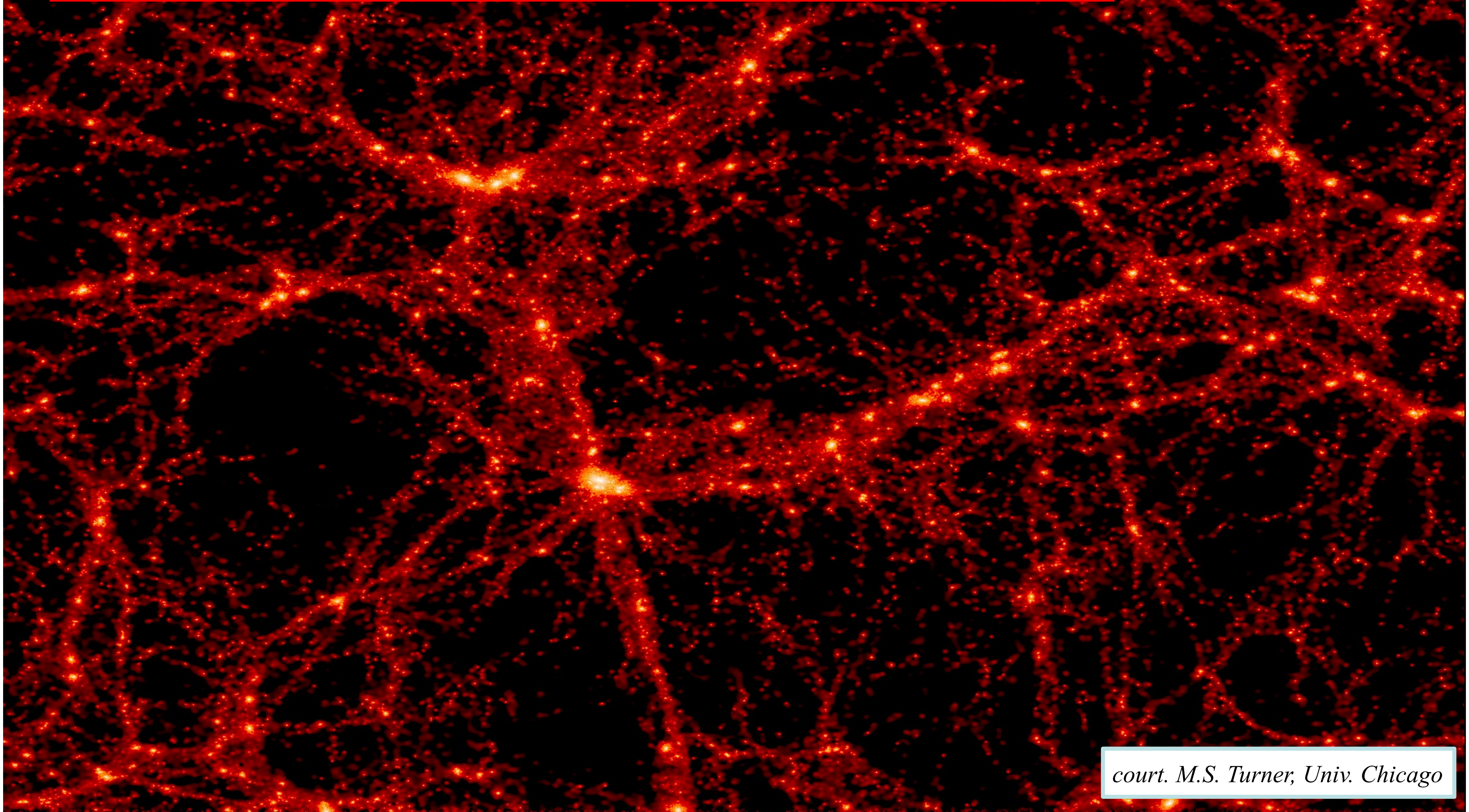
Physics beyond the Standard Model





Reconstruction of Dark Matter distribution based on observations

Budget: ***Dark Matter: 26 %***
Dark Energy: 70 %
Anything else (including us) 4 %



court. M.S. Turner, Univ. Chicago

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