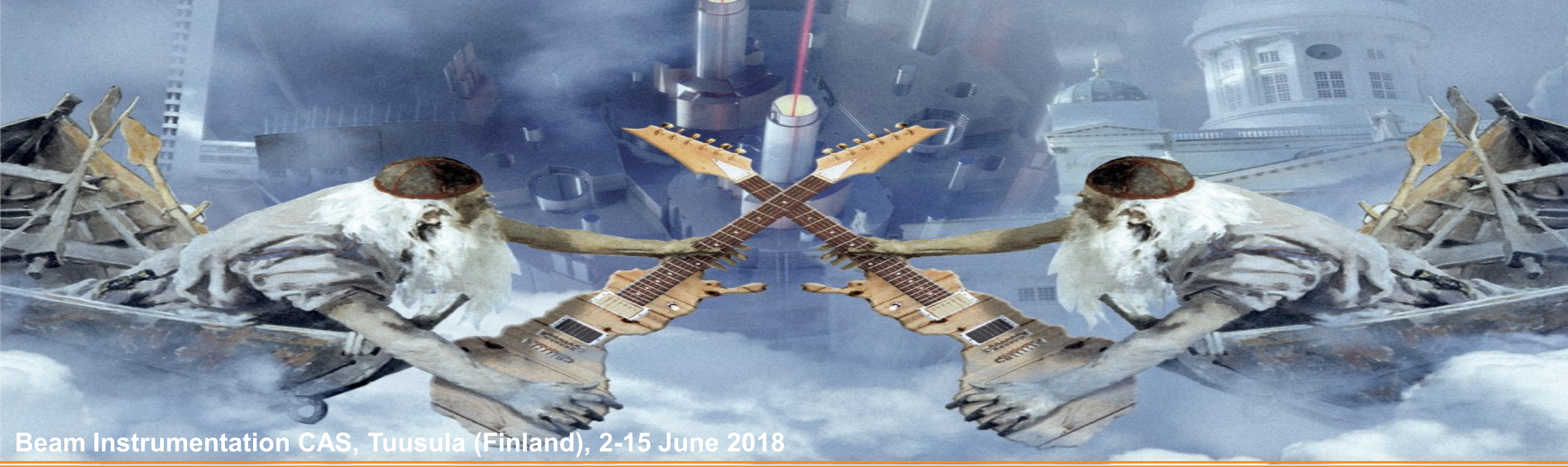


Transverse Phase Space Emittance Diagnostics

B. Walasek-Höhne, Kay Wittenburg,
Gero Kube

GSI (Darmstadt) & DESY (Hamburg)

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Accelerator Key Parameters

- light source: spectral brilliance
  - measure for phase space density of photon flux

$$B = \frac{\text{Number of photons}}{[\text{sec}][\text{mm}^2][\text{mrad}^2][0.1\%\ \text{bandwidth}]}$$

  - user requirement: high brightness
    $\rightarrow$ lot of monochromatic photons on sample
  - connection to machine parameters

$$B \propto \frac{N_\gamma}{\sigma_x \sigma_{x'} \sigma_z \sigma_{z'}} \propto \frac{I}{\varepsilon_x \varepsilon_z}$$

- collider: luminosity
  - measure for the collider performance

$$\dot{N} = \mathfrak{L} \cdot \sigma$$

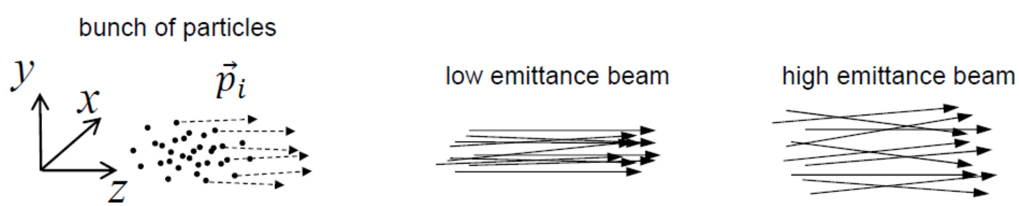
relativistic invariant proportionality factor between cross section σ (property of interaction) and number of interactions per second

  - user requirement: high luminosity
    $\rightarrow$ lot of interactions in reaction channel
  - connection to machine parameters

$$\mathfrak{L} = \frac{I_1 \cdot I_2}{\varepsilon}$$

for two identical beams with emittances $\varepsilon_x = \varepsilon_z = \varepsilon$

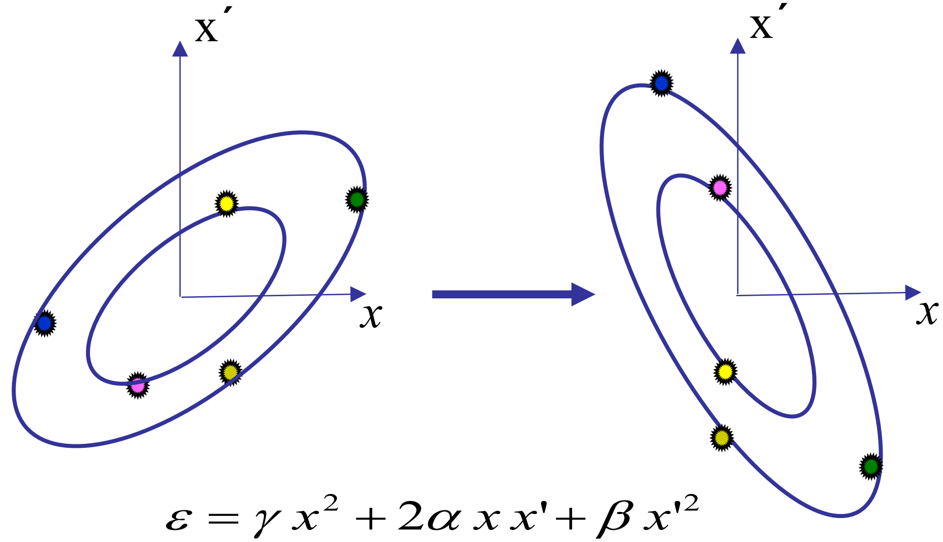
- requirements
  - design of small emittance machine
    $\rightarrow$ proper choice of magnet lattice
  - preserve small emittance
    $\rightarrow$ question of stability
    $\rightarrow$ require active feedback systems / careful design considerations



Transverse Emittance

- projection of phase space volume
  - separate horizontal, vertical and longitudinal plane
- accelerator key parameter
  - defines **luminosity** / **brilliance**
- linear forces
  - any particle moves on an ellipse in trace space (x,x')
  - ellipse rotates in magnets and shears along drifts

    → but area is preserved: **emittance**



$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

(α, β, γ: Courant-Snyder or Twiss parameters)

- transformation along accelerator
  - knowledge of the magnet structure (beam optics) → transformation from initial (i) to final (f) location

→ single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

→ transformation of Courant-Snyder/Twiss parameters

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & 1+m_{12}m_{21} & -m_{12}m_{22} \\ m_{21}^2 & -2m_{21}m_{22} & m_{22}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

Transverse Emittance Ellipse

- propagation along accelerator
  - change of ellipse shape and orientation → area is preserved

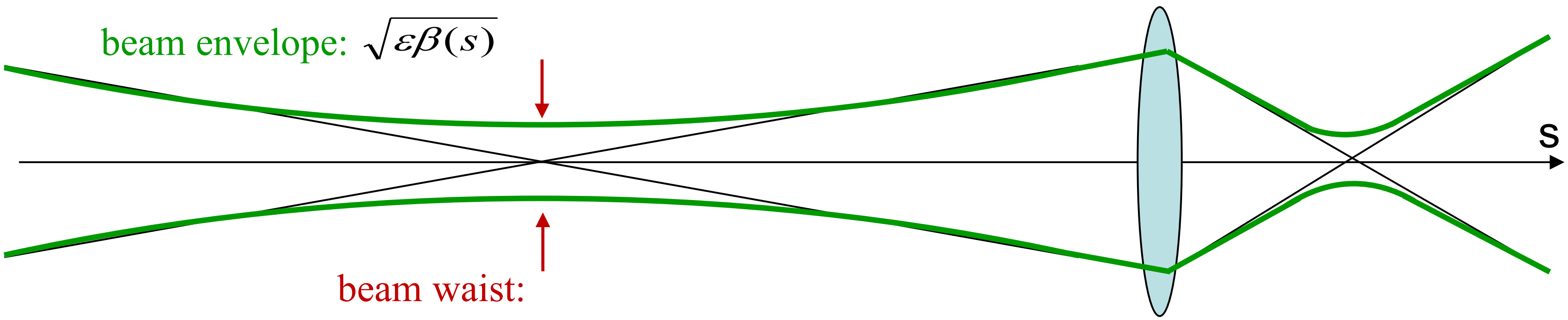
$$\alpha(s) = -\frac{\beta'(s)}{2}$$

$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

$$\boxed{\varepsilon = \gamma(s)\,x(s)^2 + 2\alpha(s)\,x(s)\,x'(s) + \beta(s)\,x'(s)^2}$$

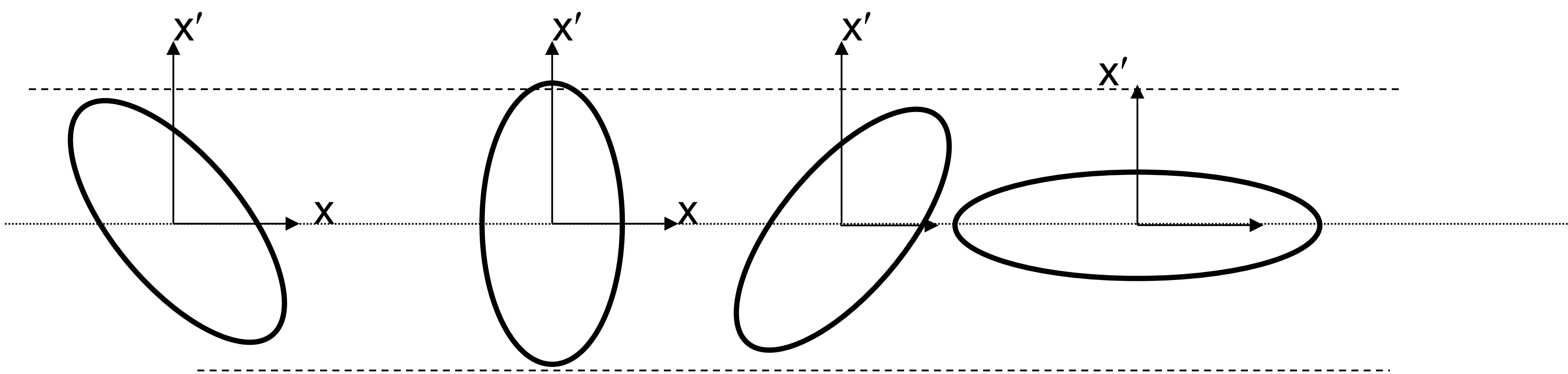
$$x(s) = \sqrt{\varepsilon\beta(s)}\cdot\cos[\Psi(s)+\Phi]$$

beam envelope: $\sqrt{\varepsilon\beta(s)}$

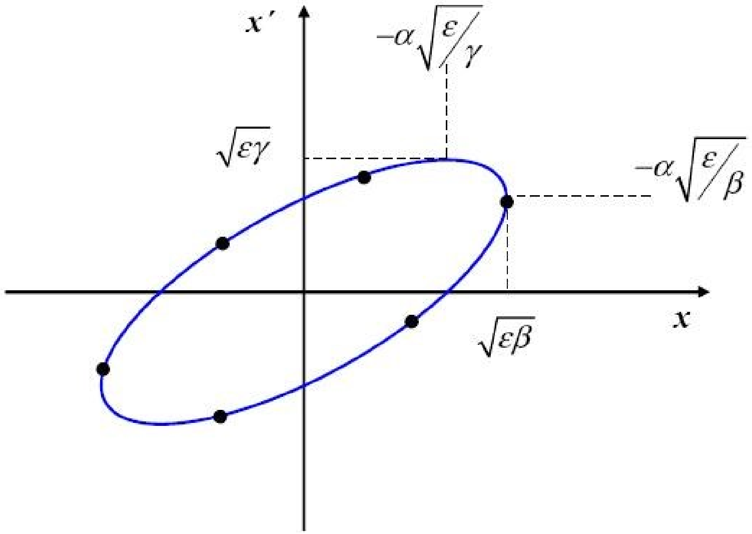


beam waist:

→ minimum in envelope → minimum in β → β′ = 0 → α = 0



Emittance and Beam Matrix



- via Twiss parameters

$$\boxed{\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2}$$

- statistical definition

P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No.3 (1971) 1101

$$\boxed{\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}}$$

2nd moment of beam distribution ρ(x)

$$\langle x^2\rangle = \frac{\int_{-\infty}^{\infty} \mathrm{d}x\, x^2 \cdot \rho(x)}{\int_{-\infty}^{\infty} \mathrm{d}x\, \rho(x)}$$

- ε_{rms} is measure of spread in phase space
- root-mean-square (rms) of distribution

$$\sigma_x = \langle x^2\rangle^{1/2}$$

- ε_{rms} useful definition for non-linear beams

  → usually restriction to certain range

  (c.f. 90% of particles instead of [-∞,+∞])

- beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2\rangle & \langle xx'\rangle \\ \langle xx'\rangle & \langle x'^2\rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

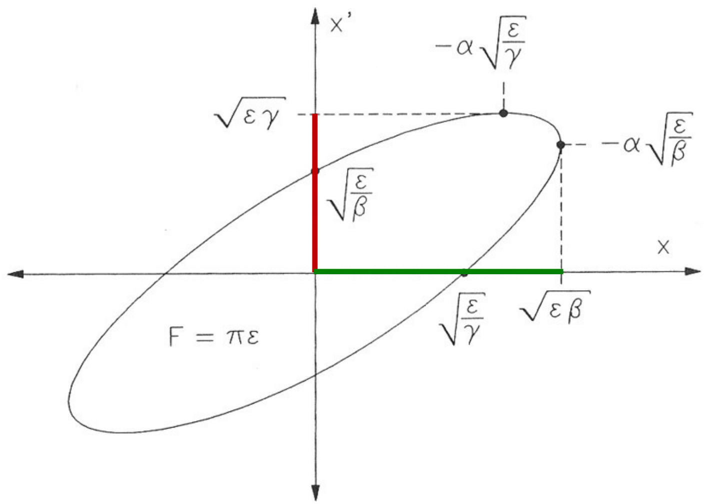
$$\boxed{\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11}\cdot\Sigma_{22} - \Sigma_{12}^2}}$$

- transformation of beam matrix

$$\Sigma^1 = \mathrm{M}\Sigma^0\mathrm{M}^T \qquad M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

Emittance Measurement: Principle

- emittance: projected area of transverse phase space volume
- not directly accessible for beam diagnostics



- measured quantity
  - beam size $\sqrt{\Sigma_{11}} = \sqrt{\langle x^2 \rangle} = \sqrt{\varepsilon\,\beta}$
  - beam divergence $\sqrt{\Sigma_{22}} = \sqrt{\langle x'^2 \rangle} = \sqrt{\varepsilon\,\gamma}$
  - divergence measurements seldom in use

    → restriction to profile measurements

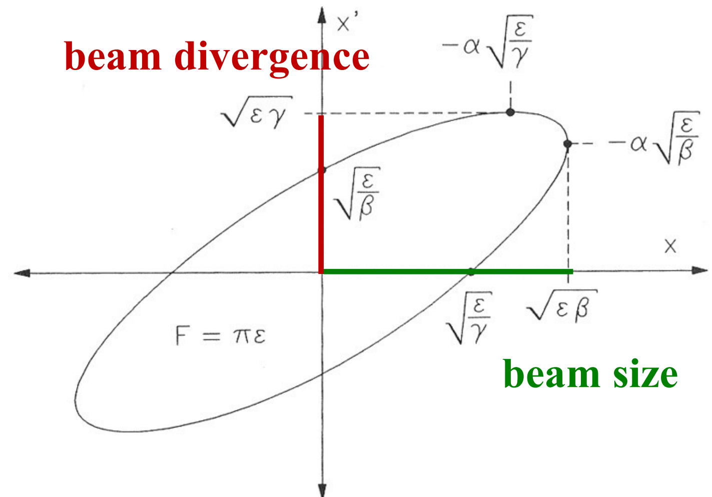
- measurement schemes
  - beam matrix based measurements

    → determination of beam matrix elements: $\varepsilon = \sqrt{\det\Sigma} = \sqrt{\Sigma_{11}\cdot\Sigma_{22} - \Sigma_{12}^2}$
  - mapping of phase space

    → restrict to (infenitesimal) element in space coordinate, convert angles x‘ in position

Circular Accelerators

- emittance diagnostics in circular accelerators
  - circular accelerator: periodic with circumference C_0
    - → one-turn transport matrix: $M(s+C_0) = M(s)$
    - → Twiss parameters $\alpha(s), \beta(s), \gamma(s)$ uniquely defined at each location in ring
  - measurement at one location in ring sufficient to determine ε
    - → measured quantity: beam profile / angular distribution



- classification

beam spot | wavefront manipulation | spatial resolving detector (CCD)

  - imaging
    - → beam size

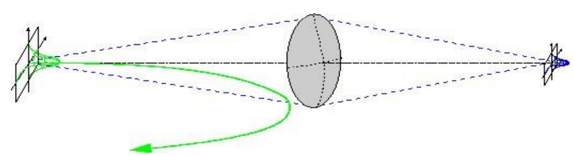
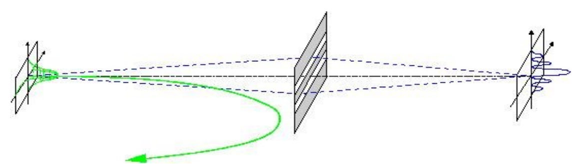
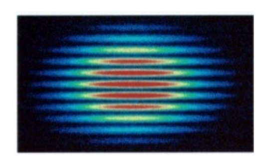


image size

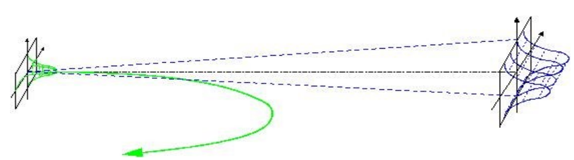
  - interference
    - → beam size

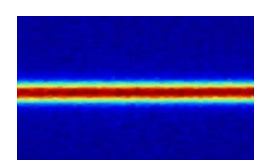




interference pattern

  - projection
    - → beam divergence





angular distribution

Beam Matrix based Measurements

- starting point: beam matrix $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$

- emittance determination
  - measurement of **3** matrix elements ***Σ_{11}, Σ_{12}, Σ_{22}***
  - **remember:** beam matrix *Σ* depends on location, i.e. ***$\Sigma(s)$***

    $\rightarrow$ determination of matrix elements at same location required

$$\boxed{\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}}$$

- access to matrix elements
  - profile monitor determines only $\sigma = \sqrt{\Sigma_{11}}$
  - other matrix elements can be inferred from beam profiles taken under various transport conditions

    $\rightarrow$ knowledge of transport matrix M required $\Sigma^b = M \cdot \Sigma^a \cdot M^T$ $\quad M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$

- measurement of at least 3 profiles for 3 matrix elements

$$\Sigma_{11}^a$$

$$\Sigma_{11}^b = m_{11}^2 \cdot \Sigma_{11}^a + 2m_{11}m_{12} \cdot \Sigma_{12}^a + m_{12}^2 \cdot \Sigma_{22}^a$$

$$\Sigma_{11}^c = \overline{m}_{11}^2 \cdot \Sigma_{11}^a + 2\overline{m}_{11}\overline{m}_{12} \cdot \Sigma_{12}^a + \overline{m}_{12}^2 \cdot \Sigma_{22}^a$$

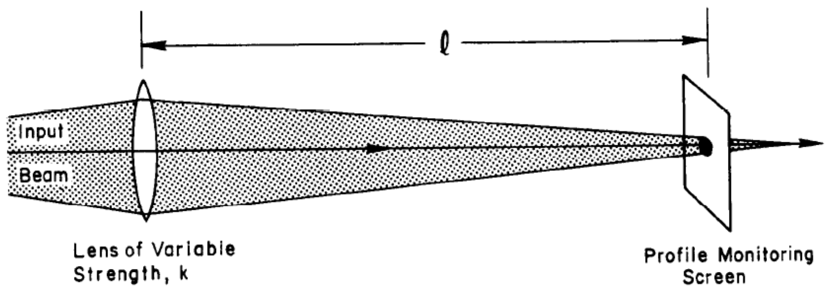
  - **measurement :** profiles $\sigma^{a,b,c} = \sqrt{\Sigma_{11}^{a,b,c}}$
  - **known:** transport optics $M, \overline{M}$
  - **deduced:** matrix elements $\Sigma_{11}^a, \Sigma_{12}^a, \Sigma_{22}^a$

$\rightarrow$ more than 3 profile measurements favourable, data subjected to least-square analysis

Beam Matrix based Measurements

- „quadrupole scan“ method
  - use of variable quadrupole strengths

    → change quadrupole settings and measure beam size in profile monitor located downstream



M_{quad} (f = 1/k) M_{drift} (drift space)

quadrupole transfer matrix

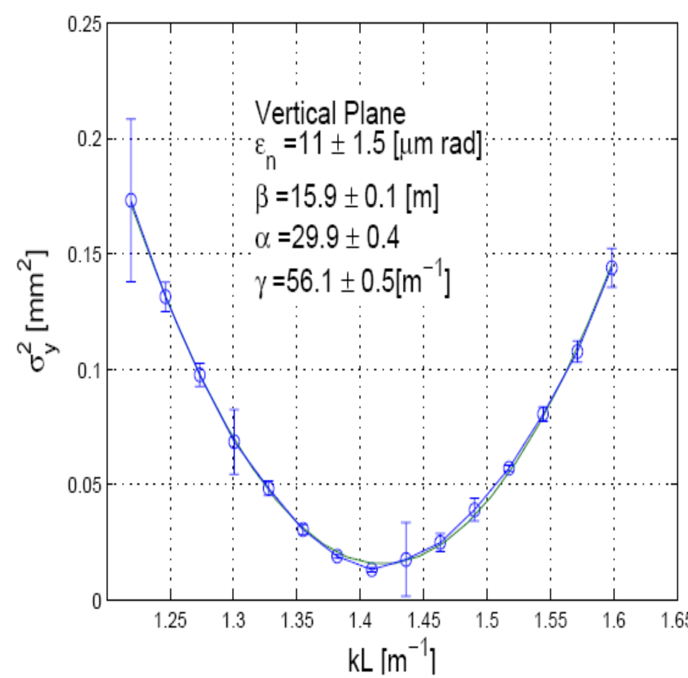
$$M_{quad} = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

drift space transfer matrix

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \quad M = M_{drift} M_{quad}$$

- Σ_{11} depends quadratically on quadrupole field strength



G. Penco et al., Proc. EPAC’08, Genoa (Italy), p.1236

Beam Matrix based Measurements

- „multi profile monitor“ method
  - fixed particle beam optics

    → measure beam sizes using multiple profile monitors at different locations

  - example:

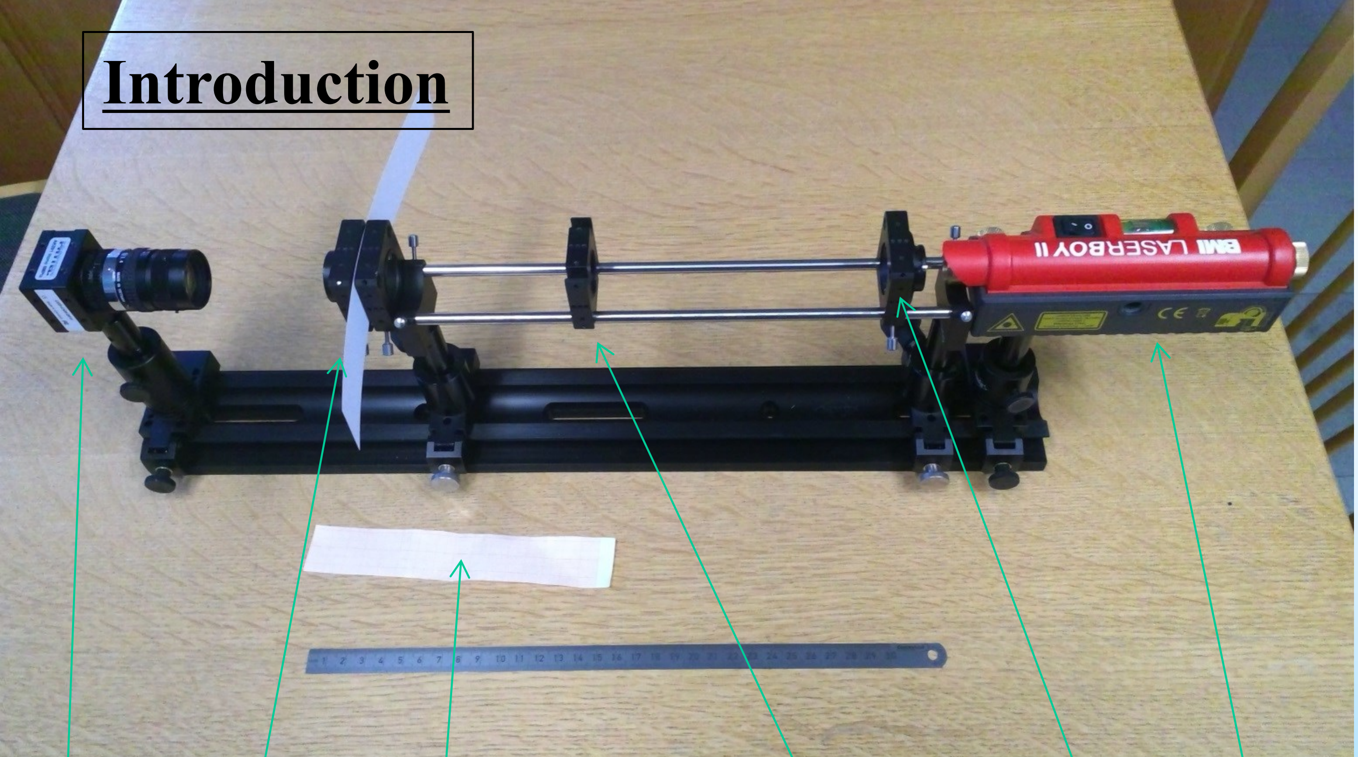
    emittance measurement

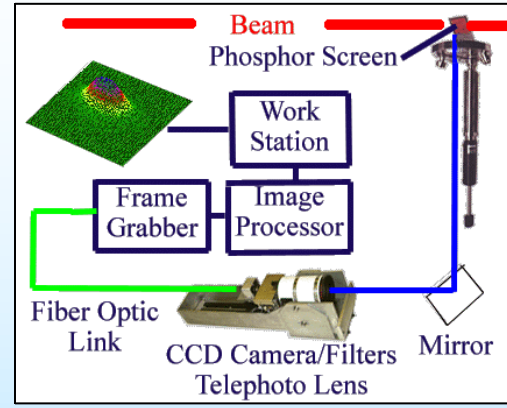
    setup at FLASH injector (DESY)

    courtesy: K. Honkavaara (DESY)

- task
  - beam profile measurement

Introduction





By moving the lens one can take pictures from the camera in the focus (not preferred due to limited resolution of the optic system) and on other positions. The distance of the lens to various screen positions can be measured by a simple ruler*. The camera is connected to a Computer where the readout software is installed. The pictures (.jpg) can be saved and can be loaded into a free software called “ImageJ” where a profile of an area can be displayed and the curser position and the value is displayed (8 bit). The σ of the profile have to be found for each screen (camera) position and the emittance have to be calculated.

<u>* Move the lens to simulate different screen positions</u>

Parameters

- CCD
  - Phytec USB-CAM 051H

Resolution	**2592 x 1944 (5 MPix),** 2048 x 1536 (3,1MPix), 1600 x 1200 (2MPix), 1280 x 960 (1,2MPix) 1024 x 768 (0,8MPix), 640 x 480 (VGA)			
Model	**USB-CAM-051H**	**USB-CAM-151H**	**USB-CAM-052H**	**USB-CAM-152H**
color / monochrom	monochrom		color	
Sensor Format	1/2,5"			
Image Sensor	Aptina MT9P031, CMOS			
Pixel Size	2,2 µm x 2,2 µm			
Color format	Y8		RGB32, RGGB (Raw)	
Lens Holder	C / CS – Mount			
fps	6 fps to 52 fps			
Dynamic Range	8 bit			
Shutter	Rolling			
Light sensitivity	1,4 V/lux-sec			
Interface	USB 2.0 High Speed			
Exposure time	1/10.000 s to 30 s			
Gain	0 dB to 18 dB			
White Balance	-		-6 dB bis +6 dB	
Power supply	4,5 V bis 5,5V DC			
Power Consumption	Circa 250 mA bei 5V			
Feature (optional)	-	ext. Trigger, Digital-Output	-	ext. Trigger, Digital-Output
Temperature range	-5°C bis +45°C			
Dimensions (B x L x H)	36 mm x 36 mm x 25 mm			
Fixing	1/4" and M6x8 on all sides			
Weight	70 g			
Connection	USB Mini-B			
Feature- Connection	-	Hirose HR10A-7R-4P	-	Hirose HR10A-7R-4P

- screen
  - material: white paper
- grid target
  - spacing: 1 mm
- Laser: LaserBoyII

BMI Bayerische Laserboy II Wasserwaage 649 015

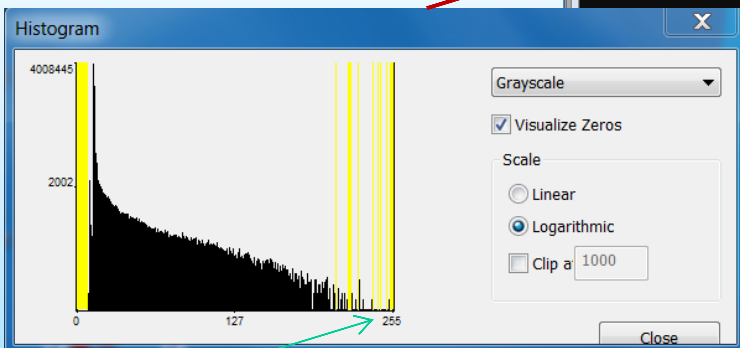
Allgemeine Informationen	
Artikelnummer	ET1117000
EAN	4007368050049
Hersteller	BMI Bayerische
Hersteller-ArtNr	649 015
Hersteller-Typ	649 015
Verpackungseinheit	1 Stück
Artikelklasse	Messlaser

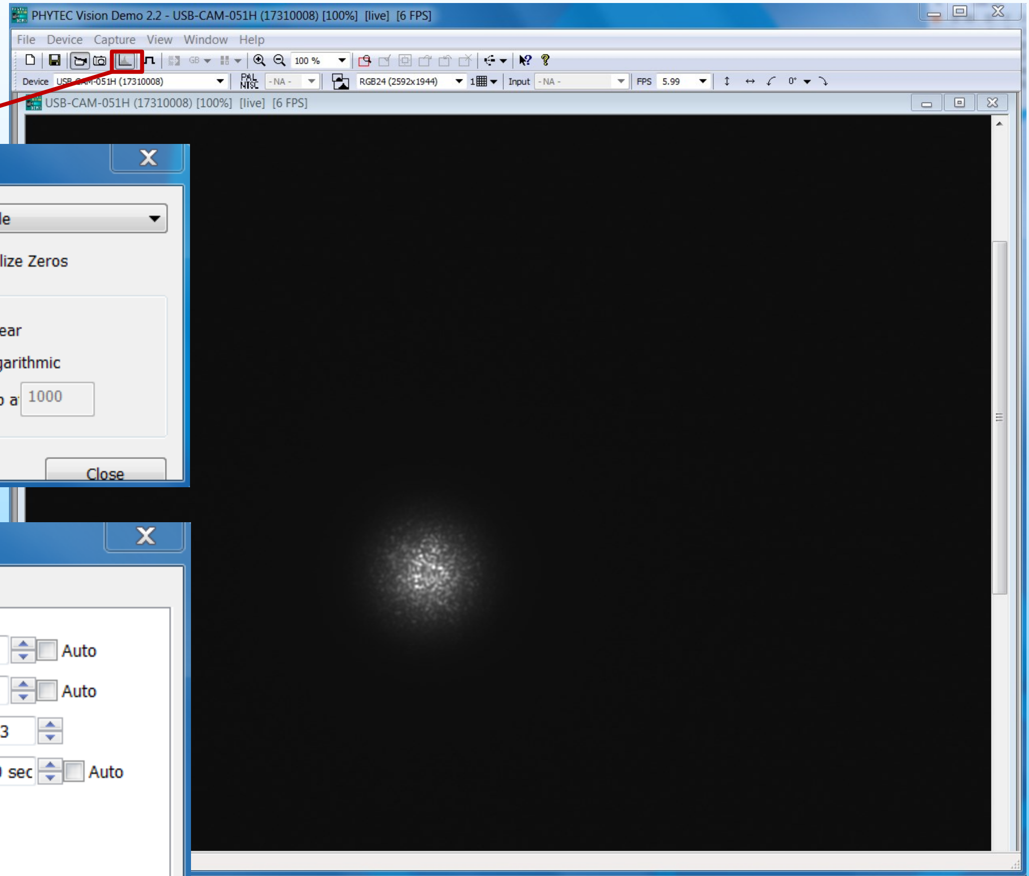
Technische Informationen	
Länge der Signalstrecke	30m
Laserklasse	
Sichtbare Signalstrecke	
Rotierende Signalstrecke	

BMI Bayerische Laserboy II Wasserwaage 649 015 Länge der Signalstrecke 30m, Laserklasse 2, Sichtbare Signalstrecke,

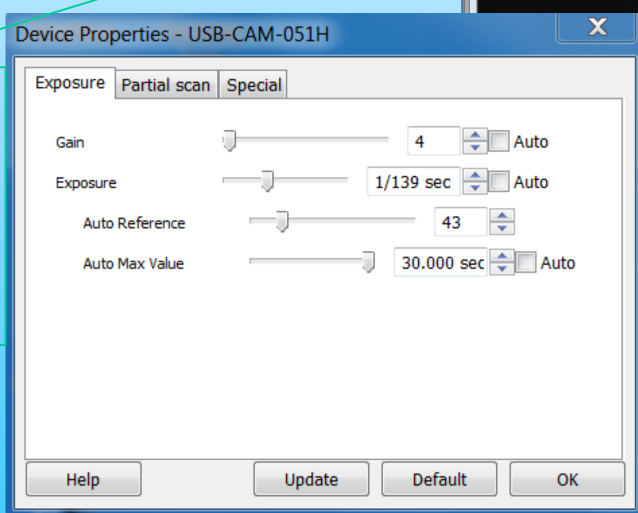
CCD Readout: Introduction

- readout program PHYTEC Vision Demo 2.2
  - histogram of grey values





Check always: Do not saturate (255)



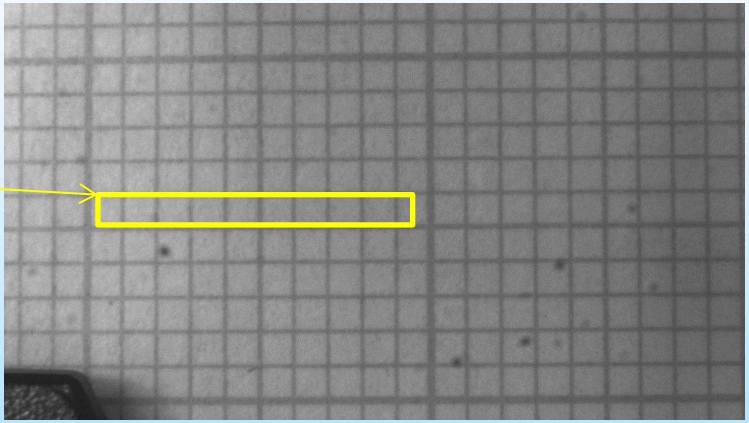
  - CCD control parameters
    → Device → Properties

Your tasks in green frames

Check: Do not saturate (255)

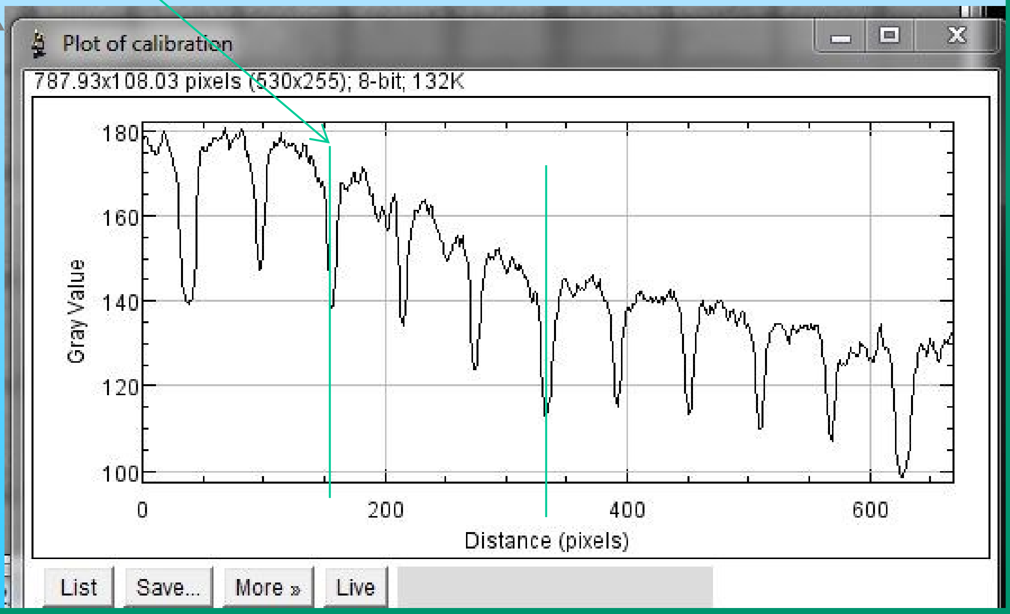
Calibration

Use mm-grid to calibrate the readout setup.
Select ROI (where beam image will appear), plot profile, use cursor and enter measurement into pre-prepared Excel sheet “Laser emittance.xlsx”



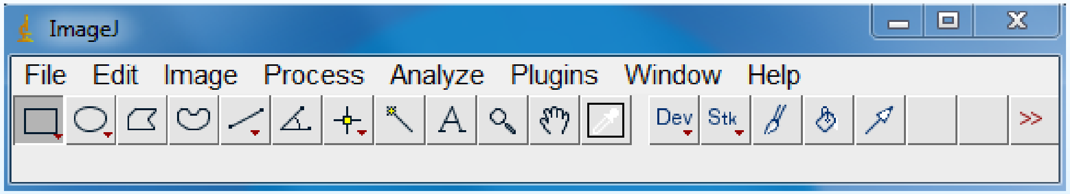
Calibration			
Line-distance [mm]	pixel 1	pixel 2	
3	66	401	
[meter]		Cal. Result	
3.00E-03	=>	111.6667	pixel/mm

All yellow cells will be calculated automatically

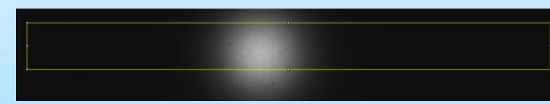


ImageJ: Introduction

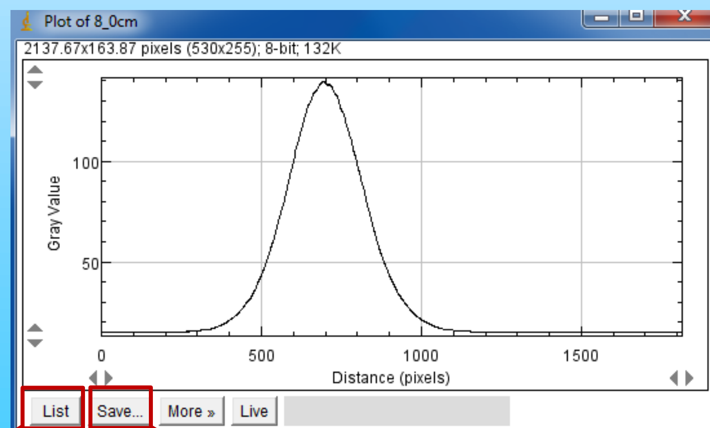
- press icon — access to start panel



- load image file → File → Open (Shortcut: Ctrl + O)
- select ROI: in start panel: select left button (below “File”), usually already pre-selected
  then with left mouse button: draw rectangular ROI



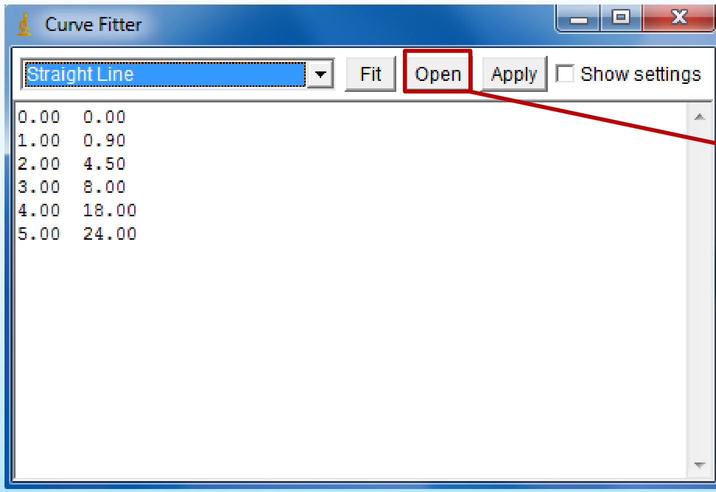
- plot horizontal projection → Analyze → Plot Profile (Shortcut: Ctrl + k)



- save data **-> list data points** **-> save data as <u>.csv file</u> (required for profile fitting)**

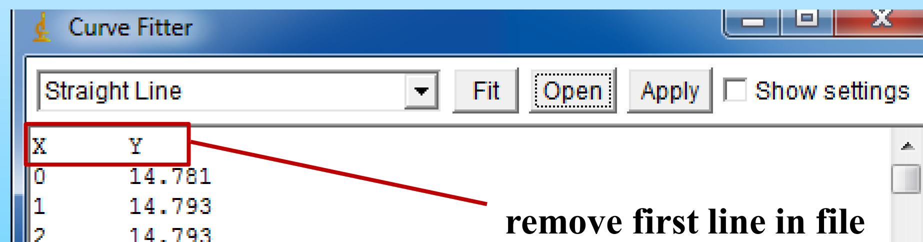
ImageJ: Introduction

- profile fitting → Analyze → Tools → Curve Fitting…
  - load profile data:



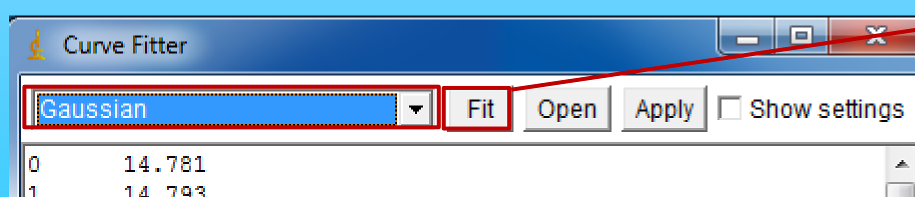
load .csv data file

  - delete bad data:



remove first line in file

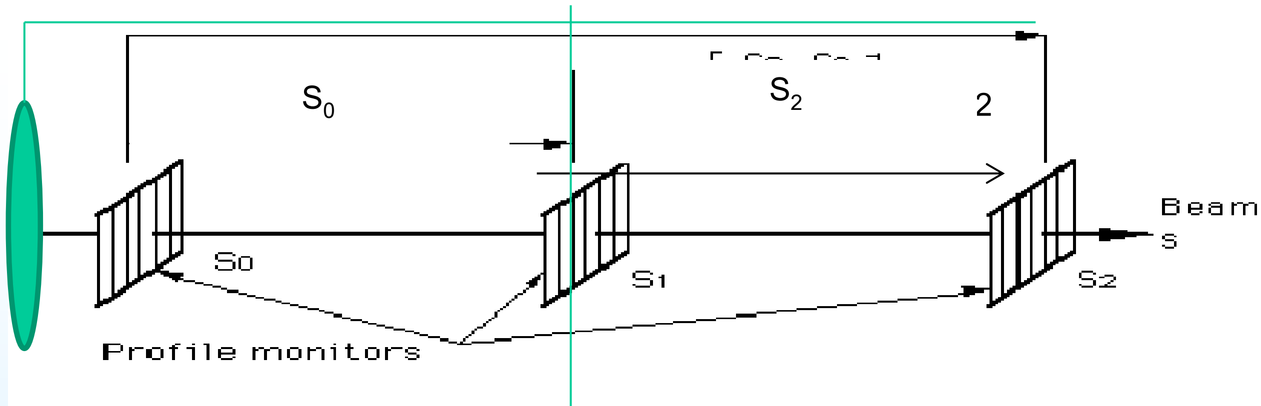
  - select fit function:



fit profile data

$$y = a + (b-a)\cdot e^{-\frac{(x-c)^2}{2d^2}}$$

Take profiles at 3 different distances lens-screen. Move lense to simulate 3 positions.



Hint1: Make the distances equal, set s_1 = 0, s_2 = - s_0
Hint 2: Avoid position at waist (why?)

S_1=0

Check: Do not saturate (255)

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\ matrix$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$\sigma_y^2{}_{measured} = M_1^2\sigma_{11} + 2M_{11} M_{12}\sigma_{12} + M_{12}^2 \sigma_{22}$$

No optical elements => $M = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$

$$\sigma^2(S1)_{measured} = \sigma_{11} + 2s_1 \sigma_{12} + s_1^2 \sigma_{22}$$
$$\sigma^2(S0)_{measured} = \sigma_{11} + 2s_0 \sigma_{12} + s_0^2 \sigma_{22}$$
$$\sigma^2(S2)_{measured} = \sigma_{11} + 2s_2 \sigma_{12} + s_2^2 \sigma_{22}$$

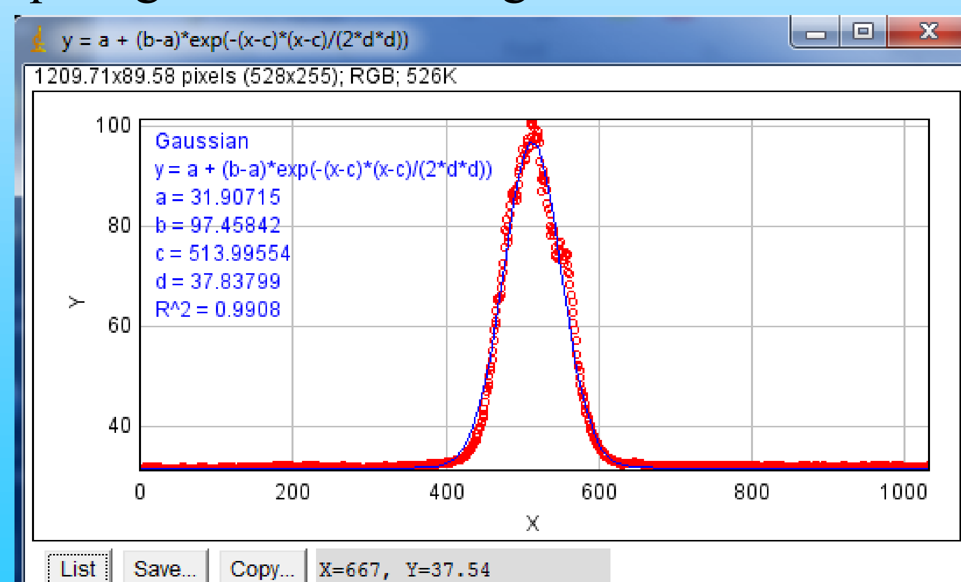
with

$$\sigma_{11} = \sigma_y^2(0) = \sigma^2(S0)$$

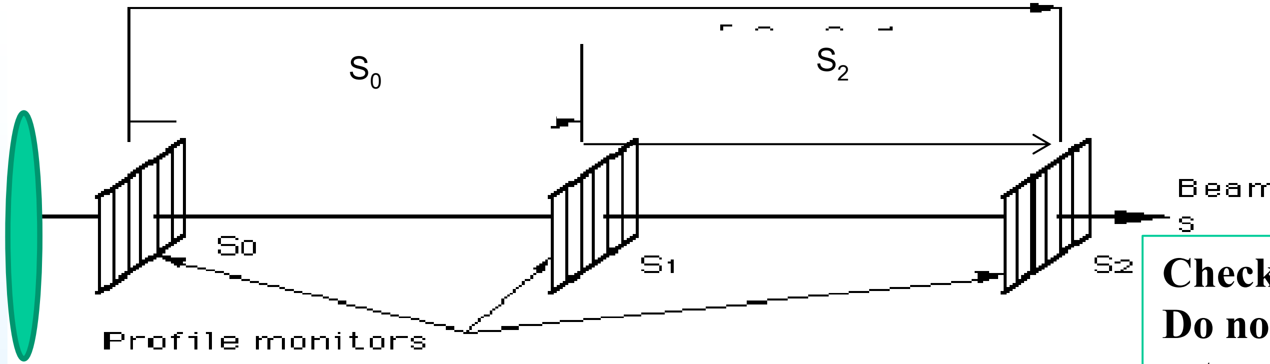
$$\sigma_{12} = \frac{\sigma_y^2(+s) - \sigma_y^2(-s)}{4s} = \sigma^2(+s)$$

$$\sigma_{22} = \frac{\sigma_y^2(+s) - 2\cdot\sigma^2(0) + \sigma_y^2(-s)}{2\cdot s^2} = \sigma^2(-s)$$

Adjust the aperture so that the image looks quite gaussian at its larges size!



Take profiles at 3 different distances lens-screen.

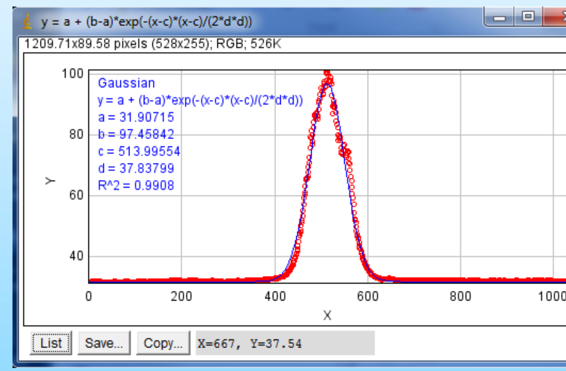


Check: Do not saturate (255)

Make the distances equal, set $s_1 = 0$, $s_2 = - s_0$

Enter the screen positons +s, 0, -s and the measured σ from fits (in pixel) at +s, 0 and –s into the Excel sheet:

	Position[m]	L [m]	sigma [p]	sigma(s) [m]	
	METER!!!		PIXEL		
+s	1.00E-02		37.8	3.38E-04	Take FWHM/2.36 if not Gaussian!
		1.00E-02			
0	2.00E-02		33.5	2.99E-04	Take FWHM/2.36 if not Gaussian!
		1.00E-02			
-s	3.00E-02		30.2	2.70E-04	Take FWHM/2.36 if not Gaussian!



distance s	[meter]		
s=	1.00E-02	(s equal between 3 positions)	
		Emitt^2	2.38E-12
sigma 11	8.95E-08	Emittance:	1.5415E-06
sigma 12	1.03E-06		
sigma 22	3.84E-05		

should not be negativ!

expected Emittance: around 1 * 10E-6 [m rad]

(often the unit is written in [π mm mrad] while π indicates that ε is the area of an ellipse)

Unfortunately it depends sometimes from one pixel in sigma only in this experiment

↑

Like Criegee: Trick: **Three screens at –s, 0, +s** with s = L

$$\sigma_{11} = \sigma_y^2(0)$$

$$\sigma_{12} = \frac{\sigma_y^2(+s) - \sigma_y^2(-s)}{4s}$$

$$\sigma_{22} = \frac{\sigma_y^2(+s) - 2\cdot\sigma^2(0) + \sigma_y^2(-s)}{2\cdot s^2}$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$\left.\begin{aligned} \hat{x}_1^2 &= \epsilon\beta_2 - 2L\cdot\epsilon\alpha_2 + L^2\cdot\epsilon\gamma_2 \\ \hat{x}_2^2 &= \epsilon\beta_2 \\ \hat{x}_3^2 &= \epsilon\beta_2 + 2L\cdot\epsilon\alpha_2 + L^2\cdot\epsilon\gamma_2 \end{aligned}\right\} \text{ with the solution } \left\{\begin{aligned} \epsilon\beta_2 &= \hat{x}_2^2 \\ \epsilon\alpha_2 &= (\hat{x}_3^2 - \hat{x}_1^2)/(4L) \\ \epsilon\gamma_2 &= (\hat{x}_1^2 - 2\hat{x}_2^2 + \hat{x}_3^2)/(2L^2) \\ \epsilon^2 &= \epsilon\beta_2\cdot\epsilon\gamma_2 - (\epsilon\alpha_2)^2 \end{aligned}\right.$$

The emittance is calculated by the formulas.
Since the Laser is not gaussian, vary a little(!) the fitted widthes. Note how sensitive the emittance behave.
=> Need of **good resolution** and good fits.