Laser Beam Physics

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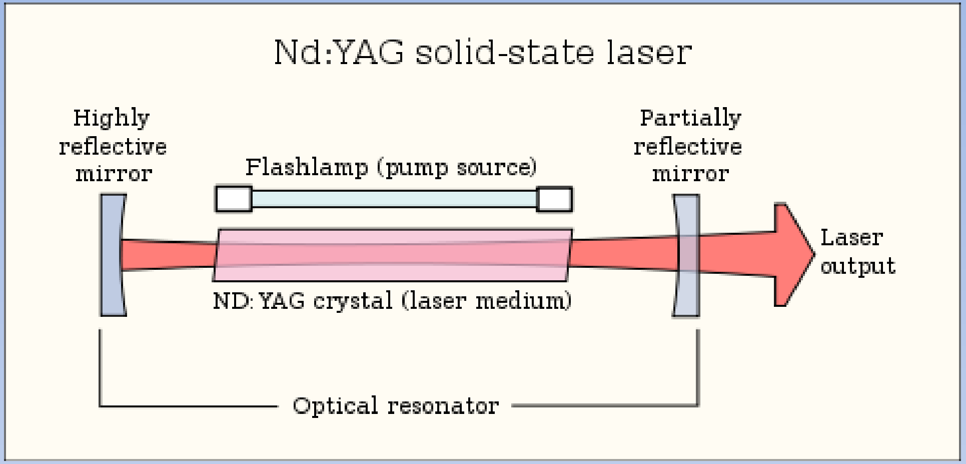
Outline

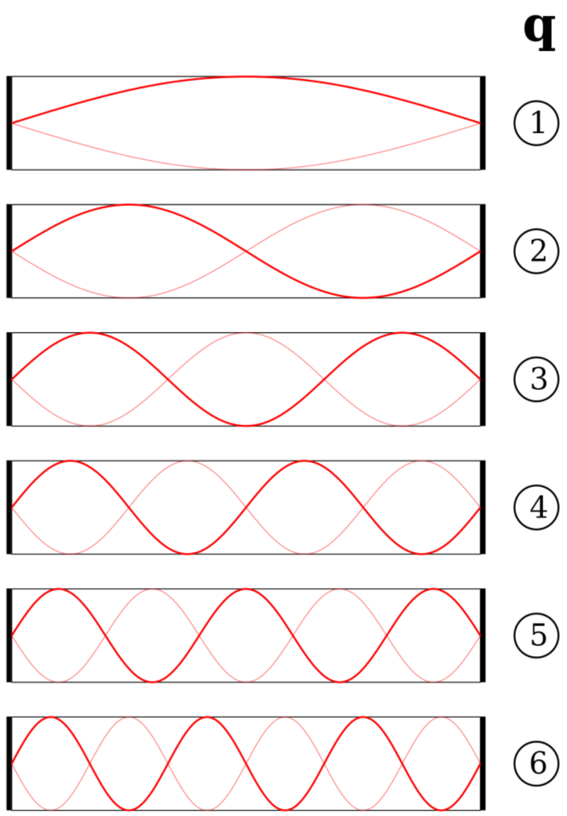
- Cavity modes – longitudinal and transverse
- Gaussian beams & the q parameter
- Ray optics & ABCD matrices
- Beam focusing

Cavity



- Pump gain medium to upper level
- A photon decays spontaneously & stimulates more emission
- The photons bounce back and forth along the cavity – if the number of photons emitted each round trip exceeds losses (mirrors etc.) laser is above threshold
- One of the mirrors allows a small amount of this light out – laser output!
- Laser output controlled by gain of medium and longitudinal & transverse modes of cavity

Longitudinal modes



- Laser oscillator is just a resonator
- Resonant cavity modes exist
- Other frequencies 'don't fit'

Resonant modes fulfil: $L = q\frac{\lambda}{2}$

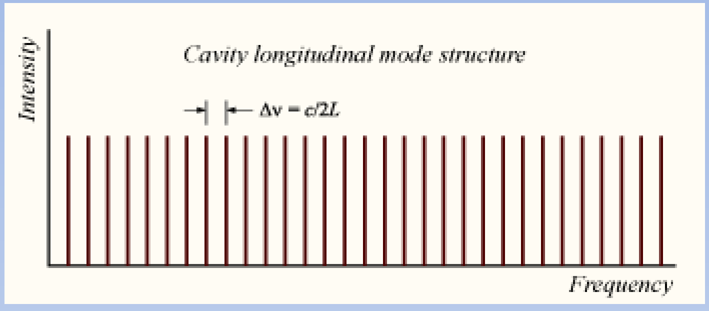
Cavity mode spacing given by:

$$\Delta\nu = \frac{c}{2nL}$$

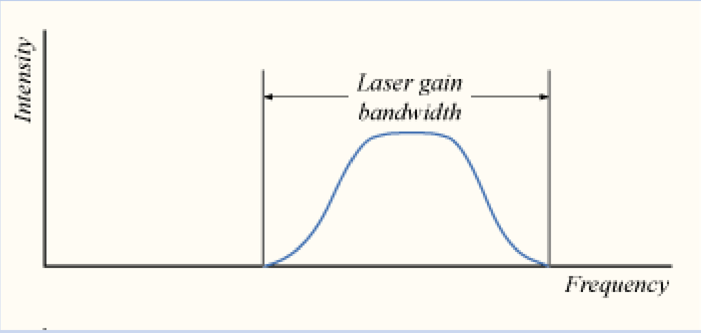
n is refractive index – may be 1

Form a 'comb' of equally spaced modes in frequency space

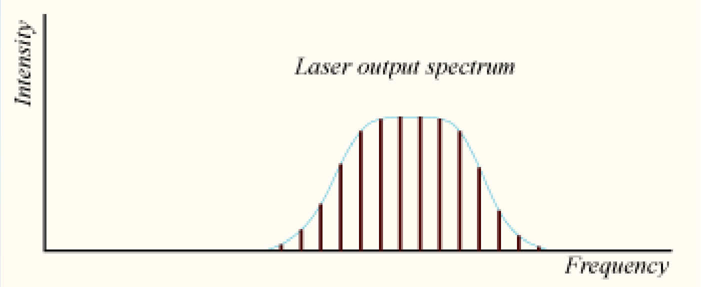
Longitudinal modes



- Length of cavity determines resonant frequencies and mode spacing



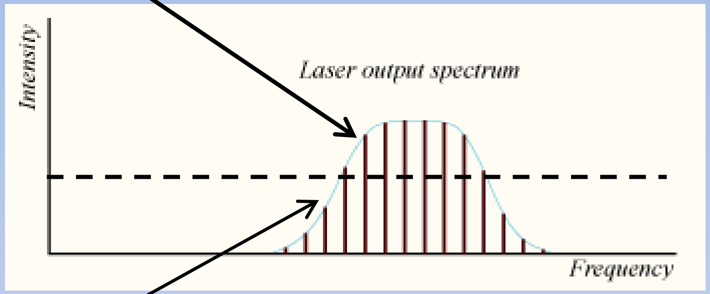
- Laser gain medium has certain bandwidth



- Combination determines what wavelengths can lase

Lasing on longitudinal modes

These modes will lase

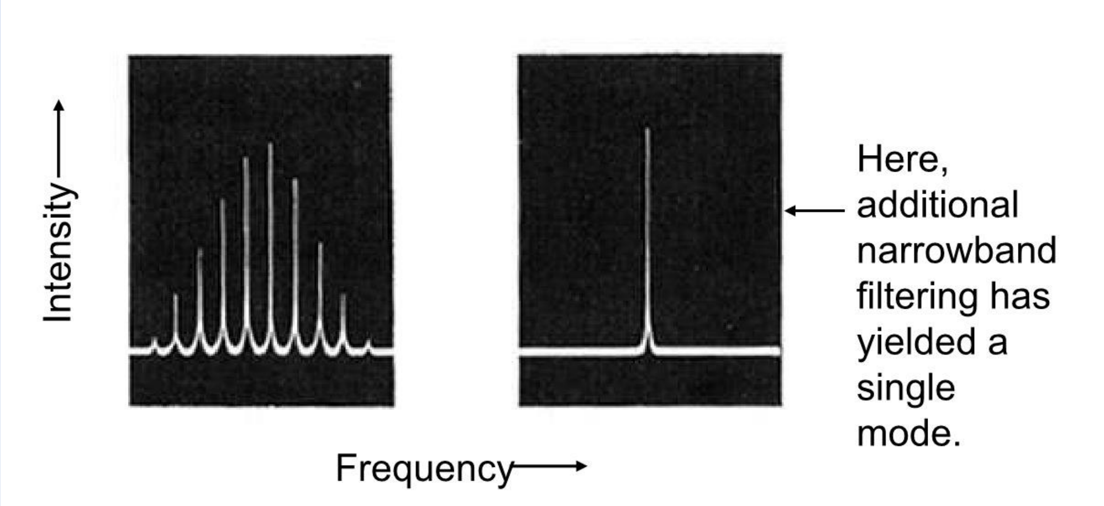


The oscillator **can** lase on these modes

Doesn't mean it will!

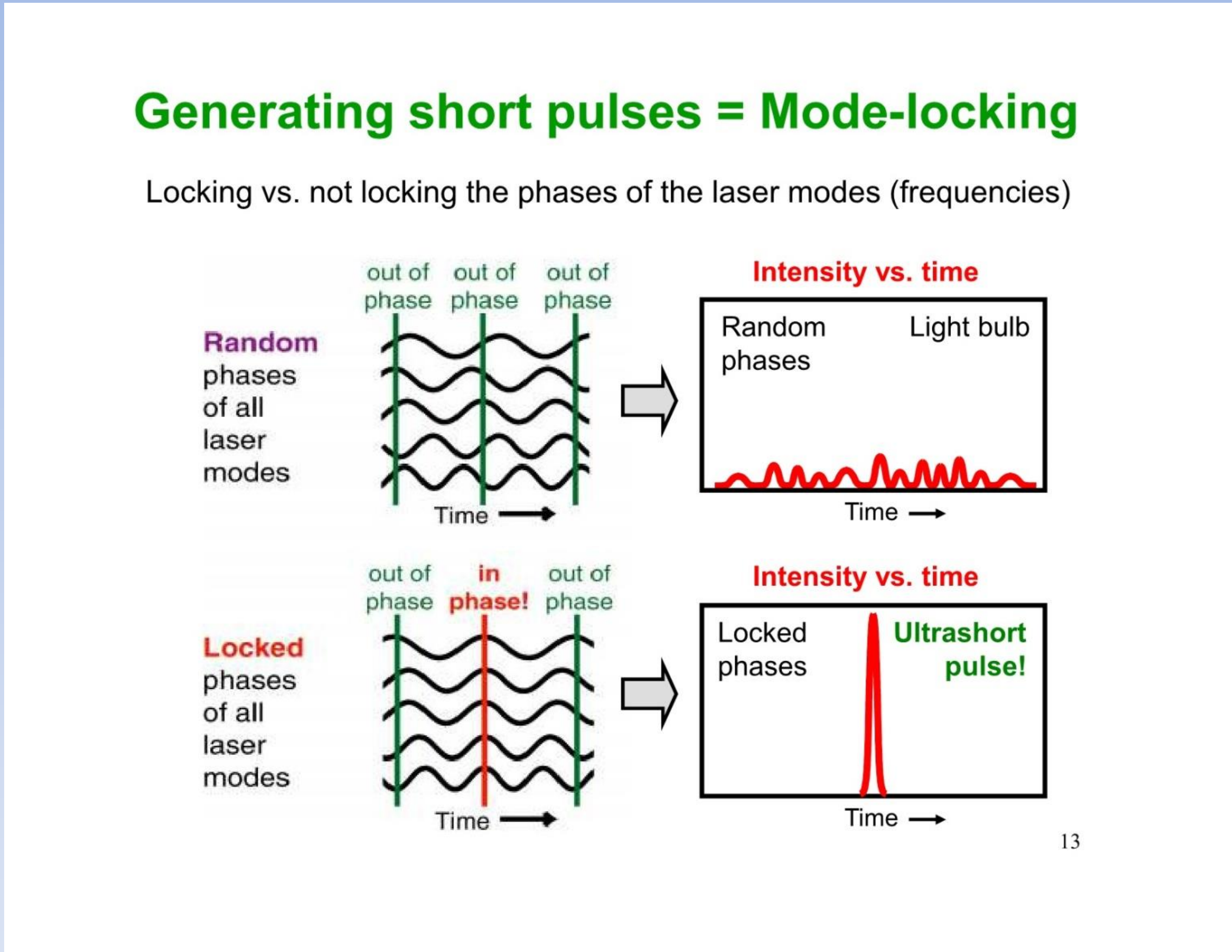
These modes won't lase

Need to be above threshold – gain > cavity losses for lasing



- Can make the laser run on a single longitudinal mode – SLM
- Very narrow bandwidth
- Spectroscopy etc.

Mode locking



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Short pulse oscillator

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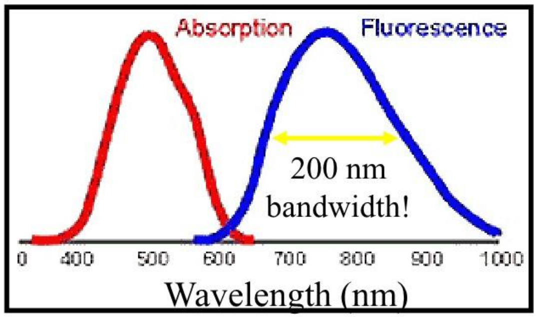
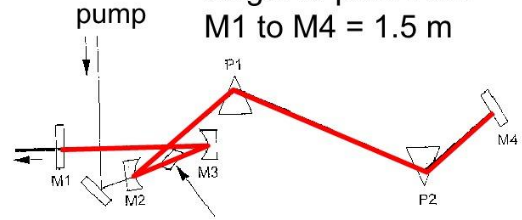
Ti:sapphire: how many modes lock?



Typical cavity layout: length of path from M1 to M4 = 1.5 m



Therefore $\Delta\nu = c/L_{rt} \approx 100$ MHz

Q: How many different modes can oscillate simultaneously in a 1.5 meter Ti:sapphire laser?

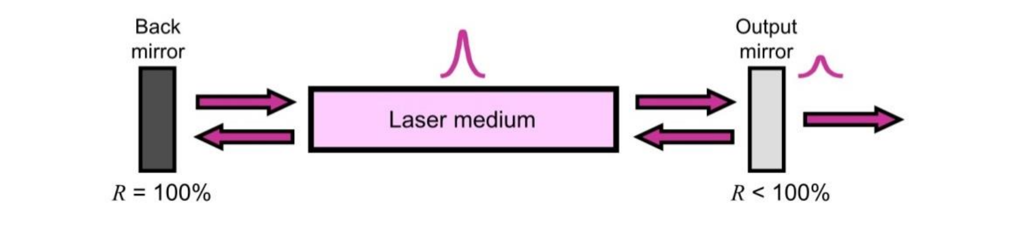
A: Gain bandwidth $\Delta\lambda = 200$ nm $\Rightarrow$ $\Delta\nu = (c/\lambda^2)\,\Delta\lambda \sim 10^{14}$ Hz

$\Delta\nu_{bandwidth}/\Delta\nu_{mode} = 10^6$ modes

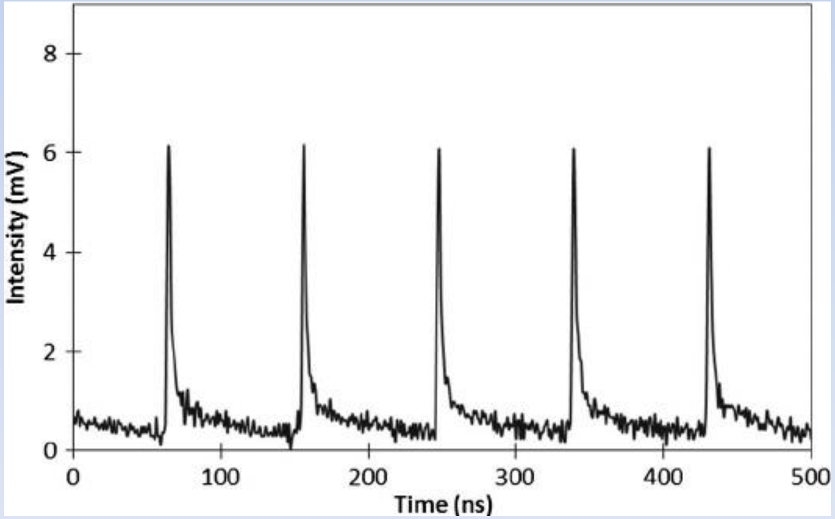
Fourier transform of comb of frequencies is train of pulses in time

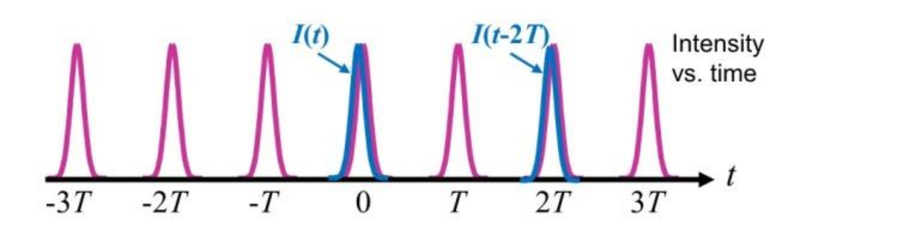
Duration of individual pulse given by total bandwidth

Pulse train output



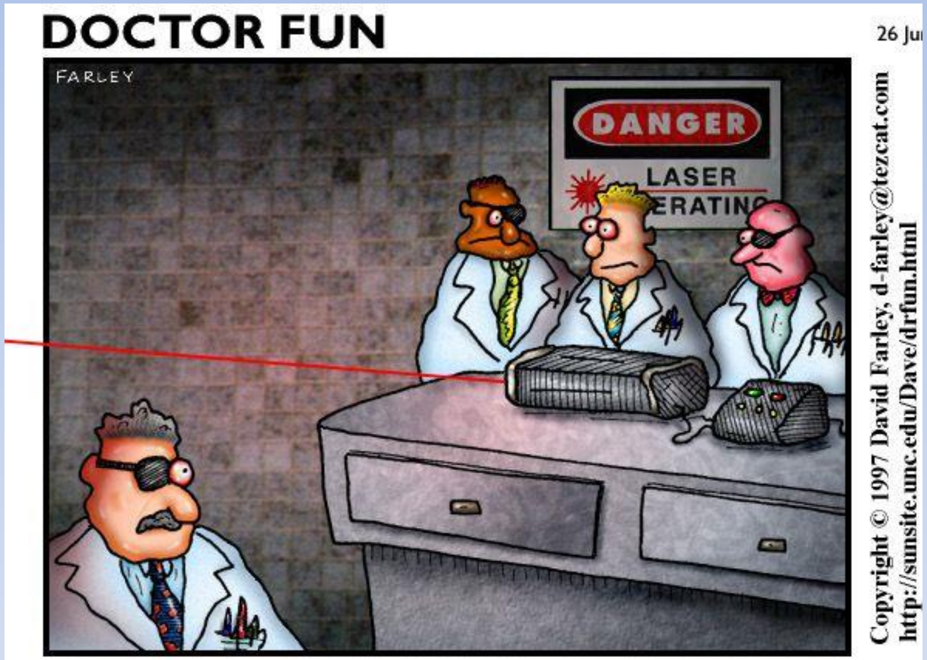
- If we can lock all the lasing cavity modes in phase we have a short pulse in the oscillator
- Each round trip a small amount is transmitted through the output coupler
- So laser output is a train of ultrashort pulses
- ‘Front end’ of chirped pulse amplification system





Transverse cavity modes

aka ‘what the laser looks like as you stare into it just before it blinds you’



Peer pressure in the laser lab

Transverse cavity modes (safely)

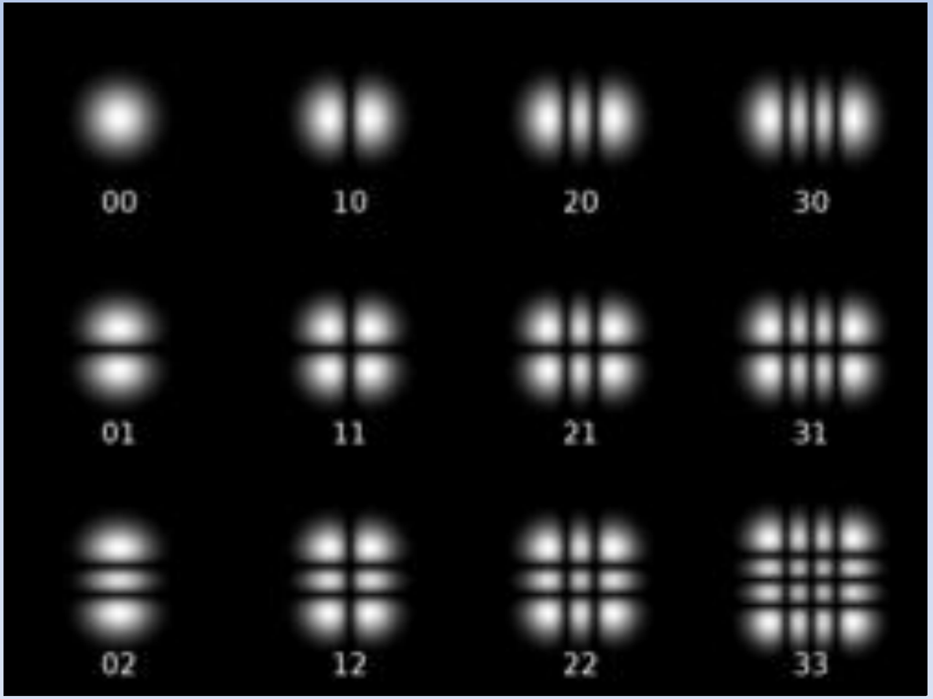
- Not an infinite plane wave – boundary conditions!
- What is the form of that wave that is self consistent after one round trip in cavity?
- Paraxial approximation

Cylindrical symmetry:
Solutions are Laguerre – Gaussian modes

Rectangular symmetry:
Solutions are Hermite – Gaussian modes

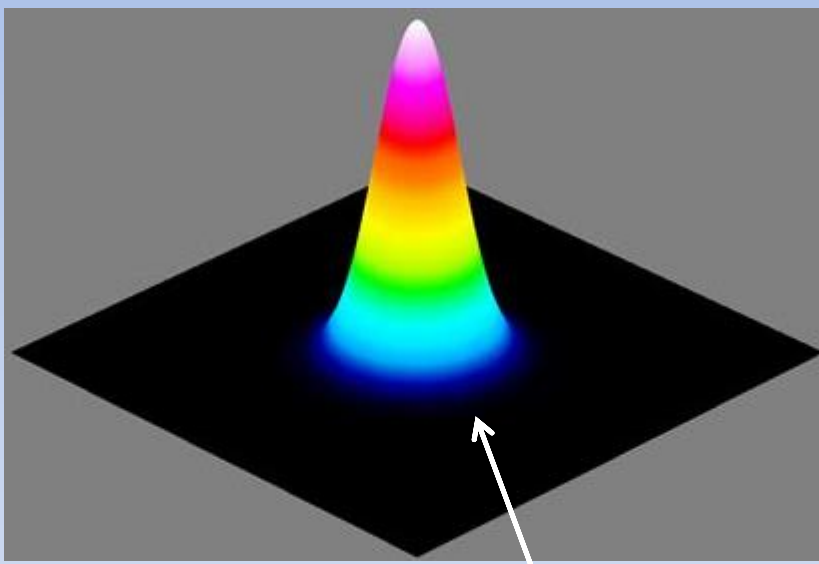
HG more common – broken symmetry in oscillator

Lowest order mode is a Gaussian



Gaussian beams

Important as lowest order lasing mode of (most) cavities



- Want to know how this beam propagates through an optical system:
- How does the beam change?
- How does it focus?

This is a simulation. You will never see a real beam this good.

Gaussian beams

A laser beam can be described by this:

$$E(x,y,z,t) = u(x,y,z)\,e^{j(kz-\omega t)}$$

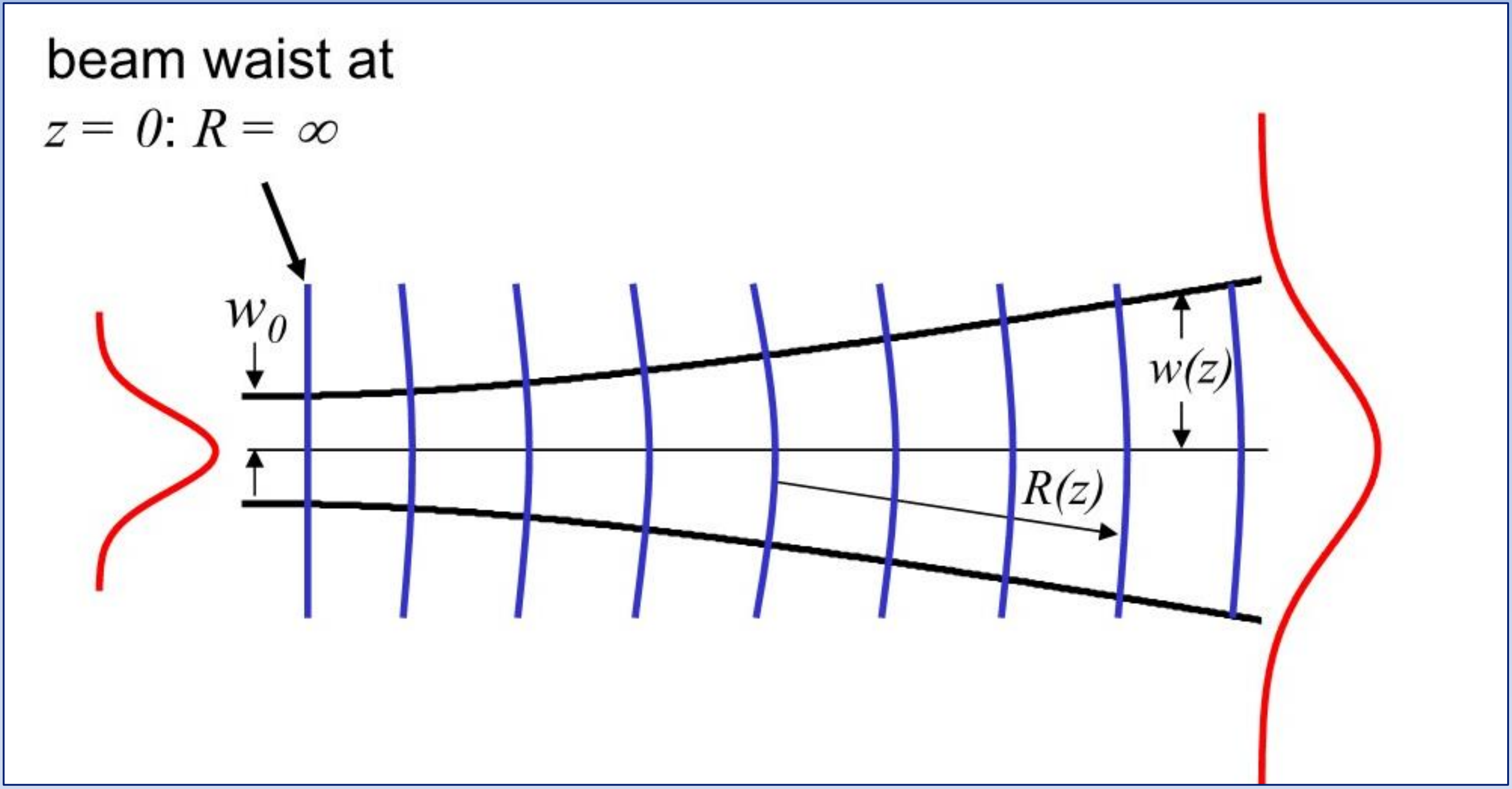
where $u(x,y,z)$ is a Gaussian transverse profile that varies slowly along the propagation direction (the z axis), and remains Gaussian as it propagates:

$$u(x,y,z) = \frac{1}{q(z)} \exp\left[-jk\,\frac{x^2+y^2}{2}\cdot\frac{1}{q(z)}\right]$$

The parameter q is called the "complex beam parameter." It is defined in terms of w, the beam waist, and R, the radius of curvature of the (spherical) wave fronts:

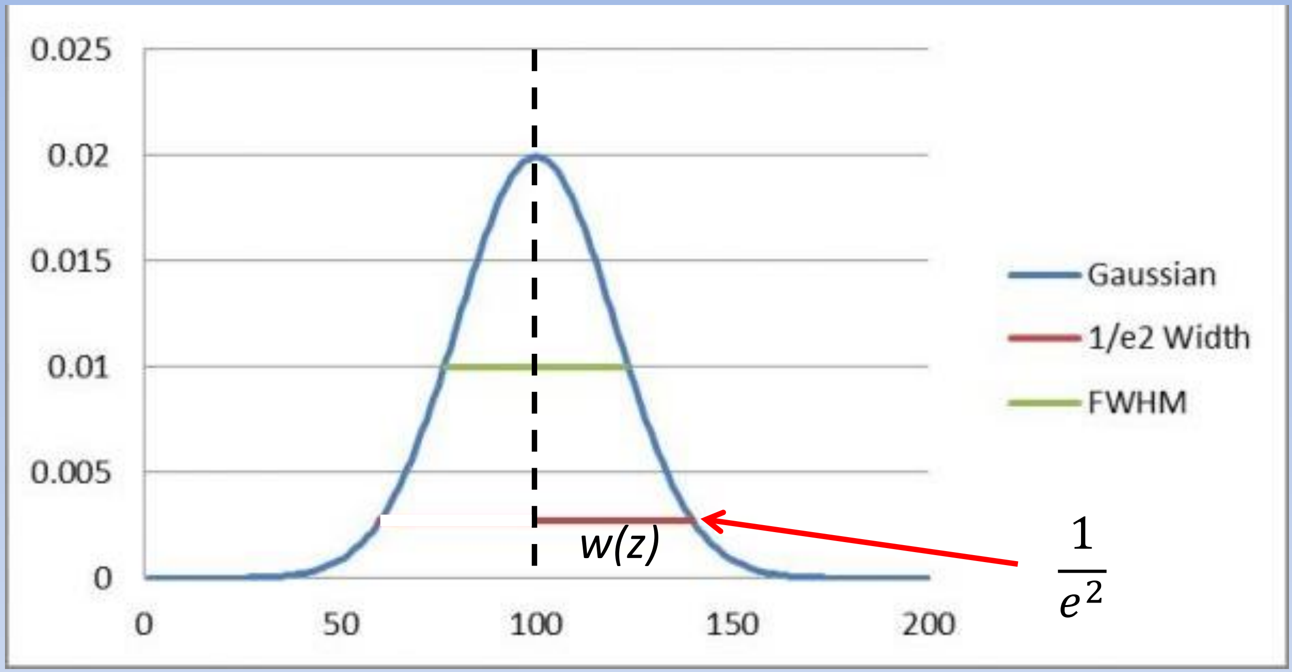
$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\,\frac{\lambda}{\pi w^2(z)}$$

Gaussian beams



- IMPORTANT: ‘spot size’ $w(z)$
  ‘beam waist’ $w(0)$

Gaussian beam size



$\frac{1}{e^2}$

- **IMPORTANT:** 'spot size' *w(z)* is $1/e^2$ **radius** of beam intensity profile

Beam diameter is $2w$

Both of these will be completely different to the way your accelerator colleagues define charged particle beam size

Propagation of Gaussian beams

Suppose a Gaussian beam (propagating in empty space, wavelength λ) has an infinite radius of curvature (i.e., phase fronts with no curvature at all) at a particular location (say, $z = 0$).

Suppose, at that location ($z = 0$), the beam waist is given by w_0.

Describe the subsequent evolution of the Gaussian beam, for $z > 0$.

We are given that $R(0)$ = *infinity* and $w(0) = w_0$. So, we can determine $q(0)$:

$$\frac{1}{q(0)} = \frac{1}{R(0)} - j\frac{\lambda}{\pi w^2(0)}$$

$$= -j\frac{\lambda}{\pi w_0^{\,2}} \longrightarrow \text{Thus} \quad q(0) = j\frac{\pi w_0^{\,2}}{\lambda}$$

NOTE: If $R = \infty$ at a given location, this implies that q is a pure imaginary number at that location: this is a focal point.

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Rayleigh range

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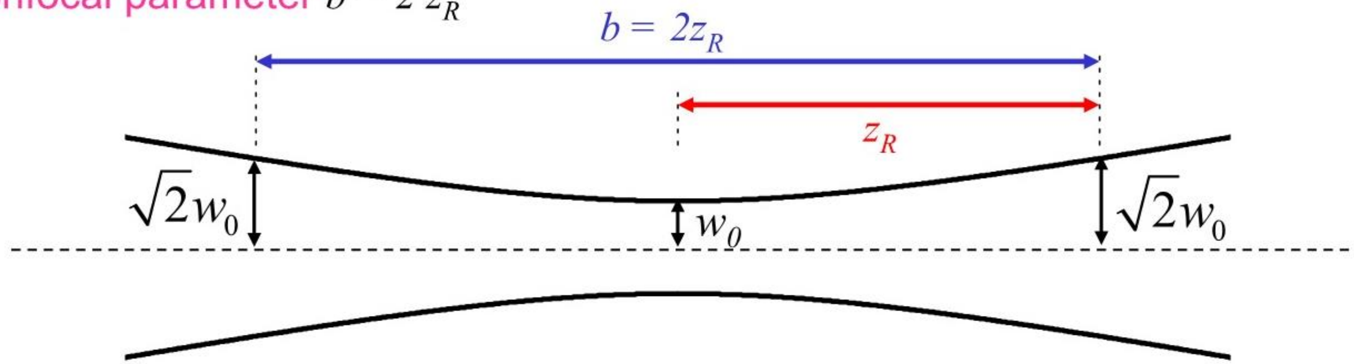
$$z_R = \frac{\pi w_0^{\ 2}}{\lambda}$$

- Can define new quantity z_R – the ‘Rayleigh range’
- Distance over which spot size w_o goes to $\sqrt{2}w_o$
- Beam area has doubled.

- Confocal parameter $b = 2\, z_R$



No such thing as a collimated beam, but $2z_R$ is a reasonable approximation

Gaussian beam propagation

A distance z later, the new complex beam parameter is:

$$q(z) = q(0) + z = jz_R + z$$

$$\frac{1}{q(z)} = \frac{1}{z + jz_R} = \frac{z - jz_R}{z^2 + z_R^{\ 2}}$$

Reminder:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$$

$$\mathrm{Re}\left\{\frac{1}{q(z)}\right\} = \frac{z}{z^2 + z_R^{\ 2}} = \frac{1}{R(z)}$$

$$R(z) = \frac{z^2 + z_R^{\ 2}}{z}$$

- At $z = 0$, R is infinite, as we assumed.
- As z increases, R first decreases from infinity, then increases.
- Minimum value of R occurs at $z = z_R$.

$$\mathrm{Im}\left\{\frac{1}{q(z)}\right\} = \frac{-z_R}{z^2 + z_R^{\ 2}} = \frac{-\lambda}{\pi w^2(z)}$$

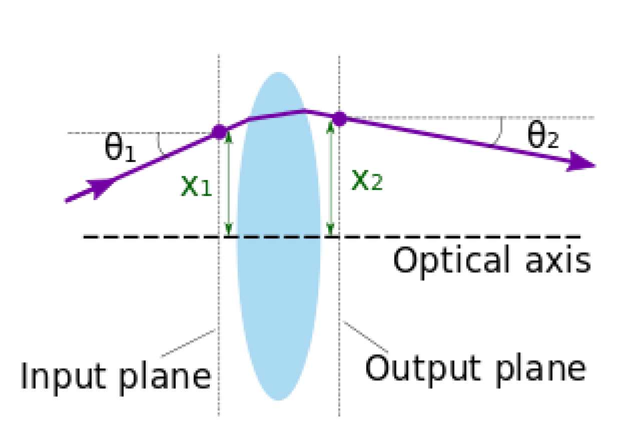
$$\boxed{w(z) = w_0\sqrt{1 + \frac{z^2}{z_R^{\ 2}}}}$$

- At $z = 0$, $w = w_0$, as we assumed.
- As z increases, w increases.
- At $z = z_R$, $w(z) = sqrt(2) \times w_0$.

ABCD matrices

This just gives free space propagation – how to model beam through optical system? Back to ray optics

- Define input light ray: position x_{in} and angle θ_{in}
- Propagate through optical system
- Have output light ray: position x_{out} and angle θ_{out}
- Related by 2 x 2 matrix



$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

Exact form of matrix depends on optical system

ABCD matrices

- Common matrices:
  - Propagation in free space $\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$
  - Focusing with thin lens $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$

Can multiply several matrices together to model e.g. cavity, telescope

Best bit – works for Gaussian beams too!

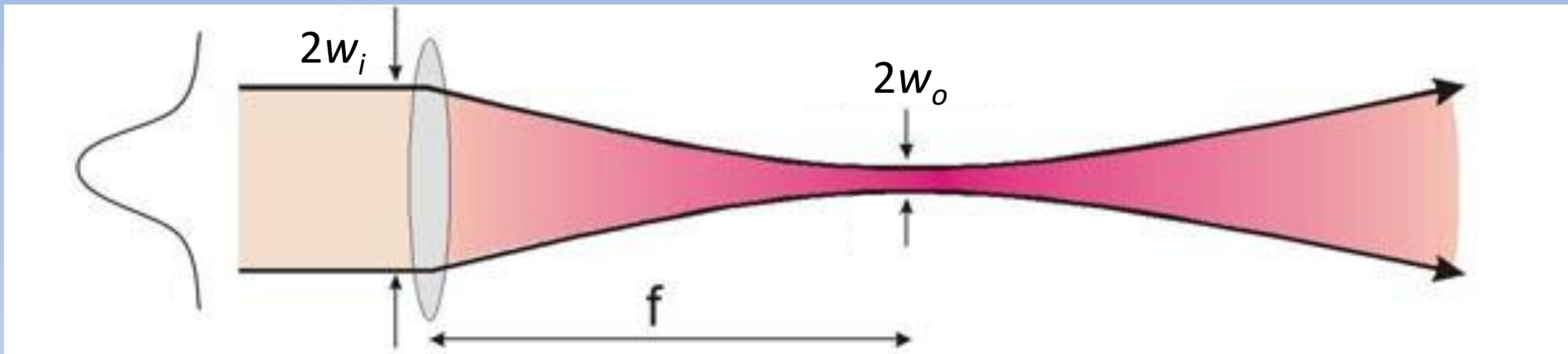
$$\boxed{q_2 = \frac{Aq_1 + B}{Cq_1 + D}}$$

Reminder:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$$

So if you know initial q parameter and ABCD matrix can find spot size and curvature anywhere

Gaussian beam focusing (theory)



- Want high intensity → small spot size
- Gaussian beam focus with lens focal length f

Beam waist

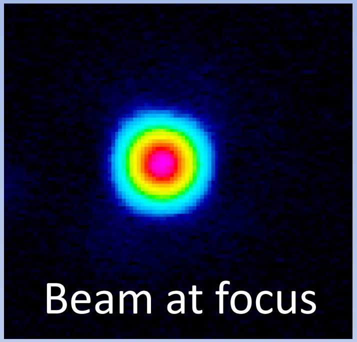
$$w_o = \frac{\lambda f}{\pi w_i}$$

Input spot size

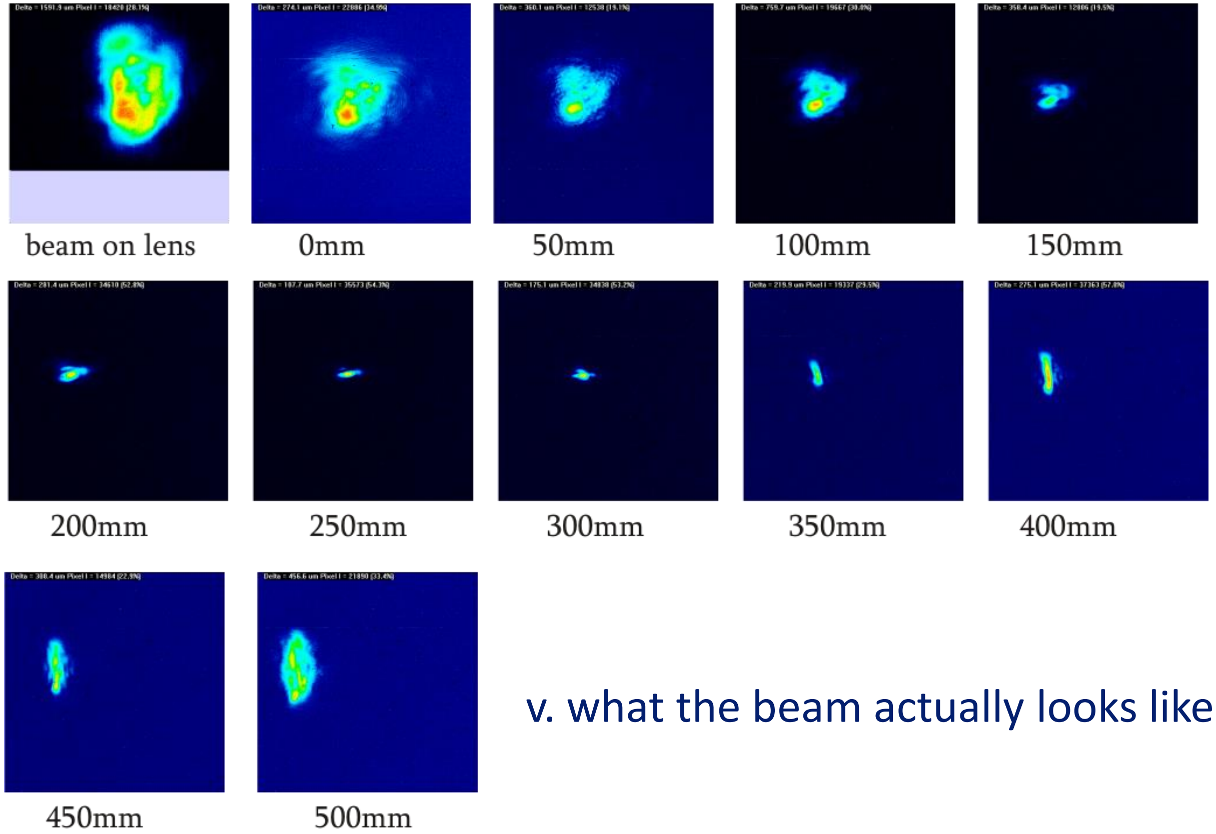
Small spot:
- Short focal length f
- Short wavelength λ
- Large input spot w_i

Gaussian beam focusing (practice)

What your supervisor thinks the beam should look like:

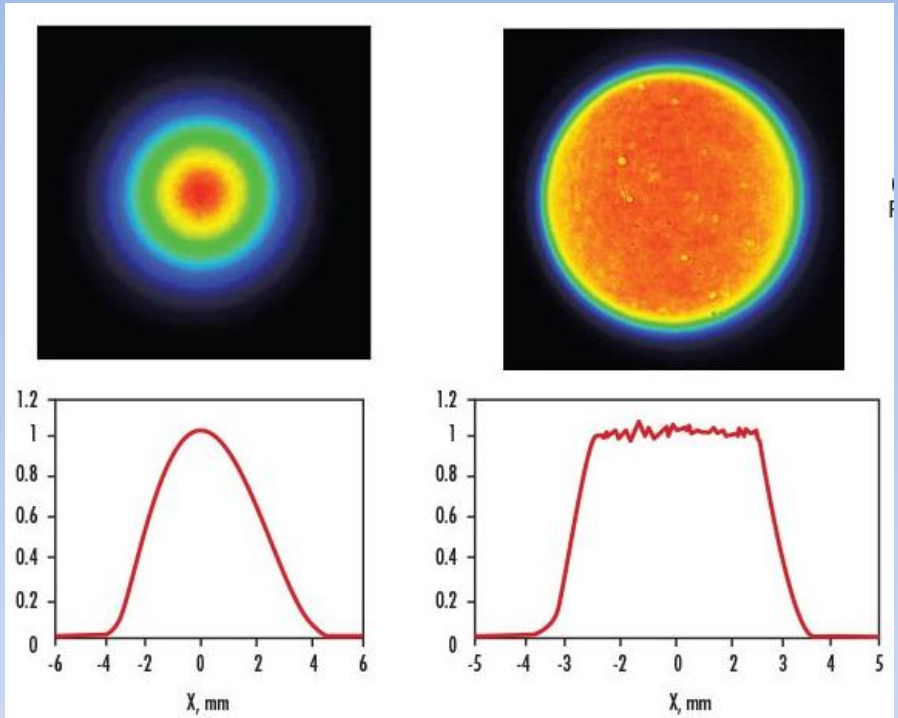


12.7 mm telescope



v. what the beam actually looks like

Top hat beams



- High power laser beams often top hat spatial profile rather than Gaussian
- More efficiently extract gain from laser amplifier
- Often use concept of f/# (f – number) for focusing

f number and focusing

- ‘f/#’ is a property of focusing (collimating) system e.g. lens, parabola
- Given by f/# = $\frac{f}{D}$
  - f – lens/parabola focal length
  - D – diameter
- We say ‘ f 10’ or f 18’

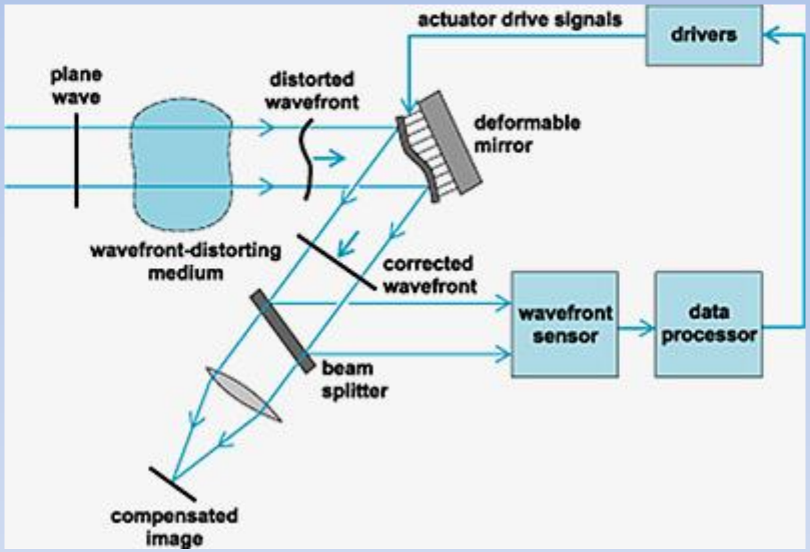
From focusing formula we find

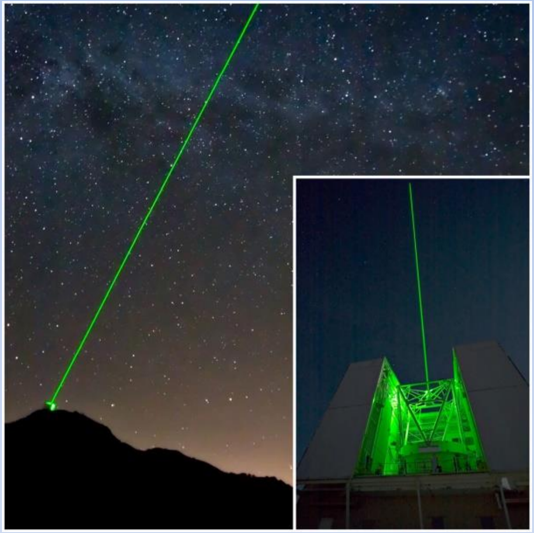
$$w \approx \lambda * f/\#$$

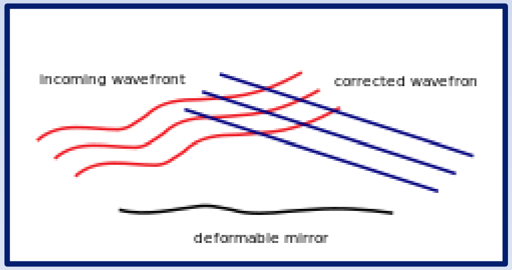
- **Caution!** D is really size of beam on optic here – ‘effective’ f/#
- e.g. f = 150mm, D = 50mm, spot diameter 40 mm
- Smaller f/# for smaller spot
- f = 150mm, D = 100mm, spot diameter 40mm – haven’t changed focus size!

Adaptive optics

Can we make the focus better?
Want to remove aberrations from the laser beam wavefront to give best focus
Use of adaptive optics to correct beam – stolen from astronomy







Use deformable mirror to correct wavefront and produce best focus

Conclusion

- Studied:
  - Longitudinal cavity modes and mode locking
  - Transverse cavity modes
  - Gaussian beam propagation and focusing
  - Adaptive optics