Linear Imperfections

Basic CAS @ online,

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Copyright statement and speaker's release for video publishing

The CERN Accelerator School

Introduction

Imperfection - sources

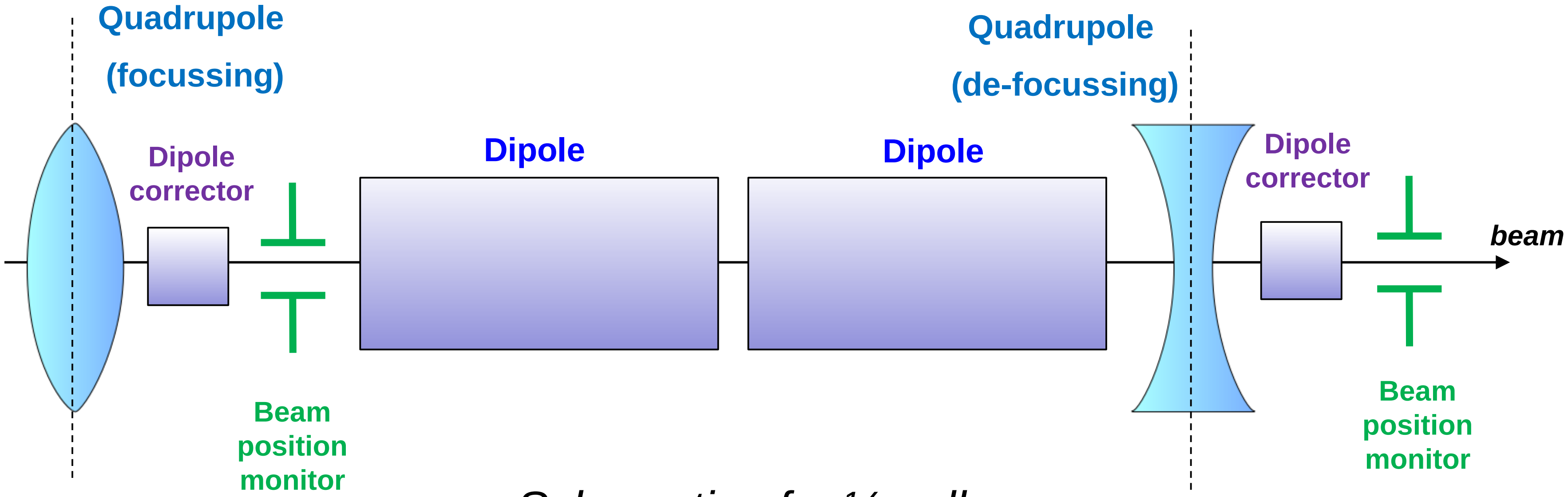
Orbit perturbations

Optics perturbations

Linear imperfections and geology

Summary

- An accelerator is typically build using a number of basic ‘cells’.
- The cell layouts of accelerators come in many variants.
- For today we consider a simple FODO cell containing:
  - **Dipole magnets** to bend the beams,
  - **Quadrupole magnets** to focus the beams,
  - **Beam position monitors** (BPM) to measure the beam position,
  - **Small dipole corrector magnets** for beam steering.



Schematic of a ½ cell

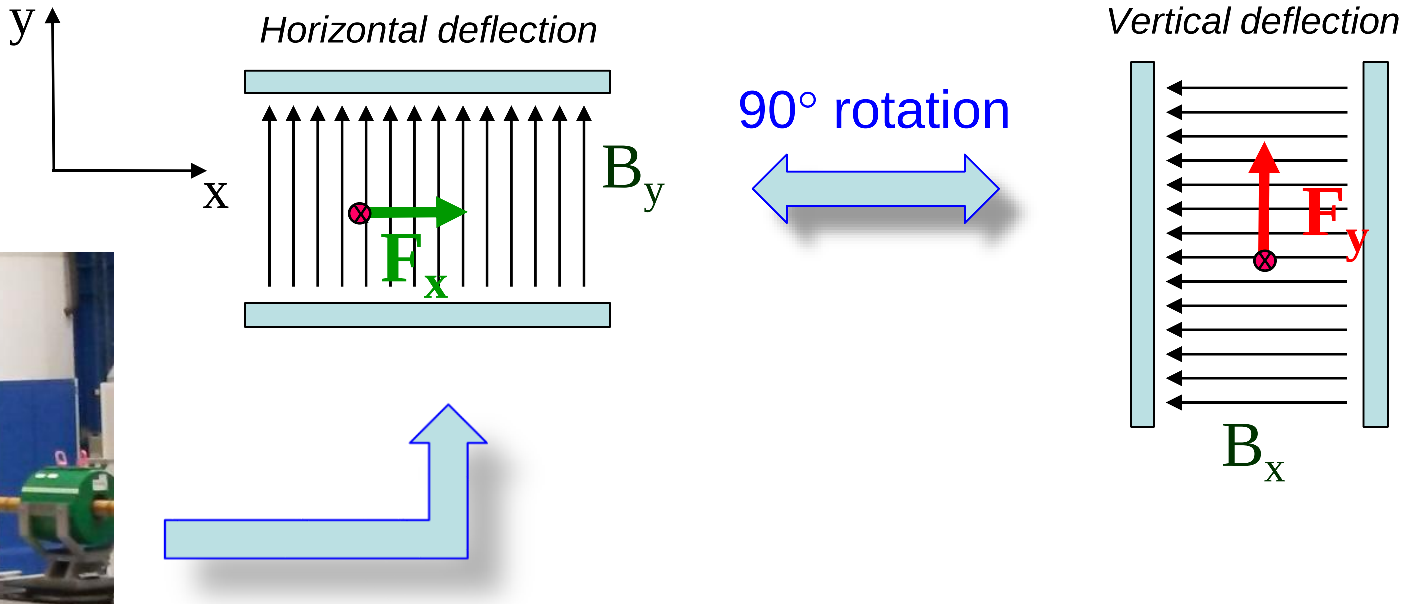
Dipole magnet

- The dipole has two magnetic poles and generates a **homogeneous field** providing a constant force on all beam particles – used to **deflect** the beam.
  - A **dipole corrector** is just a **small version** of such a magnet, dedicated to steer the beam.

Lorentz force:

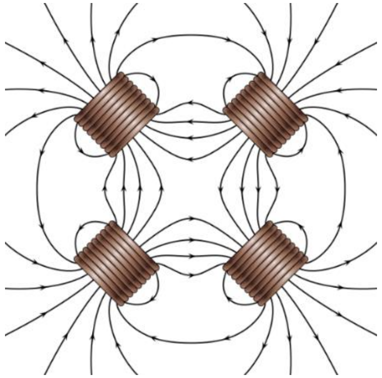
$$F = q\vec{v}\times\vec{B}$$

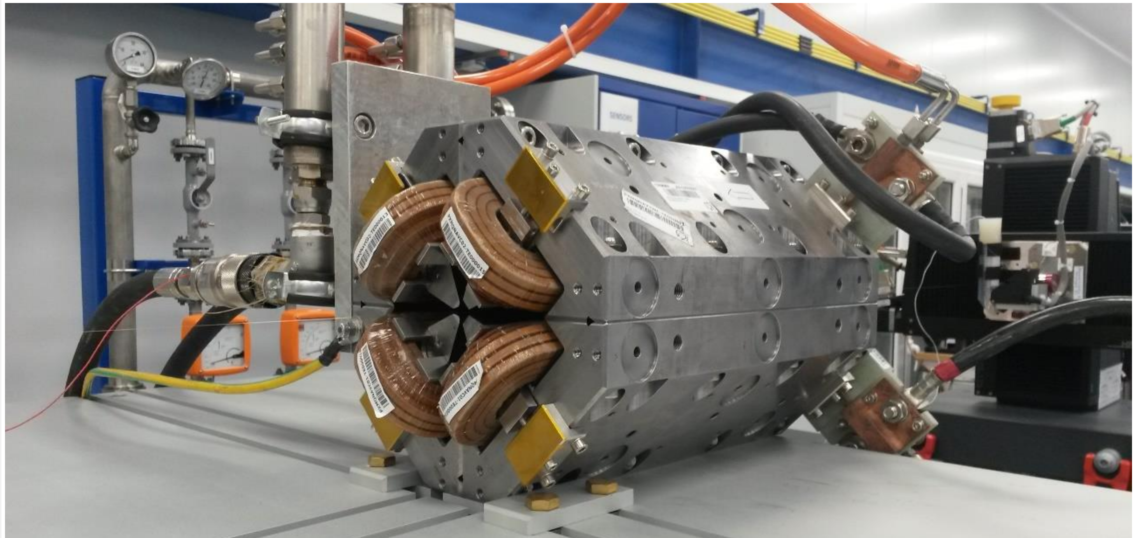
orthogonal to the speed and magnetic field directions

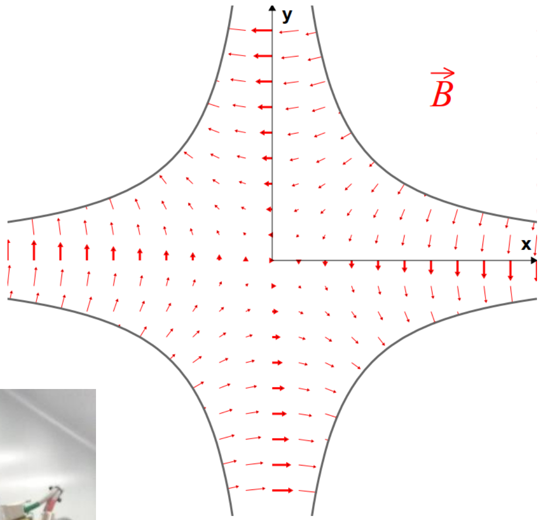


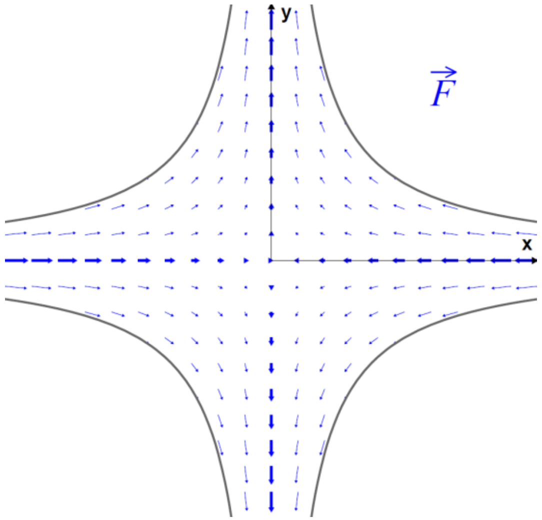
Quadrupole magnet

- A quadrupole has 4 magnetic poles.
- A quadrupole provides a field (force) that **increases linearly** with the distance to the quadrupole centre – provides **focussing** of the beam.
  - Similar to an optical lens, but a quadrupole is focussing in one plane, defocussing in the other plane.









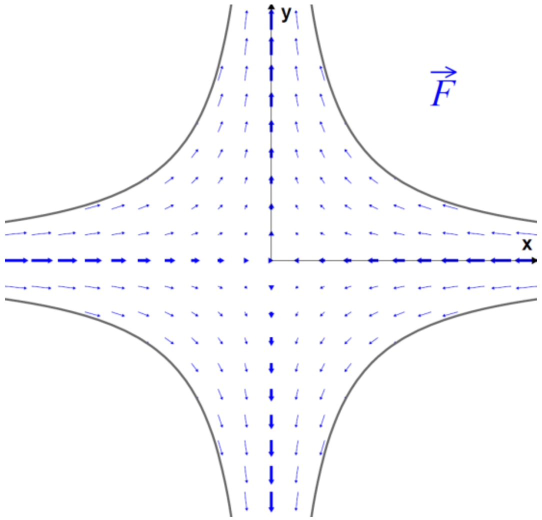
$$\vec{F} = \begin{cases} F_x = -k\,x \\ F_y = k\,y \end{cases}$$

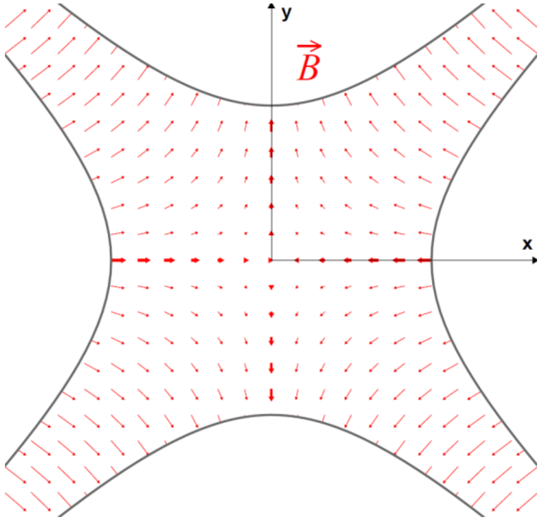
Force pushes the particle towards the center → **focussing**

Force pushes the particle away from the center → **defocussing**

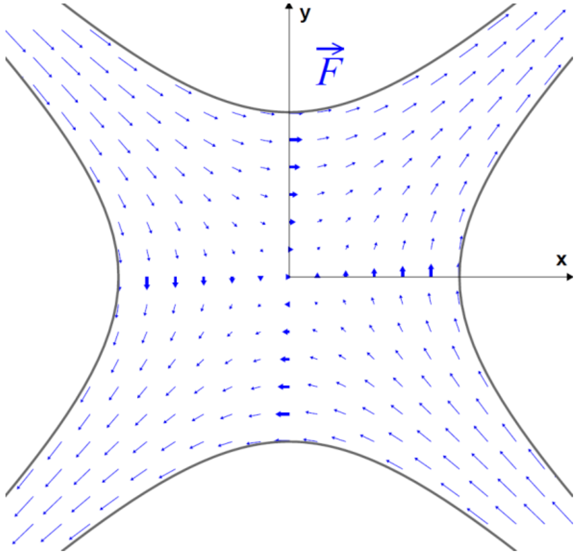
- A quadrupole **rotated by 45º** (‘skew quadrupole’) produces a force (deflection) in x that depends on y and vice-versa: such a magnet **couples** horizontal and vertical plane.

normal quadrupole





skew quadrupole



$F_x = -k\,x$

$F_y = k\,y$

No mixing of planes

Full mixing of planes

$F_x = k\,y$

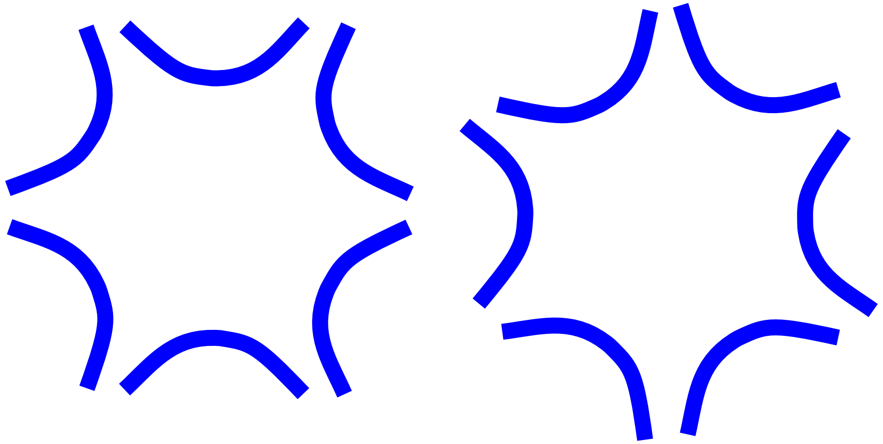
$F_y = -k\,x$

N-pole magnets

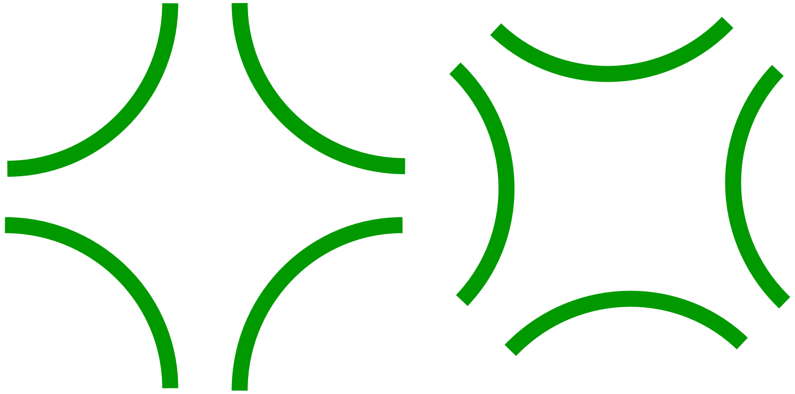
- The concept of normal / skew quadrupole can be applied to any 2N-pole magnet.
  - **Normal** variant – generally referred to as B_N,
  - **Skew** variant – generally referred to as A_N, rotated by **180°/2N** wrt B_N.
- Examples:

Dipole N =1

Sextupole N=3

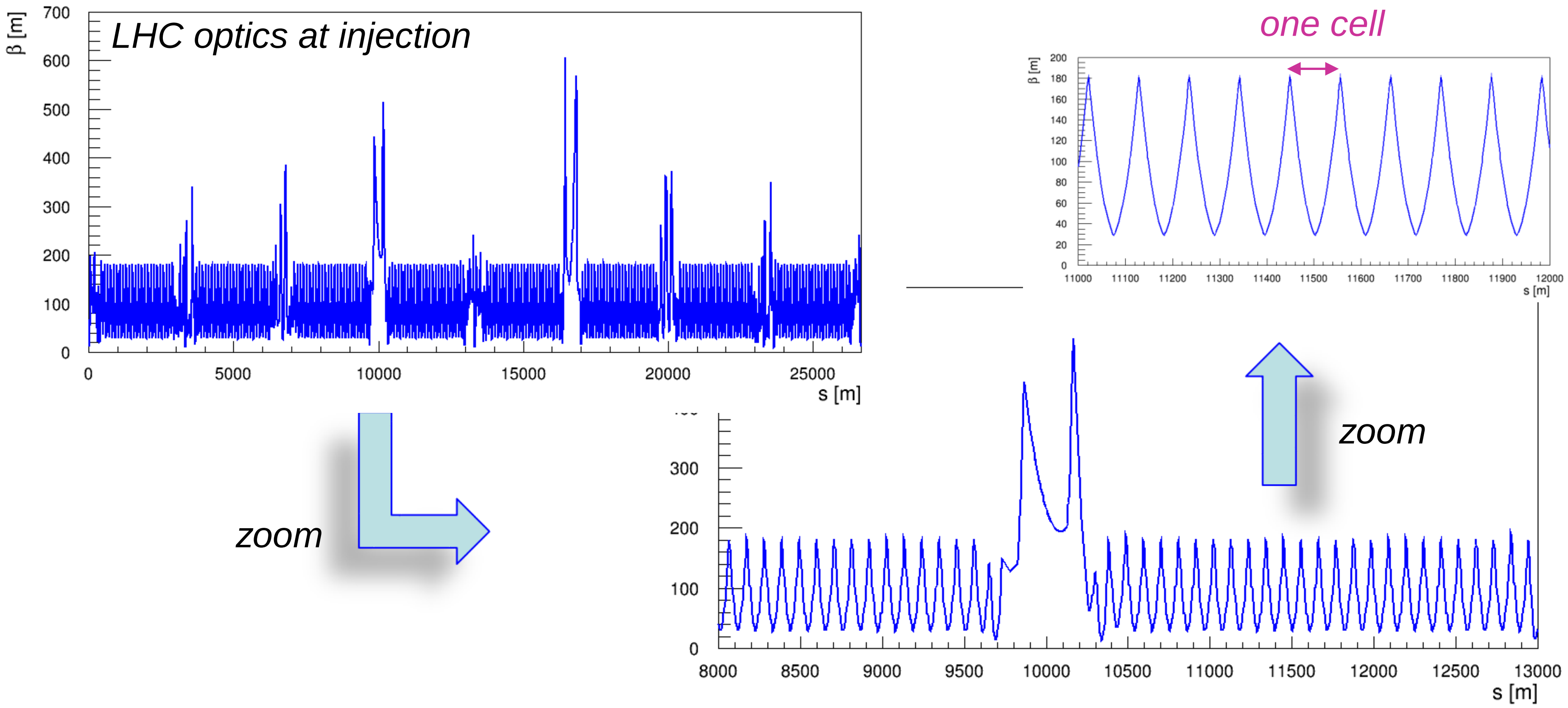


Quadrupole N=2



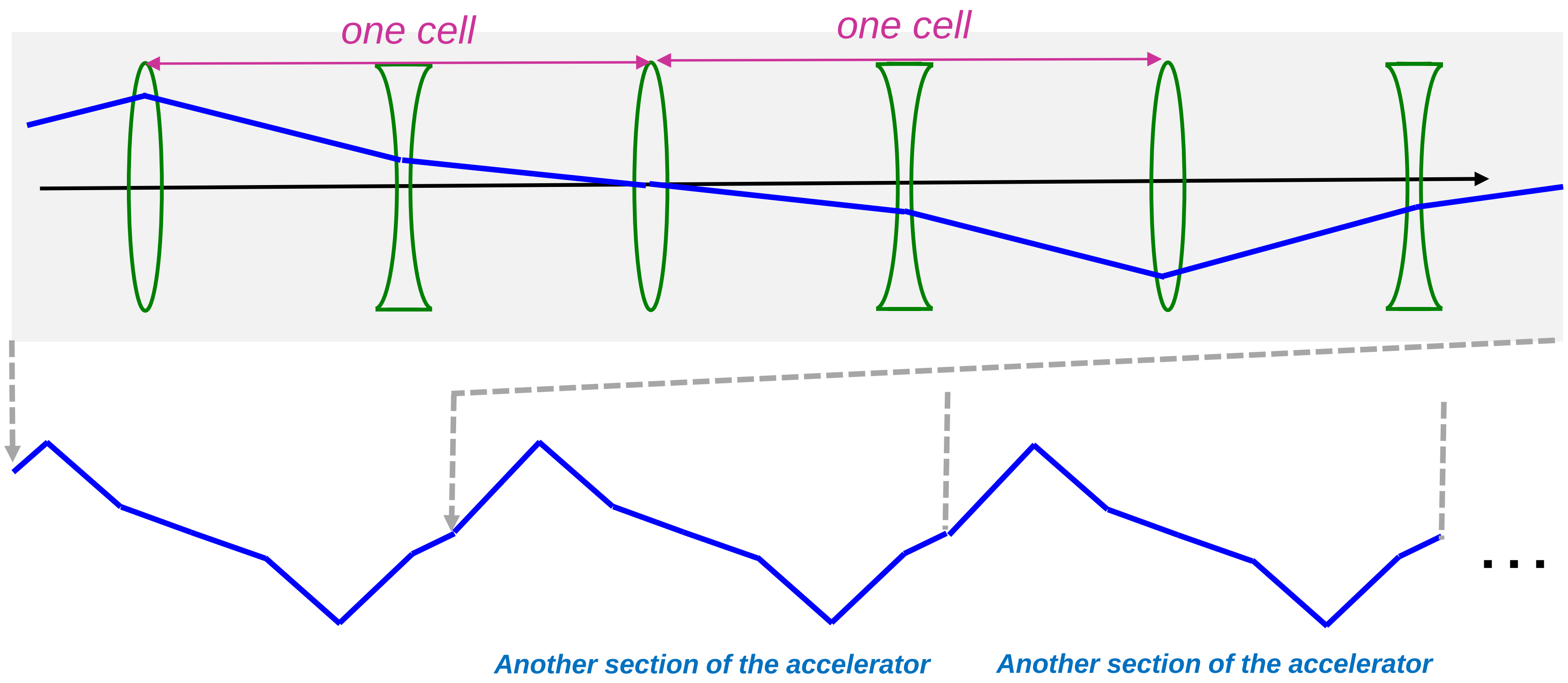
Recap on beam optics

- Quantities related to a beam optics in a circular accelerator will be needed for the lecture:
  - The **betatron function** (**β**) that defines the beam envelope,
    - **Beam size / envelope is proportional to √β**
  - The **betatron phase advance** (**μ**) that defines the phase of an oscillation.



Recap on beam optics

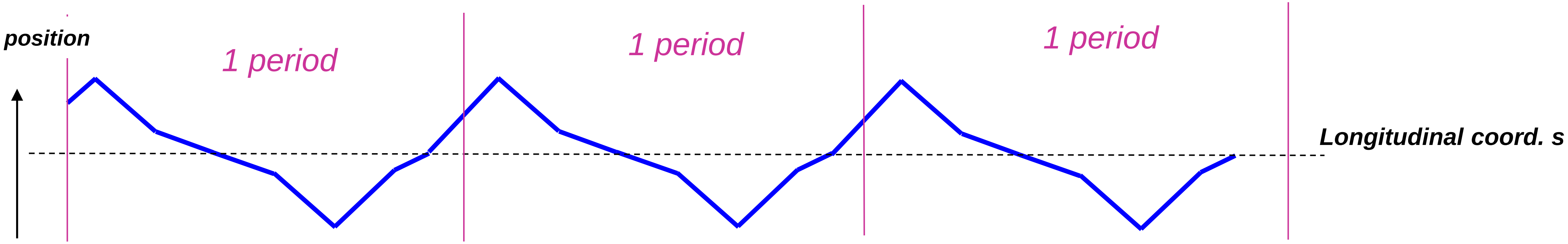
- Consider a particle moving in a section of the accelerator lattice. The focussing elements make it bounce back and forth.



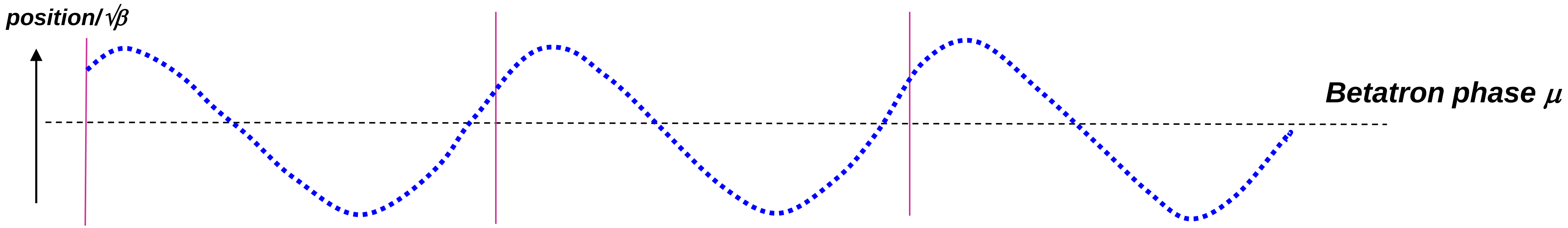
- This periodic oscillation is called the betatron oscillation.

Recap on beam optics for pedestrians

- The number of oscillation periods over one machine turn is called the machine **tune** (**Q**) or **betatron tune**.
  - In this **example** Q is around 2.75 – 2 periods and ¾ of a period.



- With **coordinate change** (from longitudinal position in meters to betatron phase advance in degrees) this 'rocky' oscillation is transformed into a sinusoidal oscillation.
  - Convenient and simple way to analyse the beam motion.



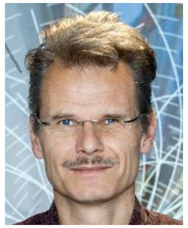
Imperfections

- The first step in the design phase of an accelerations consists in building an **“ideal” accelerator** where all magnets have nominal fields and are perfectly aligned along the design trajectory.
- But quite rapidly the designer must confront the real world, and **tolerances on errors (= imperfections) must be defined** to provide specifications for component design, manufacturing and alignment.
  - What is the precision on field quality?
  - What is the precision and stability of the power converter that feeds current into a magnet?
  - What is the tolerance on the component alignment?
  - …
- This lecture will discuss the impact of the simplest form of imperfections, the **linear imperfections**.

From model to reality - fields

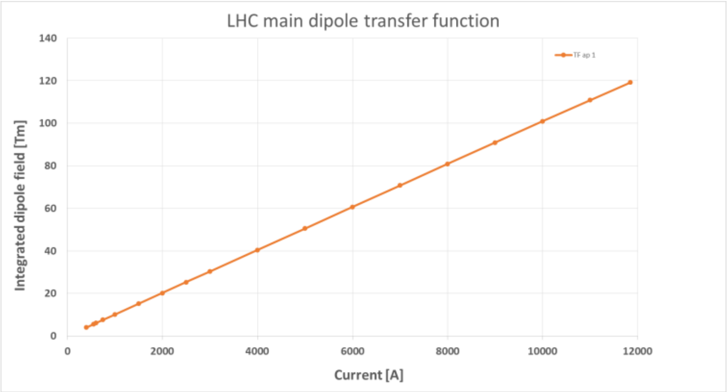
- The **physical units** of the machine model defined by the accelerator physicist must be converted into **magnetic fields** and eventually into **currents** for the power converters that feed the magnets.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by **uncertainties** or **errors** on:
  - Beam momentum, magnetic field model and power converter regulation.

Accelerator physicist

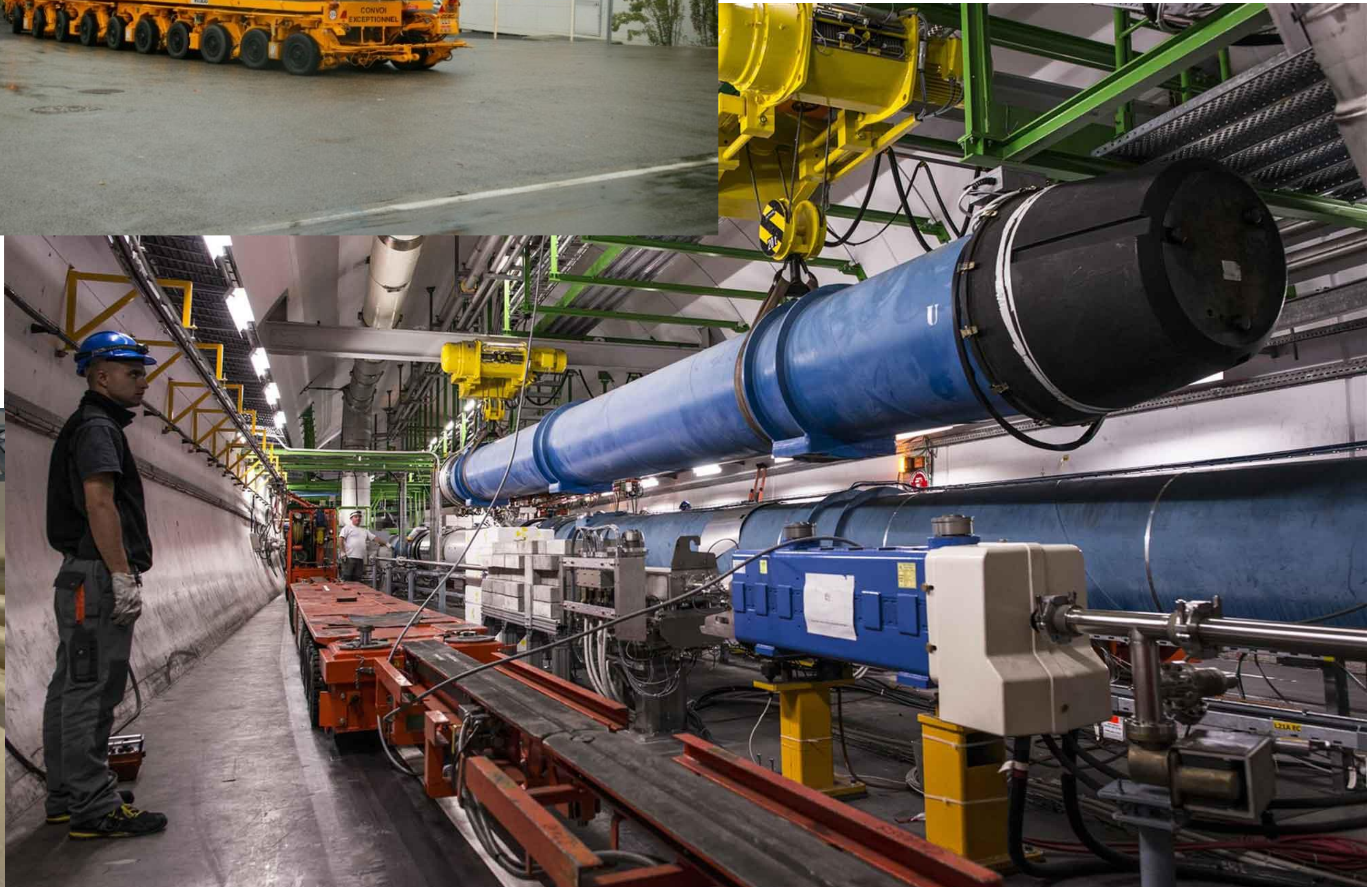


Magnet strength
(angle, focal length…)
→ **Beam momentum** →
Magnetic field or gradient
(T, T/m, T/m^2…)
→ **Magnet calibration curve (transfer function)** →
Requested current
(A)
→ **Power converter** →
Actual magnet current
(A)

Example of the LHC main dipole calibration curve

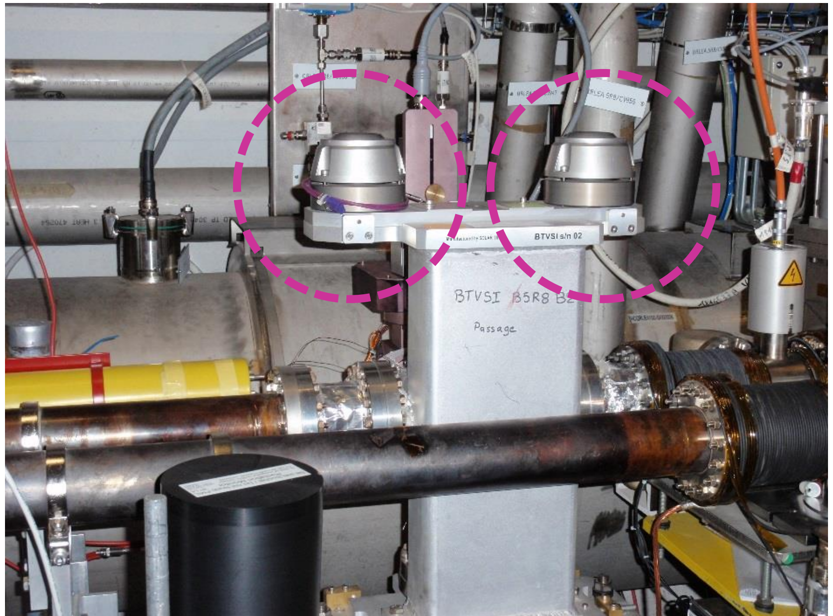


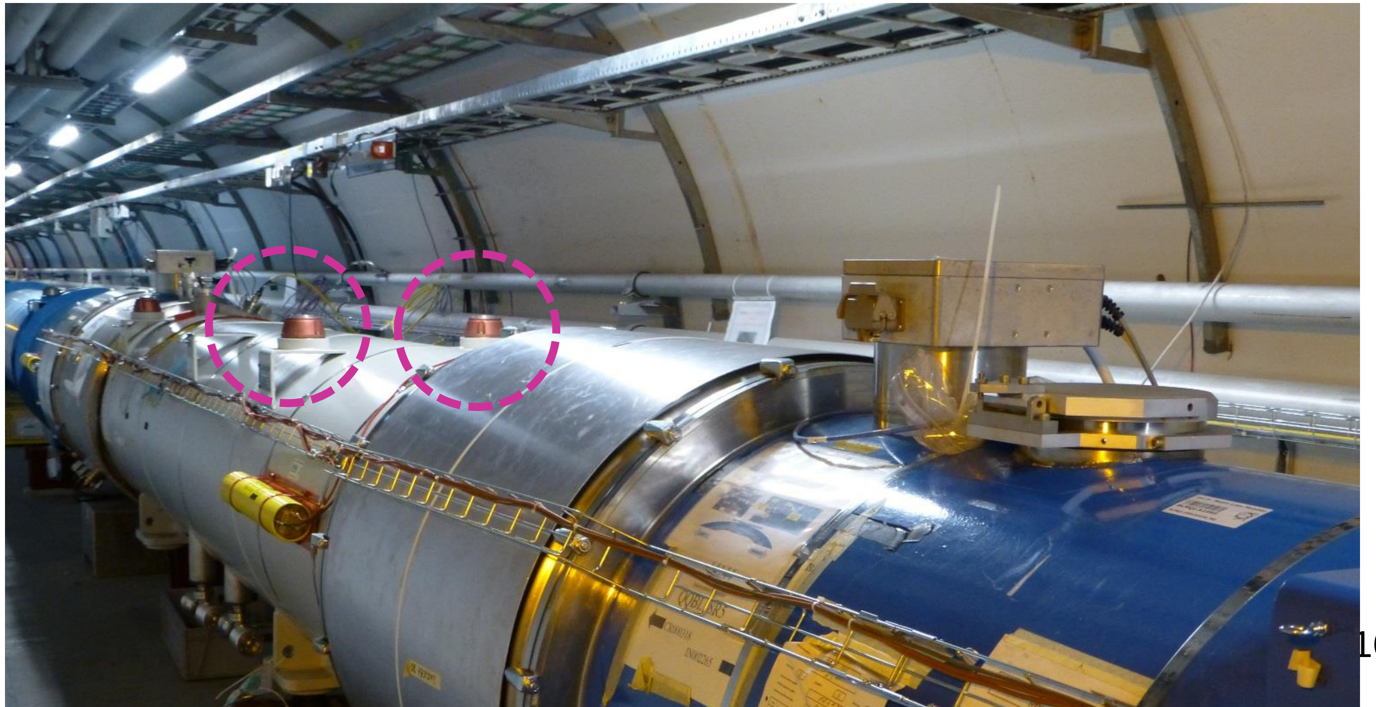
From the lab to the tunnel



From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of micrometres for a linear collider like CLIC !
  - At the CERN hadron machines we aim for accuracies of around **0.1-0.3 mm**.
- The alignment process implies:
  - Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
  - Precise in-situ alignment (position and angle) of the element in the tunnel.
- **Alignment errors** are a common source of **imperfections**.





A good attitude in the tunnel

Please remember that accelerator components in the CERN tunnels are carefully aligned – please treat with respect !

Please use the ladder !

Introduction

Imperfection - sources

Orbit perturbations

Optics perturbations

Linear imperfections and geology

Summary

Imperfection – undesired deflection

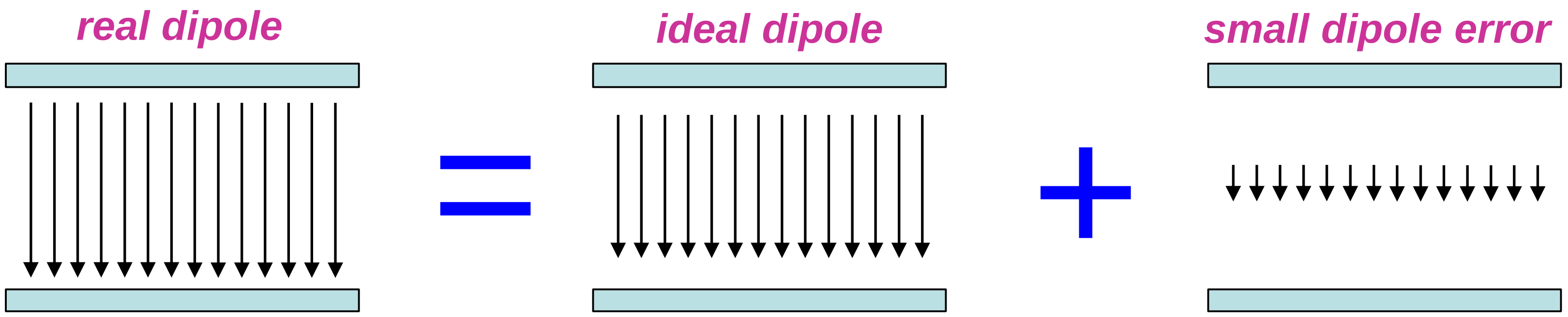
- Linear imperfections are the simplest form of machine errors involving **dipole and quadrupole fields** – let us start with dipole fields.

- The presence of an **unintended deflection** along the path of the beam is a first category of imperfections.
- This case is also in general the first one that is encountered when beam is first injected into a machine, or when a beam is launched into a linac.
- The **dipole orbit corrector** is added to the cell to **compensate** the effect of **unintended deflections**.
  - With the orbit corrector we can generate a deflection of opposite sign and amplitude that compensates locally the imperfection.

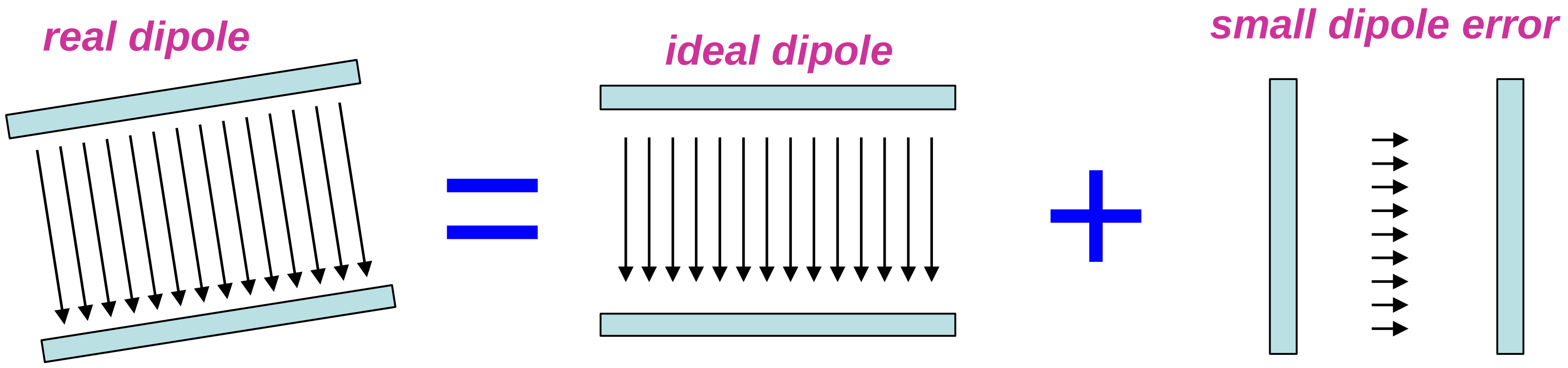
- What causes an **unintended deflection** to appear?

Unintended deflection

- The first source is a **field error** (deflection error) of a dipole magnet.
- This can be due to an **error** in the the **magnet current** or in the **calibration table** (measurement accuracy etc).
  - The imperfect dipole can be expressed as a perfect one + a small error.



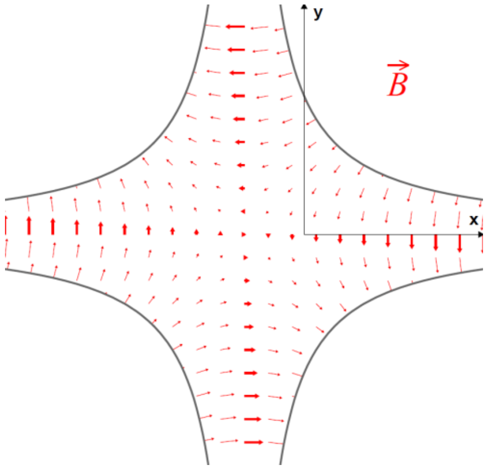
- A small rotation (misalignment) of a dipole magnet has the same effect, but in the other plane.



Unintended deflection

- The second source is a misalignment of a quadupole magnet.
  - The misaligned quadrupole can be represented as a perfectly aligned quadrupole plus a small deflection.

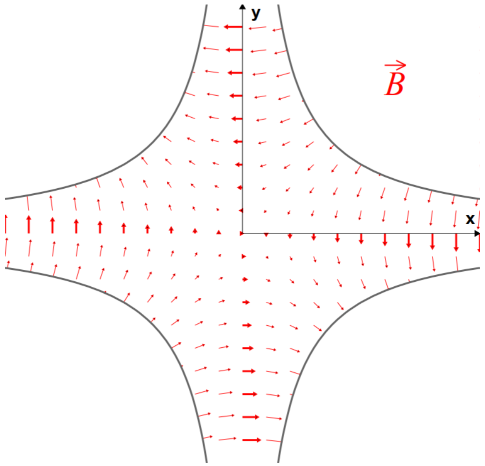
real quadrupole



Non-zero magnetic field on the beam axis !

=

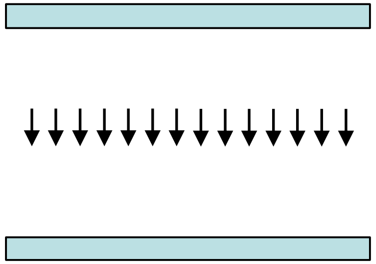
ideal quadrupole



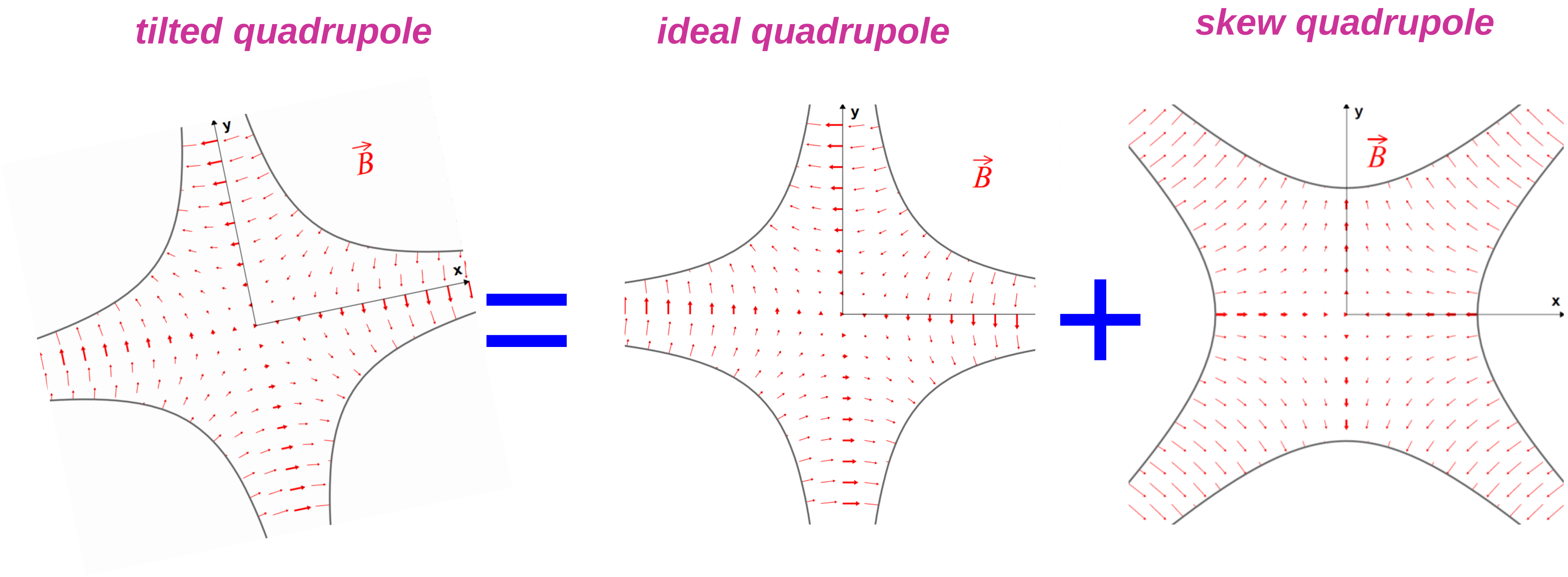
No magnetic field on the beam axis

+

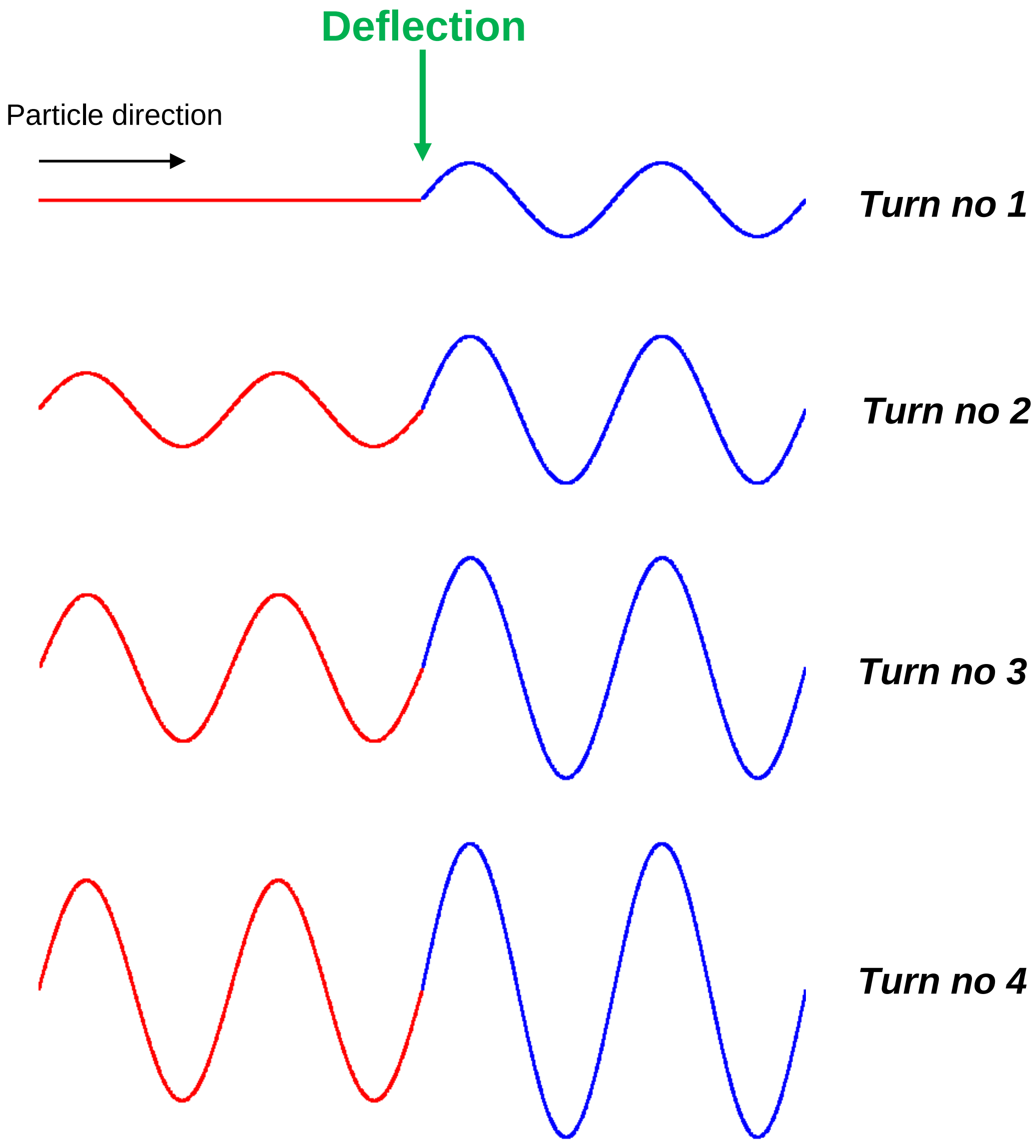
small dipole error



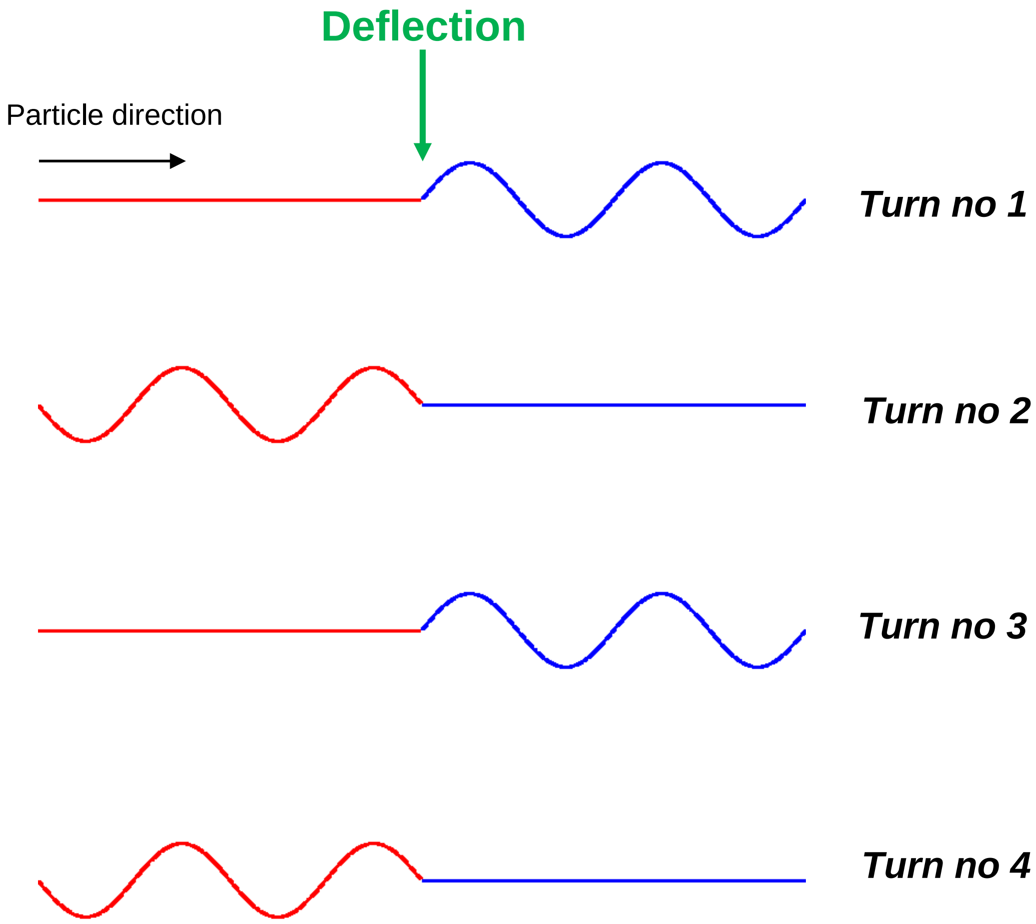
- A small rotation (misalignment) of a quadrupole leads to coupling between horizontal and vertical plane which is generally not desired.
  - The rotated quadrupole can be represented as a perfectly aligned quadrupole plus a small skew quadrupole.



Effect of a deflection

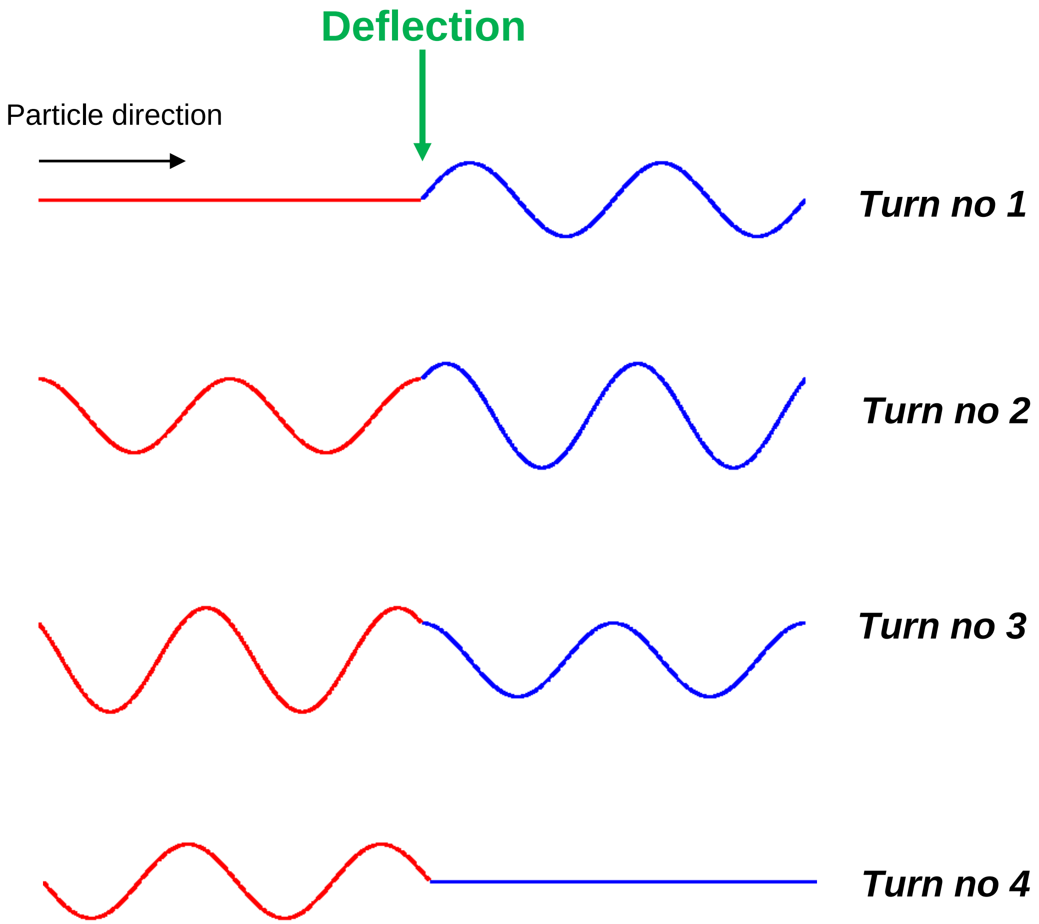


- We set the machine tune to an integer value:
  - $Q = n \in N$
- When the tune is an integer number, **the deflections add up on every turn** !
  - The amplitudes diverge, the particles do not stay within the accelerator vacuum chamber.
- We just encountered our first **resonance** – the integer resonance that occurs when $Q = n \in N$



- We set the machine tune to a half integer value:
  - $Q = n+0.5,\ n \in N$
- For half integer tune values, **the deflections compensate on every other turn** !
  - The amplitudes are stable.
- This looks like a much better working point for Q!

Effect of a deflection

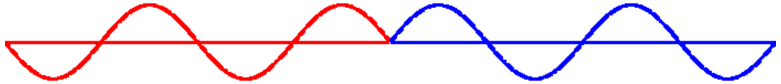


- We set the machine tune to a quarter integer value:
  - Q = n+0.25, n ∈N
- For quarter tune values, **the deflections compensate every four turns** !
  - The amplitudes are stable.
- Also a reasonable working point for Q!

Many turns reveal something

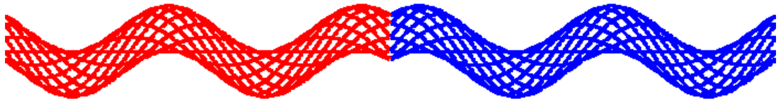
- Let's plot the 50 first turns on top of each other and change Q.
  - All plots are on the same scale

Q = n + 0.5

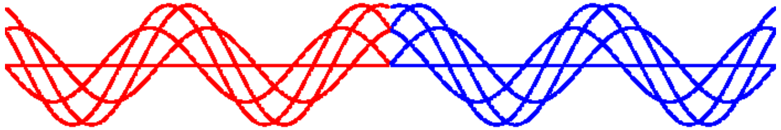


Q = n + 0.4

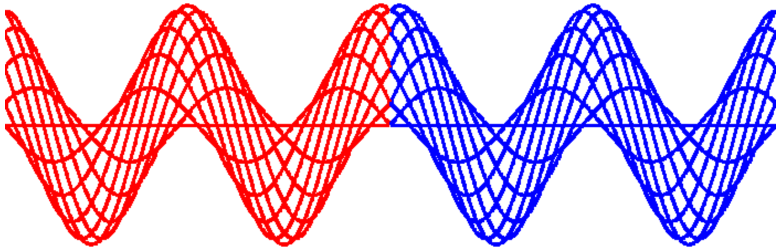
Q = n + 0.3



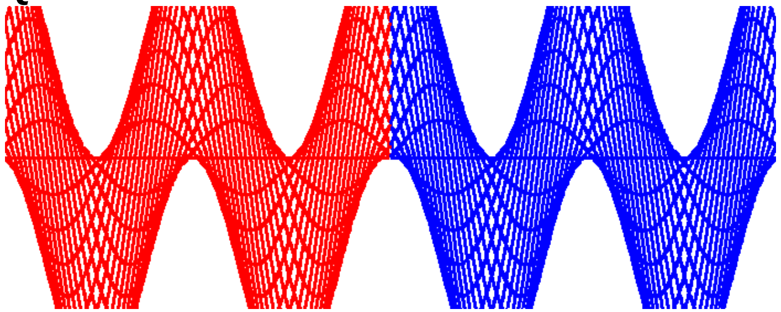
Q = n + 0.2



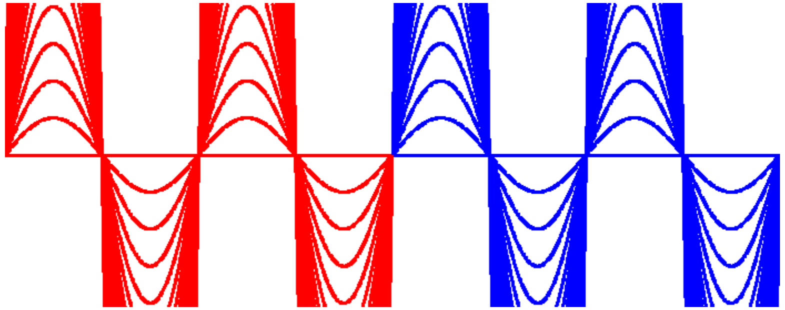
Q = n + 0.1



Q = n + 0.05



Q = n



- The particles **oscillate around a stable mean value** (Q $\neq$ n)!
- The amplitude diverges as we approach Q = n → integer resonance

- The stable mean value around which the particles oscillate is called the **closed orbit**.
  - Every particle in the beam oscillates around the closed orbit.
  - As we have seen the closed orbit 'does not exist' when the tune is an integer value.
- The general expression of the **closed orbit x(s)** in the presence of a **deflection θ** is:

amplitude modulated by the envelope β

oscillating term

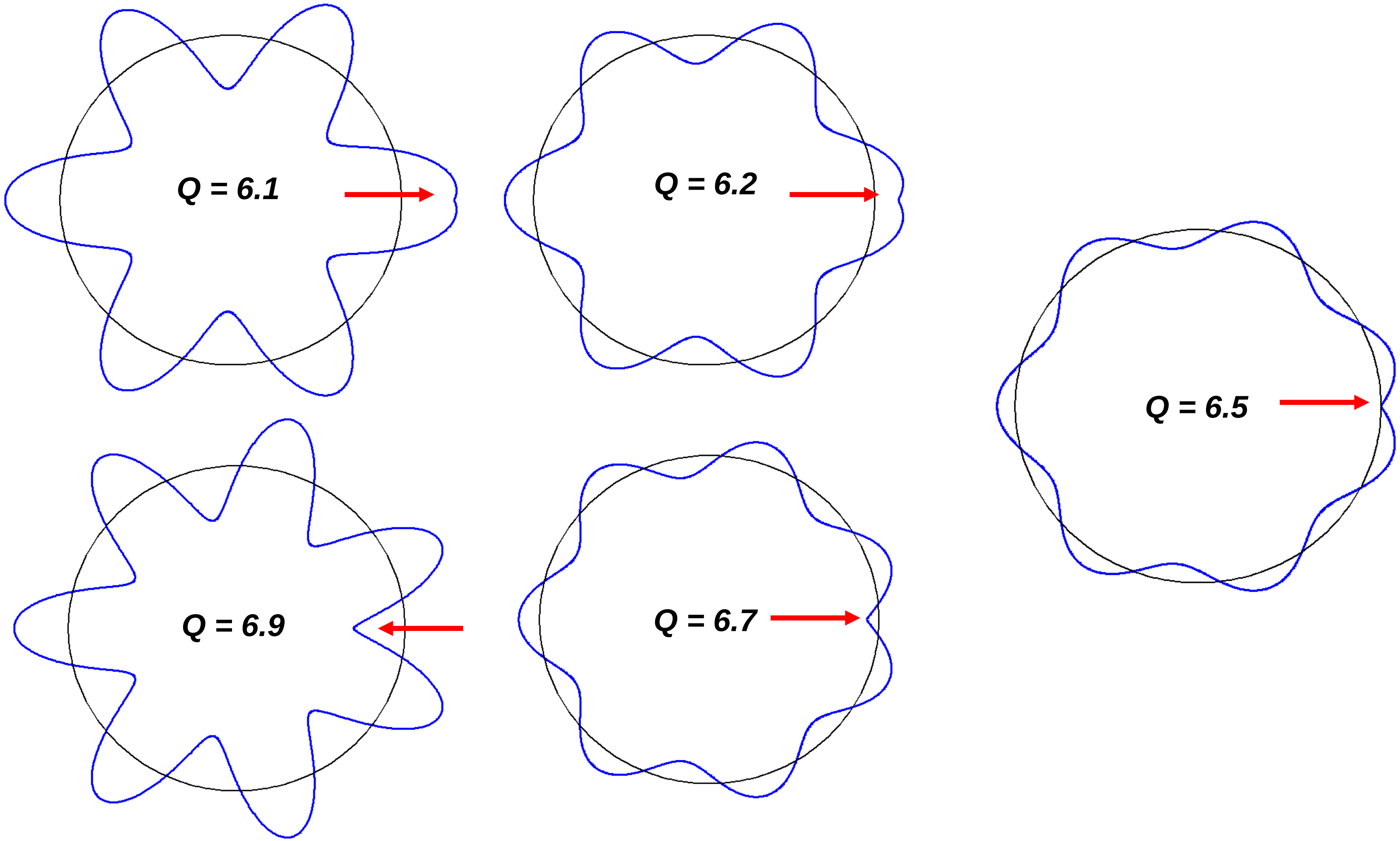
kink at the location of the deflection

$$x(s) = \frac{\sqrt{\beta(s)\beta_\theta}\cos(|\mu(s)-\mu_\theta| - \pi Q)}{2\sin(\pi Q)}\theta = R(s)\theta$$

divergence for Q = n

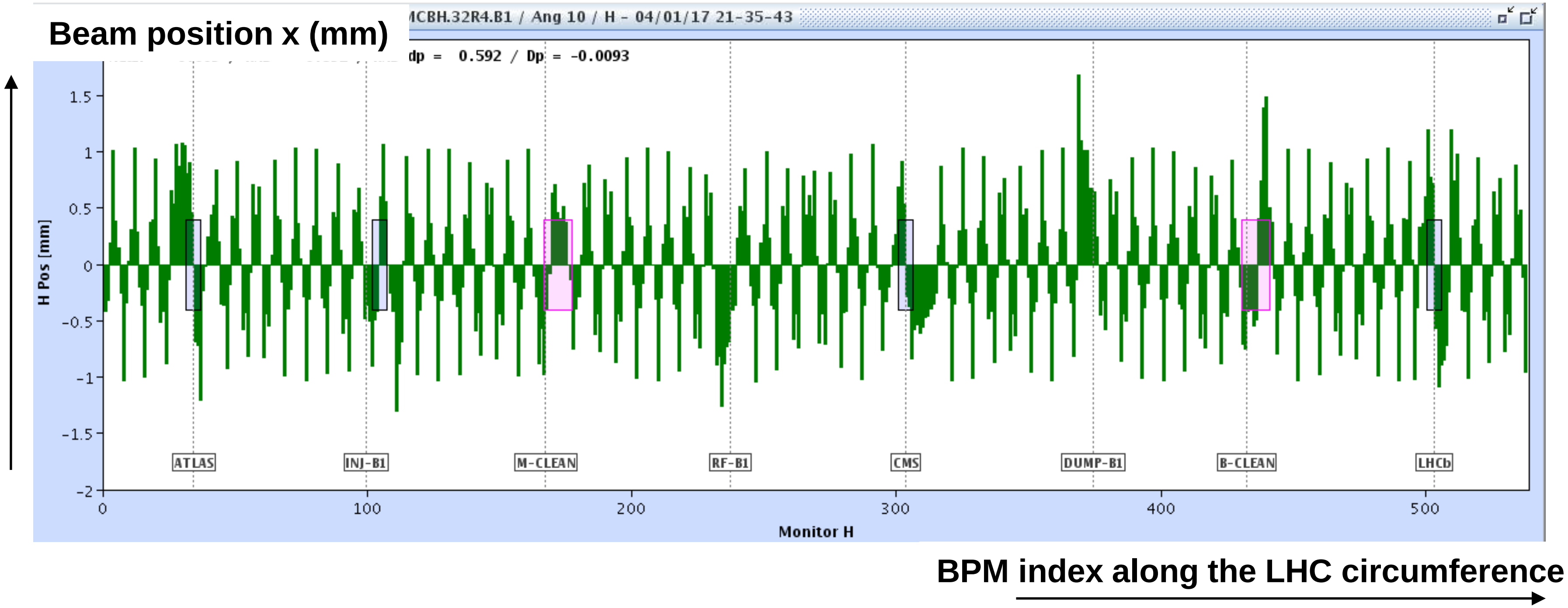
Orbit response

- Example of the horizontal closed orbit for a machine with tune Q = 6 + q.
- The **kink at the location of the deflection** (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.



A deflection at the LHC

- In the example below for the 26.7km long LHC, there is **one undesired deflection**, leading to a perturbed closed orbit.

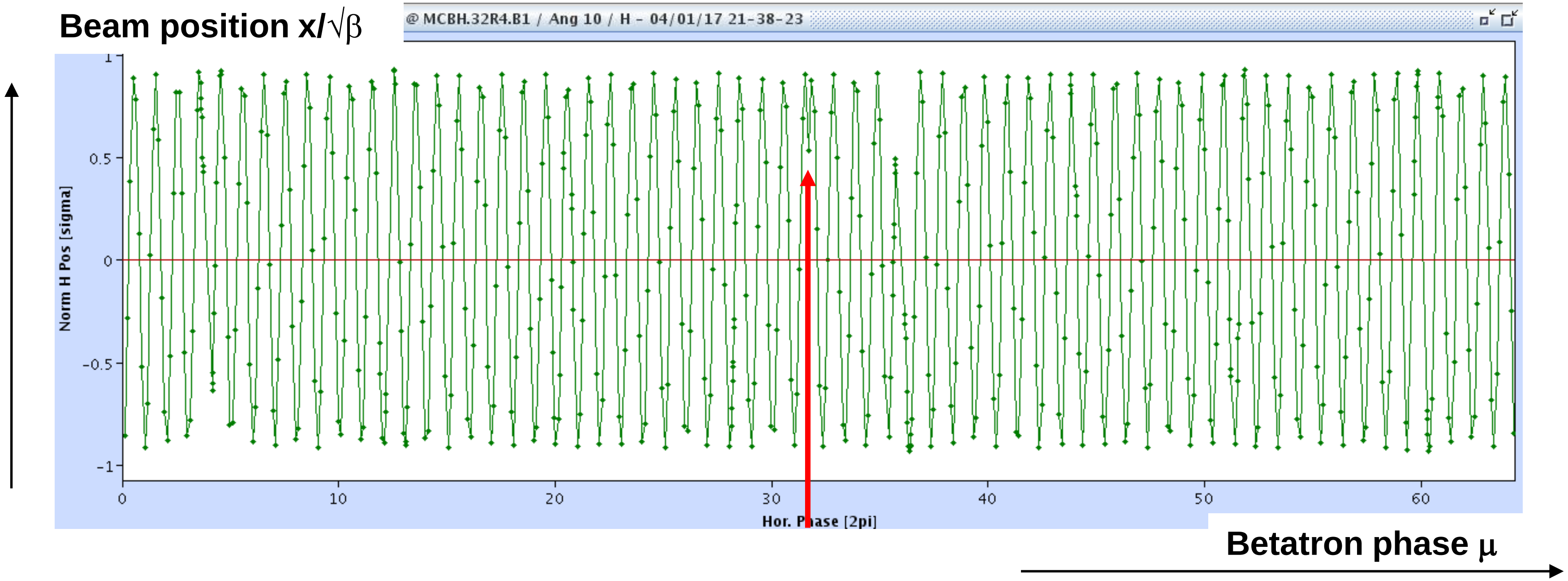


Where is the location of the deflection?

A deflection at the LHC

- To make our life easier we divide the position by $\sqrt{\beta(s)}$ and replace the BPM index by its phase $\mu(s)$.

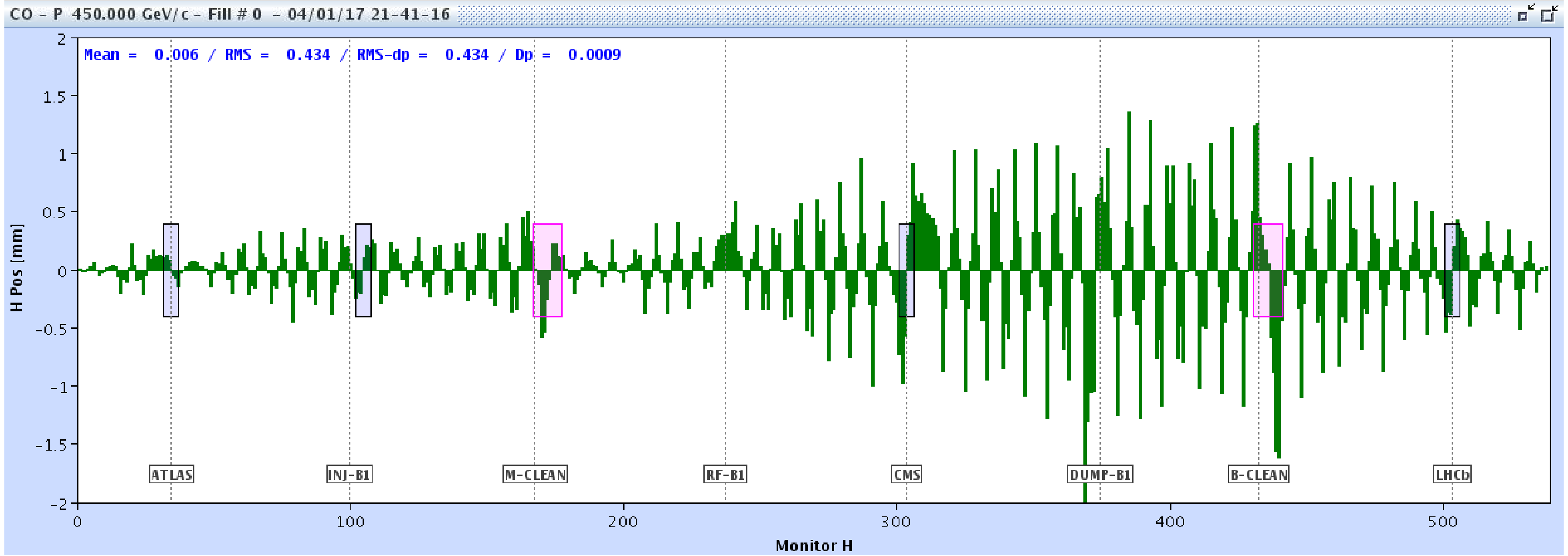
$$\frac{x(s)}{\sqrt{\beta(s)}} = \frac{\sqrt{\beta_\theta}\cos(|\mu(s)-\mu_\theta|-\pi Q)}{2\sin(\pi Q)}\theta \propto \cos(|\mu(s)-\mu_\theta|-\pi Q)$$



Can you localize the deflection?

A more realistic case at LHC

- Now a more realistic orbit with 100's of deflections.



How do we proceed to correct?

Orbit correction

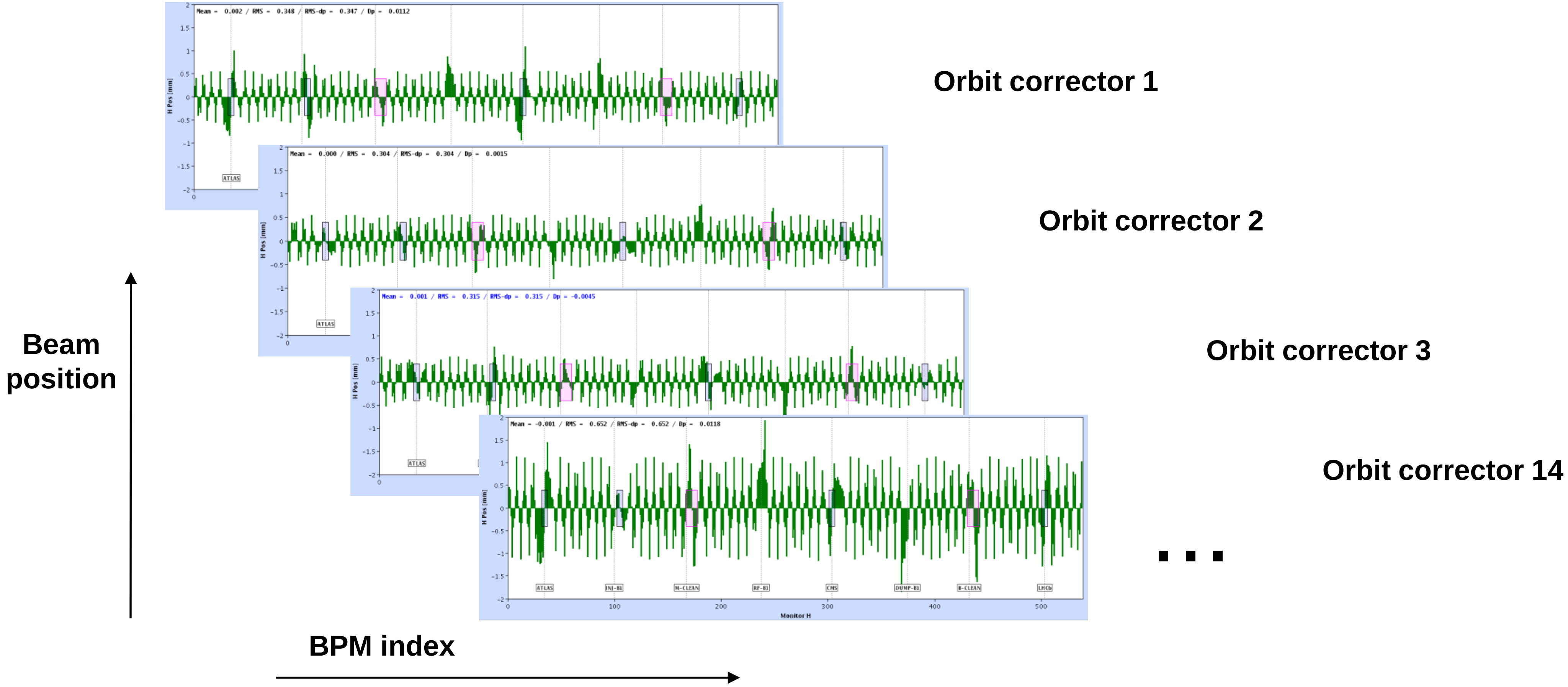
- Nowadays there are two main approaches for orbit correction:
  - **Matrix inversion algorithms** relying on the **knowledge the response R(s).**
    - R(s) is measured or calculated.
    - Popular algorithms: **MICADO** and **SVD** (Singular Value Decomposition), both come with many variants.
  - **Machine learning** with a neural network that is trained to find a solution.
    - The training may be based on data or on a model, or on continuous reward-like training.

- **Inversion algorithms** have the **advantage of higher intrinsic flexibility** (correction quality and flexibility, noise reduction,…), they can be reused at different machines without need for tuning – they are 'universal'.

- **Machine learning** based technique are adapted to situation where the models are difficult to establish, change over time, for example in low energy machines and some linacs. A model trained on a certain machine cannot be reused elsewhere.

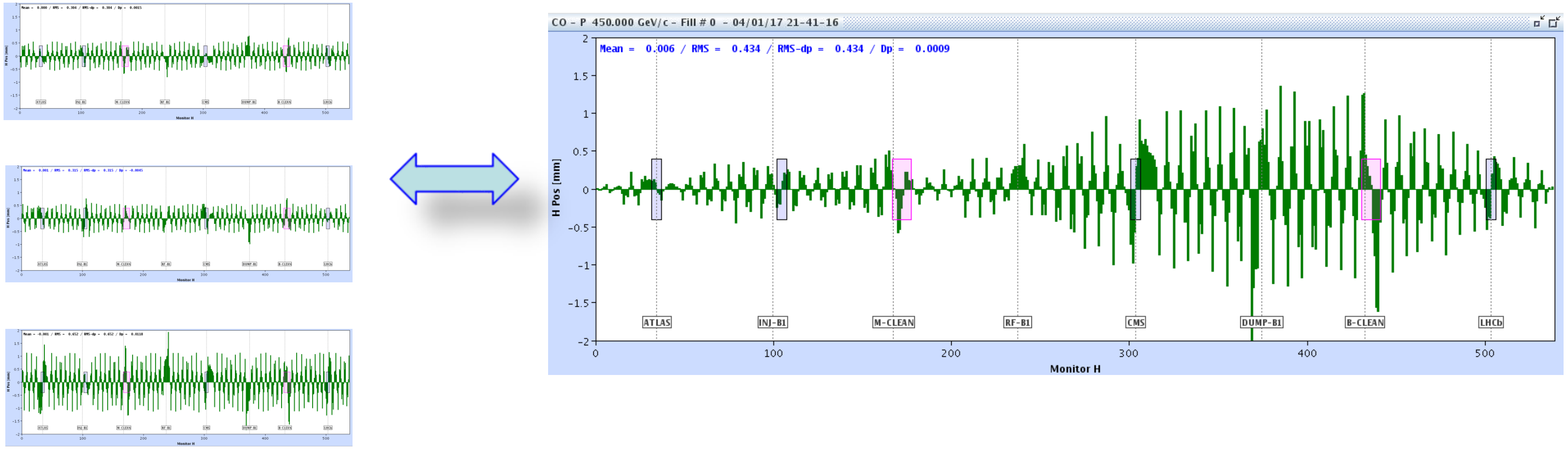
Example of model inversion

- Preparation: a model of the machine has to be obtained, i.e. for each orbit corrector the expected orbit response R(s) has to be measured or computed.



Example of model inversion - MICADO

- The MICADO algorithm compares the response of every corrector with the raw orbit.

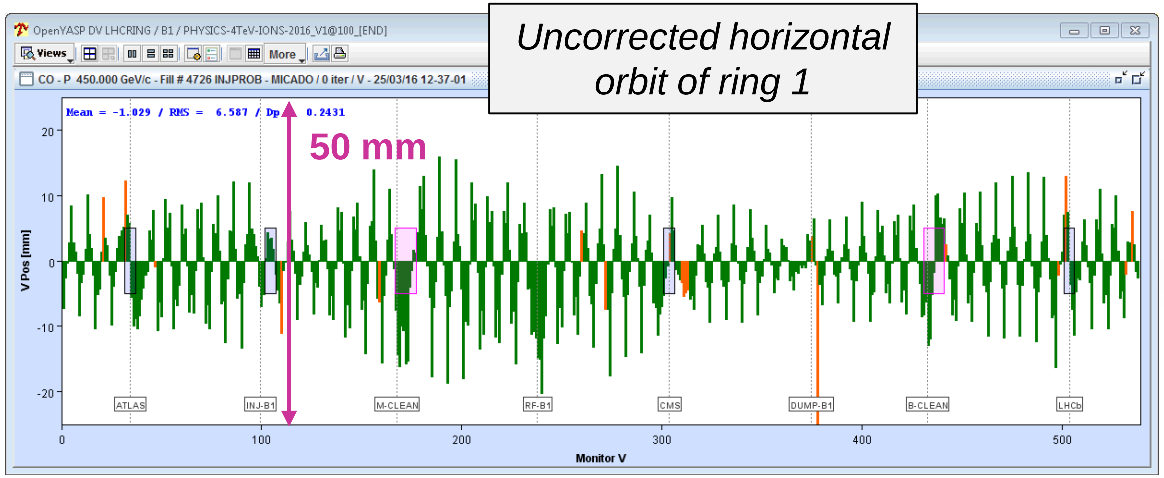


…

- MICADO picks out the corrector that hast the **best match** with the orbit, and that will give the largest improvement to the orbit deviation rms.
- The procedure can be **iterated** until the orbit is good enough (or as good as it can be).

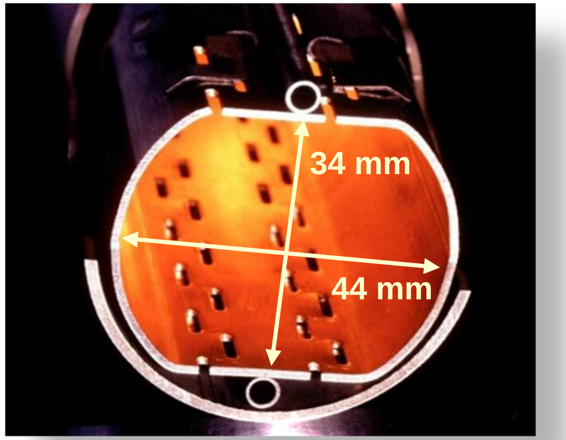
LHC orbit correction example

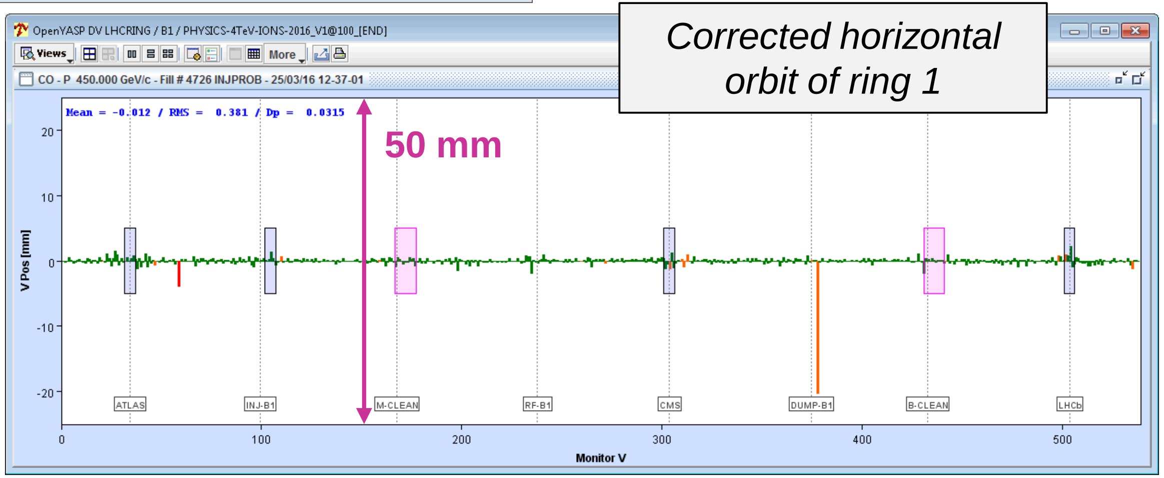
- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by more than a factor 20.



MICADO & SVD

LHC vacuum chamber





At the LHC a good orbit correction is vital !

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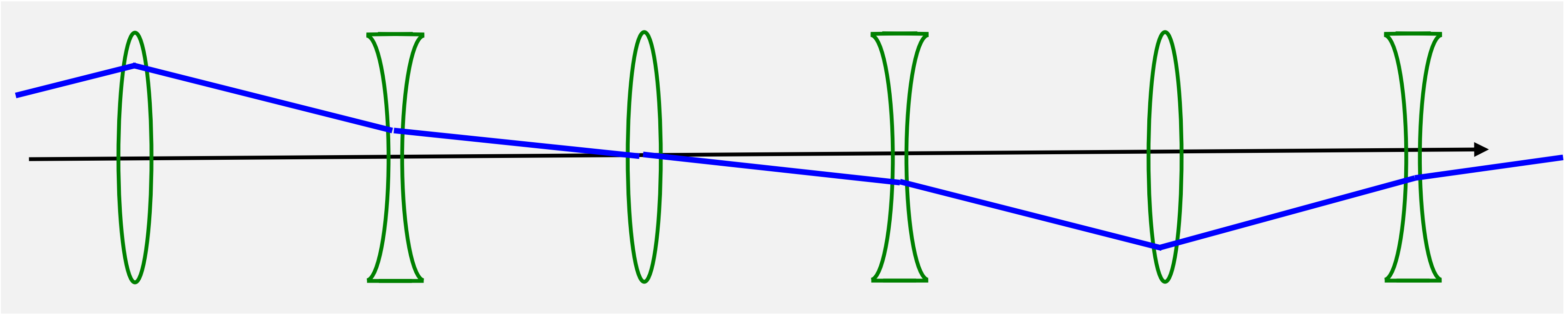
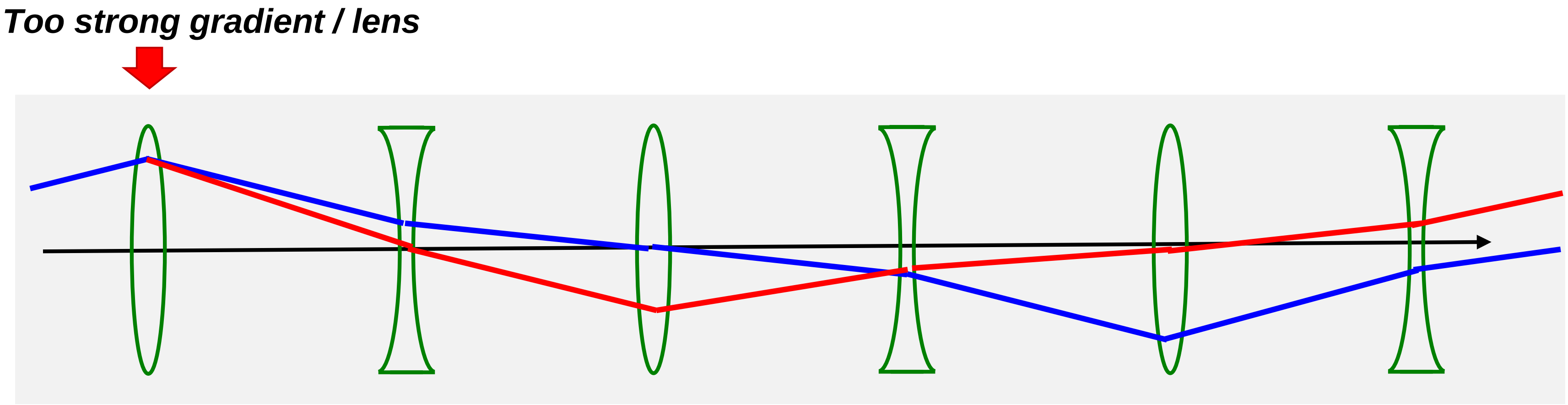
Summary

- What is the impact of a quadrupole gradient error?
  - Let us consider a particle oscillating in the lattice.

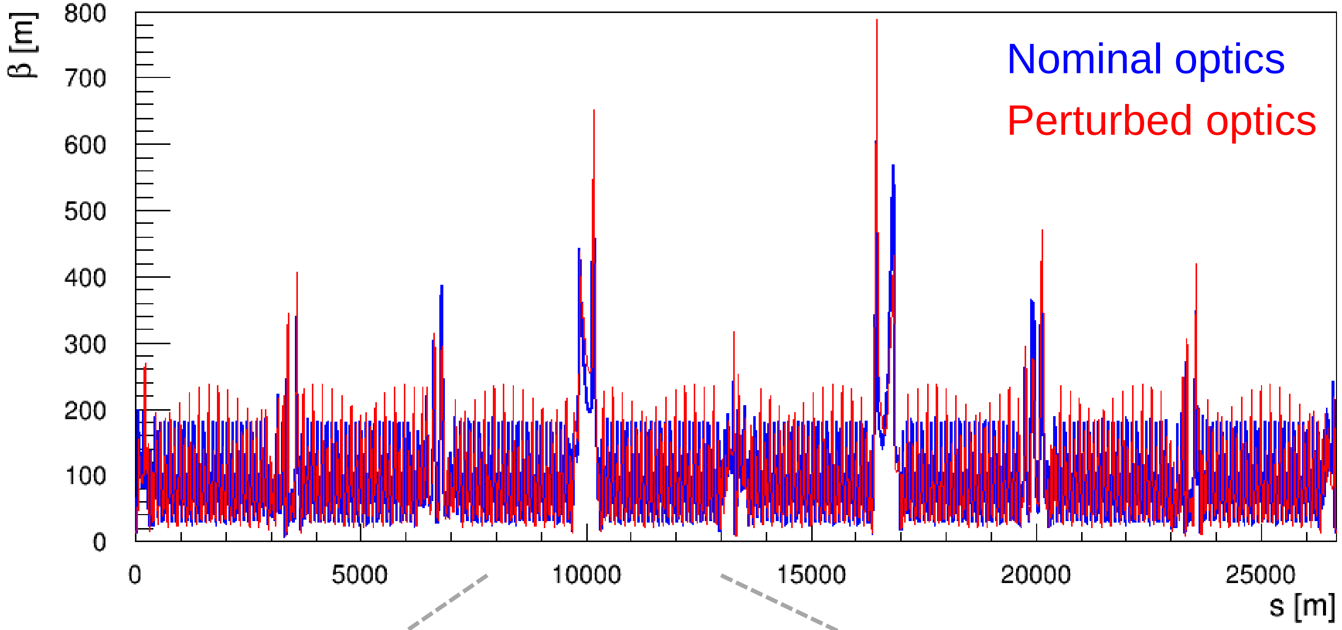


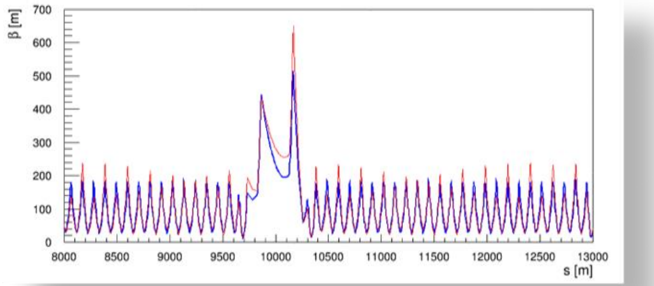


*The oscillation period is affected → **change of tune**, here Q increases !*

- In a ring a focussing error affects the beam optics and envelope (size) over the entire ring ! It also changes the tune.

Example for LHC: one quadrupole gradient is incorrect

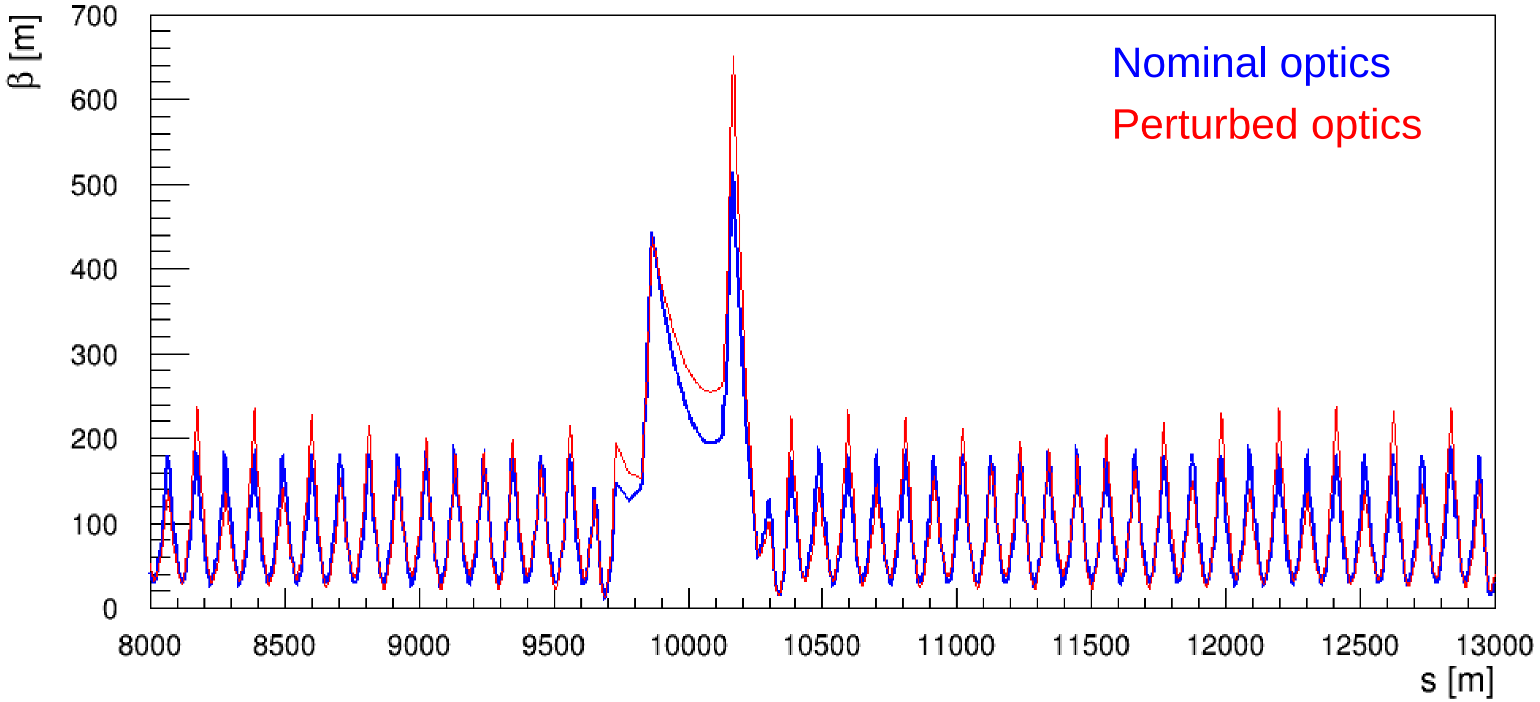




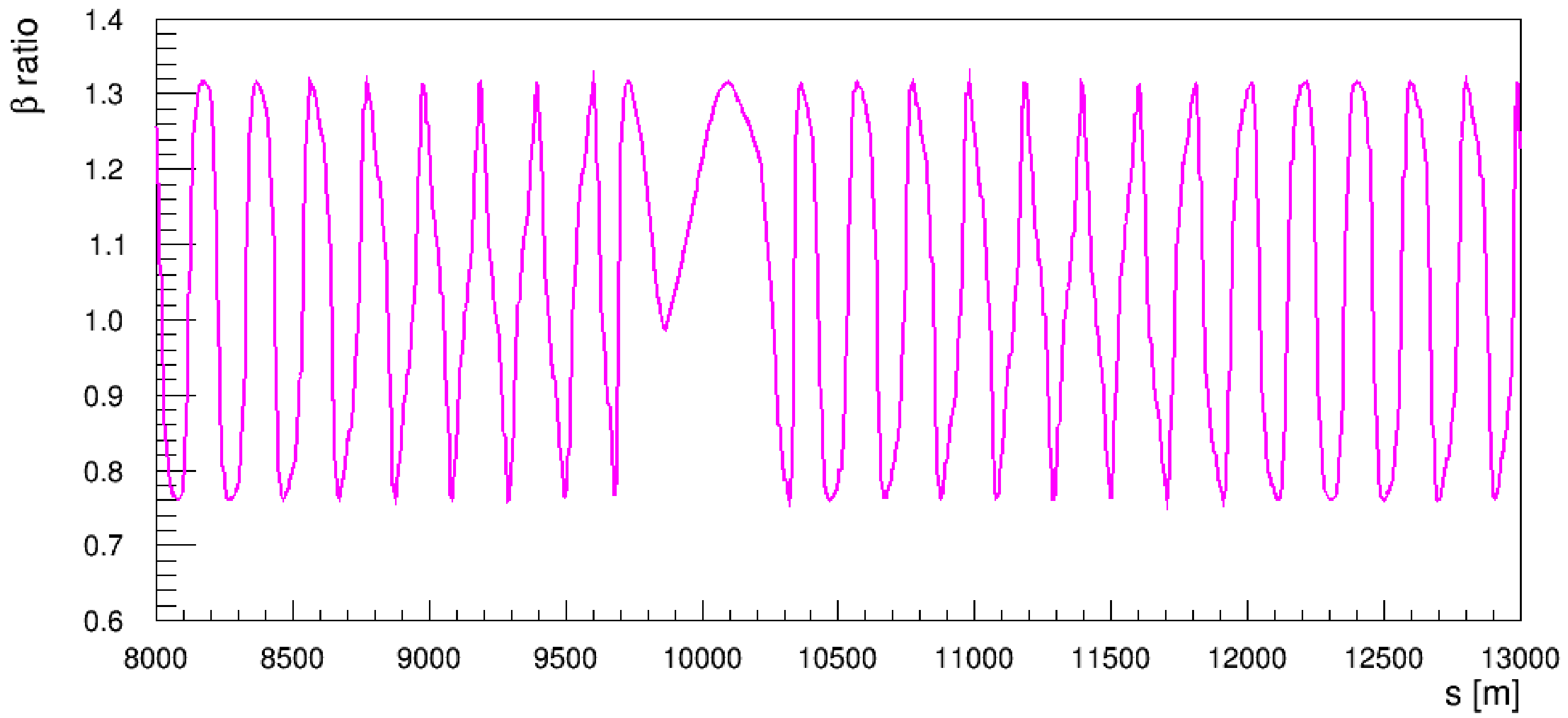
Zoom into a subsection

Optics perturbation

- The local beam optics perturbation… note the oscillating pattern of the error.

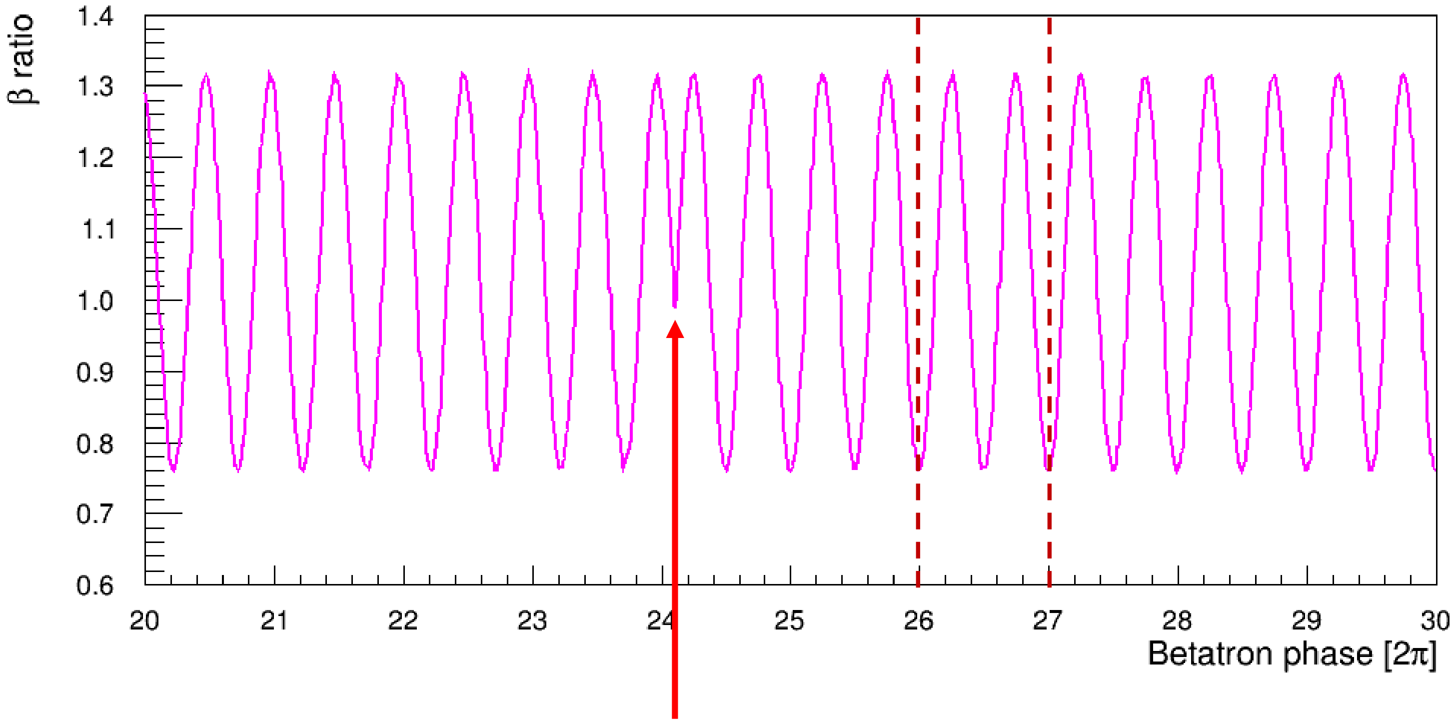


- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** (‘beta-beating’). The amplitude of the perturbation is the same all over the ring !



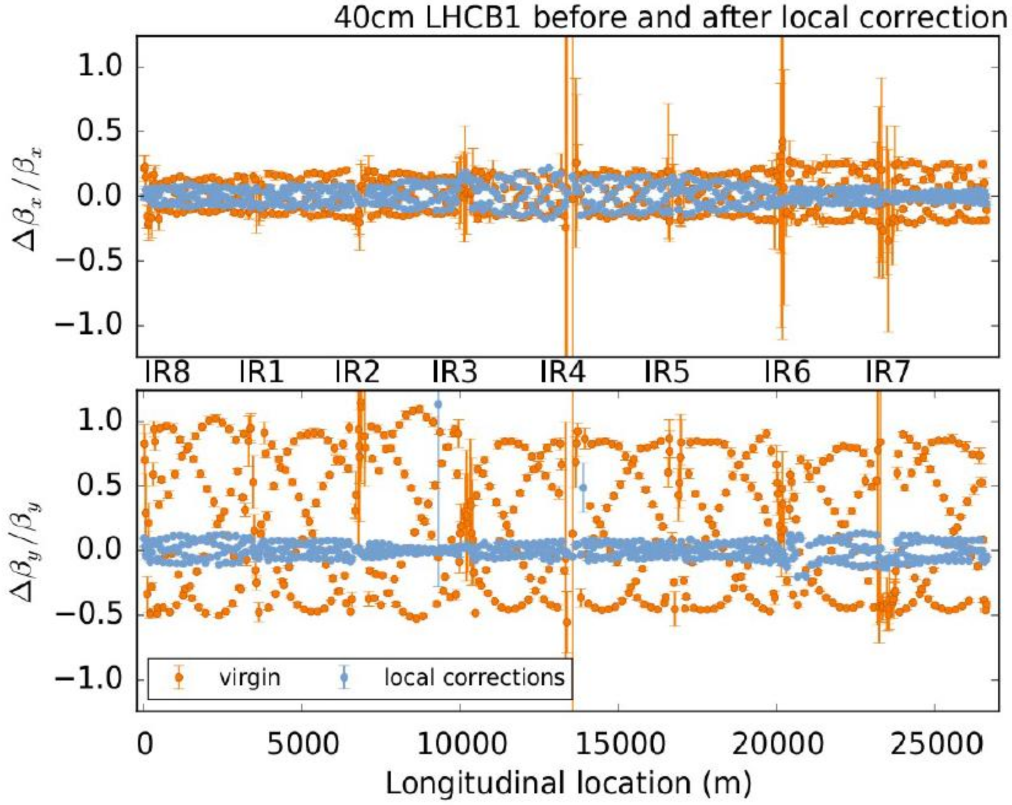
Optics perturbation

- The beta-beating pattern comes out more clearly when the longitudinal coordinate is again replaced by the betatron phase advance.
- The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink !
  - Watching closely one can observe that there are **two oscillation periods per 2π (360 deg) phase**. The beta-beating frequency is **twice** the frequency of the orbit !



LHC optics correction

- Correction strategies for optics / beta-beating rely on similar principles than for orbit correction.
  - **Inversion algorithms** – it is possible to iteratively use the same algorithms than for orbit correction,
  - **Machine learning**.
- Example for optics correction: at top energy of 6.5 TeV, the LHC optics errors cam be as large as 100% before correction.
  - Can be corrected to a few % residual error with modern correction algorithms if there are enough quadrupoles that can be individual powered.



Introduction

Imperfection - sources

Orbit perturbations

Optics perturbations

Linear imperfections and geology

Summary

Linear imperfections, geology and celestial bodies

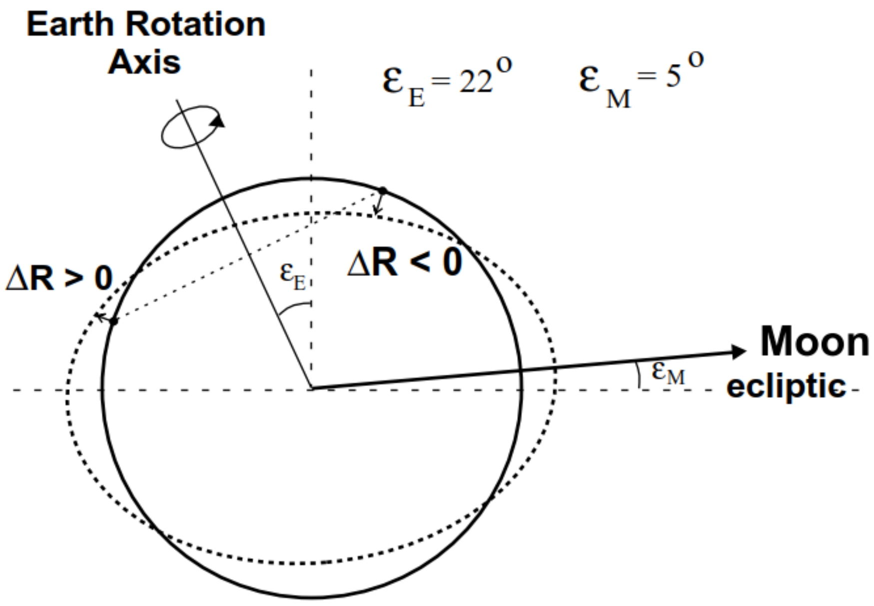
Tide bulge of a celestial body of mass M at a distance d

$$\Delta R \sim \frac{M}{2d^3}(3cos^2\theta - 1)$$

θ = angle(vertical, the celestial body)

induces surface deformations and affects the water levels of the oceans.

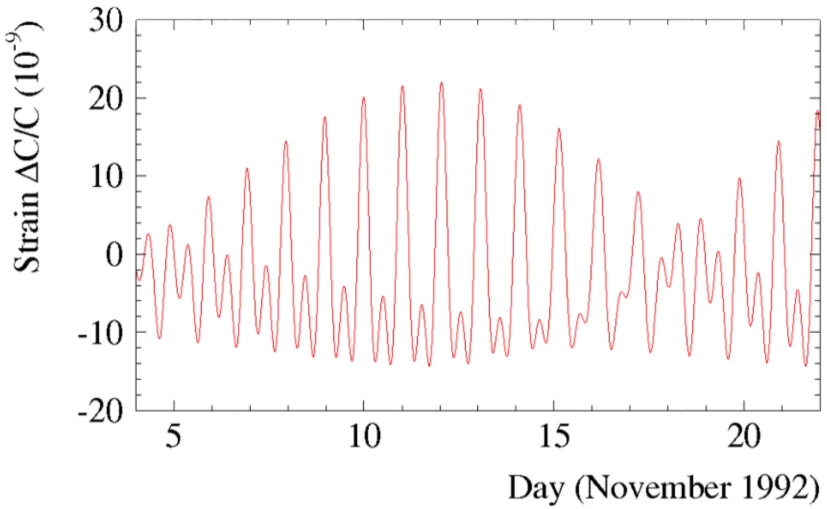
→ impacts the alignment of a large accelerator !



Such **Earth tides** alter the accelerator circumference:

- The Moon contributes 2/3, the Sun 1/3.
- Not resonance-driven (unlike Sea tides !).
- Accurate predictions possible (~%).

LEP tide predictions for Nov. 1992



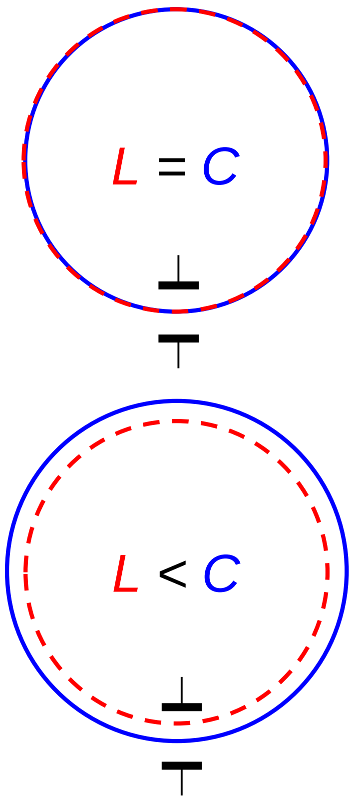
The relative circumference change amounts to ***$\sim 10^{-8}$ ~ 1 mm – resolution ~ 10^{-11}***

Gravitational wave detectors *achieve sensitivities of* ***$\sim 10^{-21}$*** *!*

Tides observation

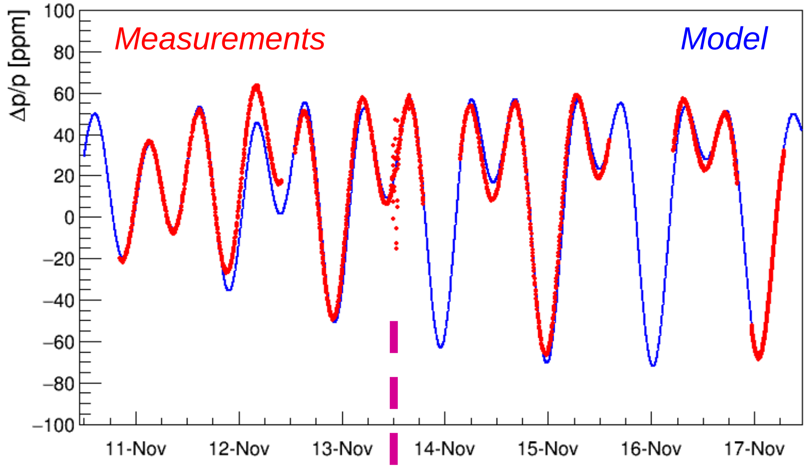
- At the LHC the beams are 'captured' by the RF system which forces the beams to remain synchronous with the RF frequency.
  - Because at LHC the speed ≅ c = constant, this fixes the length of the orbit.

- When the frequency is well adjusted, the length of the orbit ***L*** matches the circumference ***C***.
- If for any reason ***C*** varies, the beam has to move radially if ***L*** is kept constant.
- A mismatch between ***C*** and ***L*** can be observed on the mean radial orbit using the BPMs that move with the ring.
  - As a side effect it also changes very slightly the beam energy (level of 0.01%).



LHC circumference observations

- Tides are observed very clearly on the LHC circumference by measuring the mean radial (=horizontal) beam position.

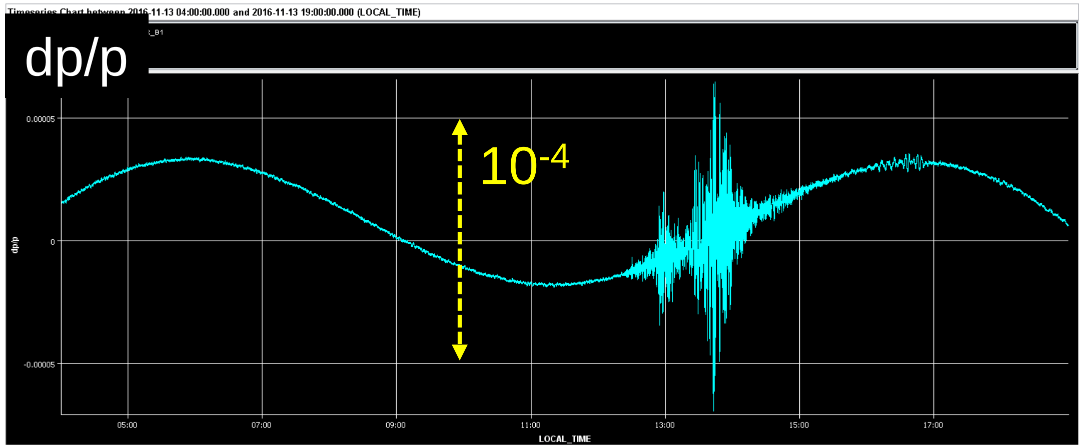


Tide observations (from orbit changes) over one week at 4 TeV in 2016

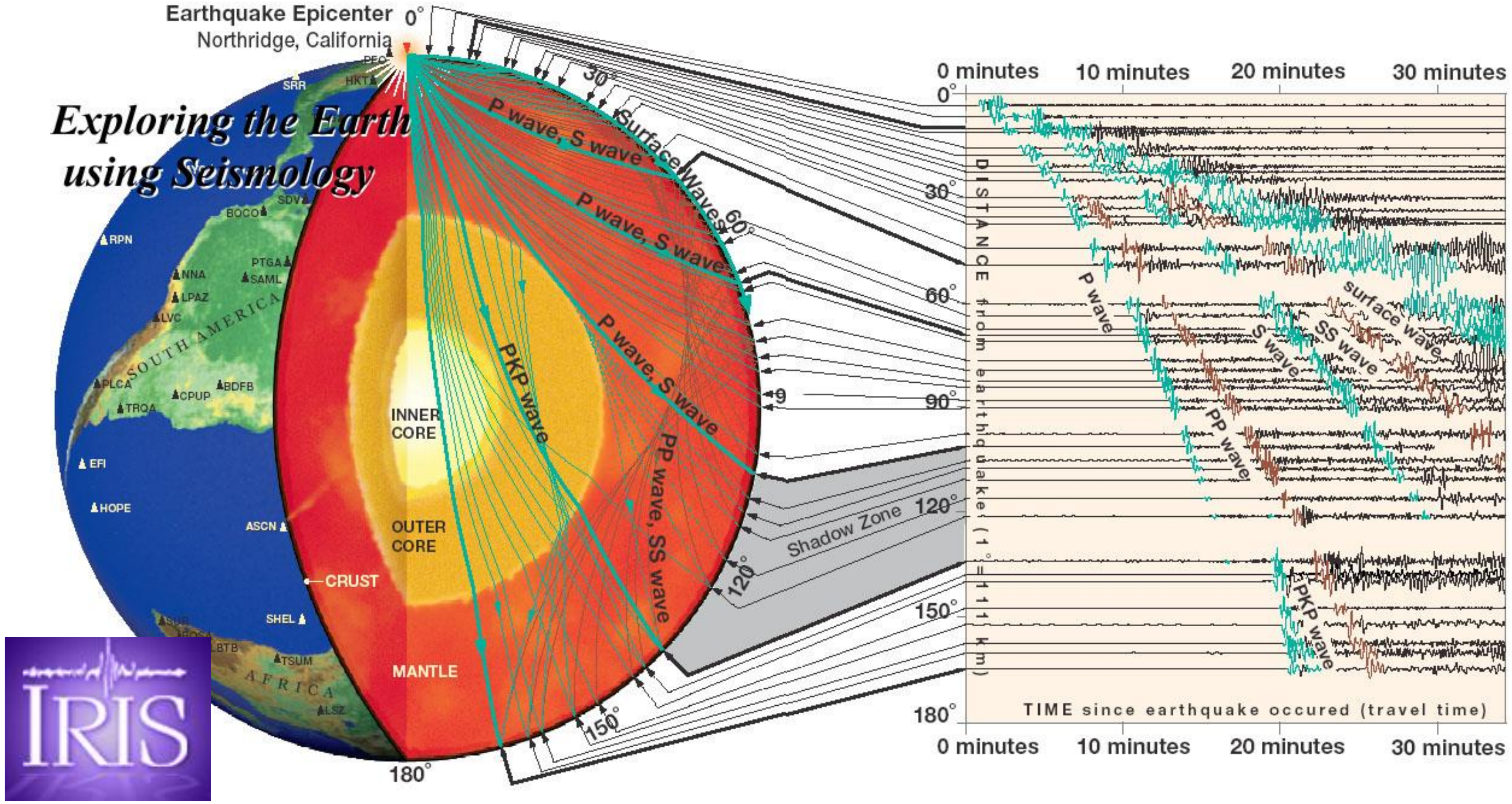
(expressed in energy change $\Delta p/p$)

Earthquake in New Zealand

The pressure waves induce a modulation of the circumference



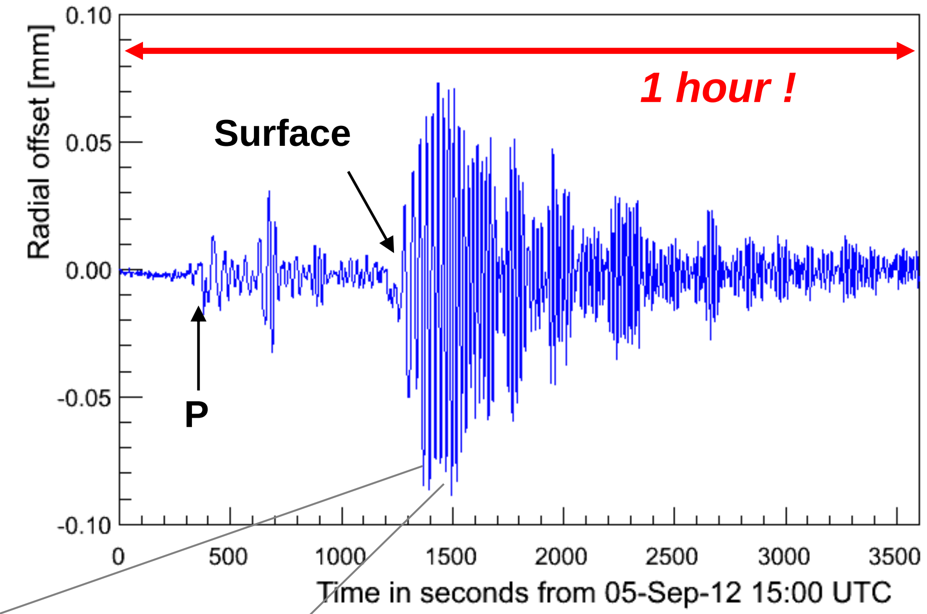
Different types of body (**P**ressure, **S**hear) and surface waves (**R**aleigh, **L**ove), multiple paths and reflections produce a complex signature of earthquakes at seismic measurement stations – also at the LHC.

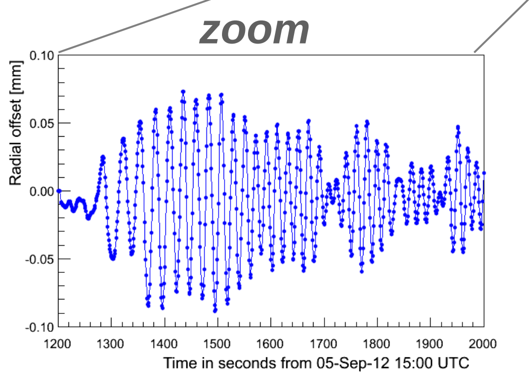


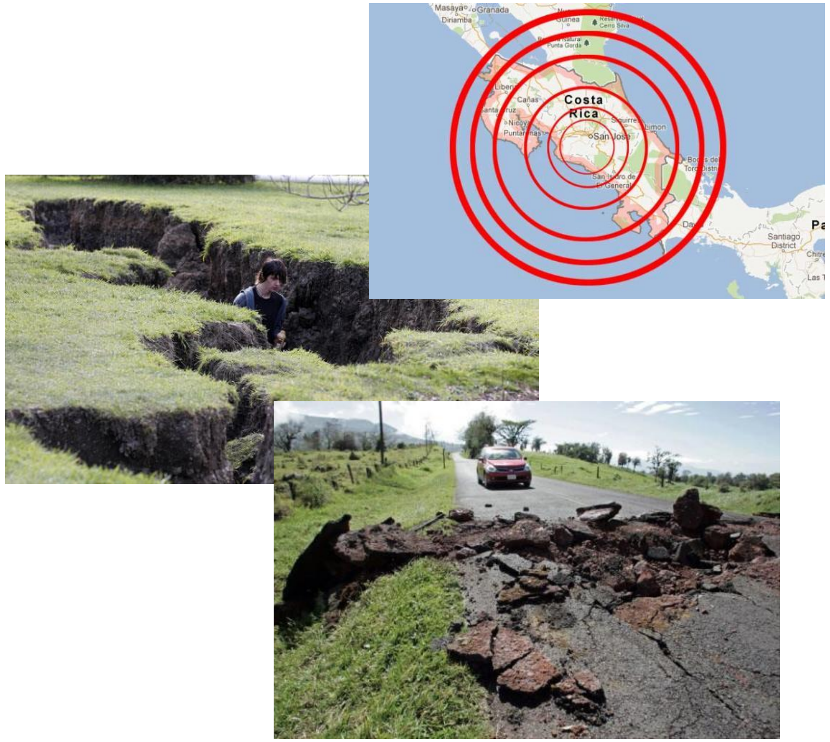
L. Braille (Purdue U.) / The IRIS (Incorporated Research Institutions for Seismology) consortium

Costa Rica earthquake - 2012

- A **magnitude 7.6** earthquake in Costa Rica (05/09/2012 @ 14:42:10 UTC) ‘struck’ the LHC in **fill 3032** with stable colliding beams.
  - Arrival of the first waves at CERN ~15:06 UTC.



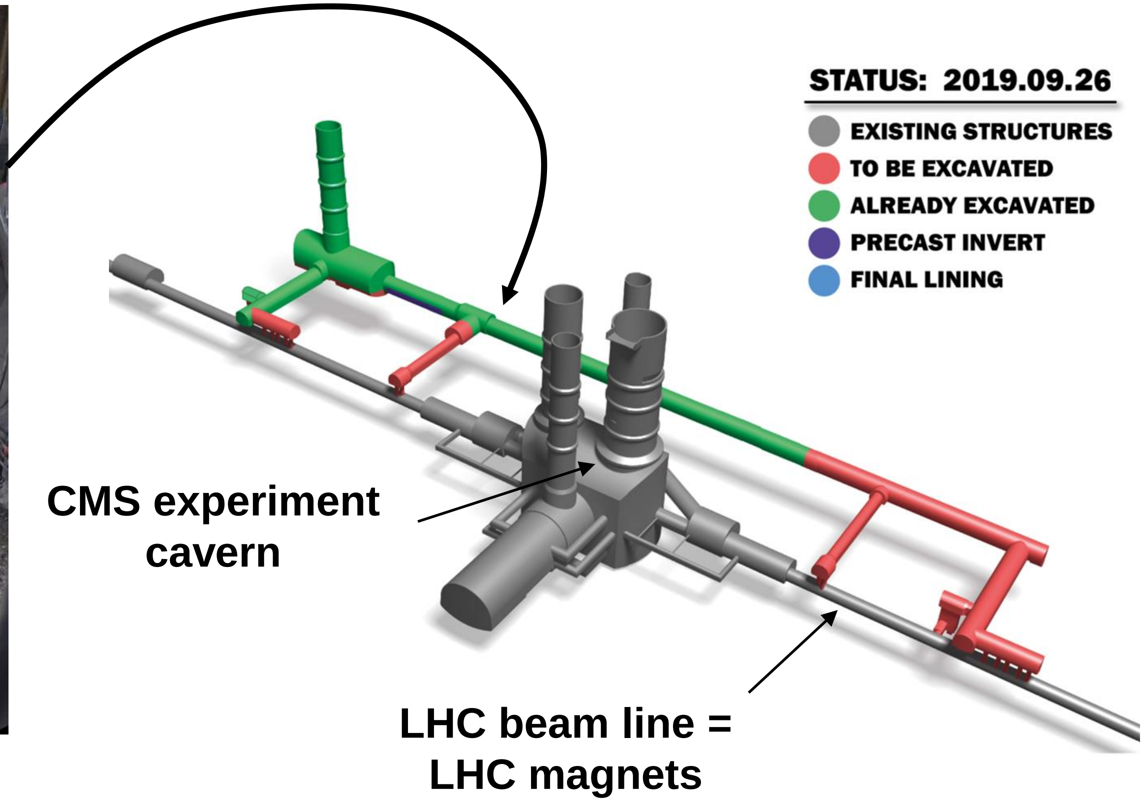




- The arrival of the different waves can be observed on the radial beam position – equivalent to largest tides.

… and there's the man-made waves

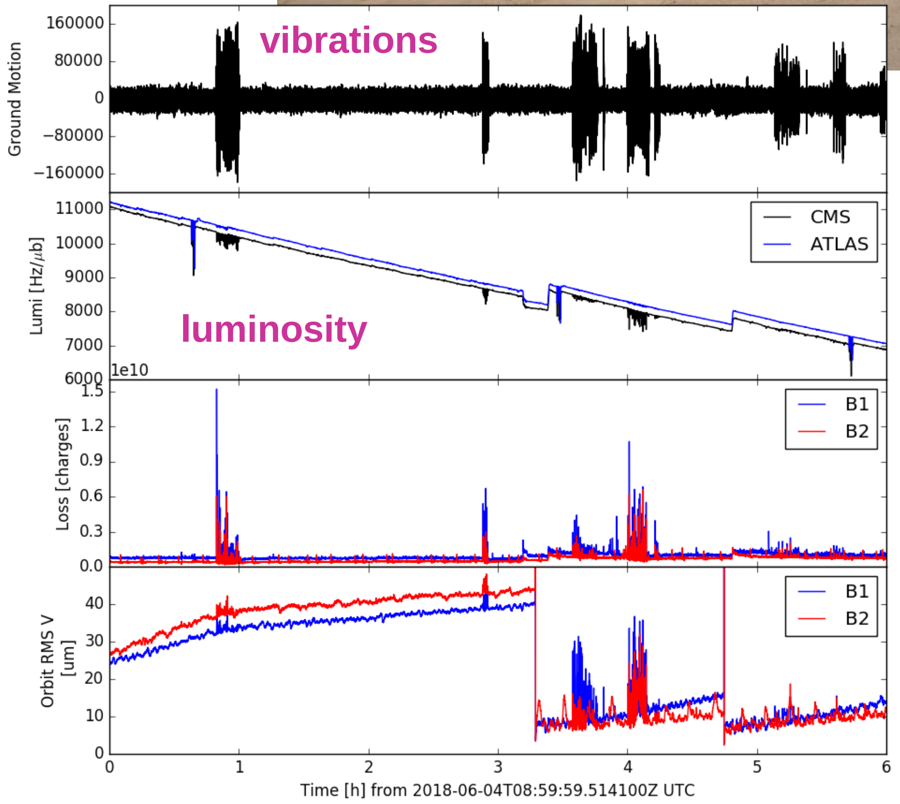
- HL-LHC has built huge underground structures in LHC points 1 and 5.
- Civil engineering is not famous for working 'quietly' !
- **Noise** also means **vibrations**, vibrations mean **moving magnets** !



- In the early part of the CE work, an important volume of soil was moved around and compacted while LHC was operating.
- Ground compactors compact soil by… **vibrating**.
- …and they managed to **shake the beams colliding at the IP ~100 m underground**.

Mechanism:

- The vibrations with **frequencies ~20 Hz** were transmitted through 100 m of rock to the tunnel magnets and their supports that resonate in the frequency range 8-22 Hz.
- The resonant excitation generated ~ **micrometer amplitude beam movements** that were clearly visible on the CMS experiments luminosity (= rate of collisions).



Summary

- At first order magnetic field and misalignments errors of accelerator components induce:
  - Errors on the beam orbit,
  - Errors on the optics and the tune.
- The errors are often sufficiently large that modern machines operate poorly or not at all.
- Fortunately ever improving tools and algorithms have been developed over the past 50 years to correct such errors.
- However to minimize the imperfections from the start it is important to have:
  - the best possible magnet (component) design,
  - well measured magnetic fields,
  - precise power converters,
  - the best possible machine alignment.

Thank you for your *perfect* attention !