CERN

Instabilities Part I: Introduction – multiparticle systems and dynamics

Giovanni Rumolo and Kevin Li

Outline

In this introductory part, we will provide a qualitative description of **collective effects** and their **impact on particle beams**.

We will introduce **multiparticle systems** and investigate **multiparticle effects**. This will be the first step towards a more involved understanding of collective effects and their effect (next lectures).

- Part 1: Introduction – dynamics of multiparticle systems
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

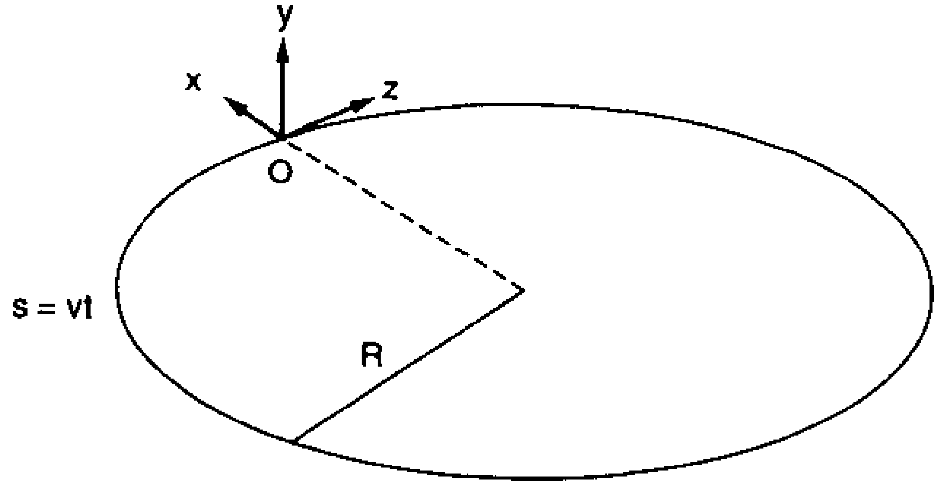
What are collective effects?

- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

What are collective effects?

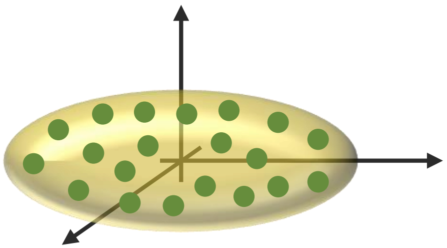
- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

- First step → **Coordinates system** we will use throughout this set of lectures
  - The origin O is moving along with the "synchronous particle", i.e. a reference particle that has the design momentum and follows the design orbit
  - Transverse coordinates x (Horizontal) and y (Vertical) relative to reference particle (x,y << R), typically x is in the plane of the bending
  - Longitudinal coordinate z relative to reference particle
  - Position along accelerator is described by independent variable s = vt



What are collective effects?

- A charged particle beam is generally described as a **multiparticle system** via the **coordinates** and the **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional phase space which can be described by a **particle distribution function**.
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):



$$\frac{\partial}{\partial s}\psi\left(x, x', y, y', z, \delta, s\right)$$

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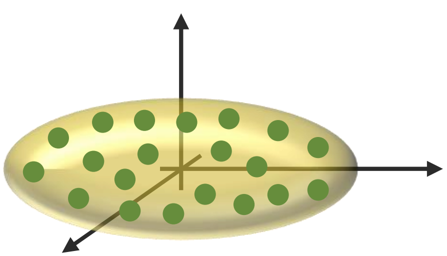
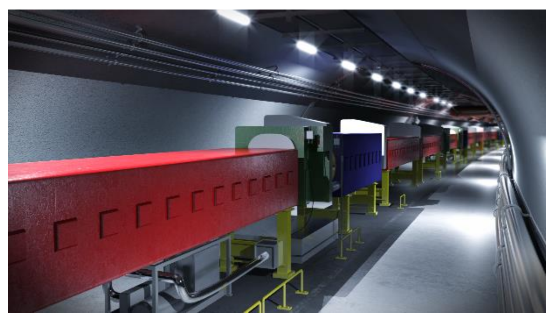
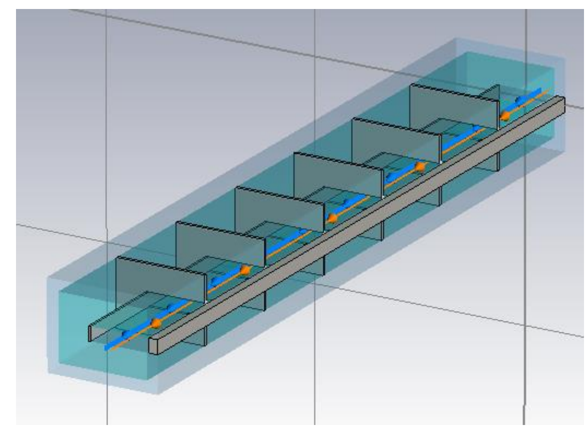
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):
  - Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
  - Collective effects add to this **distribution dependent force fields** (space charge, wake fields)



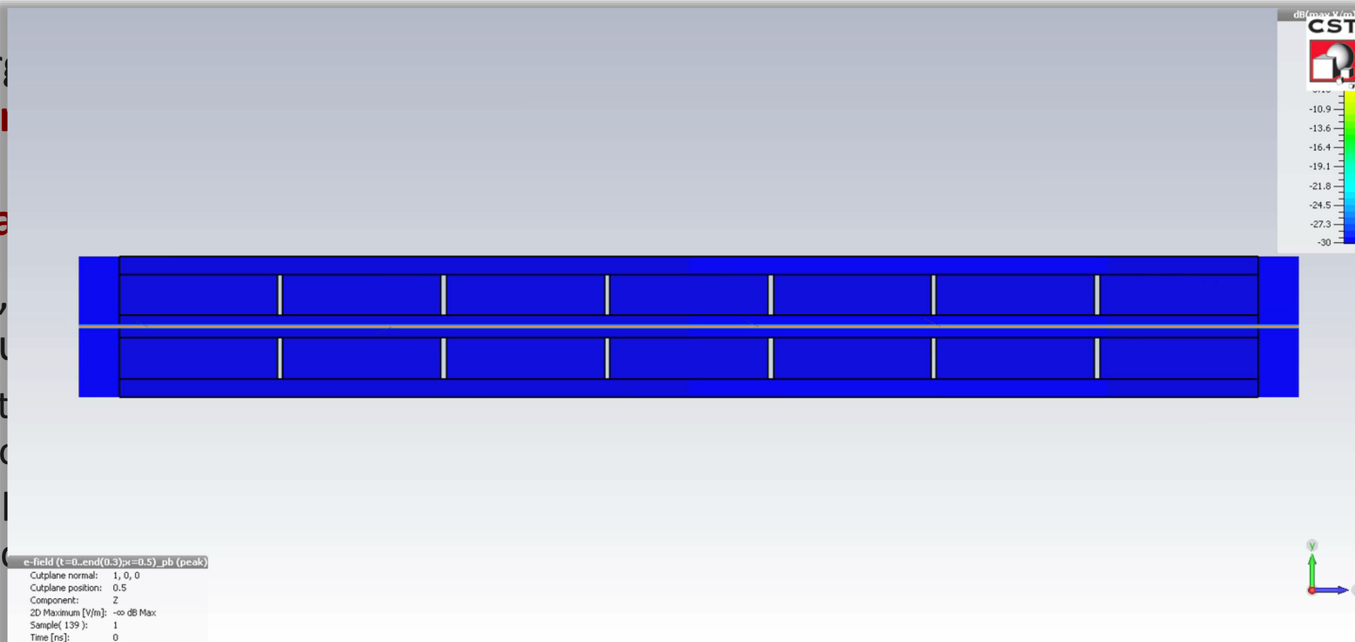


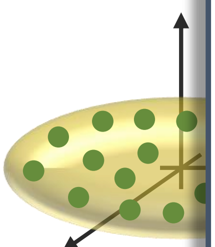


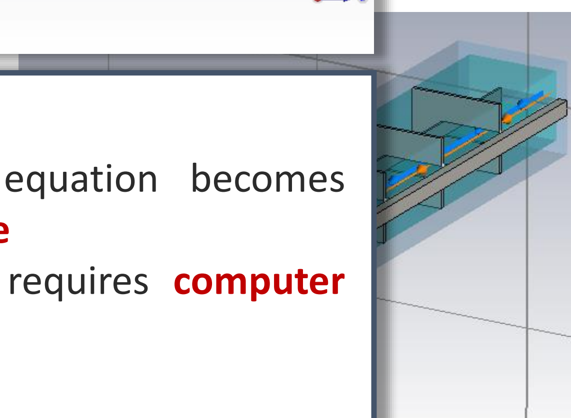
$$\frac{\partial}{\partial s}\psi\left(x, x', y, y', z, \delta, s\right) \propto f\left(F_{\text{extern}} + F_{\text{coll}}\left(\psi\right)\right)$$

What are collective effects?

- A char... ia the **coordi**... – this makes ... cribed by a **pa**...
- Hence, ... article distribu...
  - Opt... agnets, elec...
  - Coll... , wake fiel...





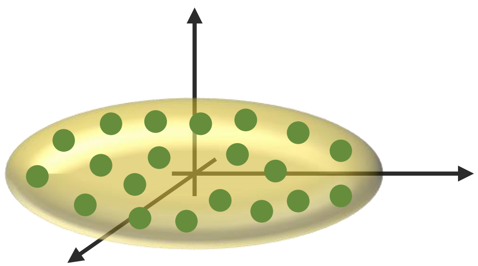


- For a **multiparticle system** this self-consistent equation becomes arbitrarily complex and practically **impossible to solve**
- Obtaining the **multiparticle dynamics** very often requires **computer simulation codes**

$$\frac{\partial}{\partial s}\psi\left(x,x',y,y',z,\delta,s\right)\propto f\left(F_{\text{extern}}+F_{\text{coll}}\left(\psi\right)\right)$$

What is a beam instability?

- A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!



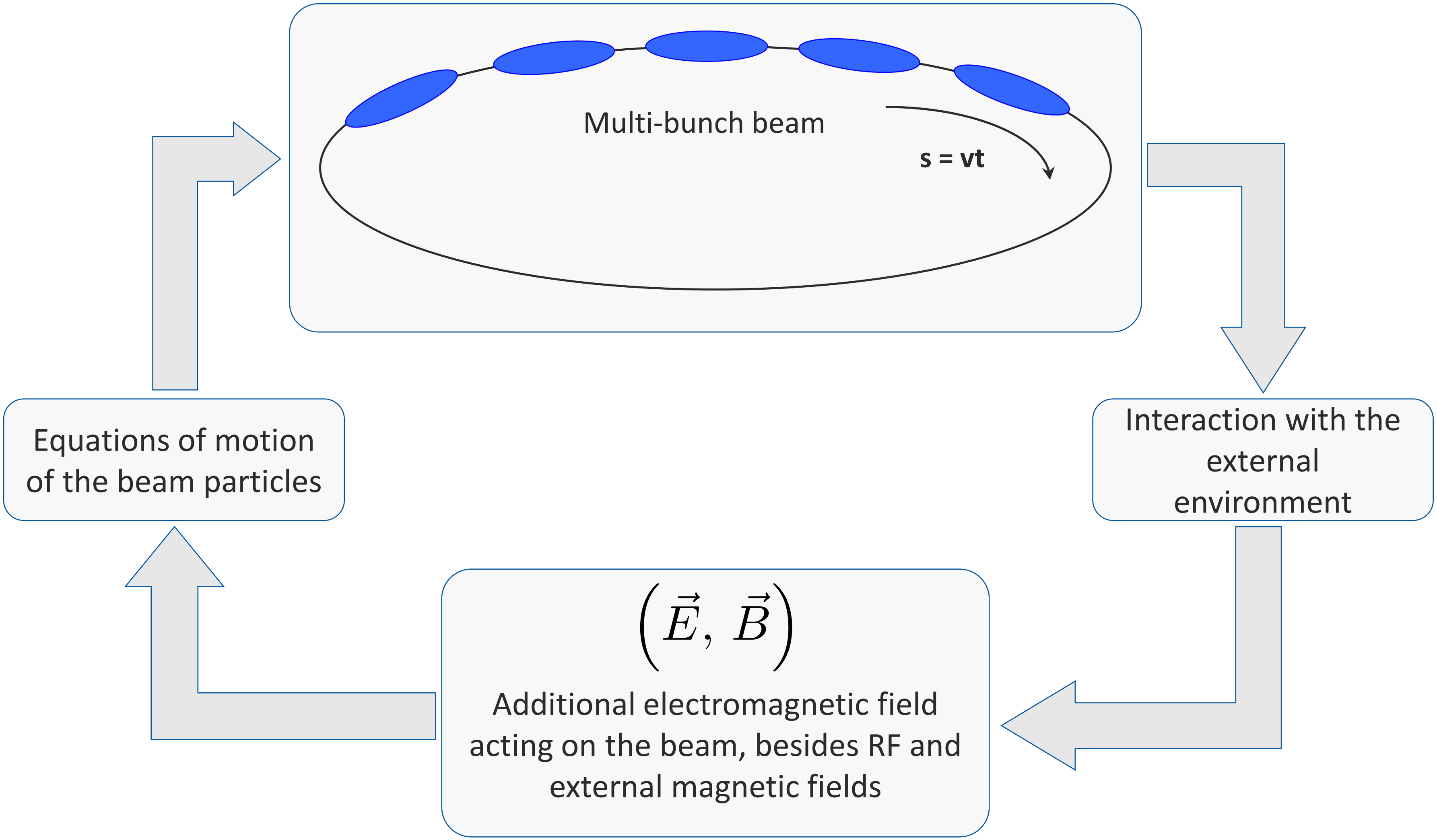
$$N = \int \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

$$\langle x \rangle = \frac{1}{N} \int x \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

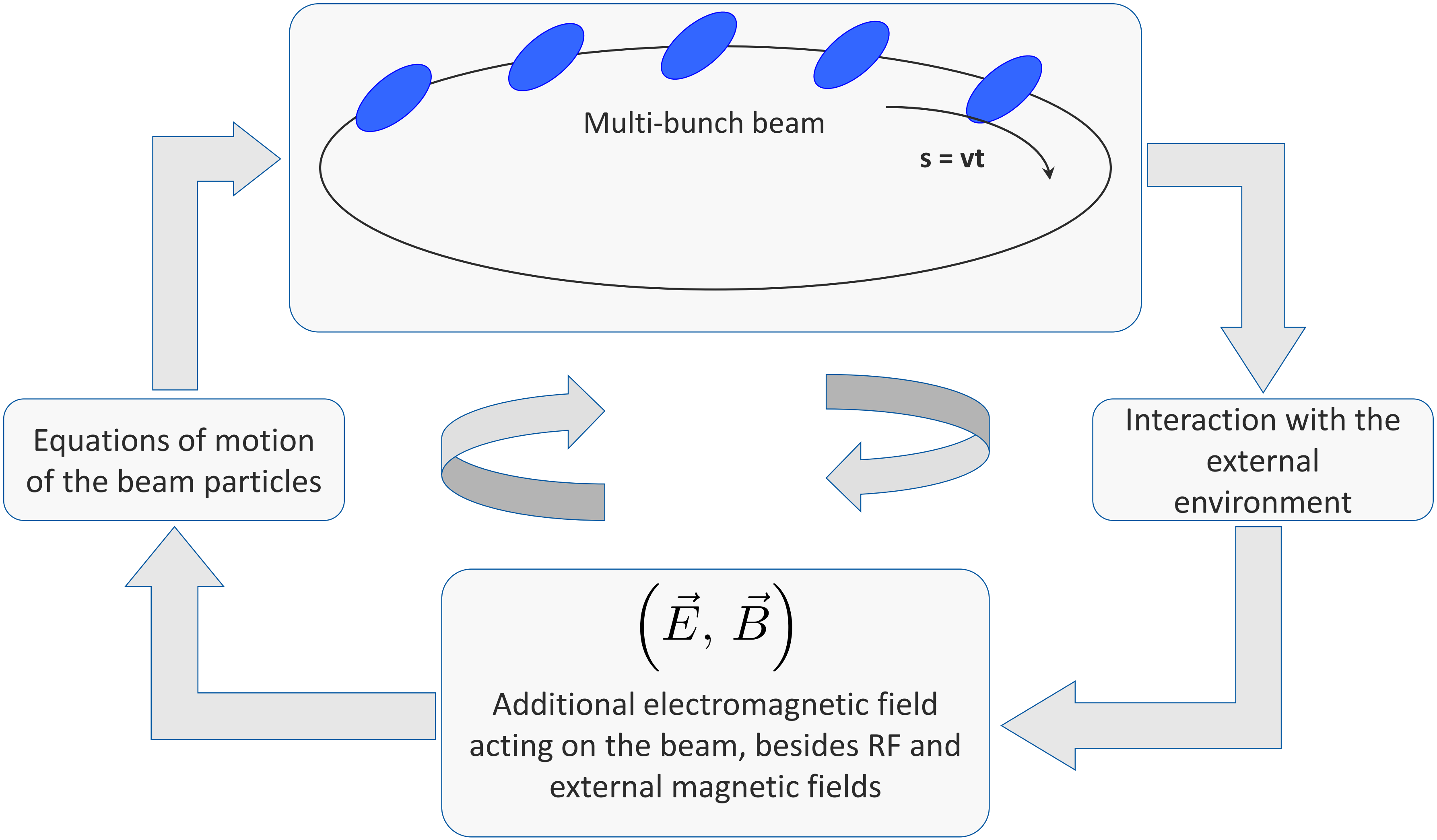
$$\sigma_x^2 = \frac{1}{N} \int \left(x - \langle x \rangle\right)^2 \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

and similar definitions for $\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z$

The instability loop

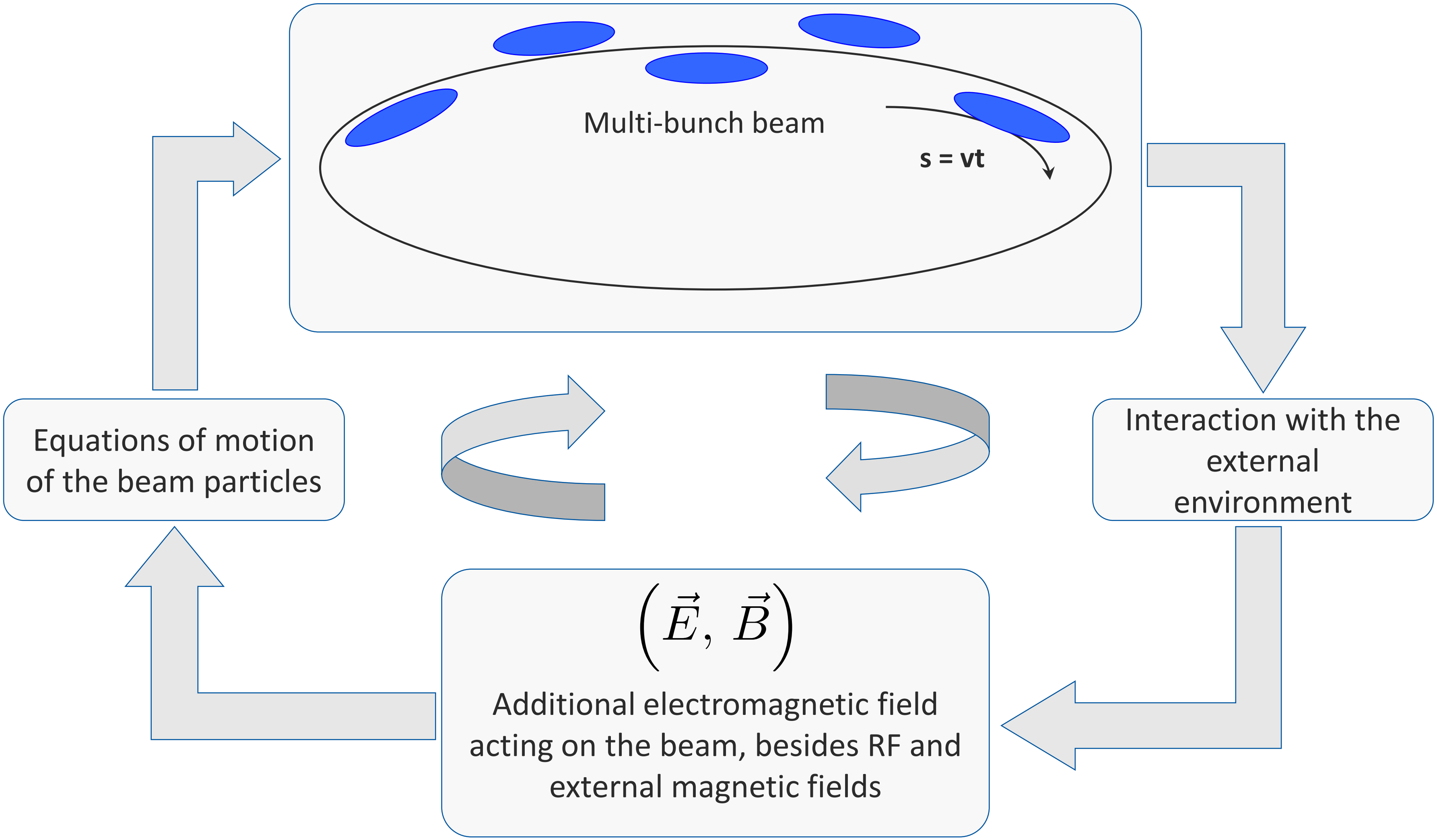


The instability loop



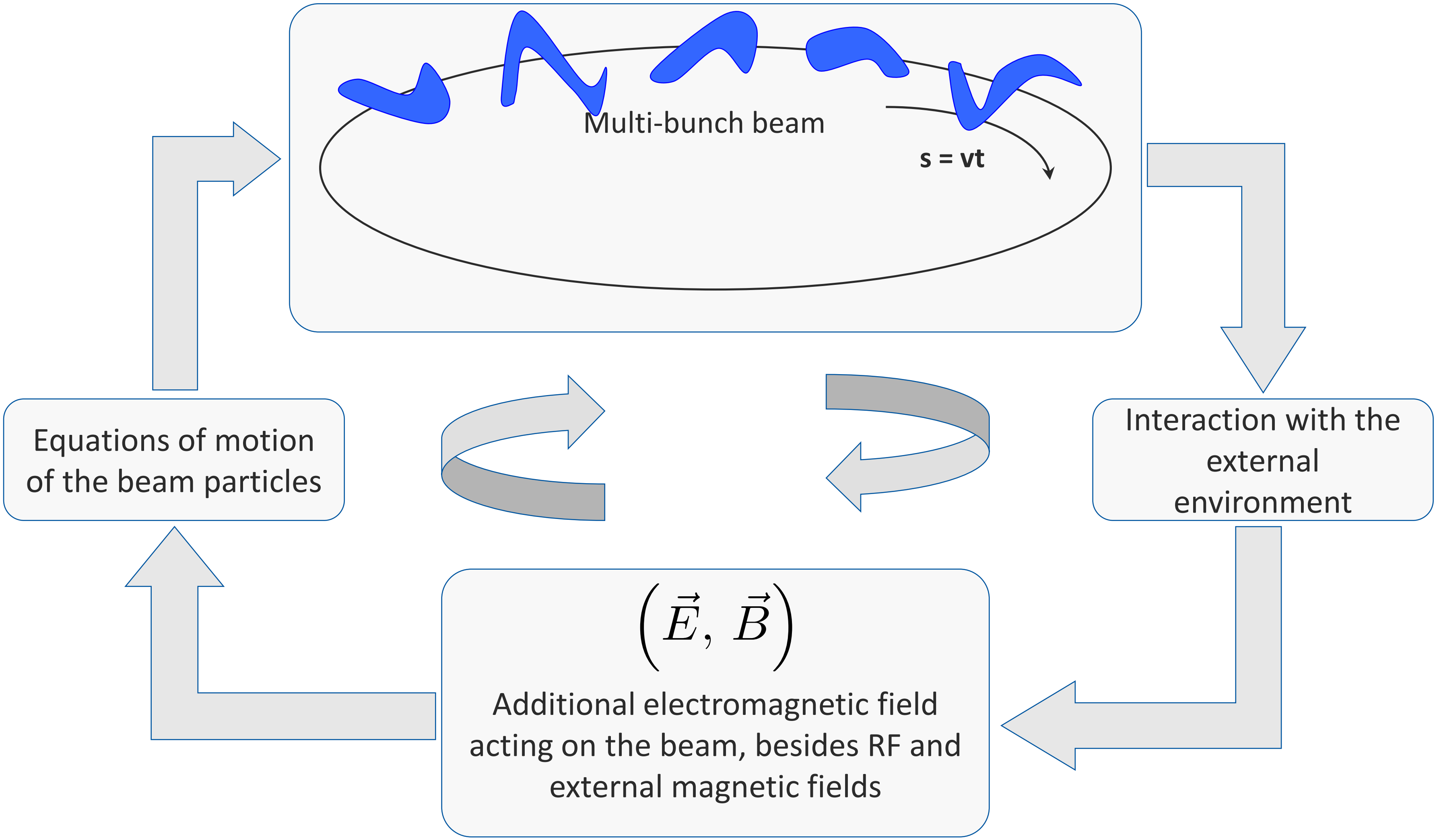
When the loop closes, either the beam will find a new stable equilibrium configuration …

The instability loop



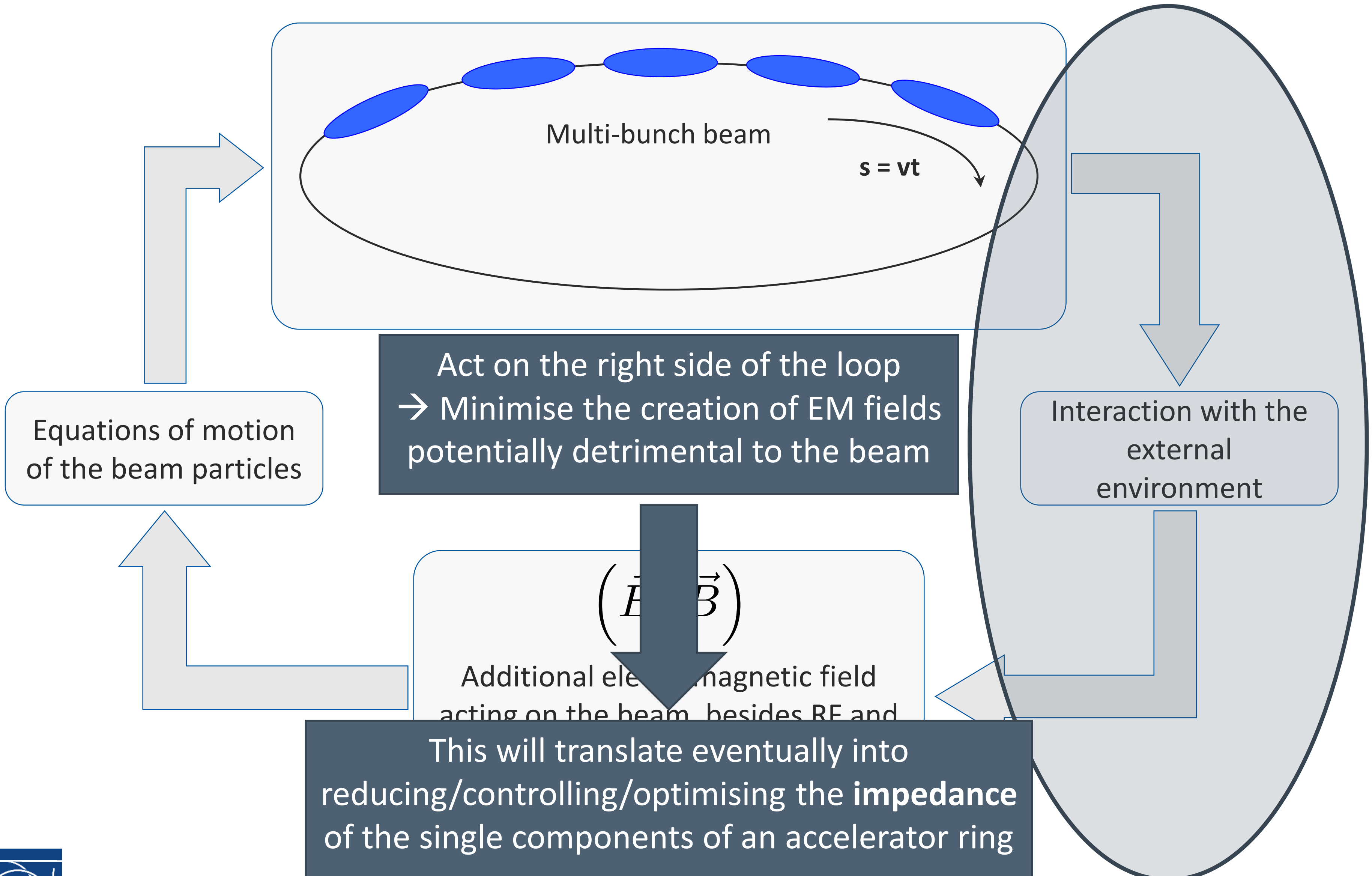
… or it might develop an instability along the bunch train …

The instability loop



… or also an instability affecting different bunches independently of each other

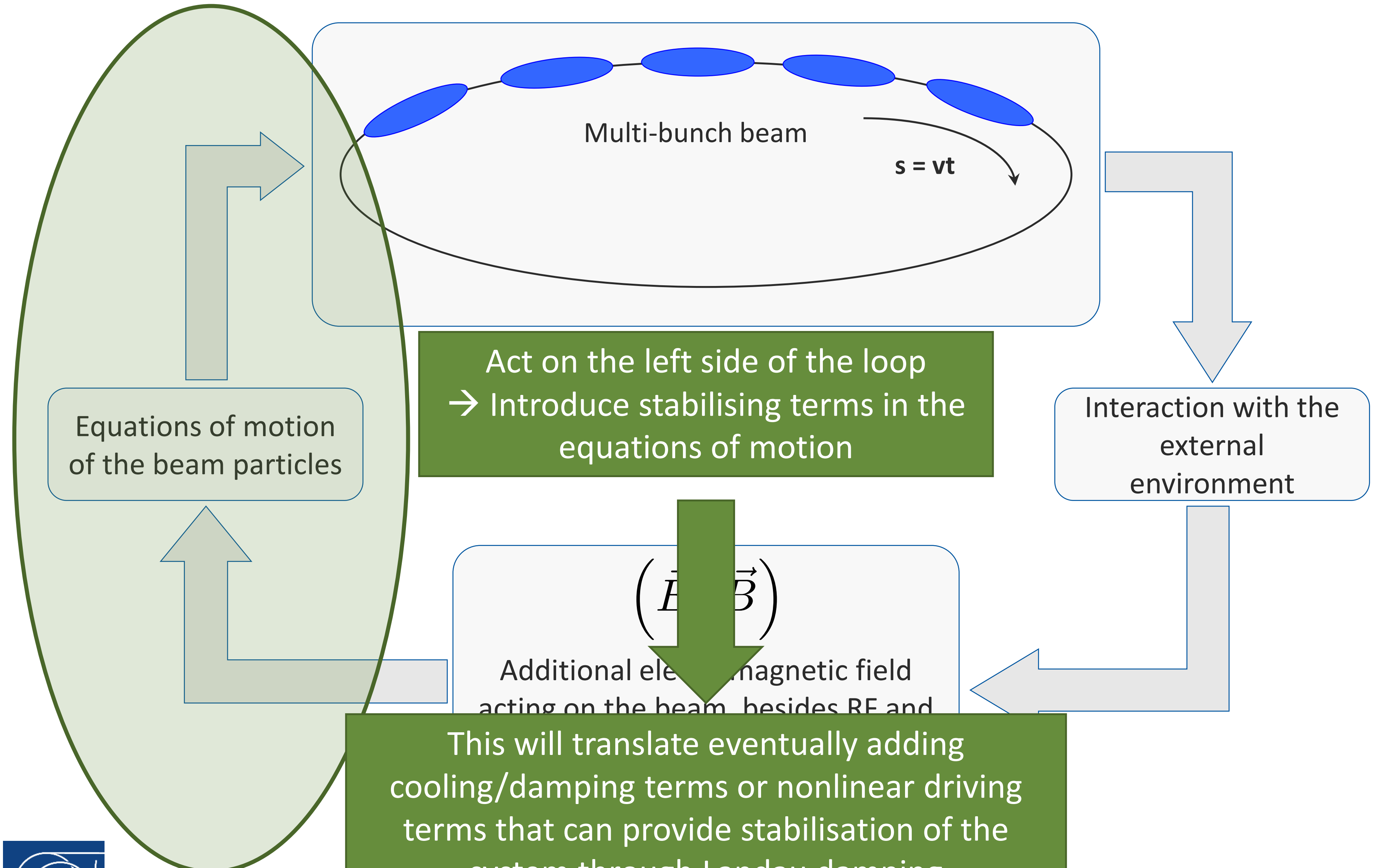
The instability loop



Act on the right side of the loop
→ Minimise the creation of EM fields potentially detrimental to the beam

This will translate eventually into reducing/controlling/optimising the **impedance** of the single components of an accelerator ring

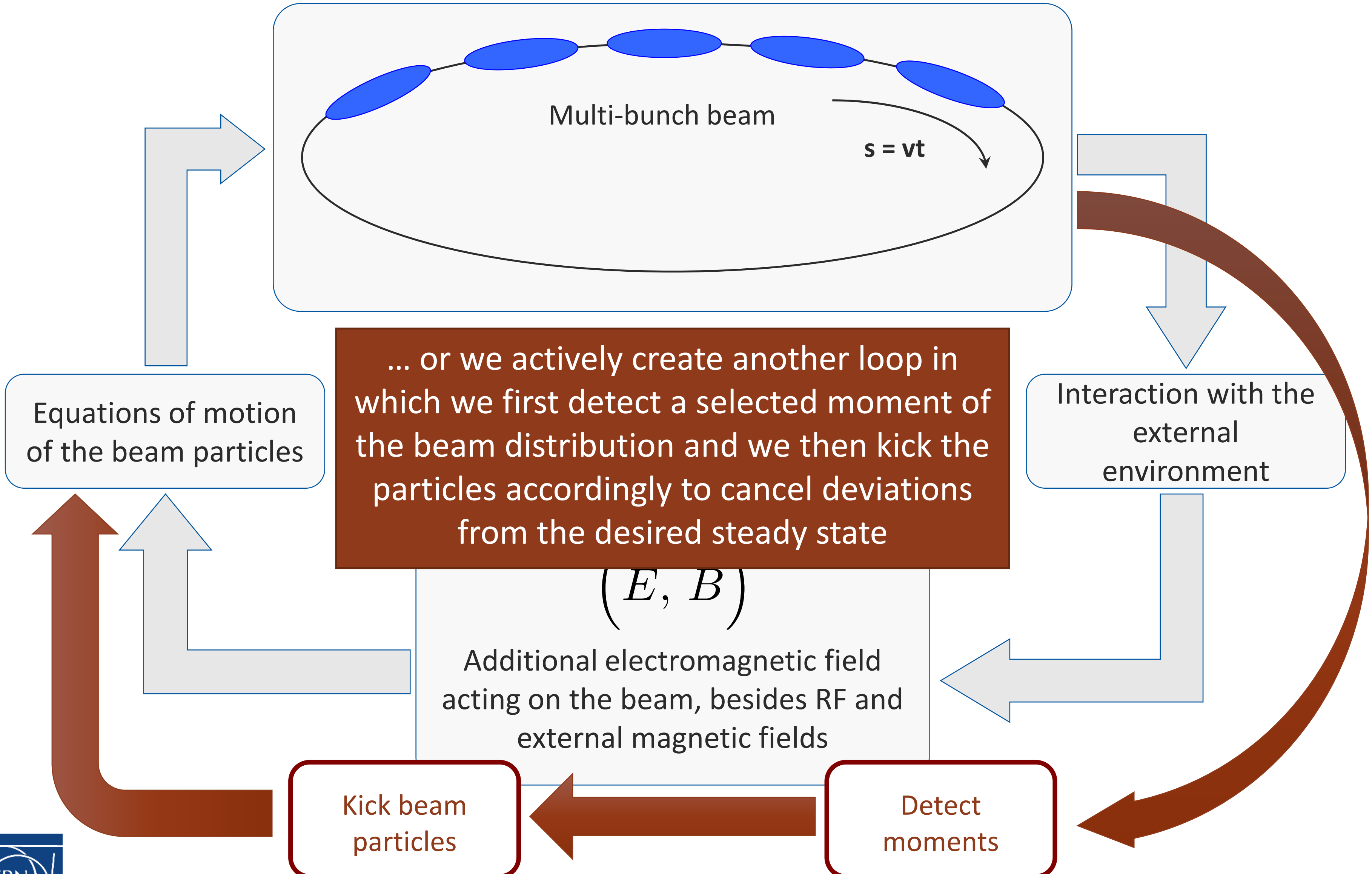
The instability loop



Act on the left side of the loop
→ Introduce stabilising terms in the equations of motion

This will translate eventually adding cooling/damping terms or nonlinear driving terms that can provide stabilisation of the system through Landau damping

The instability loop



Formal description of collective beam motion

- Formally, instead of investigating the full set of equations for a multiparticle system, we typically instead describe the latter by a **particle distribution function**:

$$\psi = \psi(x, x', y, y', z, \delta, s)$$

where

$$dN(s) = \psi(x, x', y, y', z, \delta, s)\, dx dx' dy dy' dz d\delta$$

- The accelerator environment together with the multiparticle system forms a **Hamiltonian system** for which the **Hamilton equations of motion** hold:

$$\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \qquad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x}$$

Formal description of collective beam motion

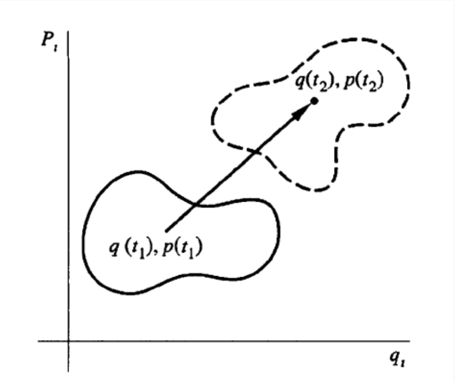
We can now derive the **Vlasov equation** which forms the **foundation of the theoretical treatment** of beam dynamics with collective effects:

- Consider an infinitesimal volume element dΩ containing a finite number of particles dN in phase space which evolve in time
  - dN is conserved as no particles can enter or leave the area (Picard-Lindelöf)
  - dΩ is conserved by means of the Hamilton equations of motion

- It follows that:

$$\frac{d}{ds}\psi = \frac{\partial \psi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial \psi}{\partial x'}\frac{\partial x'}{\partial s} + \frac{\partial}{\partial s}\psi$$

$$= \underbrace{\frac{\partial \psi}{\partial x}\frac{\partial H}{\partial x'} - \frac{\partial \psi}{\partial x'}\frac{\partial H}{\partial x}}_{[\psi, H]:\ \text{Poisson bracket}} + \frac{\partial}{\partial s}\psi = 0$$



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Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

- With the Hamiltonian composed of **an external** and **a collective part**, and the particle distribution function decomposed into **an unperturbed part** and **a small perturbation** one can write

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H_0} + \boldsymbol{H_1}, \psi_0 + \psi_1]$$

- This becomes to **first order**

$$\boxed{\frac{\partial}{\partial s}\psi_1 =} \underbrace{[\boldsymbol{H_0}, \psi_1] + [\boldsymbol{H_1}(\psi_0 + \psi_1), \psi_0]}$$

$$\text{Linearization in } \psi_1: \quad \ldots \propto \boxed{\hat{\boldsymbol{\Lambda}}\psi_1}$$

Spatial component Temporal component

$$\Longrightarrow \psi_1 = \sum_k \boxed{a_k\, \boldsymbol{v_k}}\, \boxed{\exp\left(\frac{i\Omega_k}{\beta c}s\right)}$$

We are looking for the EV of the evolution
→ **becomes an EV problem**!

Formal description of collective beam motion

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Linearization in ψ ... $\propto \hat{\Lambda}\psi_1$

We call these distinct eigenvalues ψ_k **the coherent k-mode**.

The mode and thus for example also an instability is fully characterized by a single number:

the complex tune shift Ω_k

Spatial component **Temporal component**

$$\Longrightarrow \psi_1 = \sum_k a_k \boldsymbol{v_k} \exp\left(\frac{i\Omega_k}{\beta c}s\right)$$

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Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

Remark:

- The stationary distribution ψ_0 is the distribution where

$$\frac{\partial}{\partial s}\psi_0 = [\boldsymbol{H}_0, \psi_0] = 0$$

- In particular, a distribution is always stationary if

$$\psi_0 = \psi_0(\boldsymbol{H}_0)\,, \quad \text{as} \quad [\boldsymbol{H}_0, \psi_0(\boldsymbol{H}_0)] = 0$$

Solving for or finding the stationary solution for a given H_0 (which in fact represents the machine 'potential') will be later referred to as **matching**.

Signpost

- We have seen the difference between **external forces** and **self-induced forces** and how these lead to **collective effects**.
- We have seen schematically how these collective effects can induce **coherent beam instabilities and some knobs to avoid them**.
- We have briefly sketched the **theoretical framework** within which the beam dynamics of collective effects is usually treated – we have encountered the Vlasov equation, bunch / beam eigenmodes and the complex tune shift.

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    - Multiparticle effects – filamentation and decoherence

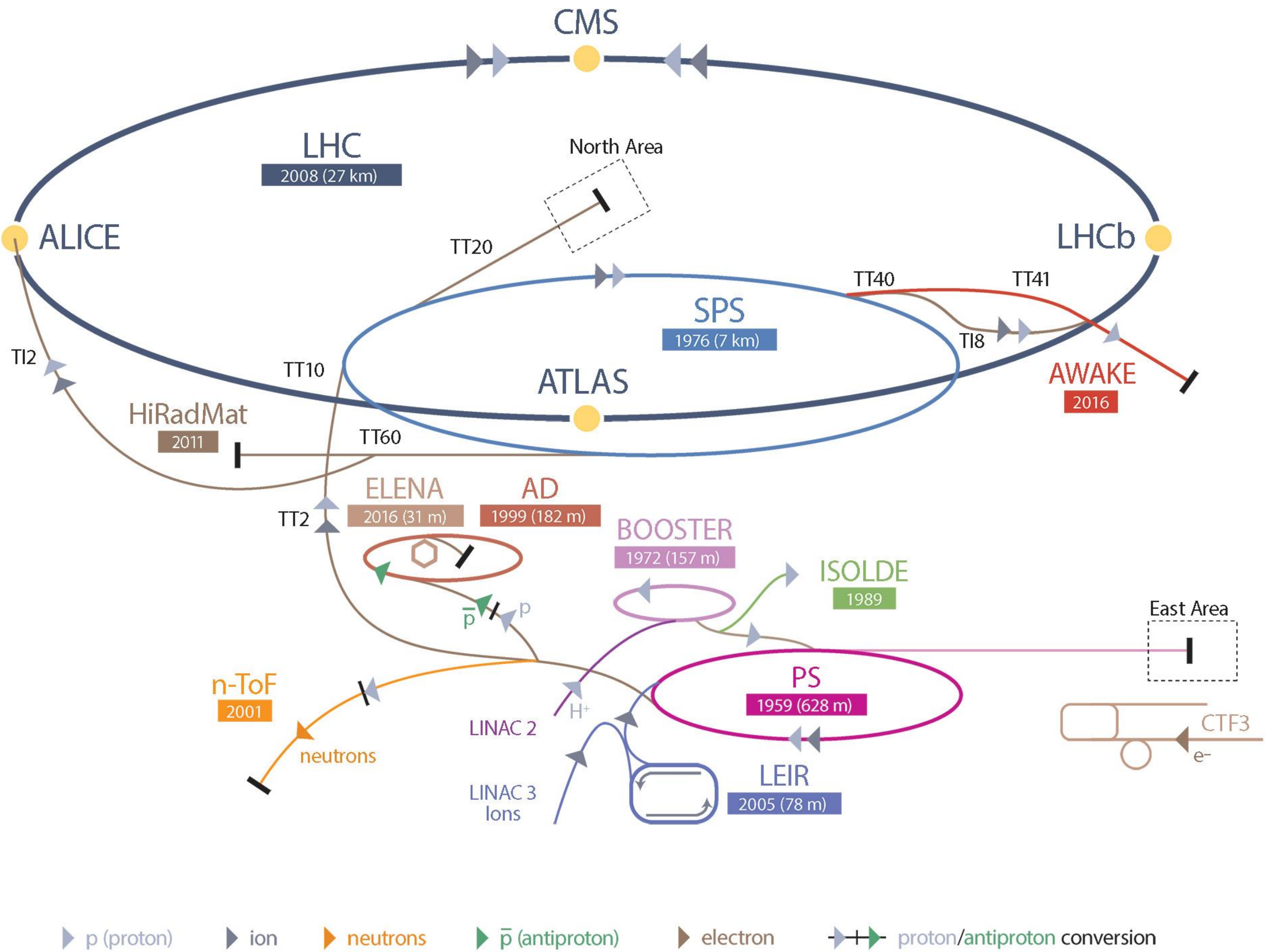
Why worry about beam instabilities?

- Why study beam instabilities?

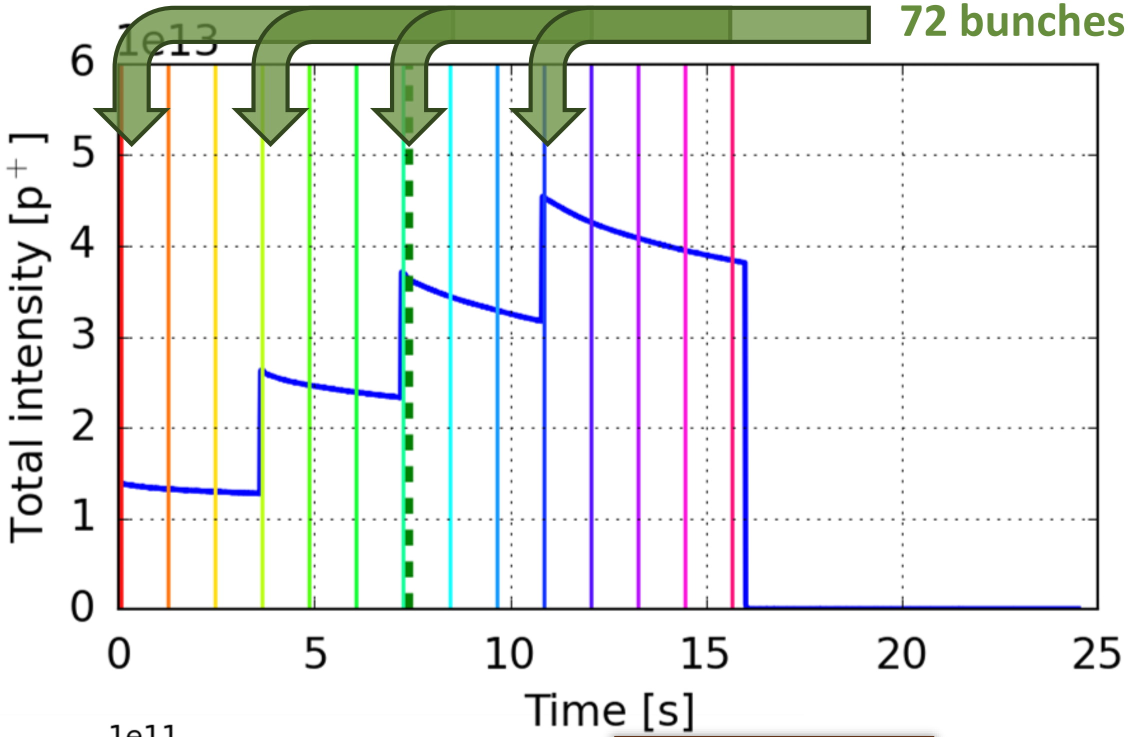
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)

  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

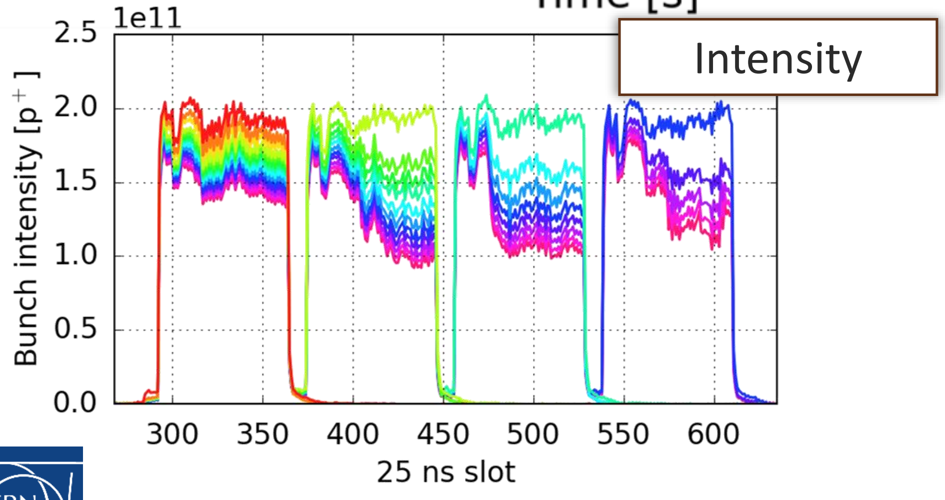
The CERN accelerator complex

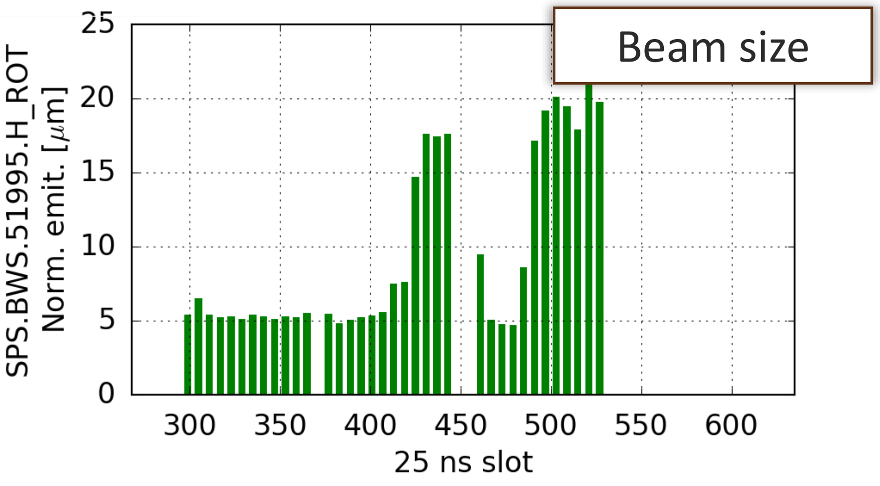


Coupled bunch instability in the SPS



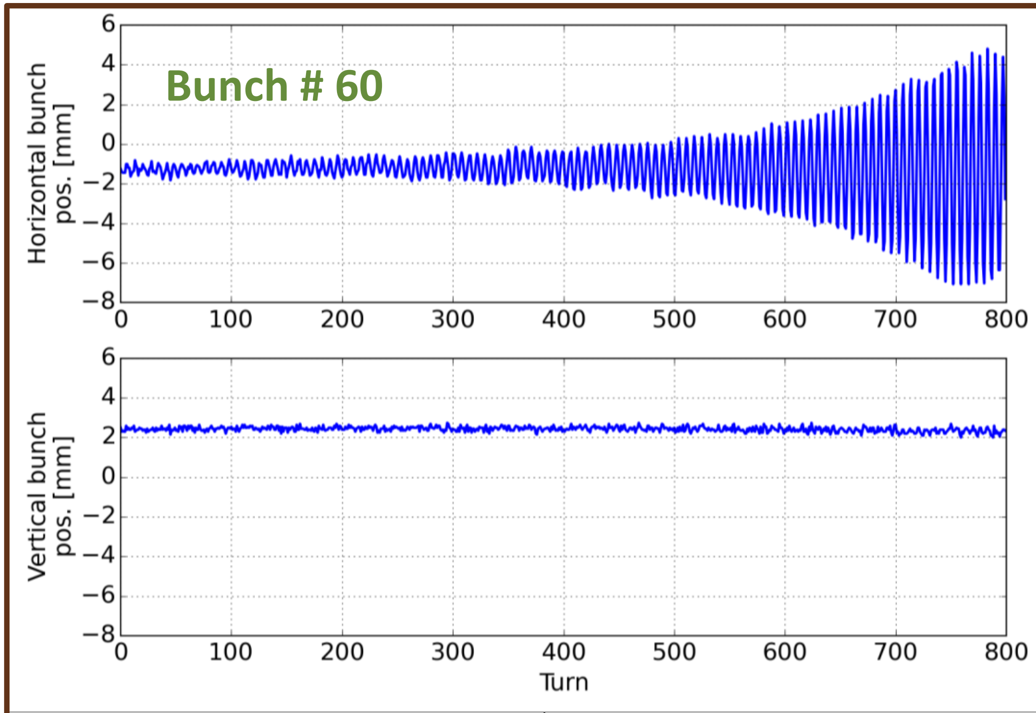
- Injection of 4 batches of 72 bunches trains into the SPS
- Later trains feature **strong losses (intensity)** and **large blow-up (emittance)** – this leads to a **strong loss of beam brightness**

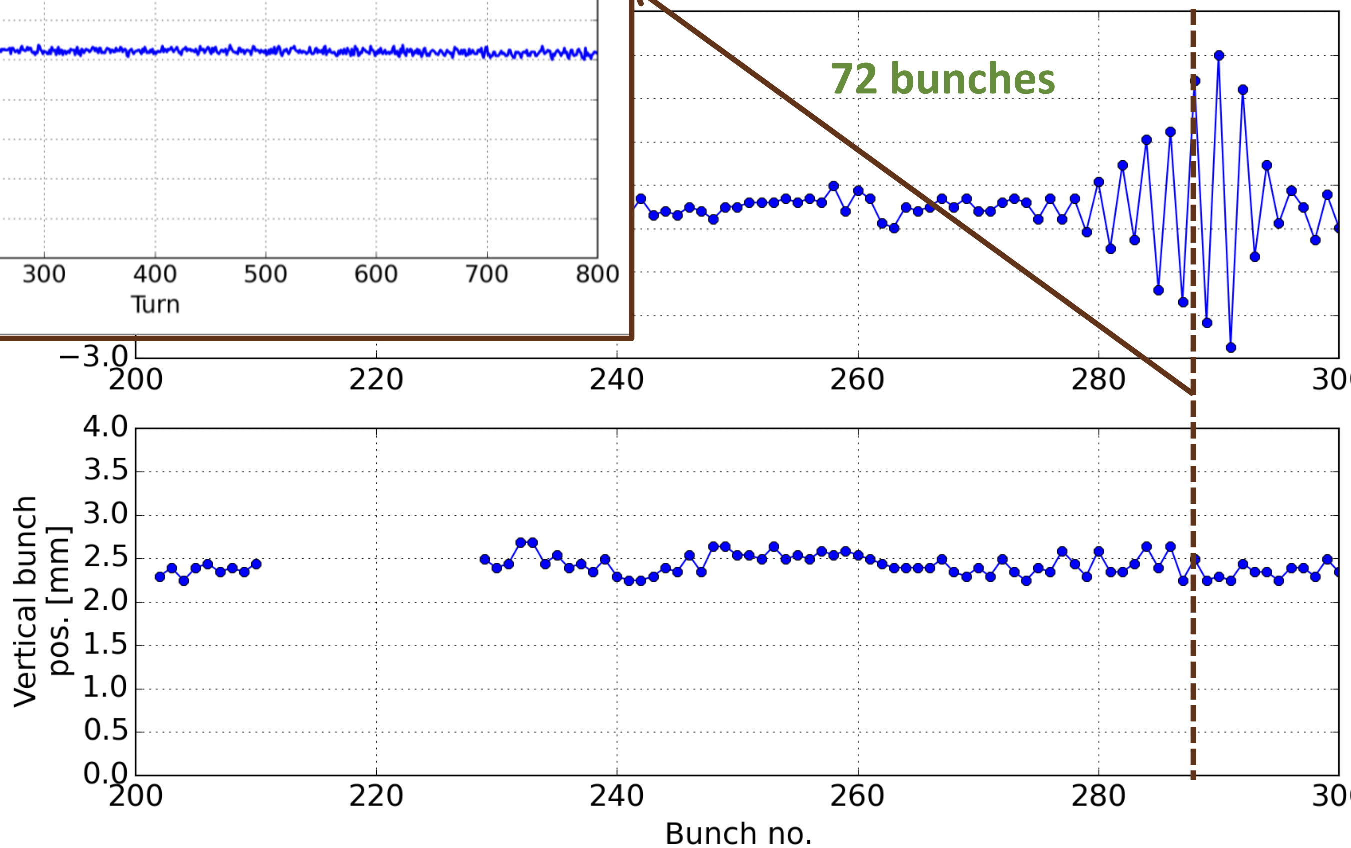




Coupled bunch instability in the SPS

- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits **strong coherent coupled bunch oscillations**

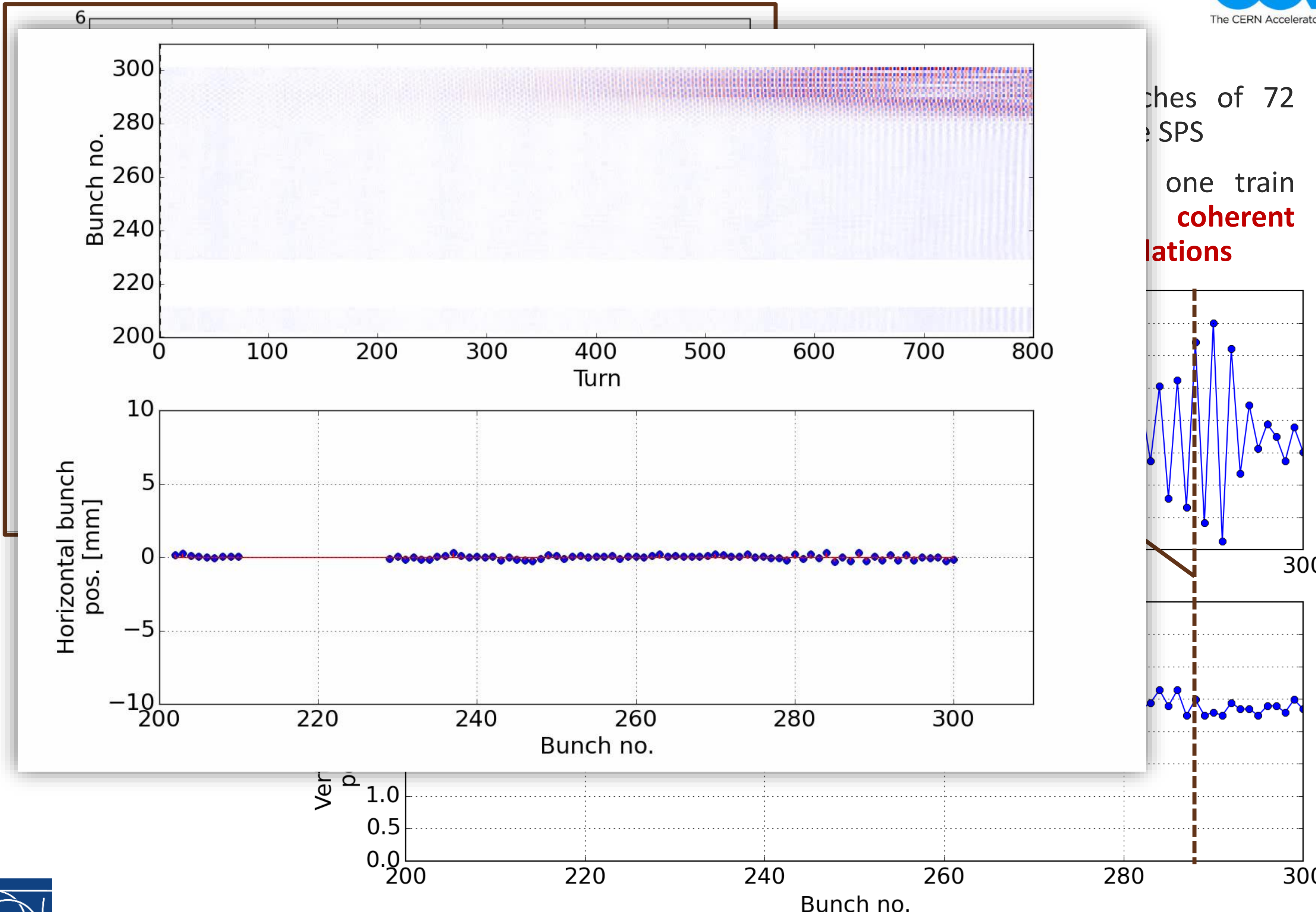




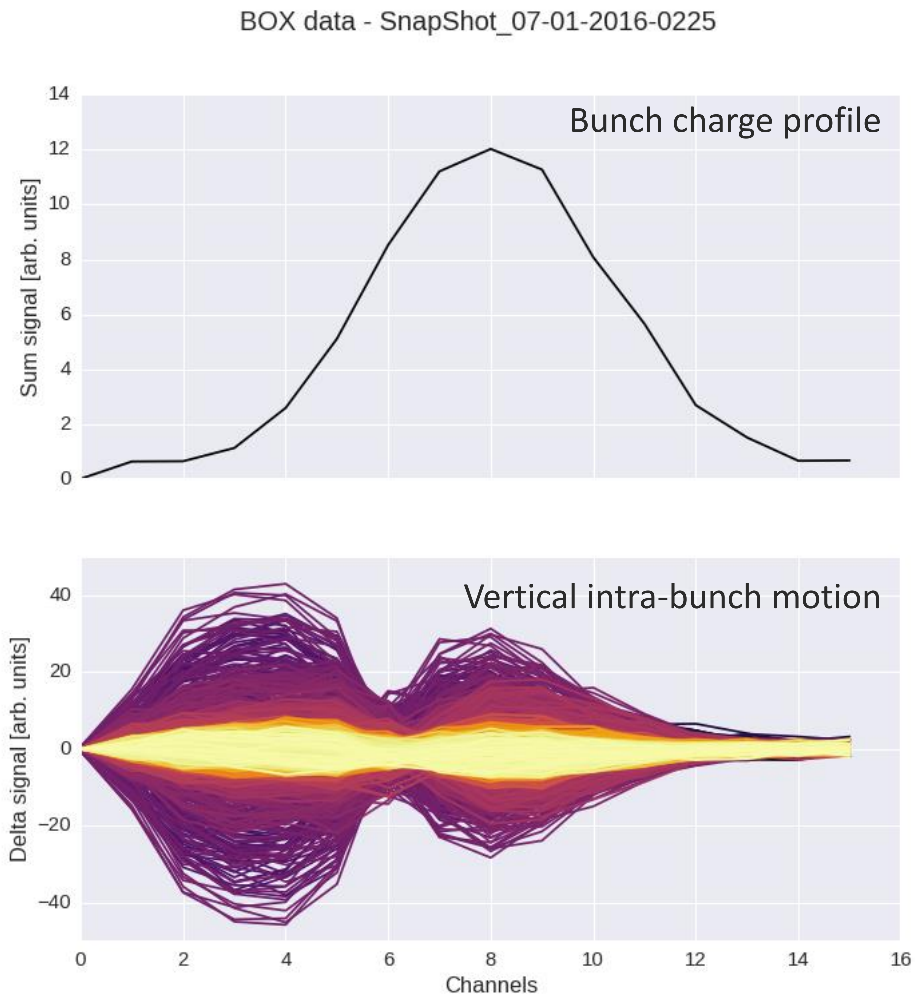
Coupled bunch instability in the SPS

ches of 72
SPS

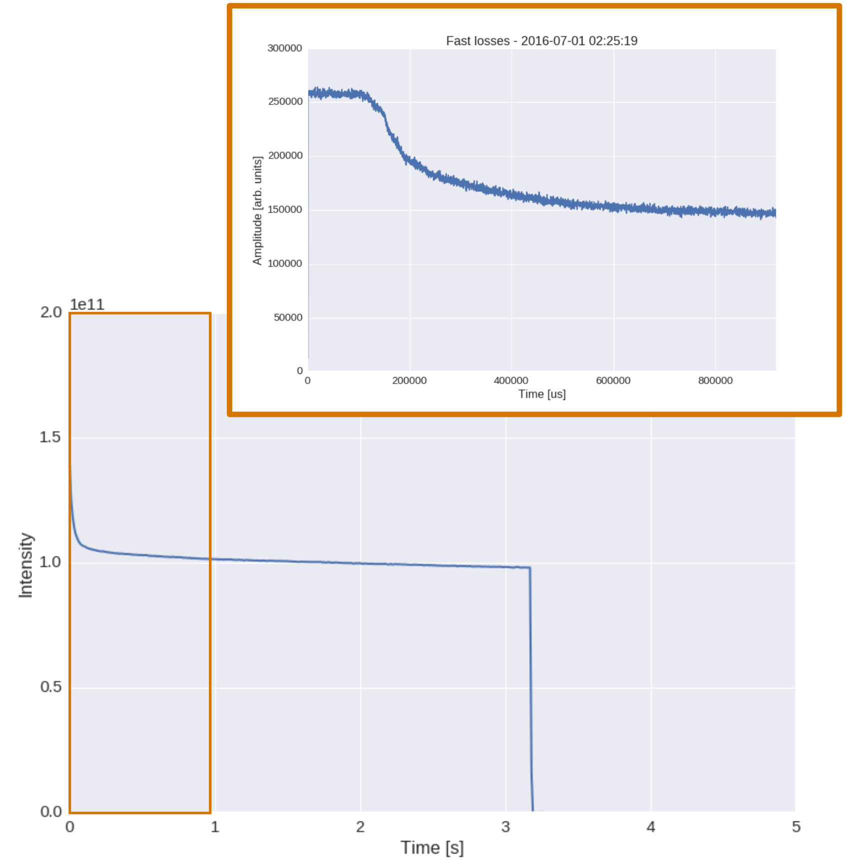
one train
coherent
lations



Single bunch instability in the SPS



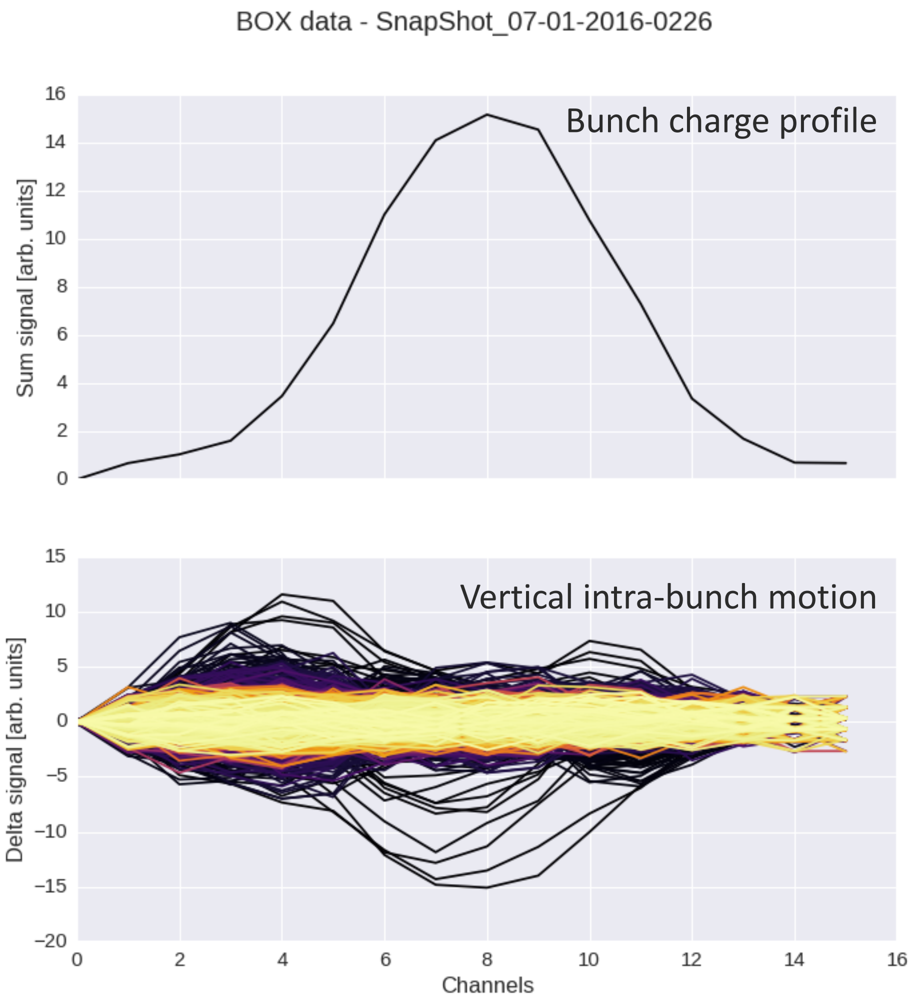
Open loop

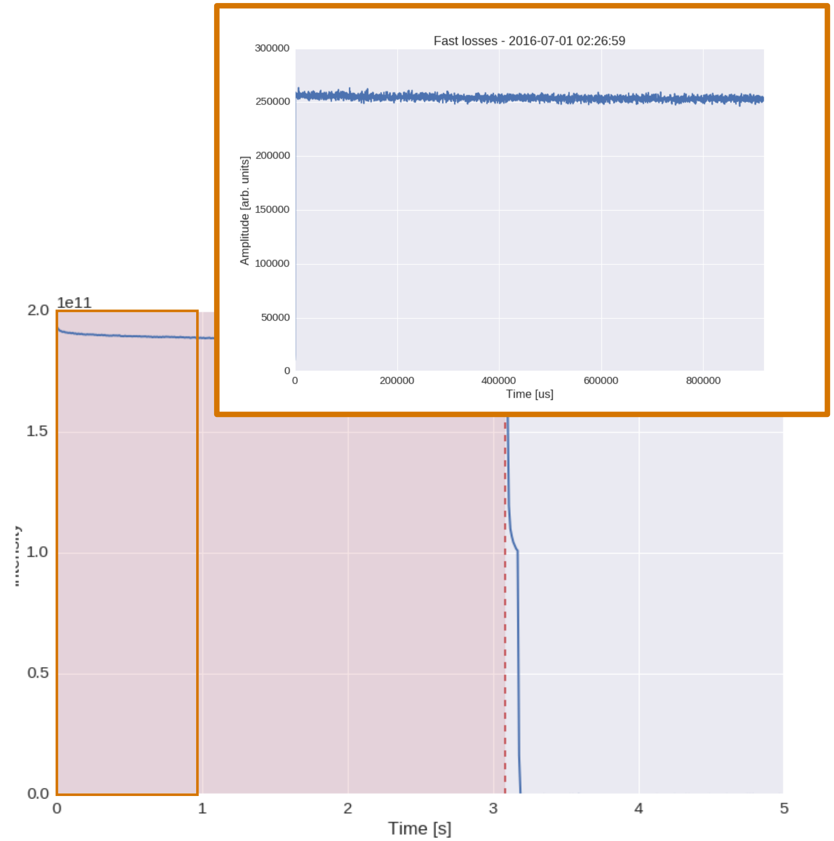


- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)**.

Single bunch instability in the SPS

Closed loop





- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)** → can be mitigated by a **wideband feedback system**.

Signpost

- We now understand that collective effects can have a **huge detrimental impact** on the machine performance and why, therefore, the study and the understanding of instabilities is important.
- We have encountered some **real world examples** of instabilities observed throughout the CERN accelerator chain.
- Before moving on to a more detail view of collective effects, we will have a quick look at some **distinct characteristics of multi-particle beam dynamics**.

- Part 1: Introduction – dynamics of multiparticle systems
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    - Multiparticle effects – filamentation and decoherence

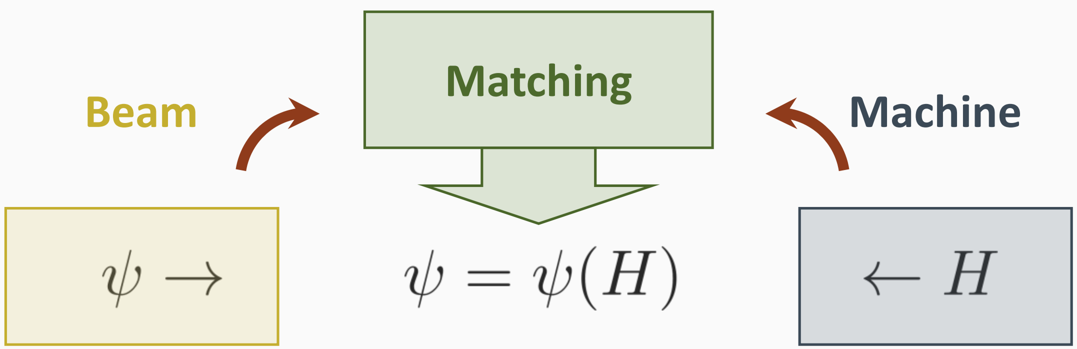
Beam matching

- As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian H) the stationary solution is given by:

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi] = 0$$

and can be constructed via matching:

- In real life, an injected beam ought to be **matched to the machine** for best performance.
- Given a **particle distribution function** and a **machine optics** locally described by a Hamiltonian we ensure matching by targeting for:



Matching examples

The CERN Accelerator School

We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

- Betatron motion

$$H = \frac{1}{2}x'^2 + \left(\frac{Q_x}{R}\right)^2 x^2$$

$$H_0 = \sigma_{x'}^2 = \left(\frac{Q_x}{R}\right)^2 \sigma_x^2 \implies \boxed{\frac{\sigma_x}{\sigma_{x'}} = \frac{R}{Q_x} = \beta_x}$$

- Synchrotron motion - linear

$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eVh}{4\pi R^2 p_0}\,z^2$$

$$H_0 = \eta\beta c\,\sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0}\sigma_z^2 \implies \boxed{\frac{\sigma_z}{\sigma_\delta} = R\eta\sqrt{\frac{2\pi\beta^2 E_0}{eV\eta h}} = \frac{R\eta}{Q_s}\sigma_\delta = \beta_z}$$

Matching examples

The CERN Accelerator School

We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

- Betatron motion

In reality the synchrotron motion is described by the Hamiltonian:

$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eV}{2\pi h p_0}\left(\cos\left(\frac{hz}{R}\right) - \cos\left(\frac{hz_c}{R}\right) + \frac{\Delta E}{eV}\left(\frac{hz}{R} - \frac{hz_c}{R}\right)\right)$$

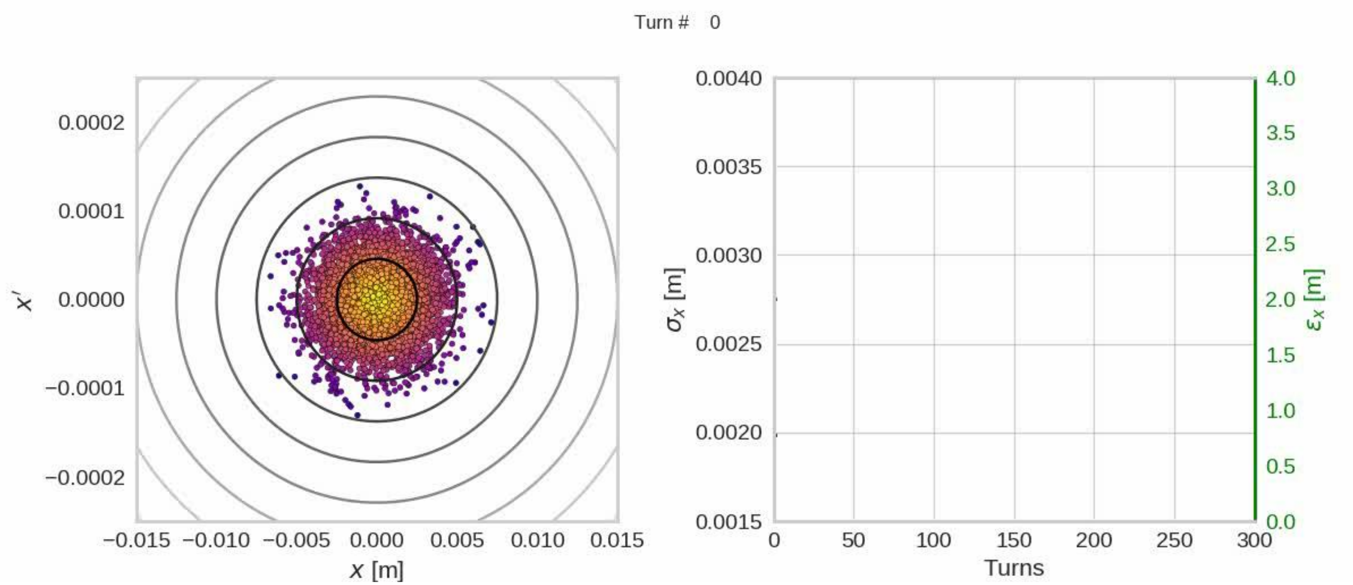
- Synchrotron motion - linear

This leads to **nonlinear equations** and the matching procedure becomes more involved.

Matching illustration – matched beams

- Betatron motion
  – **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

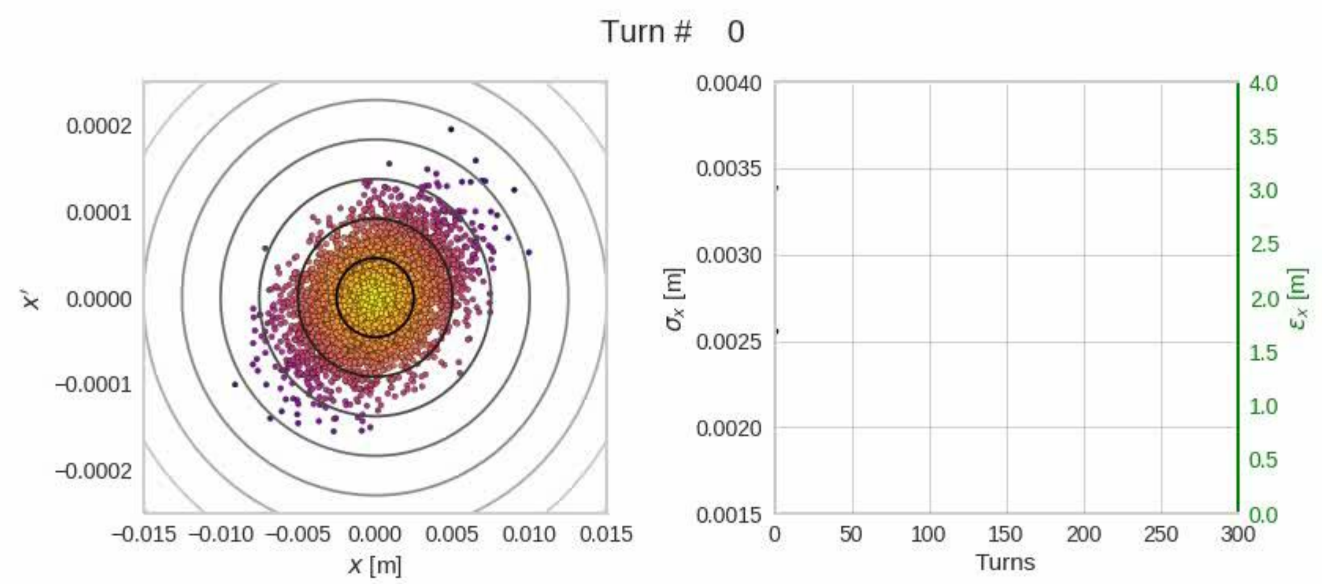


Matched beams **maintain their beam moments** and their shape in phase space

Matching illustration – mismatched beams

- Betatron motion
  - **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$



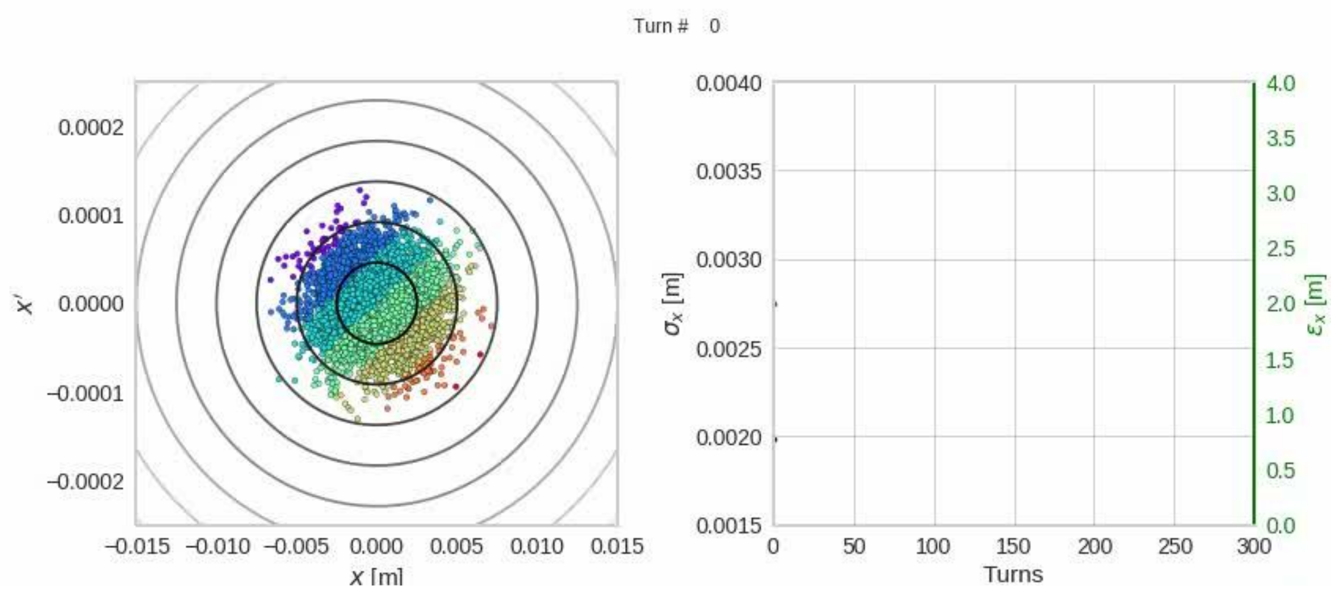
Mismatched beams show **oscillations in their beam moments** and may **change their shape due to filamentation**

Matching illustration – linear vs. nonlinear

- Betatron motion
  - **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Nonlinearities lead to **detuning with amplitude**. This is visible as the **characteristic spiraling** of larger amplitude particles.



Signpost

- We have learned about the **meaning of matching** a beam to the machine optics.
- We have seen how to **formally match a beam** to a given description of a machine.
- We have seen **examples of matched and mismatched beams** and have seen the difference between **linear and non-linear motion**.

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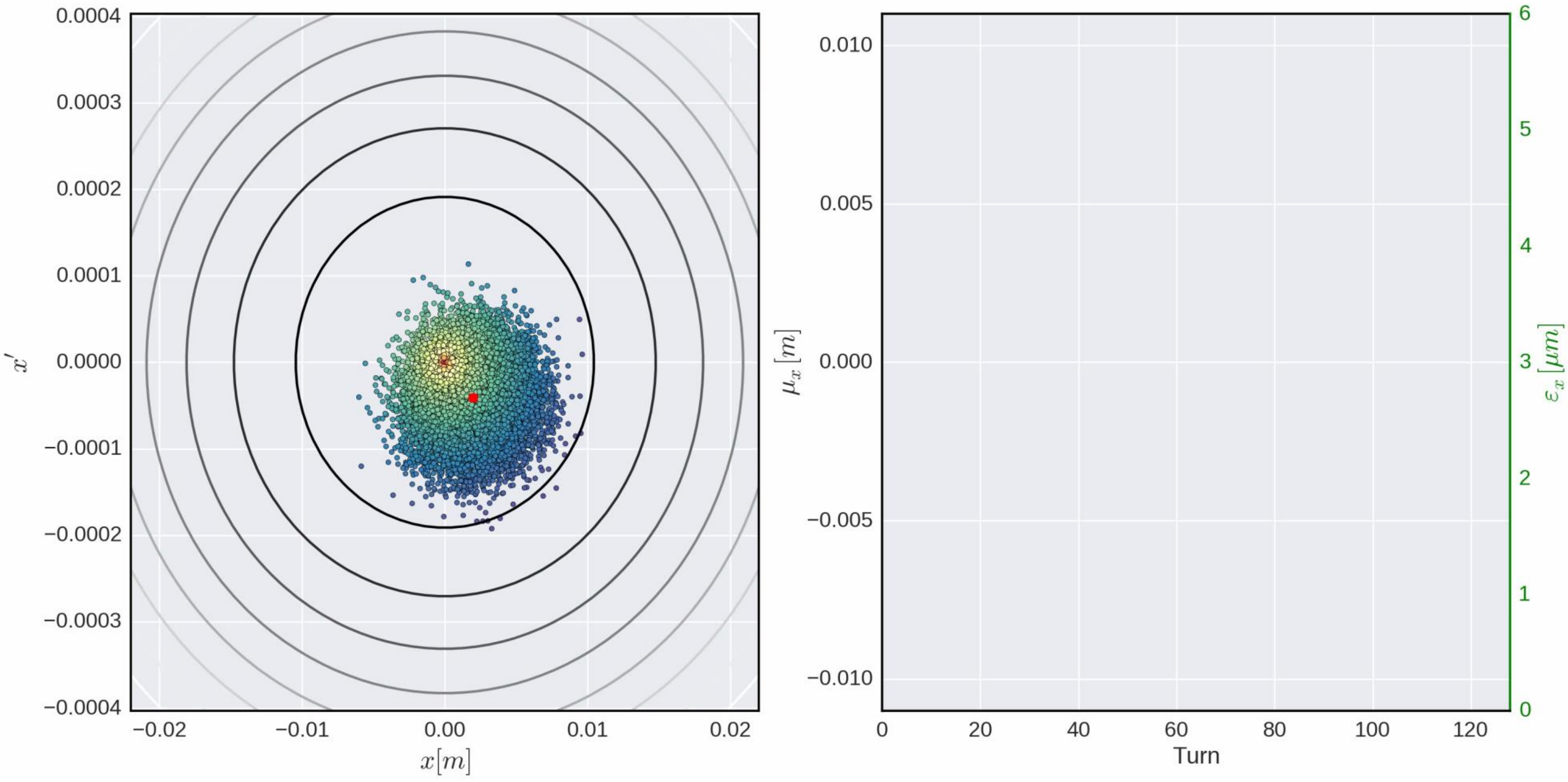
Sources and impact of transverse nonlinearities

- We have learned or we may know from operational experience that there are a set of **crucial machine parameters to influence beam stability** – among them **chromaticity and amplitude detuning**

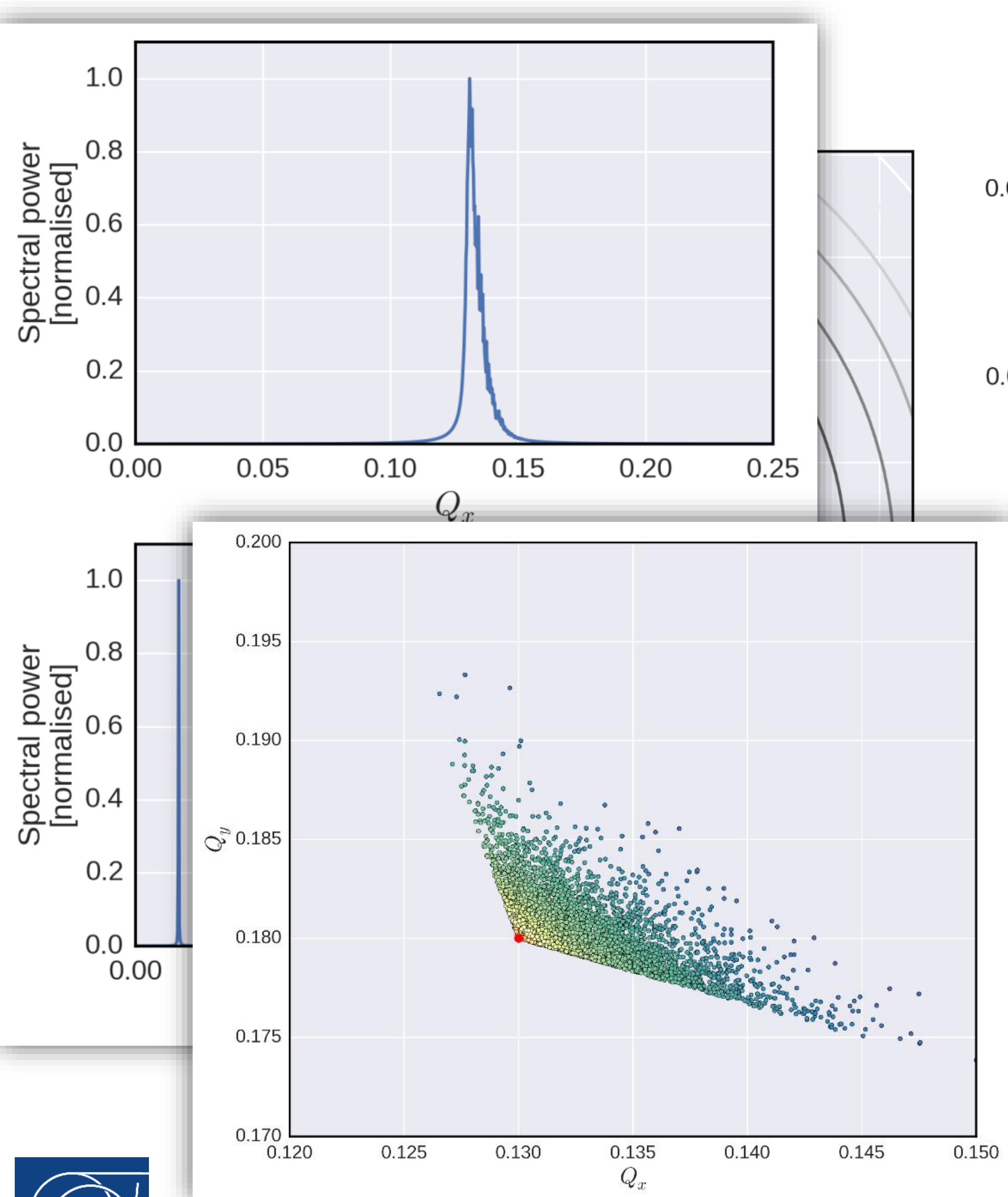
- Chromaticity
  - Controlled with sextupoles – provides **chromatic shift** of bunch spectrum wrt. impedance
  - Changes interaction of beam with impedance
  - Damping or excitation of **headtail modes**

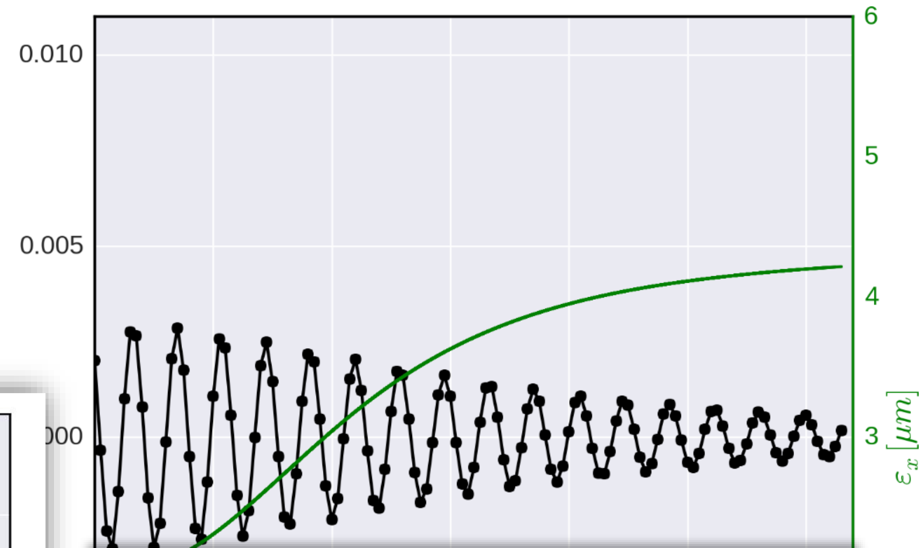
- Amplitude detuning
  - Controlled with octupoles – provides (incoherent) **tune spread**
  - Leads to absorption of coherent power into the incoherent spectrum → **Landau damping**

Example: filamentation as result of detuning



Example: filamentation as result of detuning





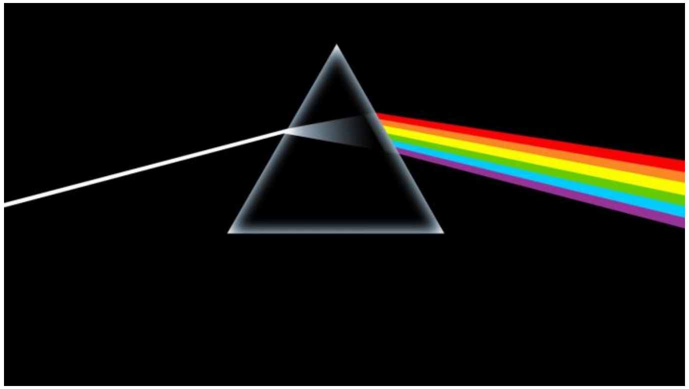
- Taking an **FFT of the centroid motion** (black curve) **reveals the tune** – interestingly there **is a spread**
- In the simulation we have access to the trajectory of **each individual particle** – we can equally perform an **FFT of every particle** and plot the horizontal vs. vertical tune to obtain the **tune footprint**

Example: chromaticity – de- & recoherence

- Chromatic detuning:

$$\Delta Q_x = Q'_x\,\delta$$

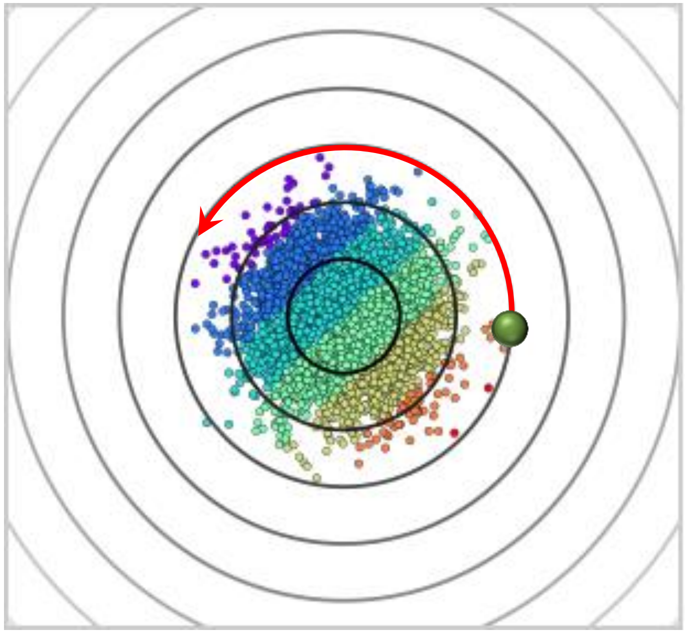
$$\delta = \hat{\delta}\,\sin(\varphi)$$



- Consider a particle in 6d phase space performing both betatron and synchrotron oscillations
- The accumulated betatron detuning after one half, resp. one full synchrotron period reads

$$\Delta Q_{x,\,\mathrm{acc}}\Big|_{T_s/2} = \hat{\delta}\int_0^{\pi}\sin(\varphi)\,d\varphi = 2\hat{\delta}$$

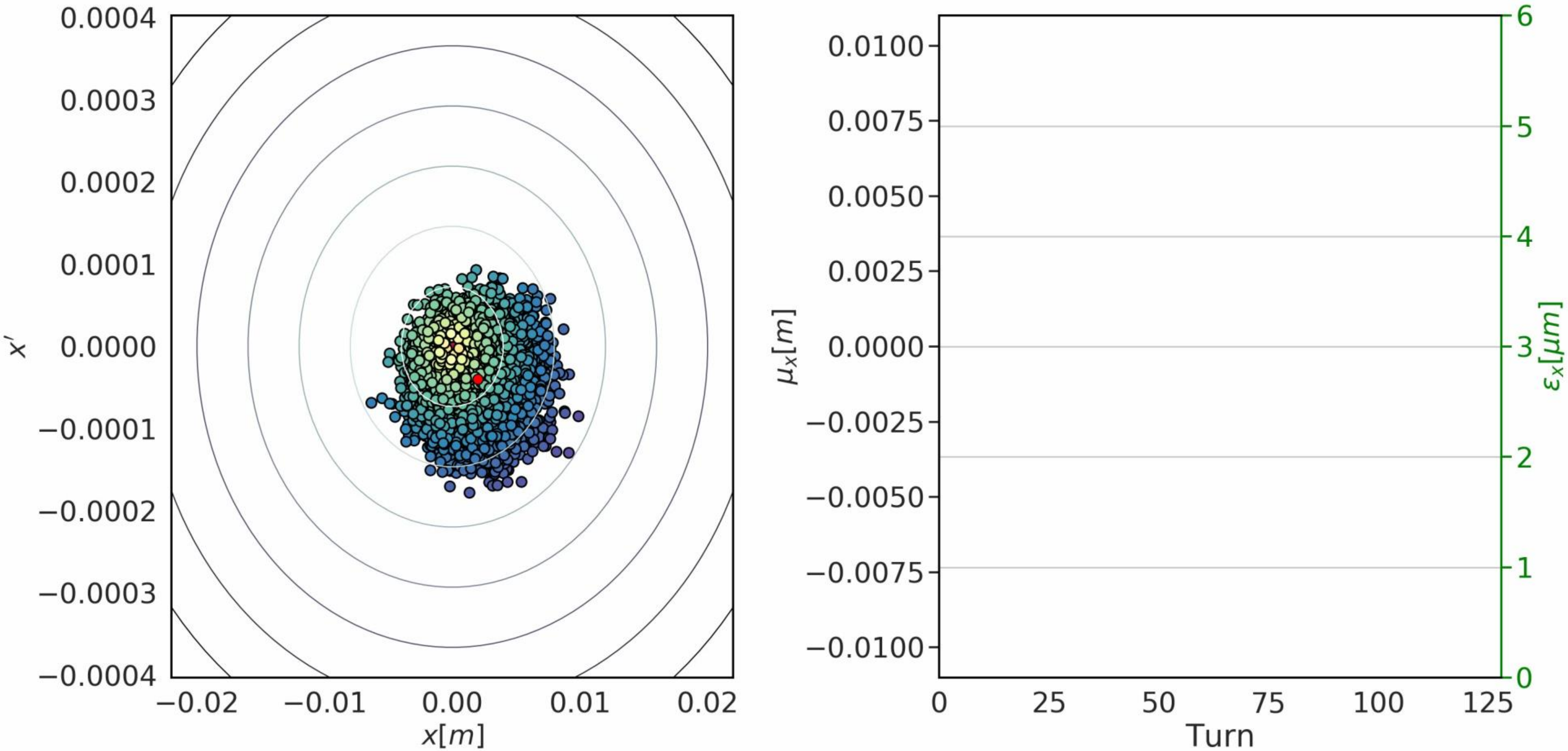
$$\Delta Q_{x,\,\mathrm{acc}}\Big|_{T_s} = \hat{\delta}\int_0^{2\pi}\sin(\varphi)\,d\varphi = 0$$



- After **one full synchrotron period** all tune shifts have vanished (i.e., also the tune spread has vanished – the **beam has re-cohered**)

Example: chromaticity – de- & recoherence

- Chromatic detuning:



spread has vanished – the **beam has re-cohered**)

Signpost

- Sources for transverse nonlinearities are, e.g., **chromaticity** and **detuning with amplitude** from octupoles.
- Transverse nonlinearities can lead to **decoherence** and **emittance blow-up**.
- The effects seen so far are **chacteristics for multiparticle systems** but are **not collective effects**.

- Part 1: Introduction – dynamics of multiparticle systems
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

Signpost

- We have learned about some of the peculiarities of **collective effects**. We have also introduced **multi-particle systems** and have seen how these can be described and treated theoretically.
- We have seen some **real-world example of collective effects** manifesting themselves as coherent beam instabilities.
- We have looked at some specific **features of multi-particle beam dynamics** such as matching, decoherence and emittance blow-up due to filamentation. These are not to be confused with collective effects.
- Part 1: Introduction – dynamics of multiparticle systems
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

End part 1

Backup

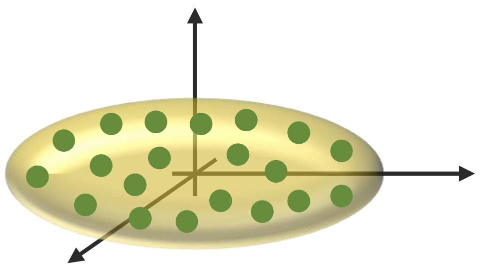
Signpost

- We have learned about the **particle description** of a beam.
- We have seen **macroparticles** and **macroparticle models**.
- We have seen how **macroparticle models** are **mapped and represented in a computational environment**.

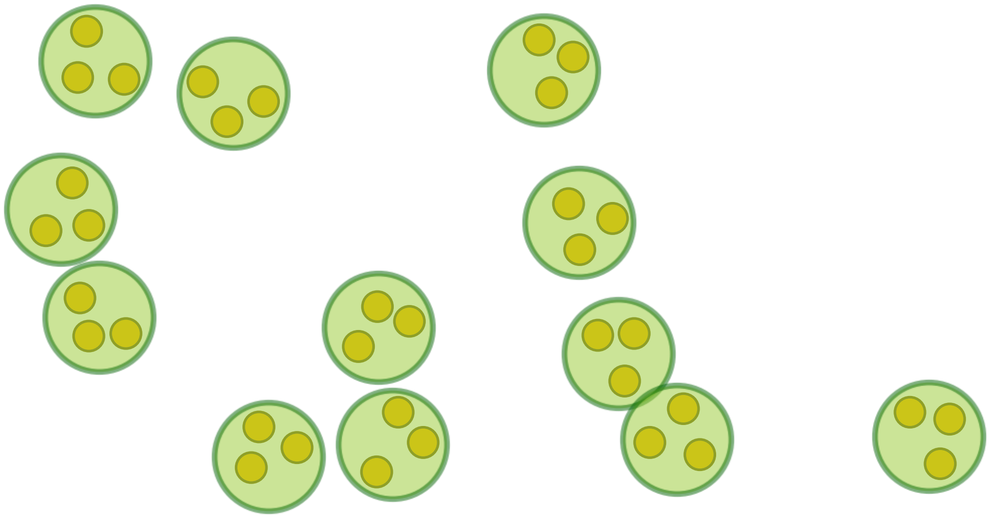
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - Basic concepts
    - Particles and macroparticles – macroparticle distributions
    - Beam matching
    - Multiparticle effects – filamentation and decoherence
    - Wakefields as sources of collective effects

The particle description

- As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a **particle distribution function**.
- In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.
- A **macroparticle** is a numerical **representation** of a **cluster of neighbouring physical particles**.
- Thus, instead of solving the system for the **N (~10^{11})** physical particles one can significantly **reduce the number of degrees of freedom** to **N_{MP} (~10^6)**. At the same time one must be aware that this **increases of the granularity** of the system which gives rise to numerical noise.



$$\Psi\left(x, x', y, y', z, \delta\right)$$



Macroparticle representation of the beam

- Macroparticle models permit a **seamless mapping** of realistic systems into a **computational environment** – they are fairly easy to implement

Beam:

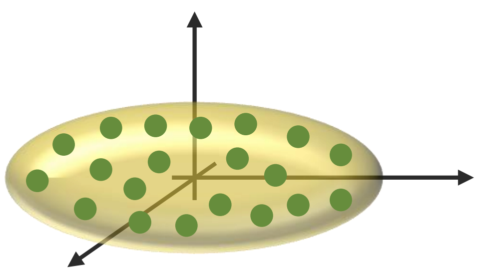
$$\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \quad \begin{pmatrix} q_i \\ m_i \end{pmatrix}, \quad i = 1, \ldots, N$$

Macroparticlenumber

$$\begin{pmatrix} y_i \\ y'_i \end{pmatrix}$$

Canonically conjugate coordinates and momenta

$$\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}$$



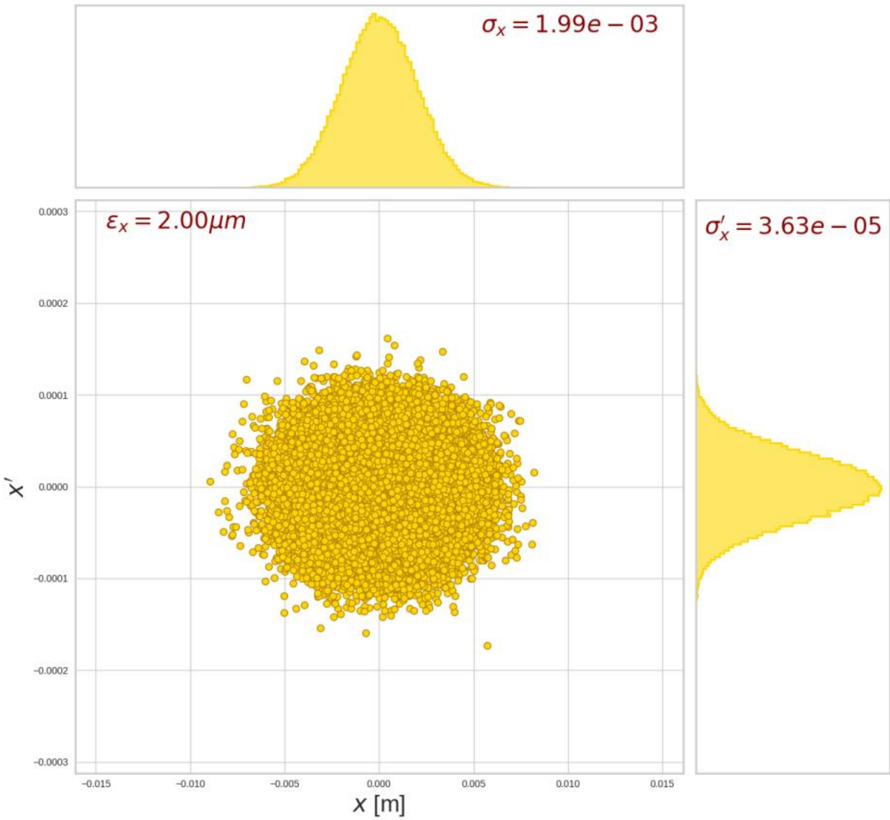
$$\Psi\left(x, x', y, y', z, \delta\right)$$

```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
        df
```

Out[6]:

	dp	x	xp	y	yp	z
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06	0.353427
1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904e-05	0.159670
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627340e-05	-0.251489
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.870926e-05	-0.038597
4	0.000572	0.000990	5.493907e-05	0.000152	-1.951051e-05	0.492968
5	-0.000418	0.001088	4.778027e-05	0.003320	-7.716856e-06	0.415582
6	-0.000114	-0.000194	1.065400e-05	0.001798	-4.984276e-07	-0.349064
7	0.001100	-0.001257	-6.873217e-05	-0.002374	5.657645e-06	-0.023157
8	0.002706	0.005351	-1.867898e-07	-0.000765	3.012523e-05	-0.291095
9	0.003508	0.000499	1.865768e-05	-0.001032	-5.363820e-05	0.211726
10	-0.001711	-0.003168	4.372560e-05	-0.001933	-2.151020e-05	-0.145358
11	-0.002150	-0.000565	-1.853825e-05	-0.003895	-6.192450e-06	0.072499
12	0.002059	0.003453	-3.808703e-05	0.000118	3.179588e-05	-0.001816
13	0.002709	0.000241	-3.457535e-05	0.000474	5.057865e-05	-0.005464
14	-0.001593	0.000711	-1.667091e-05	-0.002523	-3.804168e-05	[illegible]
15	-0.000830	-0.000393	-7.473946e-05	[illegible]		
16	-0.001743	[illegible]				

Macroparticle representation of the beam



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1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904[illegible]	
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627[illegible]	
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1[illegible]	
4	[illegible]	[illegible]	[illegible]	[illegible]		

- Initial conditions of the beam/particles

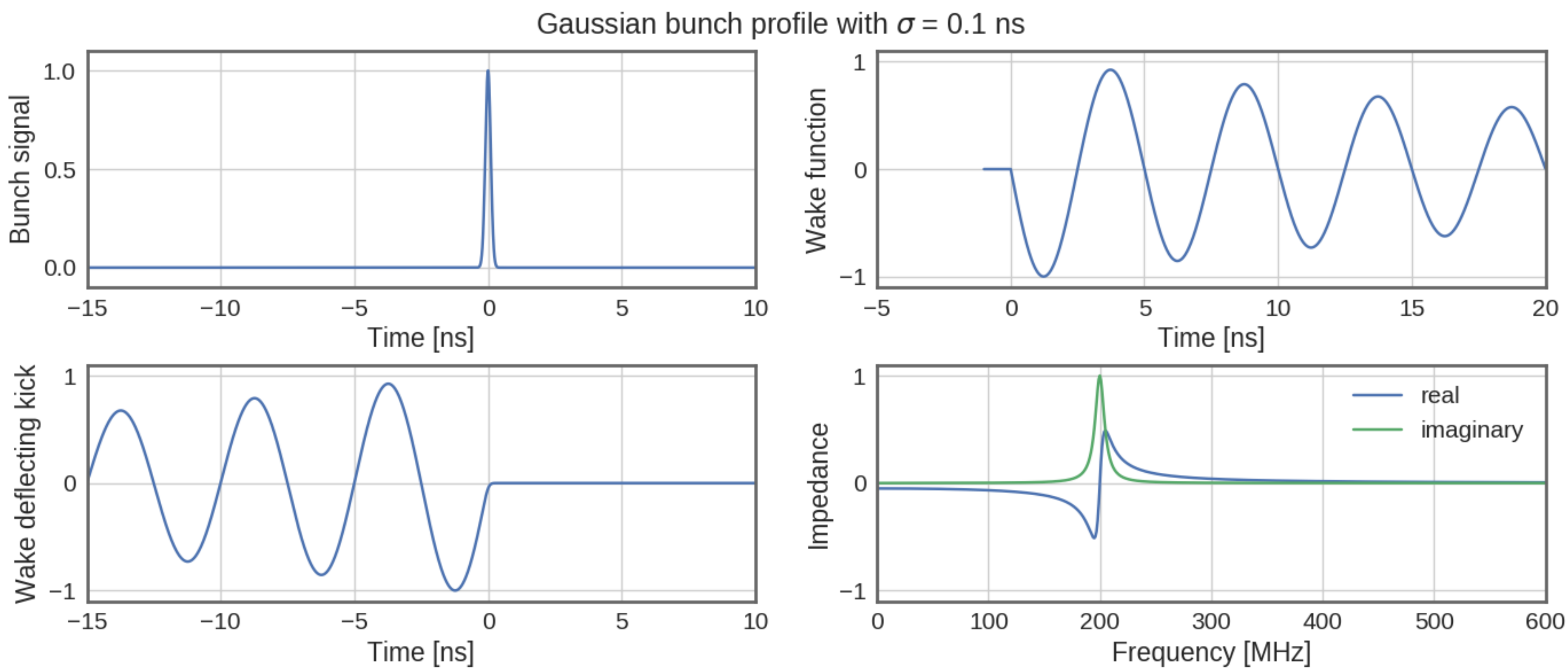
Profile	Size	Matching
Gaussian	Emittance	Optics
Parabolic		
Flat		
…		

- We use **random number generators** to obtain **random distributions of coordinates and momenta**
- Example transverse Gaussian beam in the SPS with normalized emittance of 2 um (0.35 eVs longitudinal)

$$\varepsilon_\perp = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$
$$= \beta\gamma\sigma_x\sigma_{x'}$$
$$\varepsilon_\parallel = 4\pi\sigma_z\sigma_\delta\frac{p_0}{e}$$

Wake fields illustrative examples

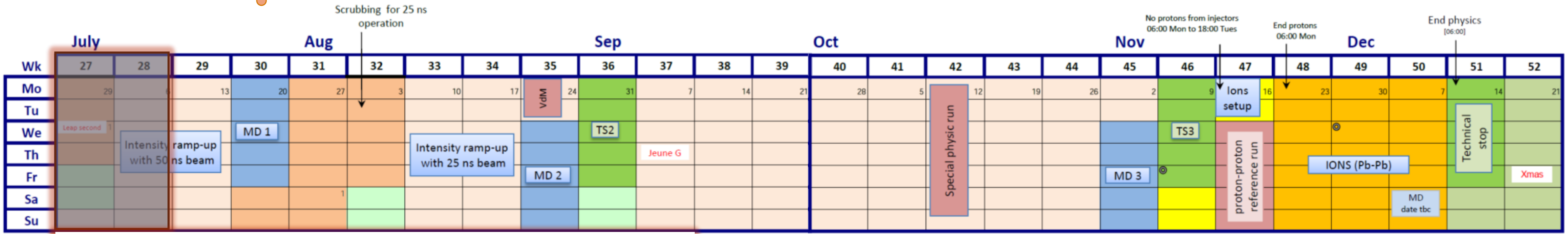
- Resonator wake: fr = 200 MHz, Q = 20 – Gaussian bunch charge profile
- The plots show how the bunch moments and the wake function **convolve into an integrated deflecting kick** at the different positions along the bunch

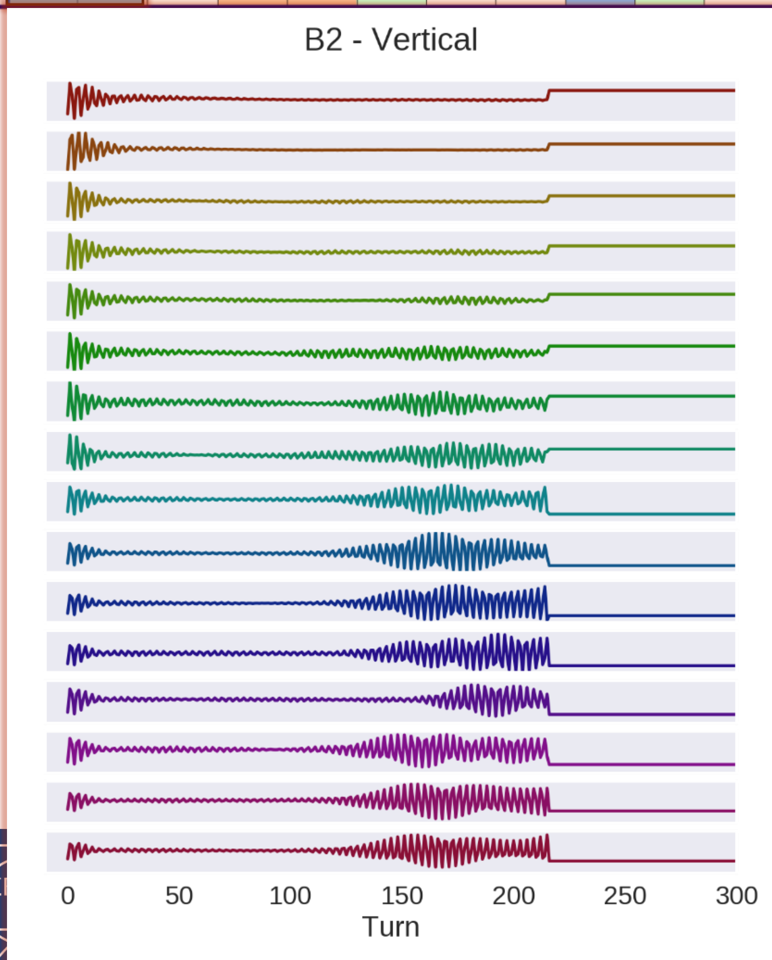


E-cloud instabilities in the LHC

The CERN Accelerator School

Scrubbing run in 2015 – early stage





Head of batch

every 4th bunch just after injection

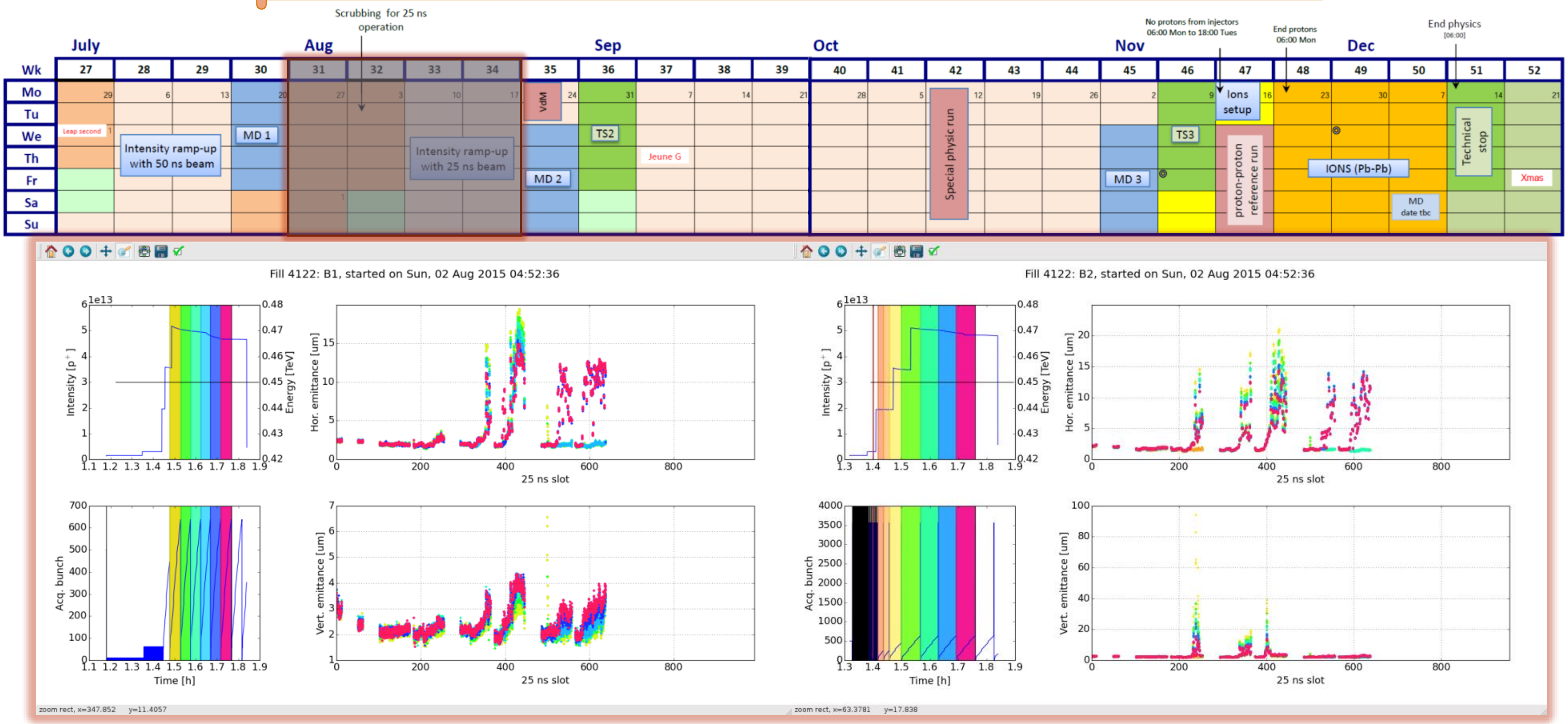
Tail of batch

- Injection of multiple bunch batches from the SPS into the LHC.
- Violent **instabilities during initial stages of scrubbing** – clear e-cloud signature
- Very hard to control in the beginning – **slow and staged ramp-up of intensity** (24 → 36 → 48 → 60 → 72 → 144 bpi)

E-cloud instabilities in the LHC

The CERN Accelerator School

Scrubbing run in 2015 – second stage



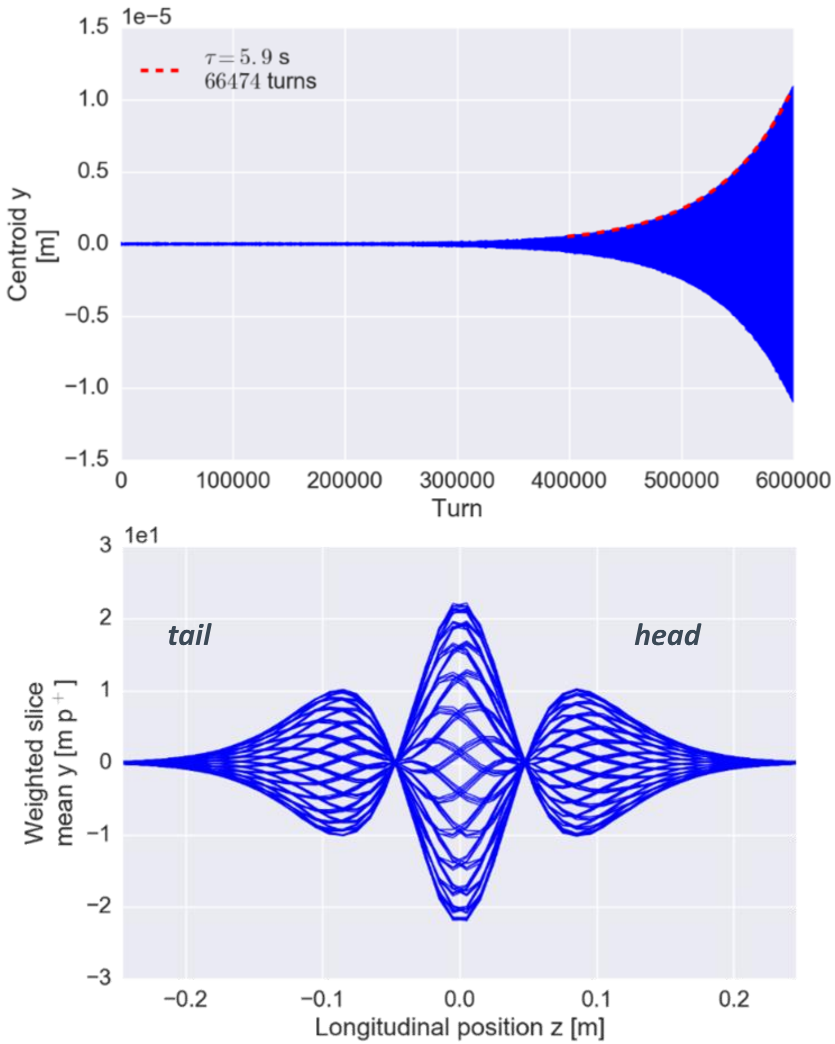
- At later stages dumps under control but still **emittance blow-up and serious beam quality degradation**.
- Beam and e-cloud induced h**eating of kickers and collimators**.

Headtail instabilities in the LHC

- The **impedance in the LHC** can give rise to coupled and single bunch instabilities which, when left untreated, can lead to **beam degradation and beam loss**.

- As an example, **headtail instabilities** are predicted from **macroparticle simulations** using the LHC impedance model.

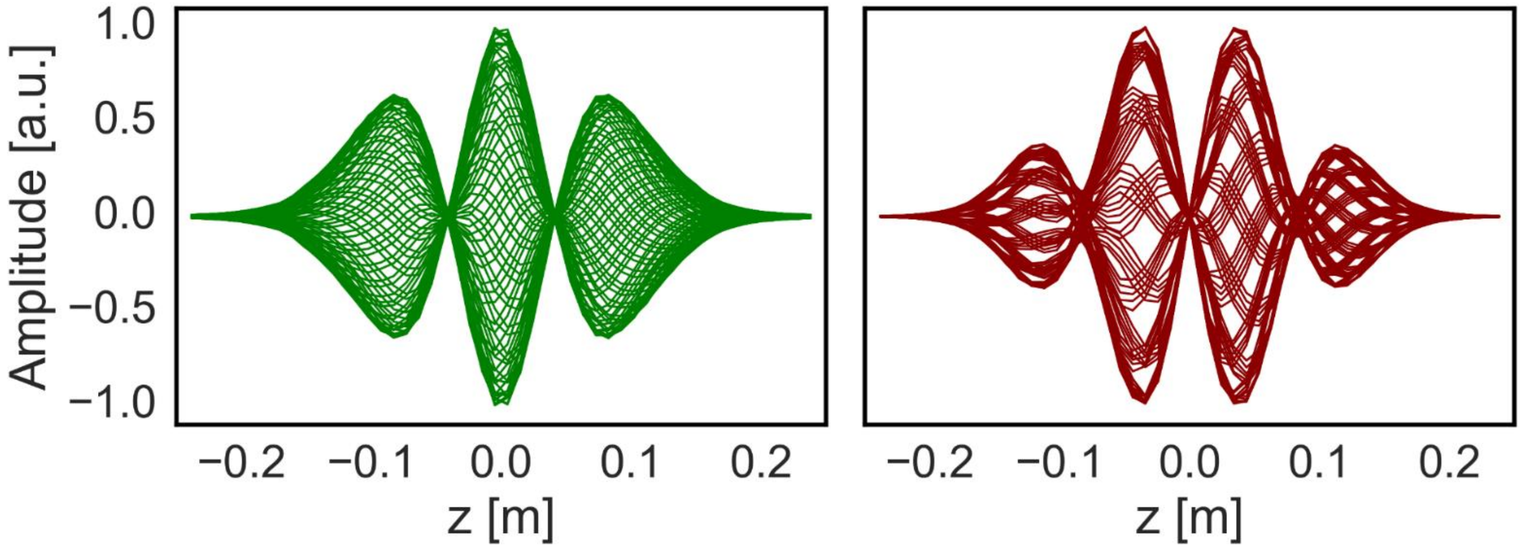
- These simulations help to understand and to predict unstable modes which are observed in the real machine.



Headtail instabilities in the LHC

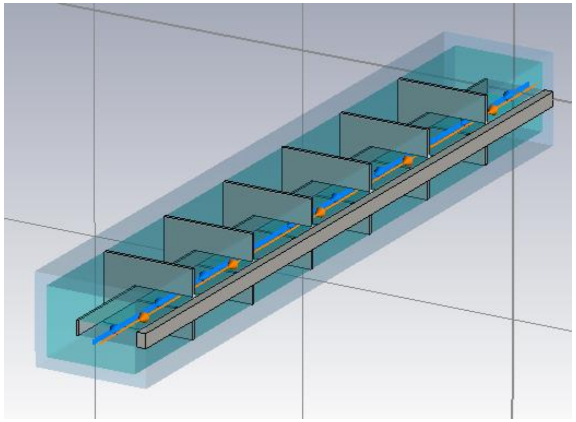
m = 0 m = -1

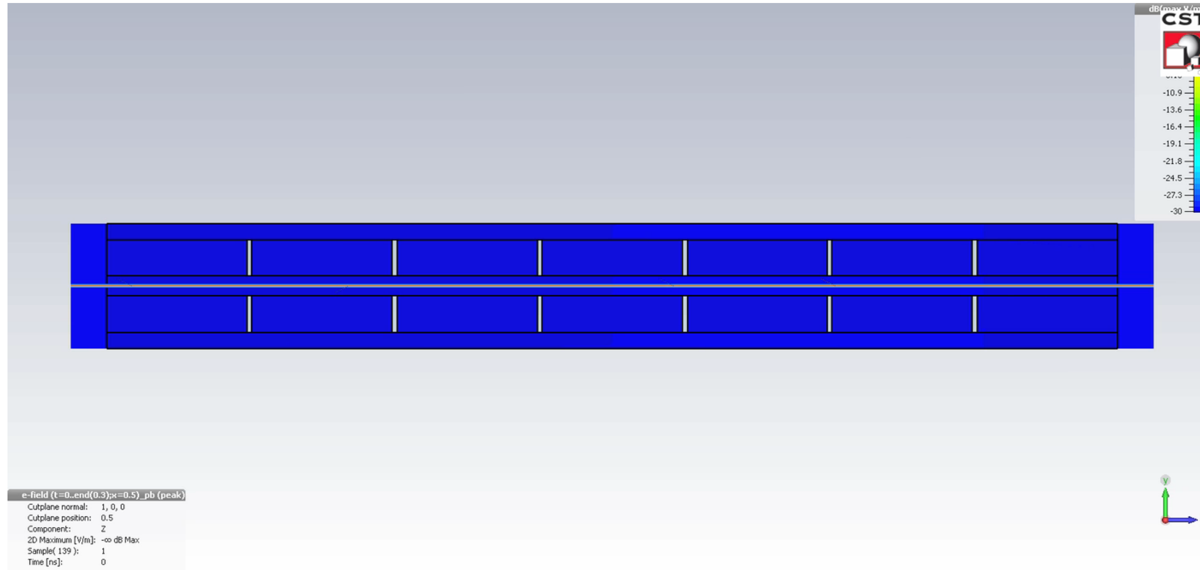
Macroparticle simulations (PyHEADTAIL)



- These simulations help to understand and to **predict instabilities** which are **observed in the real machine**.

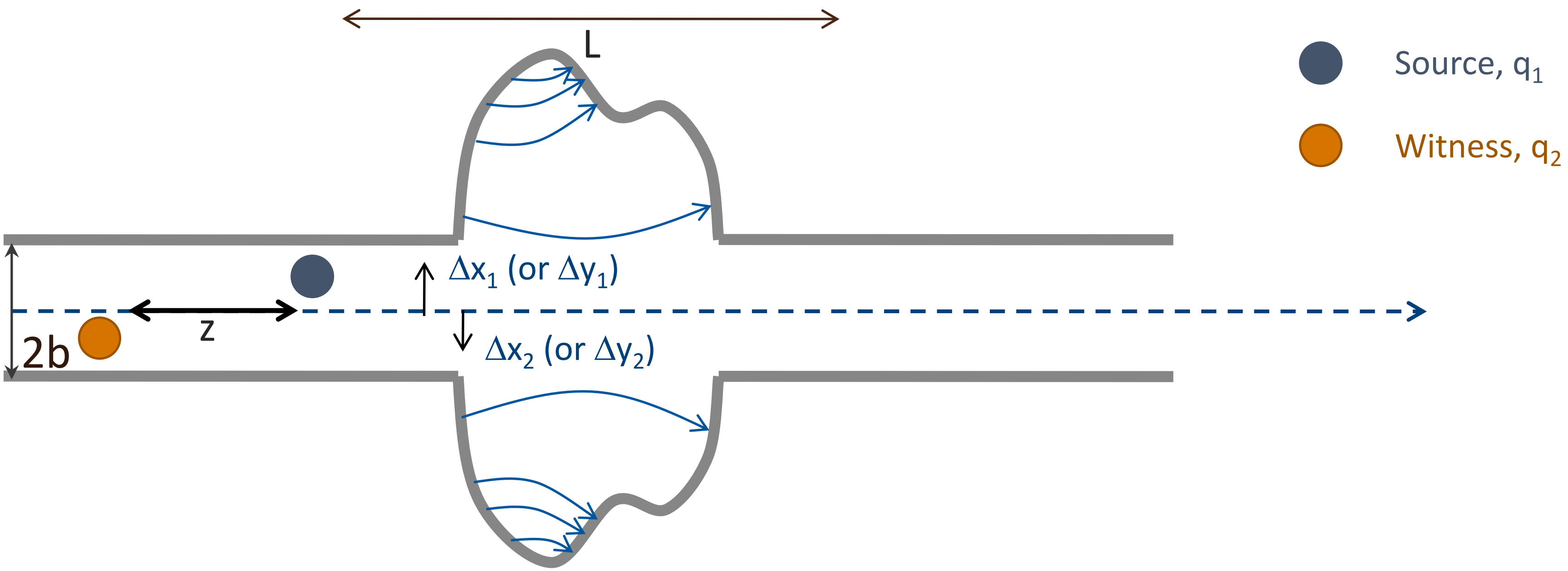
Wakefields as sources of collective effects





- The **wake function** is the **electromagnetic response** of an object to a charge pulse. It is an intrinsic property of any such object.
- The wake function **couples two charge distributions** as a function of the distance between them.
- The response depends on the boundary conditions and can occur e.g. due to **finite conductivity** (resistive wall) or more or less sudden **changes in the geometry** (e.g. resonator) of a structure.

Wake functions in general



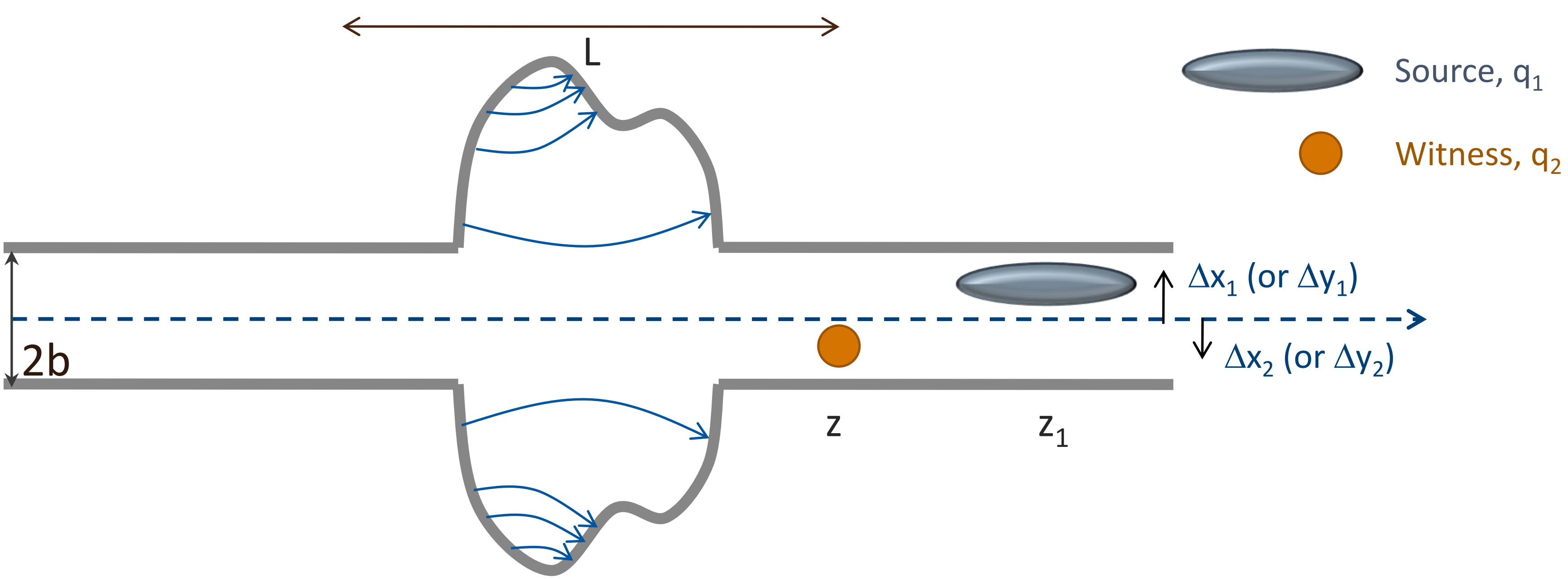
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

Wake potential for a distribution of particles



Definition as the **integrated force** associated to a change in energy:

- For an extended particle distribution this becomes

$$\Delta E_2(z) \propto \int \boxed{\lambda_1(x_1, z_1)}\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

Forces become dependent on the **particle distribution function**

Wake fields – impact on the equations of motion

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1)\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

- We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

$$H = \frac{1}{2} x' + \frac{1}{2} \left(\frac{Q_x}{R} \right)^2 x^2 + \frac{e^2}{\beta^2 E C} \int \lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)\, dx_1 dz_1 dx$$

- The equations of motion become:

$$x'' + \left(\frac{Q_x}{R} \right)^2 x + \frac{e^2}{\beta^2 E C} \int \lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)\, dx_1 dz_1 = 0$$

The presence of wake fields adds an **additional excitation** which depends on

1. The **moments of the beam distribution**
2. The **shape and the order** of the wake function

How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances

- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators", Erice, Sicily, 23-28 April, 2014

- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations