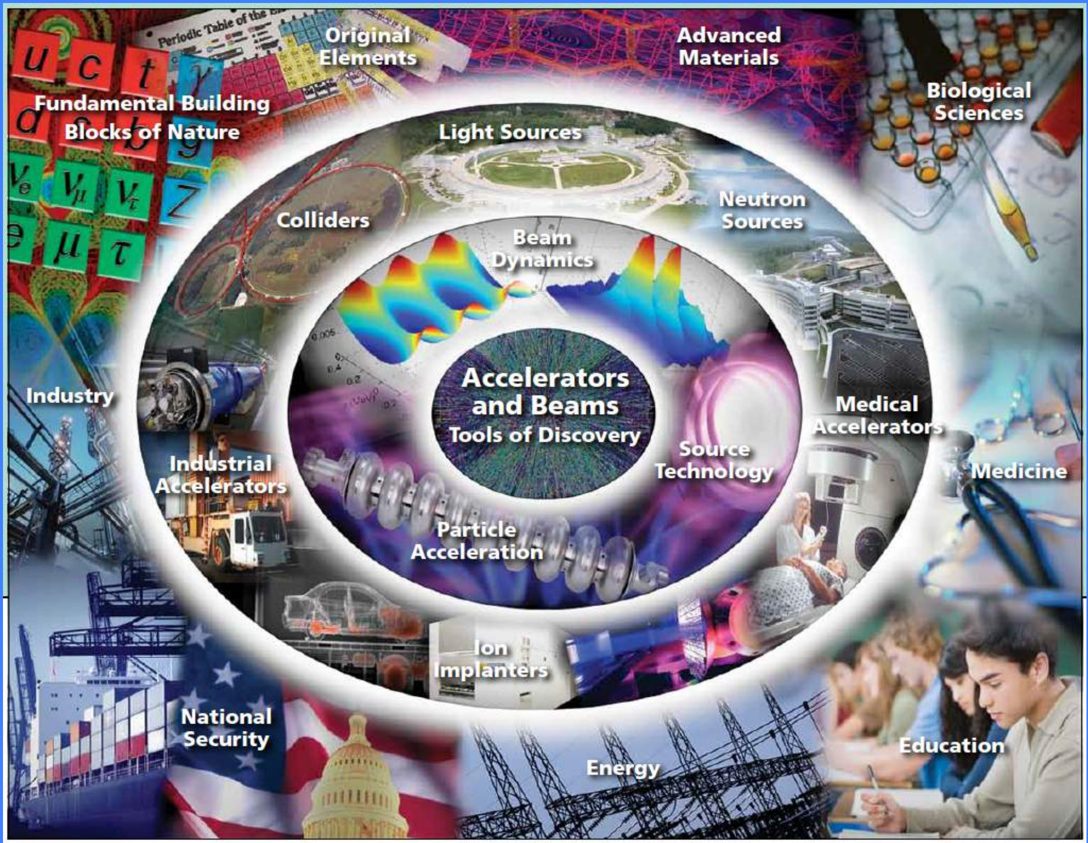
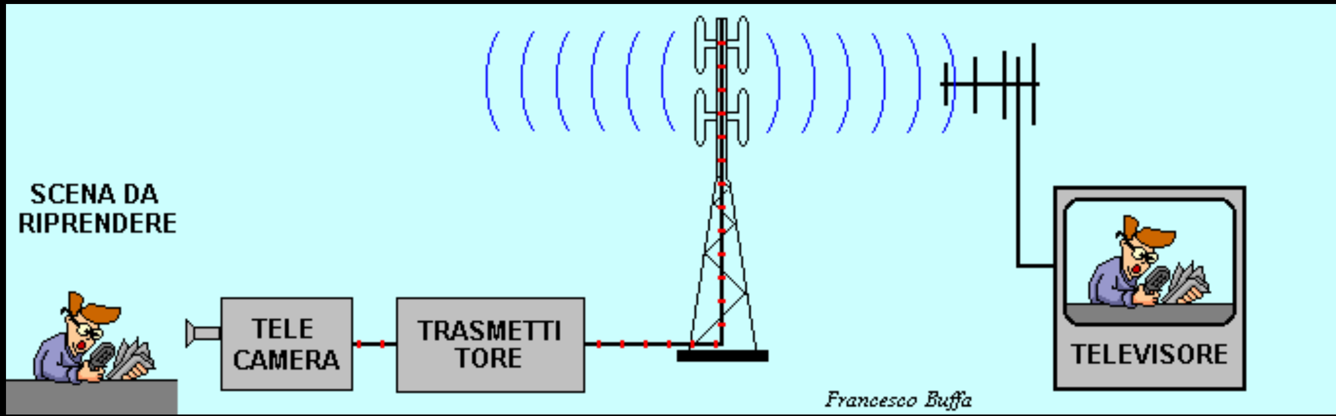
# Conventional accelerators and their limits

Massimo.Ferrario@LNF.INFN.IT

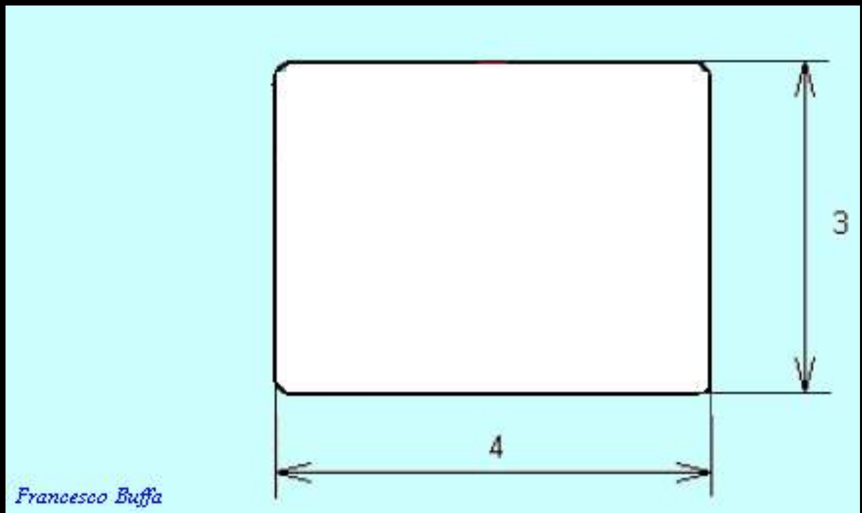



INFN Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

SPARC LAB


SCENA DA
RIPRENDERE
TELE
CAMERA
TRASMETTI
TORE
TELEVISORE
Francesco Buffa

**E = 10 keV**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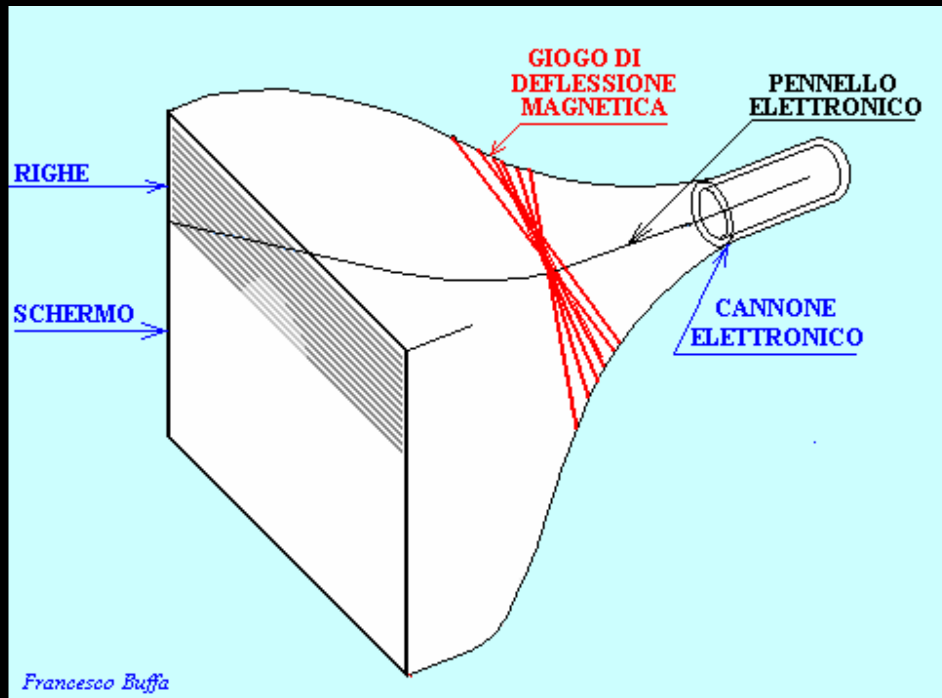
Francesco Buffa


GIOGO DI
DEFLESSIONE
MAGNETICA
PENNELLO
ELETTRONICO
RIGHE
SCHERMO
CANNONE
ELETTRONICO
Francesco Buffa

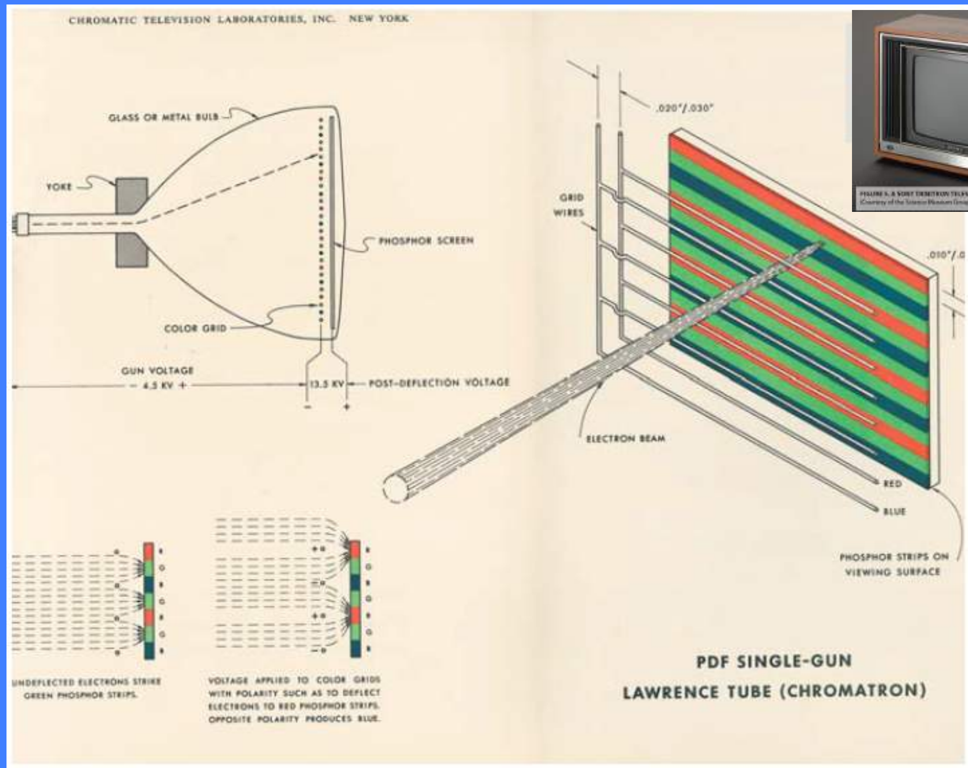
# The Chromatron

**Ernest Lawrence's brilliant failure**
Joshua Roebke

Citation: Physics Today **72**, 3, 32 (2019); doi: 10.1063/PT.3.4162
View online: https://doi.org/10.1063/PT.3.4162

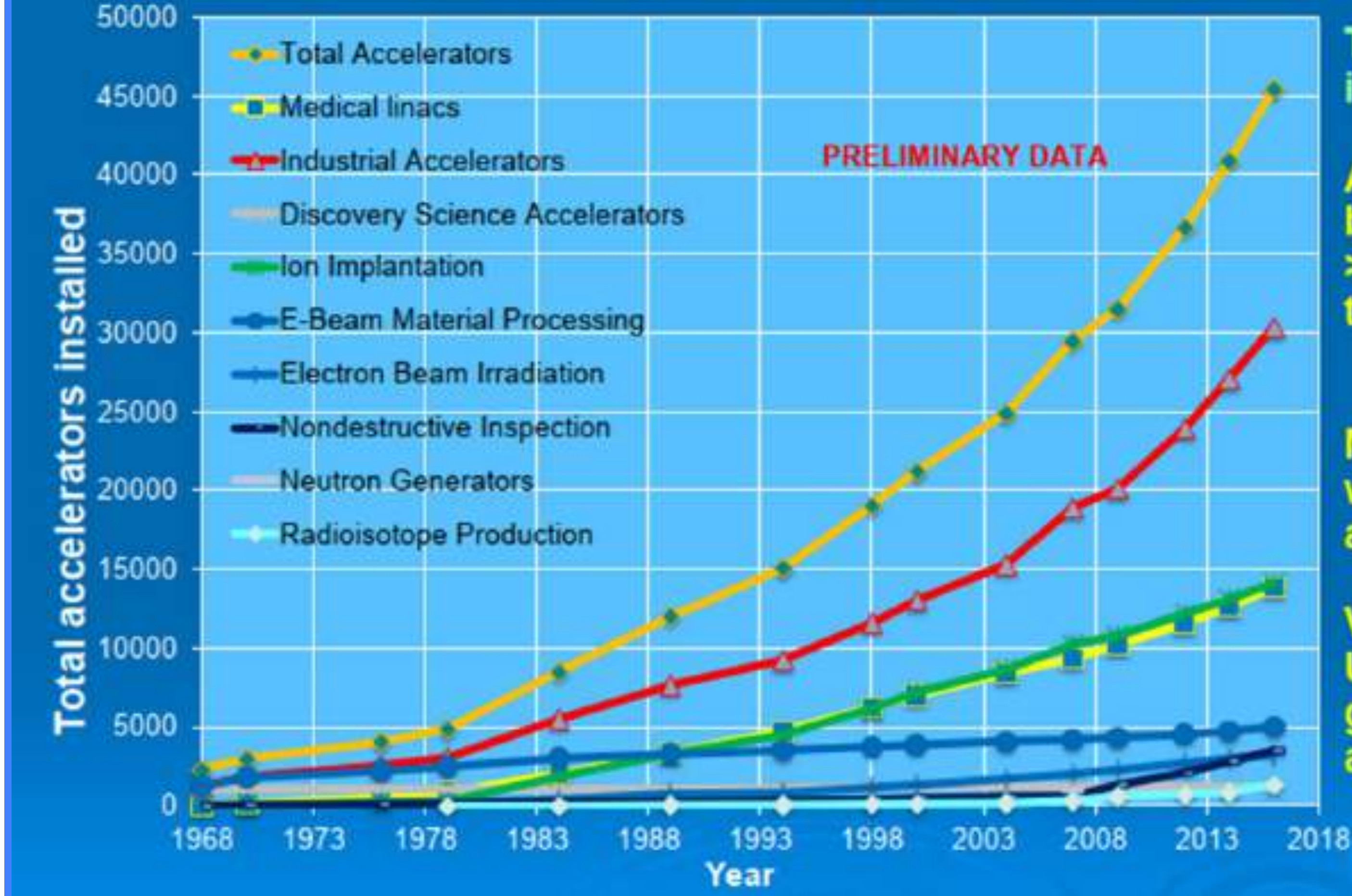
The physics of beams applied equally well to the electrons that transmitted sitcoms and to those that revealed the existence of quarks. Physicists readily adapted their expertise to improving broadcasts and sets.

FIGURE 3. ERNEST LAWRENCE, EDWIN MCMILLAN, AND LUIS ALVAREZ (left to right) admire a finished Chromatron. (Ernest O. Lawrence papers, BANC MSS 2005/200c, oversize box 3. Courtesy of the Bancroft Library, University of California, Berkeley.)




**Ernest Lawrence** founded a television "start-up" company (**Chromatic**) that employed dozens of physicists, including two future Nobel laureates—**Luis Alvarez and Edwin McMillan.**

# Accelerators installed worldwide




**Total sales of accelerators is ~US$5B annually**

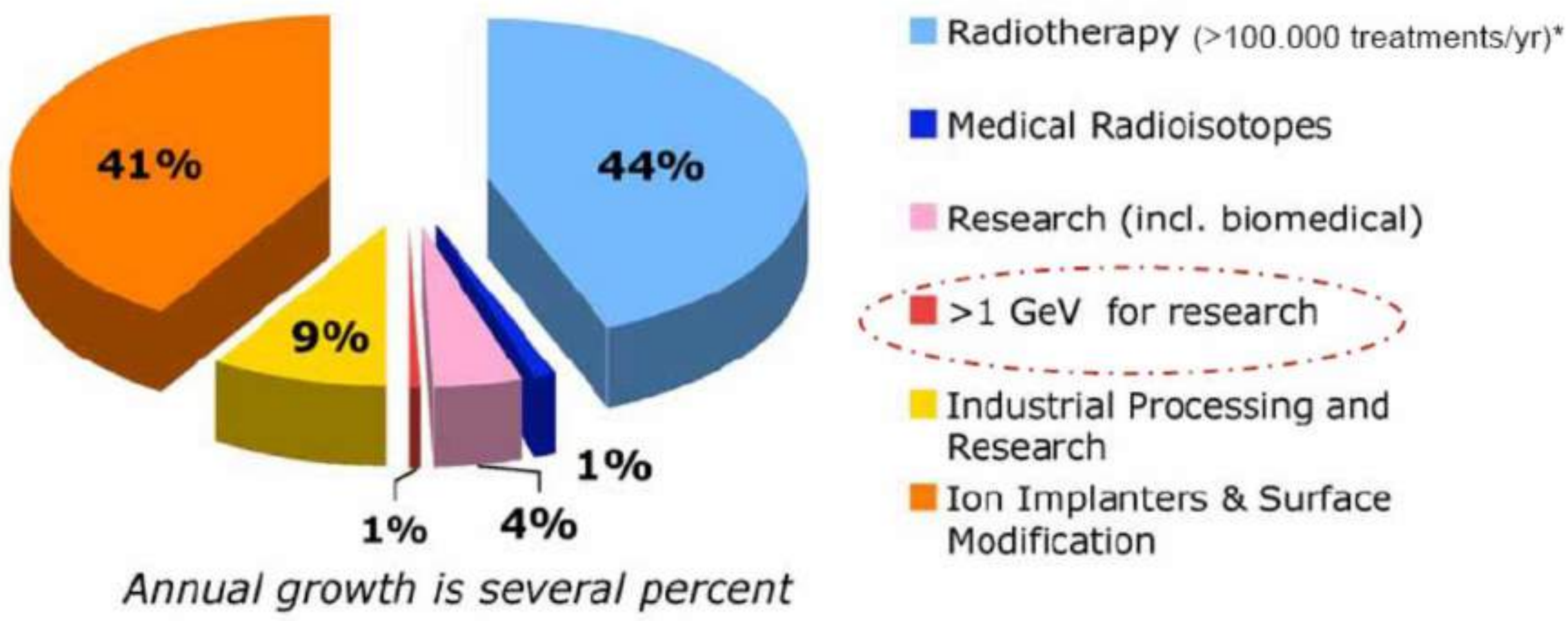
**About 47,000 systems have been sold,**
**> 40,000 still in operation today**

**More than 100 vendors worldwide are in the accelerator business.**

**Vendors are primarily in US, Europe and Japan, but growing in China, Russia and India**

-Accelerators for Americas Future Report, pp. 4, DoE, USA, 2011

# Accelerators installed worldwide



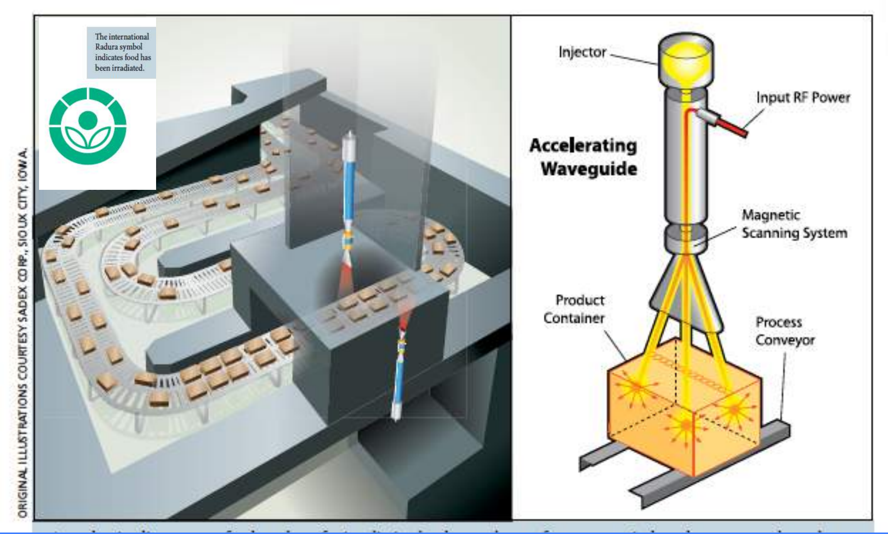

-Accelerators for Americas Future Report, pp. 4, DoE, USA, 2011

# Some application

# The Beam Business

**The beam business: Accelerators in industry**
Robert W. Hamm, and Marianne E. Hamm

Citation: Physics Today **64**, 6, 46 (2011); doi: 10.1063/1.3603918



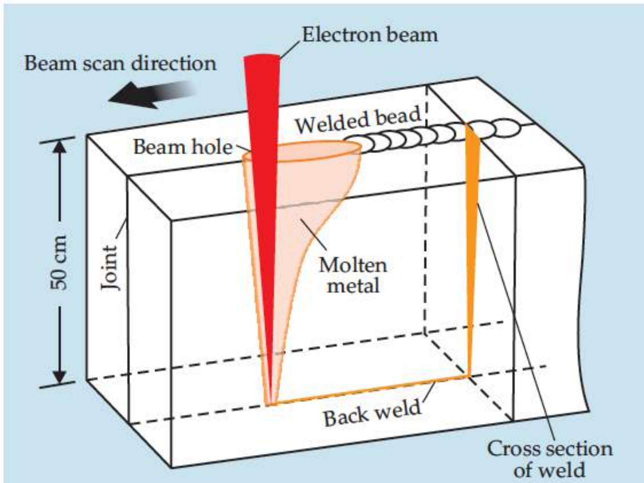
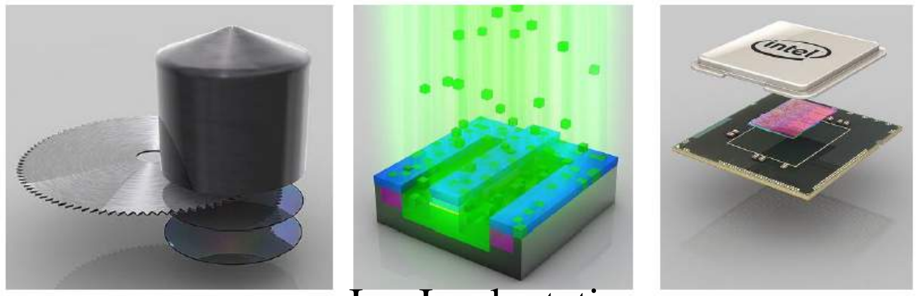




**Figure 3. Deep, precision industrial welding** of metals is done by high-energy-density electron beams. The beam is scanned along the joint to be welded. When it strikes the metal, the resultant heat instantly vaporizes the metal to form a channel as the beam penetrates into the work piece. Molten material flows around the hole and solidifies to form the weld. The method can produce welds with depth-to-width ratios up to 25:1. (Drawing courtesy of PTR-Precision Technologies, Enfield, CT.)



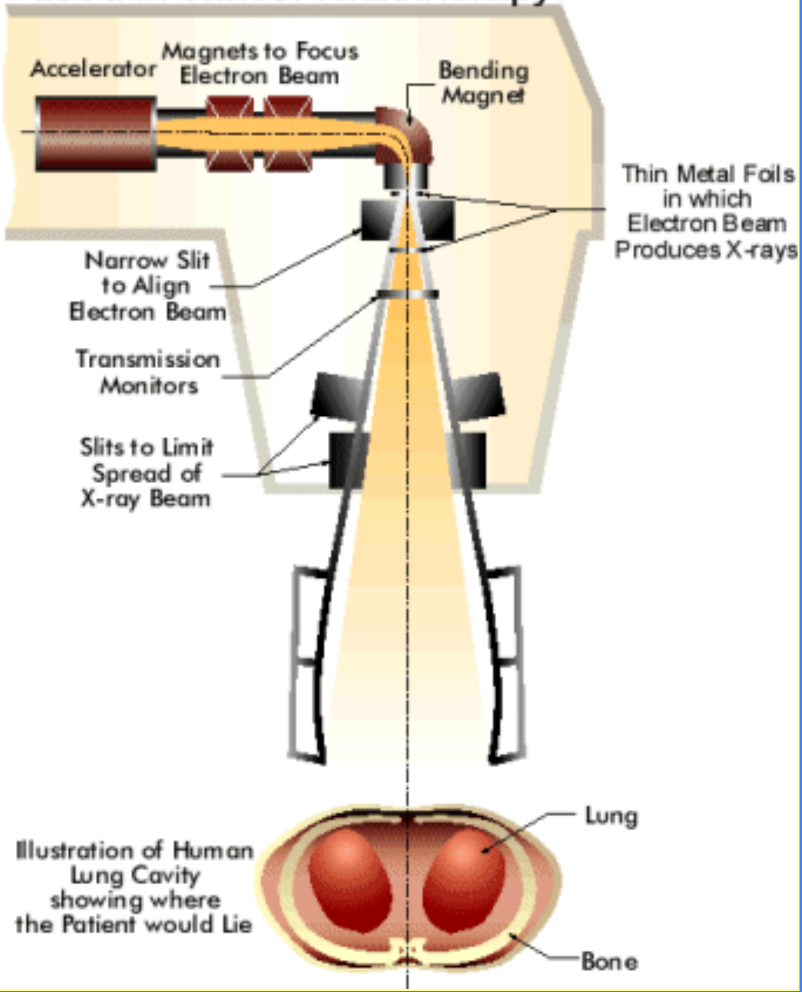

Images courtesy of Intel

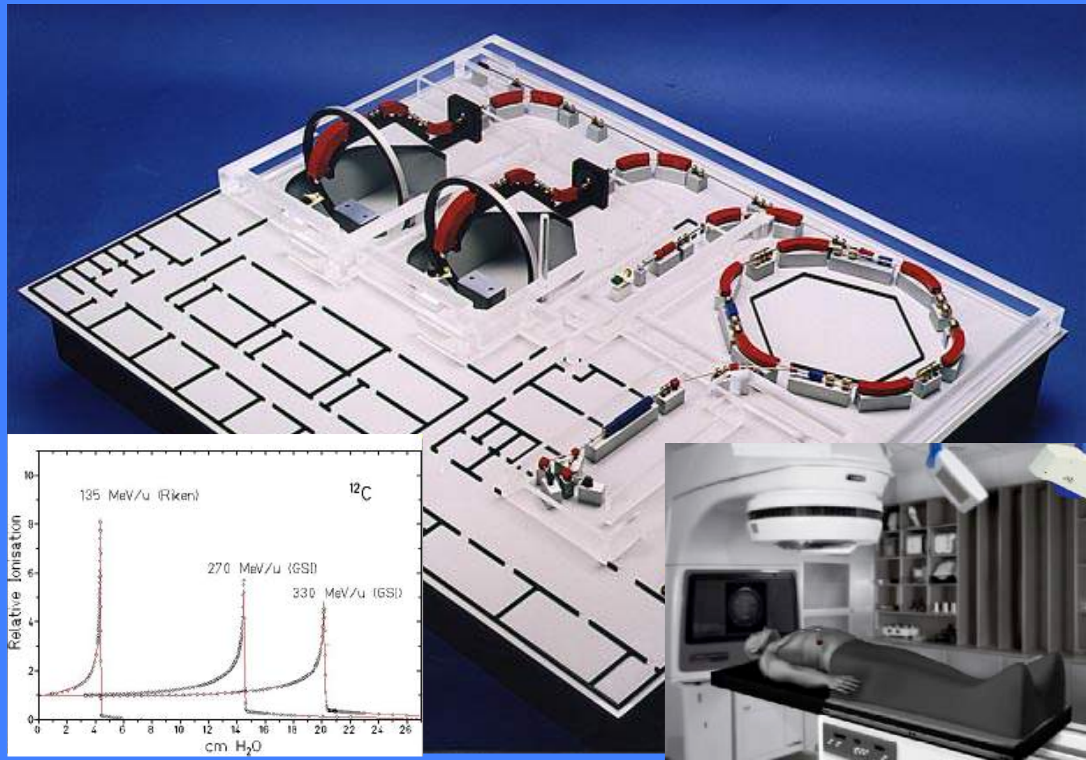
Ion Implantation

| Area | Application | Beam | Accelerator | Beam energy/MeV | Beam current/mA | Number |
|---|---|---|---|---|---|---|
| **Industrial** | Ion implantation | B, As, P | electrostatic | < 1 | 2 | >11000 |
| | Ion beam analysis | p, He | electrostatic | <5 | $10^{-4}$ | 300 |
| | Material processing | e | electrostatic, linac, Rhodatron | ≤10 | 150 | 7500 |
| | Sterilisation | e | electrostatic, linac, Rhodatron | ≤10 | 10 | 3000 |

# Medical Applications

Schematic diagram of a typical medical accelerator used in cancer radiotherapy.



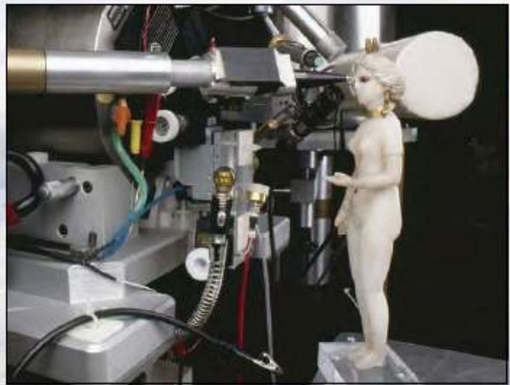

| Area | Application | Beam | Accelerator | Beam energy/MeV | Beam current/mA | Number |
|---|---|---|---|---|---|---|
| **Medical** | Cancer therapy | e | linac | 4-20 | $10^{-2}$ | >14000 |
| | | p | cyclotron, synchrotron | 250 | $10^{-6}$ | 60 |
| | | C | synchrotron | 4800 | $10^{-7}$ | 10 |
| | Radioisotope production | p | cyclotron | 8-100 | 1 | 1600 |

# National Security

# Cultural Heritage

## Why is an accelerator under the Louvre museum?

AGLAE, Accélérateur Grand Louvre d'Analyse Élémentaire in Paris, is the world's only accelerator facility fully dedicated to the study and investigation of works of art and archeological artifacts. It serves more than 1200 French museums. The 4-million-electron-volt proton beam delicately probes a large variety of materials: jewels, ceramics, glass, alloys, coins and statues, as well as paintings and drawings. These investigations provide information on the sources of the materials, the ancient formulas used to produce them, and the optimal ways to preserve these treasures.



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### The Story of Ishtar

In 1863, while excavating a tomb from the ancient Parthian civilization in Mesopotamia (200 BC – 200 AD), an amateur archeologist who was the French consul in Baghdad discovered this 5-inch-tall alabaster figurine representing the goddess Ishtar. He donated it to the Louvre. Recently a Louvre curator asked the AGLAE team to analyze the figurine's red eyes and red navel. The inlays turned out to be exquisite rubies, a great mystery since rubies are only found in remote lands like India or Southeast Asia. Analysis of rubies with known provenance from Paris jewelers yielded trace-element fingerprints showing that Ishtar's rubies originated in Burma — testifying to an unreported trade network (see map), perhaps by ship, between Babylon and Southeast Asia.


MAP BY T. CALLIGARO.

PHOTO BY OLIVIA DIAZ.

| Area | Application | Beam | Accelerator | Beam energy/MeV | Beam current/ mA | Number |
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| **Medical** | Cancer therapy | e | linac | 4-20 | $10^{-2}$ | >14000 |
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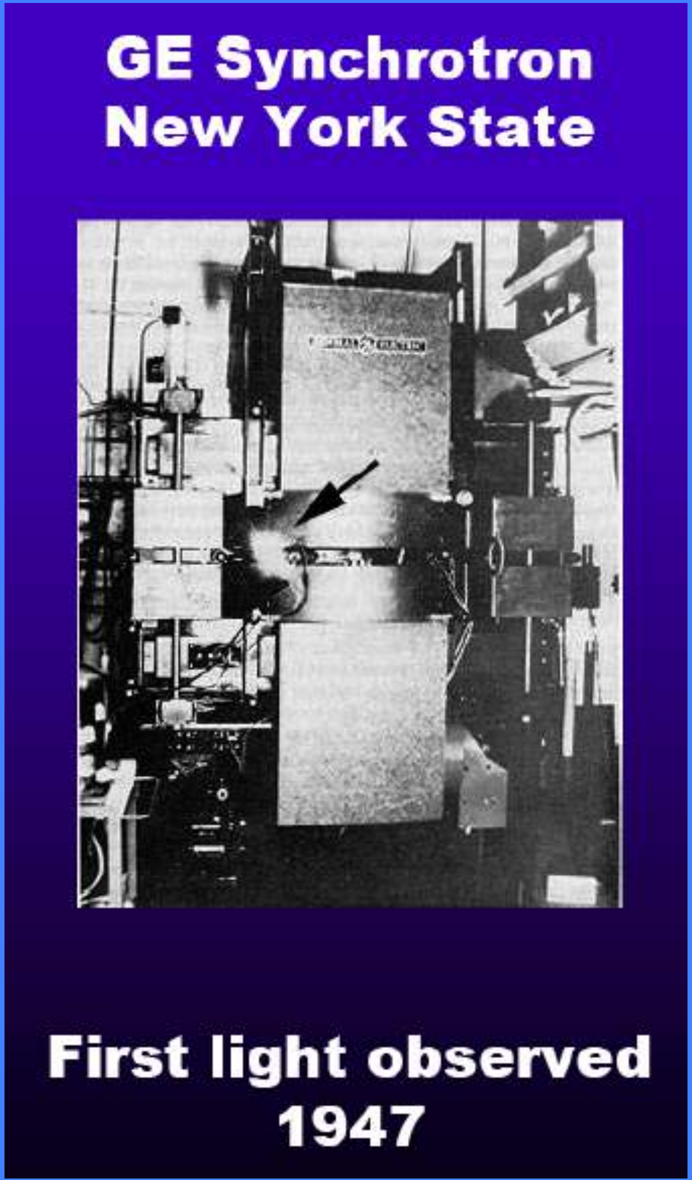
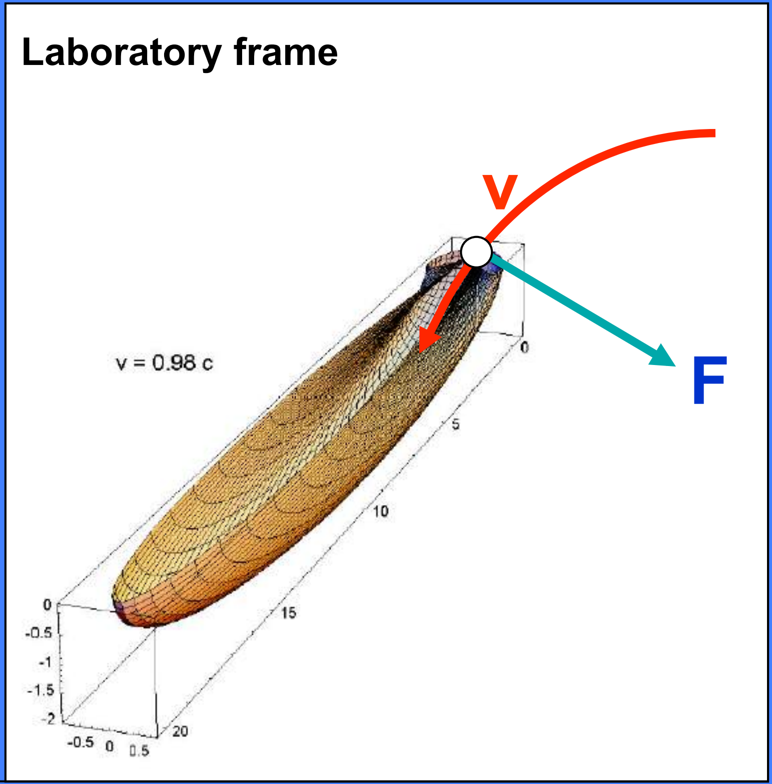
| | Sterilisation | e | electrostatic, linac, Rhodatron | ≤10 | 10 | 3000 |
| **Security** | X-ray screening of cargo | e | linac | 4-10 | ? | 100? |
| | Hydrodynamic testing | e | linear induction | 10-20 | 1000 | 5 |
| **Synchrotron light sources** | Biology, medicine, materials science | e | synchrotron, linac | 500-10000 | | 70 |
| **Neutron scattering** | Materials science | p | cyclotron, synchrotron, linac | 600-1000 | 2 | 4 |
| **Energy – fusion** | Neutral ion beam heating | d | electrostatic | 1 | 50 | 10 |
| | Heavy ion inertial fusion | Pb, Cs | Induction linac | 8 | 1000 | Under development |
| | Materials studies | d | linac | 40 | 125 | Under development |
| **Energy – fission** | Waste burner | p | linac | 600-1000 | 10 | Under development |
| | Thorium fuel amplifier | p | linac | 600-1000 | 10 | Under development |
| **Energy – bio-fuel** | Bio-fuel production | e | electrostatic | 5 | 10 | Under development |
| **Environmental** | Water treatment | e | electrostatic | 5 | 10 | 5 |
| | Flue gas treatment | e | electrostatic | 0.7 | 50 | Under development |

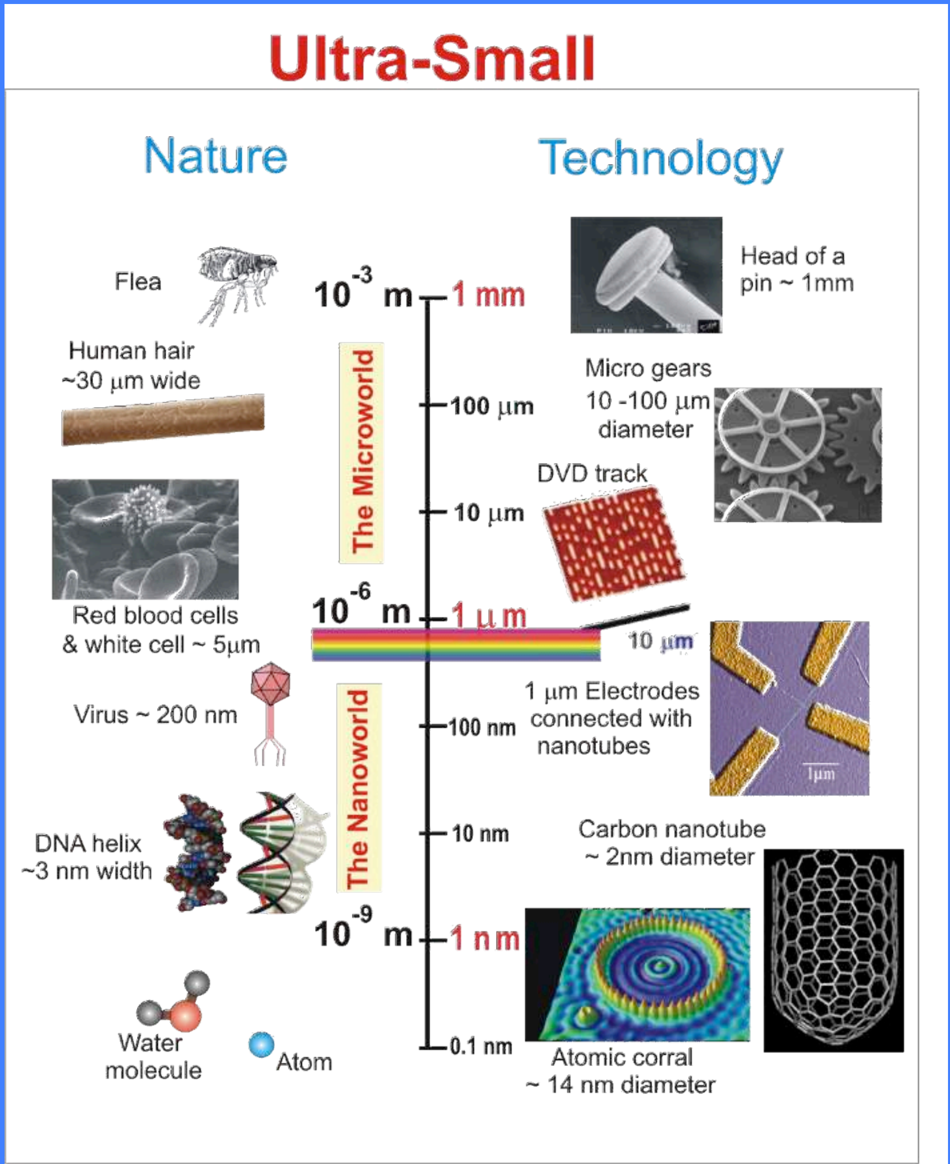
# Synchrotron Radiation

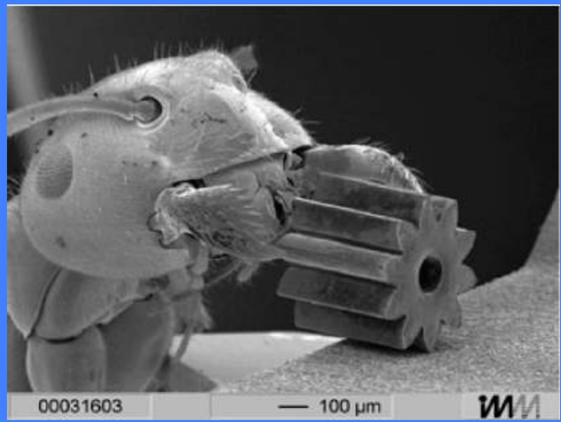


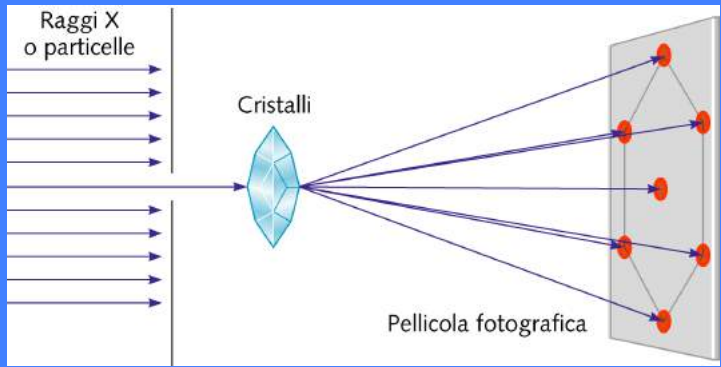

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$



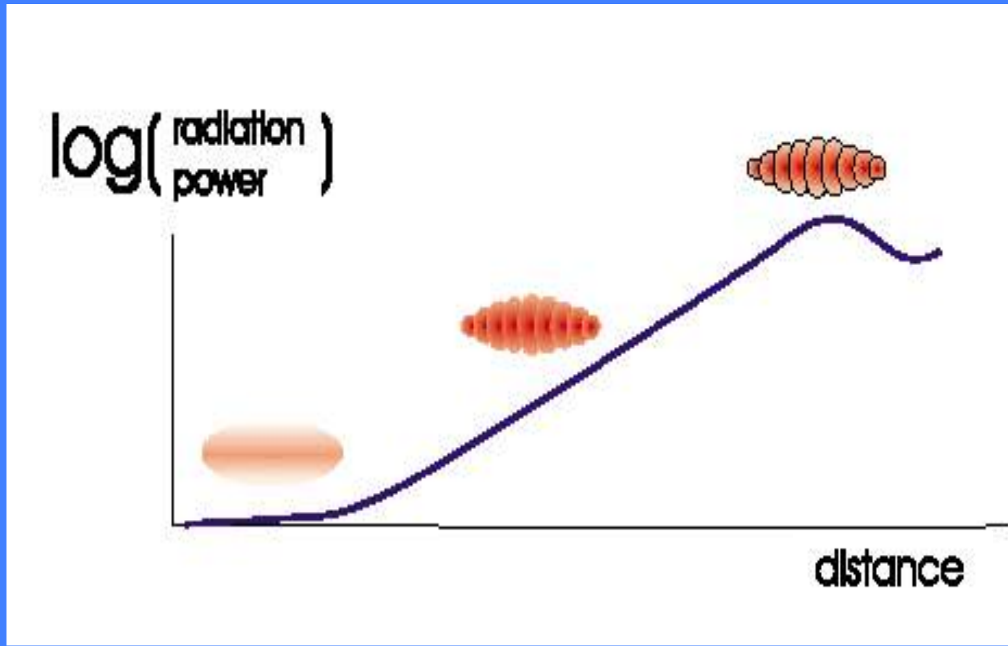
# Synchrotron Radiation



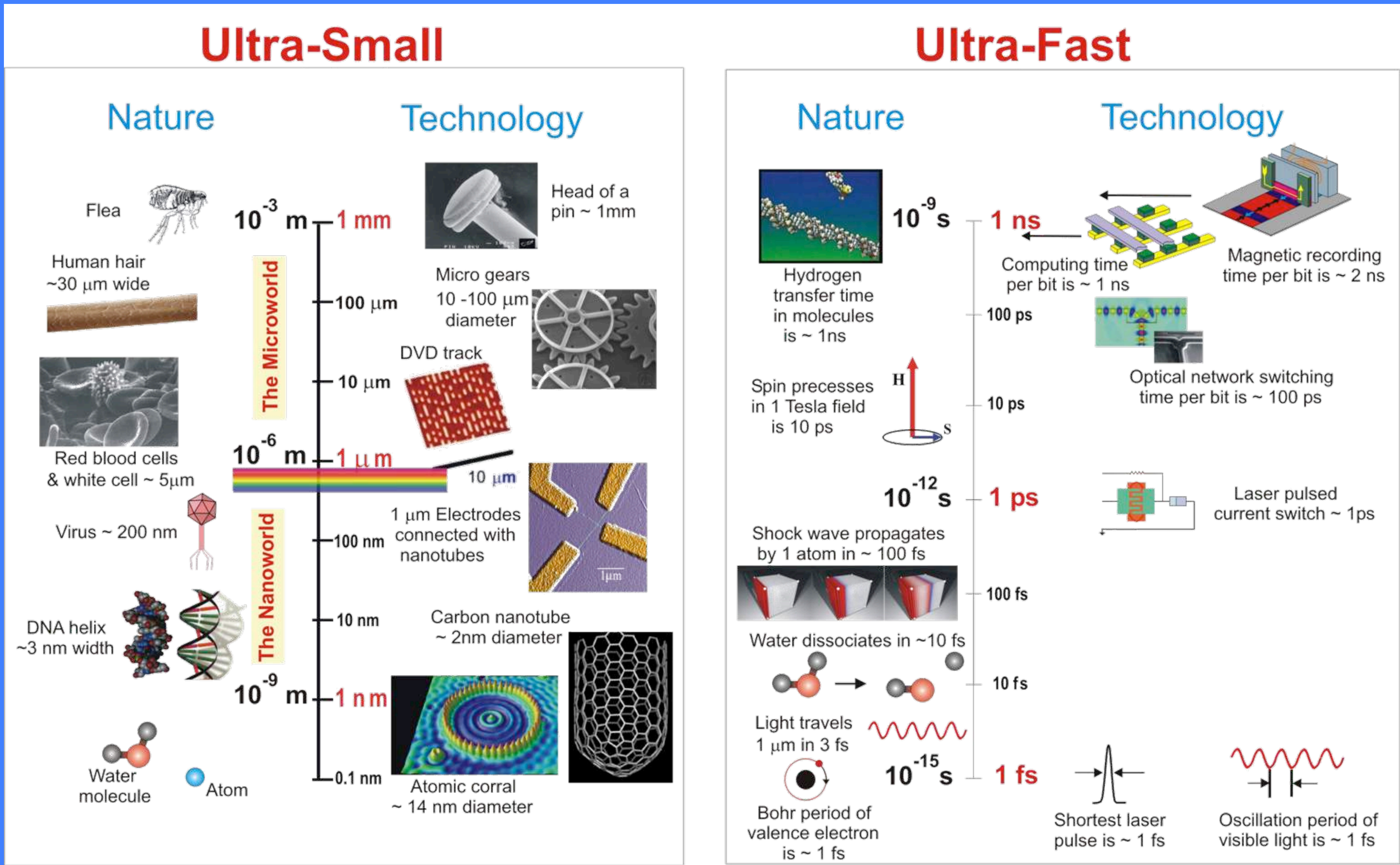
**A Free Electron Laser (FEL) is a device that converts a fraction of the electron kinetic energy into coherent radiation via a collective instability in a long undulator**




$$\lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\vartheta^2\right)$$

**(Tunability - Harmonics)**

# Free Electron Lasers



# Ultra-precise microscopy

- Probing particles are required for studies of the elementary constituents
- The associated de Broglie wavelength $\lambda$ of a probing particle defines the minimum object size that can be resolved.

$$\lambda = \frac{h}{p} = h \times \frac{c}{E} \quad \text{with} \begin{cases} h = 4 \times 10^{-15} \text{ eVs} & \text{(Plank constant)} \\ p = \text{momentum}, \; E = \text{energy} \end{cases}$$

Resolving Smaller Objects Requires Higher Momentum Probe Particles

## Example of probe wavelength

- electrons with $p$ = 1 keV/c $\Rightarrow$ $l$ = h/p = $4\times10^{-12}$ m
- photons with $E$ = 1 keV $\Rightarrow$ $l$ = $h\times c/E \sim 1.2 \times 10^{-9}$ m.
- → electrons have ~ 300 times better resolution than photons (electron-microscopy !)

## Typical microscopic sizes

- Atom $10^{-10}$ m
- Nucleus $10^{-14}$ m
- Proton $10^{-15}$ m
- Quark $10^{-19}$ m

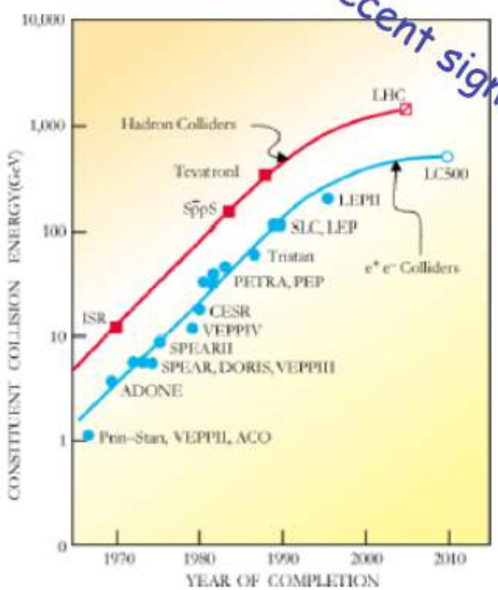
# Accelerator limitations and steps forward

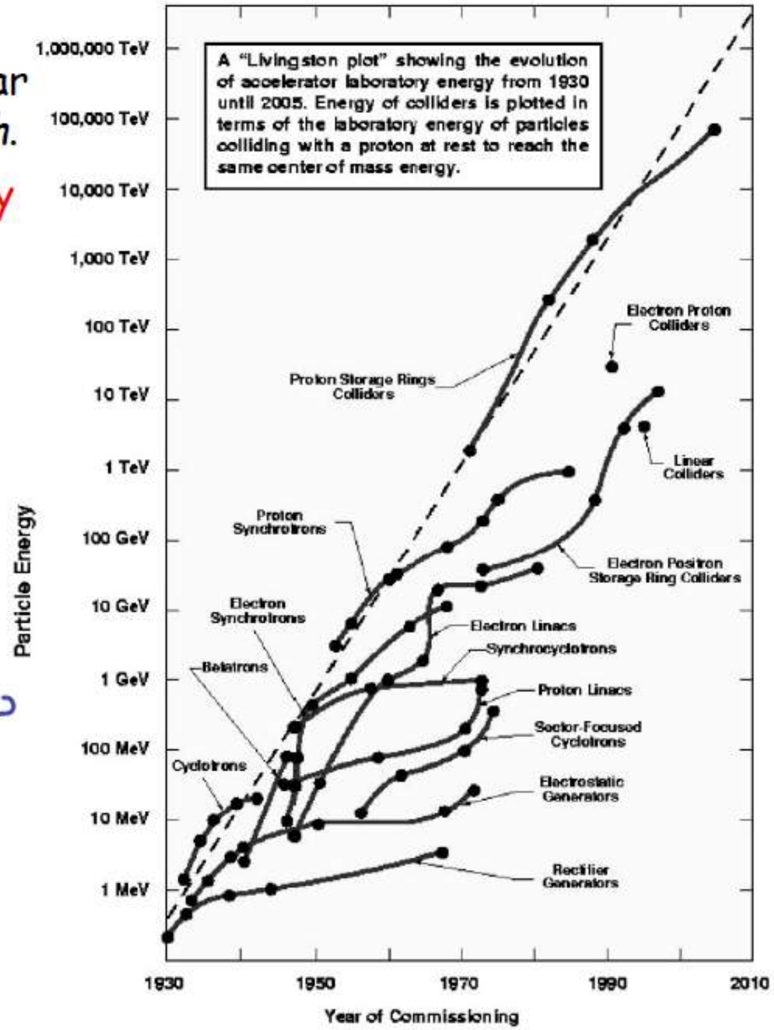
# The Livingstone's diagram

In 1950 Livingstone plotted the *accelerator energy* expressed in a semi-logarithmic scale as a function of the year of construction observing a *linear growth*.

The energy increase by a factor 33 every decade, mostly due to discoveries and technological advances.

Recent signs of saturation ?



# Historical Milestones

- 1900 to 1925 radioactive source experiments à la Rutherford -> request for higher energy beams;
- 1928 to 1932 electrostatic acceleration ->
  - Cockcroft & Walton * -> voltage multiplication using diodes and oscillating voltage (700 kV);
  - Van der Graaf -> voltage charging through mechanical belt (1.2 MV);

* Nobel 1951

- 1928 resonant acceleration -> Ising establish the concept, Wideroe builds the first linac;
- 1929 cyclotron ->
  - small prototype by Livingstone (PhD thesis), large scale by Lawrence**;

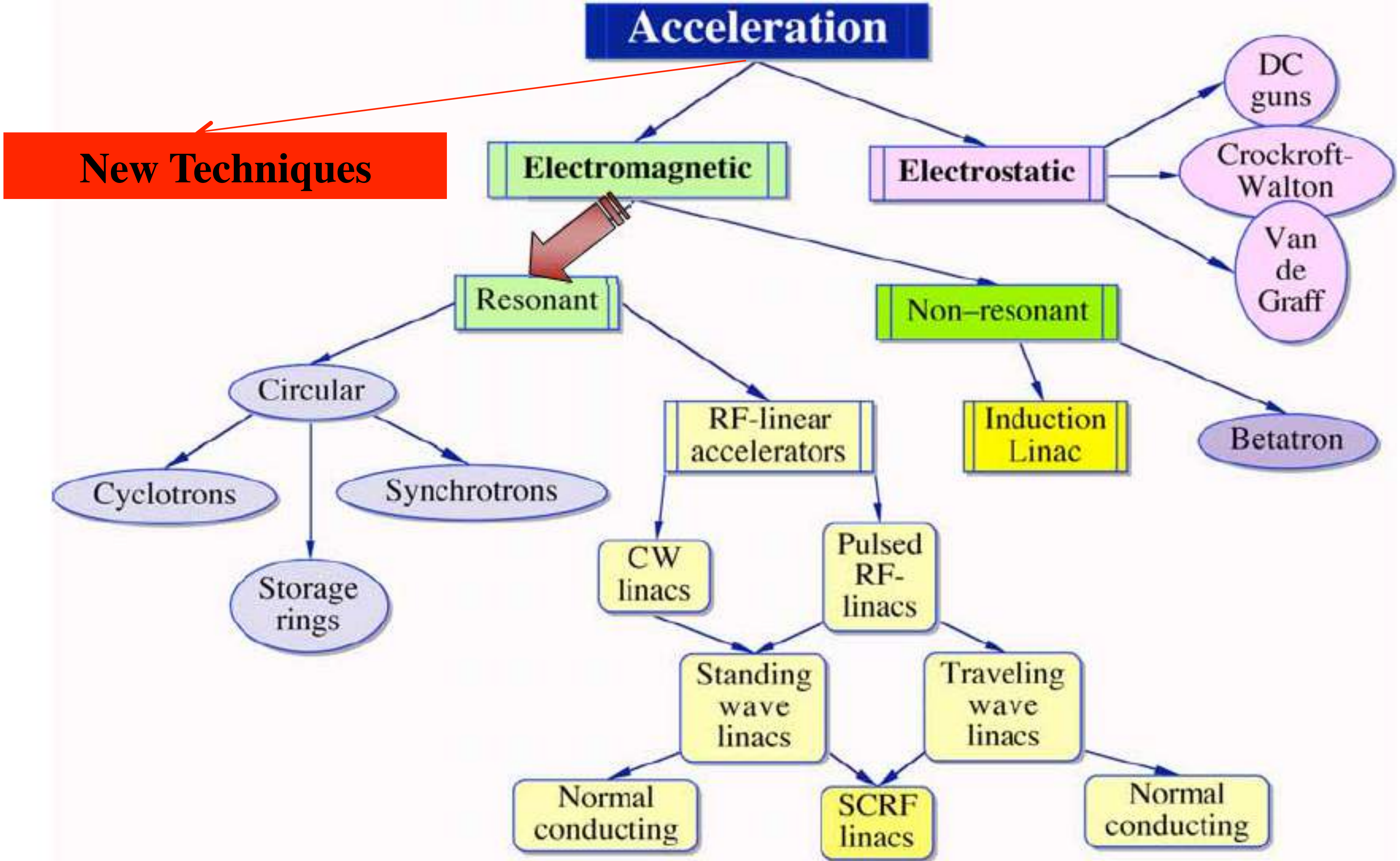
** Nobel 1939

- 1942 magnetic induction -> Kerst build the betatron;
- 1944 synchrotron -> MacMillan, Oliphant and Veksel invent the RF phase stability (longitudinal focusing);
- 1946 proton linac -> Alvarez build an RF structure with drift tubes (progressive wave in $2\pi$ mode);
- 1950 strong focusing -> Christofilos patent the alternate gradient concept (transverse strong focusing);
- 1951 tandem -> Alvarez upgrade the electrostatic acceleration concept and build a tandem;
- 1955 AGS -> Courant, Snider and Livingstone build the alternate gradient Cosmotron in Brookhaven;
- 1956 collider -> Kerst discuss the concept of colliding beams;
- 1961 $e^+e^-$ collider -> Touschek invent the concept of particle-antiparticle collider;
- 1967 electron cooling -> Budker proposes the e-cooling to increase the proton beam density;
- 1968 stochastic cooling ->
  - Van der Meer*** proposes the stochastic cooling to compress the phase space;

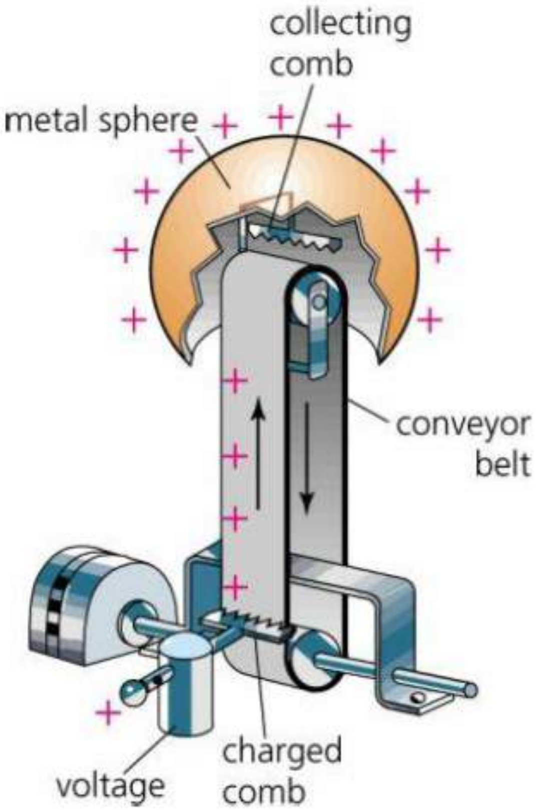
*** Nobel 1984

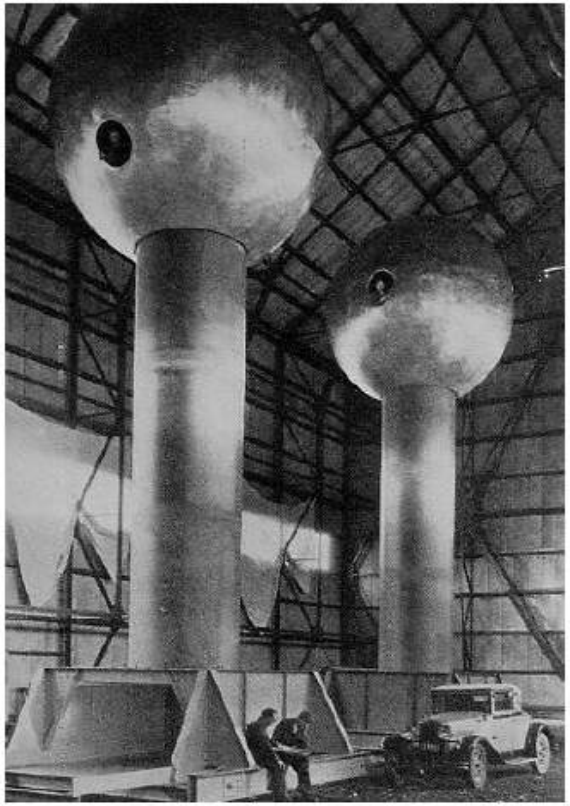
- 1970 RFQ -> Kapchinski & Telyakov build the radiofrequency quadrupole;
- 1980 to now superconducting magnets -> developed in various laboratories to increase the beam energy;
- 1980 to now superconducting RF -> developed in various lab to increase the RF gradient.

# Accelerator Configurations



# Electrostatic Accelerator: Van de Graff



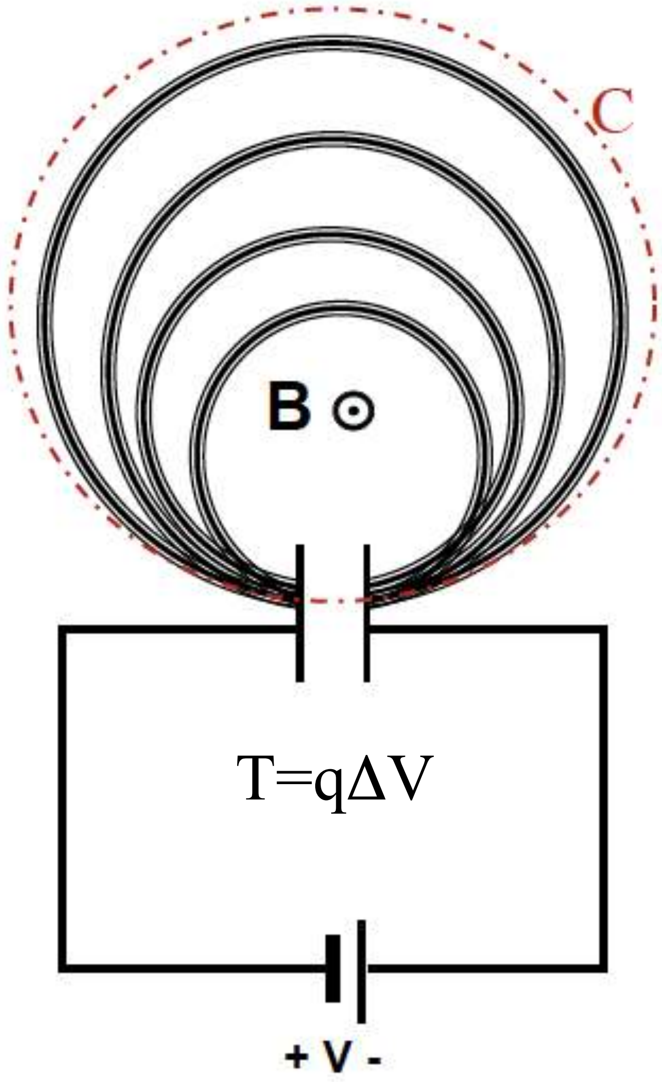



7 MV Van de Graaf at MIT (1933)

R.J. Van de Graaf

- Electric charges are transported mechanically on an insulating belt
- Stable, continuous beams, practical limit 10 - 15 MV

# Possible Higher energy DC accelerator?

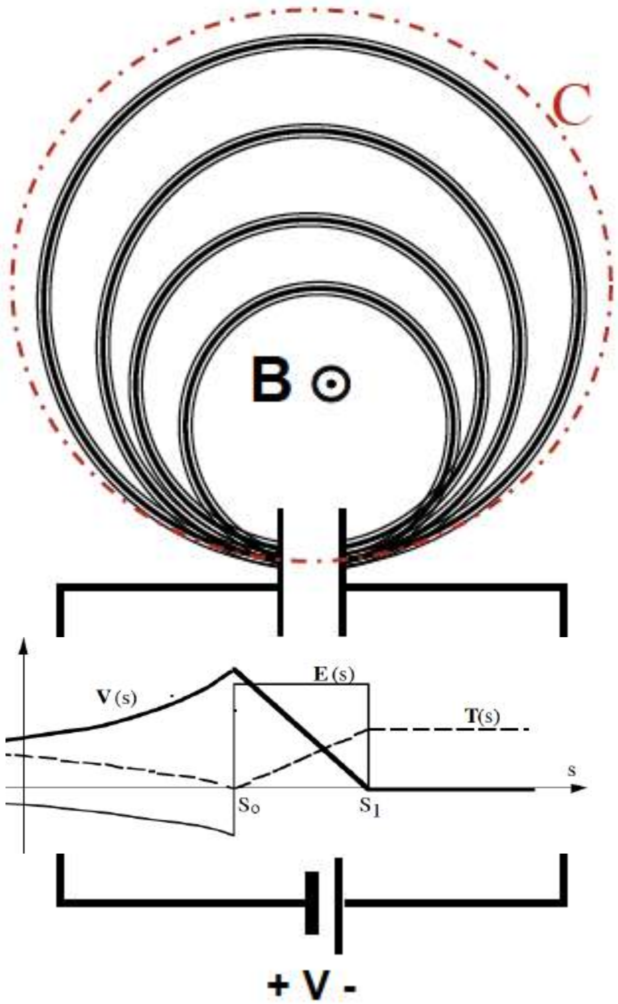



$$F_{Lorentz} = q\,v\,B = F_{centripital} = \frac{mv^2}{\rho}$$

$$\Rightarrow \rho = \frac{mv}{qB} = \frac{p}{qB}$$

$$\boxed{\rho(\mathrm{m}) = 3.34\left(\frac{p}{1\ \mathrm{GeV/c}}\right)\left(\frac{1}{q}\right)\left(\frac{1\ \mathrm{T}}{B}\right)}$$

# Forbidden by Maxwell




$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

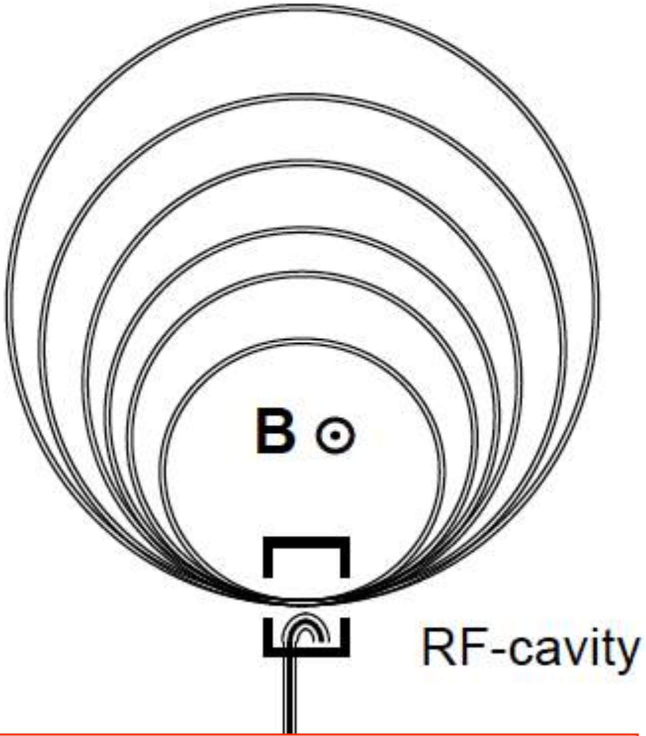
or in integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = \cancel{-\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}\, da}$$

∴ **There is no acceleration without time-varying magnetic flux**

$$\boxed{\Delta V_T = 0}$$

# B can vary in a RF cavity




$$E_z = E_0 J_0(k_r r) \cos \omega t$$

$$B_\theta = -\frac{E_0}{c} J_1(k_r r) \sin \omega t$$

**Note that inside the cavity**

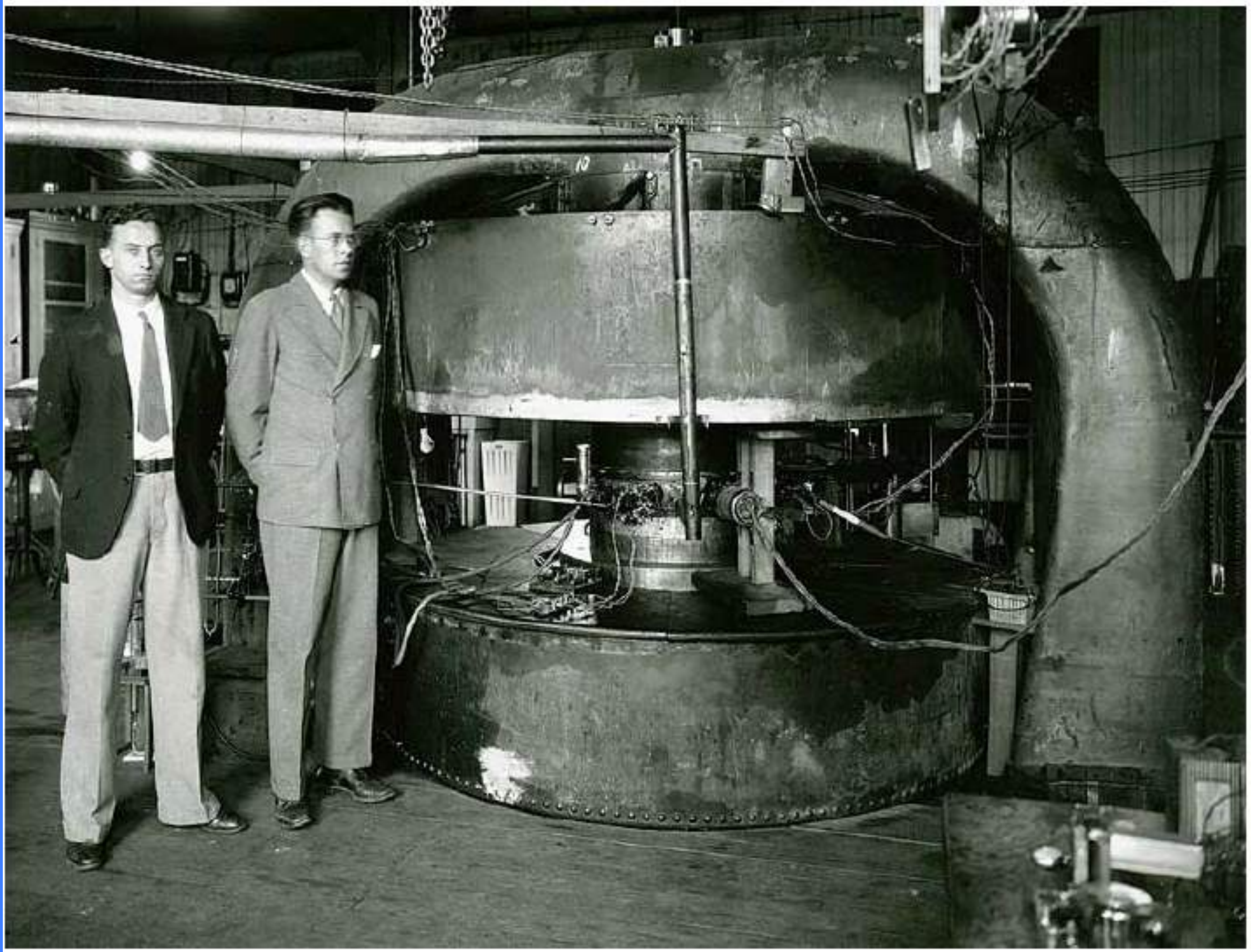
**dB/dt ≠ 0**

**However,**

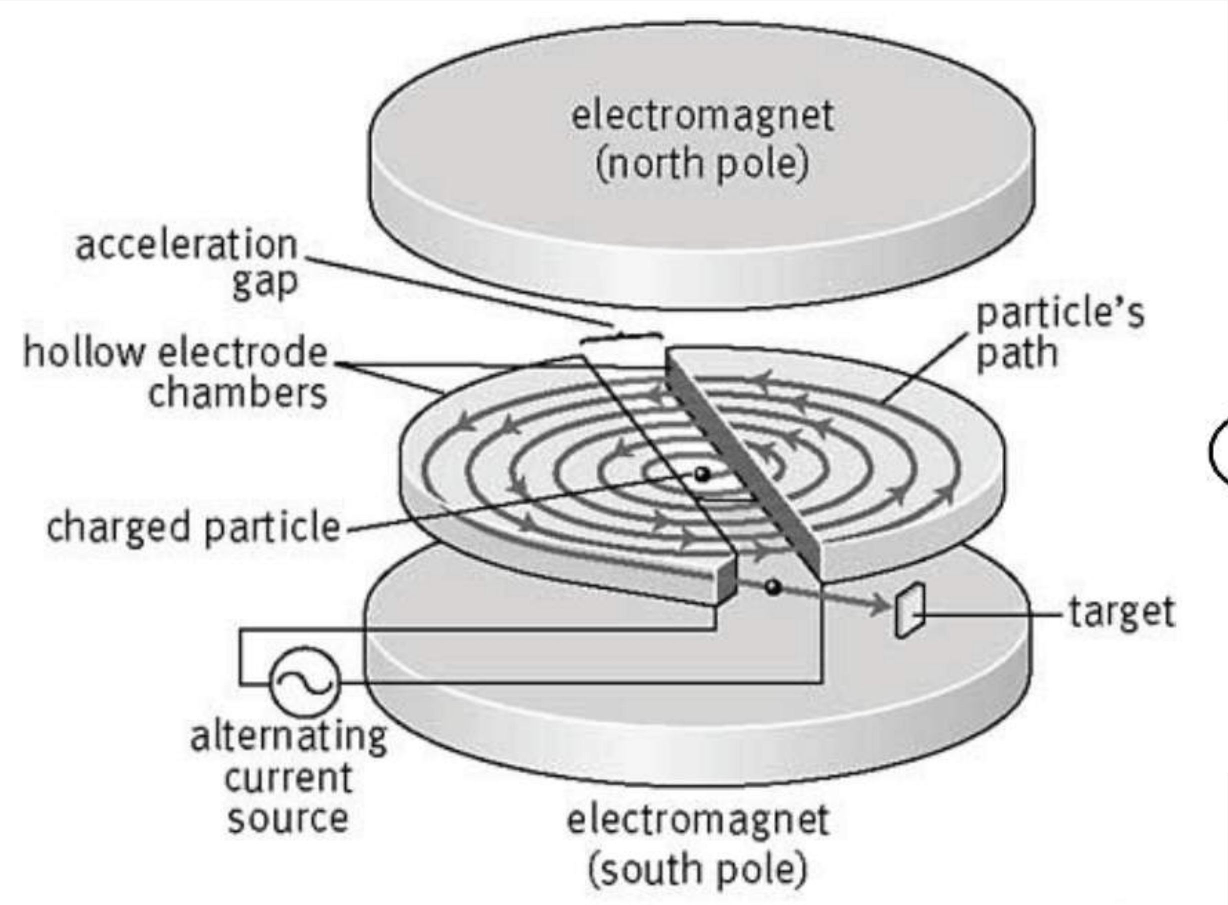
**Synchronism condition:**

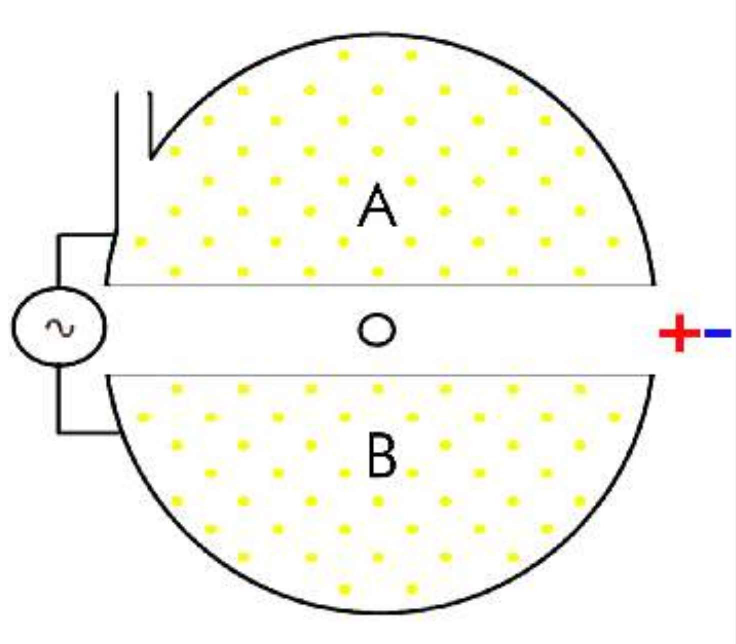
$\Delta\tau_{rev} = N/f_{rf}$

# The Lawrence Cyclotron



# The Cyclotron concept



# Cyclotron frequency

$$\dot{p}_x = m\dot{v}_x = e v_y B_z$$

$$\dot{p}_y = m\dot{v}_y = -e v_x B_z$$

$$v_x(t) = v_0 \cos(\omega_z t)$$

$$v_y(t) = v_0 \sin(\omega_z t)$$

$$\ddot{v}_x + \frac{e^2}{m^2} B_z^2 v_x = 0$$

$$\ddot{v}_y + \frac{e^2}{m^2} B_z^2 v_y = 0$$

$$\omega = \frac{e}{m} B_z$$

- **Cyclotron frequency**
- **Not dependent on the particle velocity**
- **Valid only for non relativistic case m constant**
- **=> m(E) increases**
- **=> Synchro-cyclotron -> reduce $\omega$(E)**
- **=> Iso-cyclotron ->increase B(E) to keep $\omega$ const**

# Relativistic particles

- Cyclotrons increase the energy of the particles by the same amount of energy at each turn.
- At low energy, the particles cross the gap at fixed frequency.
- At higher energy when relativistic corrections start to matter, the frequency at which they cross the gaps starts to decrease (the particles travel at the same speed ~c but follow a longer path).
- This can be addressed by varying the drive frequency but not all particles in the cyclotron are nearly at the same energy.
- The "classic" cyclotron is limited to about 20 MeV mainly due to the relativistic mass increasing

# 250 MeV proton cyclotron (ACCEL/Varian)

*Closed He system*

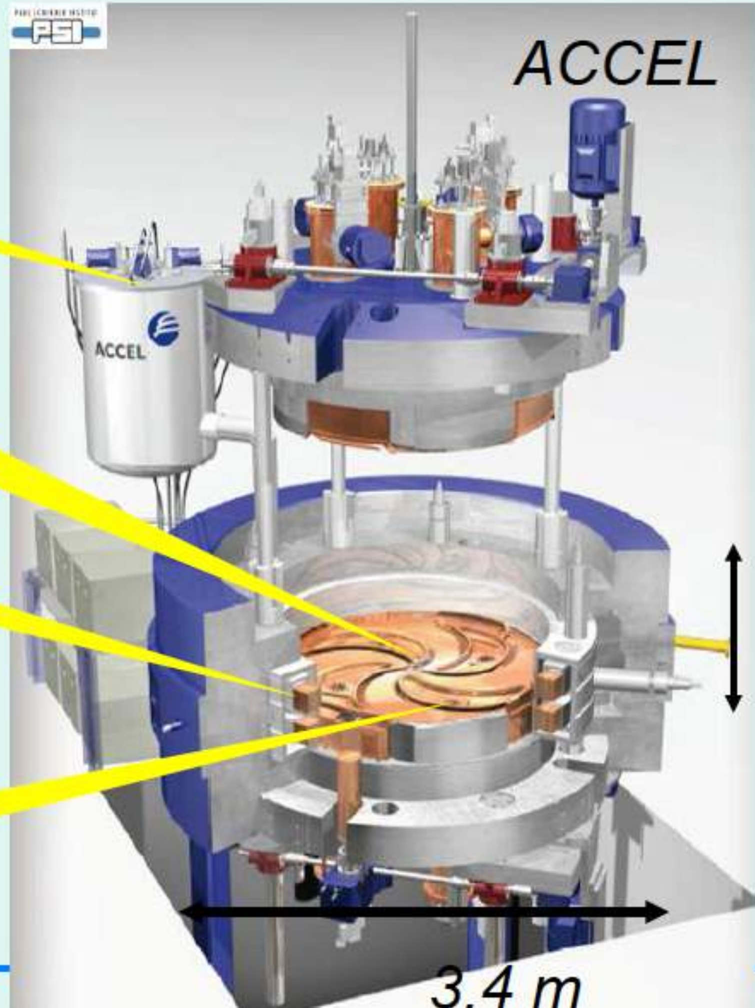
*4 x 1.5 W @4K*

*Proton source*

*superconducting coils =>*
*2.4 - 3.8 T*

*4 RF-cavities:*

*72 MHz ~80 kV*

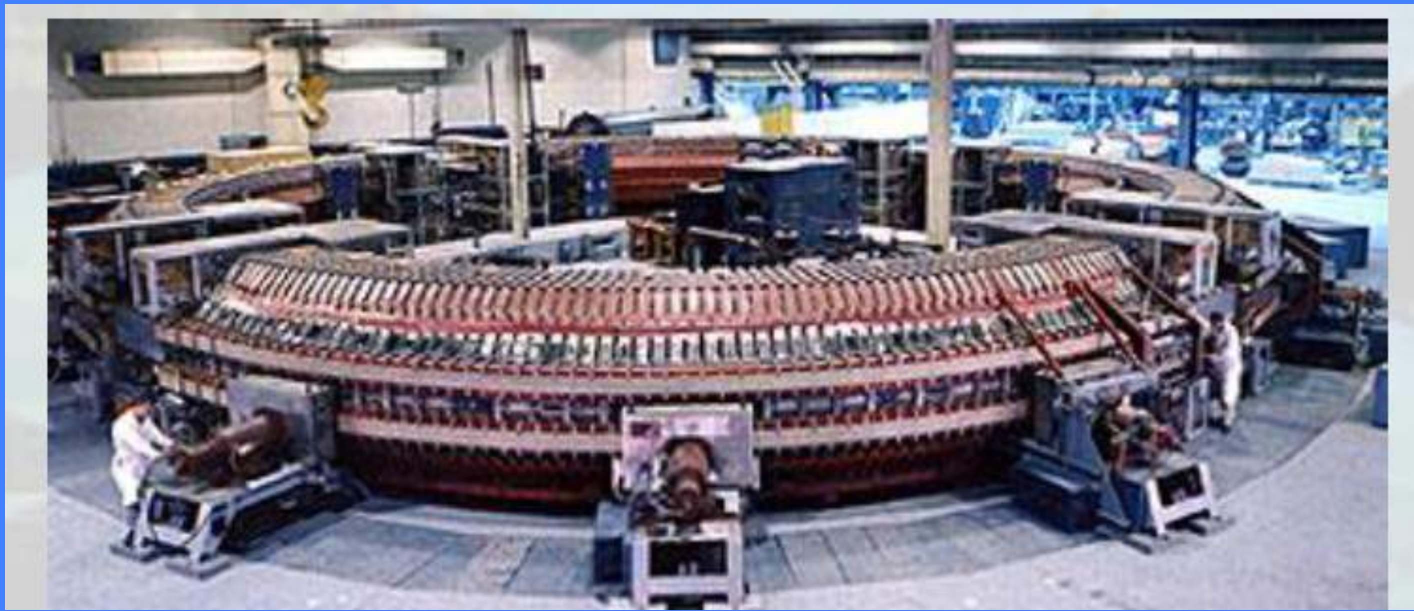



*300 kW*

*90 tons*

# The Synchrotron concept

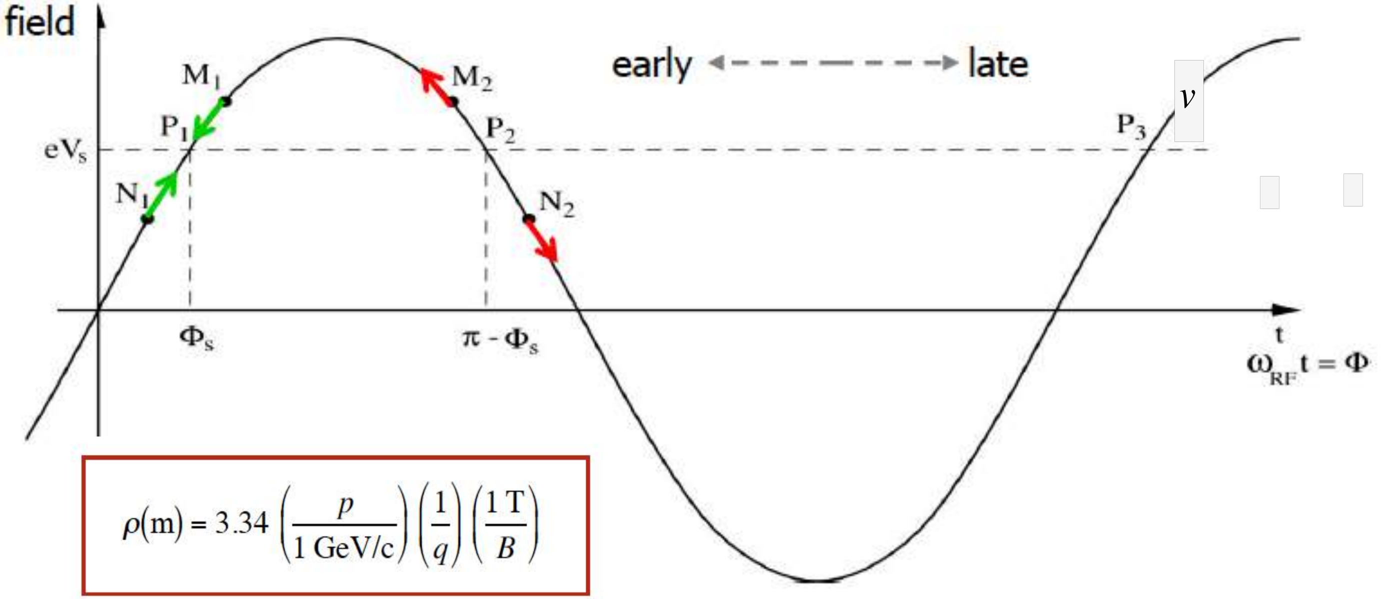
The main principle is to keep separated the bending and focusing devices (magnets of various types) from the ones that accelerates (resonant cavities).



There is main difference from cyclotrons: the particles always ride on the same orbit. Therefore:

- the cavities field must be synchronous with particle crossing and
- the bending magnet field must change in order to keep constant the radius of curvature.

# Phase stability and longitudinal focusing

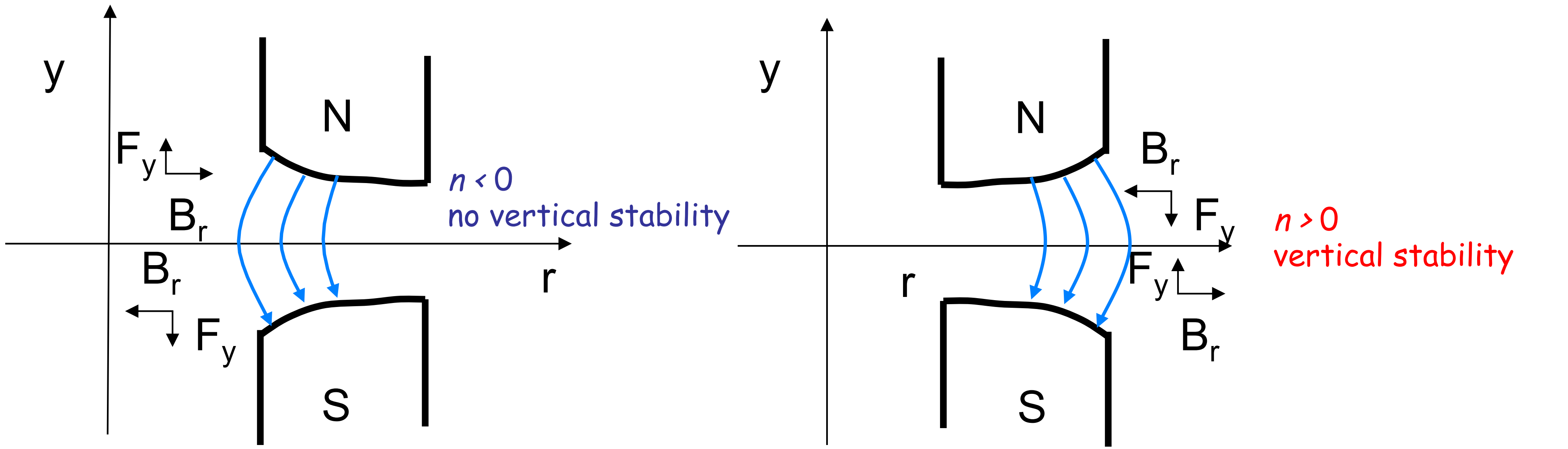



$$\rho(\mathrm{m}) = 3.34 \left(\frac{p}{1\ \mathrm{GeV/c}}\right)\left(\frac{1}{q}\right)\left(\frac{1\ \mathrm{T}}{B}\right)$$

- In a certain energy range, acceleration by RF field results in early arrival of particle at next turn: for stability, this particle should undergo less acceleration
- Operating point P2 is unstable
  - Late particle N2 sees lower acceleration and gets even later
  - Early particle M2 sees higher acceleration and gets even earlier
- Operating point P1 is stable

# Weak focusing and transverse stability

- to get vertical stability, the bending field should decrease with $\rho$, as in cyclotrons,
- to get horizontal stability the the decrease of $B$ with $\rho$ should be moderate, so that, for $\rho > \rho_0$, the Lorenz force exceeds the centripetal force.




$$B_y = B_0 \left( \frac{\rho_0}{\rho} \right)^n \qquad \text{guiding field}$$

$$n = -\frac{\rho}{B_y} \left( \frac{\delta B_y}{\delta \rho} \right)_{\rho=\rho_0} \qquad \text{field index}$$

horizontal stability:

centripetal force $\leq$ Lorenz force

$$\frac{mv^2}{\rho} = \frac{mv^2}{\rho_0 + x} \approx \frac{mv^2}{\rho_0}\left(1 - \frac{x}{\rho_0}\right) \leq evB_y = evB_0 \left( \frac{\rho_0}{\rho_0 + x} \right)^n \approx evB_0 \left( 1 - n\frac{x}{\rho_0} \right)$$

weak focusing

$$\boxed{\begin{array}{ll} \text{horizontal stability} & \Rightarrow n \leq 1 \\ \text{vertical stability} & \Rightarrow n > 0 \end{array}}$$

# From Weak focusing To Strong Focusing

- The principle of weak focusing has one serious drawback: when the trajectory oscillation wavelength is larger than the circumference of the machine one gets large deviations from the orbit if the circumference is large.
- The magnet apertures must be very big.
- The apertures can be drastically reduced if one applies strong focusing (n much larger than 1).
- This is impossible in a machine which has a guide and focusing field independent of the azimuthal angle, since in that case the condition $0 < n < 1$ has to hold, as we have just shown.
- It is, however, possible if we split up the machine into a series of magnetic sectors in which in alternating order the magnetic field increases strongly with increasing radius ( $n << 1$) or decreases strongly with increasing radius.

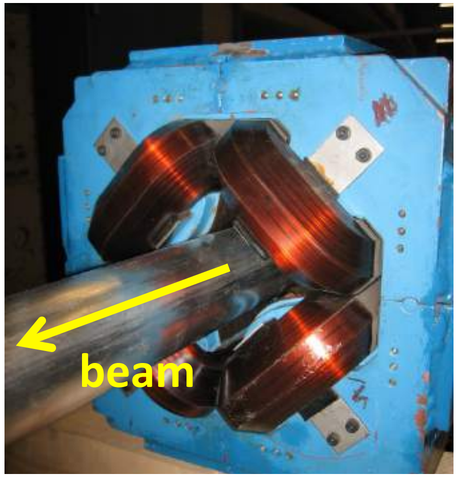
# MAGNETIC QUADRUPOLE

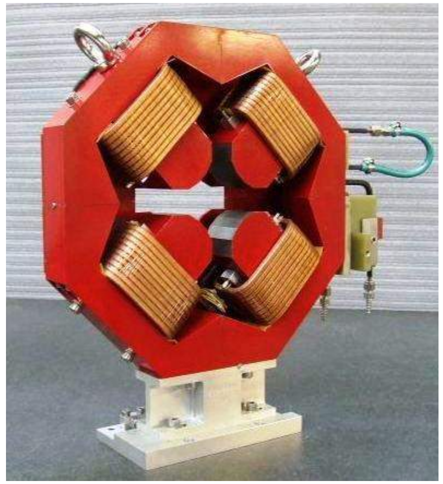
Quadrupoles are used to **focalize the beam in the transverse plane**. It is a **4 poles magnet**:

⇒**B=0 in the center of the quadrupole**

⇒The **B intensity increases linearly** with the off-axis displacement.

⇒If the quadrupole is **focusing in one plane is defocusing in the other plane**



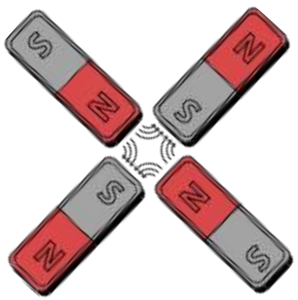



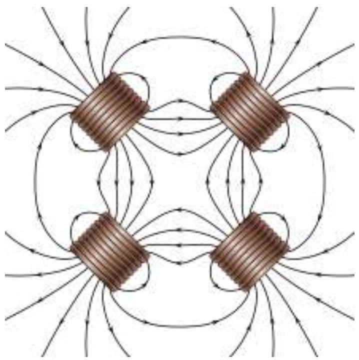
$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases} \Rightarrow \begin{cases} F_y = qvG \cdot y \\ F_x = -qvG \cdot x \end{cases}$$

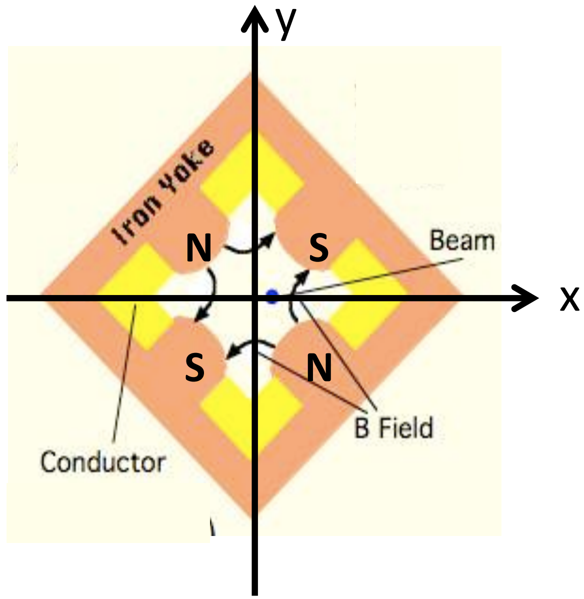
$$G = \text{quadrupole gradient} \left[\frac{T}{m}\right]$$

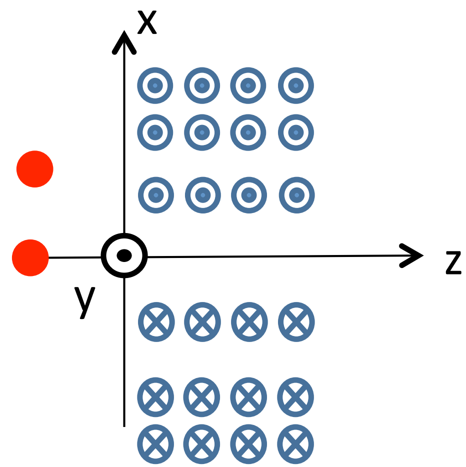
Electromagnetic quadrupoles G <50-100 T/m

$$\frac{F_B}{F_E} = v \Rightarrow \begin{cases} F_B(1T) = F_E\left(300\frac{MV}{m}\right) @ \beta = 1 \\ F_B(1T) = F_E\left(3\frac{MV}{m}\right) @ \beta = 0.01 \end{cases}$$

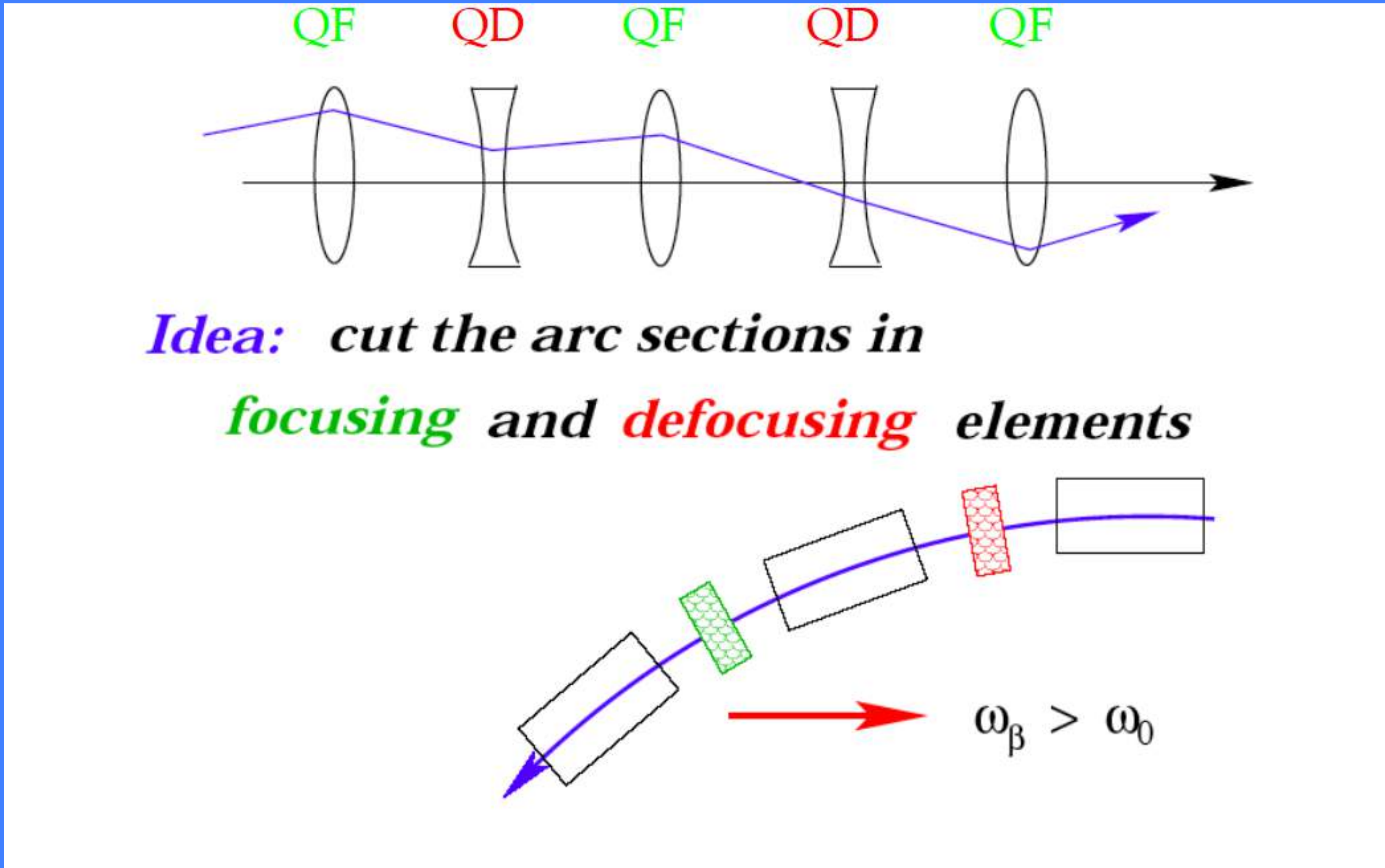


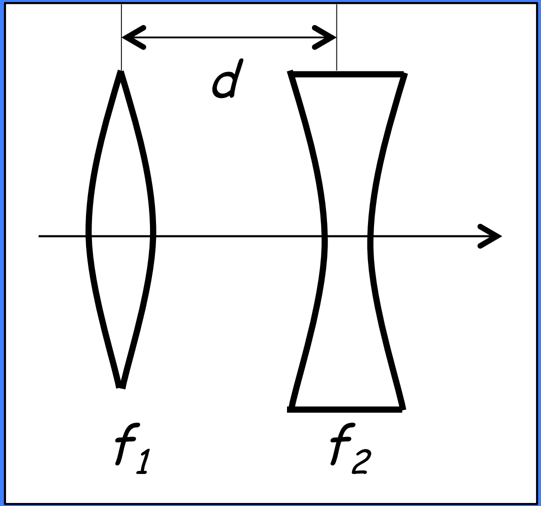


# Alternate Gradient Focusing



# Strong focusing




$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

⇒ Horizontal and vertical focusing for a large range of $f_1$ $f_2$ and $d$

- separated functions: the alternate gradient is made with quadrupoles of opposite focusing strength
- combined functions: the alternate gradient is made with dipoles with radial shape of opposite sign

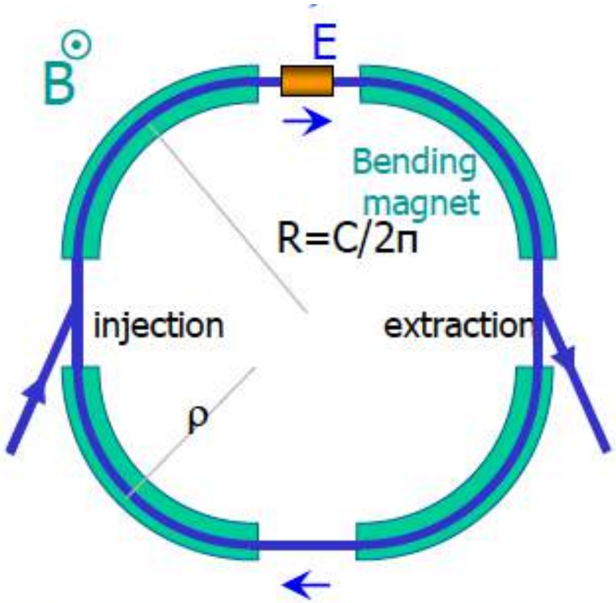
normalized quadrupole gradient $$K = \frac{eB'}{p} = \frac{B'}{B\rho} \Rightarrow K\left[m^{-2}\right] = 0.2998 \frac{B'\left[Tm^{-1}\right]}{p\left[GeV/c\right]}$$

quadrupole strength $$\frac{1}{f} = KL$$

Examples

| Ring | p [GeV/c] | $B_0$ [T] | ρ [m] | Bρ [Tm] | $1/\rho^2$ [$m^{-2}$] weak focus | $L_{quad}$ [m] | B' [T/m] | K [$m^{-2}$] |
|---|---|---|---|---|---|---|---|---|
| CERS | 5.2 | 0.18 | 96.4 | 17.3 | $10^{-4}$ | 0.5 | 5 | 0.298 |
| Tevatron | 1000 | 4.4 | 758 | 3335 | $1.7 \cdot 10^{-6}$ | 1.7 | 76 | 0.0228 |

3 GeV Cosmotron at BNL
weak focussing, combined function magnets



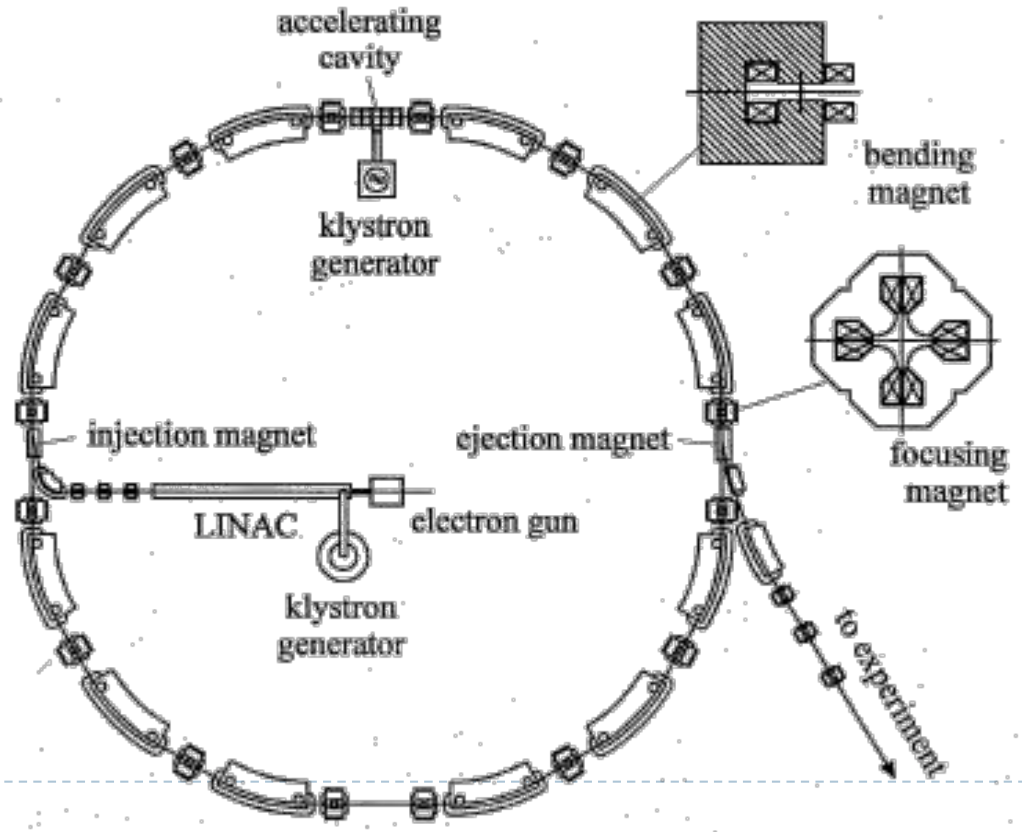

28 GeV PS at CERN
strong focussing, combined function magnets

400 GeV SPS at CERN
strong focussing, separated function magnets

# Synchrotron

- Increasing the energy, and so the radius, the size and the cost of the magnet are unaffordable
- The solution is to keep the radius constant and to ramp B
- The magnetic field is generated by **small** magnetic element around the ring and the central part is not needed anymore

$$\frac{1}{R} = \frac{eB}{p}$$



# Maximum energy

$$\Delta E_{\mathrm{beam}} = eU_{\mathrm{max}} \sin \Psi_0 - \Delta E_{\mathrm{loss}}.$$

$$\Delta E_{loss} = \frac{4}{3} \pi \alpha \hbar c \frac{\gamma^4}{\rho}$$

- The energy loss is very important mainly for electrons
- It is not possible to accelerate starting to E=0 MeV
- So a linac is needed in order to fill the ring
- For more stable system another rings is needed (booster)

# Fermi's Globatron: ~5000 TeV Proton beam

## 1954 the ultimate synchrotron

$B_{max}$ 2 Tesla

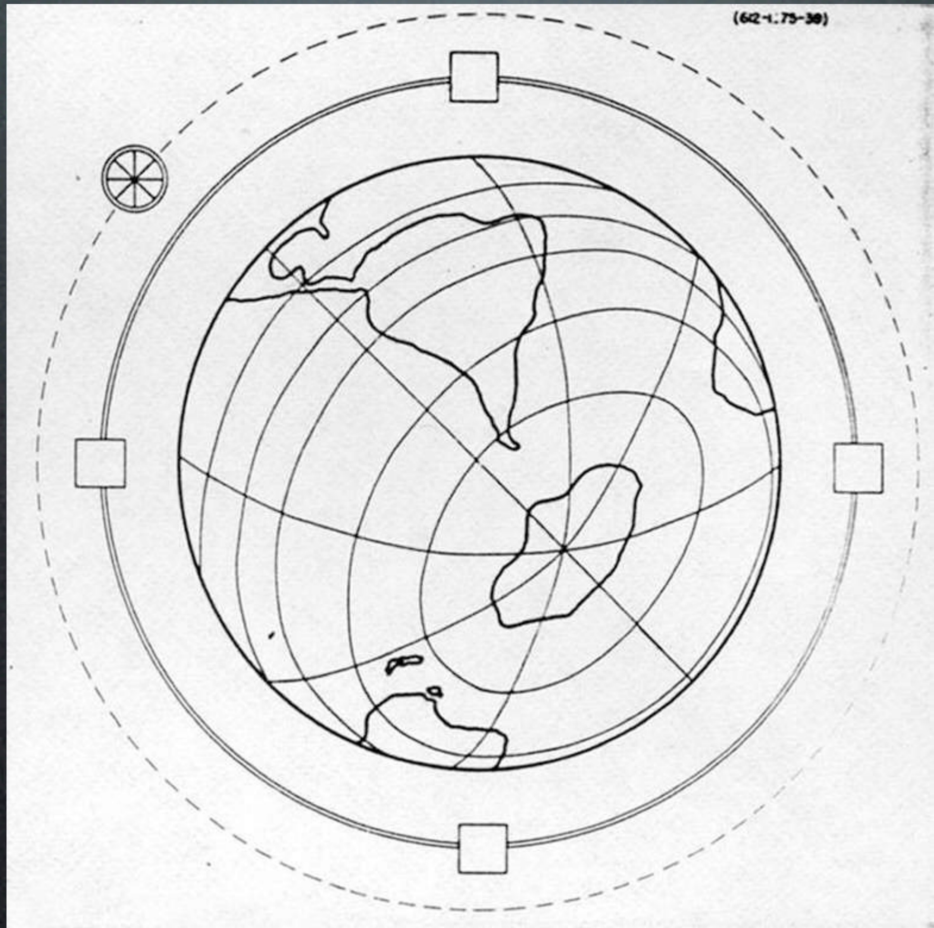
$\rho$ 8000 km

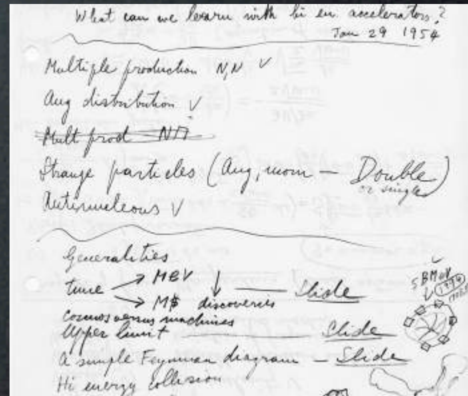
fixed target

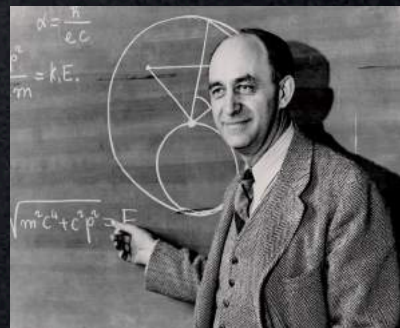
3 TeV c.m.

170 G$

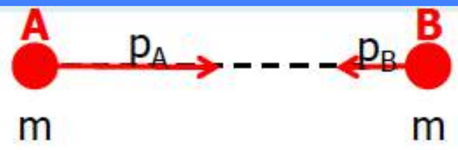
1994





# From Fixed Target to Colliders




- Relativistic invariant $(\Sigma m)^2c^4 = (\Sigma E)^2 - (\Sigma p)^2c^2$
- In the laboratory frame $4m^2c^4 = (E_A + E_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2c^2$

- Let $E^*$ be the total energy available in the collision
- In the center-of-mass frame $\overrightarrow{p^*} = \overrightarrow{p_A}* + \overrightarrow{p_B}* \equiv 0$

$$4m^2c^4 = E^{*2}$$

$$\boxed{E^{*2} = (E_A + E_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2c^2}$$

- Fixed-target $p_B = 0\ ;\ E_B = mc^2$

$$E^{*2} = E_A{}^2 - p_A{}^2c^2 + m^2c^4 + 2E_Amc^2$$

$$E^{*2} = 2m^2c^4 + 2E_Amc^2 \approx 2E_Amc^2$$

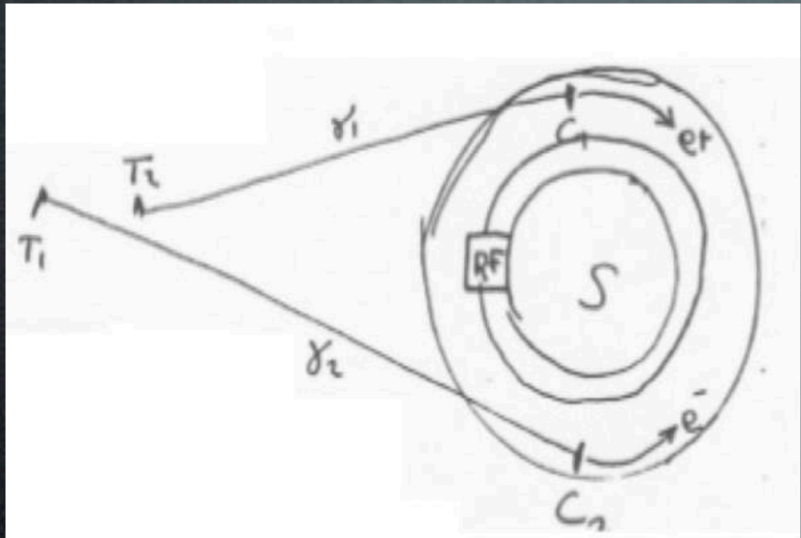
$$\boxed{E^* \approx \sqrt{2E_Amc^2}}$$

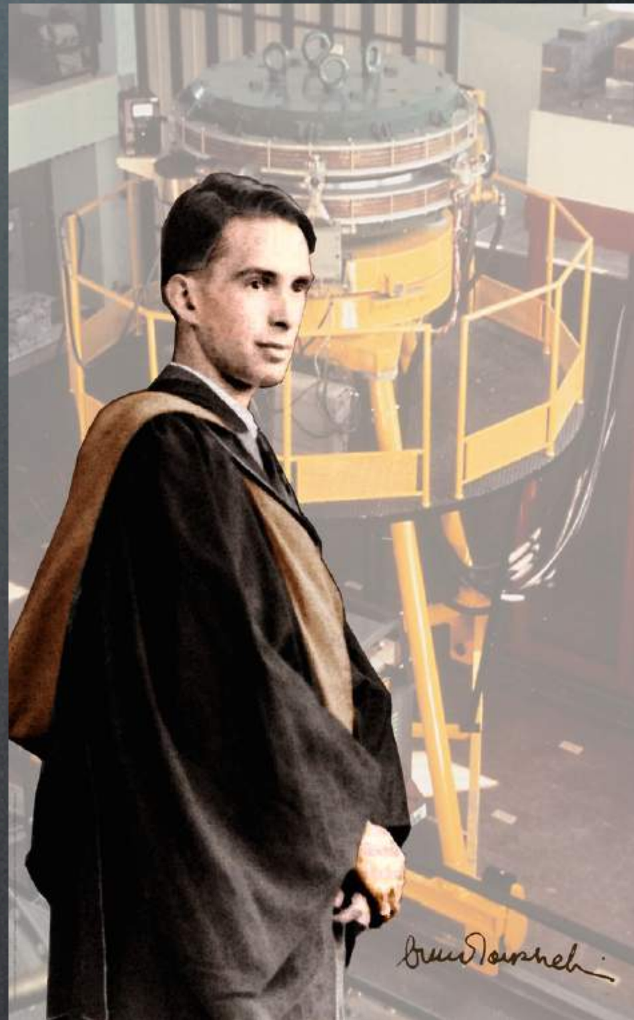
- Head-on collision $\boxed{E^* = E_A + E_B}$

# Touschek's Anello Di Accumulazione (ADA)

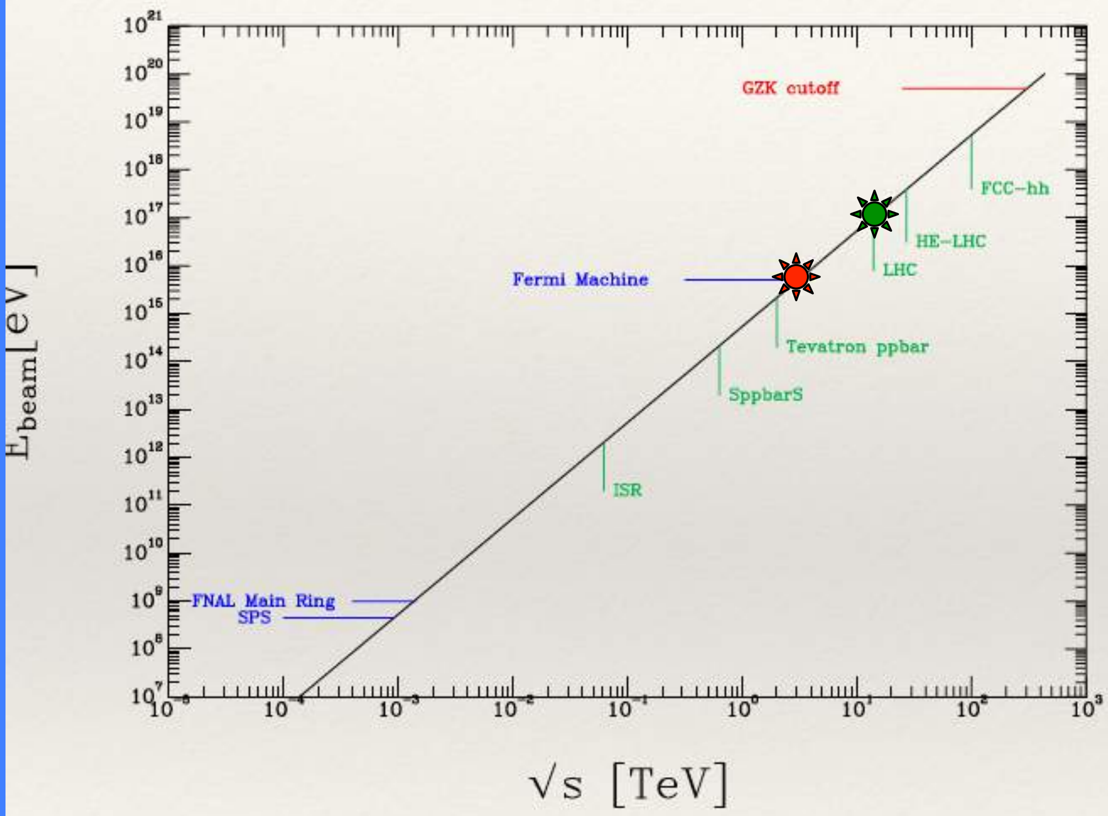
## 1943 Wideroe idea

## 1961 the first e+e- Collider ADA



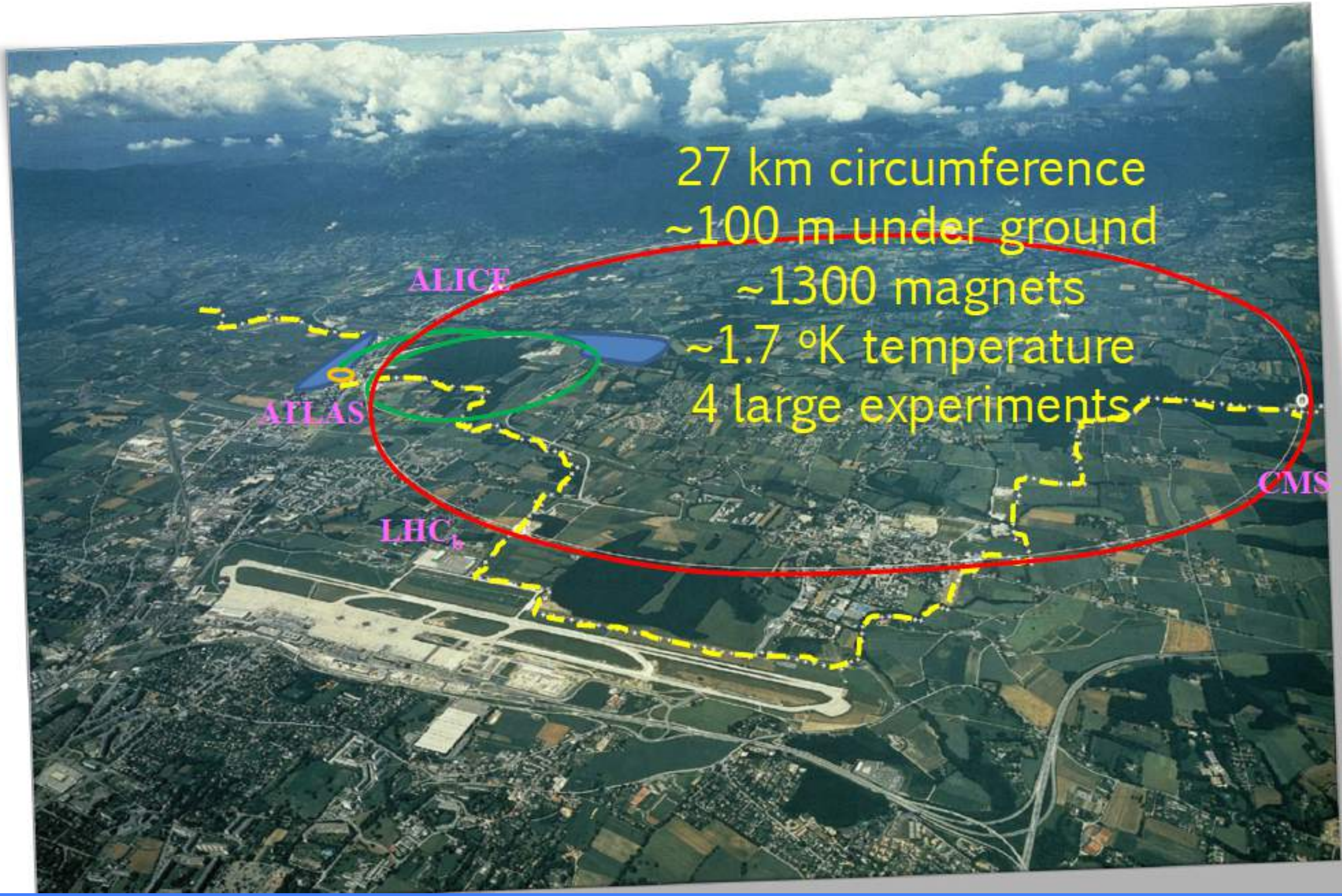
# Fixed Target equivalent accelerator energy




(The Greisen–Zatsepin–Kuzmin limit (GZK limit) is a theoretical upper limit on the energy of cosmic ray protons travelling from other galaxies through the intergalactic medium to our galaxy. The limit is set by slowing-interactions of the protons with the microwave background radiation over long distances (~160 million light-years)).

Current and future colliders have c.o.m energies fixed above that of the Fermi Machine, thanks to the colliding beam technique and the development of superconducting magnets.

# LHC few data


27 km circumference
~100 m under ground
~1300 magnets
~1.7 ºK temperature
4 large experiments
ALICE
ATLAS
LHCb
CMS

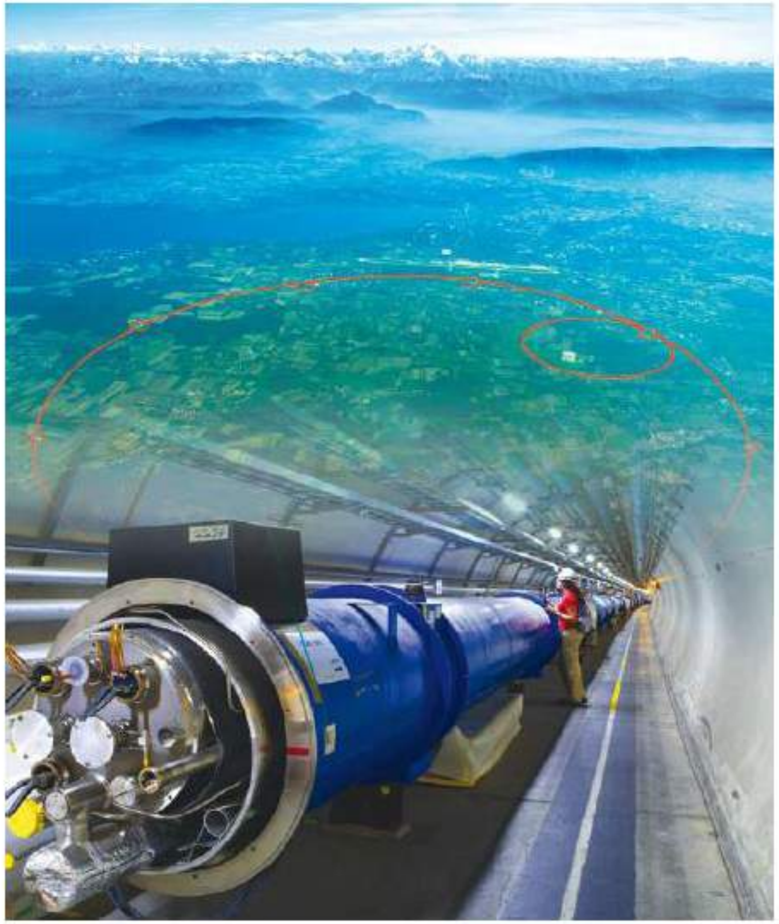
# LHC (Large Hadron Collider) – a brief history

*F. Bordry / CERN*

## 14 TeV proton-proton accelerator-collider built in the LEP tunnel

Lead-Lead (Lead-proton) collisions

| | |
|---|---|
| **1983** | **: First studies for the LHC project** |
| **1988** | **: First magnet model (feasibility)** |
| **1994** | **: Approval of the LHC by the CERN Council** |
| **1996-1999** | **: Series production industrialisation** |
| **1998** | **: Declaration of Public Utility & Start of civil engineering** |
| **1998-2000** | **: Placement of the main production contracts** |
| **2004** | **: Start of the LHC installation** |
| **2005-2007** | **: Magnets Installation in the tunnel** |
| **2006-2008** | **: Hardware commissioning** |
| **2008-2009** | **: Beam commissioning and repair** |
| **2009-2035** | **: Physics exploitation** |



**~ 30 years between the first studies and the start for the Physics**

# What next?

# Hawking: the Solartron

## Towards the Planck scale

Without further novel technology, we will eventually need an accelerator as large as Hawking expected.

“The Universe in a Nutshell”, by Stephen William Hawking, Bantam, 2001

# "How to advance?"

**Hadron (p) circular collider**

$$p = e \cdot R \cdot B_y$$

Increase bending field
SC bend magnet work (FCC-hh)

Increase radius = size (FCC-hh)

**Lepton (e-,e+) circular collider**

$$p \propto E_0 \cdot \sqrt[4]{\rho \cdot U_0}$$

Increase supplied RF voltage (FCC-ee)

Increase mass of acc. particle (muon)

Increase radius = size (FCC-ee)

**Lepton (e-,e+) linear collider**

$$p = L \cdot G_{acc}$$

Increase accelerating gradient
(a) Pushing existing technology (ILC, CLIC)
(b) New regime of ultra-high gradients (plasma, dielectric accelerators)

Increase length (ILC, CLIC)

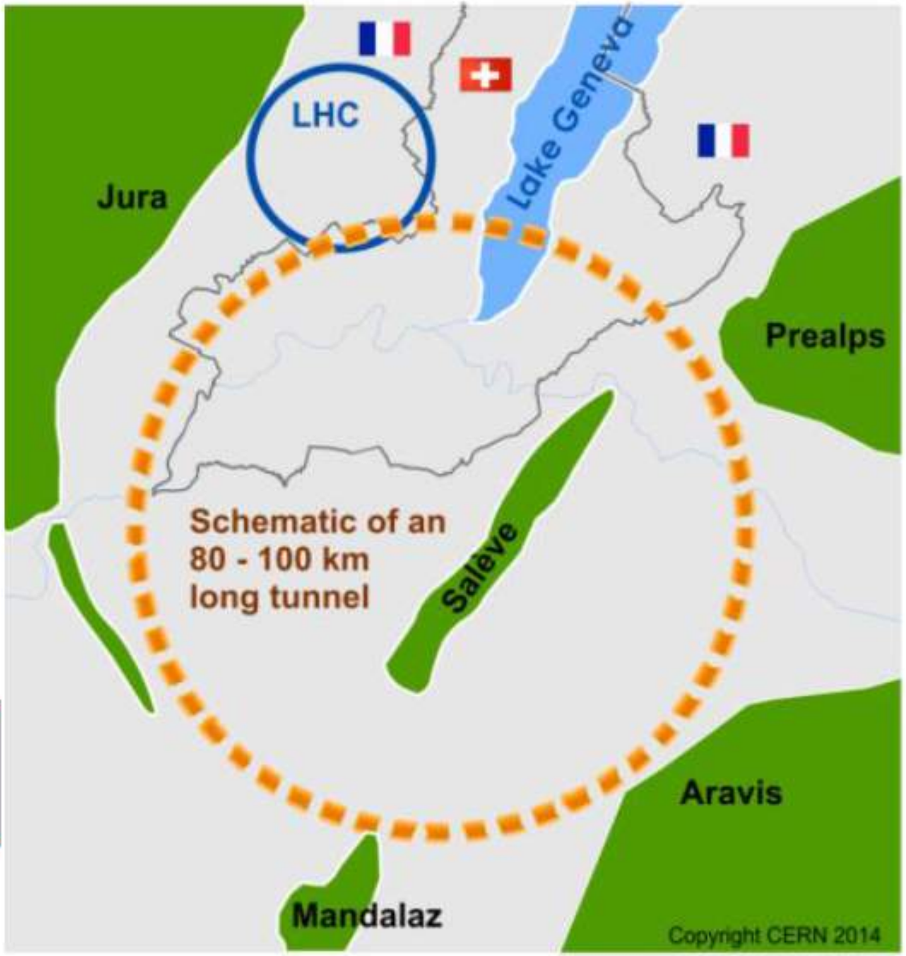
Courtesy R. Assmann

# Today FCC (Future Circular Collider) study

**International collaboration to study:**

- ***pp*-collider (*FCC-hh*)**
- ***$e^+e^-$* collider (*FCC-ee*)**
  as potential intermediate step
- ***p-e* (*FCC-he*)**
  as an option

**~16 T ⇒ 100 TeV *pp* in 100 km**
**~20 T ⇒ 100 TeV *pp* in 80 km**




**FCC: 80-100 km infrastructure in Geneva area**

# A Possible Middle Step for AAC towards Colliders

## Plasma Based Injector for 100km CEPC

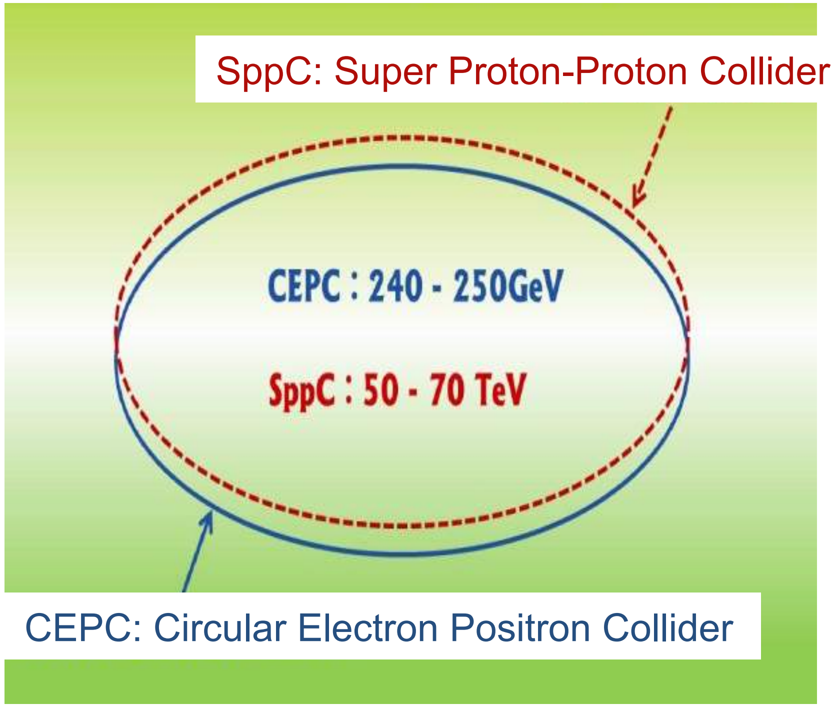
- CEPC (**Circular Electron Positron Collider**) is a major high energy physics plan under strong promotion in China to build a 100km circular machine for a Higgs Factory.

- A high energy injector (40GeV level) is needed to inject e+/e- beams into the main ring.

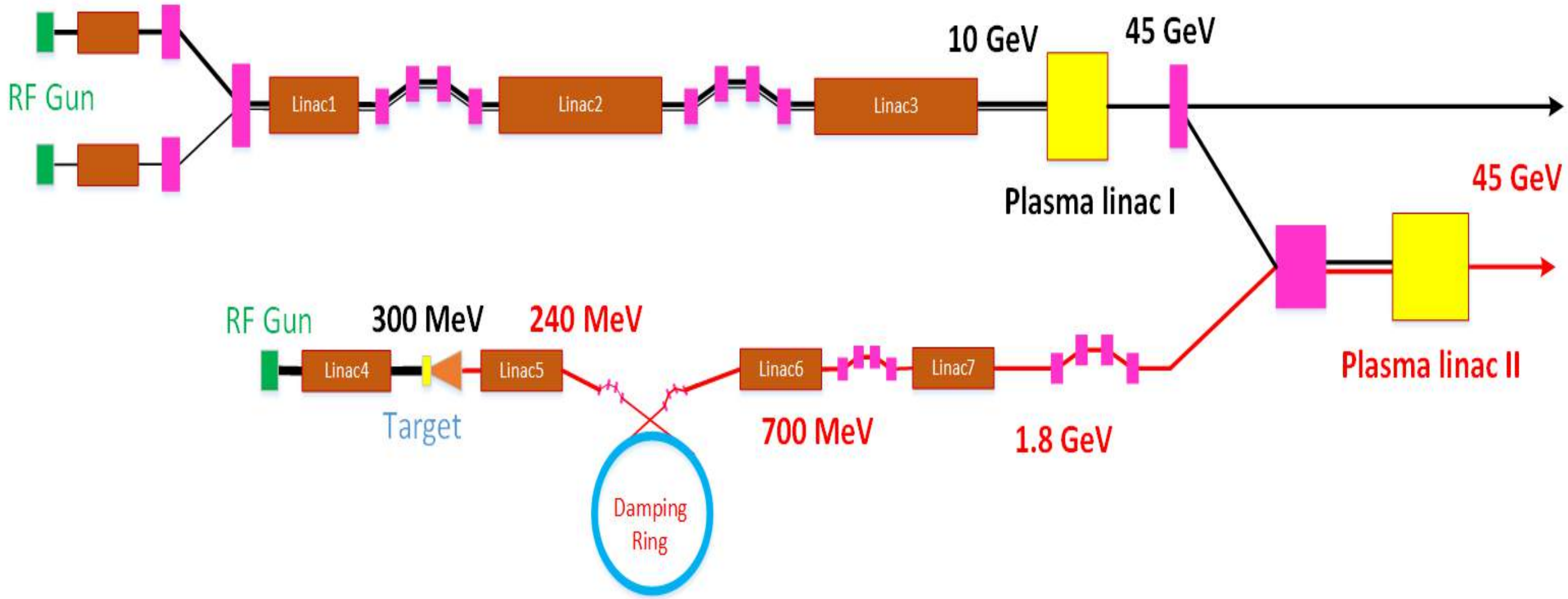
- **Plasma based schemes (PWFA) may provide a novel and cost effective solution for this injector.**

- A joint research group of Tsinghua Univ. and IHEP has been formed since 2017 to study the feasibility of using plasma based acceleration as a novel solution for CEPC injector.

Circular Higgs Factory (Phase I)
+SppC (Phase II) at same tunnel



# A preliminary design of CEPC plasma based high energy injector




- Driver/trailer beam generation through Photo-injector
- HTR PWFA with good stability (single stage TR=3-4, Cascaded stage 6-12, high efficiency)
- Positron generation and acceleration in an electron beam driven PWFA using hollow plasma channel (TR=1)

*Ref: CEPC CDR*

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SC bend magnet work (FCC-hh)

Increase radius = size (FCC-hh)

**Lepton (e-,e+) circular collider**

$$p \propto E_0 \cdot \sqrt[4]{\rho \cdot U_0}$$

Increase supplied RF voltage (FCC-ee)

Increase mass of acc. particle (muon)

Increase radius = size (FCC-ee)

**Lepton (e-,e+) linear collider**

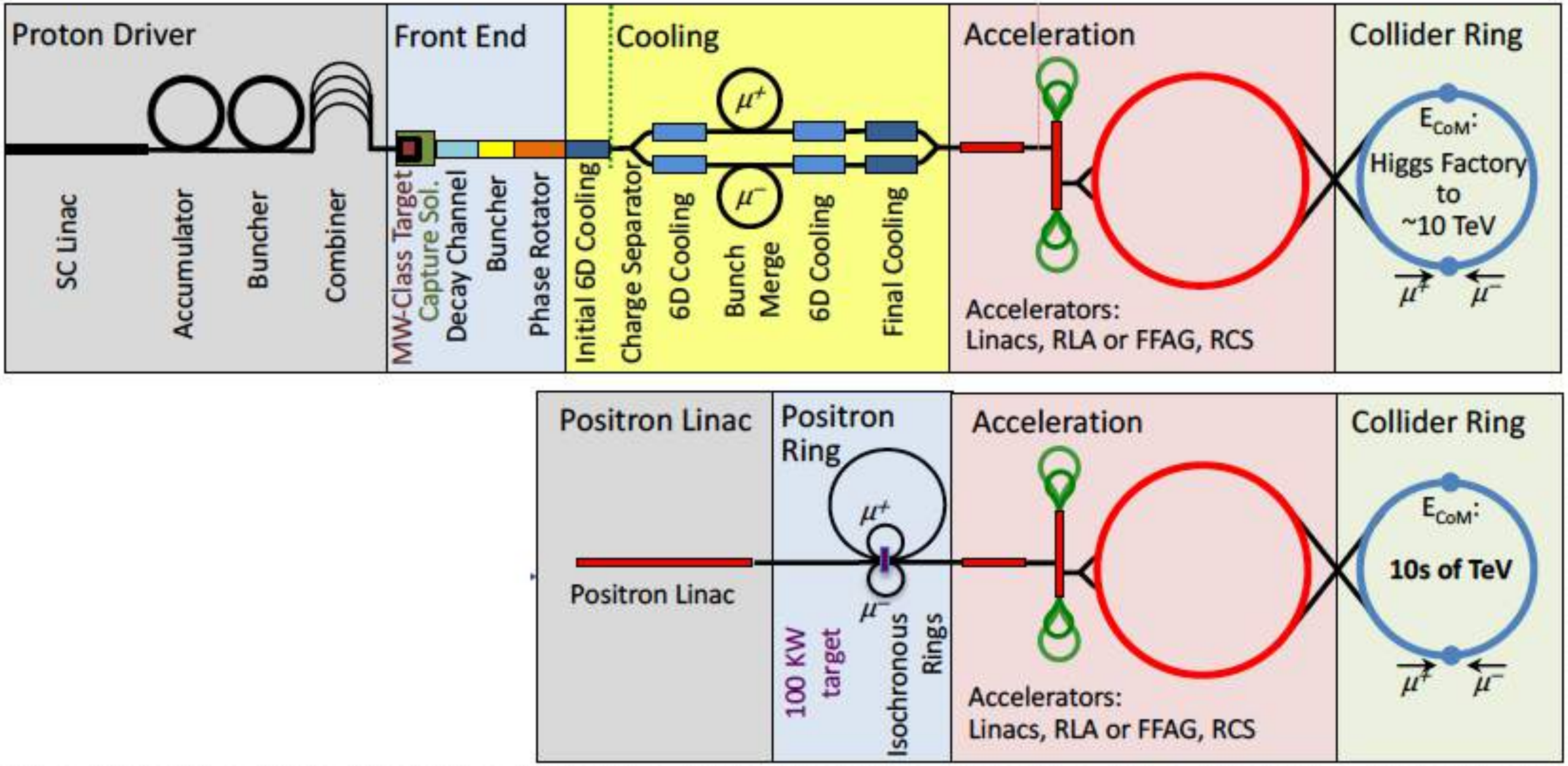
$$p = L \cdot G_{acc}$$

Increase accelerating gradient
(a) Pushing existing technology (ILC, CLIC)
(b) New regime of ultra-high gradients (plasma, dielectric accelerators)

Increase length (ILC, CLIC)

Courtesy R. Assmann

# Muon Collider



# "How to advance?"

**Hadron (p) circular collider**

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SC bend magnet work (FCC-hh)

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$$p \propto E_0 \cdot \sqrt[4]{\rho \cdot U_0}$$

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Increase length (ILC, CLIC)

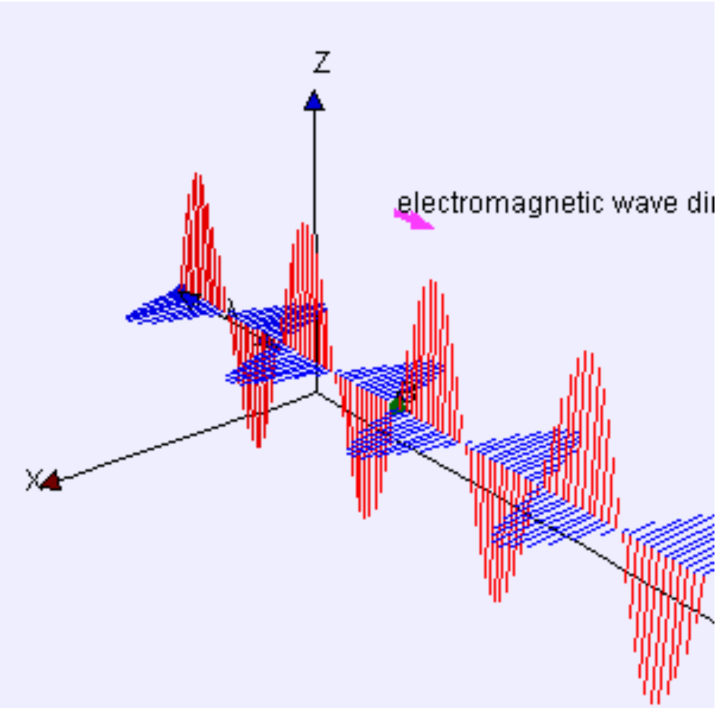
Courtesy R. Assmann

# Lawson-Woodward Theorem

(J.D. Lawson, IEEE Trans. Nucl. Sci. NS-26, 4217, 1979)

The net energy gain of a relativistic electron interacting with an electromagnetic field **in vacuum** is zero.

The theorem assumes that
(i) the laser field is in vacuum with no walls or boundaries present,
(ii) the electron is highly relativistic (v ≈ c) along the acceleration path,
(iii) no static electric or magnetic fields are present,
(iv) the region of interaction is infinite,



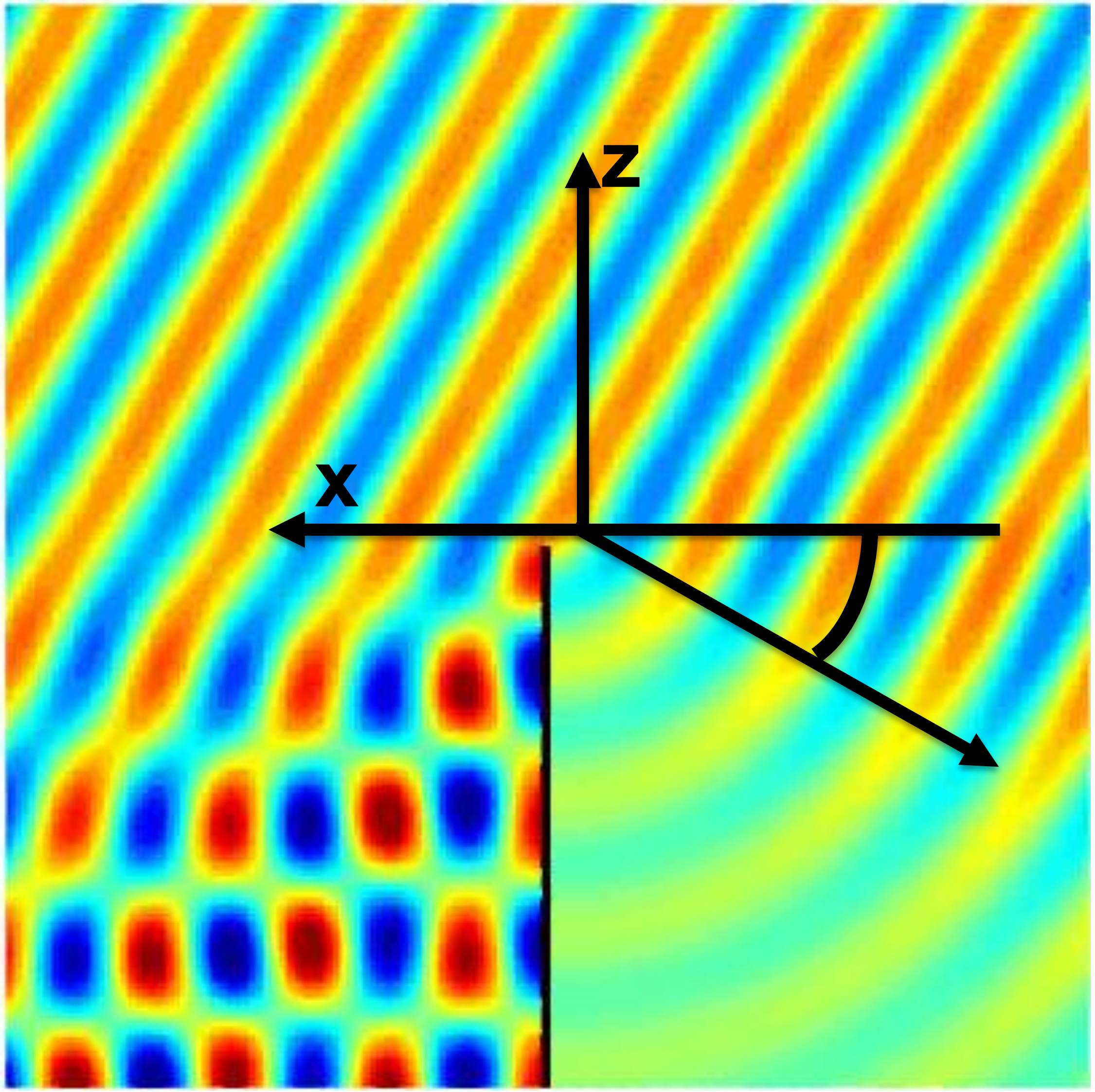

$$\Delta\mathcal{E} = e\int_{-\infty}^{\infty} \boldsymbol{v}\cdot\boldsymbol{E}(\boldsymbol{r}(t),t)\,dt, \qquad \boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}t,$$

$$\boldsymbol{E}(\boldsymbol{r},t) = \int d^3k\,\tilde{\boldsymbol{E}}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t}, \qquad \omega = ck.$$

$$\begin{aligned}\Delta\mathcal{E} &= e\boldsymbol{v}\cdot\int_{-\infty}^{\infty} dt\int d^3k\,\tilde{\boldsymbol{E}}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_0+\boldsymbol{v}t)-i\omega t}\\ &= 2\pi e\int d^3k\,\boldsymbol{v}\cdot\tilde{\boldsymbol{E}}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}) \equiv 0\end{aligned}$$

$$\omega - \boldsymbol{k}\cdot\boldsymbol{v} = ck(1-\beta\cos\alpha) > 0, \quad \Rightarrow \delta \equiv 0$$

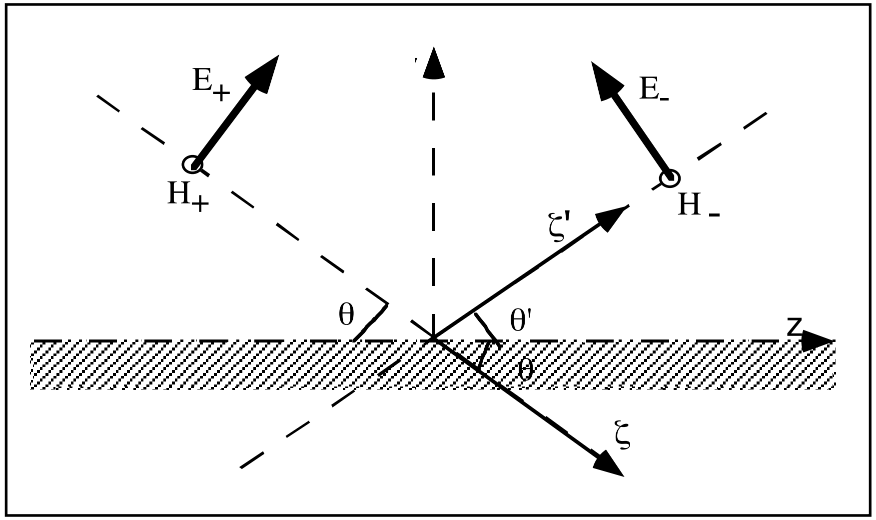
# Reflection of plane waves



# Reflection of plane waves

Plane wave reflected by a perfectly conducting plane

$$\sigma = \infty$$




In the plane xz the field is given by the superposition of the incident and reflected wave:

$$\boxed{E(x,z,t) = E_+(x_o,z_o,t_o)e^{i\omega t - ik\zeta} + E_-(x_o,z_o,t_o)e^{i\omega t - ik\zeta'}}$$

$$\zeta = z\cos\theta - x\sin\theta \qquad \zeta' = z\cos\theta' + x\sin\theta'$$

And it has to fulfill the boundary conditions (no tangential E-field)

# Reflection of plane waves (a first boundary value problem)

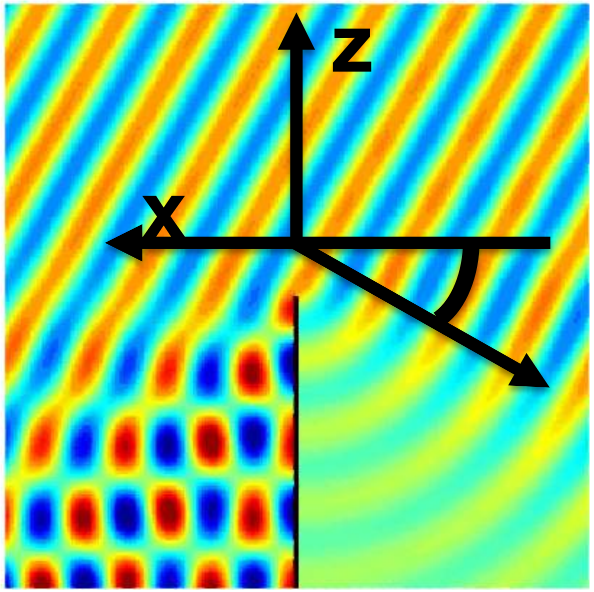
Taking into account the boundary conditions the longitudinal component of the field becomes:

$$E_z(x,z,t) = (E_+ \sin\theta)e^{i\omega t - ik(z\cos\theta - x\sin\theta)} - (E_+ \sin\theta)e^{i\omega t - ik(z\cos\theta + x\sin\theta)}$$

$$= 2iE_+ \sin\theta \sin(kx\sin\theta)e^{i\omega t - ikz\cos\theta}$$

Standing Wave pattern (along x)

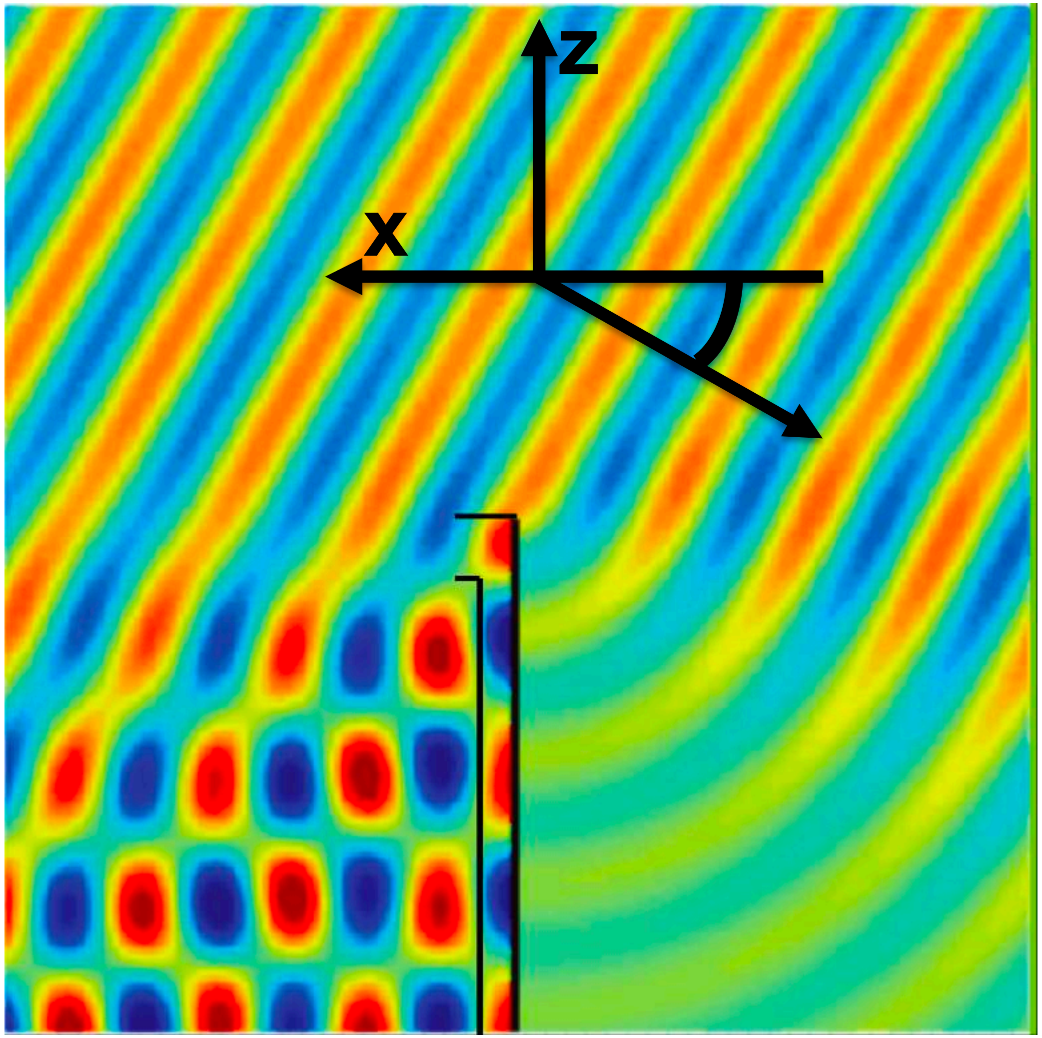
Guided wave pattern (along z)




The phase velocity is given by

$$v_{\phi z} = \frac{\omega}{k_z} = \frac{\omega}{k\cos\theta} = \frac{c}{\cos\theta} > c$$

# From reflections to waveguides



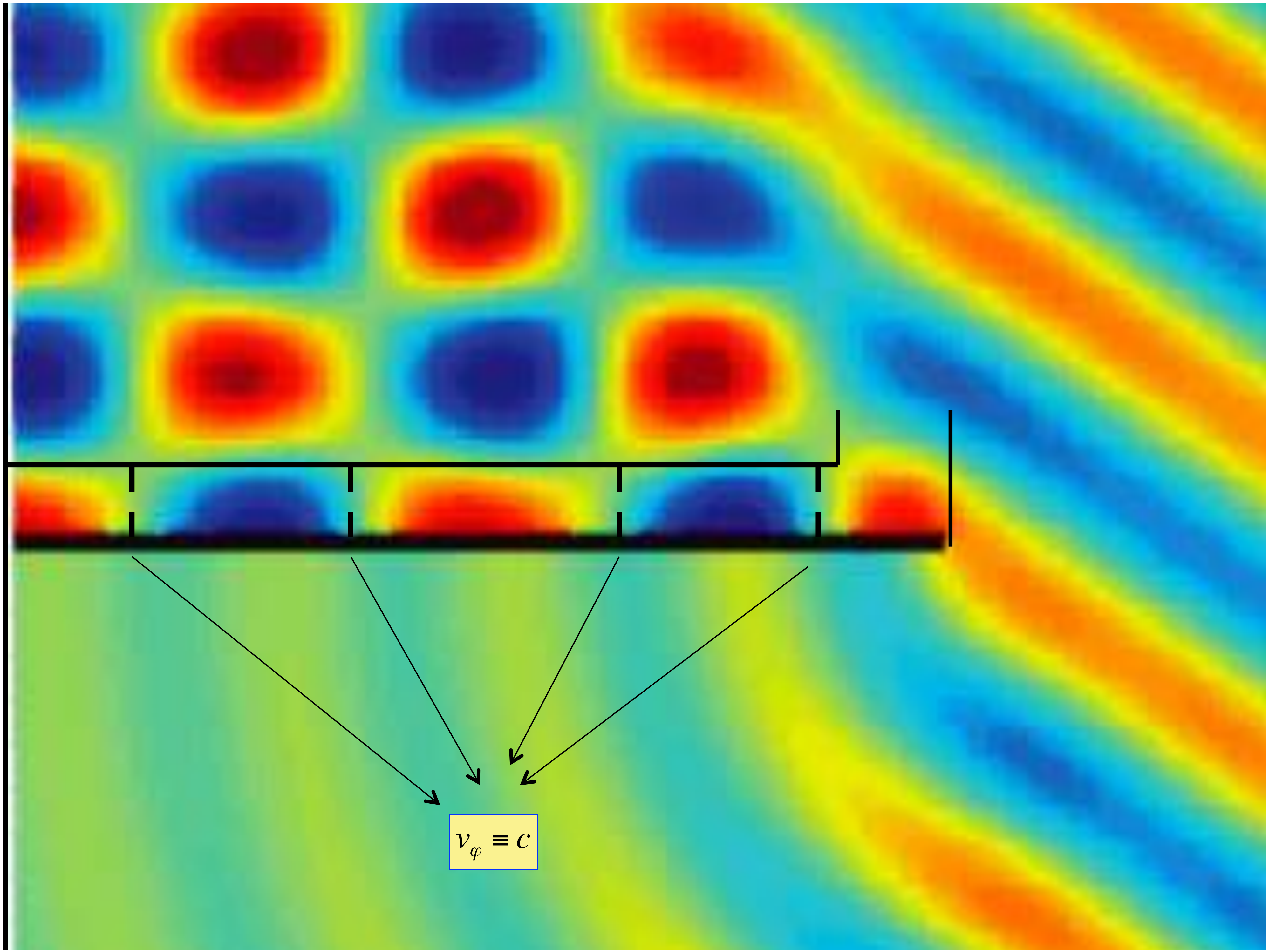

Put a metallic boundary where the field is zero at a given distance from the wall.

Between the two walls there must be an integer number of half wavelengths (at least one).

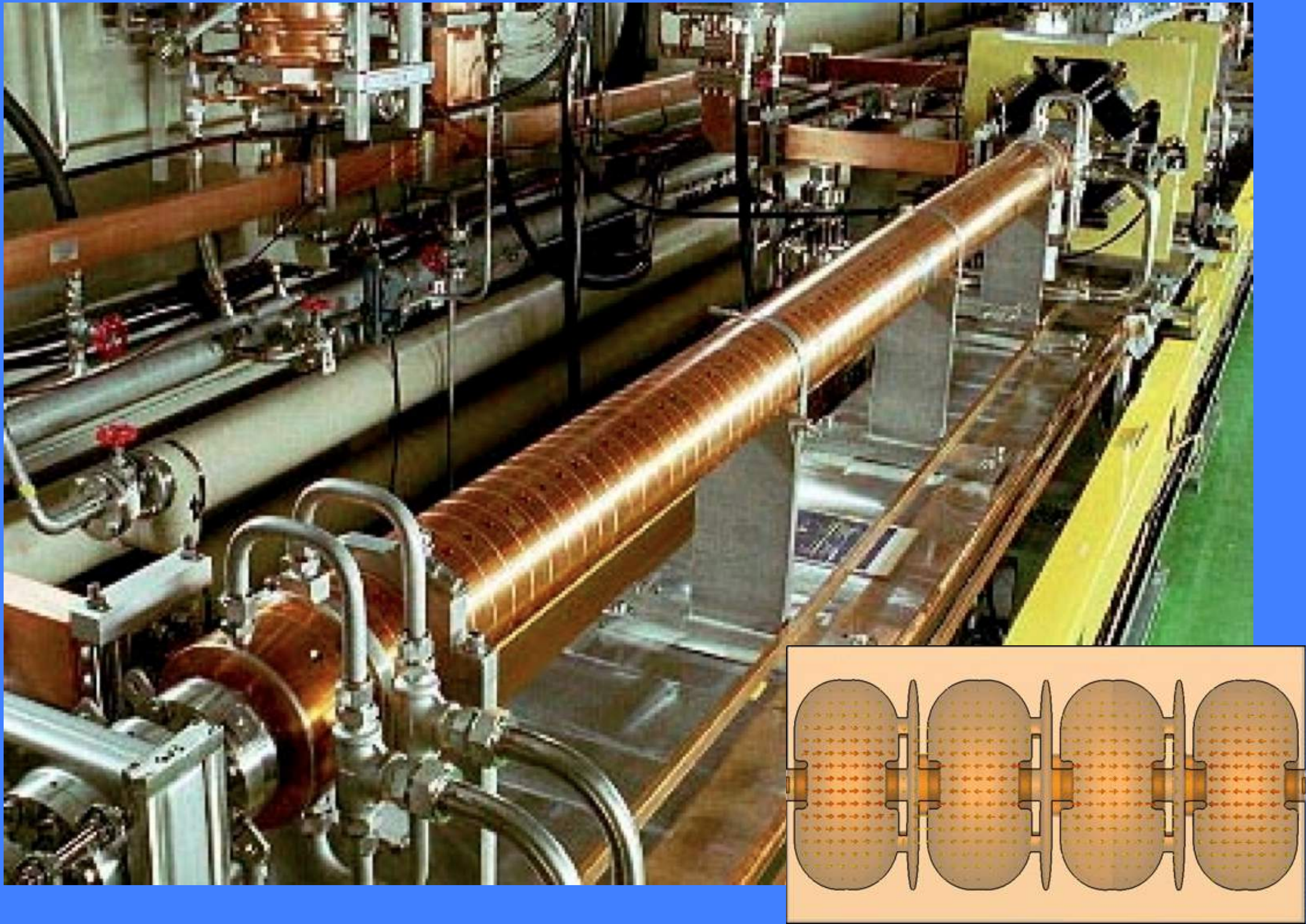
For a given distance, there is a maximum wavelength, i.e. there is **cut-off frequency**.

$$v_{\phi z} = \frac{\omega}{k_z} = \frac{\omega}{k\cos\theta} = \frac{c}{\cos\theta} > c \longrightarrow$$

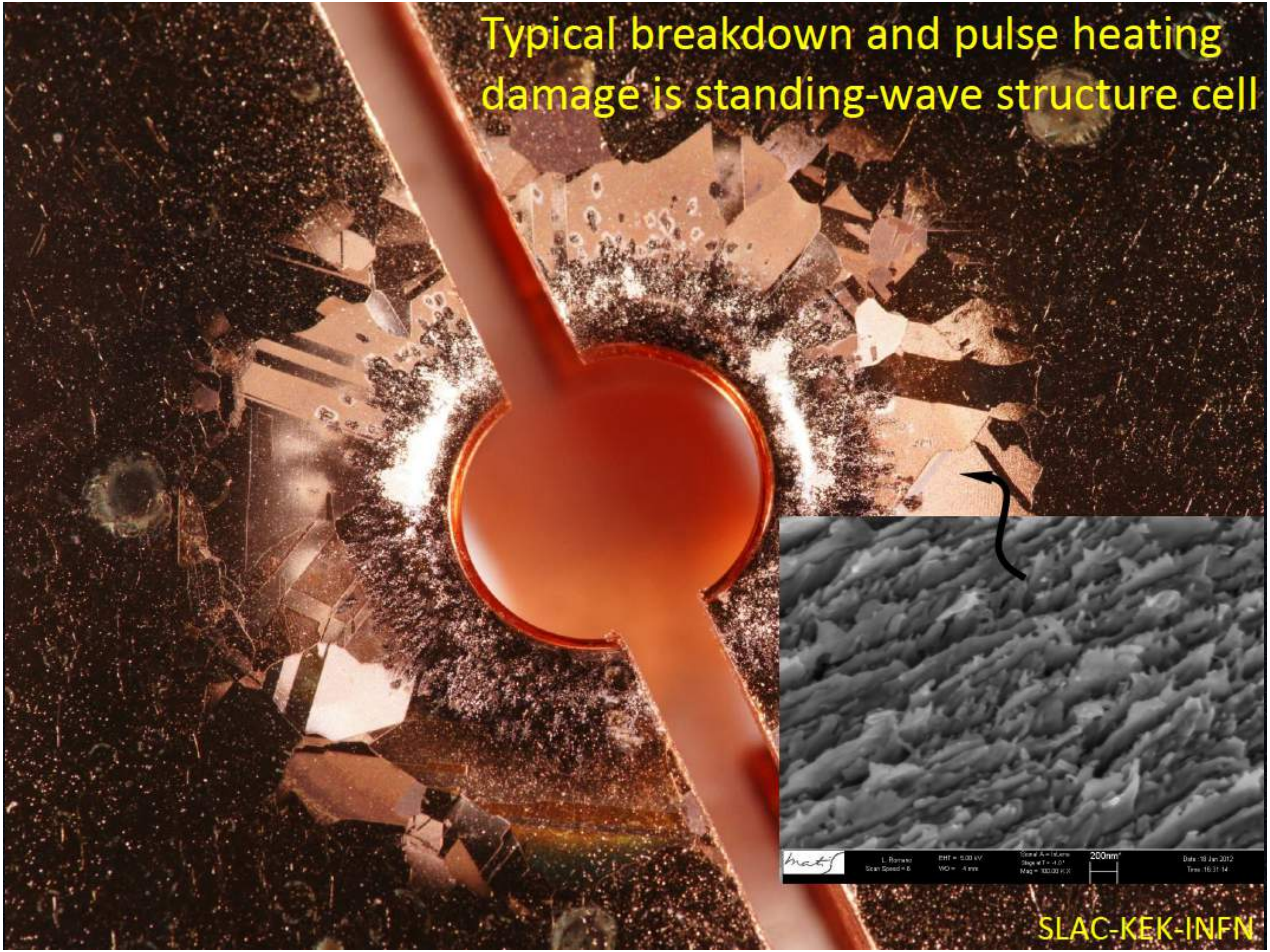
It can not be used as it is for particle acceleration

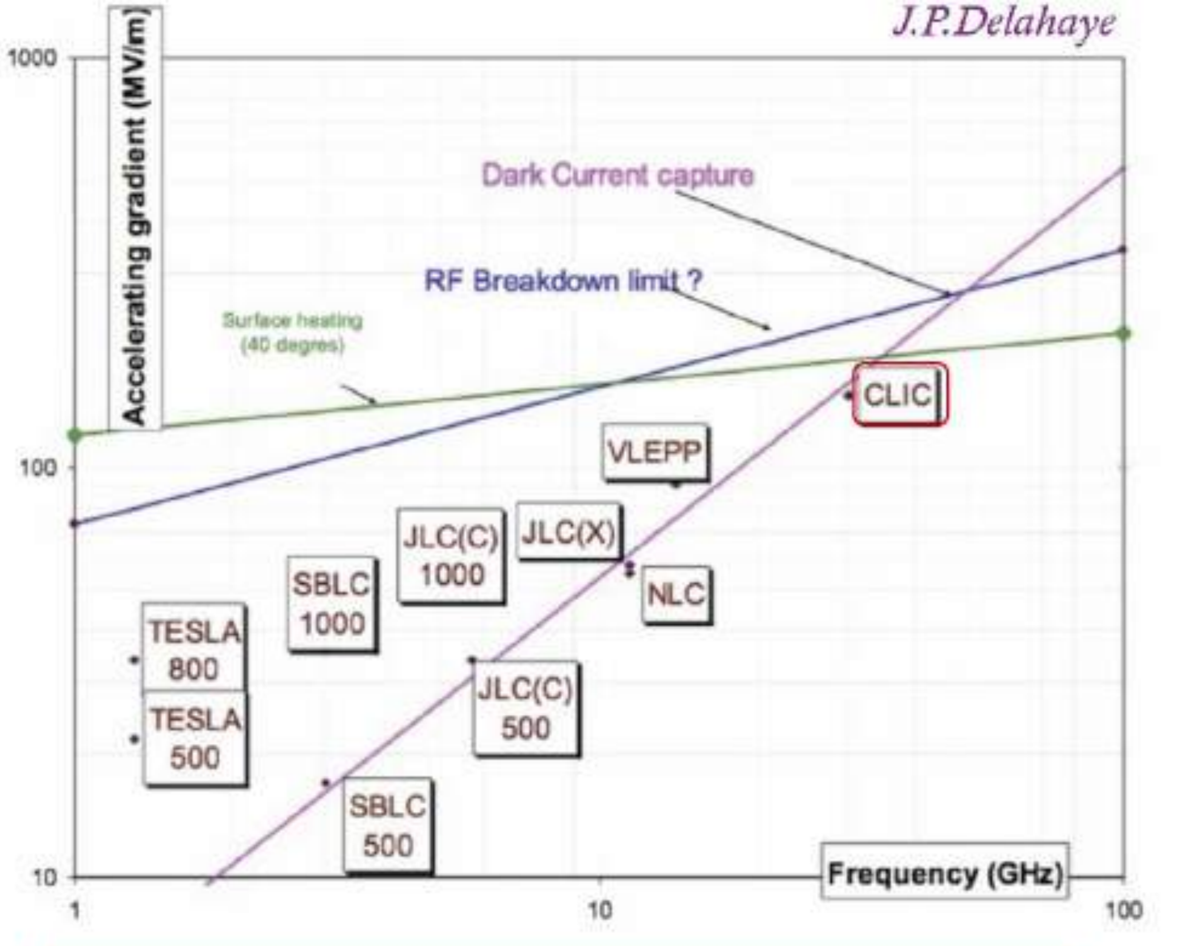

$v_\varphi \equiv c$

# Conventional RF accelerating structures



# Typical breakdown and pulse heating damage is standing-wave structure cell



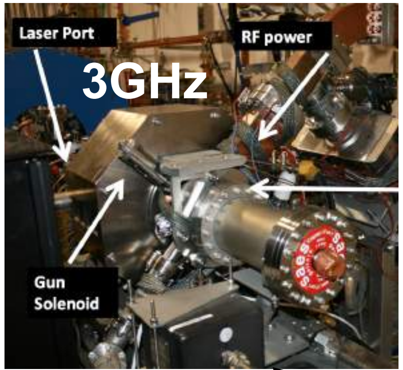


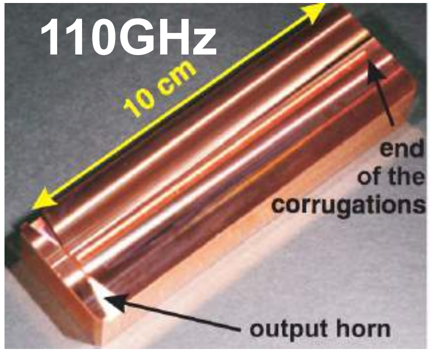

Breakdown limits metal:

$$E_s = 220(f[\mathrm{GHz}])^{1/3}\ \mathrm{MV/m}$$

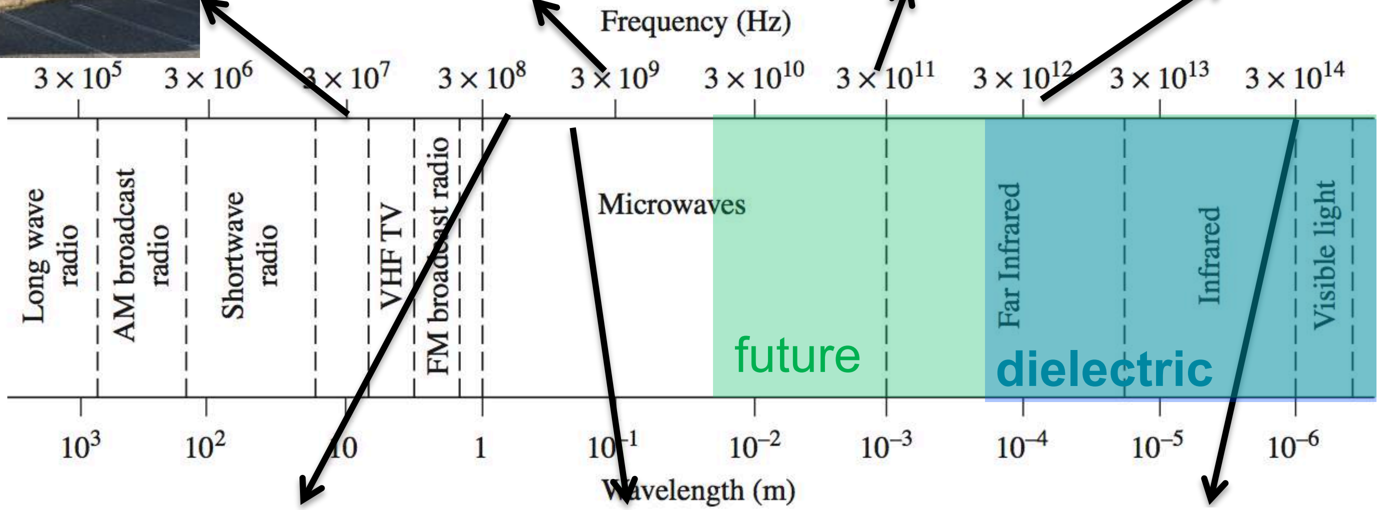
High field ->Short wavelength->ultra-short bunches-> low charge

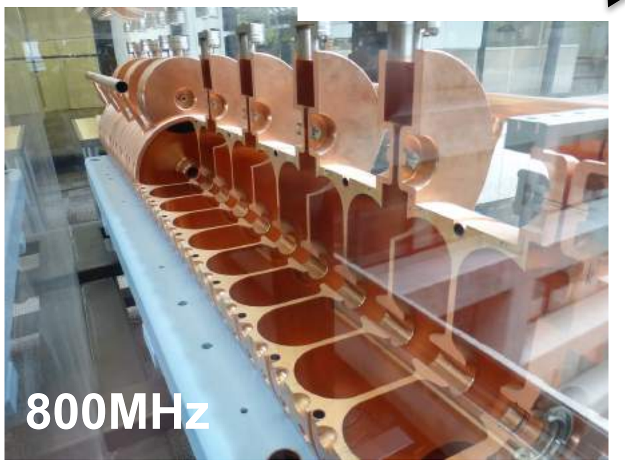
# The RF spectrum and particle accelerator devices

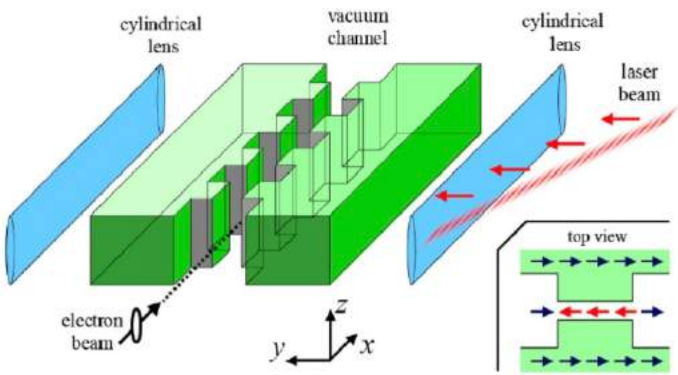







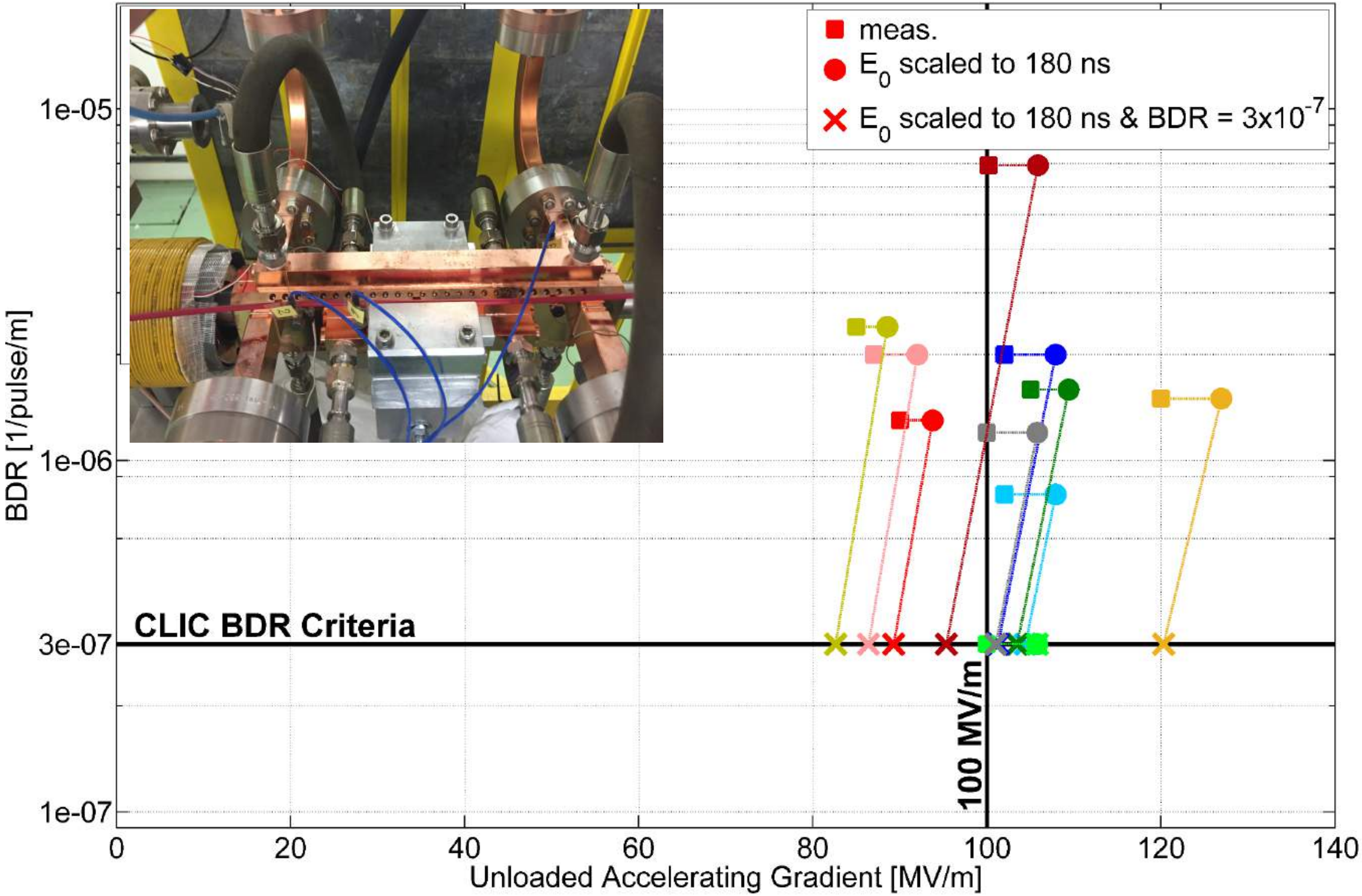







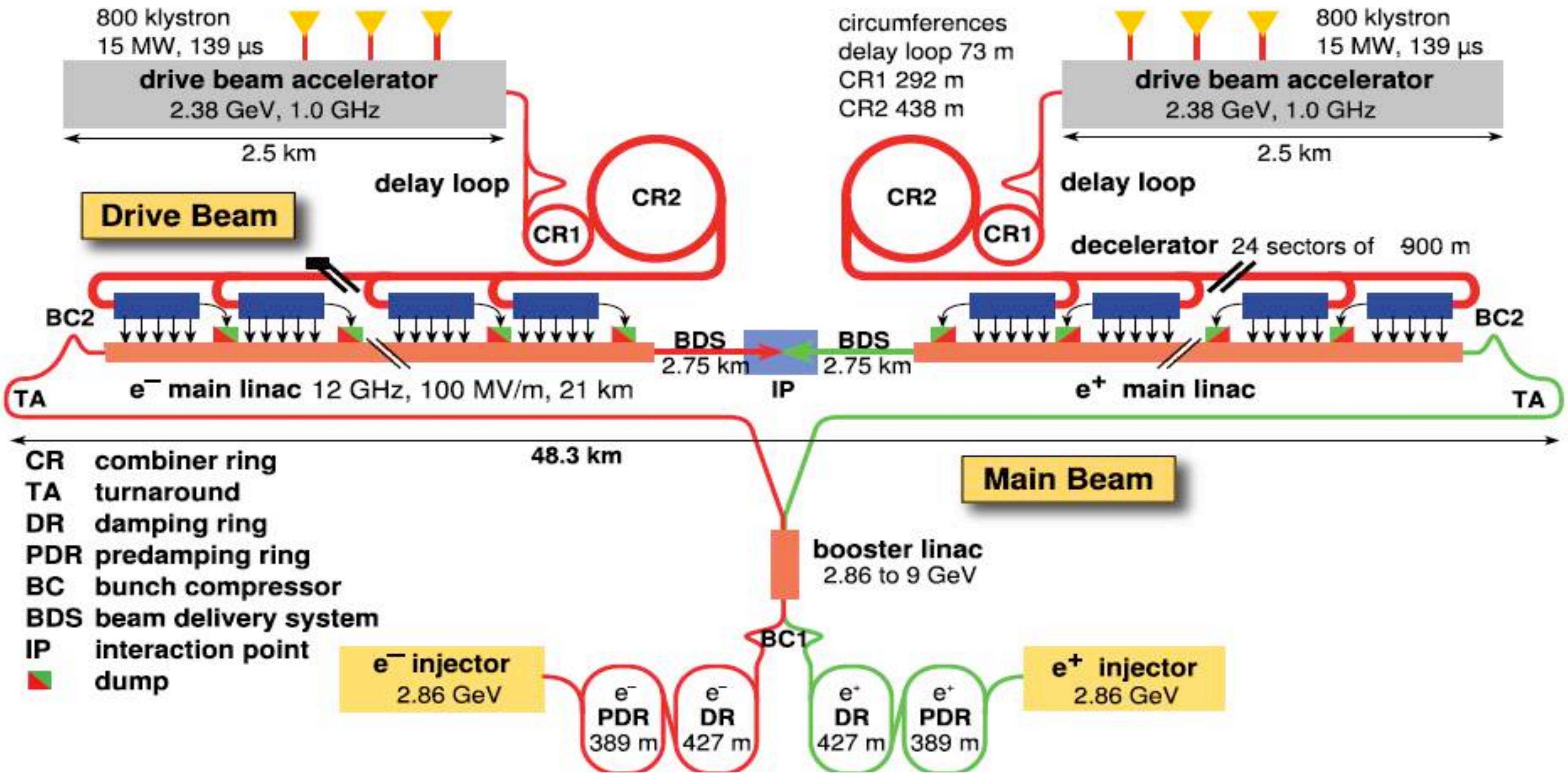

Andrea.Mostacci@uniroma1.it

# X-band RF structures best performances

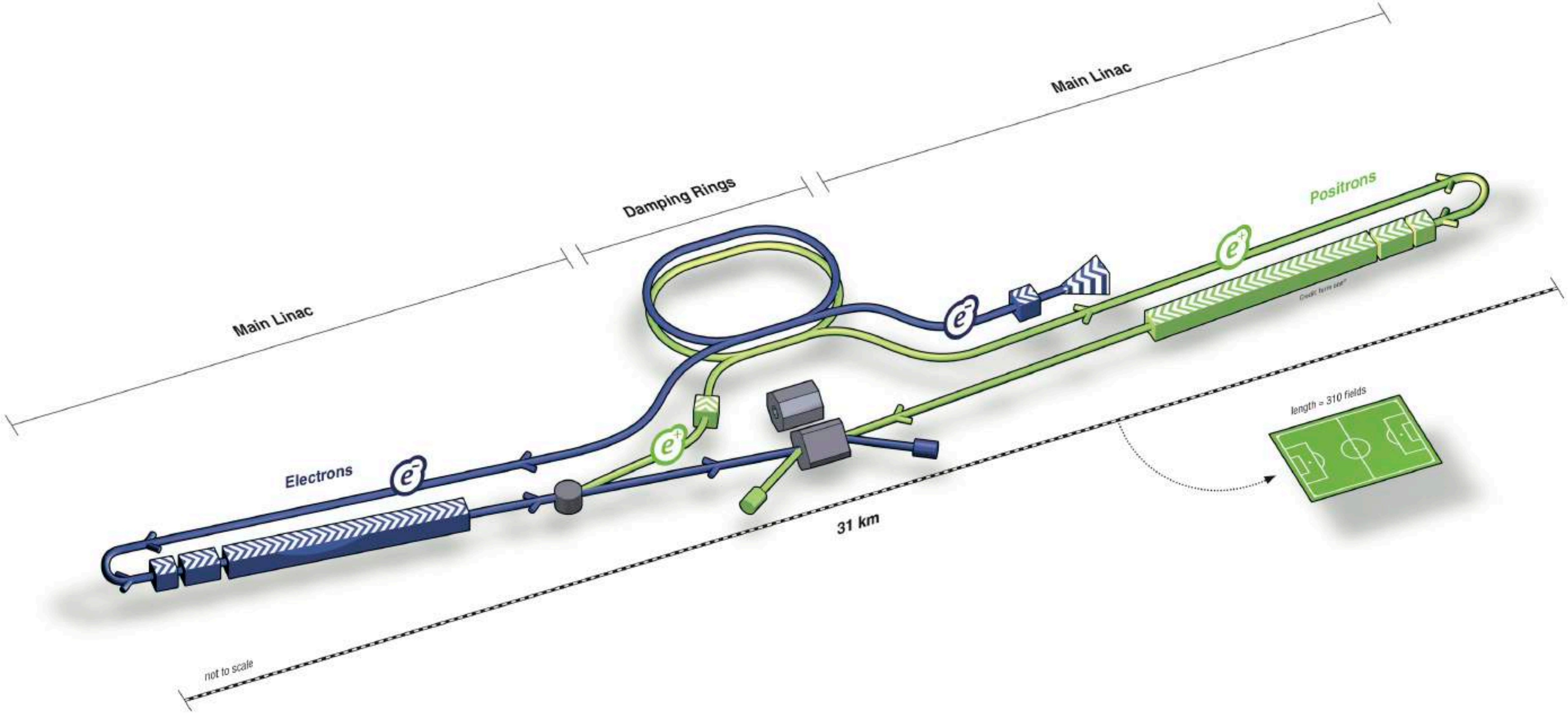


# CLIC ... a future Linear e+/ e- Accelerator

**Avoid bending magnets => no synchrotron radiation losses**
**=> energy gain has to be obtained in ONE GO**

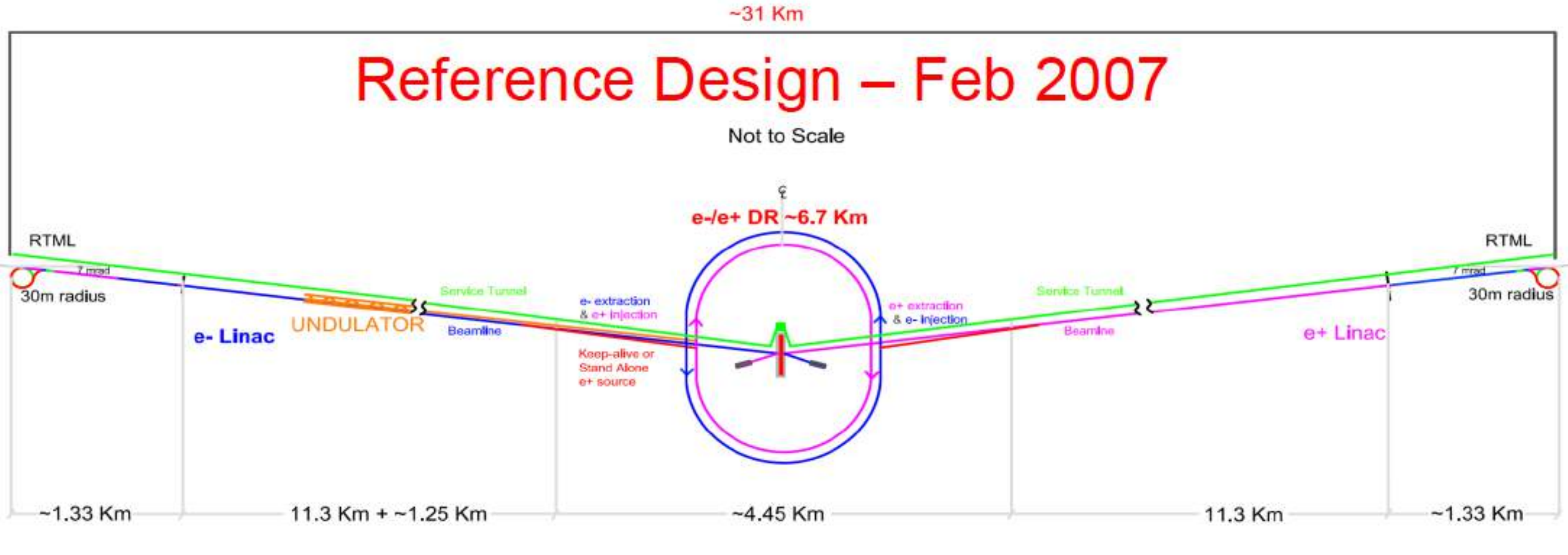


# ILC – International Linear Collider



# ILC (International Linear Collider) $e^+ e^-$

- 11km SC linacs operating at 31.5 MV/m for 500 GeV
- Centralized injector
- Single IR with 14 mrad crossing angle
- Dual tunnel configuration for safety and availability

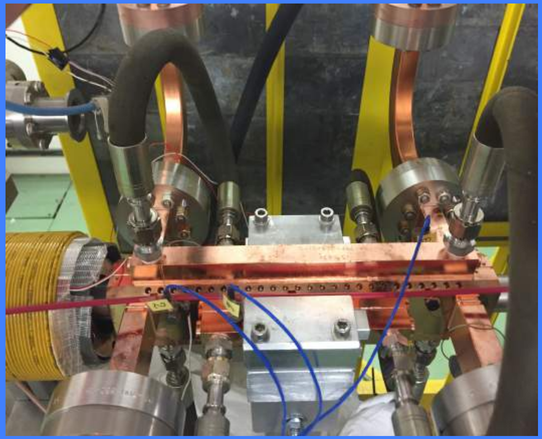



Schematic Layout of the 500 GeV Machine

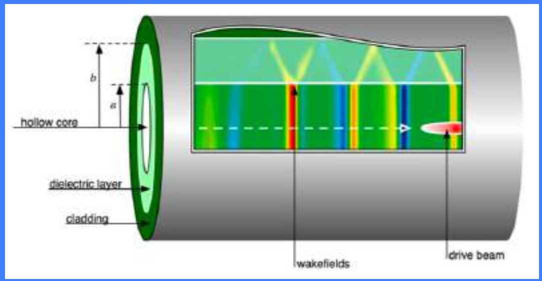
**Documented in Reference Design Report**

# Next High Gradient Options

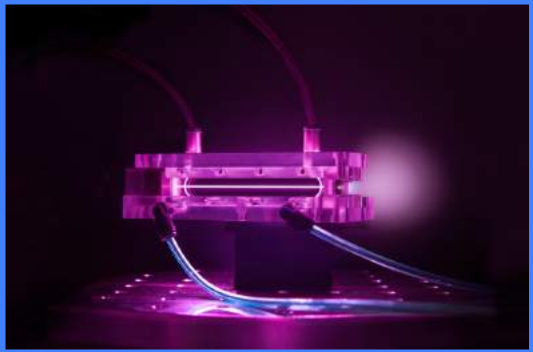
- RF accelerating structures, from X-band to K-band => 100 MV/m < $E_{acc}$ < 1 GV/m



- Dielectrict structures, laser or particle driven => 1 GV/m < $E_{acc}$ < 5 GV/m




- Plasma accelerator, laser or particle driven => 1 GV/m < $E_{acc}$ < 100 GV/m



# Beam Quality

## Future accelerators require high quality beams:

==> High Luminosity & High Brightness

==> High Energy & Low Energy Spread

$$L = \frac{N_{e+} N_{e-} f_r}{4\pi\sigma_x^* \sigma_y^*}$$

- N of particles per pulse => $10^9$
- High rep. rate $f_r$ => bunch trains

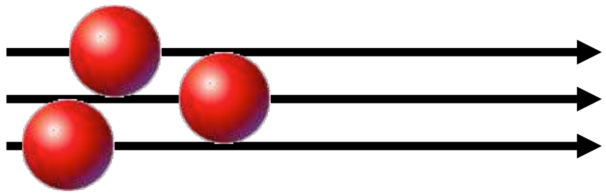
- Small spot size => low emittance

$$B_n \approx \frac{2I}{\varepsilon_n^2}$$

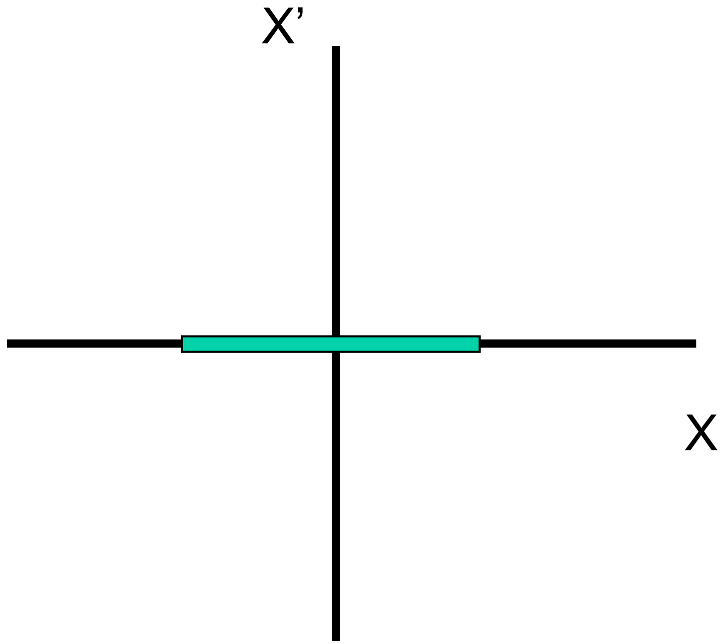
- Short pulse (ps to fs)

- Little spread in transverse momentum and angle => low emittance

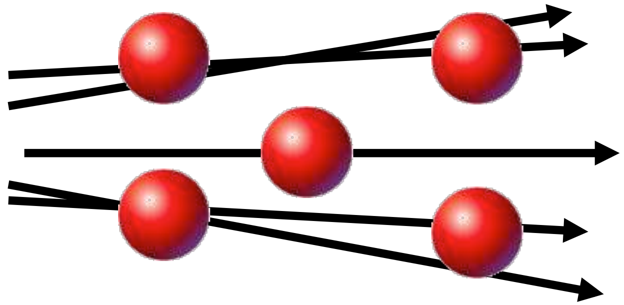
# Trace space of an ideal beam

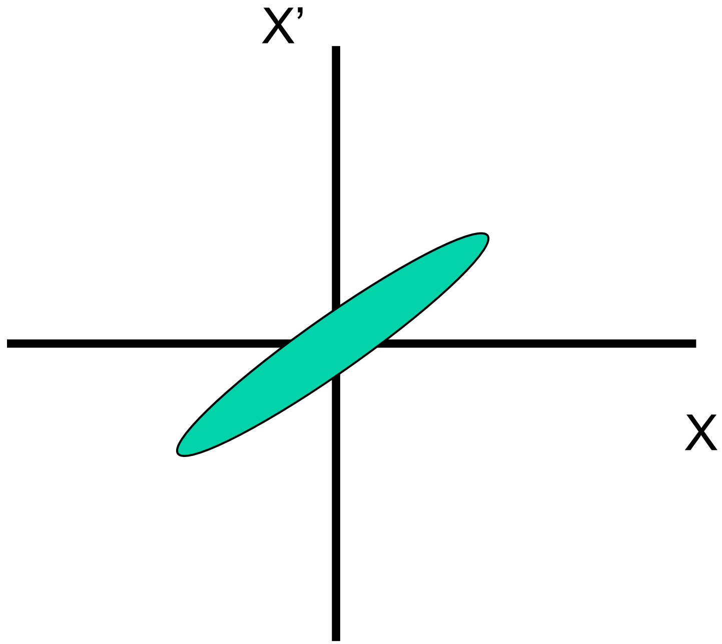


$$\begin{cases} x \\ x' = \dfrac{dx}{dz} = \dfrac{p_x}{p_z} \end{cases} \qquad p_x << p_z$$



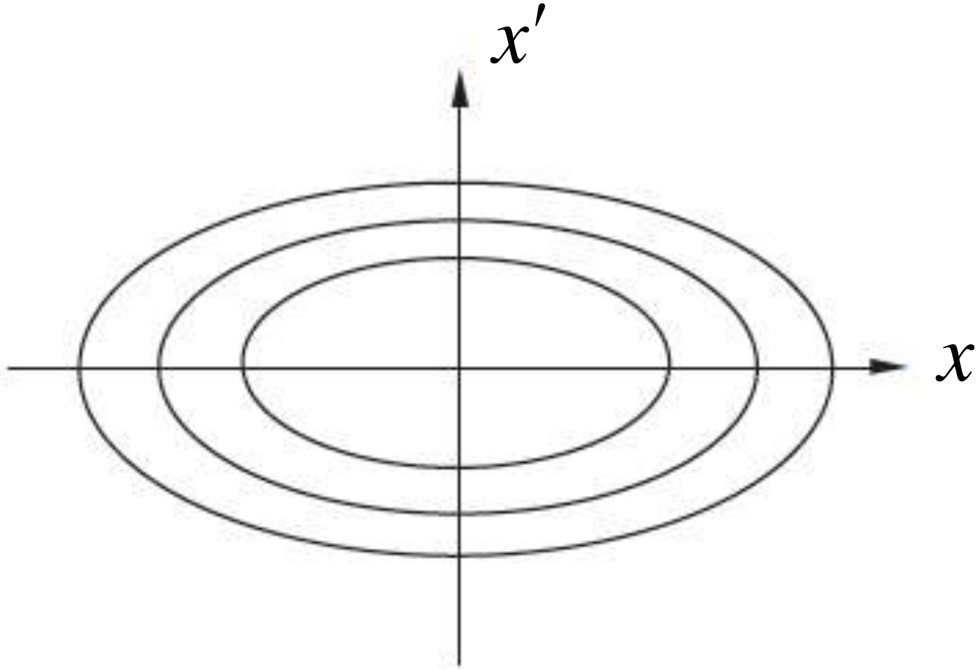
# Trace space of non laminar beam





In a system where all the forces acting on the particles are linear (i.e., proportional to the particle's displacement x from the beam axis), it is useful to assume an elliptical shape for the area occupied by the beam in x-x‘ trace space.

$$\boxed{\ddot{x} + k^2 x = 0}$$



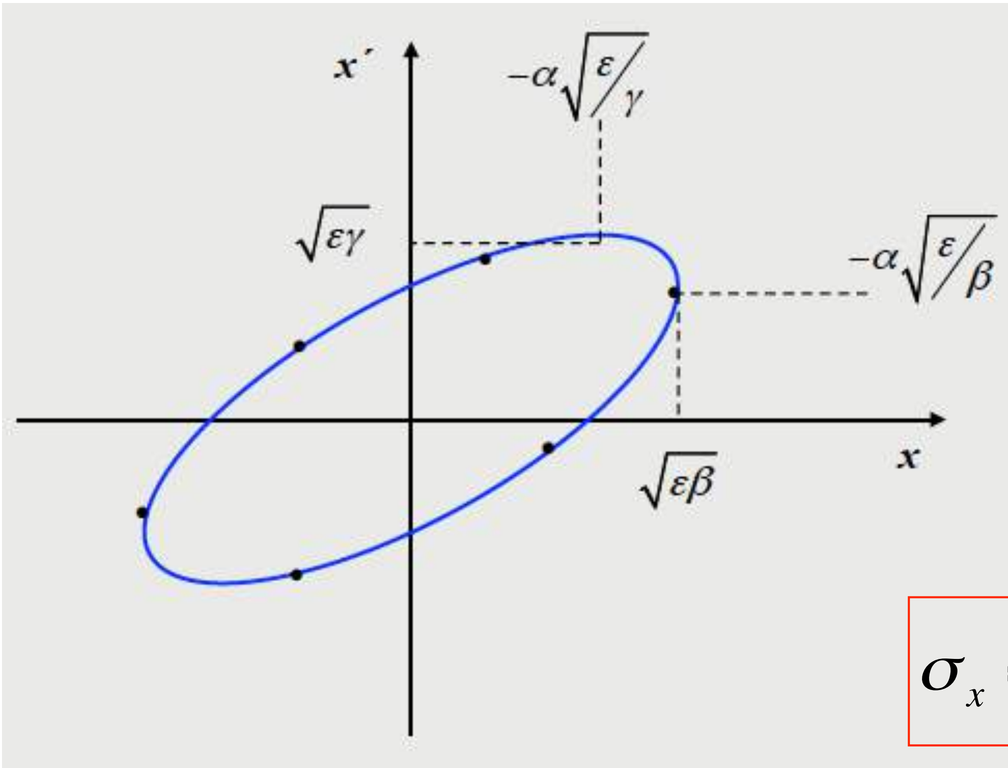

➔ Emittance Concept

Geometric emittance: $\boxed{\varepsilon_g}$

**Ellipse equation:** $\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon_g$

Twiss parameters: $\beta\gamma - \alpha^2 = 1$ $\qquad \beta' = -2\alpha$

**Ellipse area:** $A = \pi \varepsilon_g$




$$\boxed{\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\beta \varepsilon_{rms}}}$$

Liouville theorem: the density of particles $n$, or the volume V occupied by a given number of particles in phase space $(x,p_x,y,p_y,z,p_z)$ remains invariant under the effect of conservative forces.

$$\boxed{\frac{dn}{dt} = 0}$$

It hold also in the projected phase spaces $(x,p_x),(y,p_y)(,z,p_z)$ provided that there are no couplings

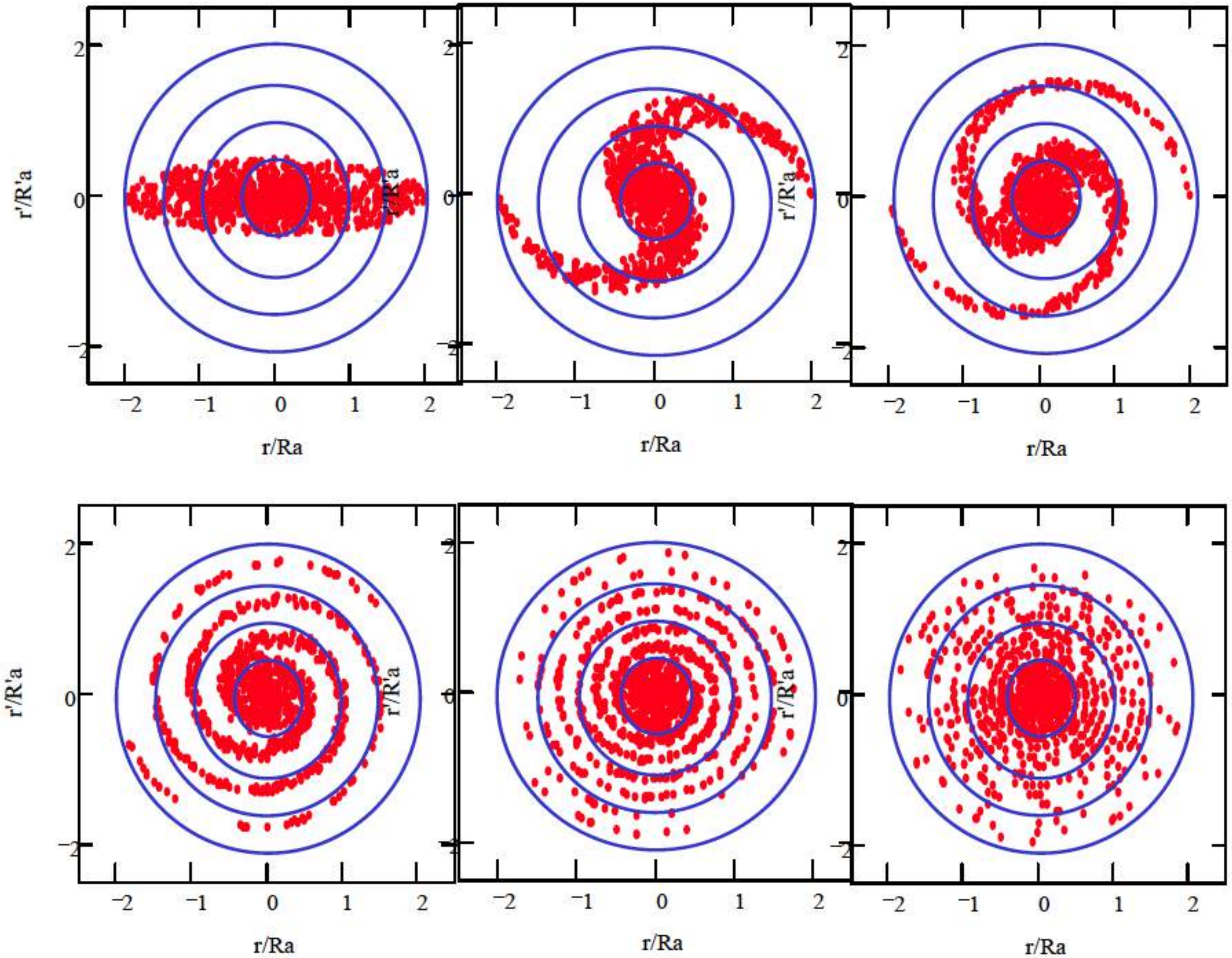
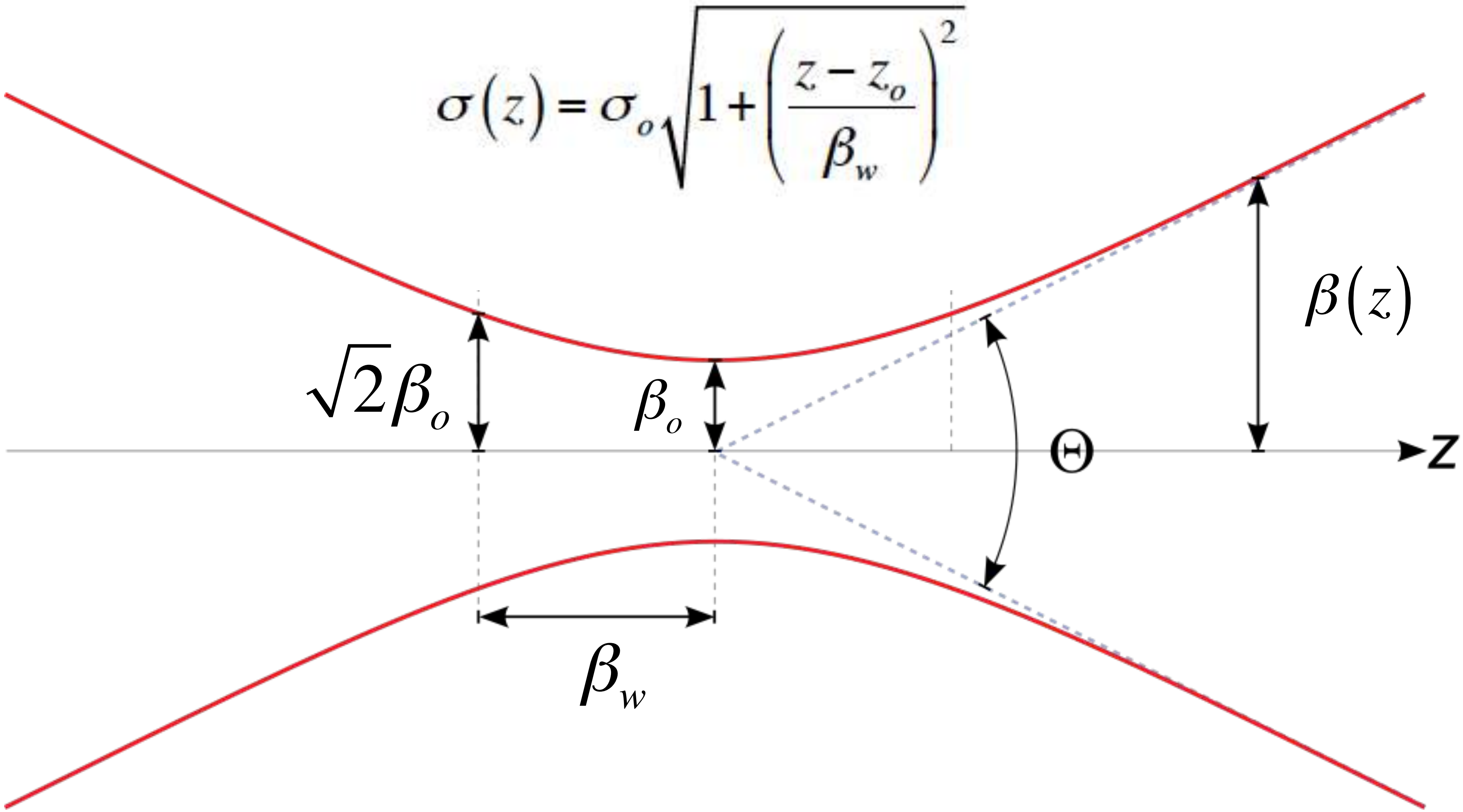
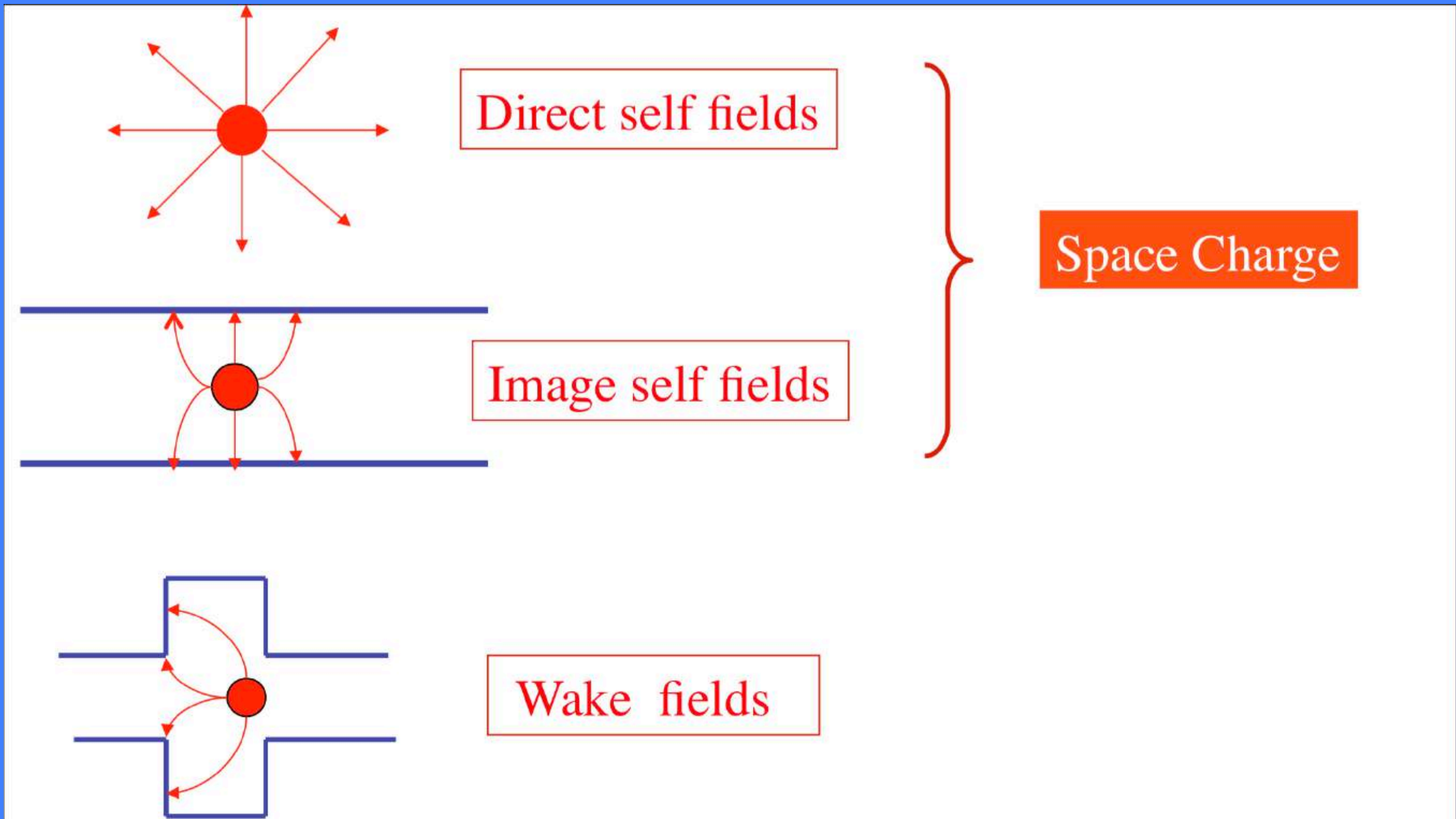



**Fig. 17:** Filamentation of mismatched beam in non-linear force

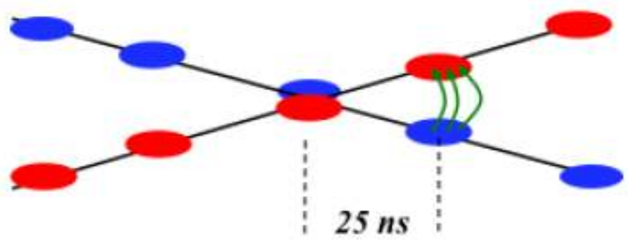



For an effective transport of a beam with finite emittance is mandatory to make use of some external force providing beam confinement in the transport or accelerating line.

# Space Charge Induced Instabilities



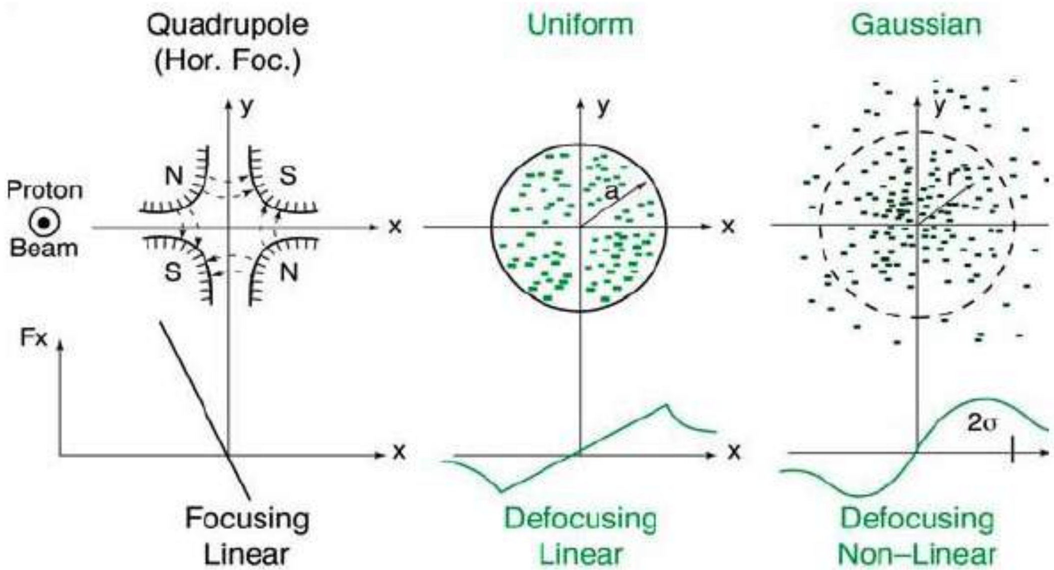
# Beam-Beam effect




*the colliding bunches influence each other*

*=> change the focusing properties of the ring !!*

*for LHC a strong non-linear defoc. effect*




court. K. Schindl

*most simple case:*

*linear beam beam tune shift*

$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi\gamma_p(\sigma_x+\sigma_y) * \sigma_x}$$

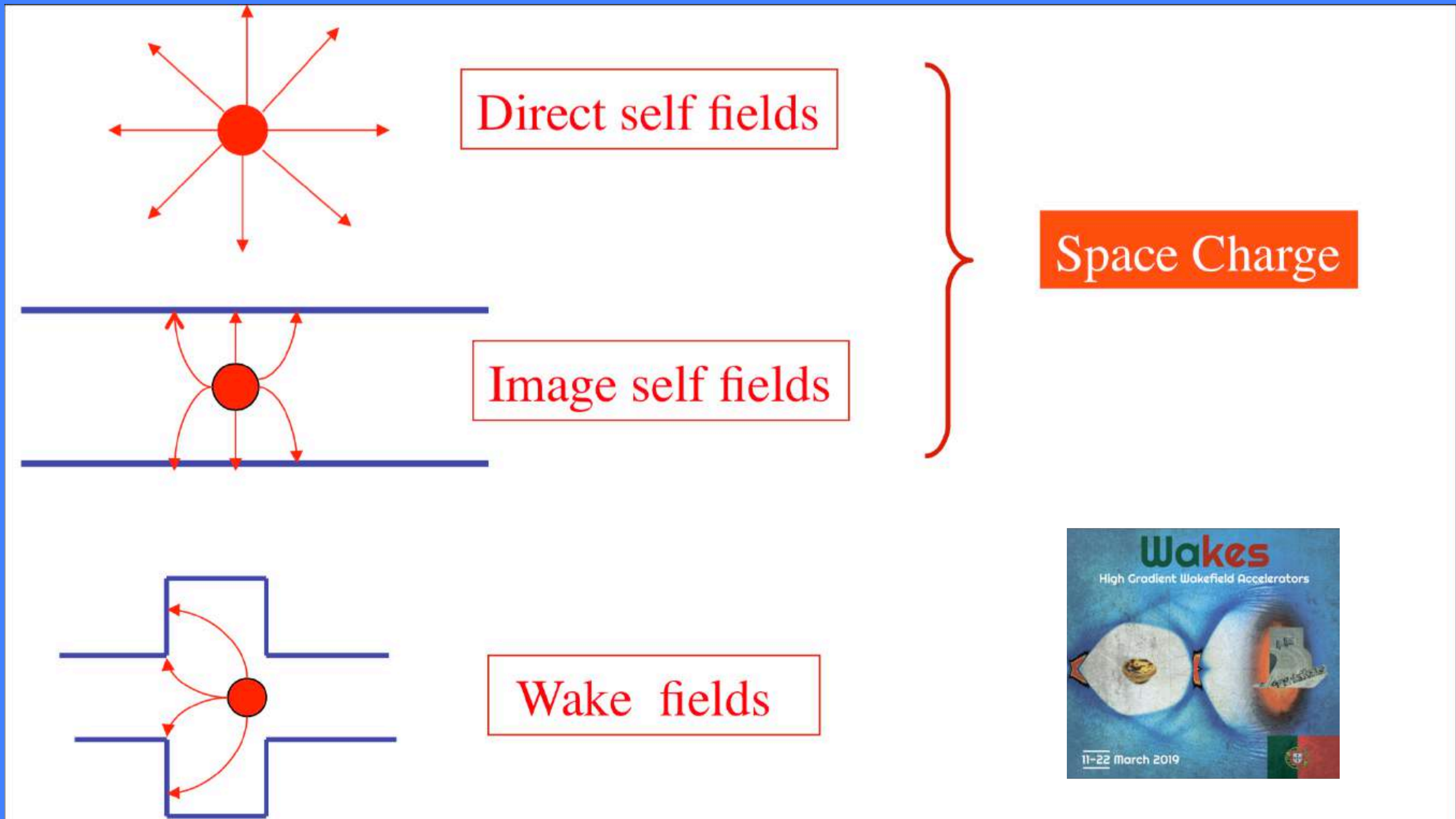
*$\Rightarrow$puts a limit to $N_p$*

*Eigenfrequency of the paticles is changed due to the beam beam interaction*

*Particles are pushed onto resonances and are lost.*

Courtesy B. Holzer

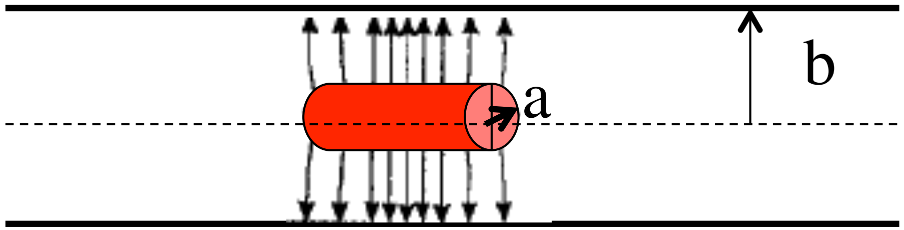
# Wake Field Induced Instabilities



# Bunched beam - Circular Perfectly Conducting Pipe

# - Beam at Centre- Static Approximation $\gamma \to \infty$

$$\rho = \frac{I}{\pi a^2 v}$$




$$\boxed{\varphi(b) = 0}$$

$$E_r = \frac{I}{2\pi\varepsilon_o a^2 v} r \quad \textbf{\textit{for}} \quad r \le a$$

$$E_r = \frac{I}{2\pi\varepsilon_o v} \frac{1}{r} \quad \textbf{\textit{for}} \quad r > a$$

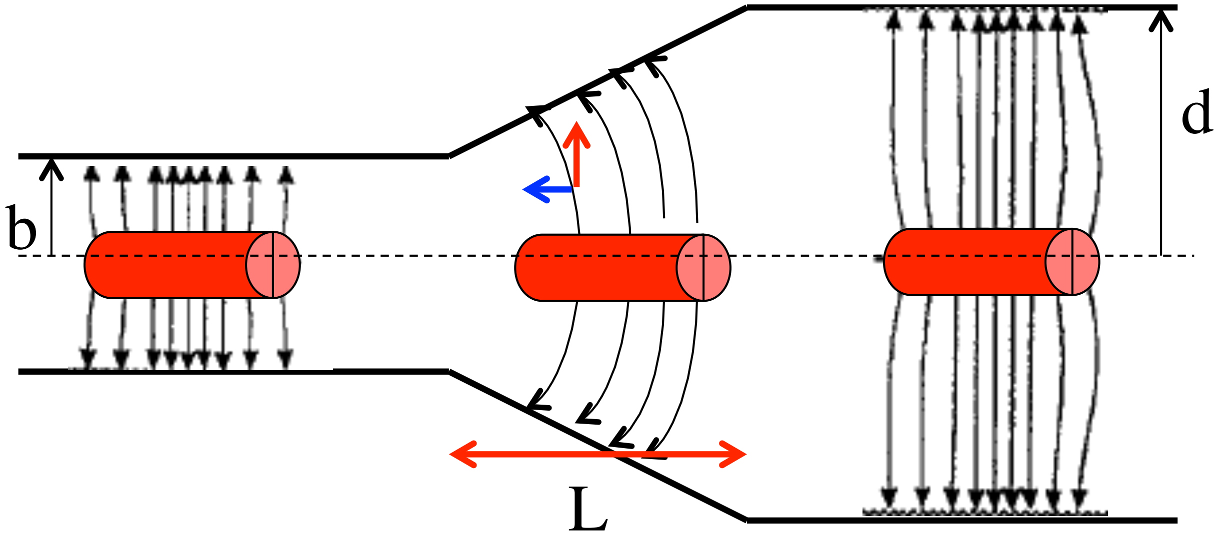
$$B_\vartheta = \frac{\beta}{c} E_r$$

$$\varphi(r) = \int_r^b E_r(r')dr'$$

$$= \frac{I}{4\pi\varepsilon_o v}\left(1 + 2\ln\frac{b}{a} - \frac{r^2}{a^2}\right) \quad \textbf{\textit{for}} \quad r \le a$$

$$= \frac{I}{2\pi\varepsilon_o v} \ln\frac{b}{r} \quad \textbf{\textit{for}} \quad a \le r \le b$$

# Circular Perfectly Conducting Pipe with Transition




There is a longitudinal $E_z(r,z)$ field in the transition and a test particle experience a voltage given by:

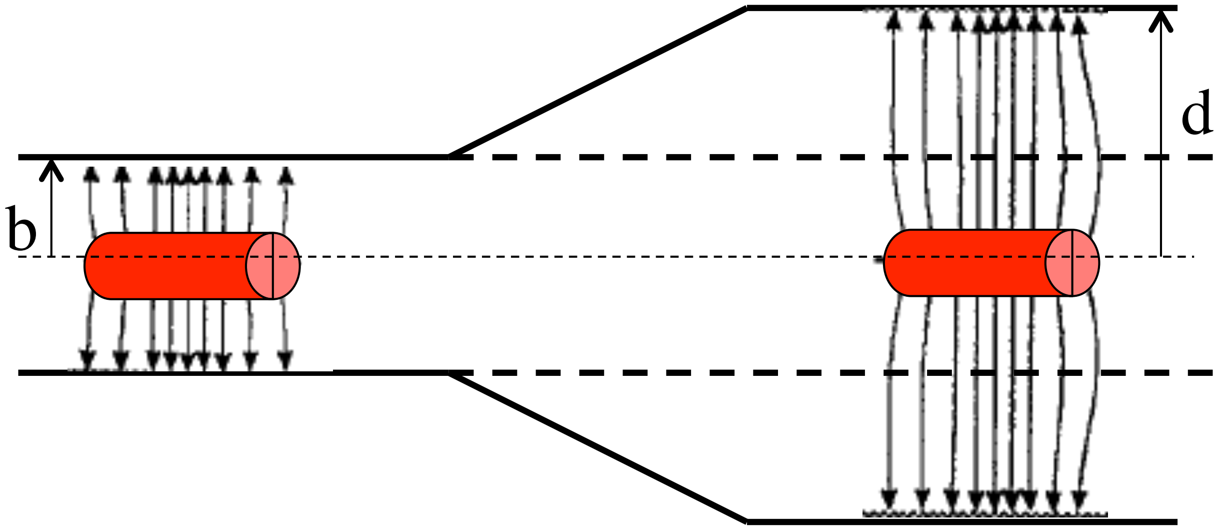
$$V = -\int_0^L E_z(r,z)dz = -\left(\varphi(r,L) - \varphi(r,0)\right) = -\frac{I}{2\pi\varepsilon_o v} ln\frac{d}{b}$$

decelerating if d > b

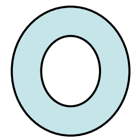
$$P_b = VI = \frac{I^2}{2\pi\varepsilon_o v} ln\frac{d}{b}$$

Power lost by the beam

## For d > b the power is deposited to the energy of the fields: moving from left to right the beam induces the fields in the additional space available



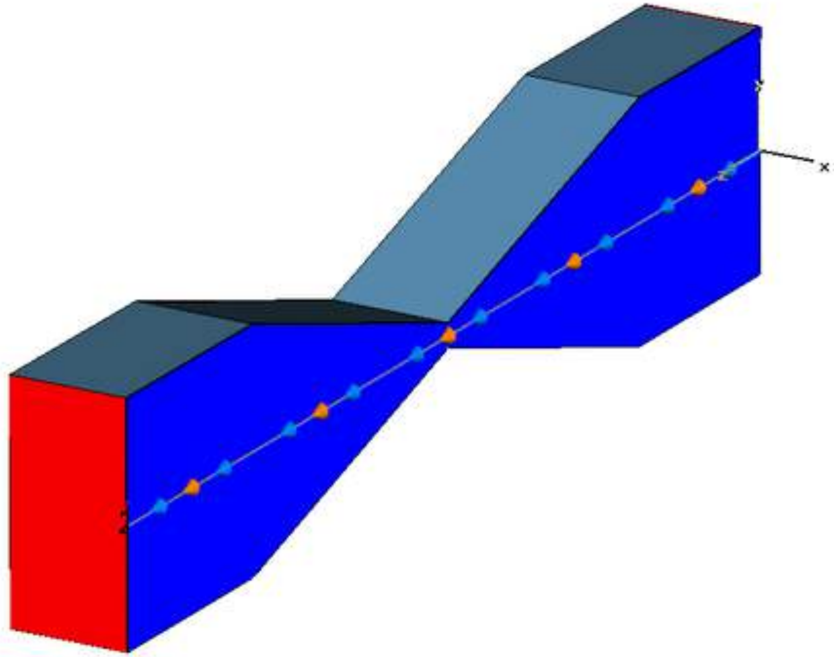

The additional power passing through the right part of the beam pipe is obtained by integrating the Poynting vector throught the sourface $\Delta S = \pi\left(d^2 - b^2\right)$



$$P_{em} = \int_{\Delta S} \left( \frac{1}{\mu} \vec{E} \times \vec{B} \right) \cdot d\vec{S} = \int_b^d \frac{E_r B_\vartheta}{\mu} 2\pi r\, dr = \frac{I^2}{2\pi \varepsilon_o v} \ln \frac{d}{b}$$

Notice that if d<b the beam gains energy. If d-->∞ the power goes to infinity, such an unphysical result is nevertheless consistent with the original assumption of an infinite energy beam (γ->∞).

# Reflected and Diffracted fiels



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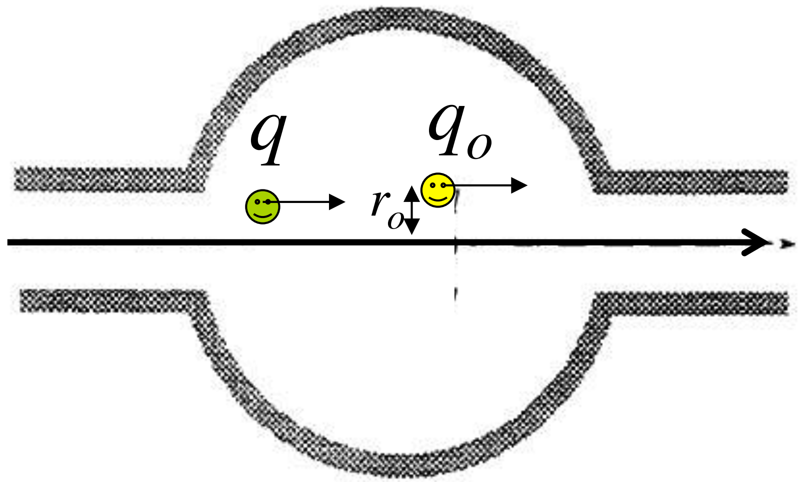
# Wake Potentials




$$\boldsymbol{F} = q\left[E_z\hat{z} + \left(E_x - vB_y\right)\hat{x} + \left(E_y + vB_x\right)\hat{y}\right] \equiv \boldsymbol{F}_{//} + \boldsymbol{F}_{\perp}$$

there can be two effects on the **test charge** :

1) a longitudinal force which changes its energy,

2) a transverse force which deflects its trajectory.

If we consider a device of length L:

the Energy Gain is:

$$\boxed{U = \int_0^L F_z ds}$$

the Transverse Deflecting Kick is:

$$\boxed{\boldsymbol{M} = \int_0^L \mathbf{F}_\perp ds}$$

These quantities, normalised to the charges, are called ***wake-potentials*** and are both function of the distance z.

Note that the integration is performed over a given path of the trajectory.

***Longitudinal wake potential***

*[V/C]*

$$\boxed{w_{//} = -\frac{U}{q_o q} = -\frac{V}{q_o}}$$

**Energy Loss**

***Transverse wake potential***

*[V/Cm]*

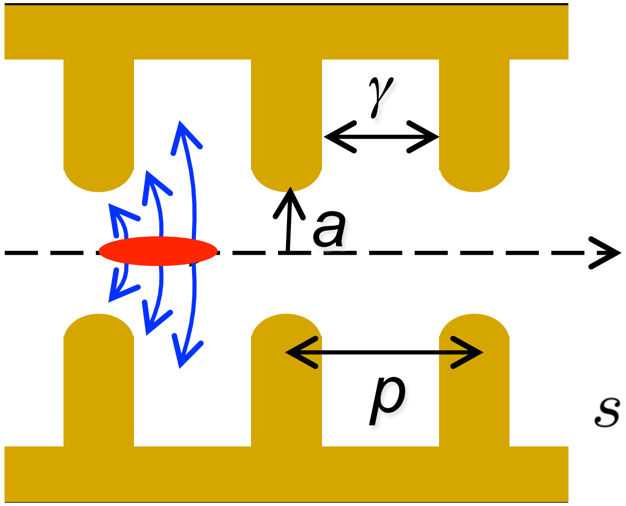
$$\boxed{\boldsymbol{w}_{\perp} = \frac{1}{r_o}\frac{\boldsymbol{M}}{q_o q}}$$

**Transverse Kick**

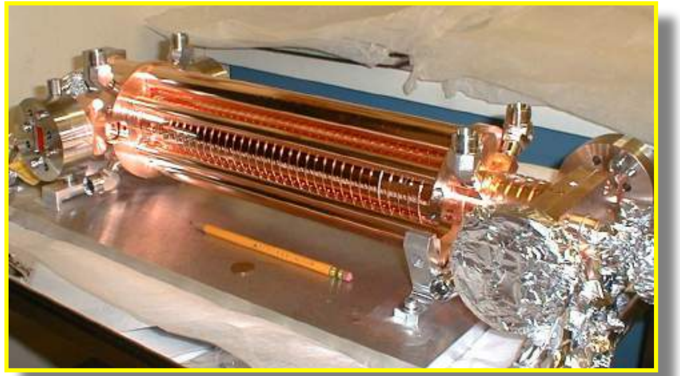
The sign minus in the longitudinal wake-potential means that the test charge loses energy when the wake is positive.

Positive transverse wake means that the transverse force is defocusing.

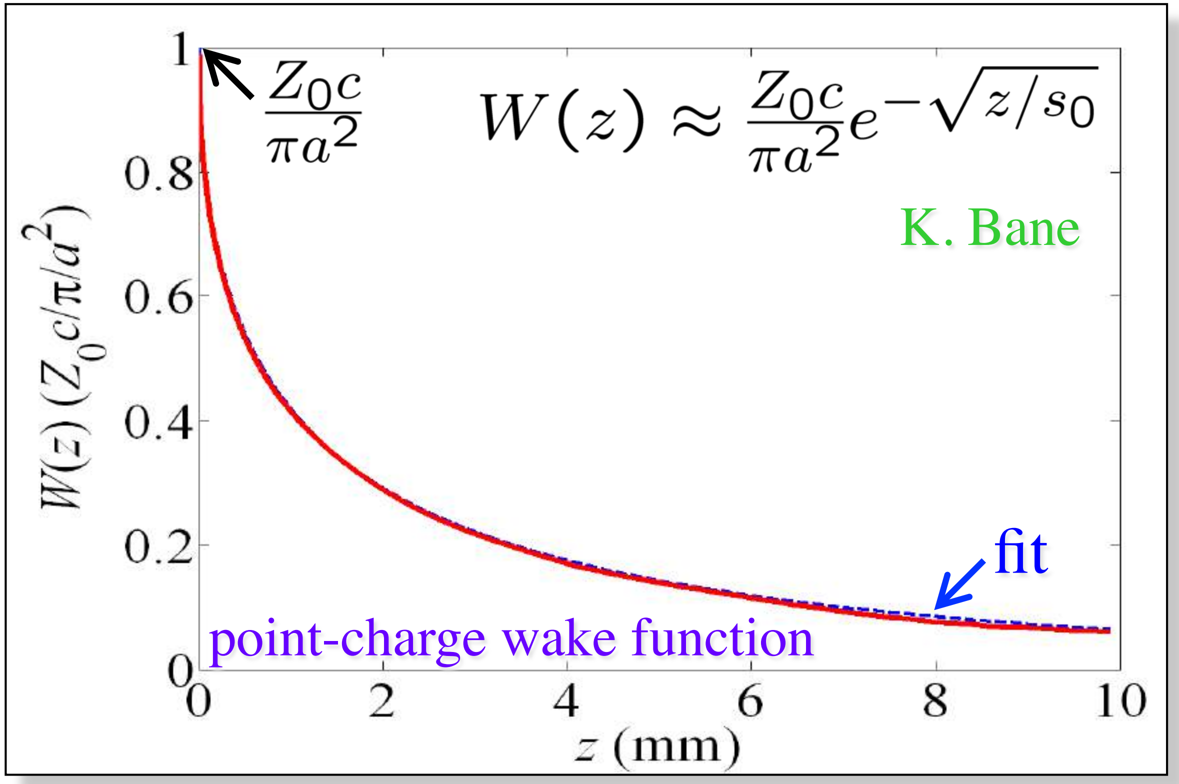
# Longitudinal Wakefields of RF Structures




***SLAC*** *S*-band:
$a \approx 11.6$ mm
$\gamma \approx 29.2$ mm
$p \approx 35.0$ mm

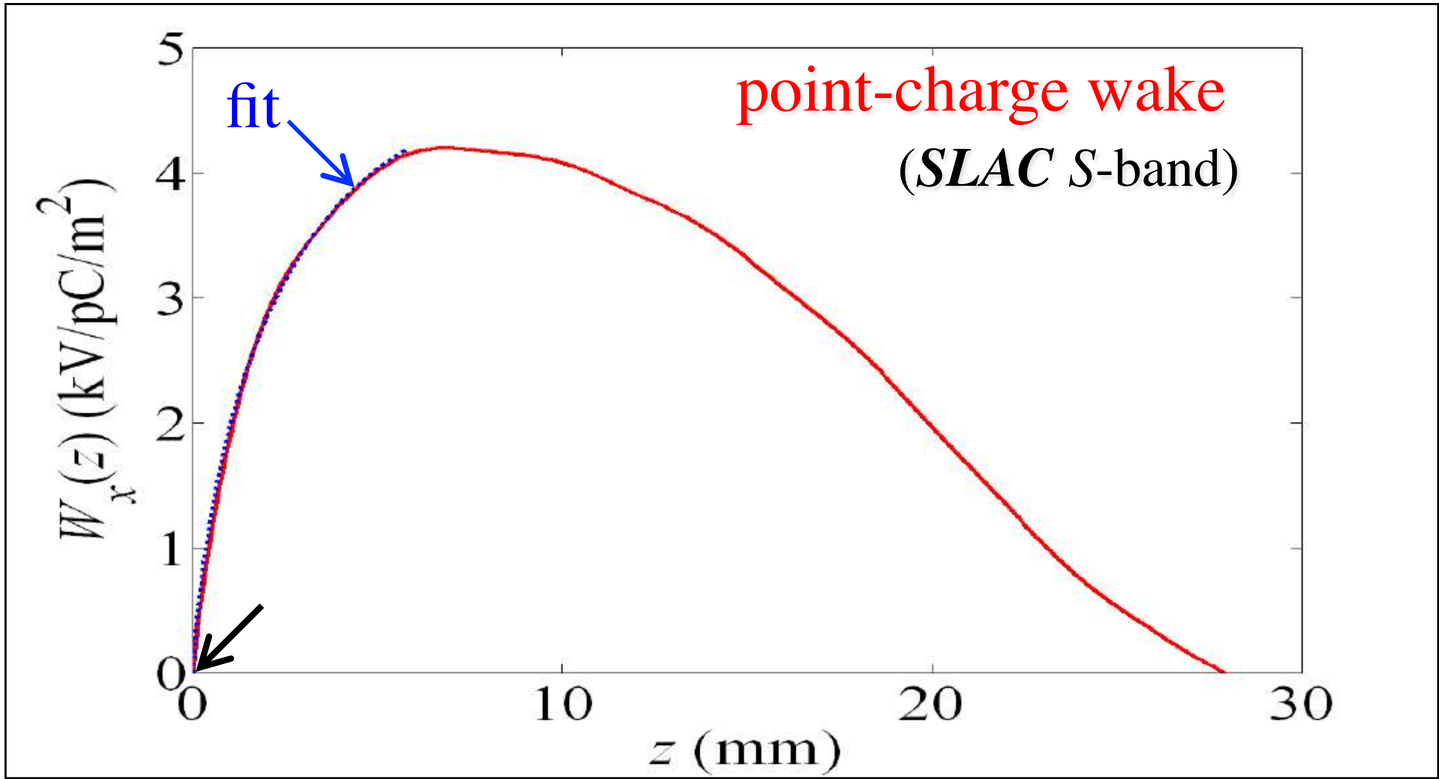


$$s_0 \approx 0.41 \frac{a^{1.8} g^{1.6}}{p^{2.4}}$$




$$\boxed{W_{//} \propto \omega^2}$$

# Transverse Wakefields




**SLAC** S-band
$s_1 \approx 0.56$ mm
$a \approx 11.6$ mm
$A \approx 1.13$
$z < \sim 6$ mm

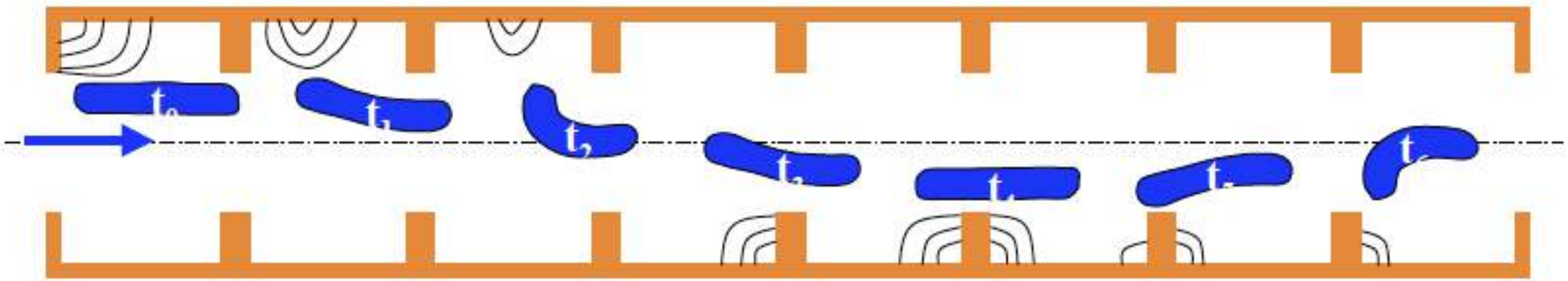
$$W_\perp \propto \omega^3$$

transverse point-charge wakefield function and short-range fit:

$$W_x(z) \approx A\frac{4Z_0 c s_1}{\pi a^4}\left[1-\left(1+\sqrt{z/s_1}\right)e^{-\sqrt{z/s_1}}\right] \quad , \quad z < 6 \text{ mm}$$

# Beam Break Up

A beam injected off-center in a LINAC, because of the focusing quadrupoles, execute betatron oscillations. The displacement produces a transverse wake field in all the devices crossed during the flight, which deflects the trailing charges.



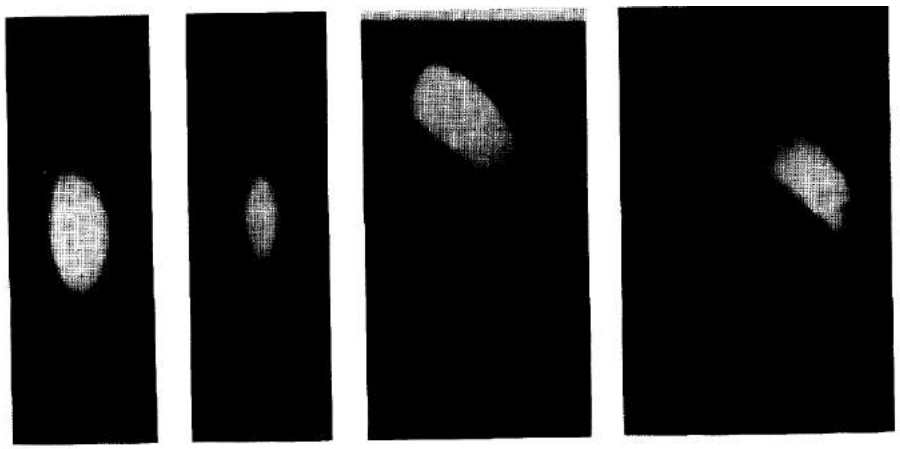


**Figure 3.4.** Four transverse beam profiles observed at the end of the SLAC linac are shown when the beam was carefully injected and injected with 0.2, 0.5, and 1 mm offsets. The beam sizes $\sigma_x$ and $\sigma_y$ are about 120 $\mu$m. (Courtesy John Seeman, 1991.)

# “How to advance?”

**Hadron (p) circular collider**

$$p = e \cdot R \cdot B_y$$

Increase bending field
SC bend magnet work (FCC-hh)

Increase radius = size (FCC-hh)

**Lepton (e-,e+) circular collider**

$$p \propto E_0 \cdot \sqrt[4]{\rho \cdot U_0}$$

Increase supplied RF voltage (FCC-ee)

Increase mass of acc. particle (muon)

Increase radius = size (FCC-ee)

**Lepton (e-,e+) linear collider**

$$p = L \cdot G_{acc}$$

Increase accelerating gradient
(a) Pushing existing technology (ILC, CLIC)
(b) New regime of ultra-high gradients (plasma, dielectric accelerators)

Increase length (ILC, CLIC)

Courtesy R. Assmann

V. Shiltsev, arXiv:1404.4097

# Phenomenological Model of Accelerator costs

- Total project cost [TPC] divided into three components

$$\mathrm{TPC} = \alpha\left(\frac{L}{10[\mathrm{km}]}\right)^{\frac{1}{2}} + \beta\left(\frac{E}{1[\mathrm{TeV}]}\right)^{\frac{1}{2}} + \gamma\left(\frac{P}{100[\mathrm{MW}]}\right)^{\frac{1}{2}}$$

- Civil Engineering and construction
- Accelerator components
- Facility Infrastructure
- Phenomenological formula parametrised in terms of tunnel length[L], centre-of-mass Energy[E] and total site AC power [P]

$$\begin{aligned}
\alpha &= \$2B \\
\gamma &= \$2B \\
\beta &= \$1\mathrm{B}\ (\mathrm{NCmagnets}), \\
\beta &= \$2\mathrm{B}\ (\mathrm{SCmagnets}), \\
\beta &= \$8\mathrm{B}\ (\mathrm{NCRF}), \\
\beta &= \$10\mathrm{B}\ (\mathrm{SCRF})
\end{aligned}$$

- Coefficient beta is technology dependent

V. Shiltsev, arXiv:1404.4097

# Estimated costs of future facilities

- Costs are in American accounting, i.e. including all labour costs. In European accounting this would be a factor ~2-2.5 smaller.

| | $E$[TeV] | $L$[km] | $P$[MW] | $\alpha\beta\gamma$ TPC [$B] | Civil construction cost [$B] |
|---|---|---|---|---|---|
| $Ce^+e^-C$ | 0.25 | 54 | $\sim 500$ | 10.2 | 4.6 |
| FCC-ee | 0.25 | 100 | $\sim 300$ | 10.9 | 6.3 |
| ILC | 0.5 | 36 | 163 | 13.1 | 3.8 |
| CLIC | 3 | 60 | $\sim 560$ | 23.5 | 4.9 |
| $\mu$ collider | 6 | 20 | $\sim 230$ | 12.9 | 2.8 |
| LHC-33 | 33 | 0 | $\sim 100$ | 4.8 | 0? |
| SppC(China) | 50 | 54 | $\sim 300$ | 25.5 | 4.6 |
| FCC-pp | 100 | 100 | $\sim 400$ | 30.3 | 6.3 |

These are Shiltsev's numbers, in no way approved by any of the proponents of these machines.

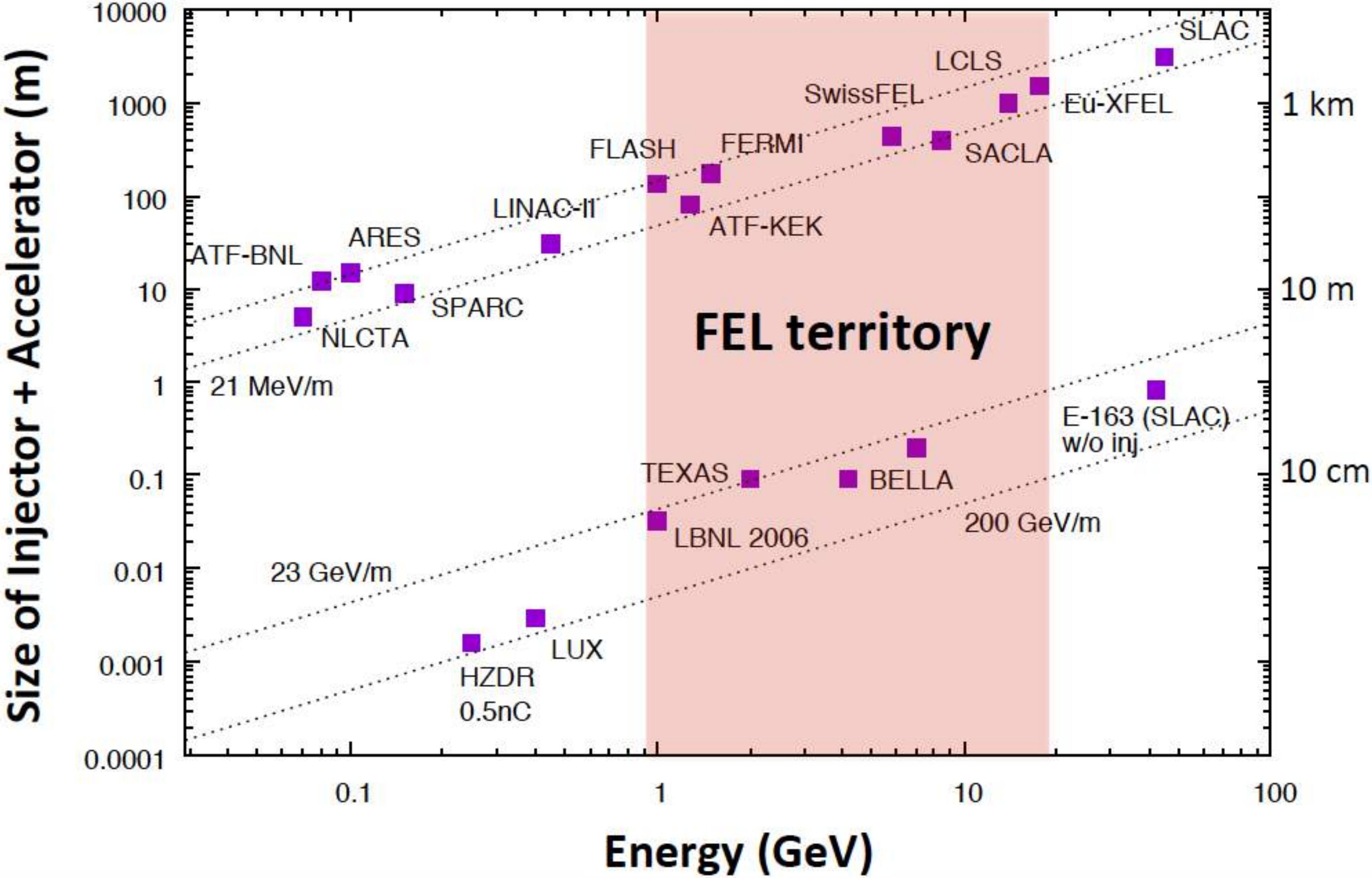
- Power usage is substantial. Rate of energy usage,~1 kW/person.
- A small nuclear power station gives 500MW of power.

Paramters for CepC,ILC,HE-LHC,SppC have now changed.

# Next challenges for PWA

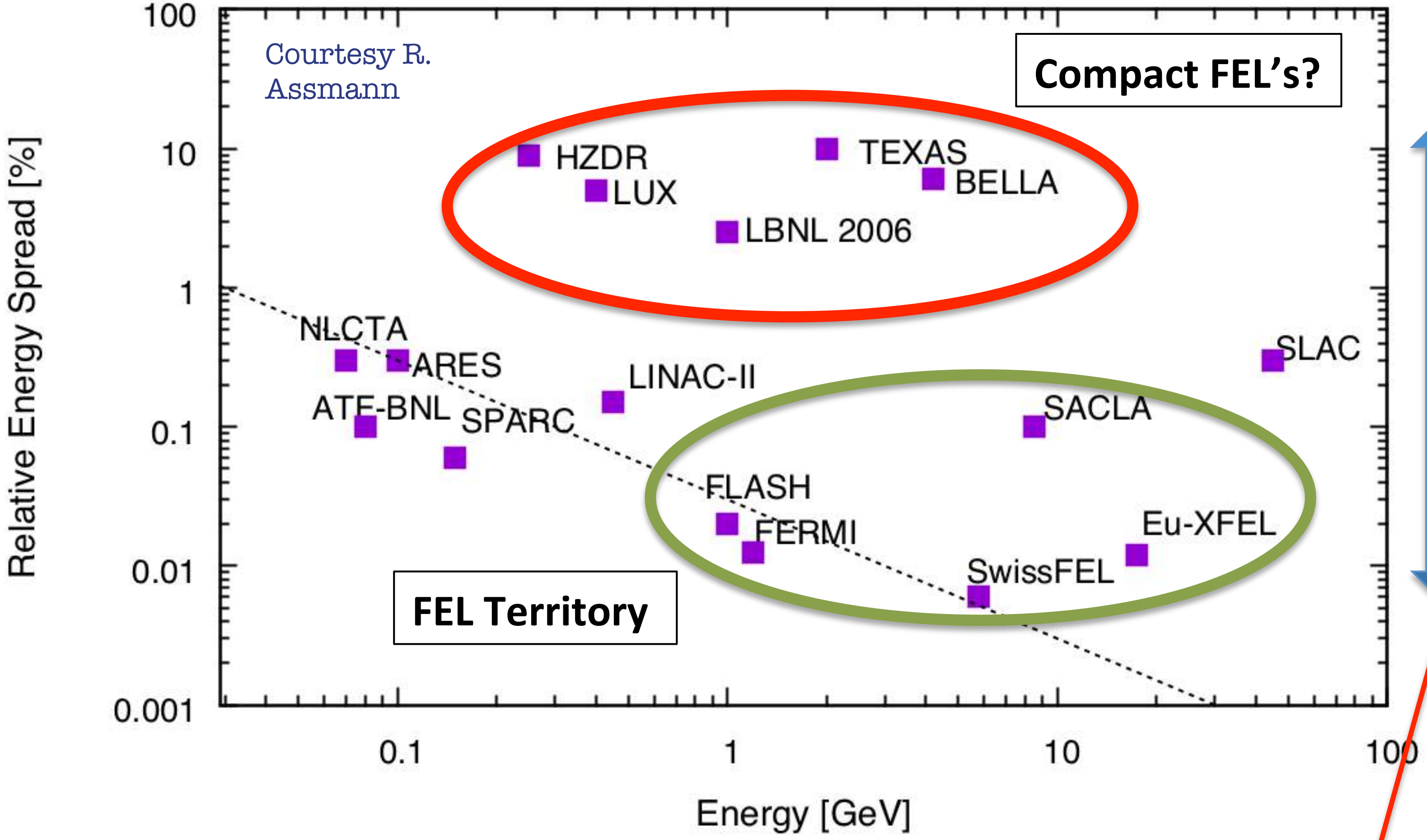
# Size versus Energy

*electron linear accelerators*



# Quality: Example Energy Spread






**Lower quality**

**Higher quality**

$$\frac{\sigma_\gamma}{\gamma} = \frac{\sqrt{3}}{2} k_p^2 \sigma_z^2$$

$$\sin\varphi_w \propto \frac{q_w}{Q_d}$$

$$\varepsilon_{n,rms} = \sqrt{\left\langle \gamma^2 \right\rangle \left( \sigma_\gamma^2 \sigma_x^2 \sigma_{x'}^2 + \varepsilon_{rms}^2 \right)}$$

M. Migliorati et al, Physical Review Special Topics,Accelerators and Beams 16, 011302 (2013)
K. Floettmann, PRSTAB,6, 034202 (2003)


4th EAAC 15-22 September, 2019
Isola d'Elba
TALKS
BREAKFAST
COFFEE
MEALS
DISCUSSIONS
WAVE-BREAKING
https://agenda.infn.it/event/17304/


Thank for your attention