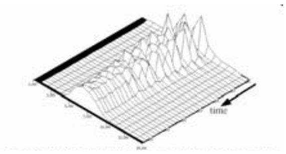
Injection mismatch

by Kay Wittenburg –DESY-



Injection mismatch:

As a rule, proton/ion accelerators need their full aperture at injection, thus avoiding mismatch allows a beam of larger normalized emittance ε* and containing more Protons. In proton/ion ring accelerators any type of injection mismatch will lead to an emittance blow-up. Off axis injection can be detected easily by (????) turn-by-turn BPMs in the ring (before Landau damping occurs).
The orbit mismatch can be corrected by a proper setup of the steering magnets, kickers and septas. However, any mismatch of the optical parameters α, β (and therefore γ) will also lead to an emittance blow-up (and beam losses) and is not detectable by BPMs.
Fig. 1a shows the phase ellipse at a certain location in a circular accelerator. The ellipse is defined by the optics of the accelerator with the emittance ε and the optical parameters β = beta function, $\gamma = (1 + \alpha)/\beta$ and the slope of the beta function $\alpha = -\beta'/2$. Fig. 1b-d shows the process of filamentation after some turns.

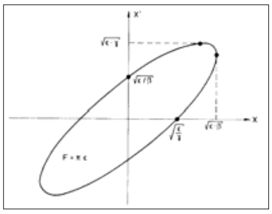


Fig M1a: A phase space ellipse of a circular accelerator, defined by α, β, γ, ε

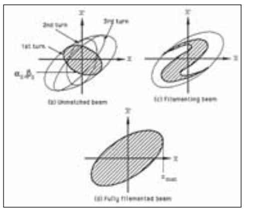


Fig. M1 b-d: Filamentation of an unmatched beam (from Ref. 2)

Assuming a beam is injected into the circular machine, defined by β_0 and α_0 (and therefore γ_0) with a given emittance ε_0. For each turn i in the machine the three optical parameters will be transformed by

$$\begin{pmatrix} \beta_{i+1} \\ \alpha_{i+1} \\ \gamma_{i+1} \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_i \\ \alpha_i \\ \gamma_i \end{pmatrix} \quad (Starting \;\; with \;\; i=0)$$

where C and S are the elements of the Twiss matrix ($\mu = 2\pi q$, q = tune, see B. Holzer's talk):

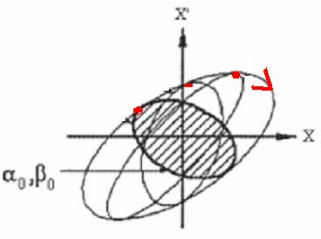
$$\begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} \cos\mu + \alpha_0 \cdot \sin\mu & \beta_0 \cdot \sin\mu \\ -\gamma_0 \cdot \sin\mu & \cos\mu - \alpha_0 \cdot \sin\mu \end{pmatrix} \quad (1)$$

and $\gamma = (1+\alpha^2)/\beta$

Without any mismatch, the three parameters will be constant while a mismatch will result in an oscillation of the parameters.

Exercise M1: Show the constant β without mismatch and the oscillation of β for the mismatch.
What is the oscillation frequency?
Explain by formula (resolving β_{i+1}) and by picture

During 1 turn the whole ellipse rotates with Q, but
the projection on the x-axis oscillates with 2 Q. One turn
gives two periods.



$$\boxed{\beta_{i+1} = C^2\beta_i - 2SC\alpha_i + S^2\gamma_i}$$

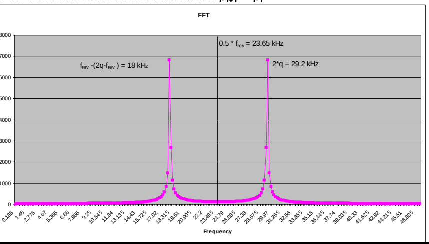
$$\boxed{(\alpha_0^2\beta_i - 2\alpha_0\beta_0\alpha_i + \beta_0^2(1+\alpha_i^2)/\beta_i)\cdot\sin^2\mu = \beta_0\cdot\sin^2\mu}$$

with (1) and some transformations

$$\boxed{\beta_{i+1} = \beta_i\cdot\cos^2\mu + (\alpha_0^2\beta_i - 2\alpha_0\beta_0\alpha_i + \beta_0^2\gamma_i)\cdot\sin^2\mu + ((\alpha_0\beta_i - \beta_0\alpha_i)\cdot 2\cdot\sin\mu\cdot\cos\mu}$$

with
$\sin\mu\cdot\cos\mu = ½ \sin|\mathbf{2\mu}|$, $\cos^2\mu = ½ (1+\cos\mathbf{2\mu})$, $\sin^2\mu = ½ (1-\cos\mathbf{2\mu})$

one gets **twice the betatron tune. Without mismatch $\beta_{i+1} = \beta_i$**



<u>Exercise M2: Discuss how to measure a 10% betatron mismatch at injection between a transport line and a storage ring , for example in the HERAp accelerator. How large is the emittance blow-up?</u>

Some important HERAp parameters
Circumference circ = 6.3 km
Tune q = 0.31 or f = 13.8 kHz
Momentum E_p = 40 GeV/c at injection
Normalized emittance $\varepsilon_n = 20\,\pi$ mm mrad, $\varepsilon_0 = 5\cdot 10^{-7}$
β_0 = 238 m, α_0 = -2.2, => γ_0 = 0.0245 at the injection point ($\beta\gamma-\alpha^2=1$). => Parameters of the ring
β = 214 m, $\alpha_0 = \alpha$, => γ = 0.0272 at the injection point. (10% mismatch)=> parameters of the injected beam

The emittance blow-up due to the betatron mismatch ($\alpha_0 = \alpha$) can be calculated with the following formula derived from Ref. 2, 3 (gaussian beams):

$$\boxed{\varepsilon_{filamented} = \varepsilon_0\cdot(1 + 0.5\cdot|\det(\Delta J)|) \quad with \quad \Delta J = \begin{pmatrix} \alpha_0 - \alpha & \beta_0 - \beta \\ -(\gamma_0 - \gamma) & -(\alpha_0 - \alpha) \end{pmatrix}}$$

$|\det \Delta J| = (\alpha_0 - \alpha)^2 + (\gamma_0 - \gamma)\cdot(\beta_0 - \beta) = 0.066$

In this example a 10% β-mismatch leads to an emittance blow up

$\Delta\varepsilon = (\varepsilon_{filamented} - \varepsilon_0)/\varepsilon_0 \cdot 100\,\%$ = <u>3.3%.</u>

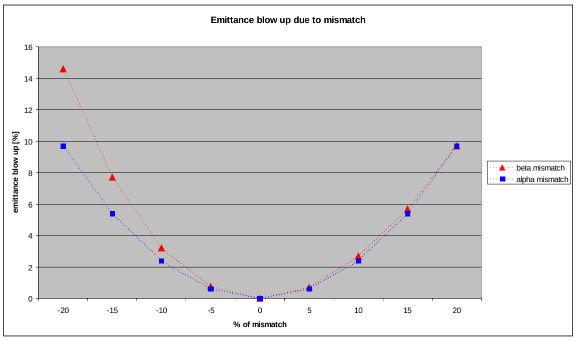


Fig. M3: Emittance blow up $\Delta\varepsilon = (\varepsilon_{filamented} - \varepsilon)/\varepsilon \cdot 100\,\%$ due to mismatch

Exercise M2a:
What is the beam size after filamentation? What kind of measurement will you propose to determine the β-mismatch? Which monitor do you propose to use for this measurement?

β_1=238 m, Normalized emittance $\varepsilon_n = 20\,\pi$ mm mrad, $\varepsilon_0 = 5 \cdot 10^{-7}$, $\varepsilon_{filamented} = 5.162 \cdot 10^{-7}$
$\Delta\varepsilon = (\varepsilon_{filamented} - \varepsilon_0)/\varepsilon_0 \cdot 100\,\% =$ 3.3%.

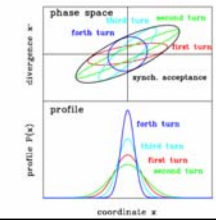
A simple beam width measurement after filamentation at (for example) β_1=238 m results in:

$$\sigma_0 = \sqrt{\varepsilon_0 \cdot \beta_1} = 10.91\,mm$$

$$\sigma_{0\,filamented} = \sqrt{\varepsilon_{filamented} \cdot \beta_1} = 11.08\,mm$$

⇨ **The effect is hard to detect for a typical measurement (Take the resolution of the instrument into account)**

A mismatch of the phase space will result in transverse shape oscillations, at least for some ten turns, before the filamentation of the beam. =>
Observation of the width-oscillation at one location.



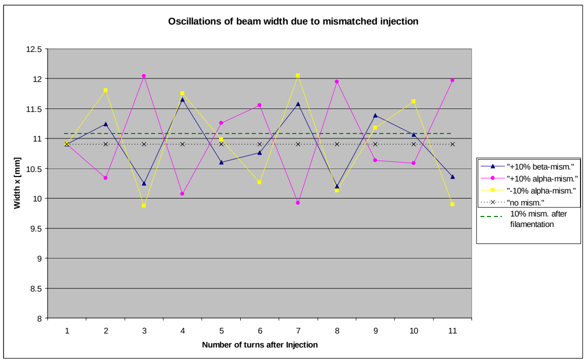


Figure M4: Oscillations of the beam width due to mismatched injection. Note also the small difference of the beam width after filamentation.

A measurement of width oscillations at injection is a very efficient method to detect an optical mismatch that increases the emittance in the circular accelerator.
Measurement of the turn-by-turn shape oscillation is possible with a fast (turn by turn) readout of:

Proposed Monitors?

1. Thin screen (OTR, Phosphor)
2. SEM grids,
3. IPM,
4. QP-Pickup
5. Synchrotron Radiation (SR) -Monitor (electrons).

examples in Refs. 4-8

Exersice M2b: What is the effect of the proposed monitor(s) on the beam?

- Screen/Grid: Emittance blow-up and losses
- IPM: Very small, a sufficient signal at each turn needs a pressure bump => emittance blow-up and losses
- QP-Pickup: None (see Rodri's talk), but very difficult to suppress the dipole mode.
- SR-Monitior: None, but no light from protons!

Blow-up:
A screen/grid or IPM pressure bump will give an additional constant increase of the emittance, but it can easily be separated from the oscillation observation. The protons receive a mean kick at each traverse through a screen resulting in an additional angle θ.

$$\theta = \frac{0.014}{p \cdot \beta} \cdot Z \cdot \sqrt{\frac{d}{l_{rad}}}\left[1 + \frac{1}{9}\log_{10}\left(\frac{d}{l_{rad}}\right)\right] \quad in\ radians$$

where p is the momentum in GeV/c and Z=1 the charge number of the proton, $\beta = v/c$ the velocity, d the thickness of the foil and l_{rad} the radiation length of the material of the foil. This formula describes the gaussian approximation of the mean scattering angle of the protons after one traverse. The change of the emittance $\delta\varepsilon$ for every turn can be calculated by:

$$\delta\varepsilon_{rms} = \sqrt{2 \cdot \pi} \cdot \theta^2 \cdot \beta$$

which adds quadratically to the 1 σ − emittance of the previous turn.

D. Möhl, P.J. Bryant,CAS: $\delta\varepsilon_{rms} = \frac{\pi}{2} \cdot \theta^2 \cdot \beta$ $\quad \delta\,\varepsilon_{rms} = \frac{1}{2} \cdot \theta_{rms}{}^2 \cdot \beta_x$

M. Giovannozzi: CAS 2005: $\delta\varepsilon_{rms} = \frac{\pi}{4} \cdot \theta^2 \cdot \beta$

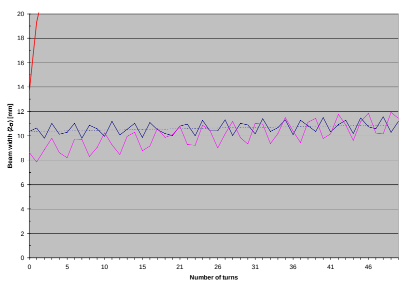
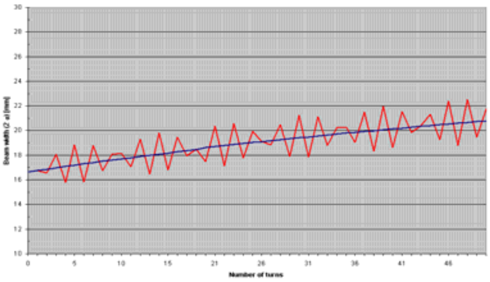


Fig. M5: Emittance growth due to a d = 10 µm Titanium foil at injection energy of HERA and PETRA ($\alpha = -2$ m, $\beta = 40$ m, $\varepsilon_n = 14\,\pi$ mm mrad, q = 0.14, p = 7.5 GeV/c) and DESY III (with a foil-thickness of d = 1 µm (!))

The emittance blow-up is shown in Fig. M5 for a 10 µm thick titanium foil as the source of OTR radiation. In addition a betatron mismatch of 10% is assumed. The figure shows a small growth of the beam width due to the foil, which does not affect the beam width oscillation. The growth rate is small compared to the oscillation amplitude. The faster growth rate in PETRA is a result of the smaller momentum of the injected protons and therefore a larger scattering angle in the foil. This angle will become much larger in DESY III (p=310 MeV/c, β = 0.3), so that the beam width will become unacceptably large within one turn and the loss rate will increase drastically (in Fig. M5 the line for DESY III extend the border of the figure within 3 turns even with a 1 µm screen).

SEM-Grids

The emittance blow up in DESY III due to a thin foil is much too large. A harp of thin wires produces less emittance blow up. Assuming a harp of 20 µm titanium wires at a separation of 1 mm, the blowup can be calculated like a 0.2 µm foil. Fig. 9 shows the beam oscillation due to a 10% mismatch in DESY III together with the blowup due to these wires. The secondary emission (SEM) current created in the wires can be read out by fast ADCs turn by turn (315 kHz). Such a readout schema is applied in the PS-Booster at CERN (Ref.4).



Simulation of the beam width versus turns as measured by SEM grid with and without a +10% beta mismatch in DESY III (α = -1.7 m, β = 14.3 m, ε_n = 6 π mm mrad, q = 0.28, p = 310 MeV/c)

Losses:

The relative proton losses per turn dN/N_0 in the foil (thickness d) is given by the nuclear interaction length L_{nuc}:

$$\frac{dN}{N_0} = \frac{d}{L_{nuc}} \quad with \quad L_{nuc} = \frac{A}{\rho \cdot N_A \cdot \sigma_{nuc}}$$

L_{nuc} depends on the total nuclear cross section of the nuclear interaction σ_{nuc}, the density ρ of the foil and the Avogadro constant $N_A = 6.0225 \cdot 10^{23}$ mol^{-1}. The nuclear cross section σ_{nuc} depends on the proton momentum and on the material of the foil and is shown for different materials in Tab. 1 between a momentum of 0.3 < p < 40 GeV/c:

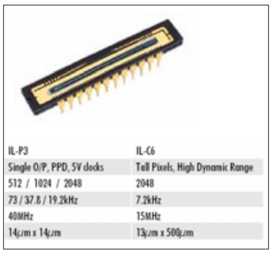
Material A [g/mol] ρ [g/cm^3]	Momentum [GeV/c]	σ_{nuc} [mb]	L_{nuc} [cm]	relative loss/turn $dN/N_0 \cdot 100$ [%] with d = 10 µm
Carbon	0.3	280	31.5	$3 \cdot 10^{-3}$
12.01	7.5	360	24.5	$4 \cdot 10^{-3}$
2.26	40	330	22.5	$4.4 \cdot 10^{-3}$
Aluminum	0.3	550	30.2	$3.3 \cdot 10^{-3}$
26.98	7.5	700	38.4	$2.6 \cdot 10^{-3}$
2.70	40	640	35.1	$2.8 \cdot 10^{-3}$
Copper	0.3	950	12.4	$8.1 \cdot 10^{-3}$
63.546	7.5	1350	17.6	$5.7 \cdot 10^{-3}$
8.96	40	1260	16.4	$6.1 \cdot 10^{-3}$

Tab. 2: Nuclear total cross sections, interaction length and particle losses

The loss rate is negligible small at the injection energies of proton machines and will not influence the mismatch measurement.

Some notes to the readout:

The optical readout of screens/IPM is slow. A turn by turn observation needs a 100 kHz (3 km) data collection of the whole image. Line sensors with a larger pixel size (for better sensitivity) may have a readout frequency of 15 MHz/pixel. Assuming 128 pixel will give a maximum readout frequency of 117 kHz for a 1 dim image.



A SEM signal as well as the QP-Pickup signal (H. Schmickler's talk) can be picked up with very high frequencies, even bunch by bunch (100 MHz) and is therefore preferred for smaller ring diameters with a higher revolution frequency and smaller beam momentum to avoid emittance blow-up

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That's the end of the mismatch session

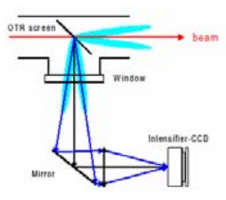


Fig.5: Matching monitor set-up in the SPS

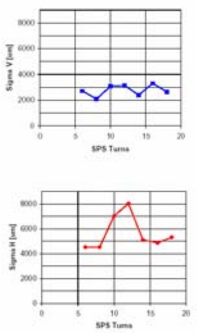


Fig.8: Horizontal and vertical beam size variation measured over 12 turns in the SPS: the matching is acceptable in the vertical direction and worse in the horizontal one.

Ref. 4: FIRST RESULTS FROM BETATRON MATCHING MONITORS INSTALLED IN THE CERN PSB AND SPS.
By C. Bovet, R. Colchester, C. Dutriat, G. Ferioli, J.J. Gras, R. Jung, P. Knaus, U. Raich, J.M. Vouillot (CERN). CERN-SL-98-037-BI, CERN-SL-98-37-BI, Jun 1998. 4pp. 6th European Particle Accelerator Conference (EPAC 98), Stockholm, Sweden, 22-26 Jun 1998.

The following parameters can be obtained by fitting the data points:

- The emittance of the injected beam (1.82 π µm). The advantage of this method, as compared to the standard 3-profile method, lies in the fact that only the beta function has to be taken into account and good statistics are obtained for the beam width due tomultiple measurements on the same beam.
- Geometric betatron mismatch (~ 50 %) which leads to an RMS blow-up of 8 %.
- The contribution of the beam width due to scattering on the SEM-Grid wires is barely visible. The RMS scattering angle is estimated to 0.04 mrad per turn.

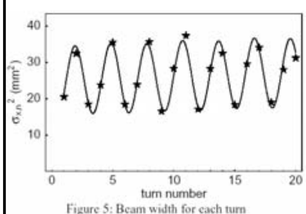


Figure 5: Beam width for each turn

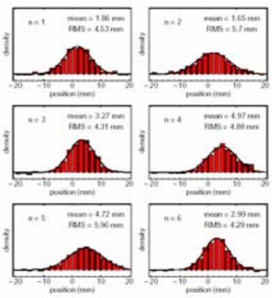
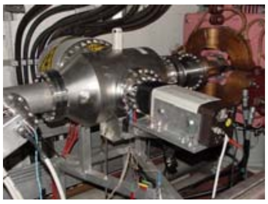


Figure 3: Turn-by-Turn Profile

INJECTION MATCHING STUDIES USING TURN BY TURN BEAM PROFILE MEASUREMENTS IN THE CERN PS
M. Benedikt, Ch. Carli, Ch. Dutriat, A. Jansson, M. Giovannozzi, M. Martini, U. Raich, CERN, Geneva, Switzerland, DIPAC 2001



IPM at DESY and CERN

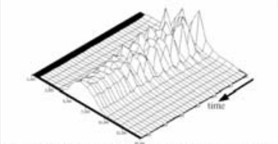
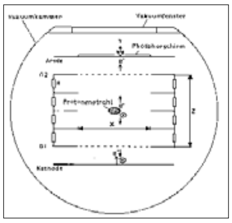
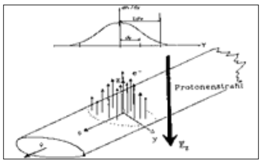


Figure 5: Profiles measured on 37 consecutive turns after injection of a beam of 1.5×10^{12} protons (40 bunches).

A Photo Multiplier Tube (PMT) with 16 anode strips and high speed acquisition electronics [4], were associated to the IPM. The phosphor used was of the P46 type, claimed to have a decay time of 0.3 µs down to 10% and 90 µs down to 1%.





A perfectly matched beam gives an emittance of $\varepsilon = x^2/\beta = \varepsilon_0$ (x = beam width, see Fig. 1a). With mismatch, the resulting phase ellipse of Fig. 1d after filamentation gives an emittance of $\varepsilon = x_{max}^2/\beta$, which is too large, because only a small fraction of the particles will fill the whole outer region of the ellipse. However, they will contribute to the measured beam width. Assuming gaussian distributions, the difference $(x_{max} - x)$ may add two times (because of the two ends of the ellipse) quadratic to x to give the beam width of the fully filamented beam:

$$\varepsilon_{filamented} = \frac{(x_{\max} - x)^2 \cdot 2 + x^2}{\beta}$$

The emittance blow up, defined by $\Delta\varepsilon = (\varepsilon_{filamented} - \varepsilon_0)/\varepsilon_0 \cdot 100\,\%$ is plotted for the upper approximation (red) and for the correct formula (blue). The agreement is quite good.