Beam Driven Systems

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Menu:

- What did we learn so far?
- Relativistic Particles/bunches
- Driving wakefields (PWFA)
- Linear wakefields theory
- Current filamentation instability
- Transformer ratio
- Beam loading
- Emittance preservation
- Focusing, propagation and emittance preservation
- Summary/conclusions

Previous CAS proceedings: http://cds.cern.ch/record/1641416/files/CERN-2016-001.pdf

PURPOSE

- Give you some basic tools and understanding of the plasma wakefield accelerator (PWFA)
- Give you some cues to approach the "real" complicated situations (theory, simulations, experiments)
- Not go into too many details …
- Learning is doing it yourself (over and over)
- Thus, many details (parameters, etc.) are left off, for you to find out …

- Show that (at least) some of the simple principles have been measured
- Theory, simulations and experiments mostly agree (to the extent of our understanding or knowledge of each of them)

- If you find my mistakes (there may be some), you understand the material …

- Apologies to those for whom this is trivial …

- Bunches go left, bunches go right, go down …

WHAT DID WE LEARN SO FAR? (DID WE?)

Plasma ⇔ collective response

Plasma is neutral: $n_{e0} = n_{i0}$

Langmuir, electro-static wave: $\vec{E} // \vec{k} \quad \vec{B} = 0$ (1D), satisfies $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}(n_{i0} - n_{e0})$

Interesting because: $E_z \approx E_{WB} = m_e c \omega_{pe} / e$ for PWFA: $E_z \approx \frac{n_b}{n_{e0}} E_{BW}$

(aka, cold plasma wave breaking field)

Plasma e$^-$ inertia ⇔ oscillate at $\omega = \omega_{pe} = \left(\frac{n_{e0} e^2}{\varepsilon_0 m_e} \right)^{1/2}$ around equilibrium position when perturbed

Plasma (e$^-$) screen fields (potentials) over scale: $c / \omega_{pe} = 1 / k_{pe} = \lambda_{pe} / 2\pi$

(aka, the cold collisionless plasma skin depth)

WHAT DID WE LEARN SO FAR? (DID WE?)

Max-Planck-Gesellschaft

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

ES wave dispersion relation: $\omega = \omega_{pe} = \left(\frac{n_{e0}e^2}{\varepsilon_0 m_e}\right)^{1/2} >> \omega_{pi} = \left(\frac{n_{e0}Z^2e^2}{\varepsilon_0 m_i}\right)^{1/2}$

$v_\varphi = \frac{\omega}{k} = ?$ Undefined, determined by excitation (pulse, bunch)

$v_g = \frac{\partial \omega}{\partial k} = 0$ Energy does not propagate (in a cold plasma)

Laser pulse: $\omega^2 = \omega_{pe}^2 + k^2c^2$ $\quad v_g = \frac{\partial \omega}{\partial k} = \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} c = \left(1 - \frac{n_{e0}}{n_{critical}}\right)^{1/2} c \overset{\frac{\omega_{pe}^2}{\omega^2} >> 1 \Leftrightarrow \frac{n_{e0}}{n_{critical}} >> 1}{\cong} \left(1 - \frac{1}{2}\frac{n_{e0}}{n_{critical}}\right) c < c$

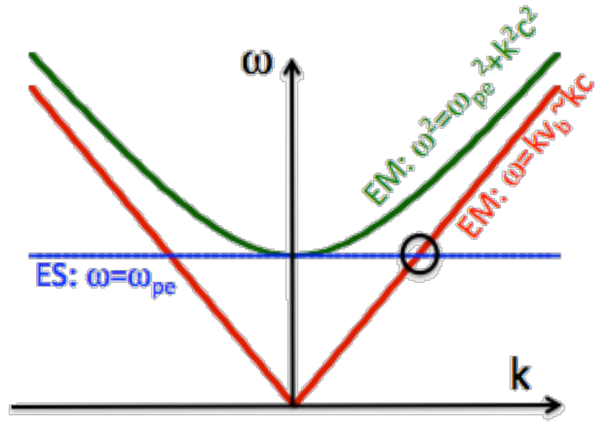
(EM: $\vec{E} \perp \vec{B} \perp \vec{k}$)

Depends on plasma density!

Particle bunch: $v_p = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} c \overset{\gamma^2 >> 1}{\cong} c$

Independent from plasma density!

Intersection ⇔ interaction ⇔ excitation



WHAT DID WE LEARN SO FAR? (DID WE?)

Plasma wakefields:

•correspond to free oscillation of plasma e⁻ (i.e., @ ω_{pe})

•are sustained by periodic plasma e⁻ density perturbation (collective!)

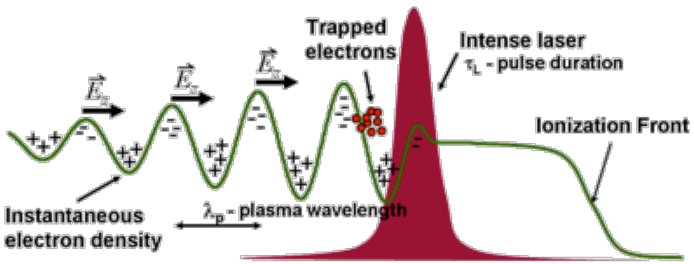
•are driven/excited by a short (<1/ω_{pe}) perturbation (laser pulse, particle bunch)

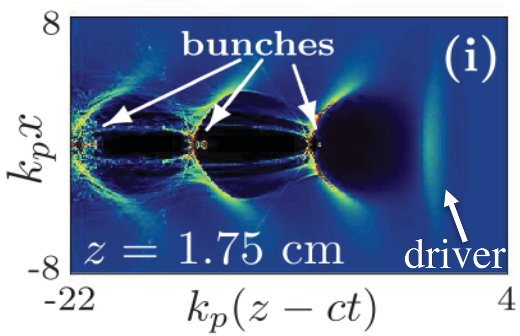
•have the same phase velocity as the driver

•are the only mode of the system (unlike RF systems TM_{01} + TM_{mn} +TE_{mn})

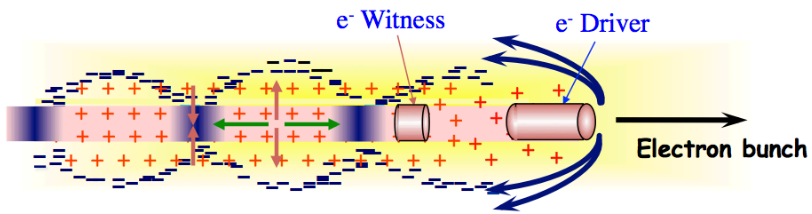
•are usually (at least) 2D

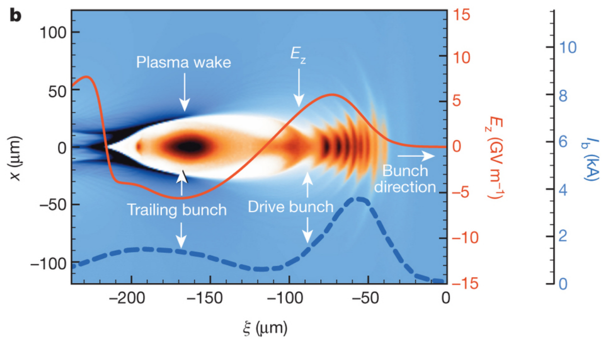
Laser WakeField Accelerator





Plasma WakeField Accelerator





✧Similarities between LWFA and PWFA!!

PLASMA WAKEFiELDS?

Analogy with PWFA:

•Water and boat are of the same “kind”

•Water “soft” (non-relativistic plasma e^-), boat “stiff” (relativistic bunch)

•Perturbation in water height provides slope for acceleration …

•Perturbation in plasma electron density sustains E_z field for acceleration

But:

•Surface wave in water, volume wave in plasma …

•Plasma can modify the bunch …

•Not a relativistic system, very small energy gain …

PLASMA WAKEFiELDS?

Analogy with PWFA:

•Water and boat are

•Water "soft" (non-r bunch)

•Perturbation in wat

•Perturbation in plas

But:

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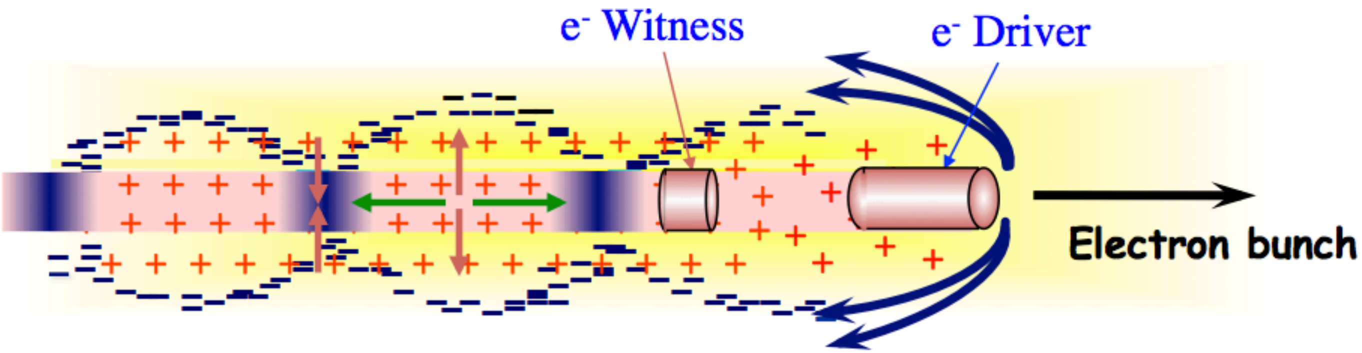
•Plasma can modify

•Not a relativistic system, very small energy gain …

PLASMA WAKEFiELDS?



✧The plasma converts the transverse E-field of the pulse or bunch into a longitudinal one

✧The plasma "filters" the ω_{pe} frequency component

✧The wakefields have the same (phase) velocity as the driver

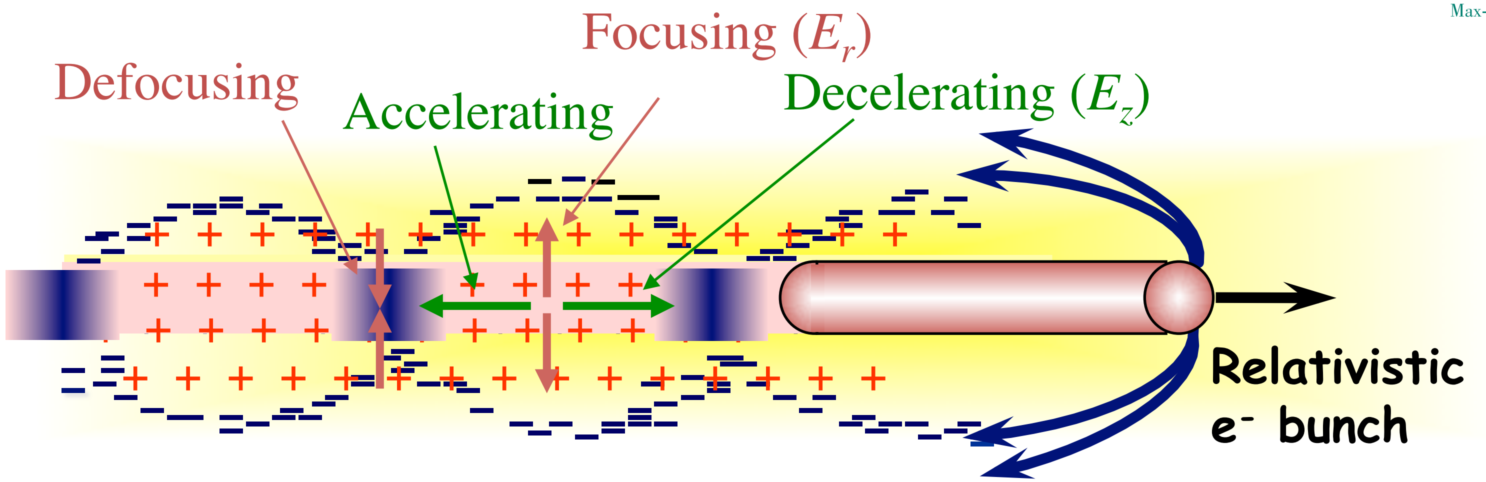
✧Plasma sustains a relativistic (relativistic driver) longitudinal electric field: acceleration

✧Energy is extracted from the drive bunch, transferred to the plasma wakefields

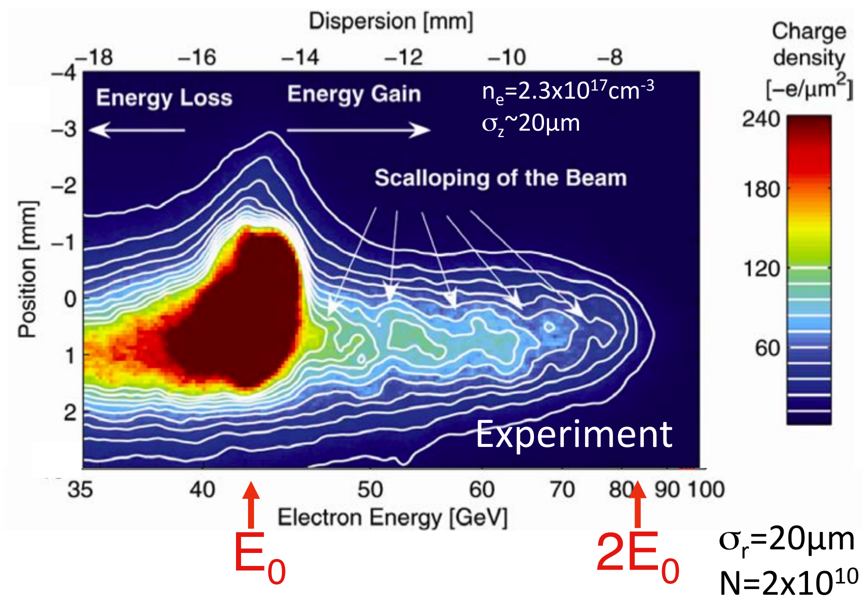
✧Energy is extracted from the wakefields by the witness bunch

✧Co-linear wakefield accelerator (CLIC = non-colinear)

PLASMA WAKEFIELD ACCELERATOR



Blumenfeld, Nature 445, 741 (2007)



σ_r=20μm
N=2x10^{10}

42 => 84GeV in 85cm! 50GeV/m

- Large energy gain (42GeV)
- Large gradient (50GeV/m)
- Long length (85cm)
- Scaling from:
  - σ_z~600μm, n_e=1.9x10^{14}cm^{-3}, G~200MeV/m
    - /30 x1400 x250
  - σ_z~20μm, n_e=2.6x10^{17}cm^{-3}, G~50GeV/m

Muggli, Phys. Rev. Lett. 93, 014802 (2004)
Hogan, Phys. Rev. Lett. 95, 054802 (2005)
Muggli, Hogan, Comptes Rendus Physique, 10(2-3), 116 (2009)
Muggli, New J. Phys. 12, 045022 (2010)

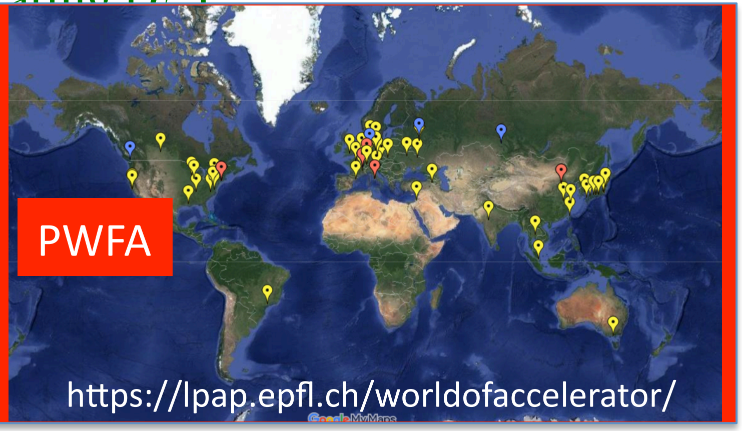
Plasma Wakefield Accelerator

Focusing (E_r)

Defocusing

Decelerating

Result that put the PWFA on the map of (possible) high-energy physics accelerators



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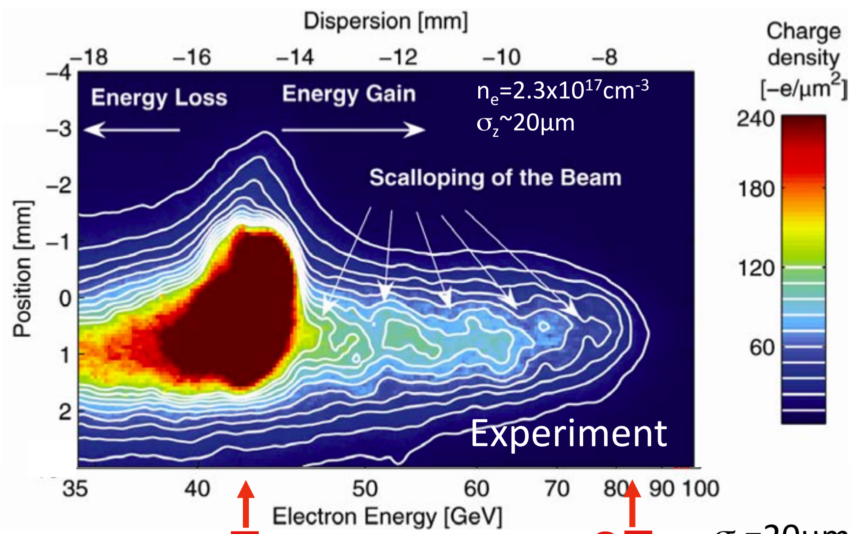

E_0 $2E_0$ σ_r=20μm N=2x10^{10}

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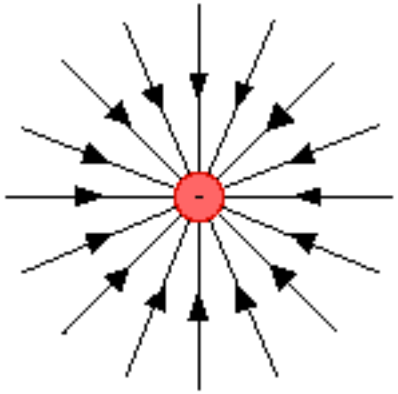
BEAM PROPAGATION IN VACUUM

Beam influenced by the transverse fields $F_{\perp} = q\left(E_r + v_b \times B_\theta\right)$ from beam and plasma (wakefields)

Beam self-fields in vacuum:

In the e^- rest frame: $\vec{E}_0 = \frac{1}{4\pi\varepsilon_0}\frac{-e}{r^2}\frac{\vec{r}}{|r|} = E_0\frac{\vec{r}}{|r|}$



The electric field from an isolated negative charge

http://physics.bu.edu/~duffy/PY106/Electricfield.html

In the lab frame (e^- moves at v_b~c)

Joules-Bernoulli equation:

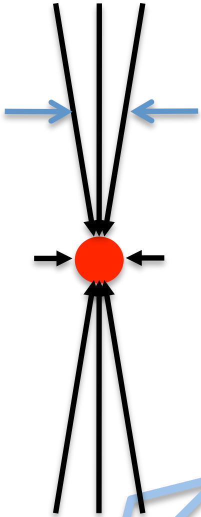
$\mathbf{E}_{\|}' = \mathbf{E}_{\|}$ $=E_0$

$\mathbf{B}_{\|}' = \mathbf{B}_{\|}$ $=0$

$\mathbf{E}_{\perp}' = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}\right)$ $=\gamma E_0$

$\mathbf{B}_{\perp}' = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right)$

$=(v/c^2)E_0$



=> $\theta = E_x/E_z = 1/\gamma$

$$F_{\perp}' = q\left(E_{\perp}' - v_{//} \times B_{\perp}'\right) = -eE_r'\left(1 - v_{//}\frac{v_{//}}{c^2}\right) = -eE_r'\frac{1}{\gamma^2}$$

(jump from single particle to beam!)

The (bunch) self-force is $1/\gamma^2$ the "space charge force" ($\sim E_r'$) ⇔ decreases as $\sim 1/\gamma^2$

1. There is (~)no space charge effect when the bunch particles are relativistic
2. Bunch particles (~)do not interact with each other
3. In vacuum the relativistic beam diverges because of its divergence, not space charge forces

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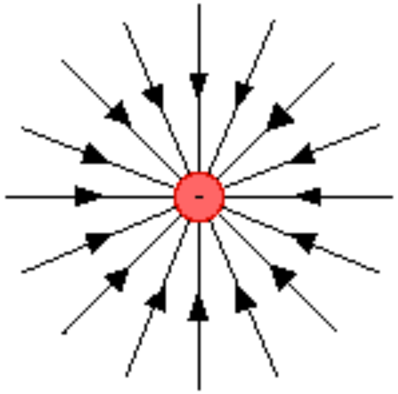

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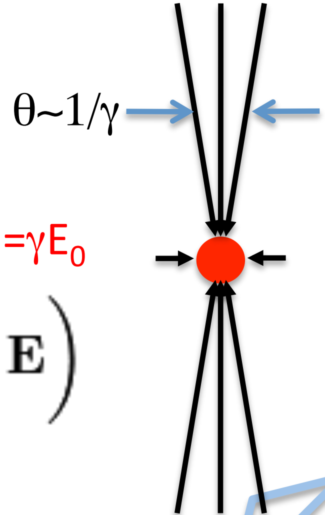
$\mathbf{E}_{\perp}' = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v}\times\mathbf{B}\right)$ $=\gamma E_0$

$\mathbf{B}_{\perp}' = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}\right)$

$=(v/c^2)E_0$



=> $\theta = E_x/E_z = 1/\gamma$

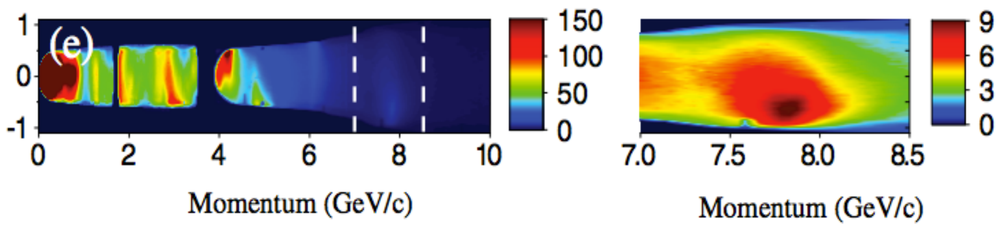
Example: SLAC e^- beam at 10, 20, 40GeV, γ~20'000,40'00, 80'000, space charge effects negligible
AWAKE injected at ~20MeV, γ~40, space charge effects may not be negligible
All depending on bunch charge
Wisdom: ~150MeV for ~nC, ~ps bunches (~100A current)

2. Bunch particles (~)do not interact with each other
3. In vacuum the relativistic beam diverges because of its divergence, not space charge forces

BEAMS

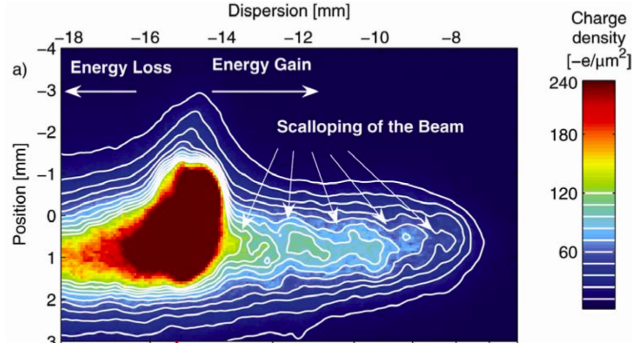
LWA: 20cm, ~8GeV

Gonsalves, Phys. Rev. Lett. 122, 084801, 2019



PWFA: 85cm, ~42GeV

Blumenfeld, Nature 445, 741 (2007)



BEAMS

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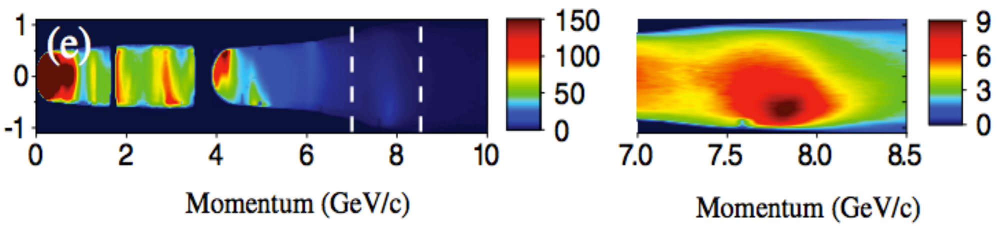

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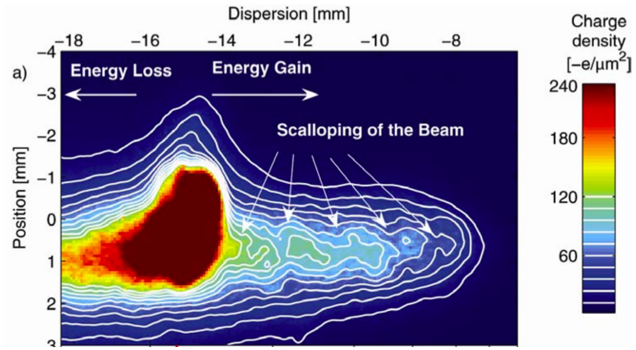


Rayleigh length of laser beam

Beta function or particle beams (at waist)

$$Z_R = \pi \frac{w_0^2}{\lambda_0} \Leftrightarrow \beta^* = \beta_0 = \frac{\sigma_0^{*2}}{\varepsilon_g} \qquad \varepsilon_g = \frac{\varepsilon_N}{\gamma}$$

Laser pulse:

W_0=20µm

λ_0=800nm

Z_R=1.6mm

$$\frac{\lambda_0}{\pi} \Leftrightarrow \varepsilon_g$$

SLAC e^- beam:

ε_N=50mm-mrad

γ=80'000 (40GeV)

ε_g=6.25x10^{-10}m-rad

σ_0=20µm

β_0=64cm

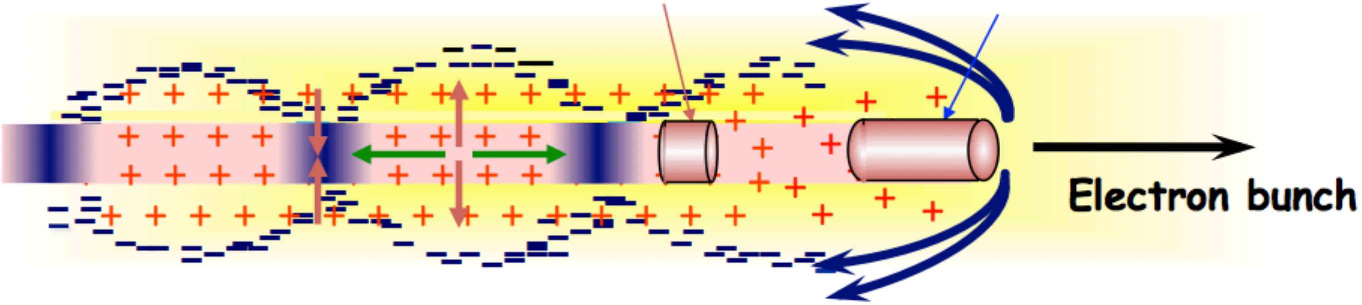
Z_R=64cm

λ_0=2nm

External guiding necessary!
$\lambda_0 \Leftrightarrow$ physics, technology

External guiding helps
$\varepsilon_g \Leftrightarrow$ technology, physics

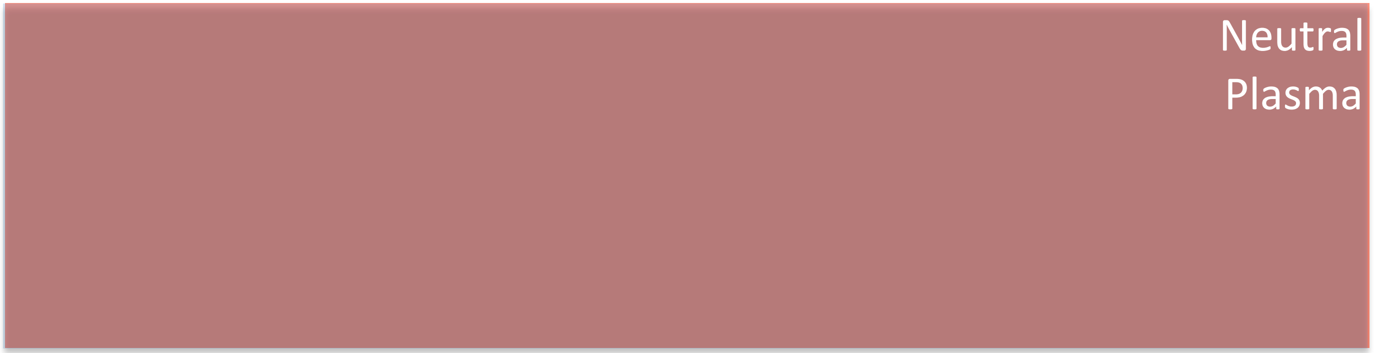
BEAMS AND PWFA



- All particles are relativistic: can lose large fraction of energy:
  - E=40GeV, γ=80'000 to E=2GeV, γ=4'000, dephasing length $\Delta L=(1/\gamma^2)(\Delta\gamma/\gamma)L$
  - Particles still "stiff"
- Plasma provides focusing (blow-out), no need for external guiding
- $\beta_0=\sigma_0^2/\varepsilon_g \gg Z_r$, can be cm's to m's
- Wakefields amplitude weakly dependent on bunch radius (for $n_b \gg n_e$)
- Not much wakefields evolution over long distances
- No plasma e^- trapping (γ_ϕ too large), good for controlled, external injection
- Bunches with many kJ's available (p^+)

DRiViNG WAKEFiELDS

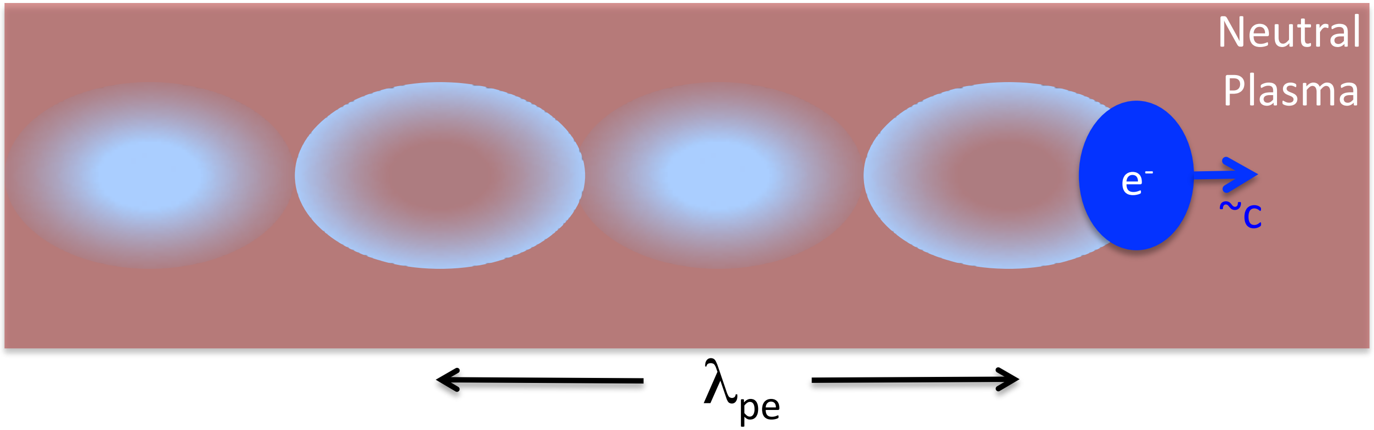
DRiViNG WAKEFiELDS



- Initially neutral plasma: $n_{e0}=n_{i0}$
- On average, no fields, no velocities, infinite, cold plasma, … ideal … simple …
- $\omega_{pe}=(n_e e^2/\varepsilon_0 m_e)^{1/2}$

DRIVING WAKEFIELDS

Short e^- bunch driver

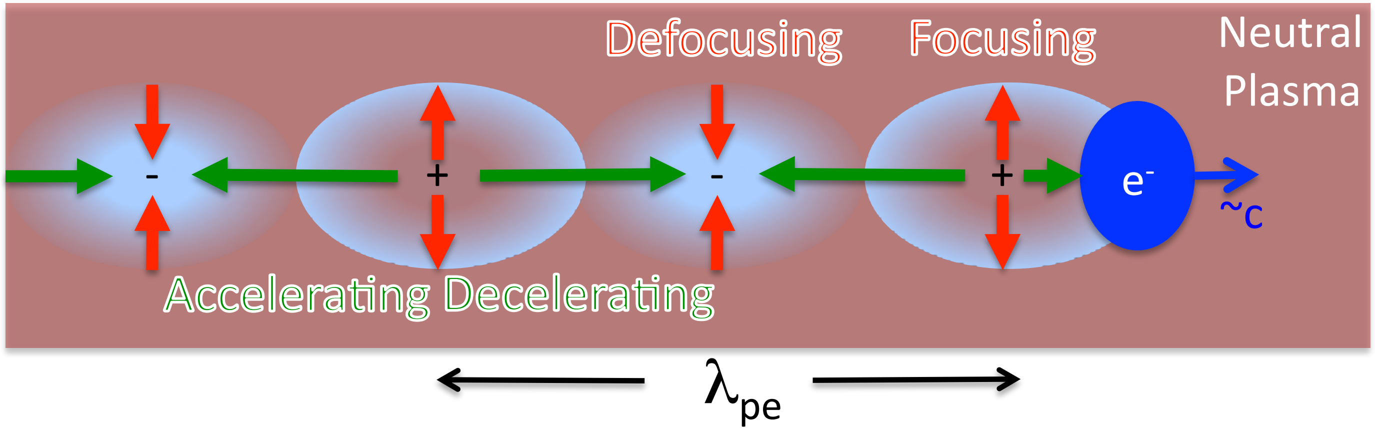


- ✧Transverse space charge field of the e^- bunch ($\sim E_{perp}$) expels plasma e^-
- ✧Leaves a (more) positively charged region behind bunch head => restoring force
- ✧Plasma e^- return towards axis, overshoot, oscillate (mostly) transversely at f_{pe}
- ✧Set-up a phased oscillation of plasma e^- => periodic density variation => wakefields
- ✧Wavelength $\lambda_{pe}=f_{pe}/v_b \sim f_{pe}/c$, $f_{pe}=\omega_{pe}/2\pi$
- ✧Linear wakefields: plasma e^- <u>do not</u> cross the beam axis
- ✧Blow-out regime wakefields: plasma e^- <u>do</u> cross axis

- ✧Plasma e^- density perturbation sustains wakefields … $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

WAKEFIELDS (FIELDS)

Short e^- bunch driver / e^- witness



✧Directions derived from electrostatic, i.e., from induced e^- charge density perturbation

✧Reality consistent with these fields, but more complicated

✧Fields by a short bunch ($<\lambda_{pe}$)

✧E-field from + to –

✧Longitudinal:

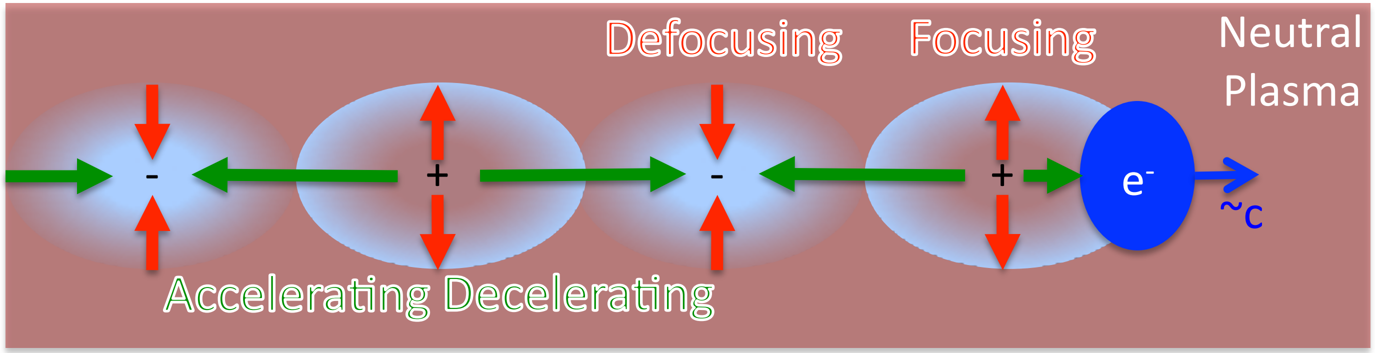
- Forward in the front: decelerating for e^-, energy loss by the drive bunch (particles)
- Backward in the back: accelerating for e^-: accelerating for witness e^- (bunch)

✧Transverse (cylindrical geometry):

- Outward in the front: focusing for e^-
- Inward in the back: defocusing for e^-

WAKEFIELDS (FIELDS)

Short e⁻ bunch driver / e⁻ witness



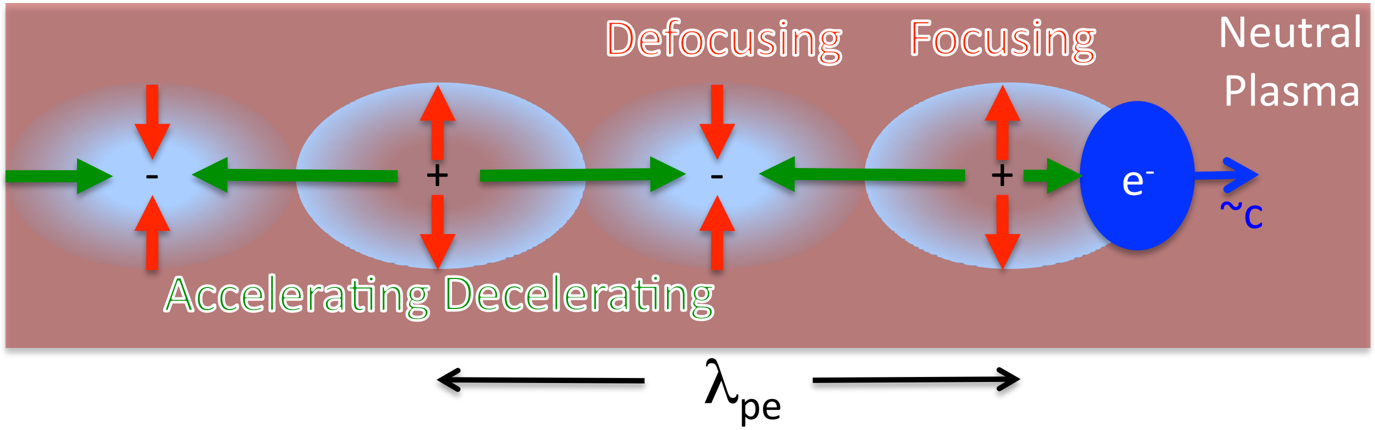
λ_{pe}

- Directions derived from electrostatic, i.e., from induced e⁻ charge density perturbation
- Reality consistent with these fields, but more complicated
- Fields by a short bunch ($<\lambda_{pe}$)
- E-field from + to –
- Longitudinal:
  - Forward in the front: decelerating for e⁻, energy loss by the drive bunch (particles)
  - Backward in the back: accelerating for e⁻: accelerating for witness e⁻ (bunch)
- Transverse (cylindrical geometry):
  - Outward in the front: focusing for e⁻
  - Inward in the back: defocusing for e⁻

Note:
- Longitudinal and transverse fields are π/2 out of phase behind the drive bunch
- Only a π/2 region is focusing and accelerating

WAKEFiELDS (FIELDS)

Short e^- bunch driver / e^- witness



✧Directions derived from electrostatic, i.e., from induced e^- charge density perturbation

✧Reality consistent with these fields, but more complicated

✧Fields with a short bunch ($<\lambda_{pe}$)

Q: Why do we consider only e^- motion?

Q: When should we?

Q: What would happen with a e^+ bunch driver?

•Forward in the front: accelerating for e^-, energy loss by the drive bunch (particles)

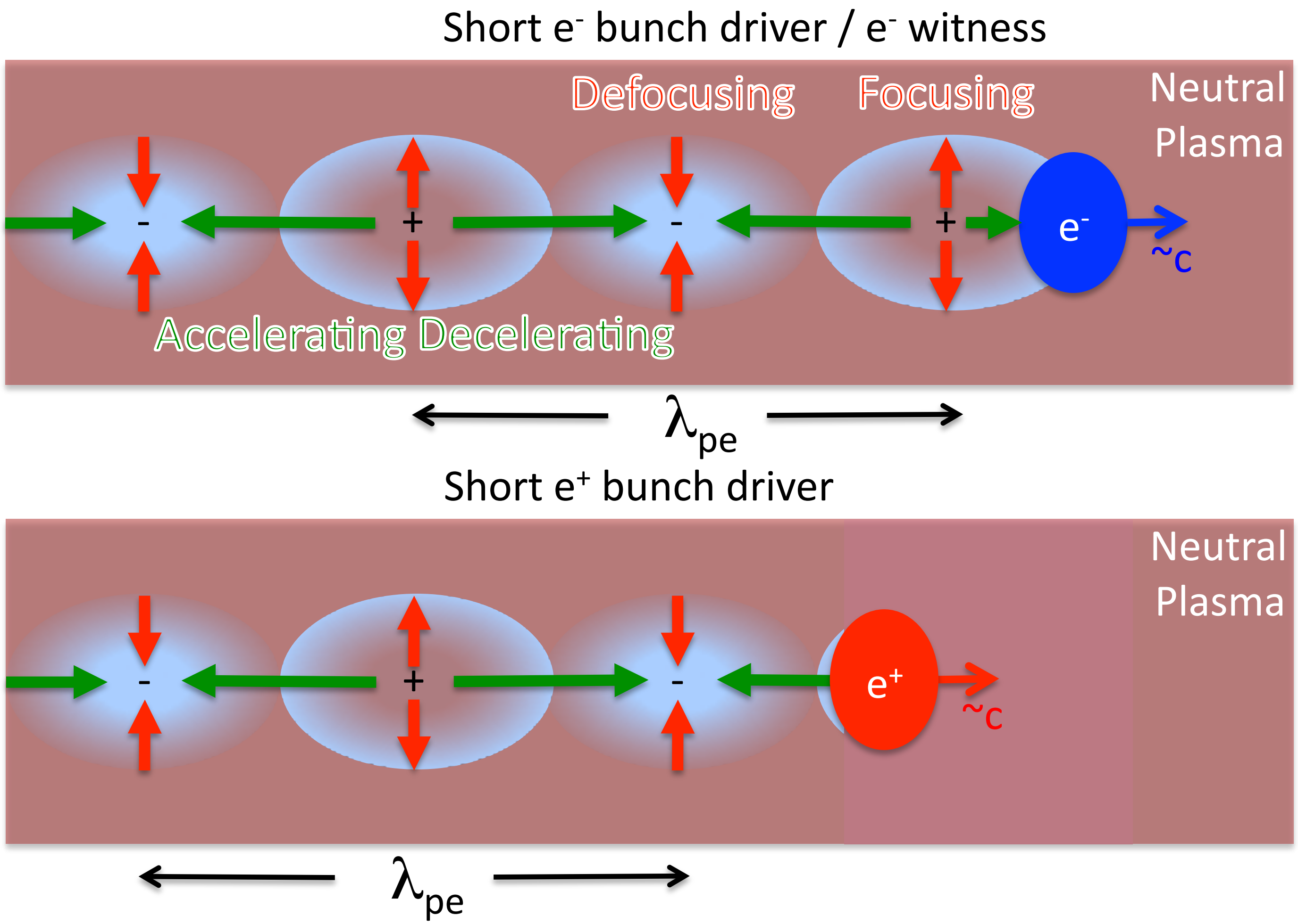
•Backward in the back: accelerating for e^-: accelerating for witness e^- (bunch)

✧Transverse (cylindrical geometry):

•Outward in the front: focusing for e^-

•Inward in the back: defocusing for e^-

WAKEFIELDS (FIELDS)



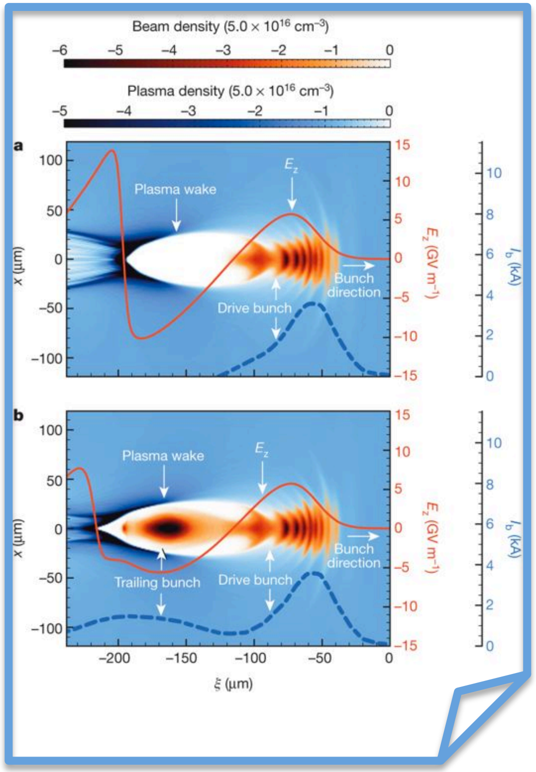
✧Plasma e- are attracted, sucked-in

✧Simple ~π/2 change of phase in the wakefields (linear regime) wrt the drive bunch

✧Nonlinear regime much more complicated (see. S. Corde's lecture)

WAKEFILEDS EXCITATION

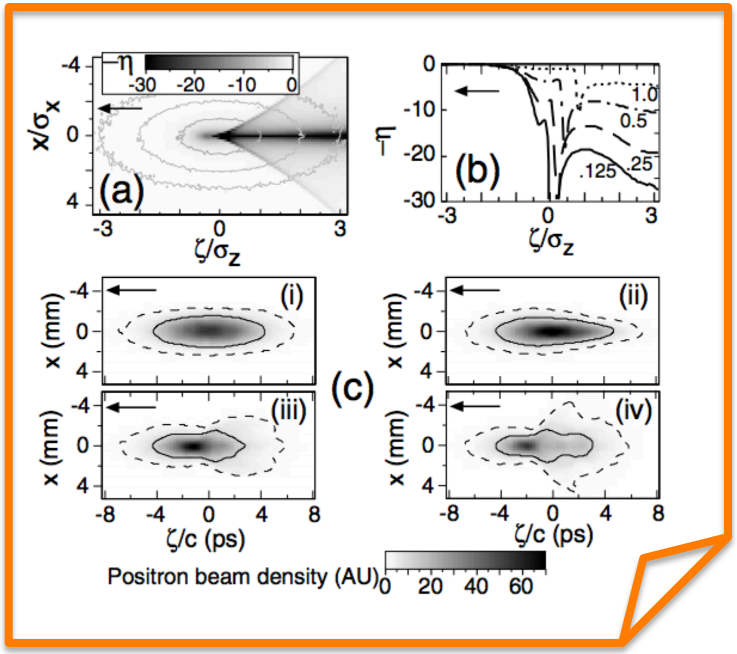
Litos, Nature 515, 92 (2014)



e-

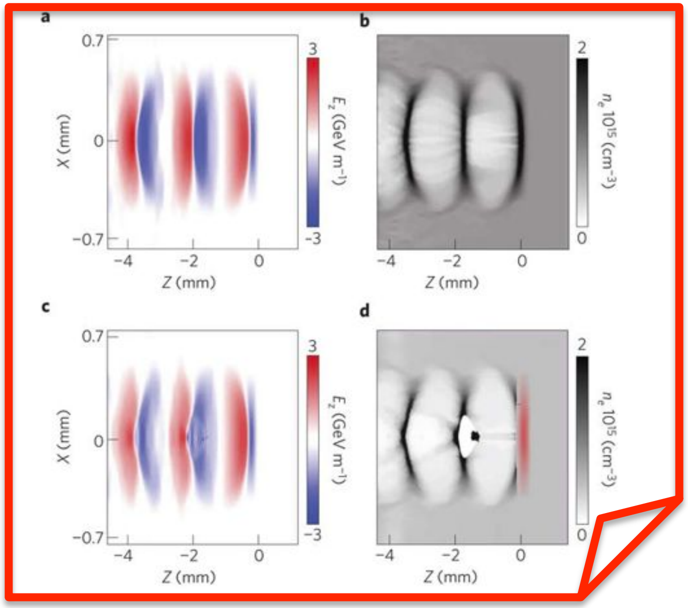
Hogan, Phys. Rev. Lett. 90, 205002 (2003)

Blue, Phys. Rev. Lett. 90, 214801 (2003)



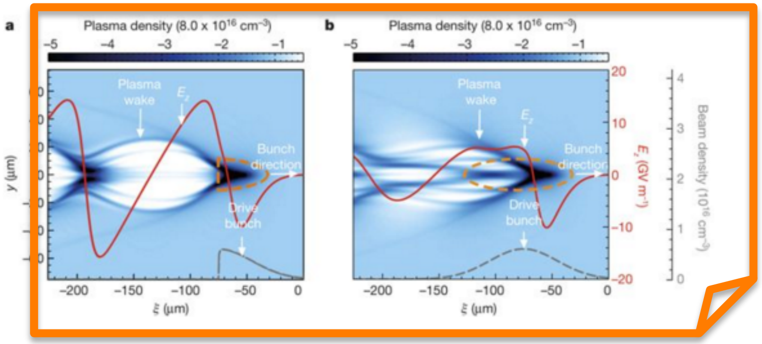
e+

Caldwell, Nature Physics 5, 363 (2009)



p+

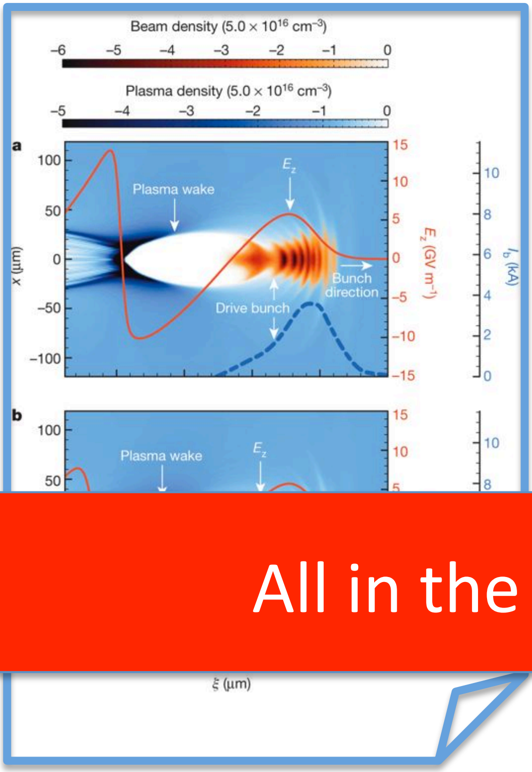
Corde, Nature 524, 442 (2015)



e+

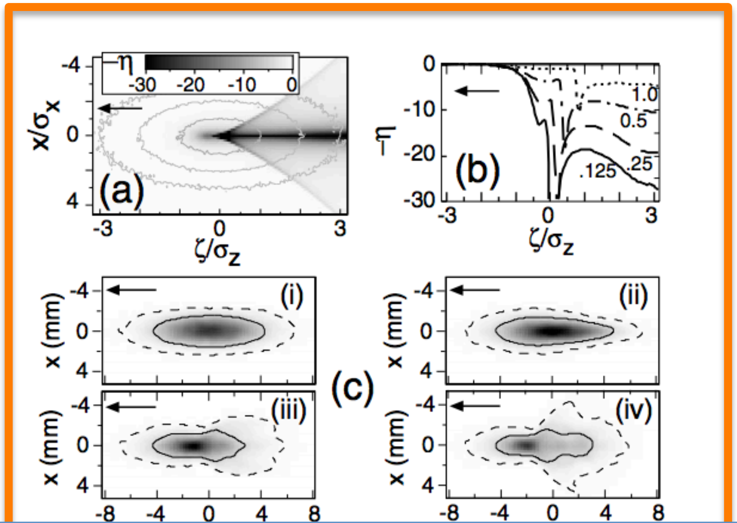
WAKEFILEDS EXCITATION

Litos, Nature 515, 92 (2014)



e⁻

Hogan, Phys. Rev. Lett. 90, 205002 (2003)

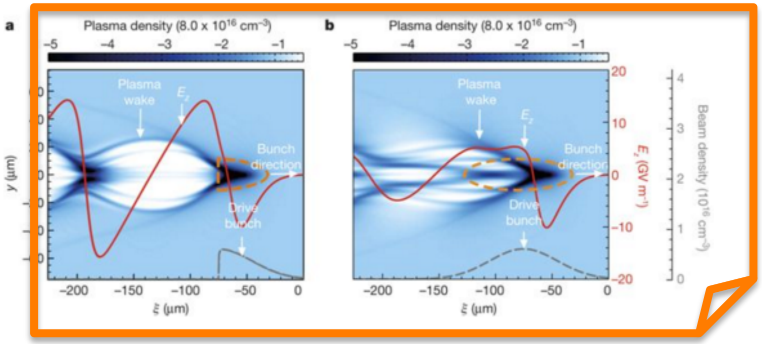


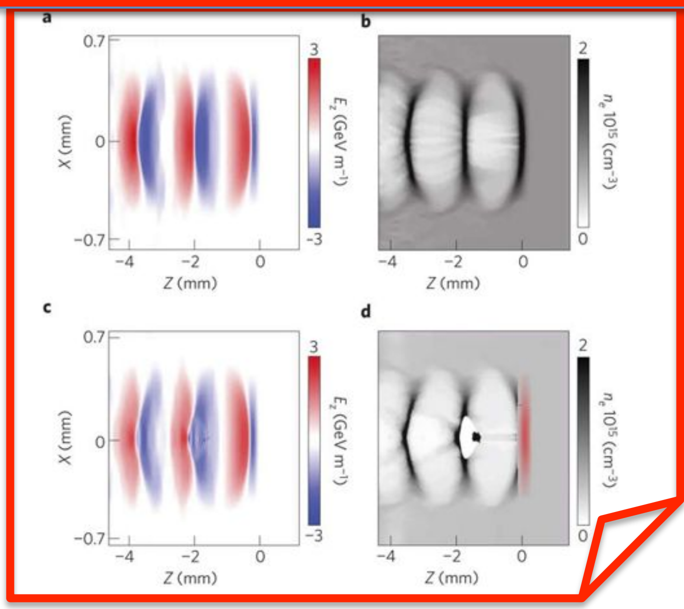
e⁺

All in the not linear, aka, non-linear regime

Corde, Nature 524, 442 (2015)

e⁺





p⁺

FIRST PWFA OBSERVATION (e^-)

Concept: Chen, et al., Phys. Rev. Lett. 54, 693 (1985)

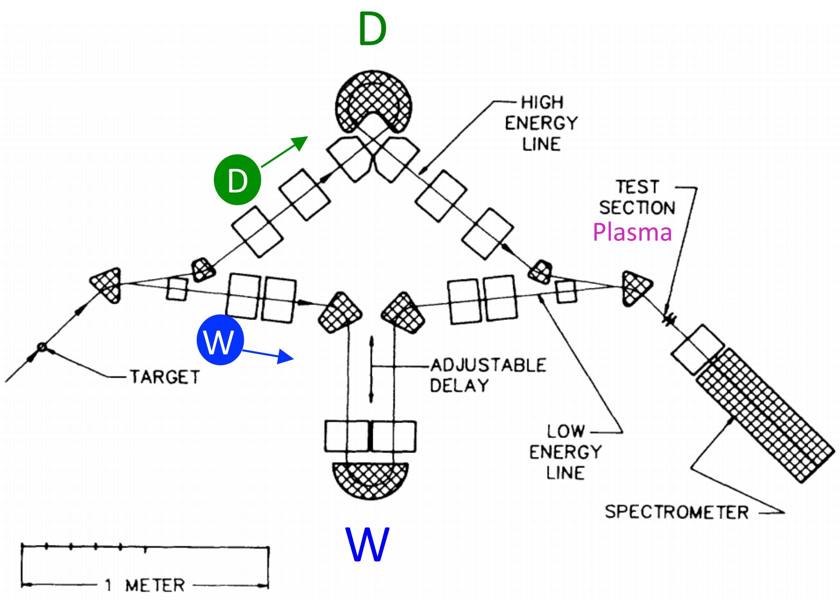


FIG. 1. Schematic of Argonne National Laboratory AATF layout.

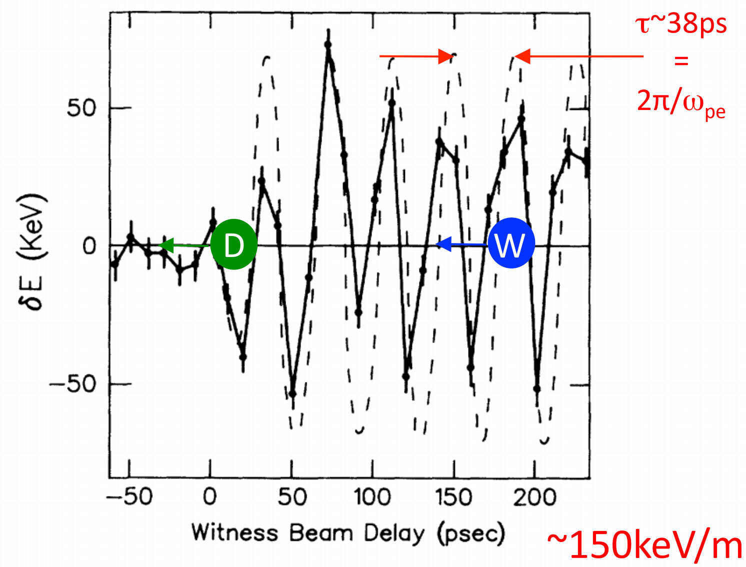


FIG. 2. Scan 1: Witness-beam energy-centroid change δE vs time delay behind driver. Total driver-beam charge $Q=2.1$ nC; plasma parameters $L=28$ cm and $n_e=8.6\times10^{12}$ cm^{-3}. Theoretical predictions are given by the dashed line.

Observation: Rosenzweig, PRL 61, 98 (1988)

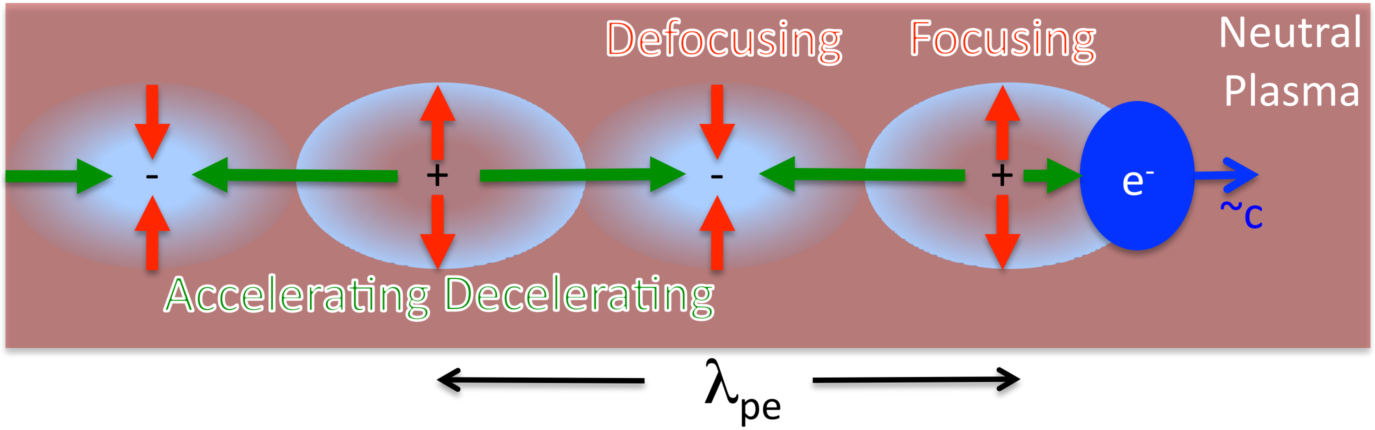
- ✧Drive/witness bunch experiment
- ✧Low wakefield amplitudes (low n_e, long bunches, …)
- ✧High-gradient acceleration: Kallos Phys. Rev. Lett. 100, 074802 (2008); Muggli, Phys. Rev. Lett. 93, 014802 (2004)
- ✧Really high gradient acceleration: Blumenfeld, Nature 445, 741 (2007); Litos, Nature 515, 92 (2014)

MAX-PLANCK-GESELLSCHAFT

WAKEFIELDS (FIELDS)

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

e^- driver / e^- witness



Its is clear that only focusing fields are interesting in practice
(most case)

Now we know how to drive (linear) wakefields

•Backward in the back: accelerating for e^-

Transverse (cylindrical geometry):

•Outward in the front: focusing for e^-

•Inward in the back: defocusing for e^-

LINEAR WAKEFIELDS THEORY (n_{b0}=cst)

Max-Planck-Gesellschaft

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Model:

- Cold, infinite, neutral ($n_{e0}=n_{i0}$) , no flow ($v_0=0$), non-magnetized ($B_0=0$) plasma fluid theory
- Drive beam causes a small perturbations in the density plasma ($n_b \sim n_{e1} << n_{e0}$, $E_1 << E_{WB}$, $v_{e1} << c$)
- Neglect ion motion ($m_i >> m_e$, $n_i = n_{i0} = n_{e0}$)
- 1D situation, then 2D

Method:

- Cold plasma, fluid theory
- Linearize equations
- Equilibrium, quantities satisfy all equations ($div \cdot E_0 = \rho/\varepsilon_0 = -e(n_{i0}-n_{e0}) = 0$)
- Keep 1st order terms only $n_b \sim n_{e1} << n_{e0}$, $E_1 << E_{WB}$, $v_{e1} << c$ (neglect eg. $n_{e1}v_{e1}$)
- Obtain Green's function for a δ "bunch", i.e., wakefields for a "single particle"
- Convolute Green's function with (finite extent) bunch distribution
- Assume parallel and transverse dependencies are separable $f(r,\xi)=R(r)X(\xi)$
- Obtain 2D wakefields from transverse equation … bunch transverse distribution dependent

Good references:

Beginner: R. D. Ruth et al., SLAC-PUB-3374, July 1984
Advanced: R. Keinigs et al., Phys. Fluids 30, 252 (1987)

LINEAR THEORY GREEN'S FUNCTION

Ruth, SLAC-PUB-3374 (1985)

Wakefields driven by a 1D sheet of particles

Here $k=k_{pe}$

$$n_1(\xi<0)=0$$

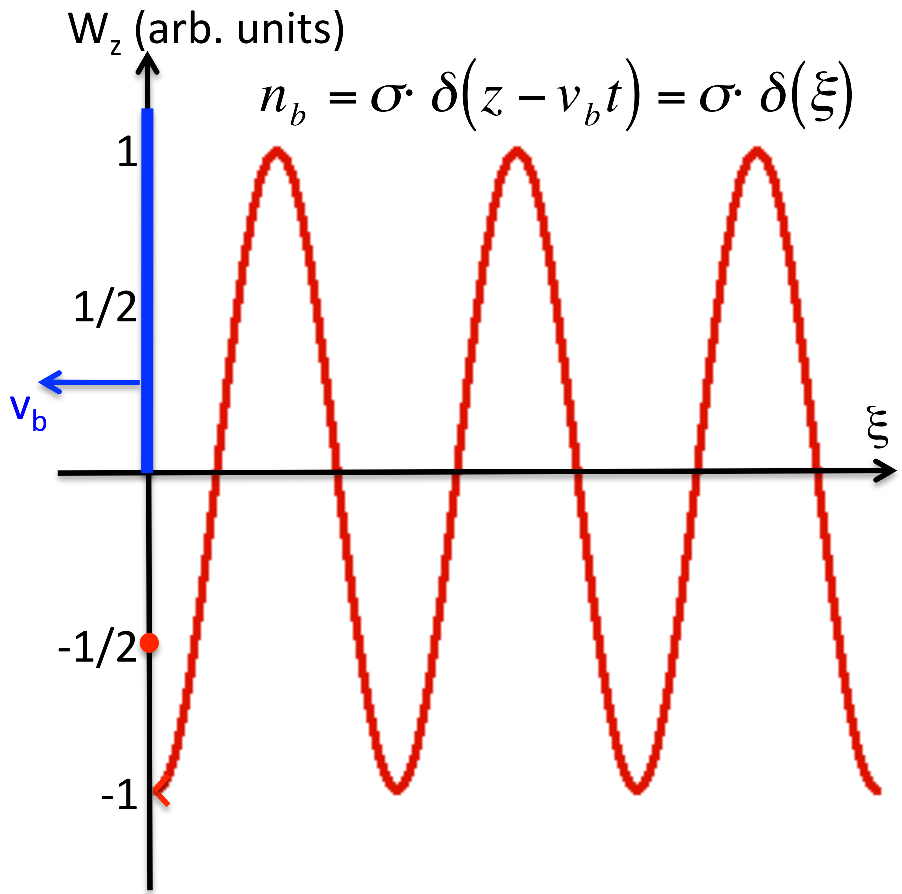
$$\frac{\partial n_1}{\partial \xi}|_{0-}^{0+}=-\frac{k^2}{2}\sigma$$

$$n_1(\xi>0)=-k\sigma sin(k\xi)$$

$$E_1(\xi<0)=0$$

$$E_1(\xi=0)=-\frac{e}{\epsilon_0}\frac{\sigma}{2}.$$

$$E_1(\xi>0)=-\frac{e}{\epsilon_0}\sigma cos(k\xi)$$



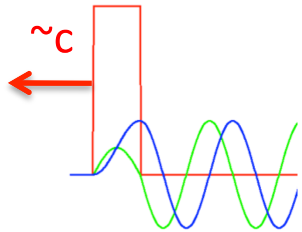
✧Satisfies fundamental beam loading theorem: R=2

✧Wakefields of a bunch from convolution of Green's fct with bunch profile

LINEAR WAKEFIELDS THEORY (2D, n_{b0}=cst)

$$W_z(\xi,r) = \frac{e n_{b0}}{\epsilon_0} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') cos[k_{pe}(\xi-\xi')] d\xi' \cdot R(r), \qquad W_z = eE_z$$

$$W_\perp(\xi,r) = \frac{e n_{b0}}{\epsilon_0 k_{pe}} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') sin[k_{pe}(\xi-\xi')] d\xi' \cdot \frac{dR(r)}{dr}, \qquad W_r = e\left(E_r - v_b \times B_\theta\right)$$

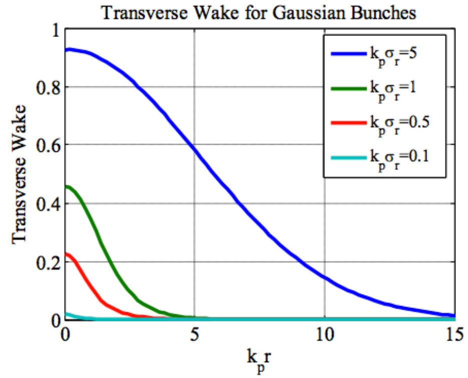


$$n_b = n_{b0} f(r,\xi) = n_{b0} n_{b\parallel}(\xi) n_{b\perp}(r)$$

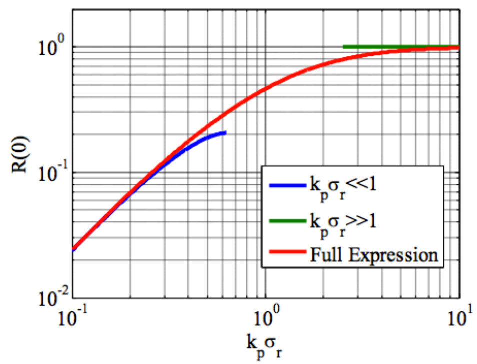
with $f(r,\xi)$, $n_{b\parallel}(\xi)$ and $n_{b\perp}(r)$ normalized to 1.

$$R(r) = k_{pe}{}^2 K_0(k_{pe}r) \int_0^r r'dr' n_{b\perp}(r') I_0(k_{pe}r') + k_{pe}{}^2 I_0(k_{pe}r) \int_r^{\infty} r'dr' n_{b\perp}(r') K_0(k_{pe}r'),$$

(for a Gaussian transverse bunch profile)



n_b=cst



Note: for a bunch moving towards –infinity …

What do these equations mean?

- The wakefields at position ξ depend on the wakefields driven by all charges at $\xi' \leq \xi$ (causality)
- The wakefields extend radially over a range ~ bunch rms size (screening ~a few c/ω_{pe})
- $W_z \sim R(r)$ is maximum at r=0 (on axis)
- $W_r \sim dR(r)/dr$ is maximum at $r \sim \sigma_r$ and $W_r=0$ at r=0

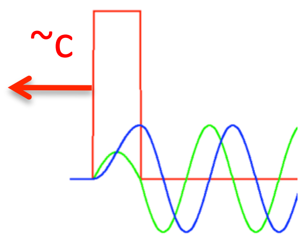
LINEAR WAKEFIELDS THEORY (n_{b0}=cst)

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

$$W_z(\xi,r) = \frac{e n_{b0}}{\epsilon_0} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') cos[k_{pe}(\xi - \xi')] d\xi' \cdot R(r), \qquad W_z = eE_z$$

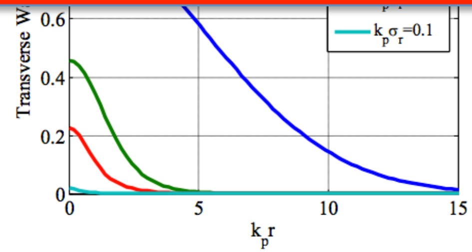
$$W_\perp(\xi,r) = \frac{e n_{b0}}{\epsilon_0 k_{pe}} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') sin[k_{pe}(\xi - \xi')] d\xi' \cdot \frac{dR(r)}{dr}, \qquad W_r = e\left(E_r - v_b \times B_\theta\right)$$

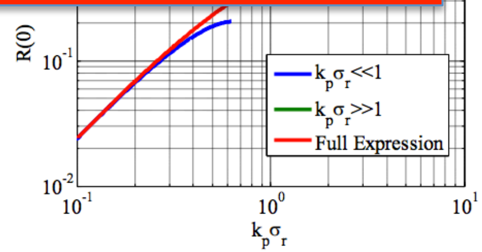


Convolution Bunch distribution Green's function Radial dependency

$$R(r) = k_{pe}{}^2 K_0(k_{pe} r) \int_0^r r' dr' n_{b\perp}(r') I_0(k_{pe} r') + k_{pe}{}^2 I_0(k_{pe} r) \int_r^\infty r' dr' n_{b\perp}(r') K_0(k_{pe} r'),$$

(for a Gaussian transverse bunch profile)





Note: for a bunch moving towards –infinity …

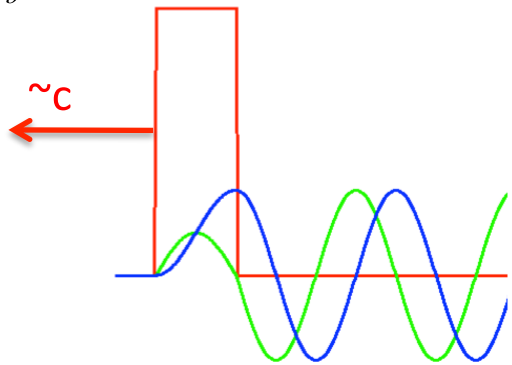
What do these equations mean?

- •The wakefields at position ξ depend on the wakefields driven by all charges at ξ'≤ξ (causality)
- •Causality: also $v_{b,p}$~c, no effect ahead of a slice
- •The $n_{b||}$ term is non-zero only within the beam … $\xi \leq \xi_b$
- •The wakefields extend radially over a range ~ bunch rms size
- •Transversely large beams are more effective at driving wakefields (n_b=cst)

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

Solutions for a bunch with constant density $n_b(\xi)$ and radius $\sigma_r(\xi)$:

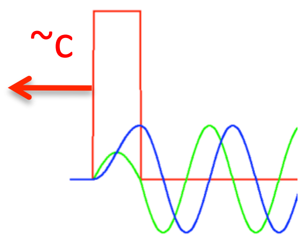
$$\begin{cases} n_{b\parallel}(\xi) = 0 & \xi < 0 \\ n_{b\parallel}(\xi) = n_{b0} & 0 \le \xi \le L \; or \; \xi_b \\ n_{b\parallel}(\xi) = 0 & \xi > 0 \end{cases}$$



LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

$$W_z(\xi,r) = \frac{e n_{b0}}{\epsilon_0} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') cos[k_{pe}(\xi-\xi')] d\xi' \cdot R(r), \qquad W_z = eE_z$$

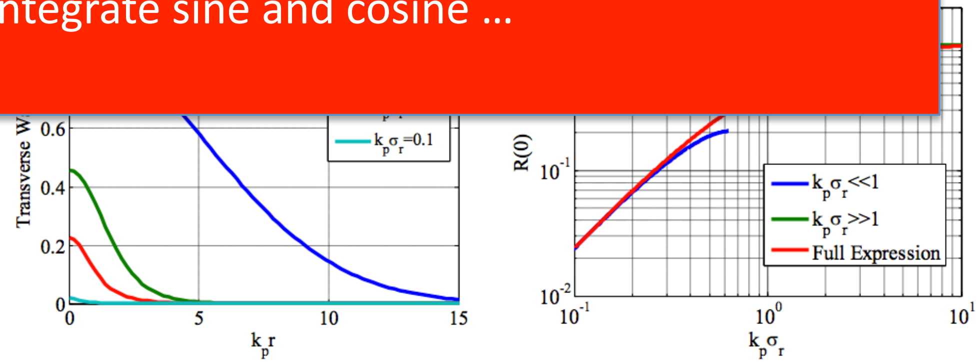
$$W_\perp(\xi,r) = \frac{e n_{b0}}{\epsilon_0 k_{pe}} \int_{-\infty}^{\xi} n_{b_\parallel}(\xi') sin[k_{pe}(\xi-\xi')] d\xi' \cdot \frac{dR(r)}{dr}, \qquad W_r = e\left(E_r - v_b \times B_\theta\right)$$



$n_{b||}$=cst term, comes out of the equations, integrate sine and cosine …
ignore r-term (cst for given $k_{pe}\sigma_r$)

$$R(r) = k_{pe}{}^2 K_0(k_{pe}r) \int_0^r r'dr' n_{b\perp}(r') I_0(k_{pe}r') + k_{pe}{}^2 I_0(k_{pe}r) \int_r^\infty r'dr' n_{b\perp}(r') K_0(k_{pe}r'),$$

(for a Gaussian transverse bunch profile)



Note: for a bunch moving towards –infinity …

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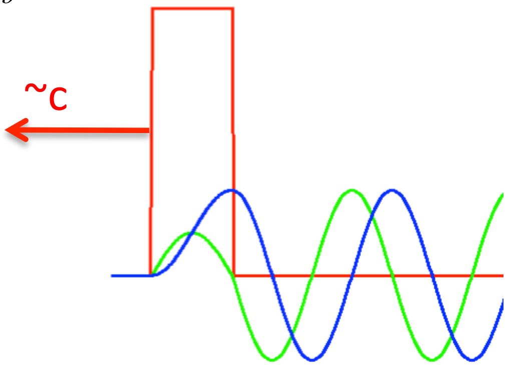
- Causality: also $v_{b,p}$~c, no effect ahead of a slice
- The $n_{b||}$ term is non-zero only within the beam … ξ≤ξ_b
- The wakefields extend radially over a range ~ bunch rms size
- Transversely large beams are more effective at driving wakefields (n_b=cst)

LiNEAR WAKEFiELDS THEORY $(n_{b0}=cst)$

Solutions for a bunch with constant density and radius $\sigma_r(\xi)$=cst:

$$
\begin{cases}
n_{b||}(\xi)=0 & \xi<0 \\
n_{b||}(\xi)=n_{b0} & 0\leq\xi\leq L \; or \; \xi_b \\
n_{b||}(\xi)=0 & \xi>0
\end{cases}
$$



In front of the bunch: $W_z(\xi<0)=W_r(\xi<0)=0.$

Inside the bunch:

$$
\begin{cases}
W_z(0\leq\xi\leq\xi_b)=\frac{e}{\epsilon_0}R(r)\frac{n_{b0}}{k_{pe}}sin[k_{pe}\xi] \\
W_\perp(0\leq\xi\leq\xi_b)=\frac{e}{\epsilon_0}\frac{dR(r)}{dr}\frac{n_{b0}}{k_{pe}^2}\left(1-cos[k_{pe}\xi]\right).
\end{cases}
$$

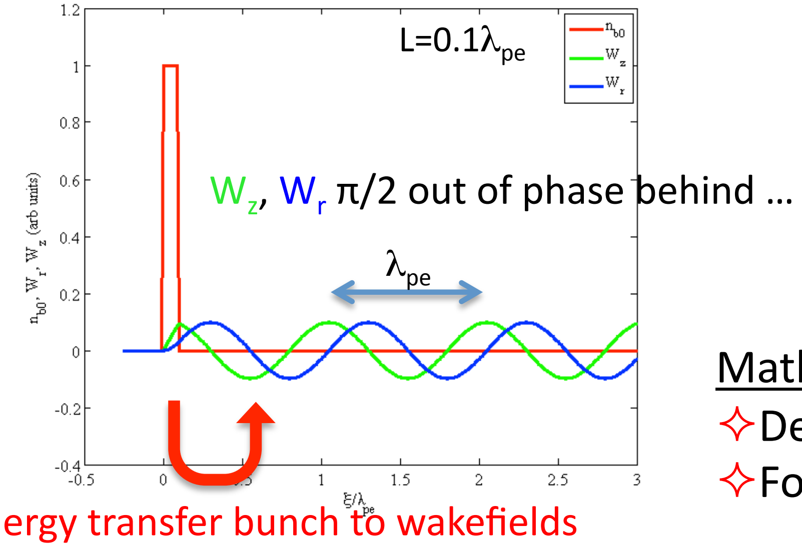
After the bunch:

$$
\begin{cases}
W_z(\xi>\xi_b)=\frac{e}{\epsilon_0}R(r)\frac{n_{b0}}{k_{pe}}\left(sin[k_{pe}(\xi-\xi_b)]-sin[k_{pe}\xi]\right) \\
W_\perp(\xi>\xi_b)=\frac{e}{\epsilon_0}\frac{dR(r)}{dr}\frac{n_{b0}}{k_{pe}^2}\left(cos[k_{pe}(\xi-\xi_b)]-cos[k_{pe}\xi]\right).
\end{cases}
$$

Note:

- W_z within the bunch <u>does</u> change sign
- W_r within the bunch <u>does not</u> change sign
- Wakefields oscillate (around zero) behind the bunch
- W_r and W_z are $\pi/2$ out of phase behind the bunch

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)



<u>Math, signs (should) tell us:</u>

✧Decelerating inside the (drive) bunch

✧Focusing inside the (drive) bunch

<u>Physics tells us:</u>

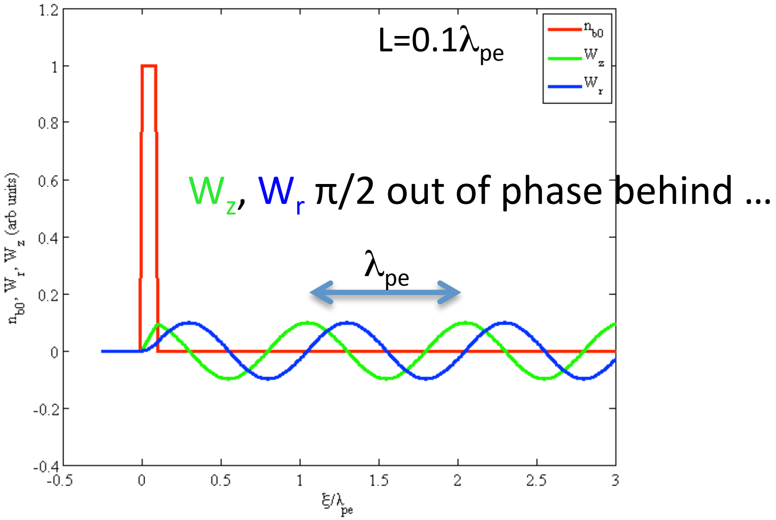
✧Decelerating inside the (drive) bunch

- ✧Losing energy to the wakefields displacing plasma e^-

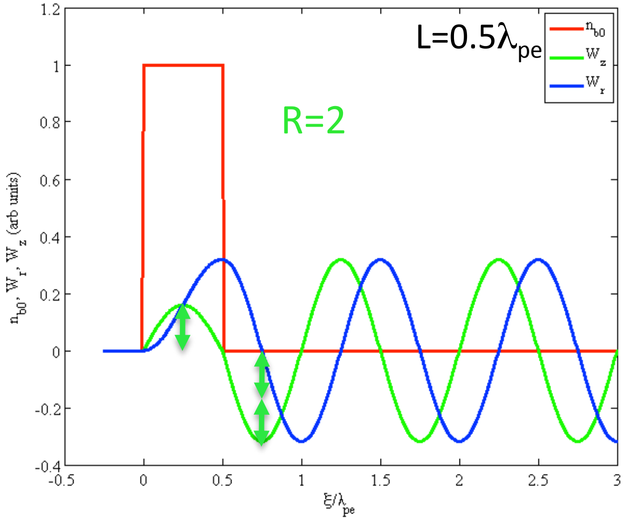
✧Focusing inside the (drive) bunch

- ✧Plasma e^- move into(out of) the e^-(e^+) bunch, partial neutralization => v_b x B_θ focusing

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)



✧R~1



✧Def: Transformer ratio R=$|W_{z,max}$(behind)$|/|W_{z,max}$(within)$|$

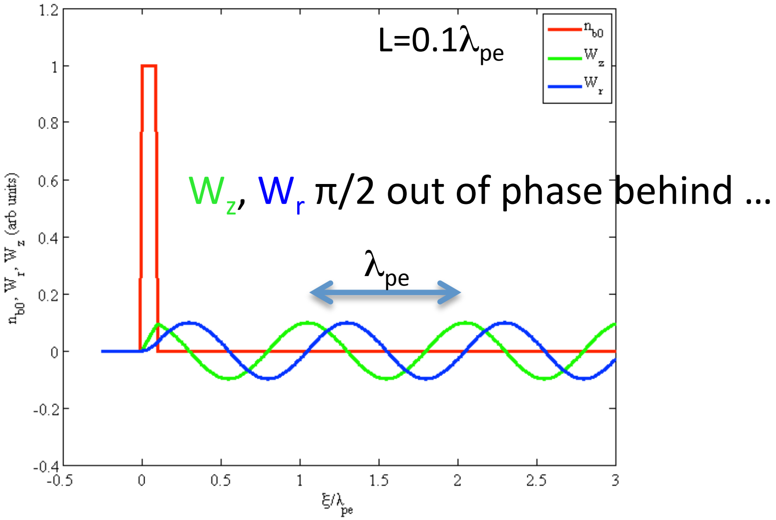
✧R≤2 for a single symmetric bunch

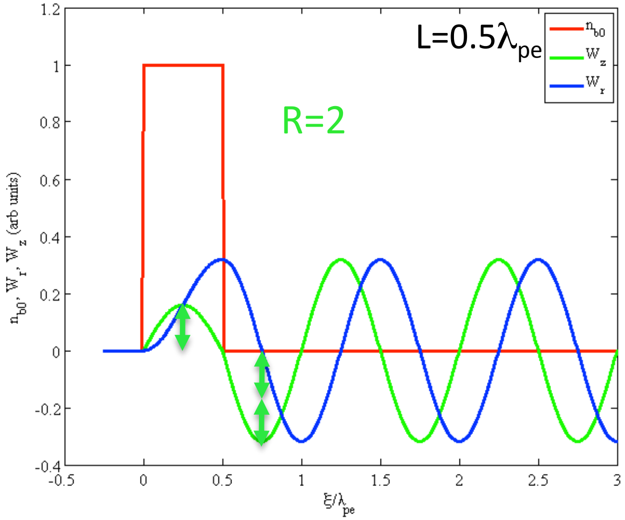
✧Field only decelerating within the bunch: only energy loss

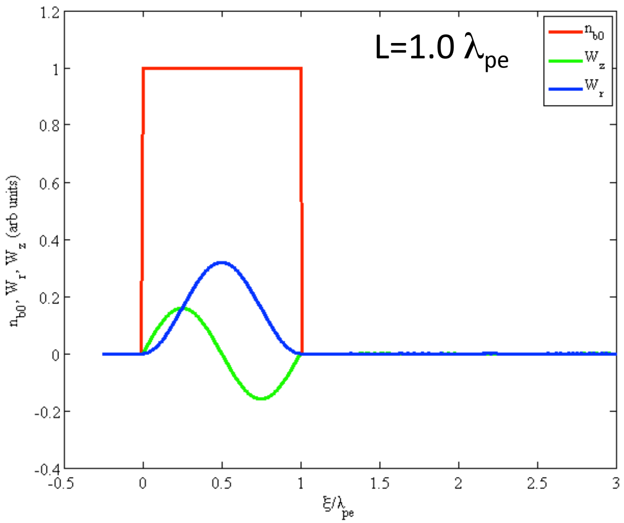
✧Field behind the bunch: net energy loss to the wakefields

✧Larger wakefield amplitudes than in the 0.1λ_{pe} case

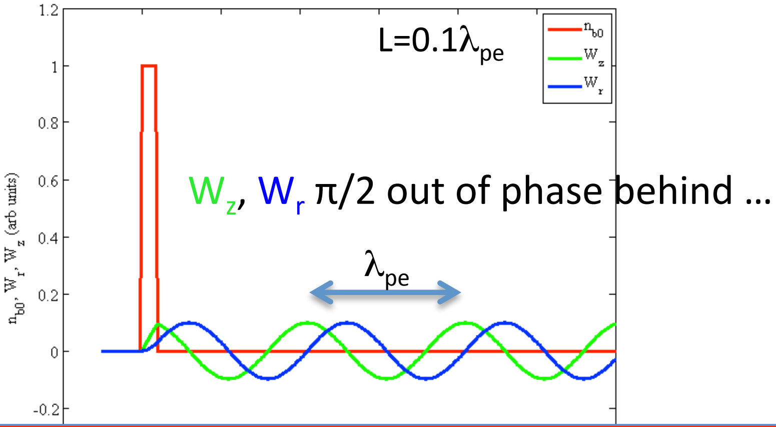
LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

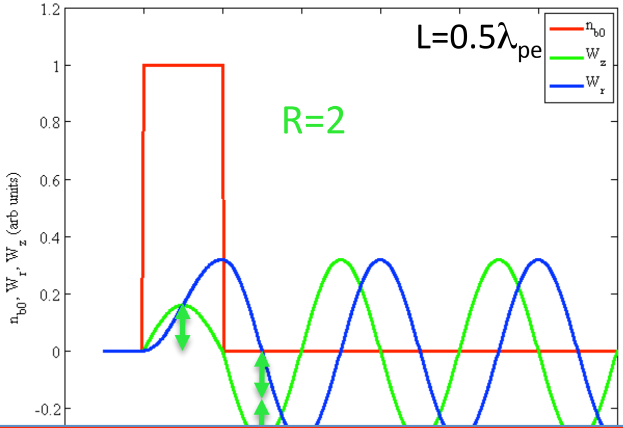




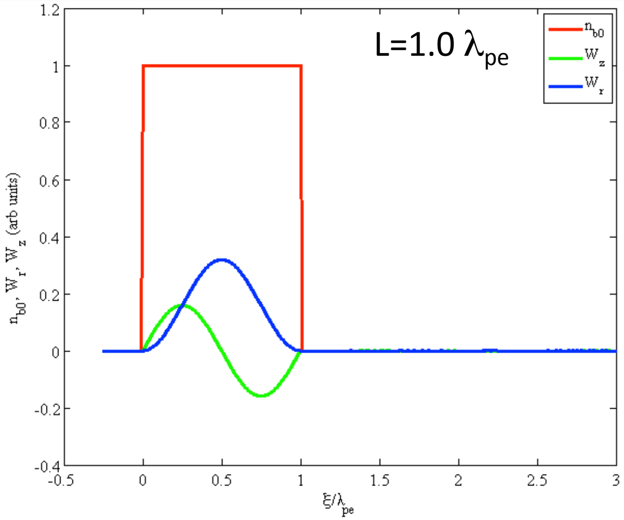


LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)



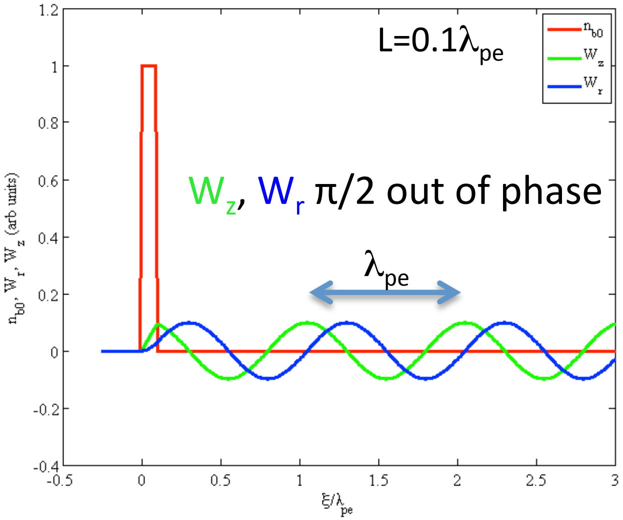


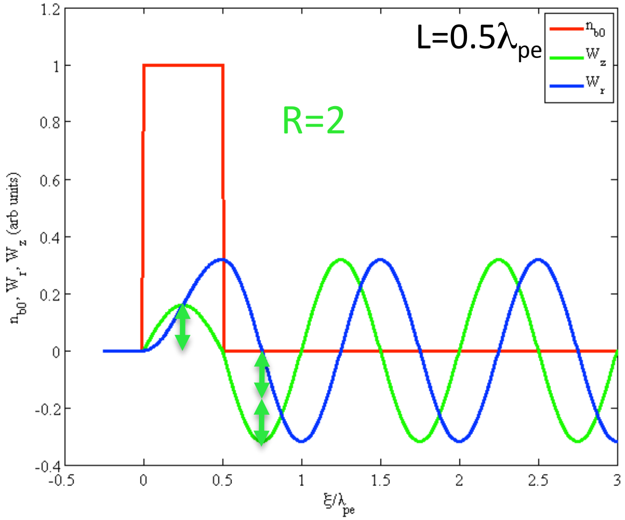
Q: why are there no wakefields behind the λ_{pe}-long bunch?



✧No fields behind the drive bunch: "perfect" accelerator …

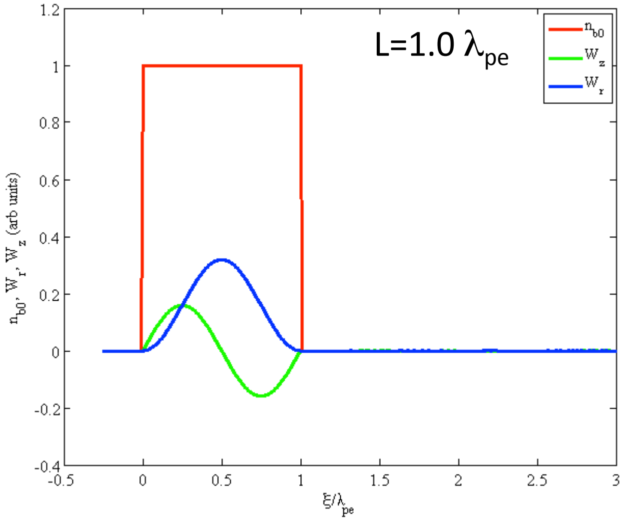
LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

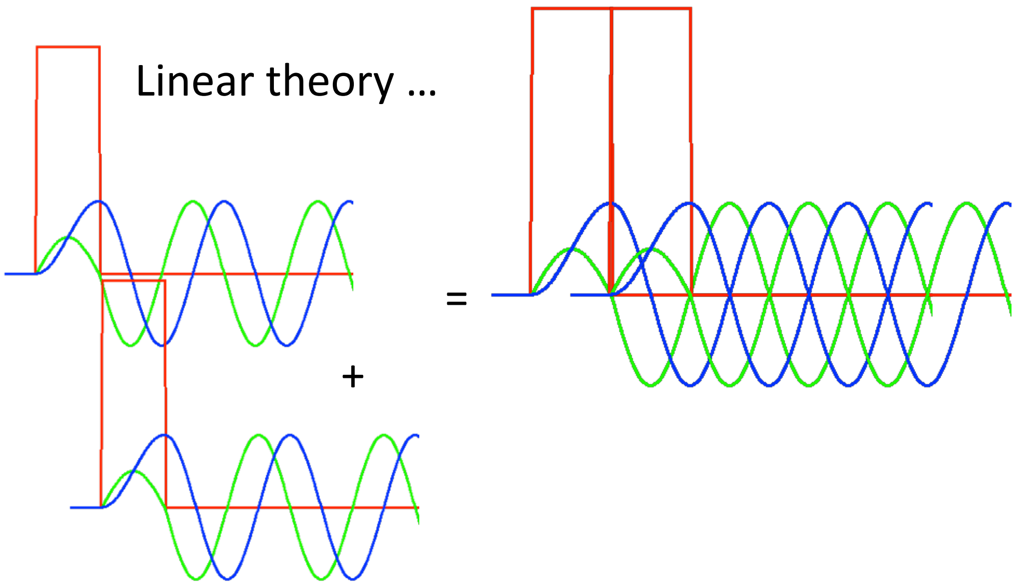




Def: Transformer ratio $R=|W_{z,max}(\text{behind})|/|W_{z,max}(\text{within})|$

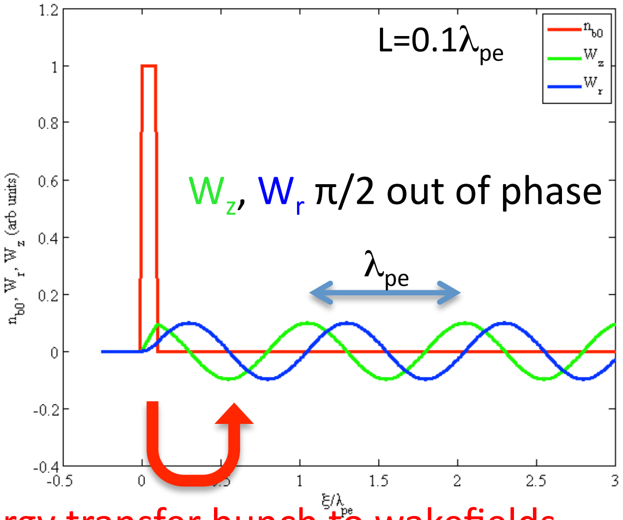
$R\leq 2$ for symmetric bunch





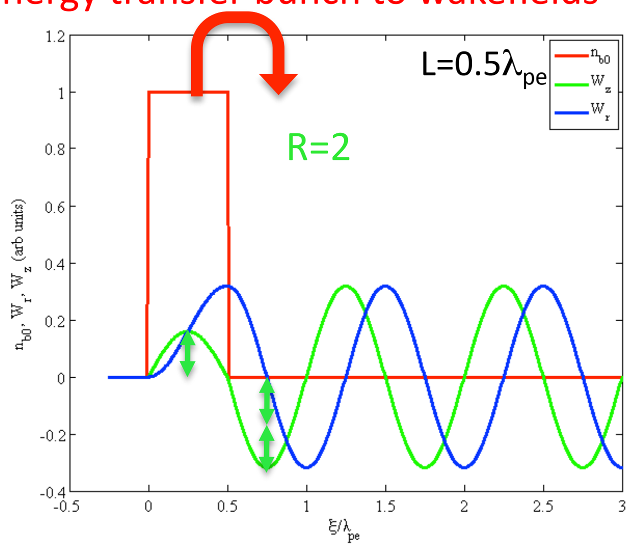
Wakefields cancel, interfere destructively behind the bunch

LINEAR WAKEFIELDS THEORY (n_{b0}=cst)



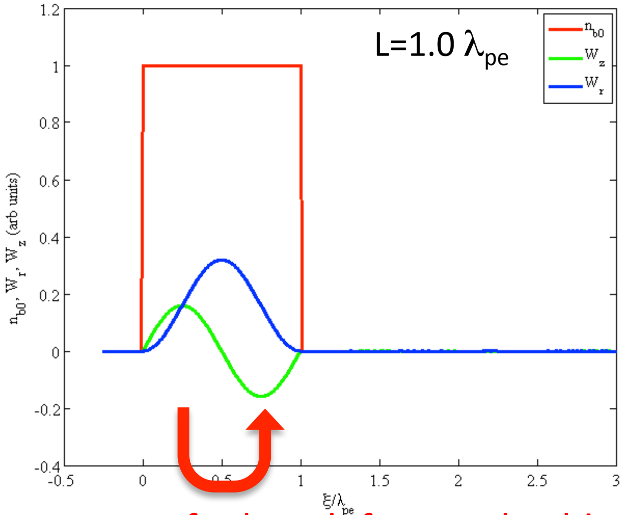
Energy transfer bunch to wakefields

Energy transfer bunch to wakefields

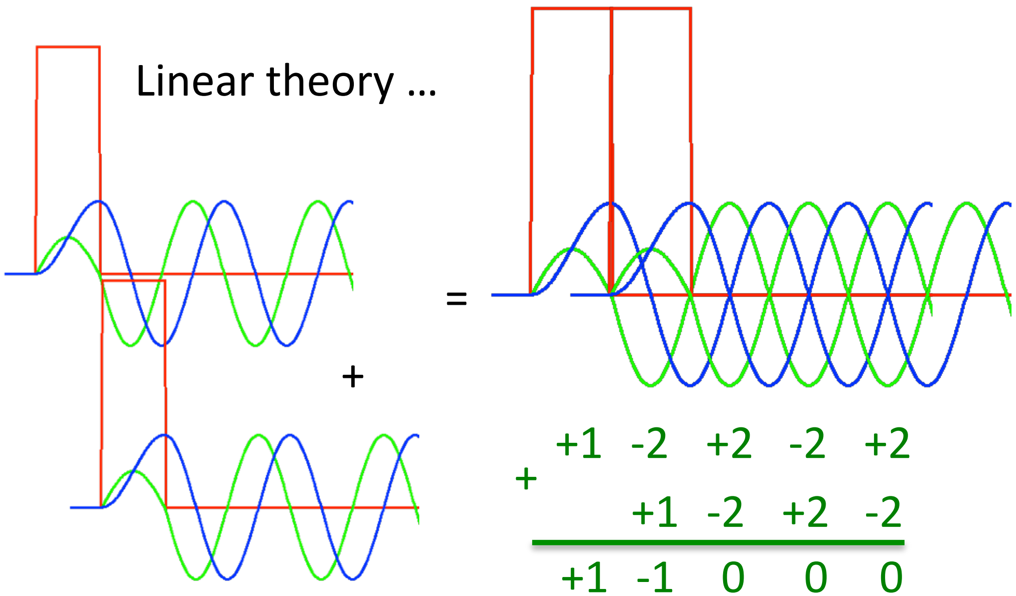


Def: Transformer ratio $R=|W_{z,max}(\text{behind})|/|W_{z,max}(\text{within})|$

$R \leq 2$ for symmetric bunch



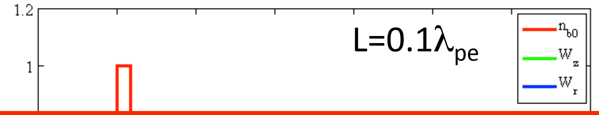
Energy transfer bunch front to back!

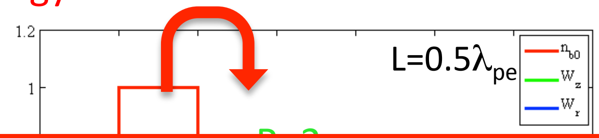


Wakefields cancel, interfere destructively behind the bunch

LINEAR WAKEFIELDS THEORY (n_{b0}=cst)

Energy transfer bunch to wakefields

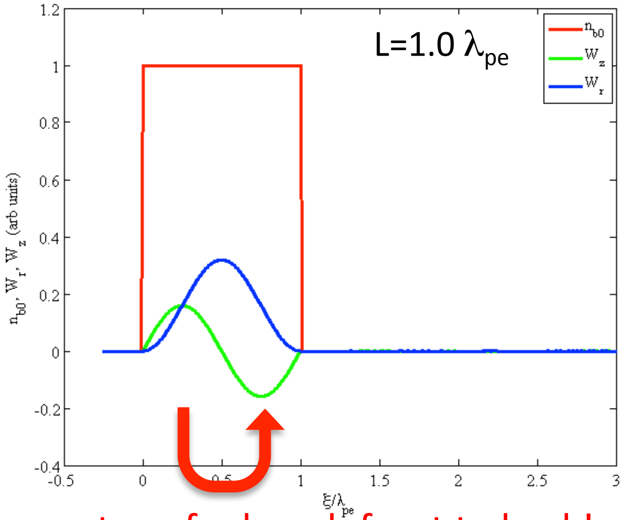




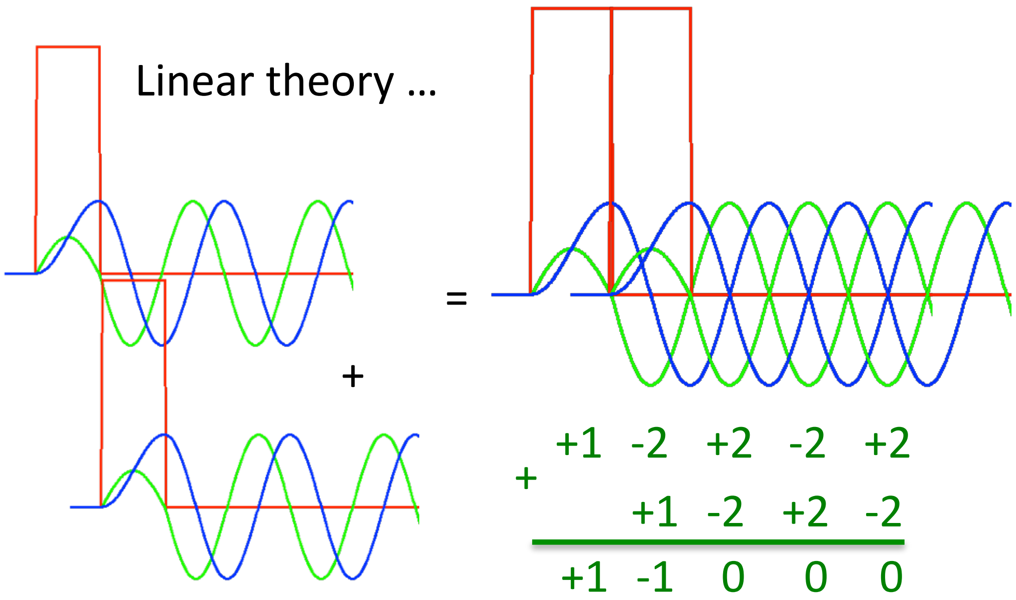
Q: from an accelerator point of view, what is <u>right</u> with the L=λ_{pe} case?

Q: from an accelerator point of view, what is <u>wrong</u> with the L=λ_{pe} case?

R≤2 for symmetric bunch



Energy transfer bunch front to back!



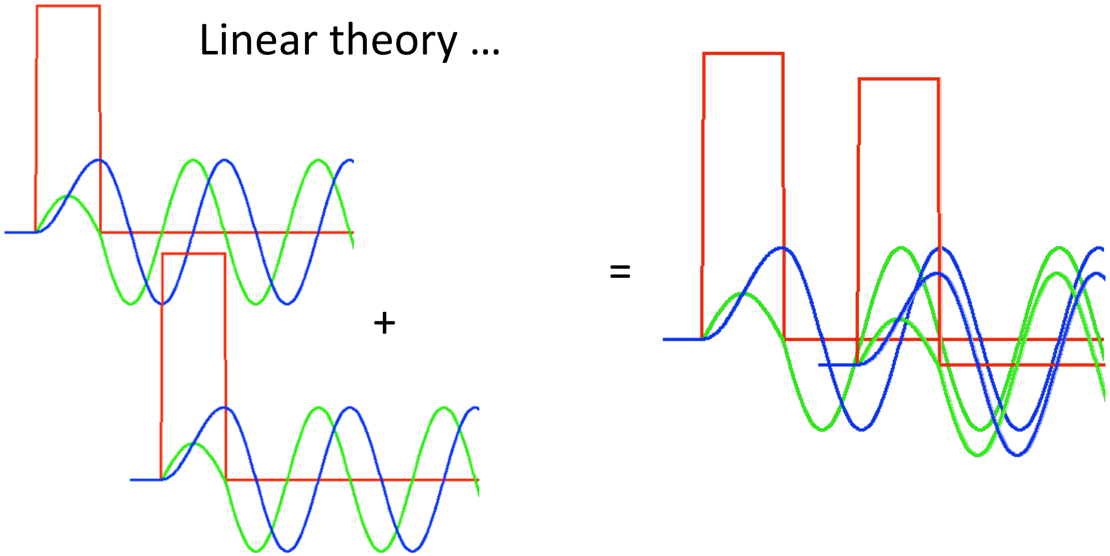
Wakefields cancel, interfere destructively behind the bunch

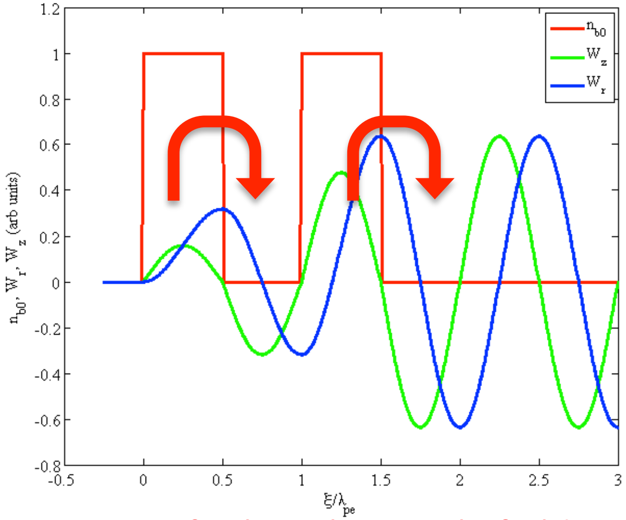
MAX-PLANCK-GESELLSCHAFT

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Linear theory …





Energy transfer bunch to wakefields

	+1	-2	+2	-2	+2
+			+1	-2	+2
	+1	-2	+3	-4	+4

R=>1!

Principle of AWAKE: AWAKE (and of bunch shaping)

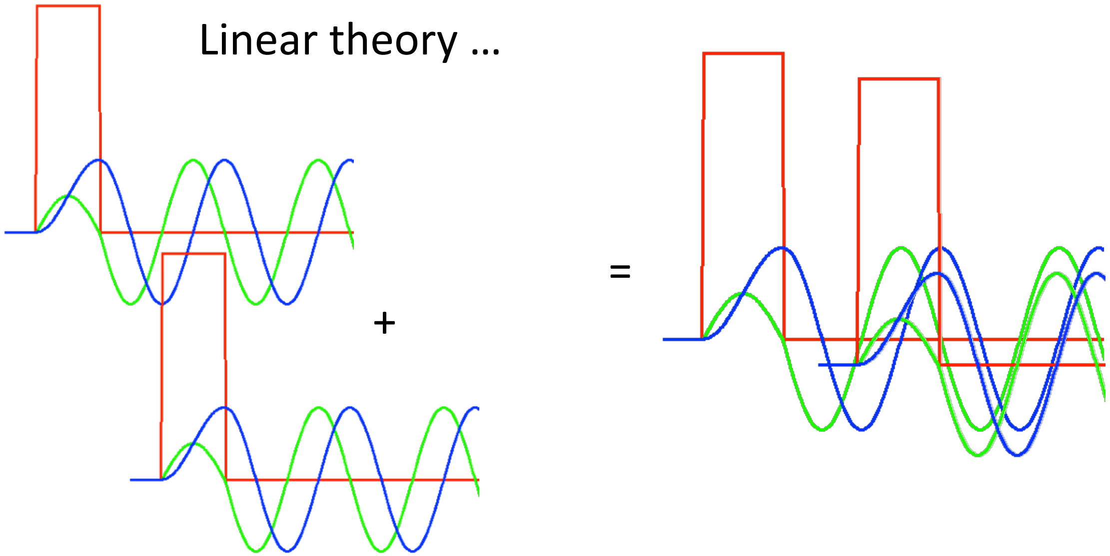
✧Let transverse wakefields (W_r) transform the long bunch (>>λ_{pe}) …

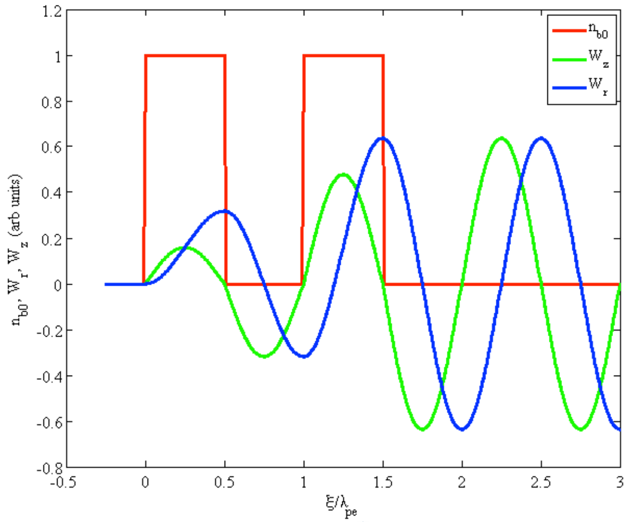
✧Into a periodic (~λ_{pe}) train of (~100) short bunches (<<λ_{pe}) …

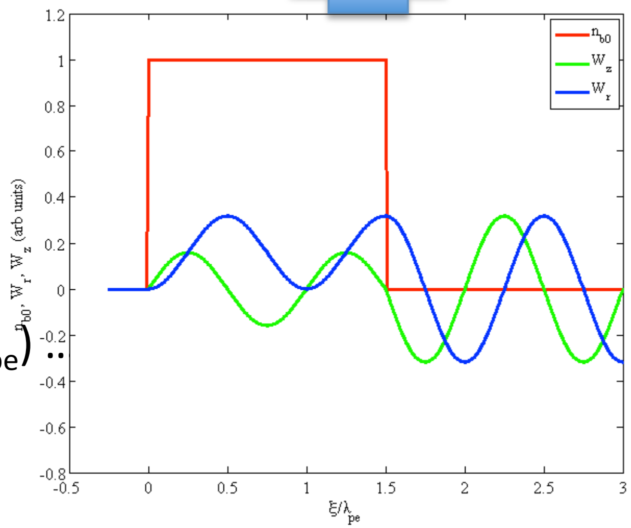
✧To drive large longitudinal wakefields (W_z) …

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

Linear theory …





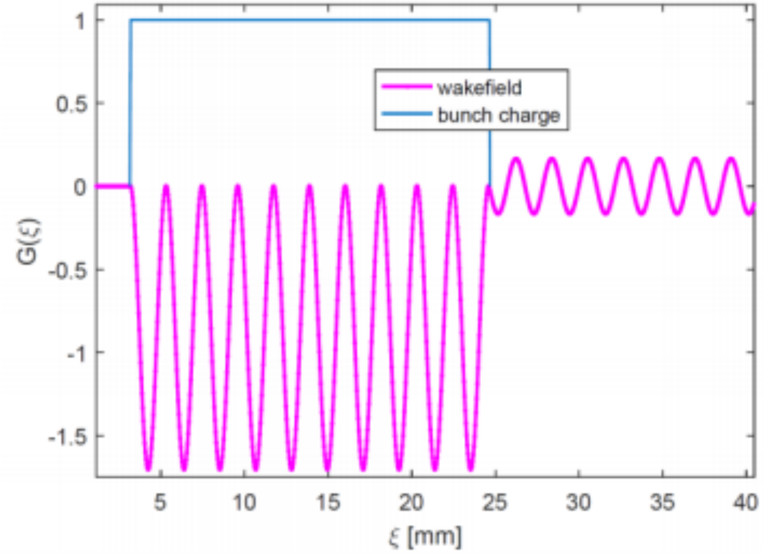


Principle of AWAKE: AWAKE

- ✧Let transverse wakefields (W_r) transform the long bunch ($>>\lambda_{pe}$) …
- ✧Into a periodic ($\sim\lambda_{pe}$) train of (~100) short bunches ($<<\lambda_{pe}$) …
- ✧To drive large longitudinal wakefields (W_z) …
- ✧Wakefields cancel, interfere destructively behind the bunch…
- ✧But … there is a self-modulation process … evolution …

LiNEAR WAKEFiELDS THEORY

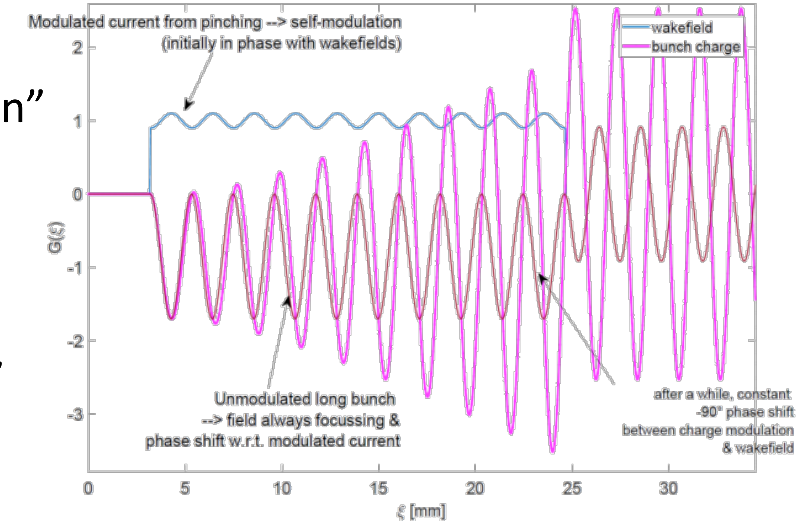
Un-modulated bunch



"Propagation"

"Evolution"

Modulated bunch



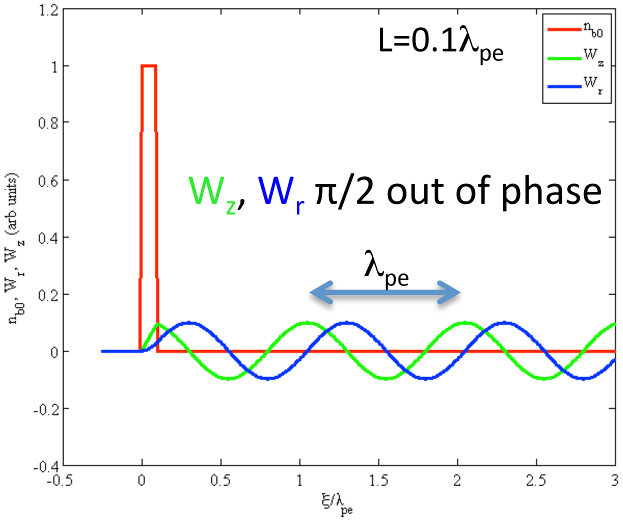
Calculation: F. Braunmueller

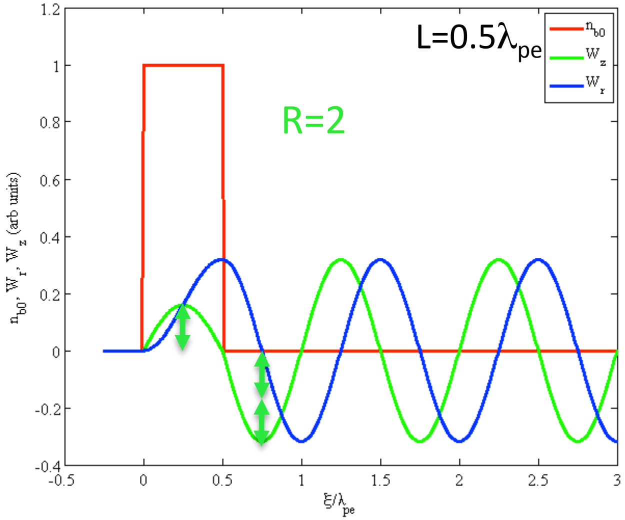
Principle of AWAKE:

$$W_{\perp}(\xi,r)=\frac{en_{b0}}{\epsilon_0 k_{pe}}\int_{-\infty}^{\xi} n_{b_{\parallel}}(\xi')sin[k_{pe}(\xi-\xi')]d\xi'\cdot\frac{dR(r)}{dr},$$

✧Modulated bunch drives larger wakefields than un-modulated bunch
✧Wakefields shift backwards from the front of the bunch
✧Growth of the wakefields, transverse evolution of the bunch …

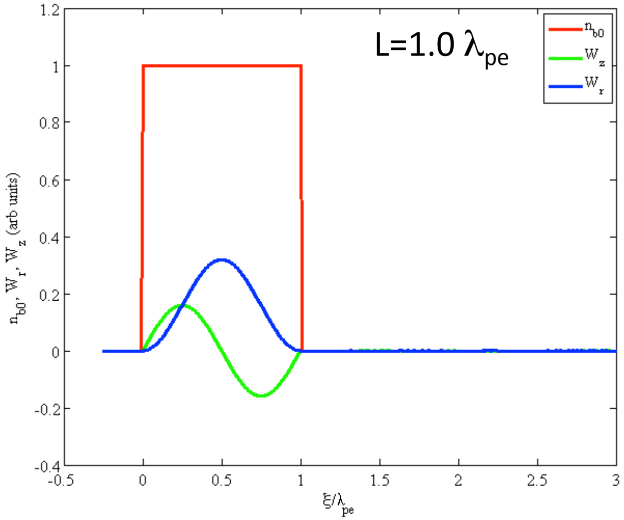
LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

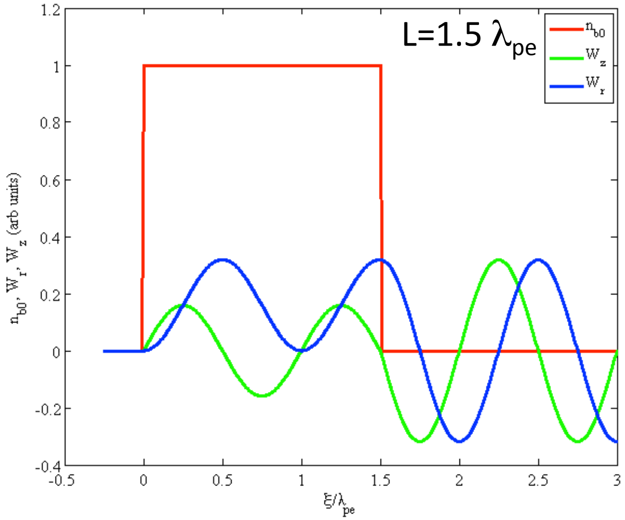




Def: Transformer ratio $R=|W_{z,max}(\text{behind})|/|W_{z,max}(\text{within})|$

$R\leq 2$ for symmetric bunch



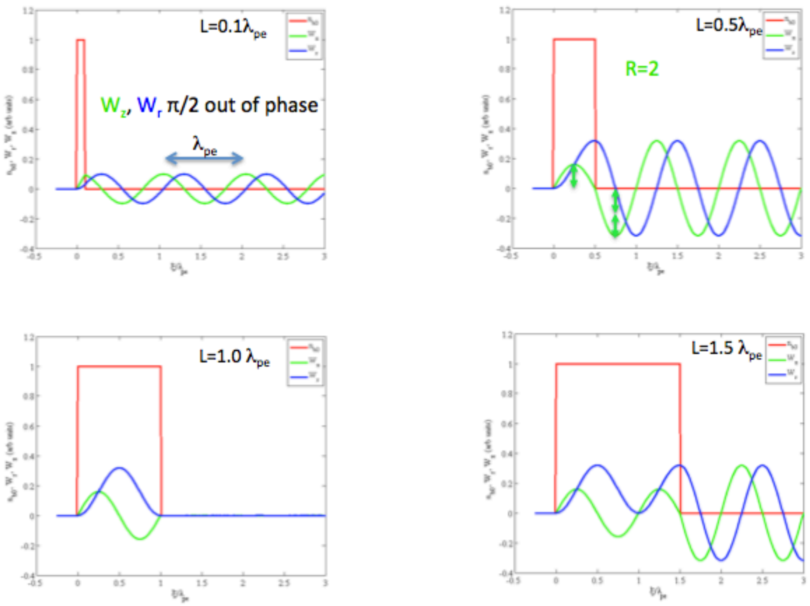


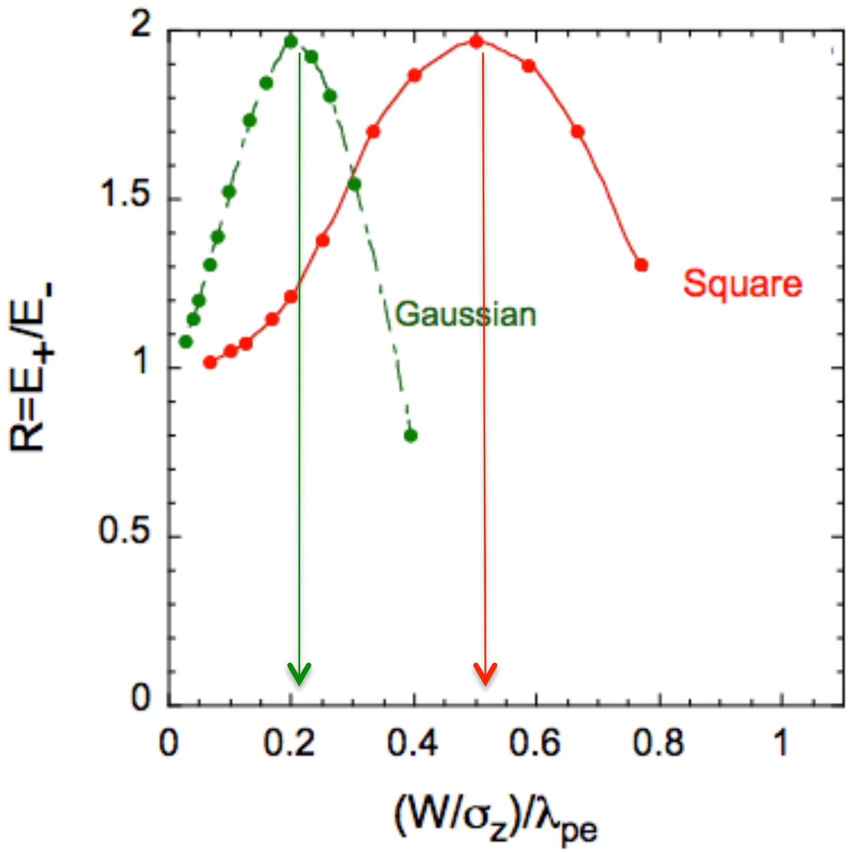
$W_r \geq 0$ (focusing) within the drive bunch

W_r-W_z the same as with $L=0.5\lambda_{pe}$

LiNEAR WAKEFiELDS THEORY (n_{b0}=cst)

Use linear fields and calculate $W_z(\xi)$ and calculate R





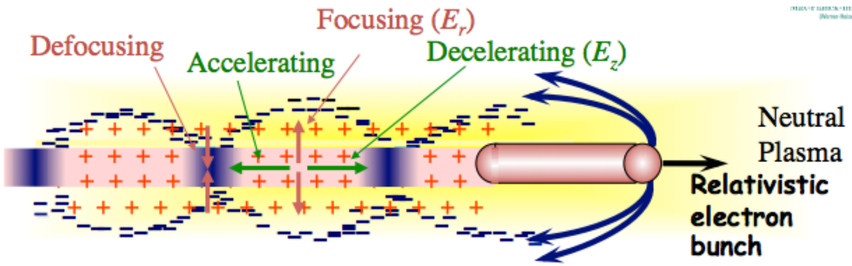
✧Suggests wakefields amplitude depends on bunch length

✧For Gaussian bunch, optimum for $k_{pe}\sigma_z \sim \sqrt{2}$ or $\sigma_z/\lambda_{pe} \sim 0.225$

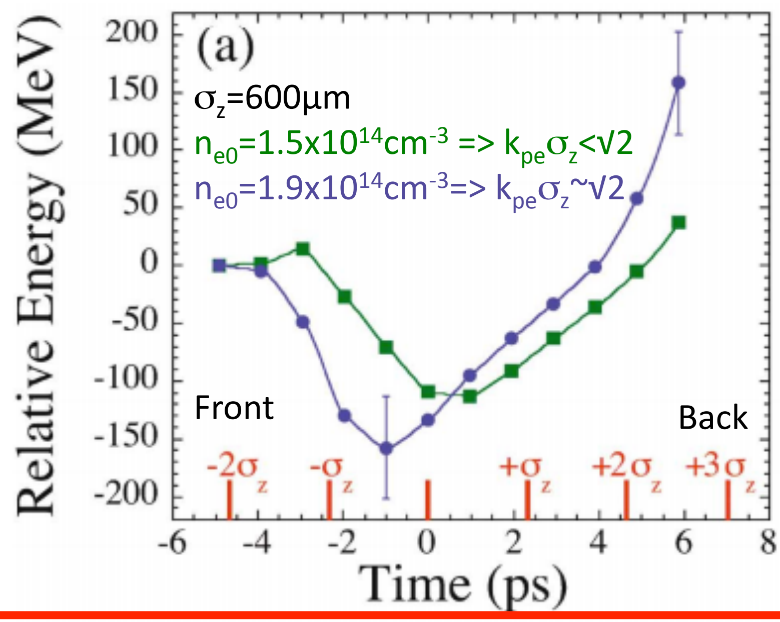
✧For a square bunch: $L=\lambda_{pe}/2$

✧R: effectiveness in driving wakefields …

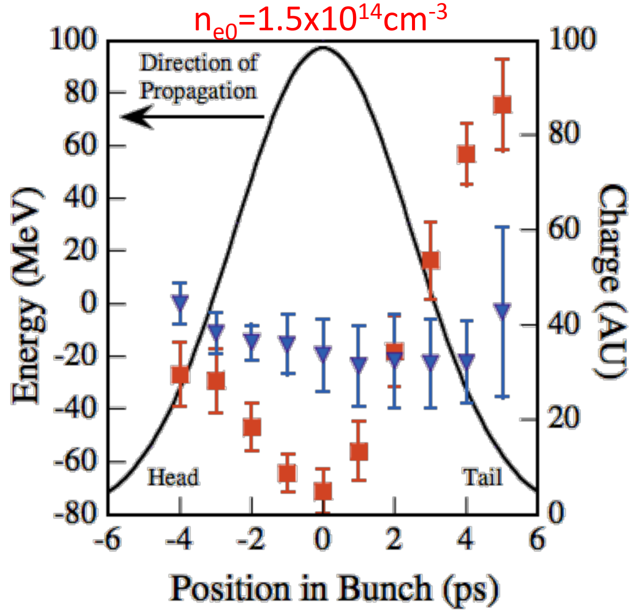
WAKEFIELDS WITHIN DRIVE BUNCH



e^- Muggli, Phys. Rev. Lett. 93, 014802 (2004)



e^+ Blue, Phys. Rev. Lett. 90, 214801 (2003)

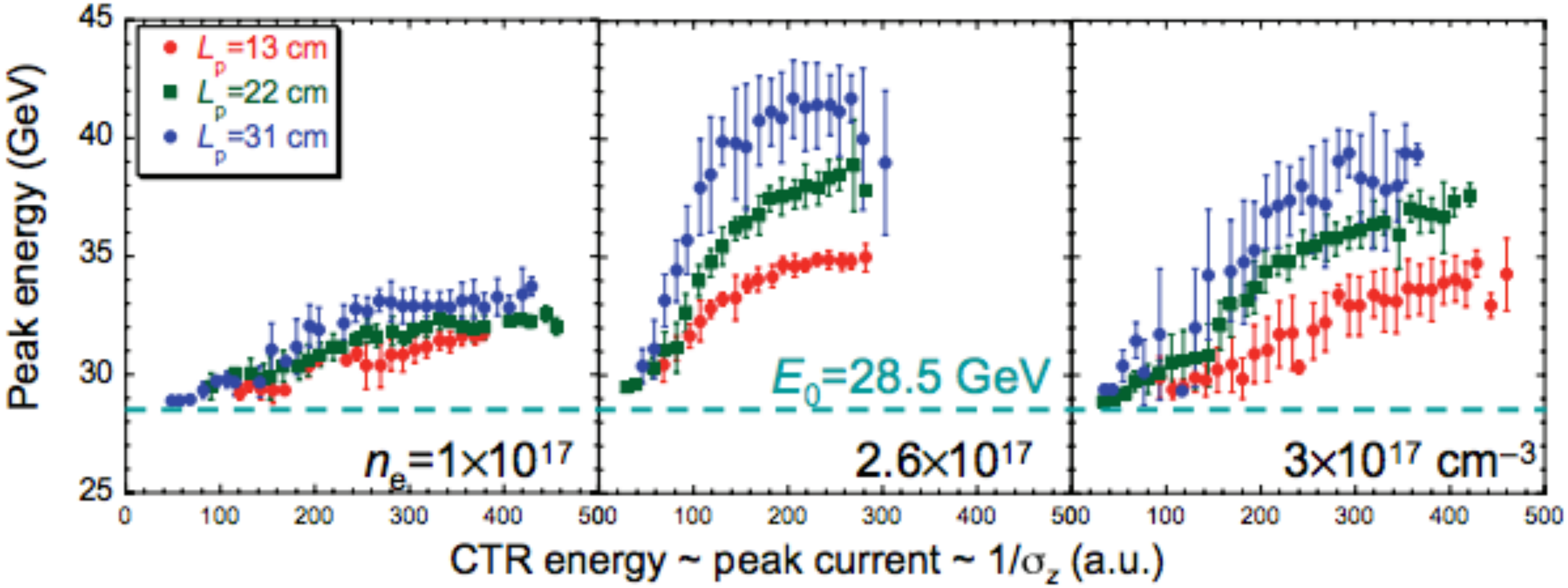


✧Verification of density scaling $k_{pe}\sigma_z \sim \sqrt{2}$

✧Negatively (e^-) and positively (e^+) charged bunches drive wakefields

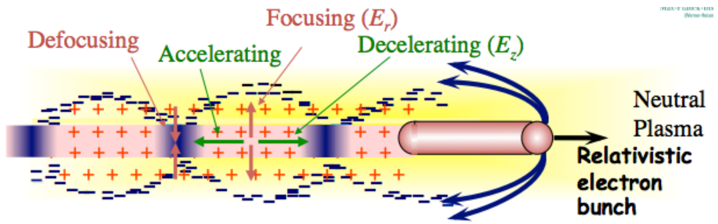
WAKEFIELDS SCALINGS

Bunch length dependency verification: in the non-linear regime ….



← longer, shorter →

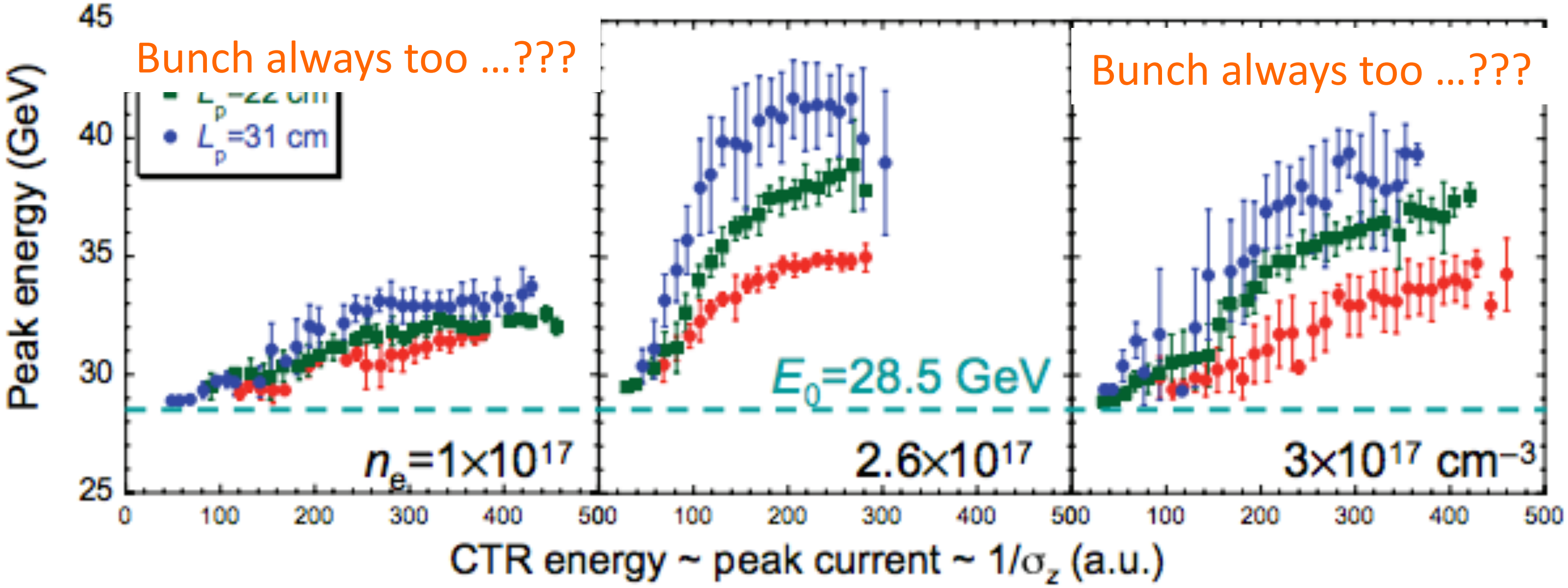
✧Dependency less clear than in simple (linear) calculations …



Muggli, New J. Phys. 12, 045022 (2010)

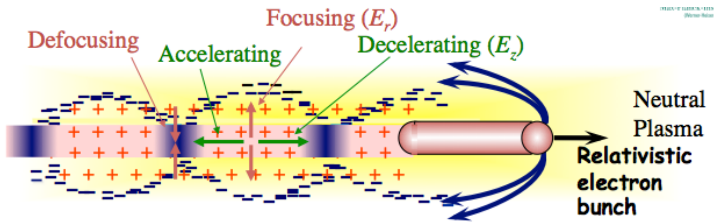
WAKEFIELDS SCALINGS

Bunch length depend — Bunch length sometimes … ??? — non-linear regime ….



← longer, shorter →

✧Dependency less clear than in simple (linear) calculations …



Muggli, New J. Phys. 12, 045022 (2010)

WAKEFIELDS SCALINGS

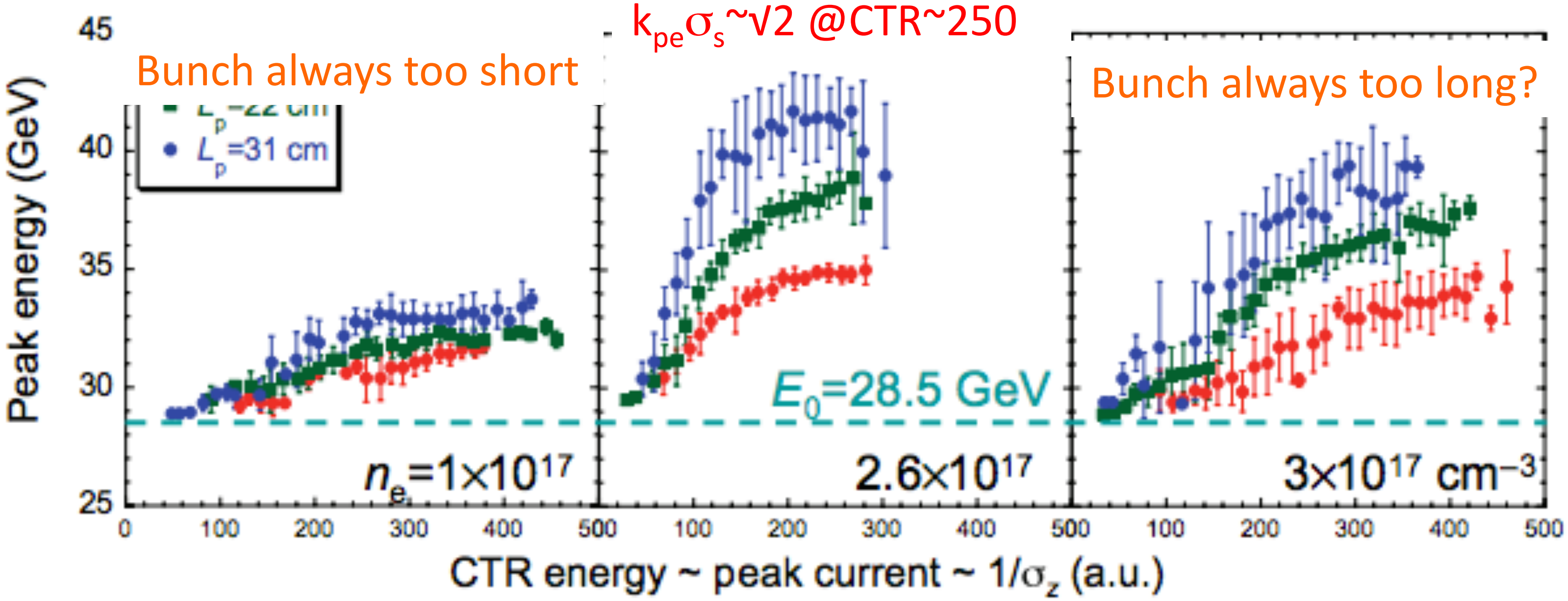
Bunch length depend

Bunch length sometimes just right: $k_{pe}\sigma_s$~√2 @CTR~250

non-linear regime ….

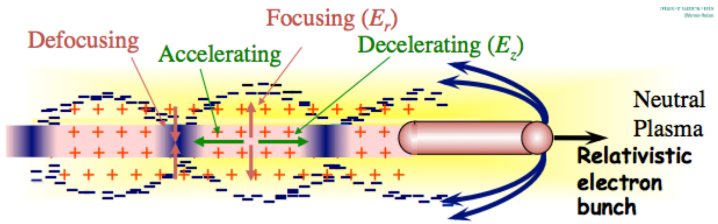
Bunch always too short

Bunch always too long?



← longer, shorter →

- ✧Dependency less clear than in simple (linear) calculations …
- ✧Maximum at σ_z~20µm for $k_{pe}\sigma_z$~√2

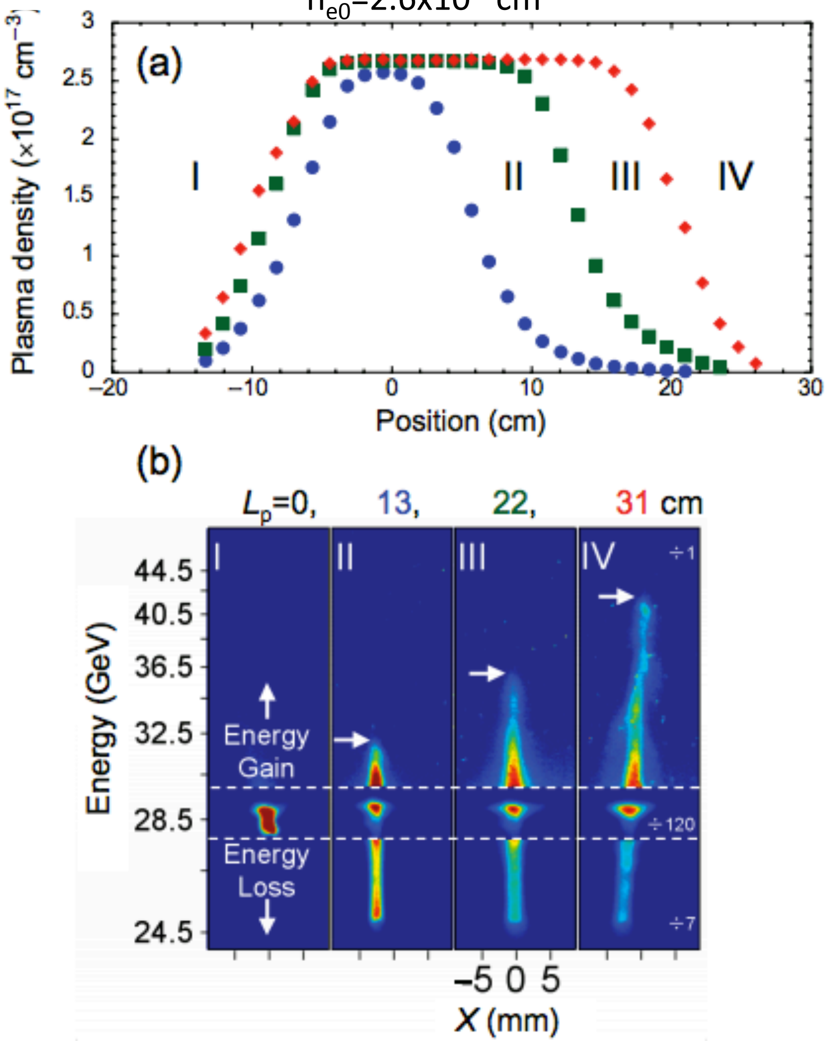


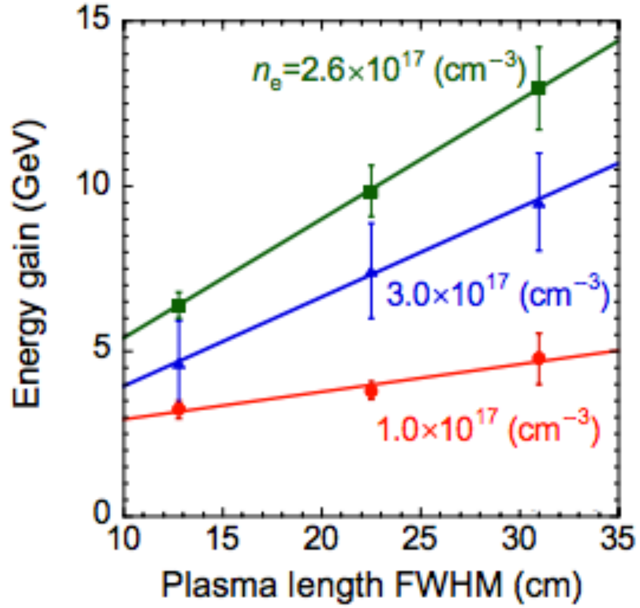
Muggli, New J. Phys. 12, 045022 (2010)

WAKEFiELDS SCALiNGS

Plasma length dependency

n_{e0}=2.6x10^{17}cm^{-3}





✧Larger gain, gradient at 2.6x10^{17}cm^{-3}

✧~$k_{pe}\sigma_z$~√2

✧Dependency is clear

✧Important for scaling to large energy gain, collider linac = looooong accelerator

Muggli, New J. Phys. 12, 045022 (2010)

LINEAR WAKEFIELDS THEORY

To efficiently drive wakefields: $k_{pe}\sigma_z \cong \sqrt{2}$ and $k_{pe}\sigma_r \leq 1$ (Gaussian bunch)

Can reach: $E_{WB} = \frac{m_e c \omega_{pe}}{2}$ or a fraction n_b / n_{e0} of that

Rewrite:

$$E_{WB} = \frac{m_e c \omega_{pe}}{2} = \frac{m_e c^2 k_{pe}}{2} \cong m_e c^2 \frac{\sqrt{2}}{\sigma_z} \cong 0.7 MeV \frac{1}{\sigma_z}$$

$$k_{pe}\sigma_z = \frac{\omega_{pe}}{c}\sigma_z \cong \sqrt{2} \quad \Rightarrow \quad \frac{n_e e^2}{\varepsilon_0 m_e c^2}\sigma_z^2 \cong 2 \quad \Rightarrow \quad n_e \cong 2\frac{\varepsilon_0 m_e c^2}{e^2}\frac{1}{\sigma_z^2} \cong 5.65 \times 10^{13} \frac{1}{\sigma_z^2}$$

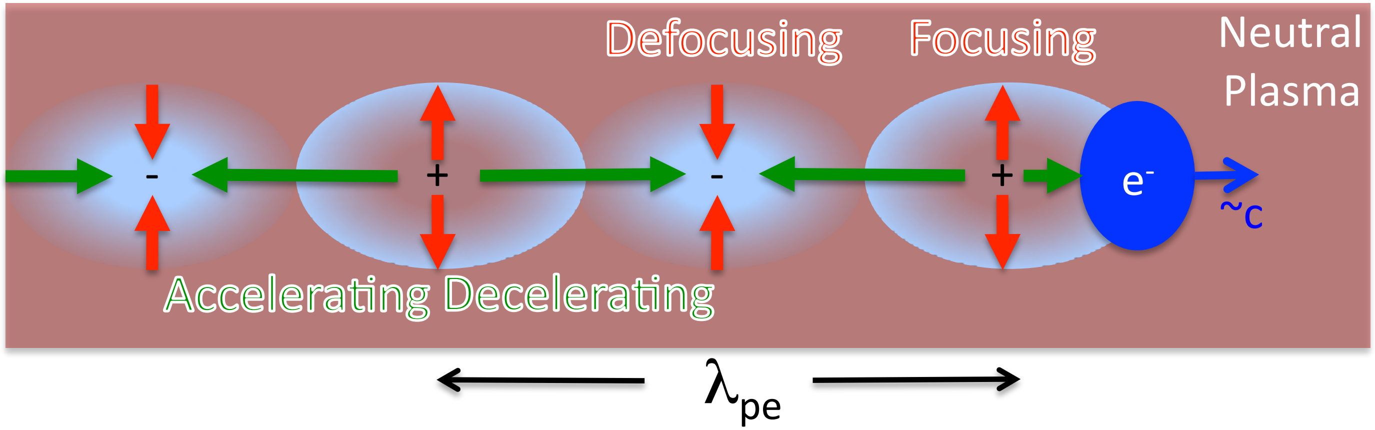
$$\sigma_r \leq \frac{1}{k_{pe}} \cong \frac{\sigma_z}{\sqrt{2}}$$

Emphasizes the need for:

- ✧Small (in all 3D) bunches to effectively drive wakefields
- ✧Large plasma density

WAKEFiELDS

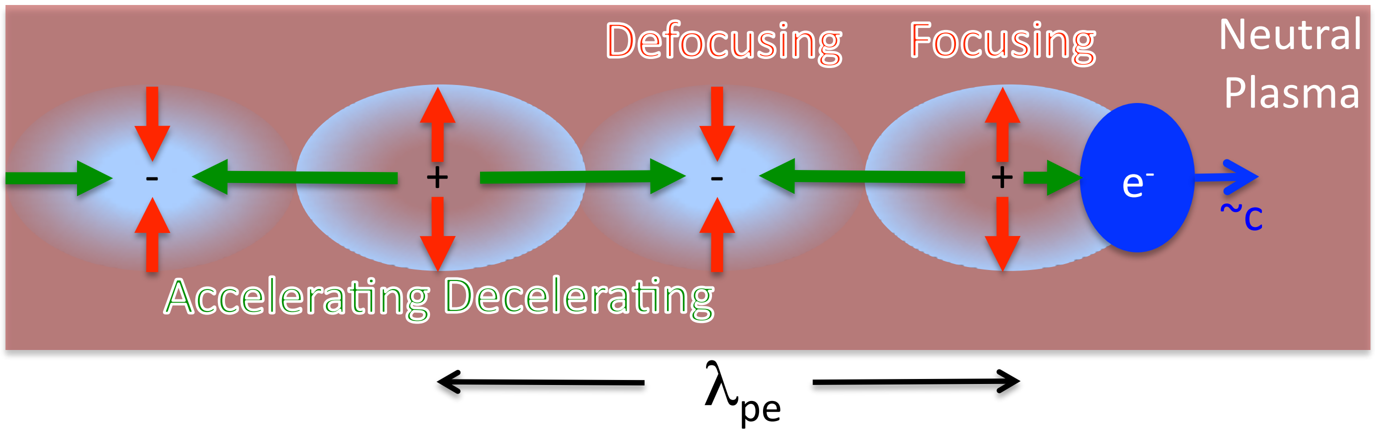
e^- driver / e^- witness



✧Cartoon, and theory and experiments suggests $k_{pe}\sigma_z \sim \sqrt{2}$
✧Cartoon suggests $k_{pe}\sigma_r \sim 1$: Why?

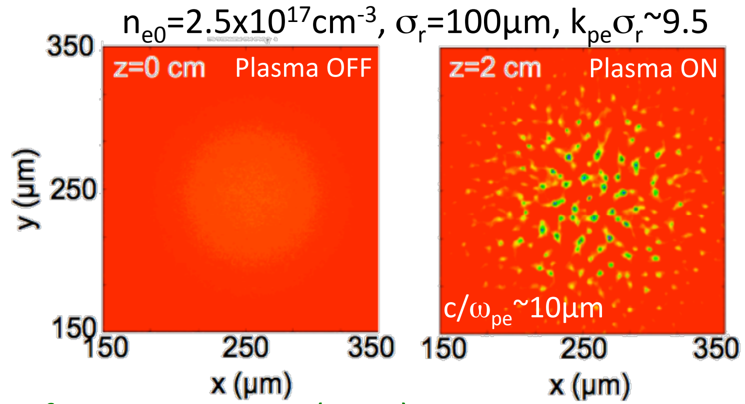
WAKEFiELDS

e^- driver / e^- witness



- Cartoon, and theory and experiments suggests $k_{pe}\sigma_z \sim \sqrt{2}$
- Cartoon suggests $k_{pe}\sigma_r \sim 1$: Why?

- Plasma (e^-) screens electric fields/potentials at the c/ω_{pe}
- Plasma (e^-) creates a (opposite) return current; $j_{return}=-en_{pe}v_e=-(-en_b v_b)=-j_b$
- Opposite currents repel each other
- Current non-uniformities amplify
- Current filamentation instability (CFI)
- Distance between current filaments $\sim c/\omega_{pe}$



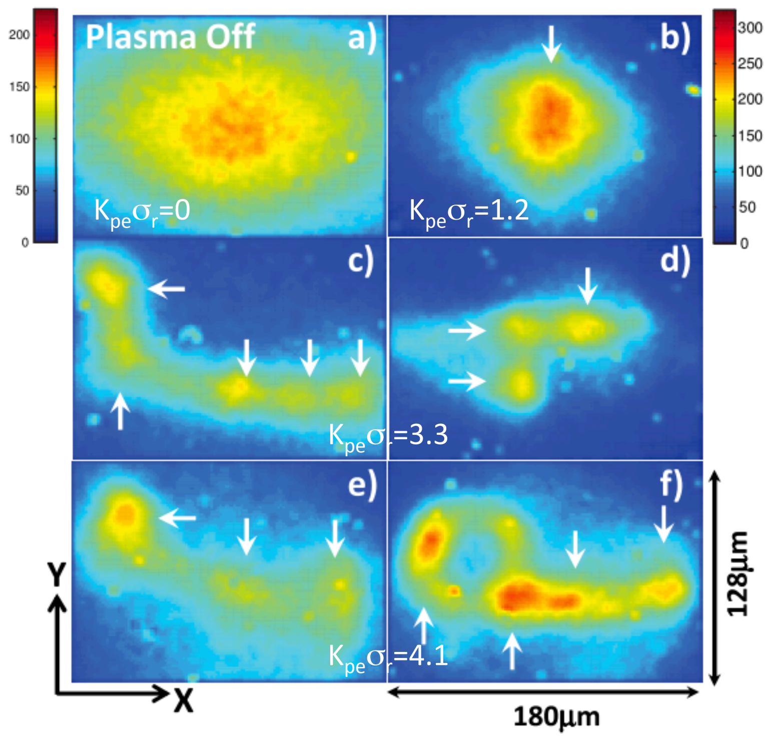
Allen, AIP Conf. Proc. 1299, 516 (2010)

TRANSVERSE EFFECT (CFI)

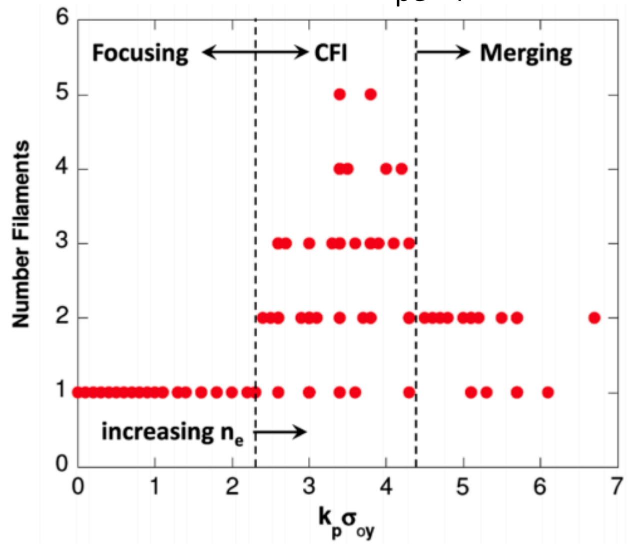
MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Beam transverse effect: $k_{pe}\sigma_r$



Transition at $k_{pe}\sigma_r$~2.2



Allen, Phys. Rev. Lett. 109, 185007 (2012)

Return current effect:

✧$k_{pe}\sigma_r$<1: plasma return current flows <u>outside</u> the bunch => global focusing, stable

✧$K_{pe}\sigma_r$>1: plasma return current flows <u>inside</u> the bunch

⇒Two inter-penetrating current: e^- bunch current and plasma return current

⇒Opposing currents (at the c/ω_{pe} transverse scale repel each other)

⇒Instability: (transverse) current filamentation instability: CFI

A. Bret, Astrophys. J. 699, 2 (2009)

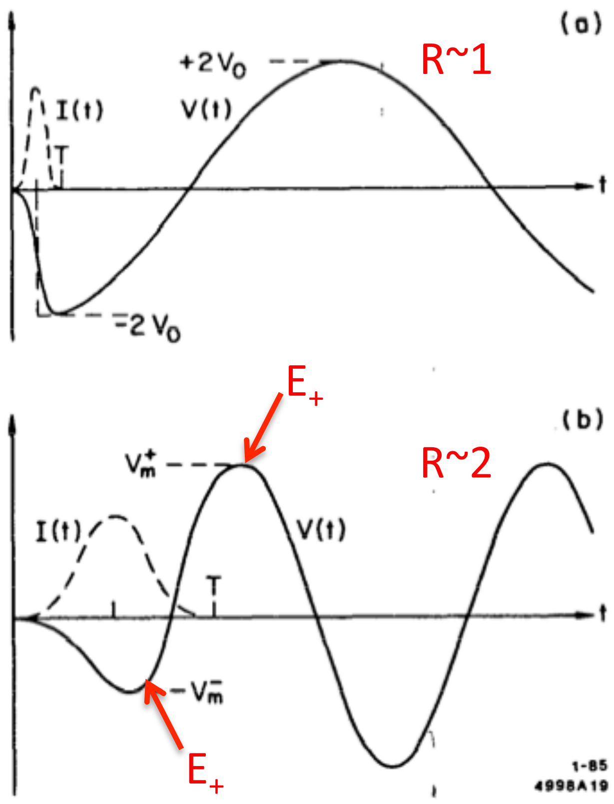
TRANSFORMER RATIO

Definition: Transformer ratio (R)

:=Peak accelerating field behind drive bunch(es) / peak decelerating field within drive bunch(es)

:=E_+/E_-

minimum



Bunch too short!

Maximum field after the bunch

Bunch just right!

Maximum field inside the bunch

For a single symmetric (in time) bunch: R≤2

Fundamental theorem of beam loading

Figure 7. Potential in and behind a charge distribution interacting with a si mode for (a) a short bunch, and (b) a long bunch.

WAKE FIELD ACCELERATORS*

P. B. WILSON

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

SLAC-R-296

PLASMA ACCELERATORS*

RONALD D. RUTH AND PISIN CHEN†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

Invited talk presented at the SLAC Summer Institute on
Particle Physics, Stanford, California, July 29 – August 9, 1985

TRANSFORMER RATIO

Adjusting the current profile or with multiple bunches R>2 is possible:

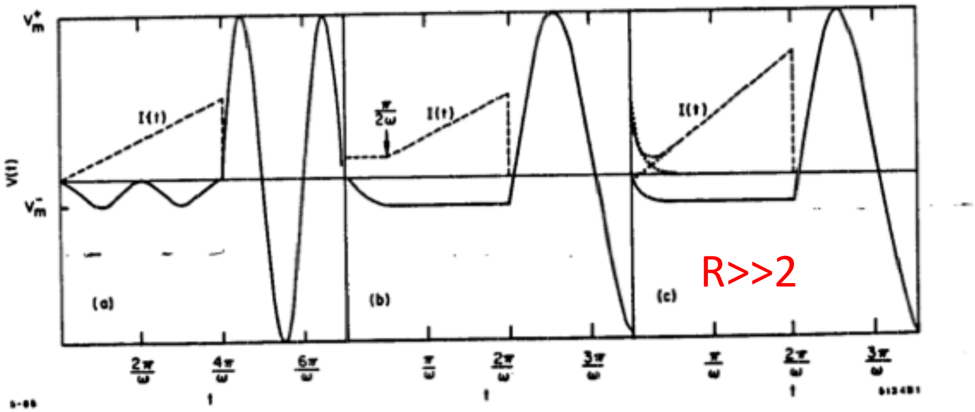
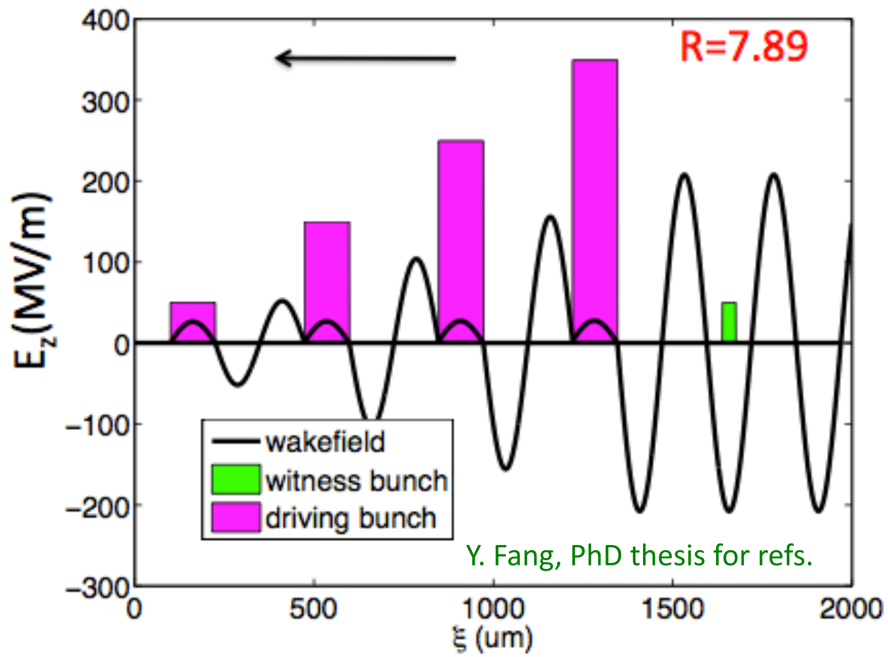


Figure 9. The voltage induced by three different asymmetric current distributions interacting with a single mode.



✧Conservation of energy => $W=Q*U$

=> $\Delta W_W=Q_W E_+ L_p \leq \Delta W_D=Q_D E_- L_p$

But $R=E_+/E_-$

=> $Q_W \leq Q_D/R$

✧R applies also to nonlinear wakes Tzoufras, PRL 101, 145002 (2008)

✧Has important implications on energy transfer efficiency, e.g, almost all particles lose energy at the same rate!

✧Can produce high energy particles out of low energy ones (single plasma)

✧W-bunch particles can gain R-times the drive bunch particles' energy

TRANSFORMER RATIO

Adjusting the current profile R>2 is possible:

Gao, Phys. Rev. Lett. 120, 114801 (2018)

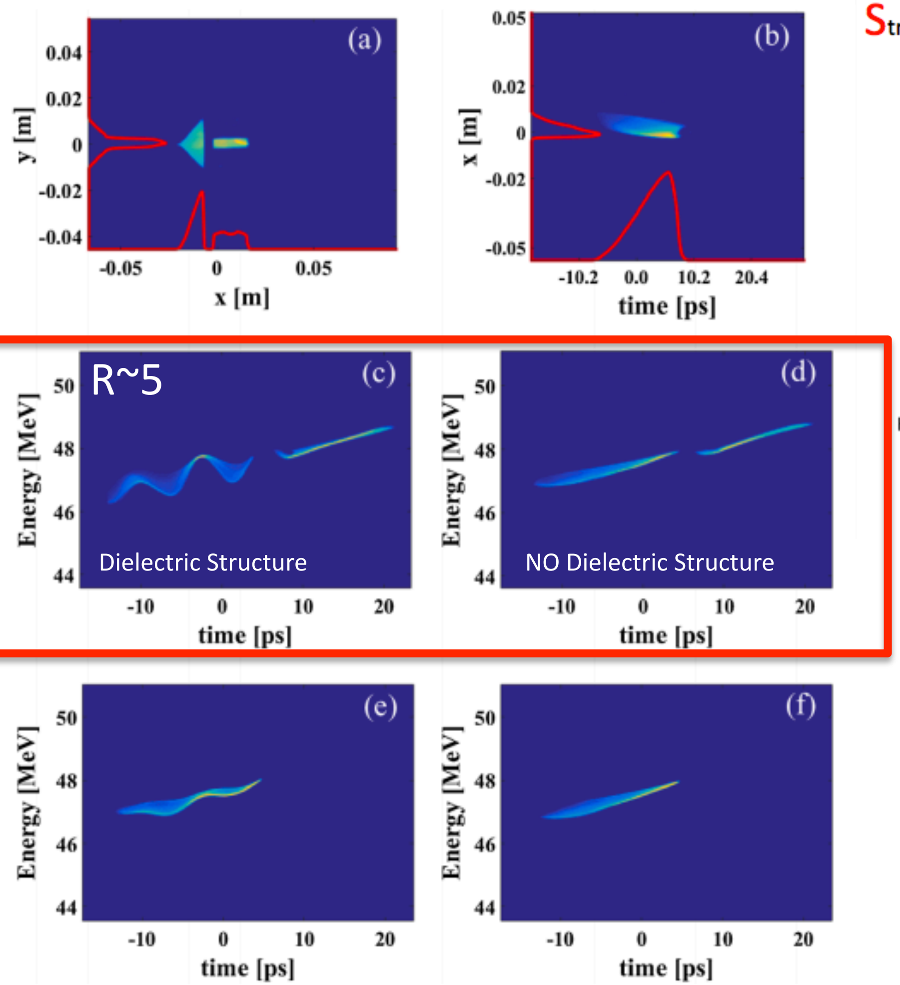
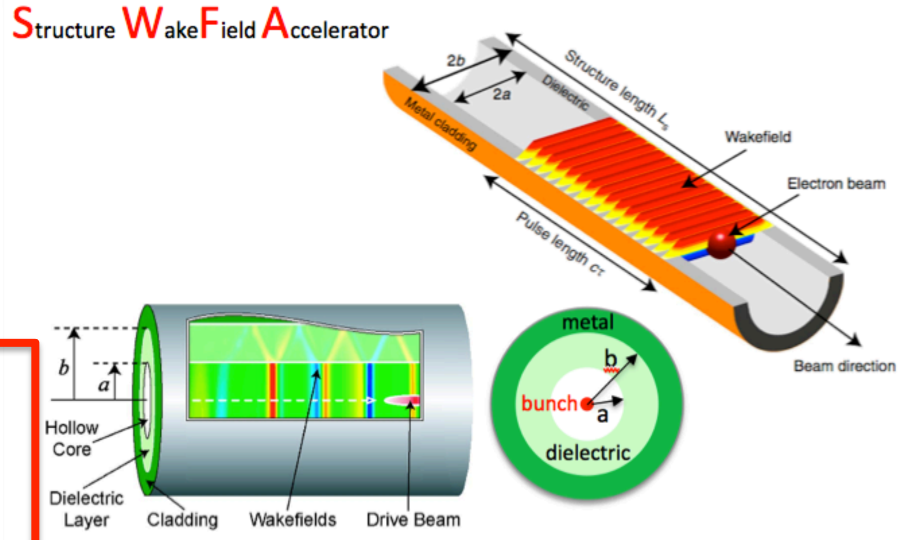


FIG. 2. Beam image from experiment: (a) Drive and witness transverse distribution at YAG1, the red line refers to projection. (b) Longitudinal profile measurement result of drive bunch at YAG2. (c)–(d) Drive and witness bunch at YAG3 with wakefield on-off. (e)–(f) Witness bunch at YAG3 with wakefield on-off.



- Dielectric wakefield accelerator (DWA), now SWFA
- Another co-linear wakefield accelerator
- Concept: W. Gai et al., Phys. Rev. Lett. 61, 2756 (1988)
- Demonstration of GV/m fields: M. C. Thompson et al., Phys. Rev. Lett. 100, 214801 (2008)

TRANSFORMER RATIO

Adjusting the current profile R>2 is possible:

Gao, Phys. Rev. Lett. 120, 114801 (2018)

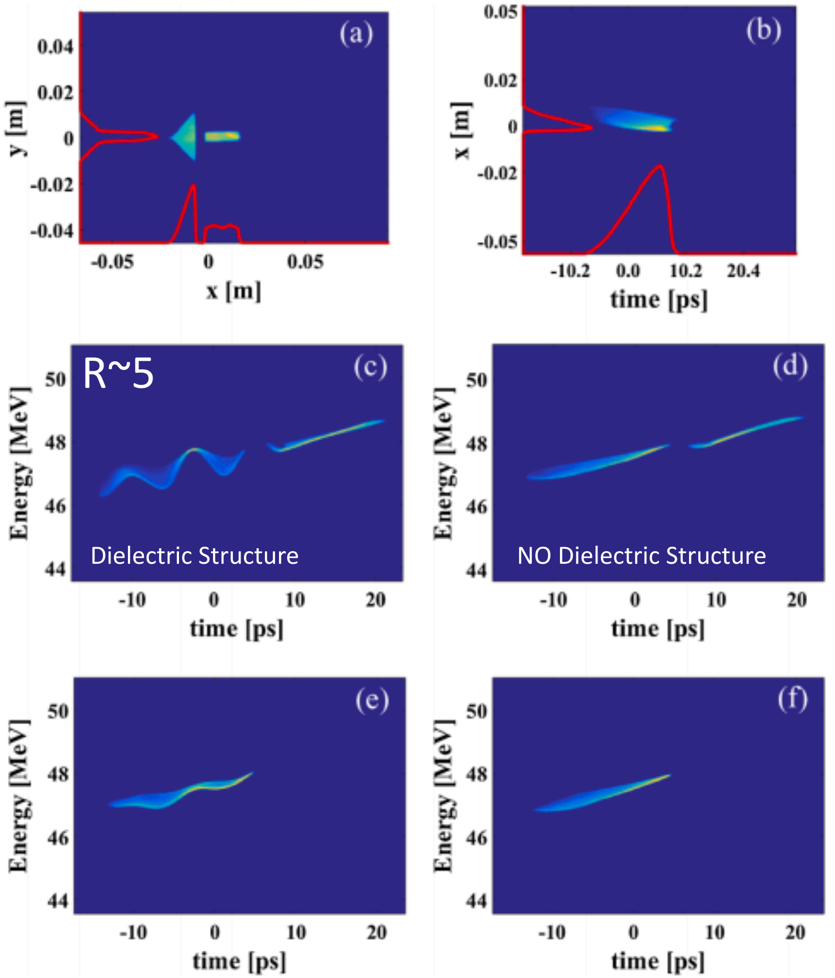
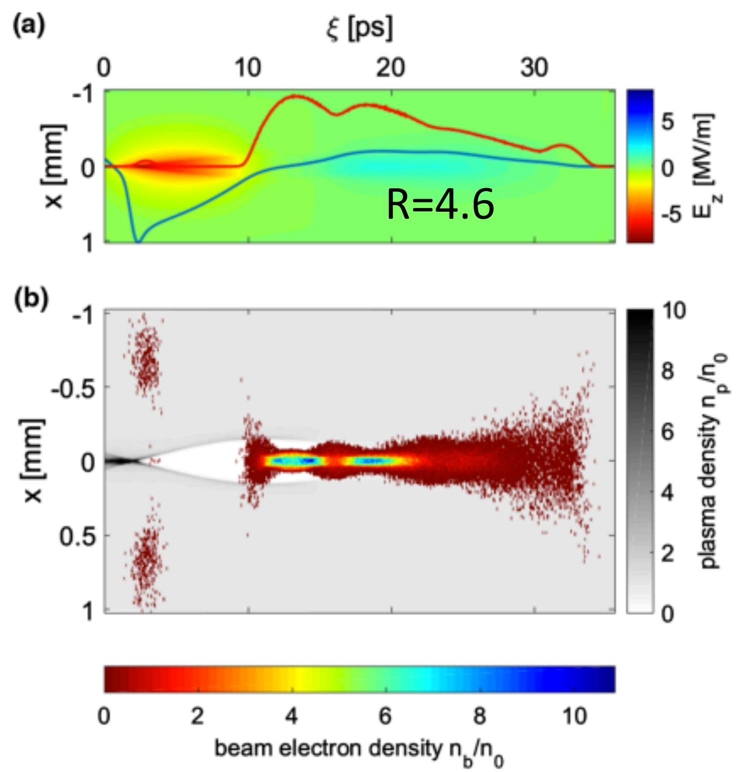


FIG. 2. Beam image from experiment: (a) Drive and witness transverse distribution at YAG1, the red line refers to projection. (b) Longitudinal profile measurement result of drive bunch at YAG2. (c)–(d) Drive and witness bunch at YAG3 with wakefield on-off. (e)–(f) Witness bunch at YAG3 with wakefield on-off.

PWFA



Loisch, Phys. Rev. Lett. 121, 064801 (2018)

TRANSFORMER RATIO

When R=2, accelerated particles can gain energy at a rate twice as large as large as that of those losing energy to the wakefields

Does that violate energy conservation?

TRANSFORMER RATIO

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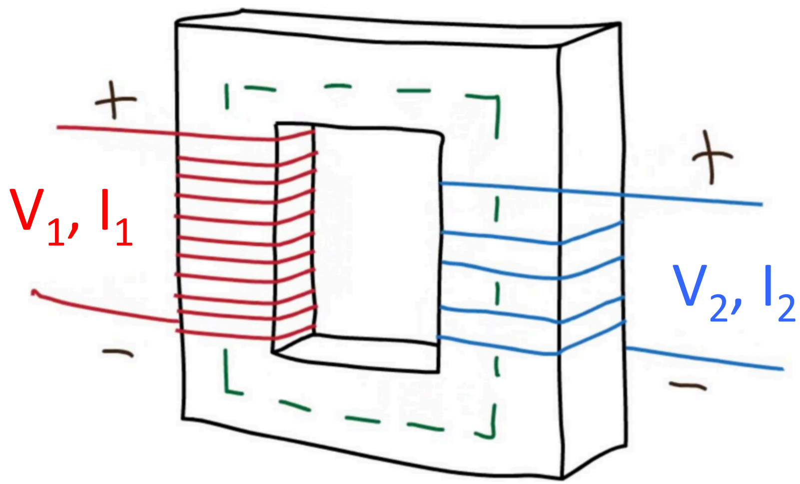
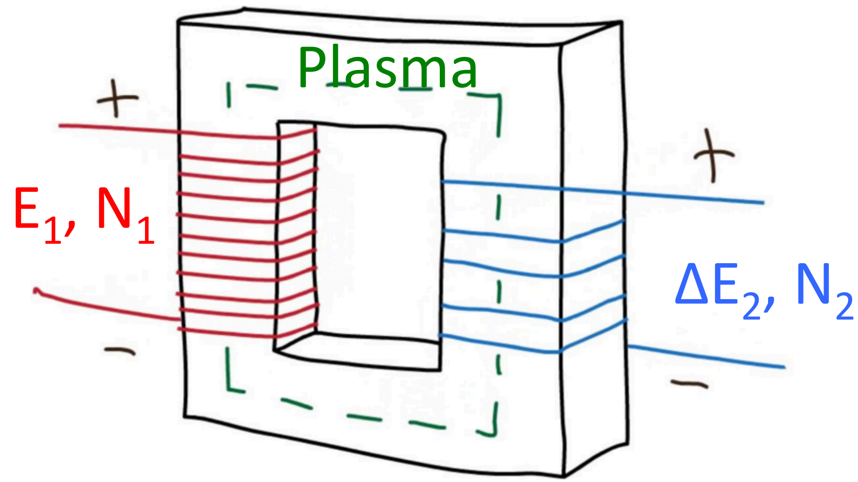
Does that violate energy conservation?





$P_2=V_2I_2\leq P_1=V_1I_1$

$\Delta E_2\leq E_1(2-N_2/N_1)$

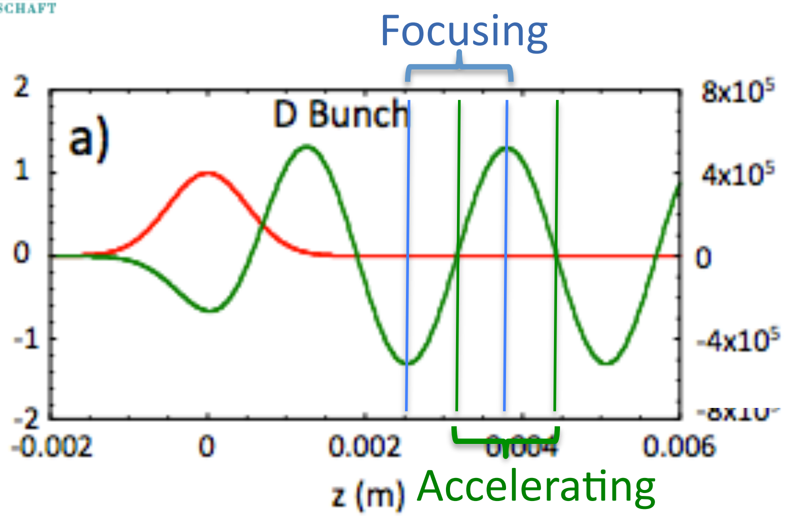
Assume full energy depletion of the D-bunch

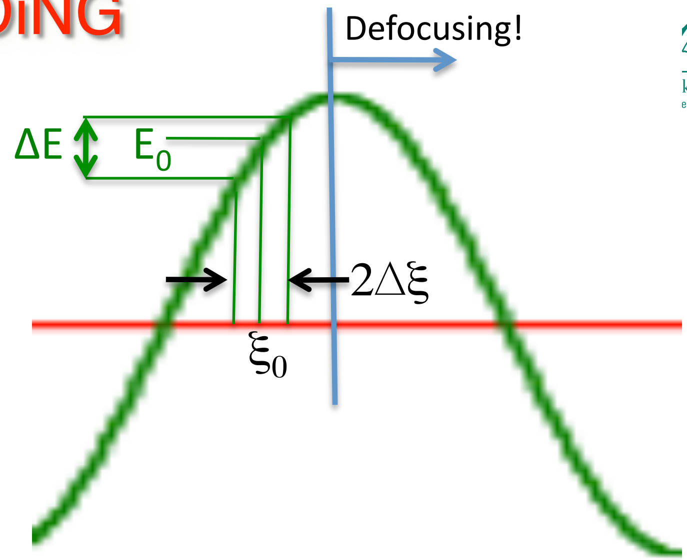
R.D. Ruth et al., SLAC-PUB-3374 (1984).

In practice, more complicated, 2D problem with finite size bunches

Q: what is wrong with the sketches???

BEAM LOADiNG





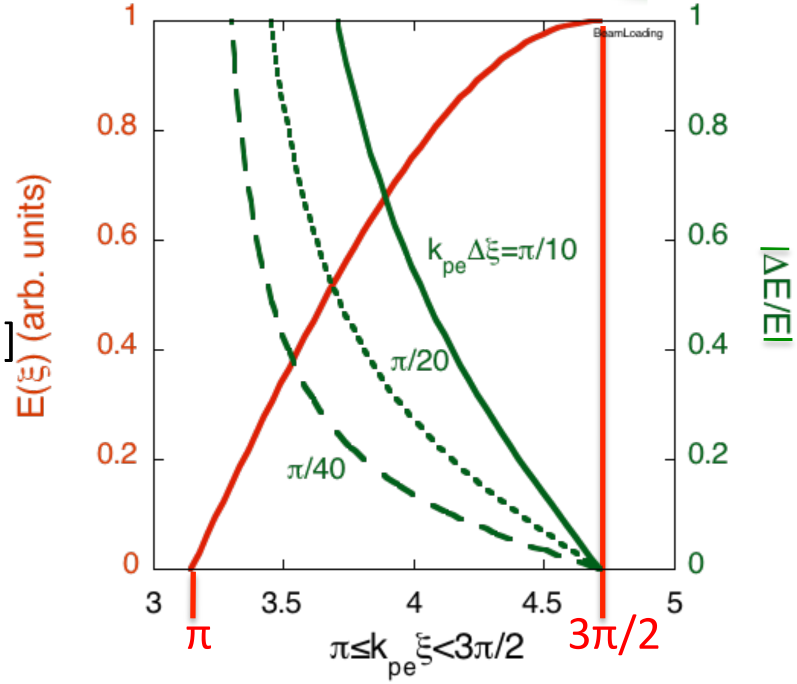
$E(\xi)=-E_0\sin(k_{pe}\xi)$ accelerating for $\pi\leq\xi\leq3\pi/2$

Bunch center at ξ_0, extends from $\xi_0-\Delta\xi$ to $\xi_0+\Delta\xi$

$E(\xi_0+\Delta\xi)=-E_0\sin(k_{pe}(\xi_0+\Delta\xi))\sim-E_0[\sin(k_{pe}\xi_0)+k_{pe}\Delta\xi\cos(k_{pe}\xi_0)]$

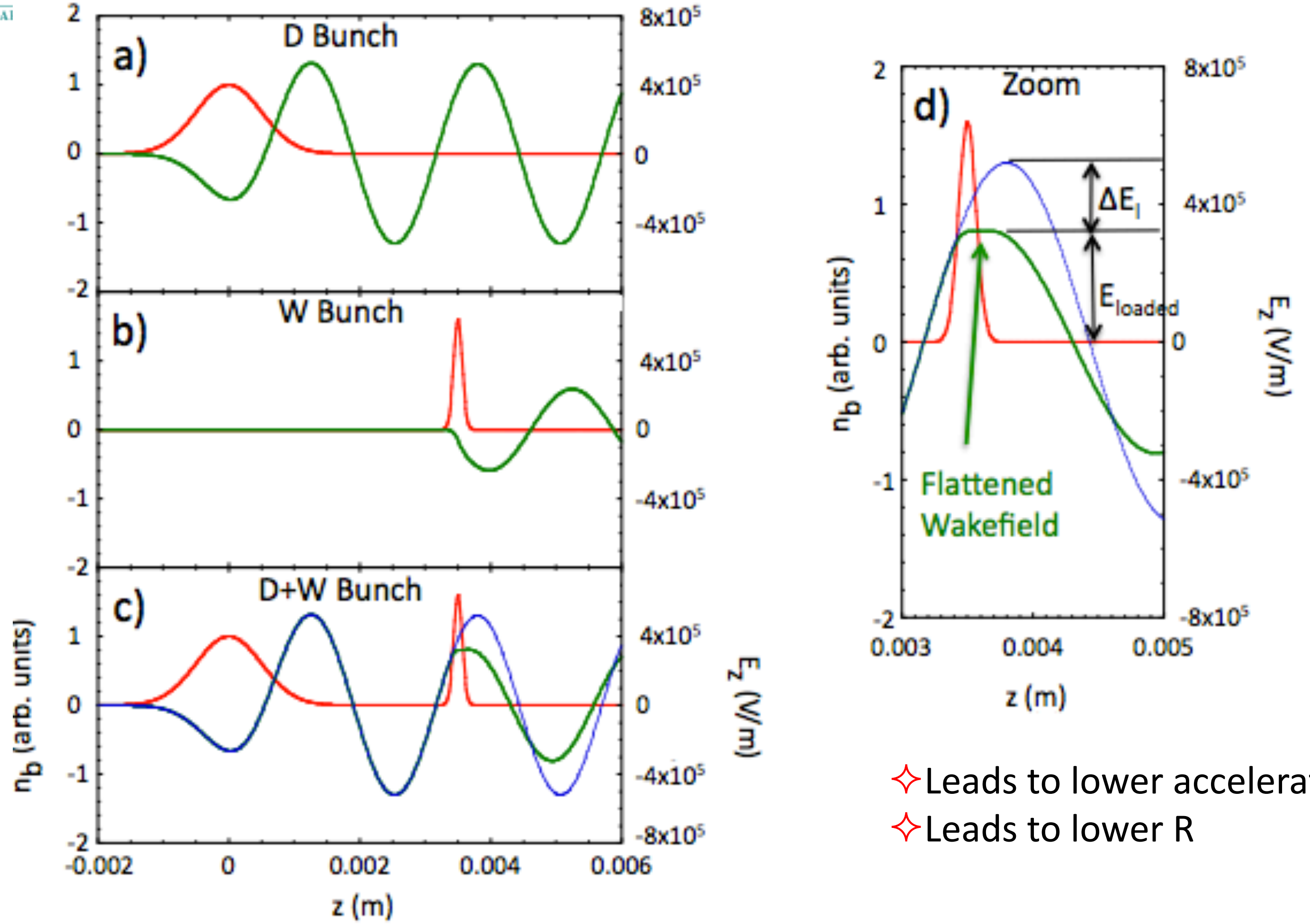
$|\Delta E|\sim2|(E(\xi_0+\Delta\xi)-E(\Delta\xi))|=|-2E_0k_{pe}\Delta\xi\cos(k_{pe}\xi_0)|$

$|\Delta E/E|(\xi_0,\Delta\xi)\sim|-2k_{pe}\Delta\xi\cos(k_{pe}\xi_0)/\sin(k_{pe}\xi_0)|$



✧Finite length witness bunch with low charge experiences energy spread

BEAM LOADiNG



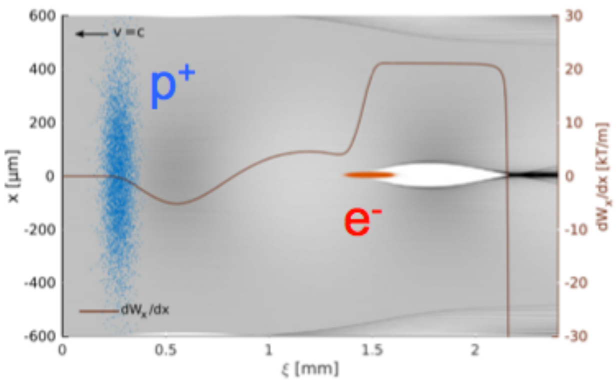
✧Leads to lower accelerating gradient
✧Leads to lower R

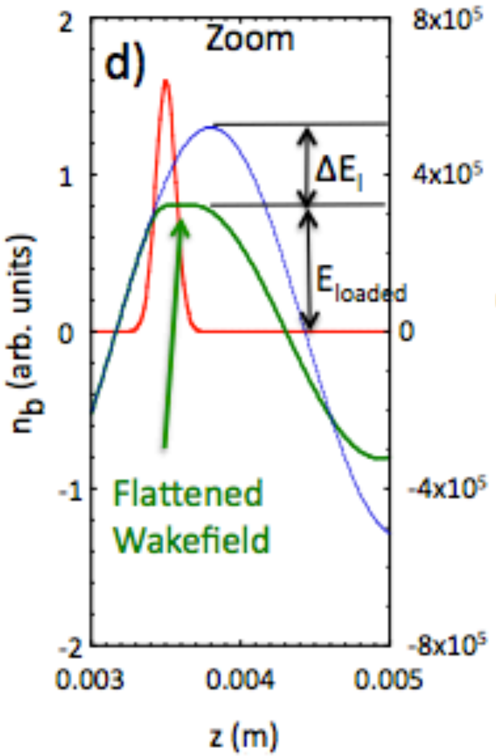
✧Beam loading occurs when the W-bunch wakefields are comparable to, and affect the D-bunch wakefields

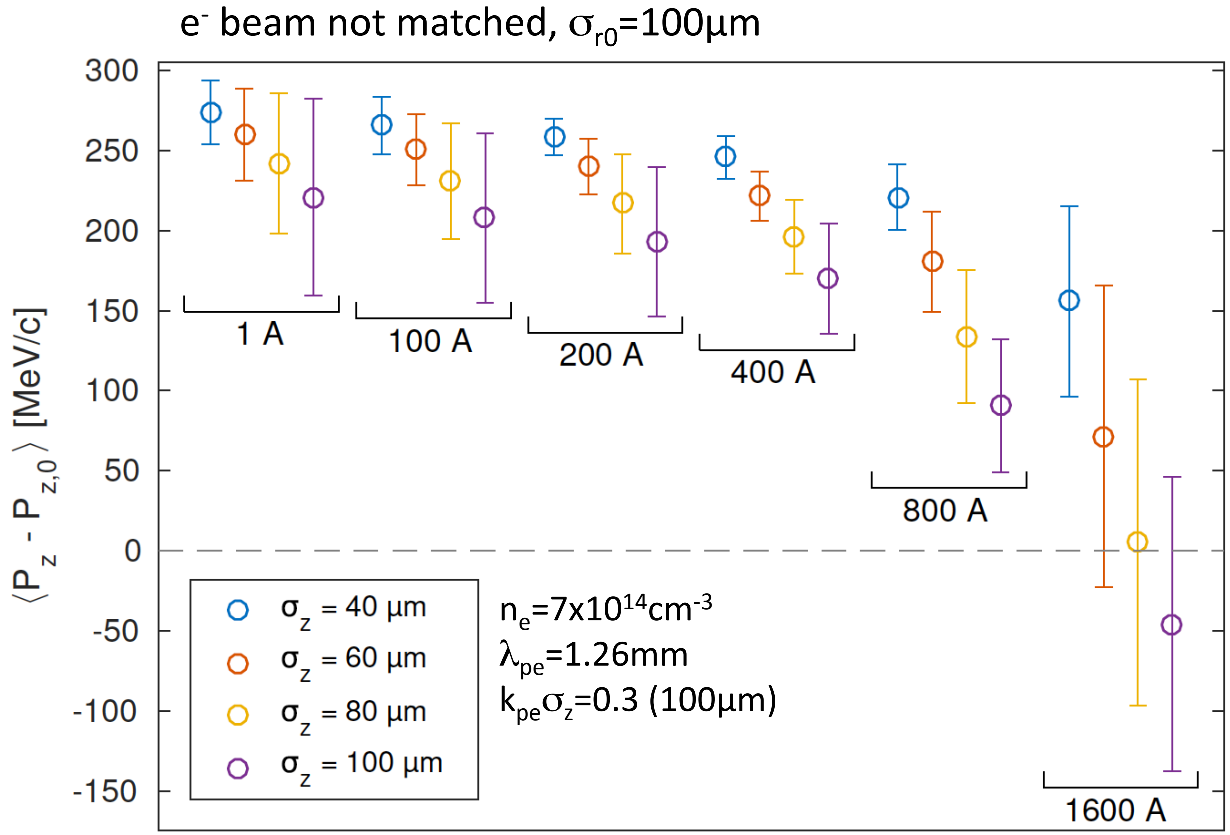
✧Adjust D/W bunch delay
✧Adjust W-bunch length: <D-bunch
✧Adjust W-bunch charge: <D-bunch
✧Adjust W-bunch current: >D-bunch
✧Shape W-bunch profile: shape?

… to best flatten the accelerating field
… to minimize field loss

BEAM LOADiNG





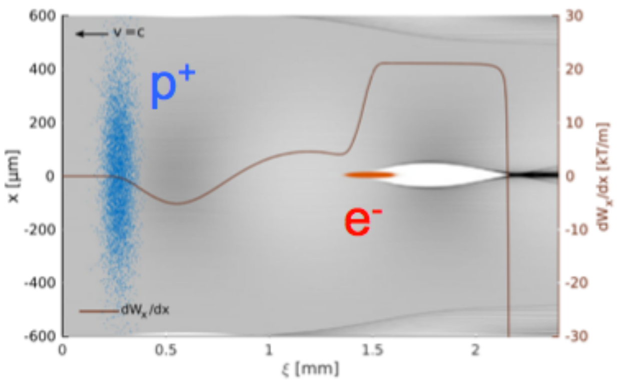


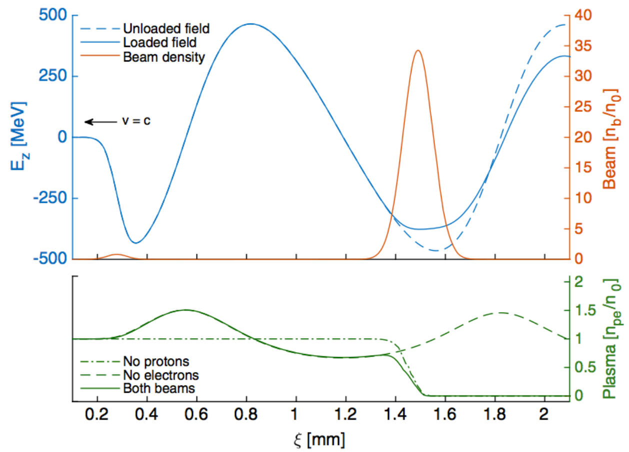
Berglyd Olsen, Proceedings of NAPAC2016, Chicago, IL, USA
Phys. Rev. Accel. Beams 21, 011301 (2018)

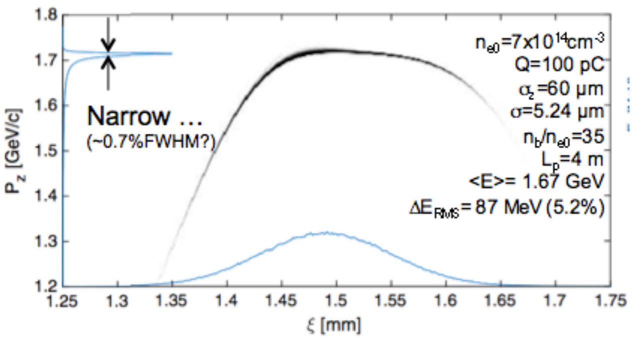
✧Energy gain decreases with more current, charge
✧Energy spread increases with longer bunch

BEAM LOADiNG

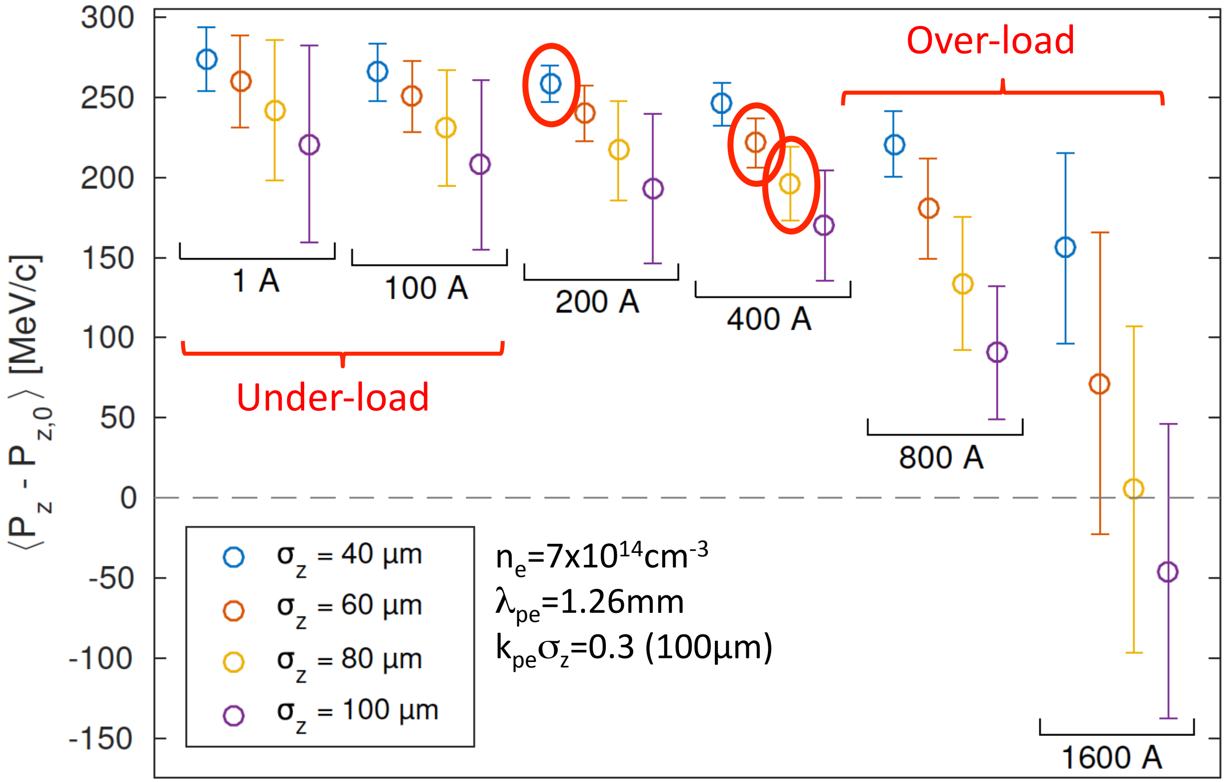
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)







e⁻ beam not matched, σ_{r0}=100μm

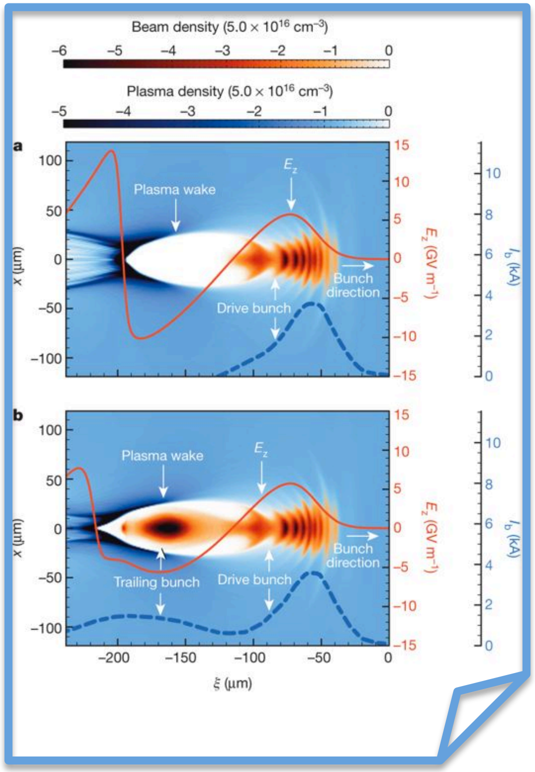


Berglyd Olsen, Proceedings of NAPAC2016, Chicago, IL, USA
Phys. Rev. Accel. Beams 21, 011301 (2018)

- ✧Energy gain decreases with more current, charge
- ✧Energy spread increases with longer bunch
- ✧"Optimum" exists

BEAM LOADiNG

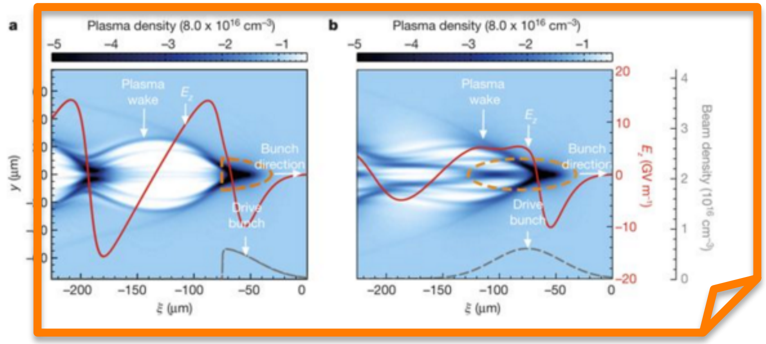
Litos, Nature 515, 92 (2014)



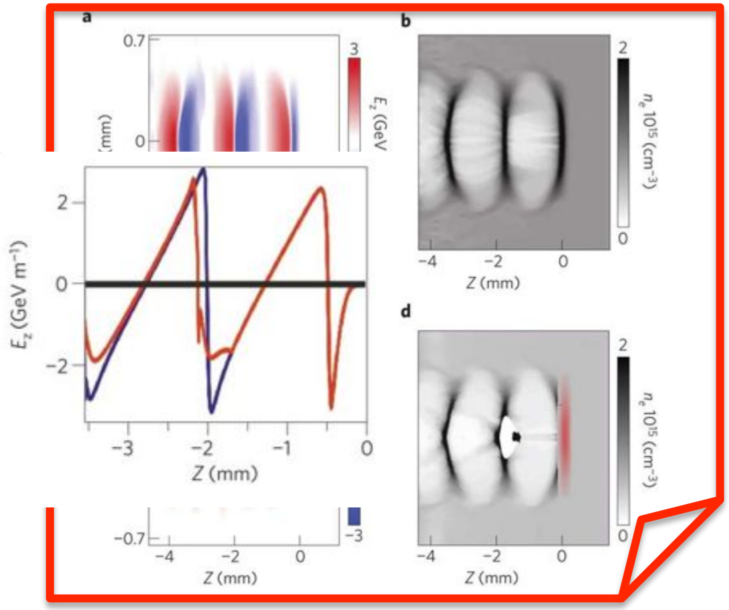
e⁻

Corde, Nature 524, 442 (2015)

e⁺



Caldwell, Nature Physics 5, 363 (2009)



p⁺

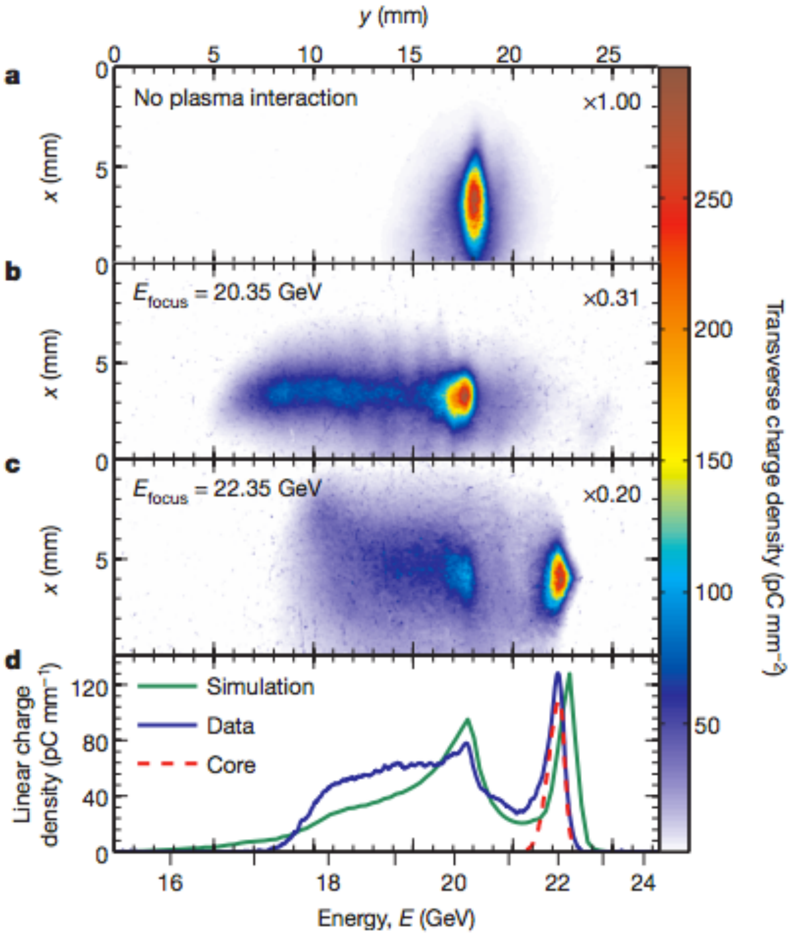
✧Used in (almost) all experiments

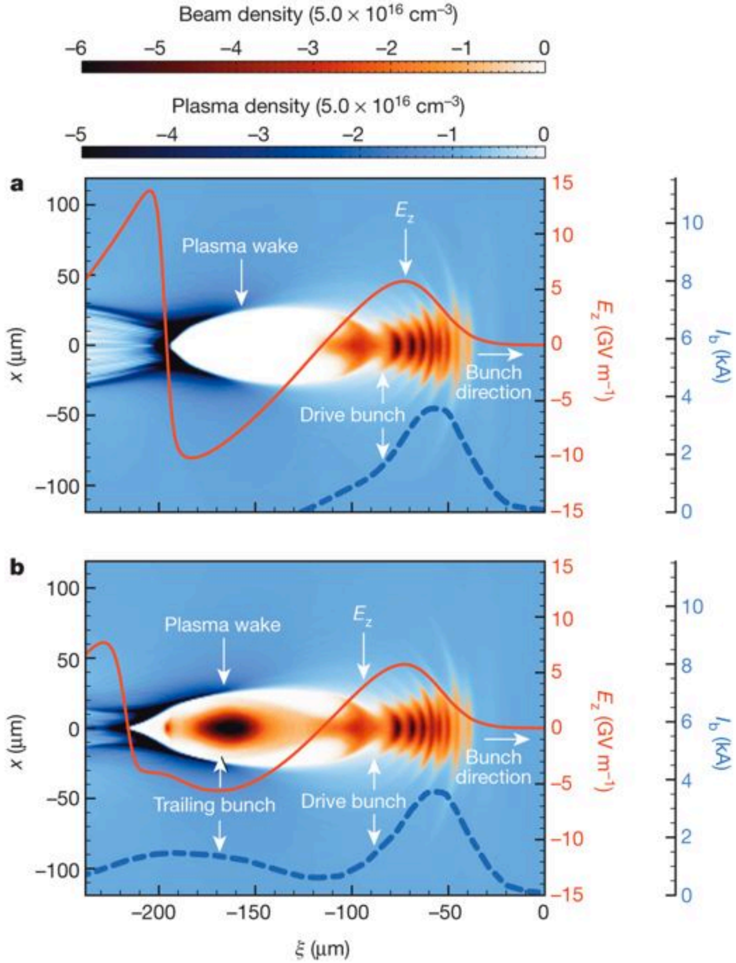
✧Needed to produce narrow ΔE/E, because W-bunch σ_z not <<λ_{pe}

✧Requires very good control of witness bunch parameters

BEAM LOADiNG e⁻

Litos, Nature 515, 92 (2014)

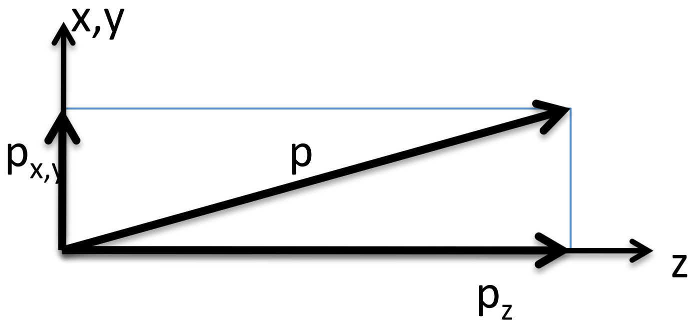




n_e=5x10^{16}cm^{-3}, L_p=34cm
ΔE_w~1.6GeV, <ΔE/E>=2% because of beam loading
G=4.4GeV/m

✧Beam loading inferred from narrow energy spread and simulation results
✧Inferred energy transfer efficiency (D->W) ~30%

BEAM EMITTANCE



Transverse particle motion defined by (x,p_x) but (x,x') is more useful (remember optics: position and angle)
Random distribution of (x,p_x) or (x,x')
$x'=p_x/p_z$

RMS emittance definition:

$$\varepsilon_{geo,x} = [\pi]\left(\langle x^2\rangle\langle x'^2\rangle - \underbrace{\langle xx'\rangle}^2\right)^{1/2}$$

Correlation term

$\langle\ldots\rangle$ average $\Rightarrow \langle x^2\rangle^{1/2}$ RMS size

$\Rightarrow \langle x'^2\rangle^{1/2}$ ~RMS transverse velocity or temperature

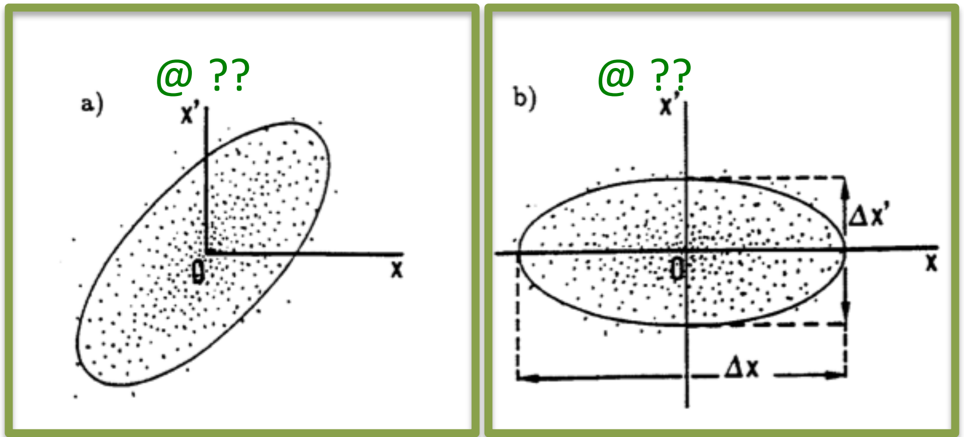
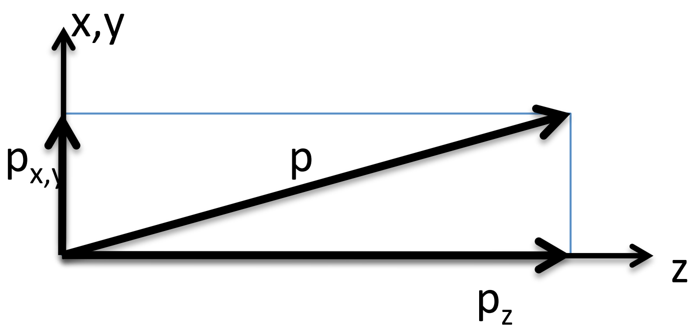


Fig. 1 : A set of points representative of a beam in the (x, x') phase space
a) Tilted emittance ellipse.
b) Upright emittance ellipse.

MAX-PLANCK-GESELLSCHAFT

BEAM EMITTANCE

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)



Transverse particle motion defined by (x,p_x) but (x,x') is more useful (remember optics: position and angle)
Random distribution of (x,p_x) or (x,x')
$x'=p_x/p_z$

RMS emittance definition:

Q: To which of the beam positions wrt a waist do the ellipses correspond to and why?

Correlation term

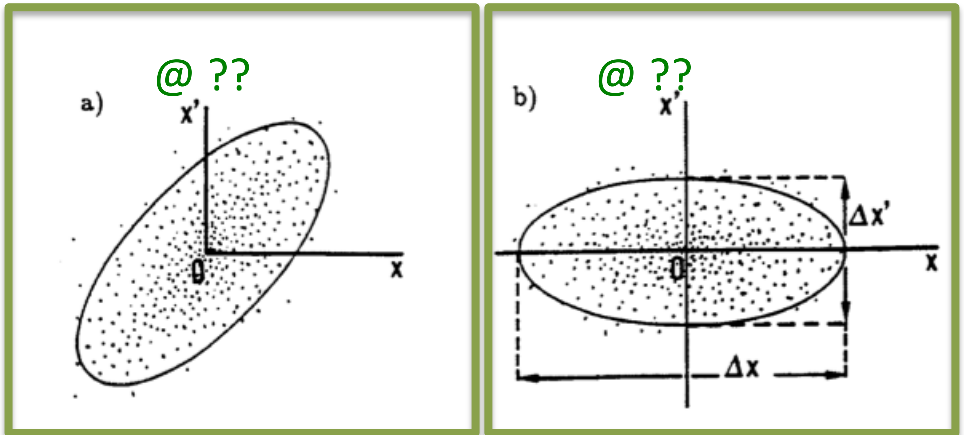
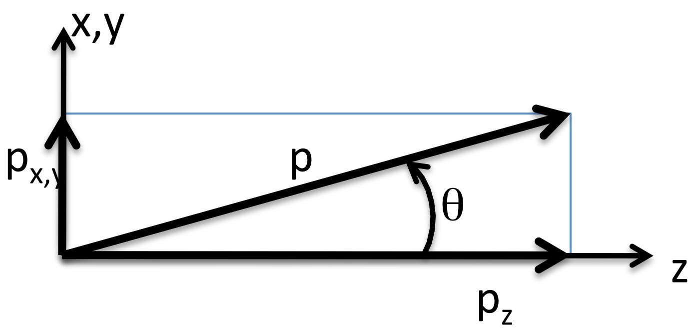


Fig. 1 : A set of points representative of a beam in the (x, x') phase space
a) Tilted emittance ellipse.
b) Upright emittance ellipse.

BEAM EMiTTANCE



- Focusing acts on transverse quantities (position, velocity, momentum)
- Transverse particle motion defined by (x,p_x) but (x,x') is more useful (remember optics: position and angle), $x'=tg(\theta)\sim\theta$

RMS emittance definition:

$$\varepsilon_{geo,x} = [\pi]\left(\langle x^2\rangle\langle x'^2\rangle - \underbrace{\langle xx'\rangle}^2\right)^{1/2}$$

Correlation term

$\langle \ldots \rangle$ average $\Rightarrow \langle x^2\rangle^{1/2}$ RMS size

$\Rightarrow \langle x'^2\rangle^{1/2}$ ~RMS transverse velocity or temperature

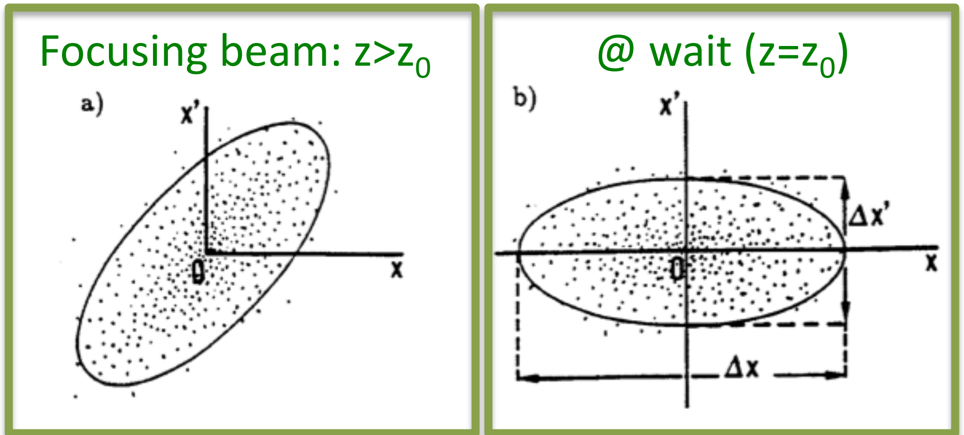


Fig. 1 : A set of points representative of a beam in the (x,x') phase space
a) Tilted emittance ellipse.
b) Upright emittance ellipse.

BEAM EMiTTANCE

$$\varepsilon_{geo,x} = \left(\langle x^2\rangle\langle x'^2\rangle\right)^{1/2} \cong \sigma_x \frac{\sigma_{p_x}}{\beta\gamma mc} \propto \frac{1}{\gamma}$$ (at a waist)

=> The geometric emittance <u>decreases</u> upon acceleration …

Define a quantity that is <u>preserved</u> upon acceleration (no other effects):

Normalized emittance: $\varepsilon_{N,x} = \gamma\varepsilon_{geo,x}$

⇒A higher energy accelerator (preserving normalized emittance) produces lower geometric emittance beams that can be focused to smaller transverse sizes …

⇒Emittance? ⇔ how well can it be focused?

EMiTTANCE PRESERVATiON

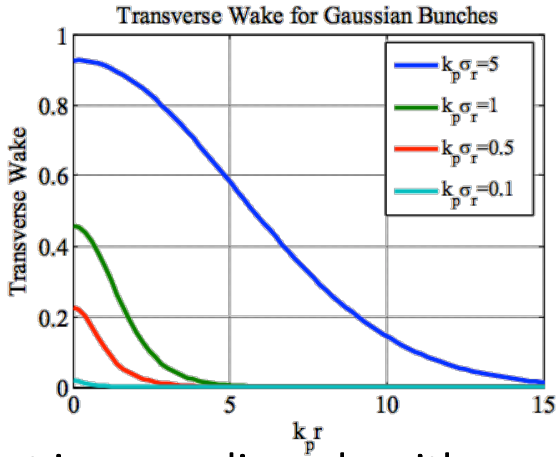
An optical system with focusing forces varying linearly with radius (magnet, plasma ion column, …) preserves the emittance of the incoming beam, with bi-Gaussian distributions (x,x'), when matched and with no varying energy gain … (otherwise: preserves slice emittance)

But in PWFA linear theory:

Inside the bunch (driver): $W_\perp(0 \le \xi \le \xi_b) = \frac{e}{\epsilon_0}\frac{dR(r)}{dr}\frac{n_{b0}}{k_{pe}^2}\left(1 - cos[k_{pe}\xi]\right).$

After the bunch (witness): $W_\perp(\xi > \xi_b) = \frac{e}{\epsilon_0}\frac{dR(r)}{dr}\frac{n_{b0}}{k_{pe}^2}\left(cos[k_{pe}(\xi-\xi_b)] - cos[k_{pe}\xi]\right).$

R(r):



⇔ dR/dr, W_r NOT ~r!

The radial wakefields do not increase linearly with r and therefore <u>do not preserve</u> emittance
(of a beam with Gaussian distributions in transverse space and velocity)

Solution in a plasma: blow-out, bubble regime, (pure) ion column focusing

EMITTANCE PRESERVATION

Physics:

Plasma focusing is the result of the (partial) cancellation of the (relativistic) bunch fields:

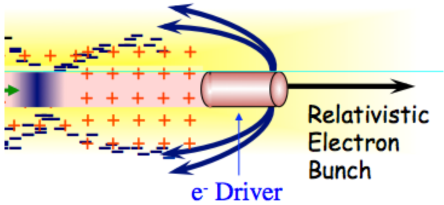
$$F_\perp = q\Big(\underbrace{E_{\perp b} - v_{//b} \times B_{\perp b}}_{-eE_r\frac{1}{\gamma^2}}\Big) + q\Big(E_{\perp p}(n_e,n_i) - v_{//b} \times B_{\perp p}(n_e,v_e)\Big) \cong q\Big(E_{\perp p}(n_e,n_i) - v_{//b} \times B_{\perp p}(n_e,v_e)\Big) \quad <<$$

This is because the plasma (total) charge density is positive (e^- drive), but not uniform across the bunch ...

Solution:

Make the radial field ~r, which is the case if n(r)=cst (in an infinitely long cylinder case, Gauss law)

Evacuate, blow-out all plasma electrons from the bunch volume



Use pure ion column with $n_i=n_{i0}=n_{e0}$=cst (immobile ions)

Physics:

In linear PWFA theory, the plasma electrons move to cancel (on average) the bunch charge:

$n_{e1}(r,\xi) \sim n_b(r, \xi)$

That means that the displaced plasma e^- charge within the bunch is on the order of the bunch charge and has an r-dependency

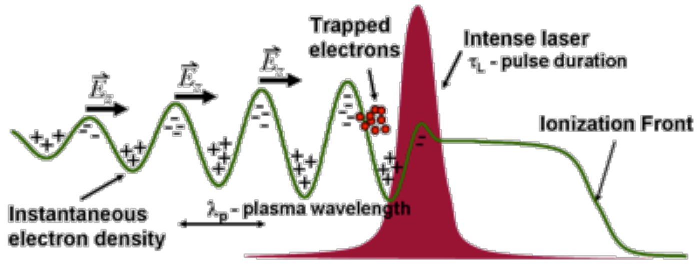
Solution:

Make the bunch density equal to, or larger than the plasma electron density: $n_b \geq n_{e0}$

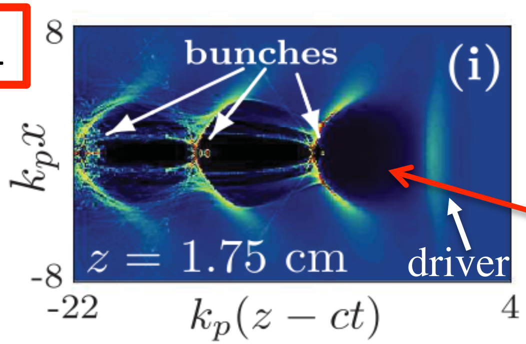
Because of plasma e^- inertia (ω_{pe}): $n_b >> n_{e0}$, blow-out within the D-bunch!

BLOWOUT REGiME

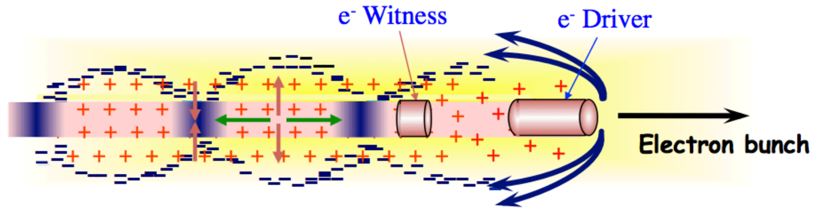
Laser WakeField Accelerator

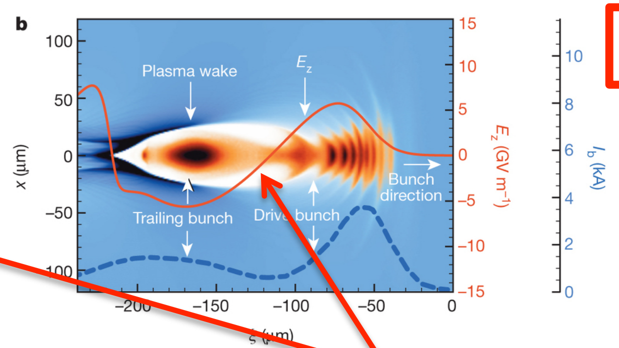


$a_0 >> 1$



Plasma WakeField Accelerator





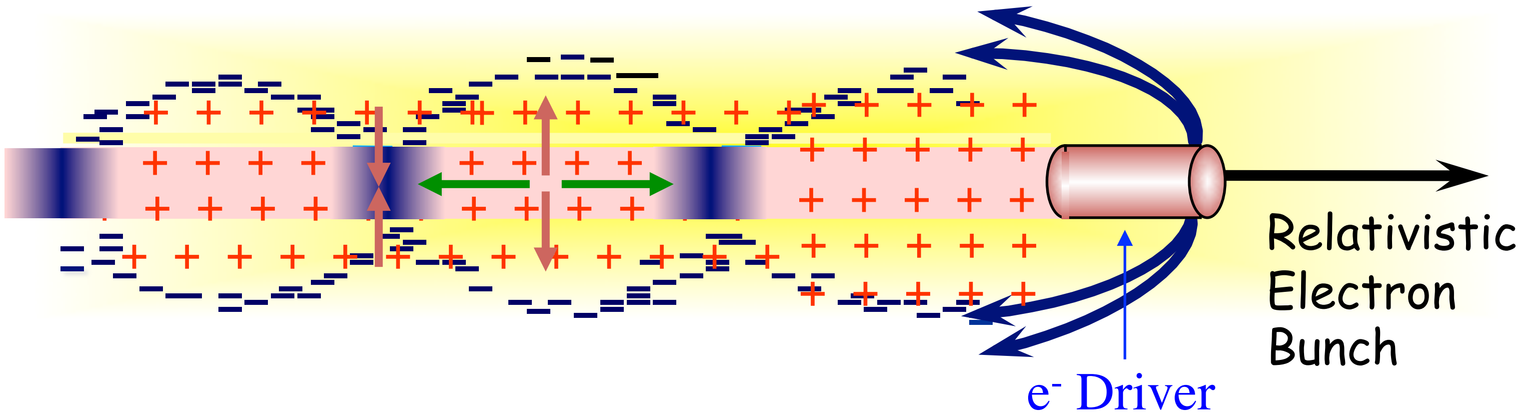
$n_b/n_{e0} >> 1$

No more plasma electrons
Pure ion column

- Regime most people are most familiar with …
- Most useful regime for e⁻ bunches acceleration …
- $E_z \sim E_{WB}$ ("linear concept")

LWFA: Pukhov, Appl. Phys. B., Lasers Opt. 74, 355 (2002)
PWFA: Rosenzweig, Phys. Rev. A 44, R6189 (1991)
Lu, PRL 96, 165002 (2006)

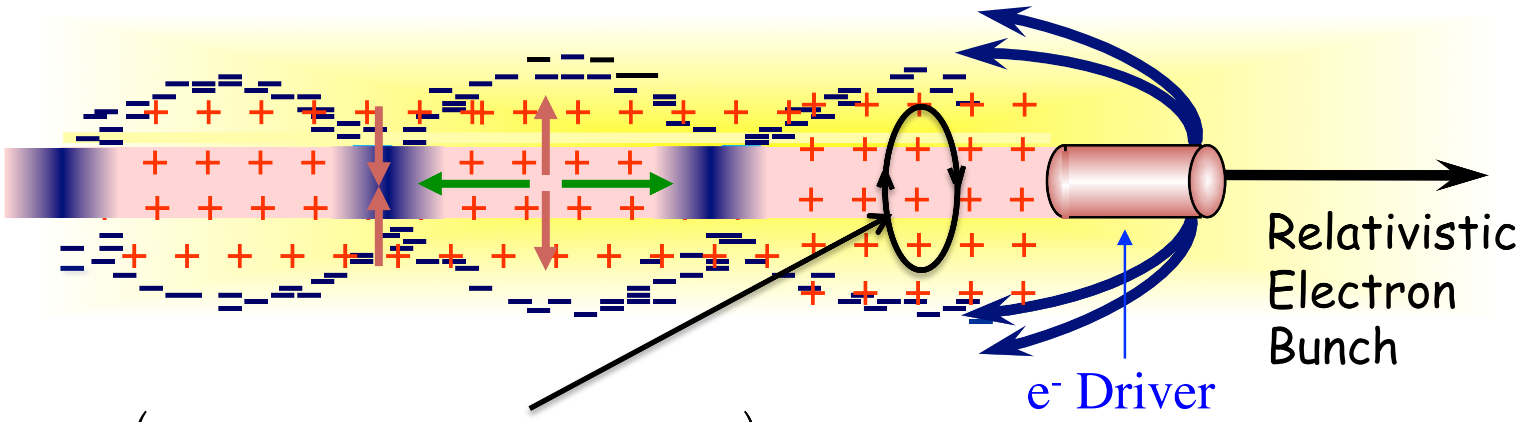
BLOWOUT/BUBBLE REGiME



Characteristics:

- $n_b >> n_{e0}$
- Head of the drive bunch (pulse) drives the blow-out => propagates in plasma e^-
- Pure (uniform) ion column reached along, behind the D-bunch
- Back of the D-bunch and the W-bunch propagate in the pure ion column
- Transverse field of the ion column varies linearly with r: preserves emittance of the W-bunch
- Transverse field is constant within the bubble: W_r independent of ξ
- Accelerating field is independent of r (narrow slice energy spread):
  - Panofsky-Wenzel theorem: $$\frac{\partial W_z}{\partial r} = \frac{\partial W_r}{\partial \xi} = 0 \Rightarrow W_z(r) = cst$$
- Beam loading and matching possible

PURE (uniform) ION COLUMN FOCUSING



$$F_{\perp tot} = q\left(E_{rb} + v_b \times B_{\theta b} + E_{rp} + v_b \times B_{\theta p}\right)$$

Cancel to $1/\gamma^2$

=0, no current inside the bunch

Gauss law for infinite cylinder (approximation)

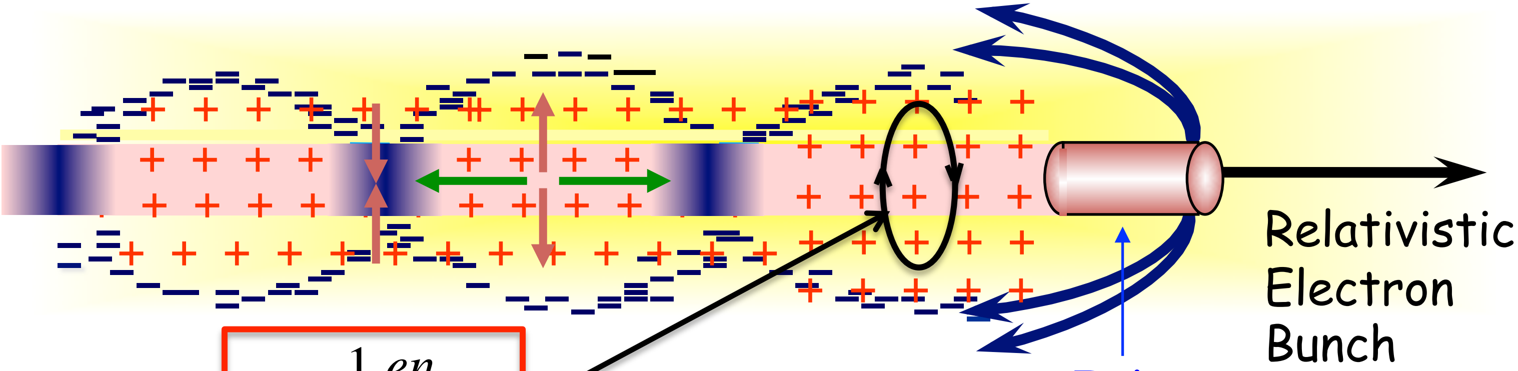
n_i is uniform and $n_i=n_{e0}$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \Rightarrow 2\pi r dz E_r = \frac{\pi r^2 e n_i}{\varepsilon_0} \Rightarrow E_r = \frac{1}{2}\frac{e n_{e0}}{\varepsilon_0} r$$

The focusing field varies linearly with radius => focusing free of geometric aberrations

=> can preserve incoming emittance ($\Delta\gamma$=0)

PURE (uniform) ION COLUMN FOCUSING



$$E_r = \frac{1}{2}\frac{en_{e0}}{\varepsilon_0}r$$

n_i is uniform and $n_{i0}=n_{e0}$

Motion of a particle in the ion column:

$$\gamma m_e \frac{dv_\perp}{dt} = F_\perp \quad \Rightarrow \quad \gamma m_e c^2 \frac{d^2 r}{dz^2} = e\frac{1}{2}\frac{en_{e0}}{\varepsilon_0}r \quad \Rightarrow \quad \frac{d^2 r}{dz^2} = \frac{1}{2\gamma c^2}\frac{e^2 n_{e0}}{m_e \varepsilon_0}r = \frac{\omega_{pe}^2}{2\gamma c^2}r = \frac{k_{pe}^2}{2\gamma}r = k_\beta^2 r$$

Harmonic motion (no energy gain/loss)

Dependent on γ!

$$\frac{d^2 r}{dz^2} = k_\beta^2 r \quad \Rightarrow \quad r(z) = r_0 e^{ik_\beta z}$$

=> emission of betatron radiation (synchrotron)

Examples: SLAC E_{kin}=28.56GeV => γ~56'000

n_e~$2\text{x}10^{14}\text{cm}^{-3}$ => KeV photons Wang, PRL 88, 135004 (2002)

n_e~$2\text{x}10^{17}\text{cm}^{-3}$ => MeV photons Johnson, PRL 97, 175003 (2006)

PURE (uniform) ION COLUMN FOCUSING

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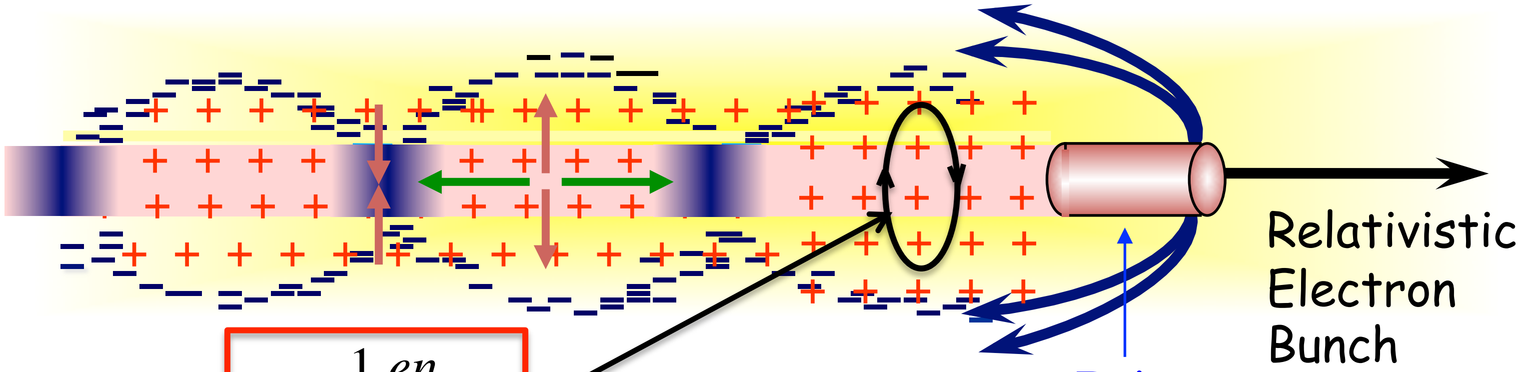


$$E_r = \frac{1}{2}\frac{en_{e0}}{\varepsilon_0} r$$

n_i is uniform and $n_{i0}=n_{e0}$

$$\frac{d^2r}{dz^2} = k_\beta^2 r \quad \Rightarrow \quad r(z) = r_0 e^{ik_\beta z}$$

=> Particles oscillate about the beam axis:

$$\begin{cases} k_\beta = k_{pe}/\sqrt{2\gamma} << k_{pe} \\ \omega_\beta = k_\beta c = \omega_{pe}/\sqrt{2\gamma} << \omega_{pe} \end{cases}$$

=> $\lambda_\beta = \sqrt{2\gamma}\lambda_{pe} >> \lambda_{pe} \Rightarrow$ the beam transverse size (envelope) oscillates over a length/ time scale much longer than the plasma e^-

$\Rightarrow$ Quasi-static approximation used in computer codes …

$$\frac{d^2r}{dz^2} = k_\beta^2 r$$

Equation for a single particle, derive envelope equation for an ensemble of particles, i.e., for $\sigma_r = \langle r^2 \rangle$

Envelope Equation Derivation

MAX-PLANCK-GESELL

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

The equation for the evolution in space (along z) of the envelope size σ of a particle beam is derived from the equation of motion for a fluid of density n with a transverse temperature T_e. The fluid equation reads, neglecting the convective term $v\nabla v$:

Add temperature in fluid eq.:

$$n\gamma m\frac{\partial v}{\partial t} = nF_{foc} - \nabla P \quad (1)$$

where $v=v(r)$ is the transverse oscillatory velocity, F_{foc} is a focusing force and P is the (transverse) pressure resulting from the (transverse) fluid temperature. The fluid relativistic in its longitudinal motion, but not in its transverse motion. This corresponds to assuming $v_x << v_z \approx c$. This also means that to first order the energy of the particles is constant along the motion, hence γ is outside of the derivative, and the mass can be replaced with γm. Assuming that the focusing force is independent of time, the time derivative can be replaced by a derivative along z ($\partial/\partial t = c\partial/\partial z$):

$$n\gamma mc\frac{\partial v}{\partial z} = nF_{foc} - \nabla P \quad (2)$$

The fluid is assumed to follow an ideal gas law with a single, uniform temperature:

Equation of state (as if T_e):

$$P = nk_BT_e \quad (3)$$

The fore the pressure gradient term becomes:

$$-\nabla P = -k_BT_e\nabla n \quad (4)$$

Considering an electron beam with a Gaussian transverse profile or density, the density gradient is:

Gaussian r-distribution:

$$n(r) = n_0 e^{-r^2/2\sigma^2} \quad \rightarrow \quad -\nabla n = \frac{r}{\sigma^2}n \quad (5)$$

Therefore Eq. 2 becomes:

$$\gamma mc\frac{\partial v}{\partial z} = F_{foc} + k_BT_e\frac{r}{\sigma^2} \quad (6)$$

Assuming that the focusing force is linear with radius, it can be written as:

Transverse force linear with r: $$F_{foc} = -kr \quad (7)$$

where k is a constant. The transverse velocity is also the derivative of the transverse position: $v=\partial r/\partial t=c\partial r/\partial z$, and Eq. 6 becomes:

$$\frac{\partial^2 r}{\partial z^2} = -\frac{k}{\gamma mc^2} r + \frac{k_B T_e}{\gamma mc^2}\frac{r}{\sigma^2} \quad (8)$$

The transverse temperature and the transverse velocity are related through the velocity distribution. For a Maxwelian distribution with a temperature T_e this relation is:

Thermal velocity: $$\frac{1}{2}\gamma m\sigma_v^2 = \frac{1}{2}k_B T_e \quad (9)$$

where σ_v^2 is the mean of the square of the thermal velocity.
The emittance of the beam can be defined as:

$$\varepsilon = \sigma\theta = \sigma\frac{v_r}{v_z} \cong \sigma\frac{\sigma_v}{c} \quad (10)$$

or:

Emittance-T_e: $$\varepsilon^2 = \sigma^2\frac{\sigma_v^2}{c^2} = \sigma^2\frac{k_B T_e}{\gamma mc^2} \quad (11)$$

Therefore, Eq. 8 becomes:

$$\frac{\partial^2 r}{\partial z^2} = -\frac{k}{\gamma mc^2} r + \frac{\varepsilon^2}{\sigma^2}\frac{r}{\sigma^2} = -\frac{k}{\gamma mc^2} r + \frac{\varepsilon^2}{\sigma^4} r \qquad (12)$$

Rewriting the linear focusing force coefficient as:

$$K = \frac{1}{\gamma mc^2}\frac{F_r}{r} \qquad (13)$$

and assuming that for a Gaussian beam the beam evolution is given by the evolution of the beam envelope with radius with $r=\sigma$, one rewrites:

Beam envelope equation:

$$\frac{\partial^2 \sigma}{\partial z^2} + K\sigma = \frac{\varepsilon^2}{\sigma^3} \qquad (14)$$

Energy gain not included

This last equation is the envelope equation.

Note that the handling of the emittance was a little bit cavalier and needs to be re-examined!

A solution to the envelope equation is the propagation in vacuum:

Solution in vacuum:

$$\sigma = \sigma_0\left(1 + \frac{\varepsilon^2 z^2}{\sigma_0^2}\right)^{1/2} \qquad (15)$$

Similar equations!

In this case, the emittance is defined from the beam size at the waste σ_o. and the envelope angle θ at large distance z from the waste:

$$\frac{\varepsilon^2 z^2}{\sigma_0^2} >> 1 \quad \sigma \cong \frac{\varepsilon z}{\sigma_0} \qquad (16)$$

So that the angle θ is approximately equal to

$$\theta = \frac{\sigma}{z} \cong \frac{\varepsilon}{\sigma_0} \qquad (17)$$

And therefore:

$$\varepsilon \cong \sigma_0 \theta \qquad (18)$$

However this is not true at any z along the beam trajectory.

Single particle equation:

$$\frac{d^2 r}{dz^2} - k_\beta^2 r = 0 \quad or \quad \frac{d^2 r}{dz^2} - \frac{1}{\gamma mc^2}\left(\frac{F_r}{r}\right) r = 0$$

ENVELOPE EQUATION

Meaning of the envelope equation: $\sigma_r'' + K^2\sigma_r = \frac{\varepsilon_g^2}{\sigma_r^3} \Leftrightarrow \sigma_r'' = \left(\frac{\varepsilon_g^2}{\sigma_r^4} - K^2\right)\sigma$

Fix: n_{e0}, ε_g, γ:

When σ_r is large … $K^2\sigma_r >> \frac{\varepsilon_g^2}{\sigma_r^3} \Rightarrow \sigma_r'' \cong -K^2\sigma_r$ the beam focuses

When σ_r is small … $K^2\sigma_r << \frac{\varepsilon_g^2}{\sigma_r^3} \Rightarrow \sigma_r'' \cong \frac{\varepsilon_g^2}{\sigma_r^3}$ the beam expands

In between … $\sigma_r'' = \frac{\varepsilon_g^2}{\sigma_r^3} - K^2\sigma_r = 0$ matched …

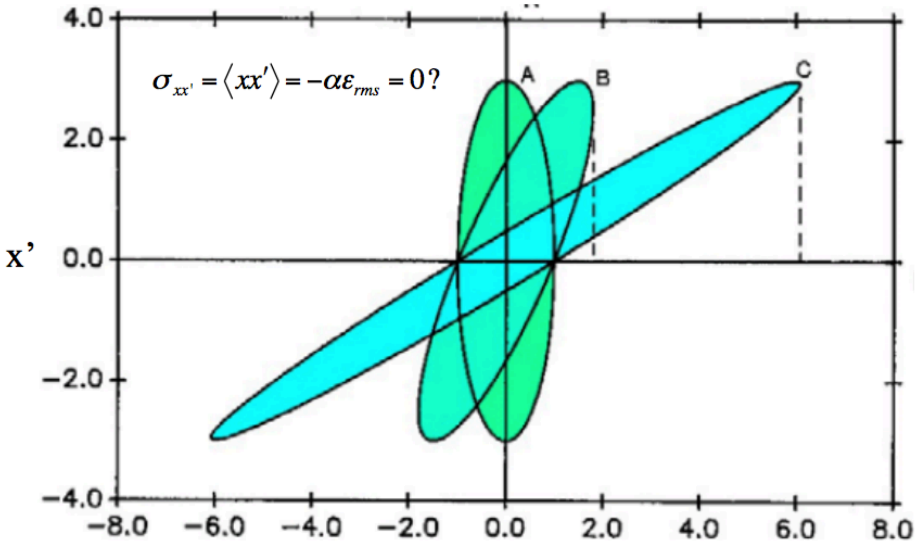
2nd order differential equation, two initial conditions

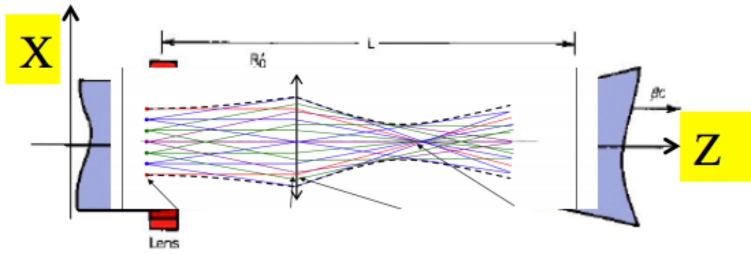
=> The beam radius does not change when $K^2(z) = cst \quad and \quad \sigma(z=0) = \sigma_0, \sigma'(z=0) = 0$

with $K^2 = k_\beta^2 = \frac{\omega_\beta^2}{c^2} = \frac{\omega_{pe}^2}{2\gamma c^2} \Rightarrow \frac{\sigma_0^4 n_{e0}}{\gamma\varepsilon_g^2} = \frac{2\varepsilon_0 m_e c^2}{e^2}$ matching condition

Note: there is a more general envelope equation that includes acceleration, …

ENVELOPE EQUATION





From Massimo's slides

In between …

$$\sigma_r'' = \frac{\varepsilon_g^2}{\sigma_r^3} - K^2\sigma_r = 0$$

matched …

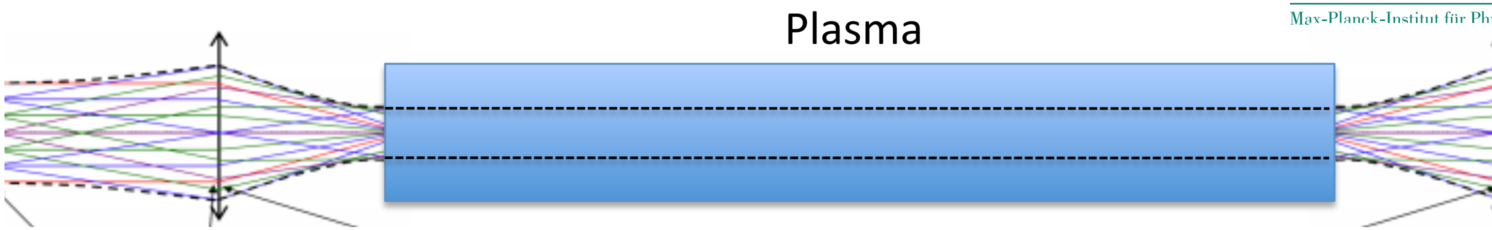
2nd order differential equation, two initial conditions

=> The beam radius does not change if $K = cst$ *and* $\sigma(z=0) = \sigma_0, \sigma'(z=0) = 0$

with $K^2 = k_\beta^2 = \omega_\beta^2 c^2 = \dfrac{\omega_{pe}^2 c^2}{2\gamma} \Rightarrow \dfrac{\sigma_0^4 n_{e0}}{\gamma\varepsilon_g^2} = \dfrac{2\varepsilon_0 m_e c^2}{e^2}$ matching condition

Note: there is a more general envelope equation that includes acceleration, …

ENVELOPE EQUATION



Mismatched

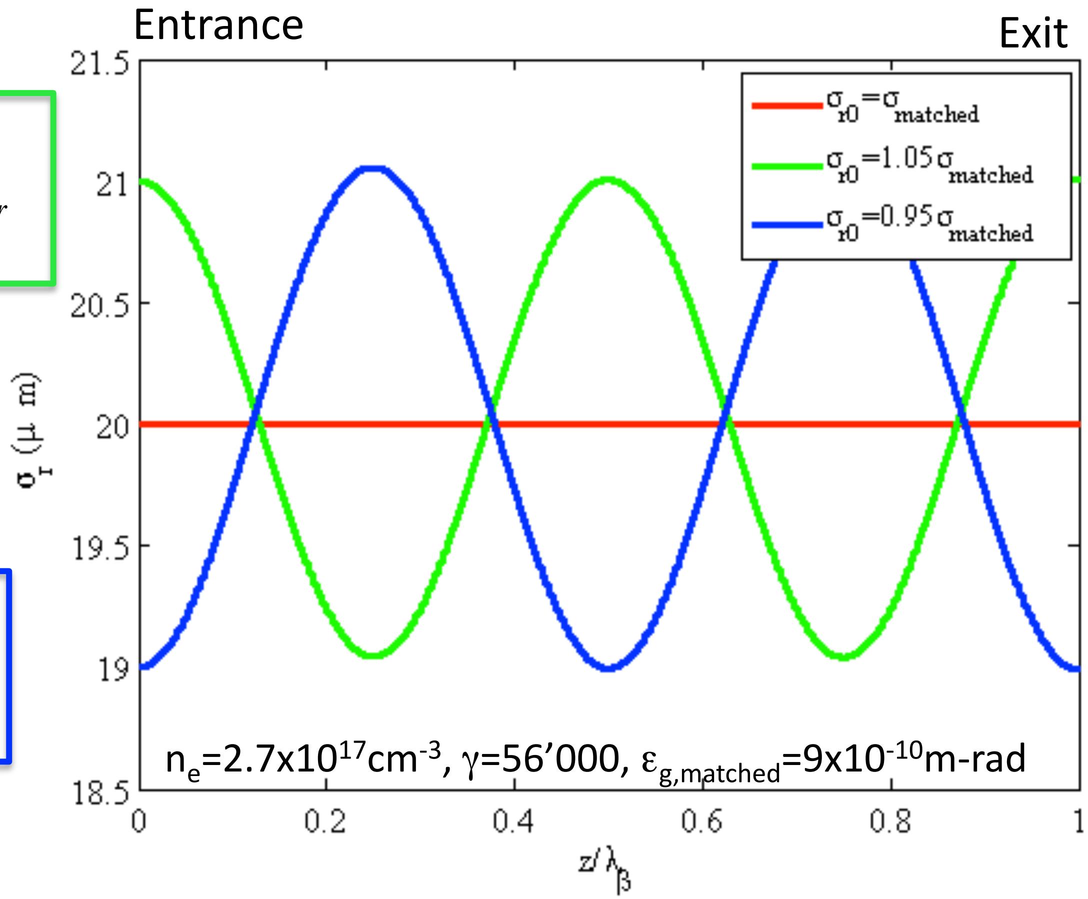
$$K^2 >> \frac{\varepsilon_g^2}{\sigma_r^4} \quad \Rightarrow \quad \sigma_r'' \cong -K^2\sigma_r$$

Matched

$$\sigma_r'' = \left(\frac{\varepsilon_g^2}{\sigma_r^4} - K^2\right)\sigma_r = 0$$

Mismatched

$$K^2 << \frac{\varepsilon_g^2}{\sigma_r^4} \quad \Rightarrow \quad \sigma_r'' \cong \frac{\varepsilon_g^2}{\sigma_r^3}$$



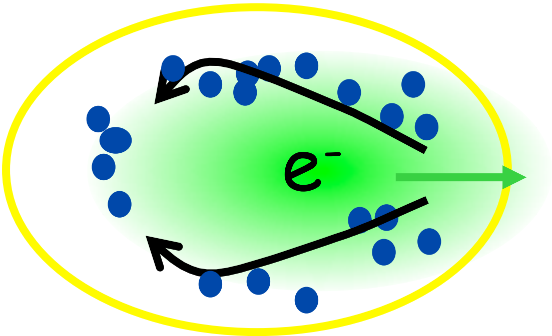
✧When matched: the beam envelope size remains cst: $\sigma_r(z=0)=\sigma_{r0}$ …

✧…because n_{e0}=cst and $K_\beta(n_{e0})$=cst and $d\sigma_r/dz=0$ at z=0

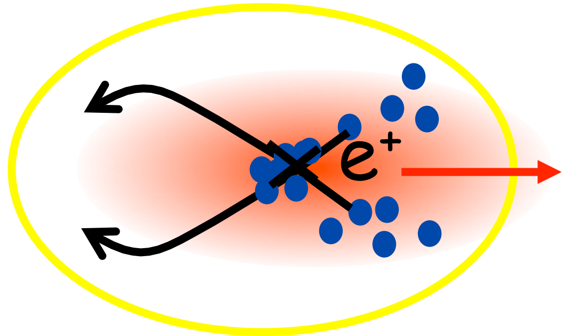
✧Note: e^- still oscillate … in all cases

e^- & e^+ BEAM NEUTRALIZATION

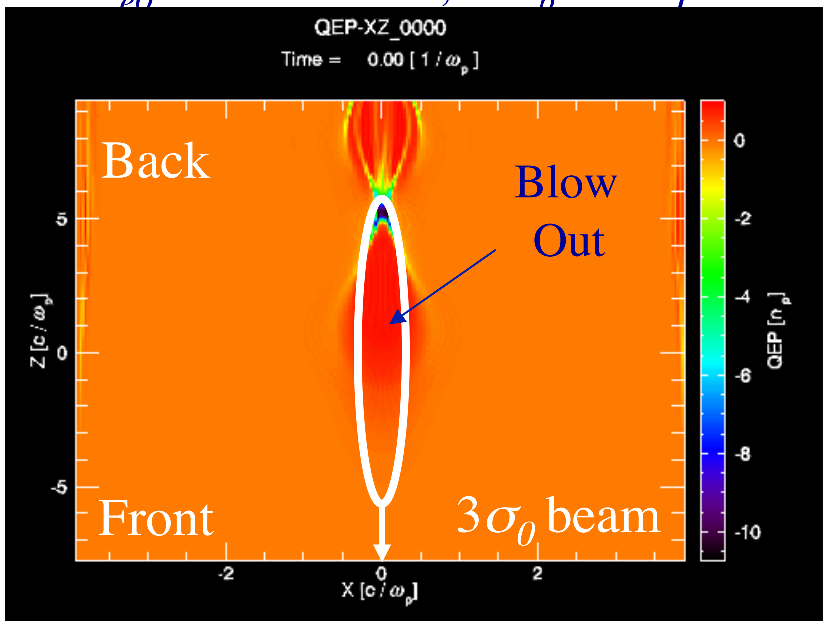
3-D QuickPIC simulations, plasma e^- density:



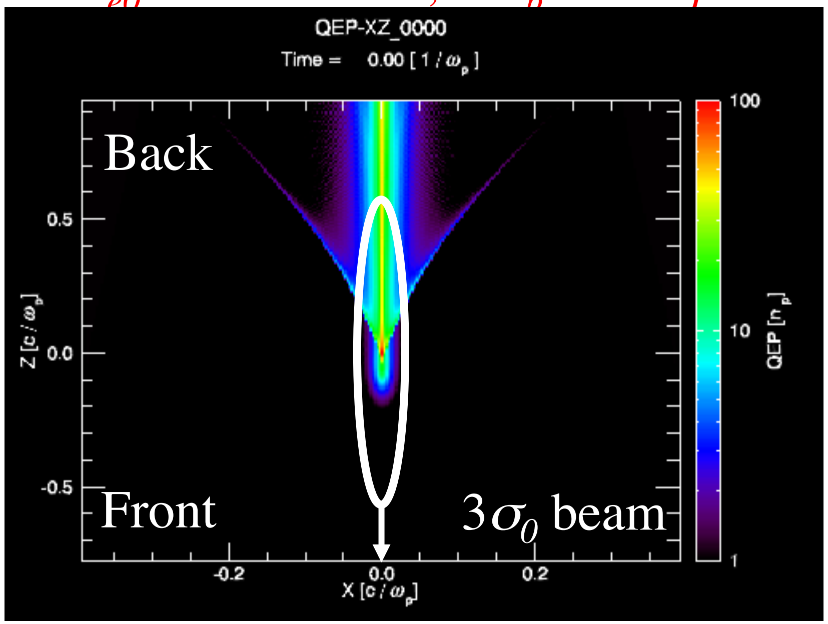
σ_r=35 μm
σ_r=700 μm
N=1.8×10^{10}
d=2 mm



e^-: n_{e0}=2×10^{14} cm^{-3}, c/ω_p=375 μm

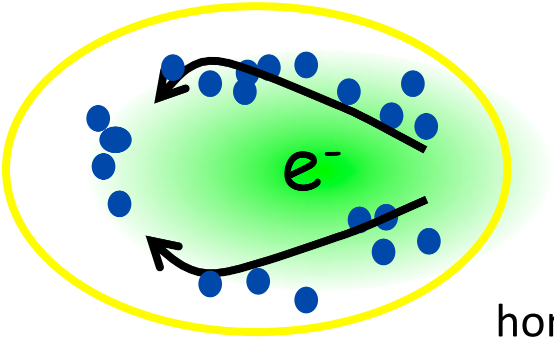


e^+: n_{e0}=2×10^{12} cm^{-3}, c/ω_p=3750 μm



- Uniform focusing force (r,z)
- Non-uniform focusing force (r,z)

WAKEFIELD FIELDS for e^- & e^+



σ_r=35 μm

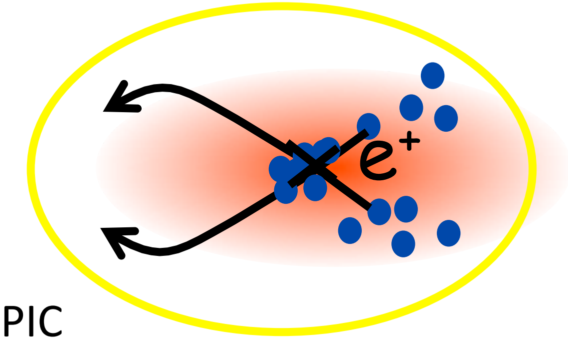
σ_r=700 μm

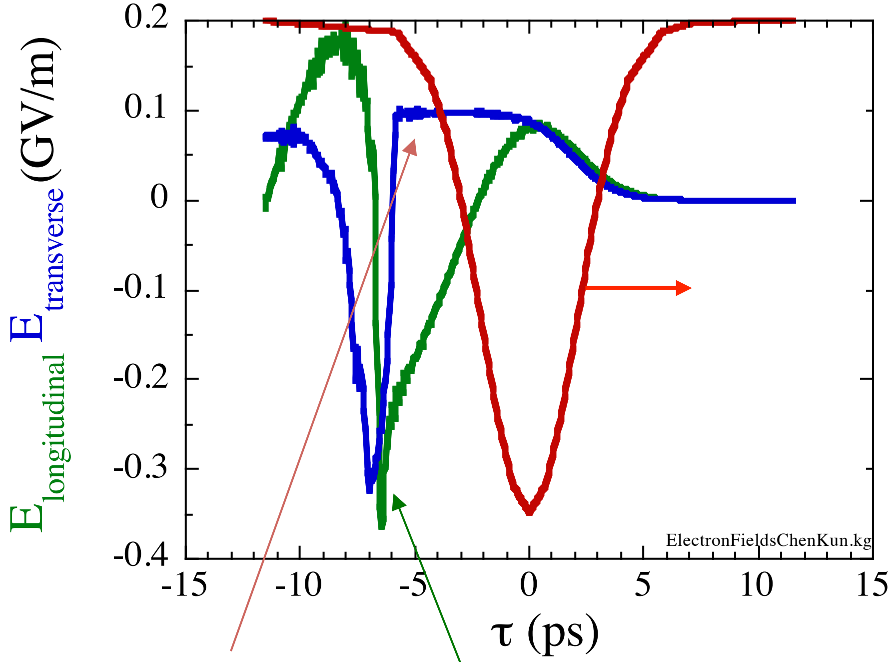
N=1.8×10^{10}

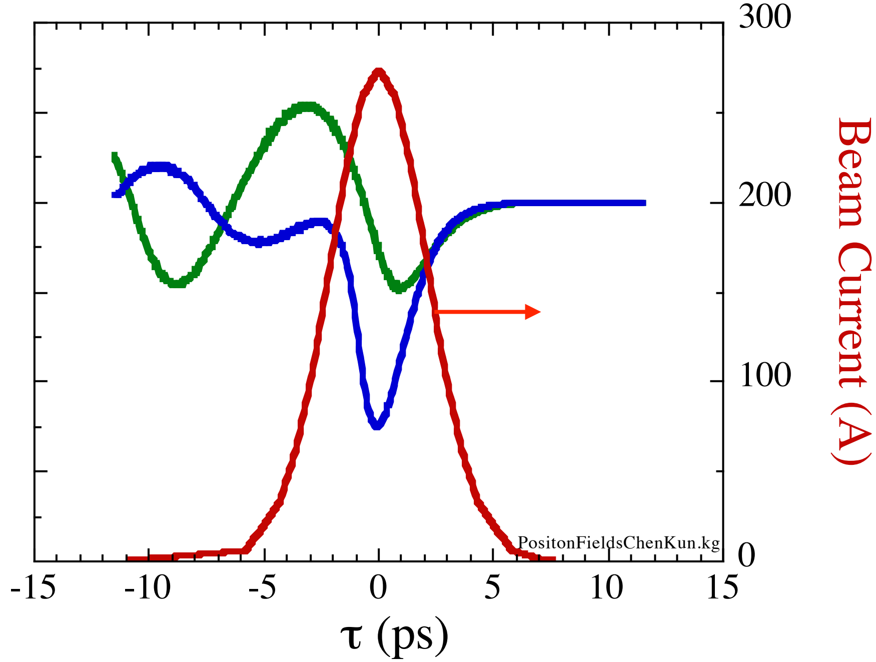
d=2 mm

n_e=1.5×10^{14} cm^{-3}

homogeneous, QUICKPIC



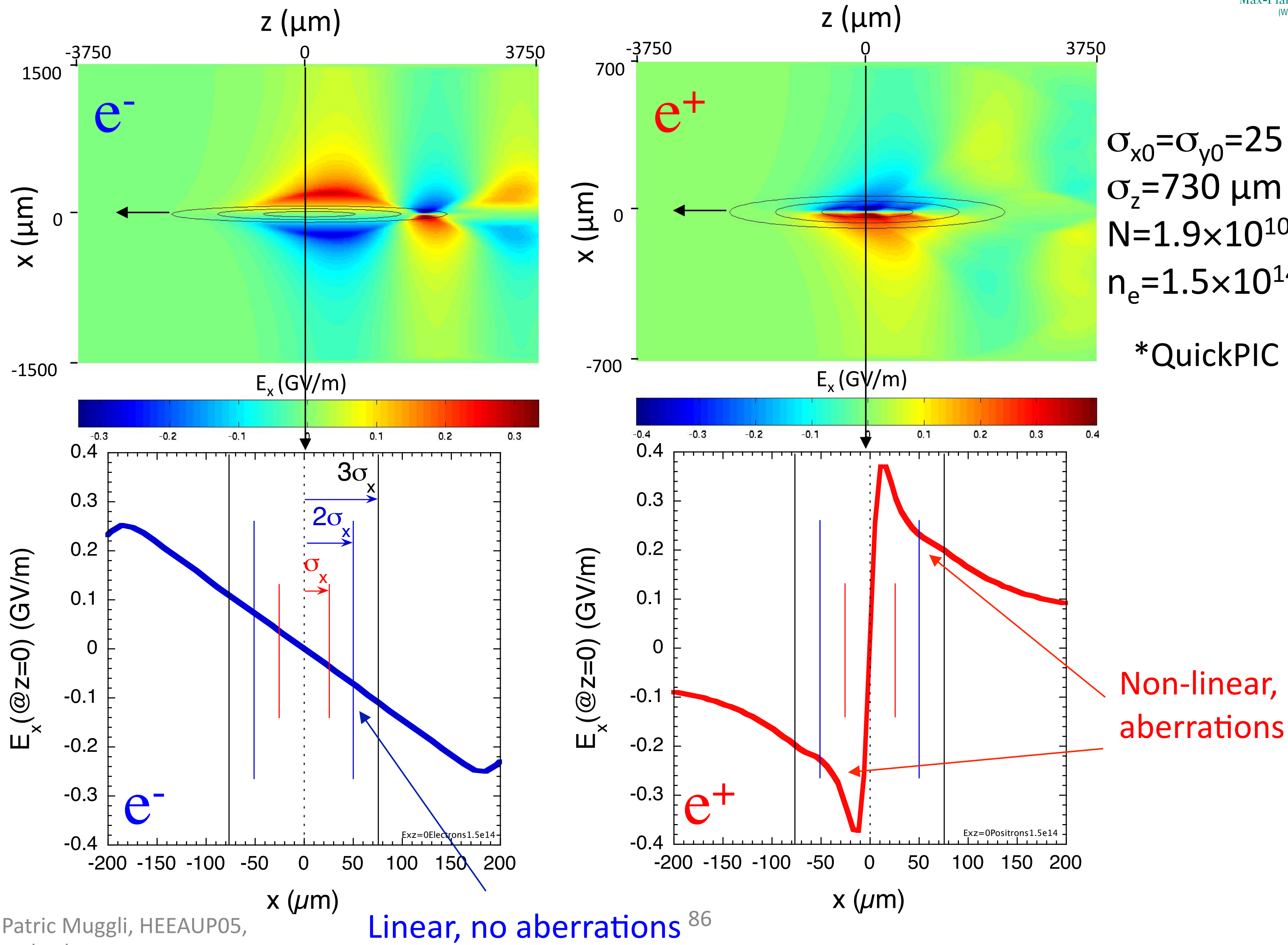




- Blow-Out
- Accelerating "Spike"

- Fields vary along r, stronger
- Less Acceleration, "linear-like"

e^- & e^+ FOCUSING FIELDS*



$\sigma_{x0}=\sigma_{y0}=25$ μm

$\sigma_z=730$ μm

$N=1.9\times10^{10}$ e^+/e^-

$n_e=1.5\times10^{14}$ cm^{-3}

*QuickPIC

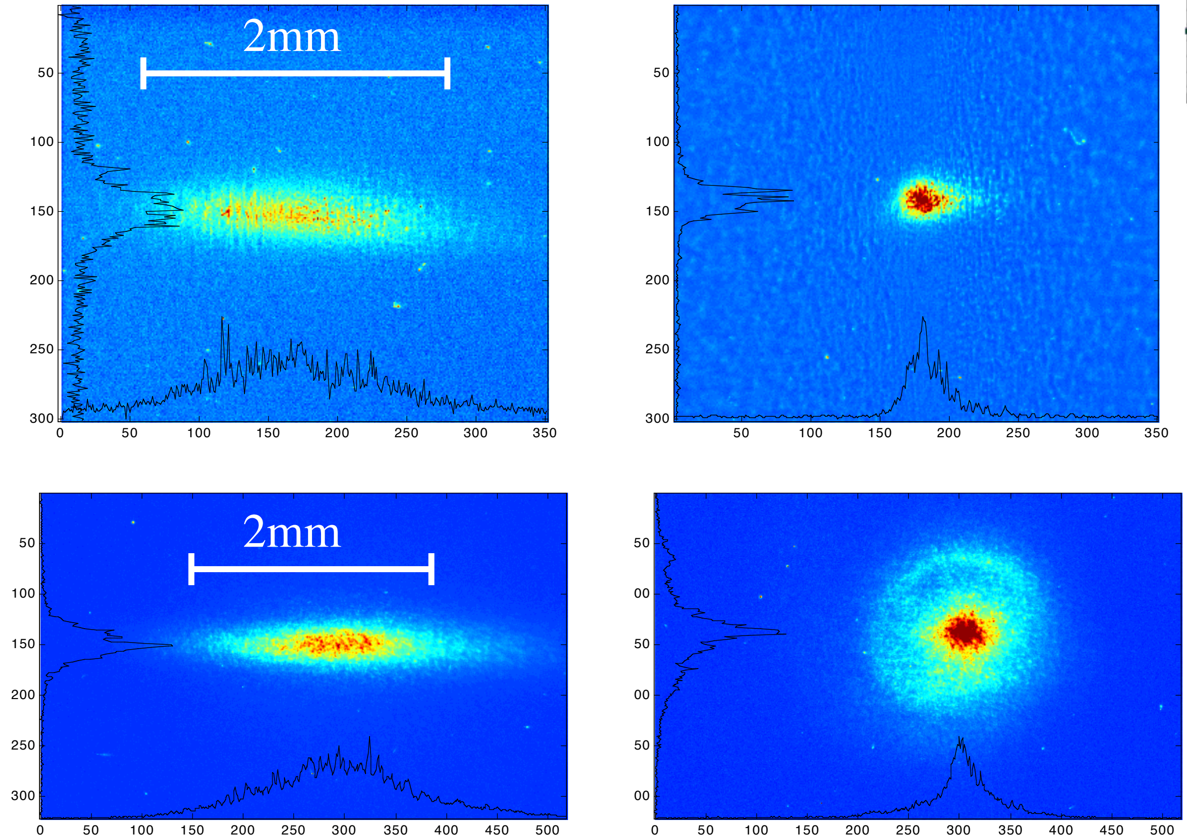
FOCUSING OF e^-/e^+

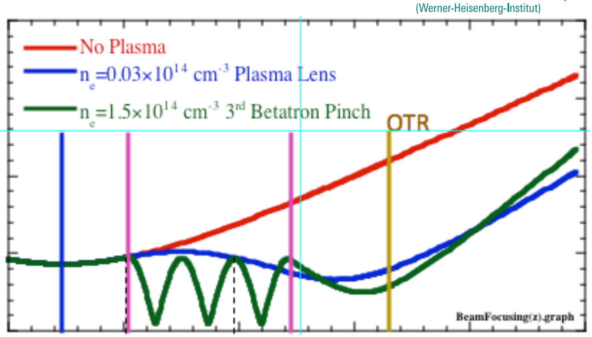
- OTR images ≈1m from plasma exit ($\varepsilon_x \neq \varepsilon_y$)

n_e=0 $\quad$ $n_e \approx 10^{14}$ cm^{-3}

e^-

e^+





- Ideal Plasma Lens in Blow-Out Regime

- Plasma Lens with Aberrations
- Halo formation

Muggli, PRL 101, 055001 (2008)

➡ Qualitative differences

➡ Input beams with unequal emittances ($\varepsilon_x > \varepsilon_y$), focused to a round size at plasma entrance

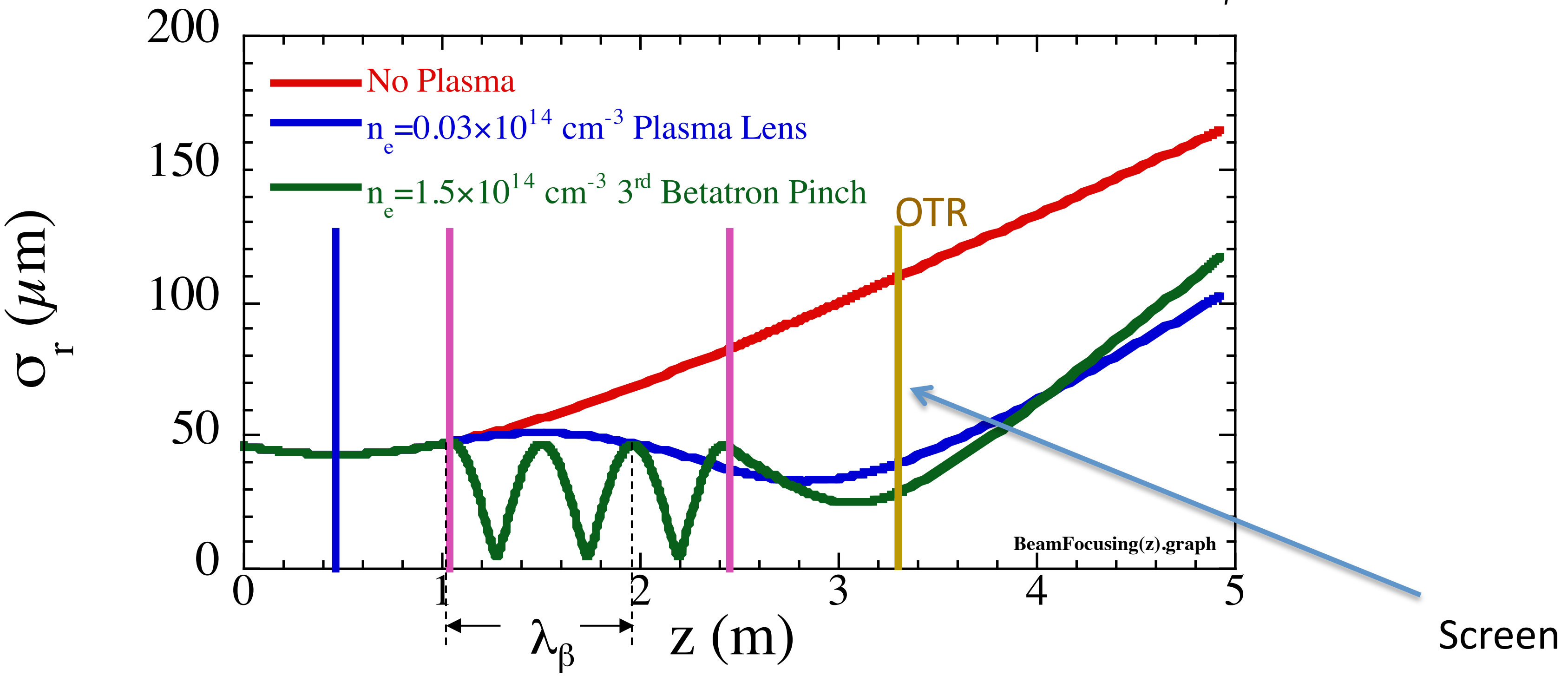
ENVELOPE EQUATION e⁻

Solution to the envelope equation (for the Gaussian beam transverse size σ_r):

$$\frac{d^2\sigma_r}{dz} + K^2\sigma_r = \frac{\varepsilon_g^2}{\sigma_r^3} \Leftrightarrow \sigma_r'' + K^2\sigma_r = \frac{\varepsilon_g^2}{\sigma_r^3} \Rightarrow \sigma_r(z) = \sigma_{r0}\left(1 + \frac{z^2}{\beta_0^2}\right)^{1/2} = \sigma_{r0}\left(1 + \frac{\varepsilon_g^2 z^2}{\sigma_0^4}\right)^{1/2}$$

$$\sigma_{r0} = \sigma_r(z=0), \quad \beta_0 = \frac{\sigma_{r0}^2}{\varepsilon_g}$$

Plasma Focusing Force > Beam "Emittance Force" ($\beta_{beam} > \beta_{plasma}$)

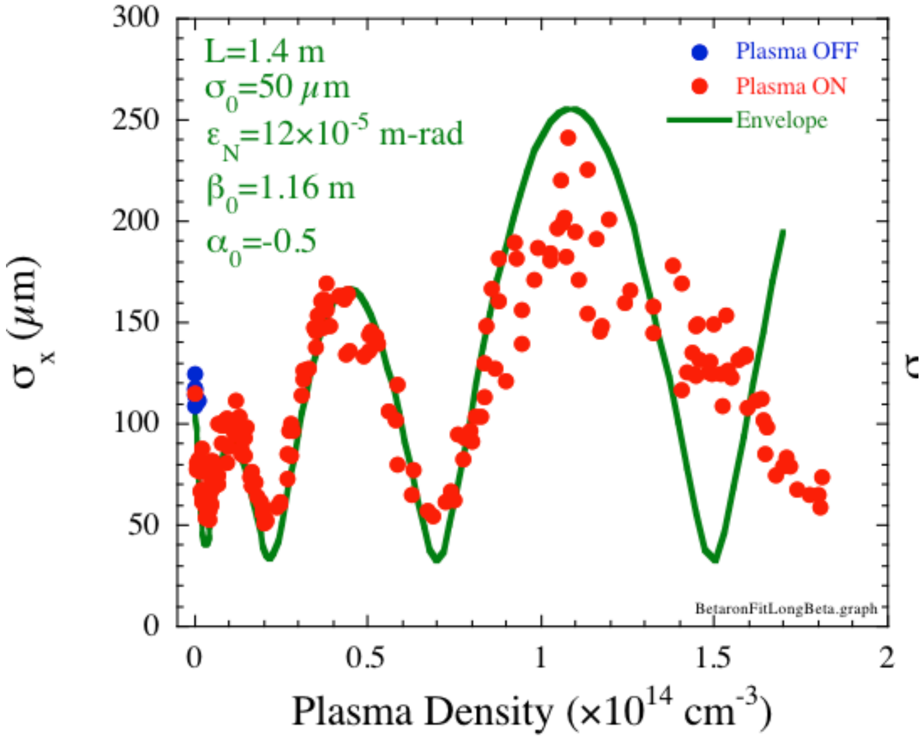


- σ at plasma exit ≤ σ at entrance
- Particles exit angle ∝ $\sigma_{x,y}/\lambda_\beta \propto \sigma_{x,y} n_e^{1/2}$ Remove!

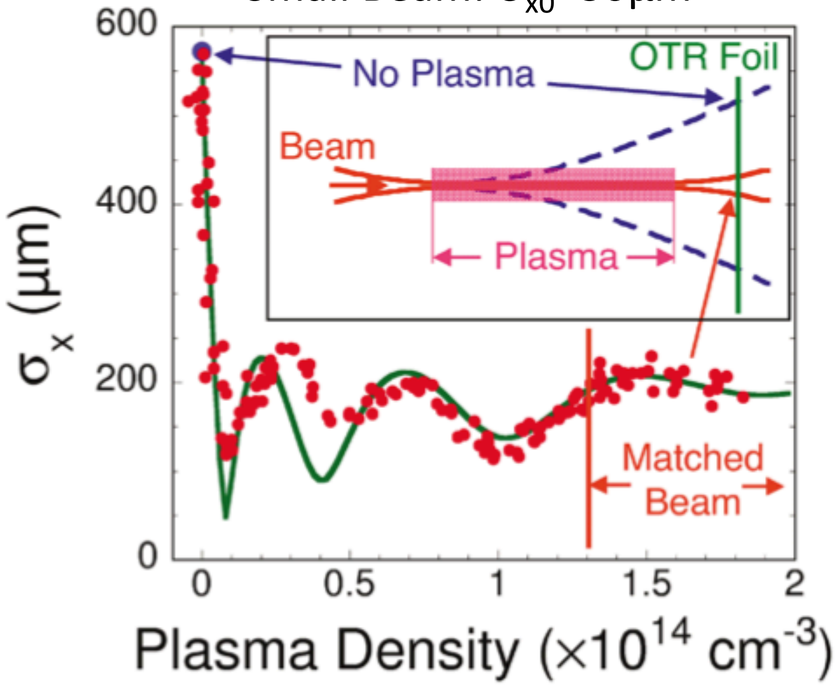
Simple Beam Envelope Model for Plasma Focusing

EMiTTANCE PRESERVATiON e-

Small Beam: σ_{x0}=50µm



Small Beam: σ_{x0}=30µm



$$K^2 >> \frac{\varepsilon_g^2}{\sigma_r^4} \Rightarrow \sigma_r'' \cong -K^2\sigma_r$$

For all n_{e0}
Focusing always stronger than ε_g

$$K^2 << \frac{\varepsilon_g^2}{\sigma_r^4} \Rightarrow \sigma_r'' \cong \frac{\varepsilon_g^2}{\sigma_r^3}$$

For all $n_{e0}<n_{e,matched}$~1.7×10^{14}cm^{-3}
ε_g stronger than focusing till …

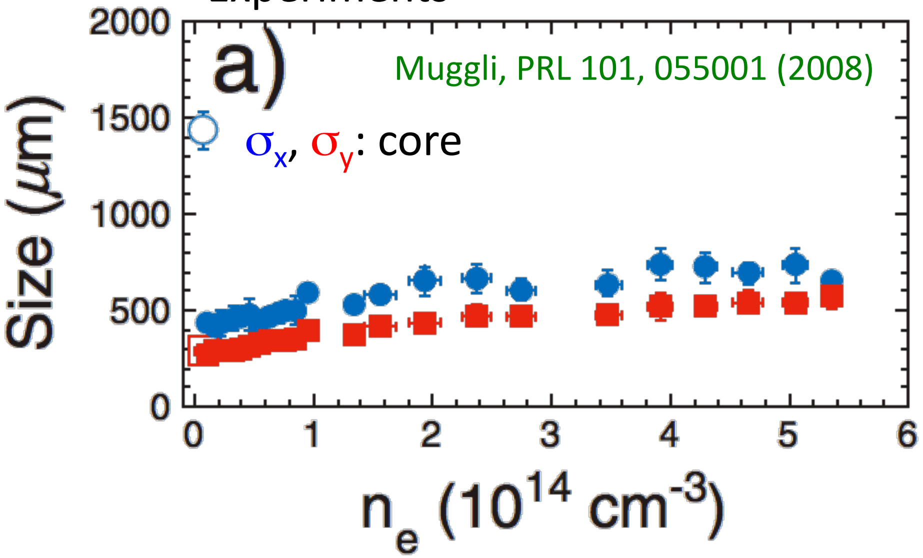
Clayton, PRL 88, 154801 (2002)

Muggli, PRL 93, 014802 (2004)

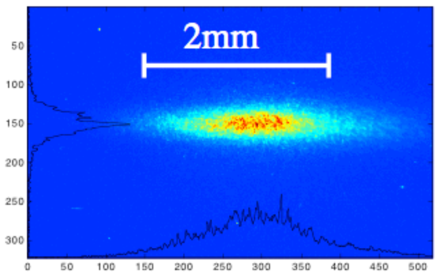
✧Fit of envelope equation consistent with preserved emittance

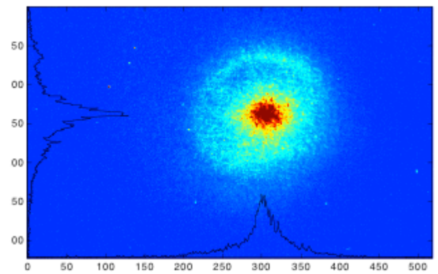
EMiTTANCE PRESERVATiON e^+

Experiments

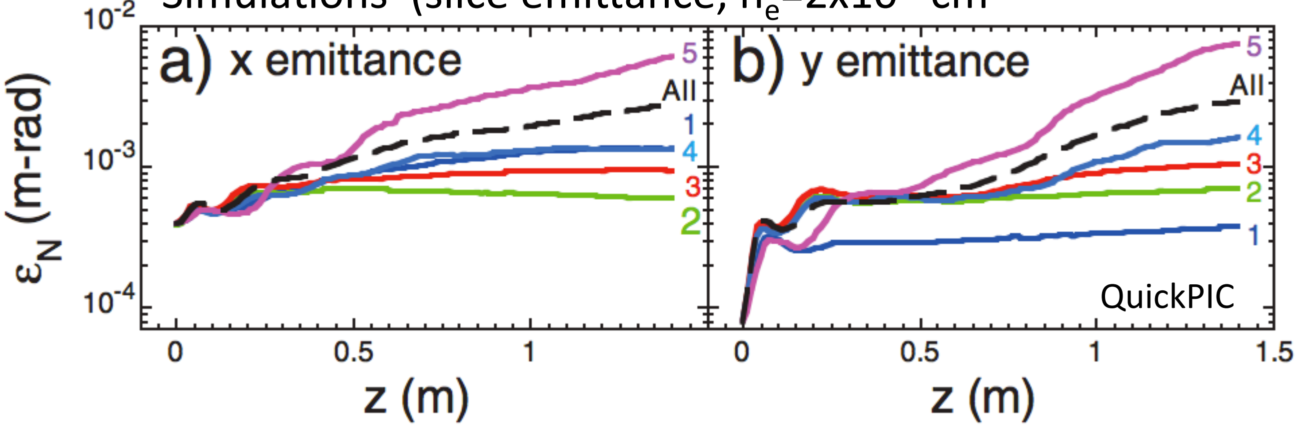


- ✧No envelope betatron oscillations
- ✧Beam "round" for n_{e0}>0
- ✧Halo => emittance growth





Simulations (slice emittance, n_e=2x10^{14}cm^{-3})

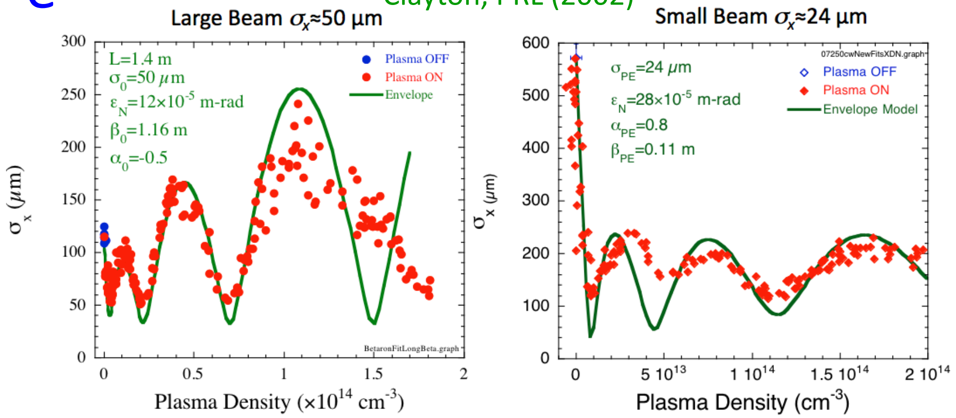


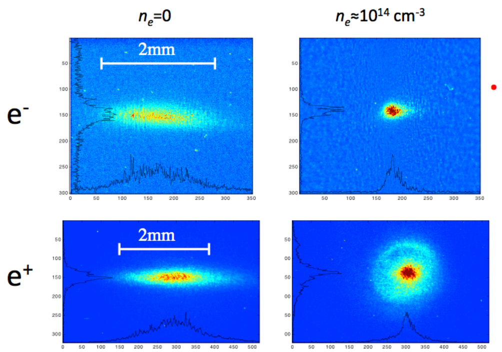
- ✧Emittances grow
- ✧Similar emittances, n_{e0}>0

✧Simulation results indicate emittance growth with e^+ beam, agreement with experimental results

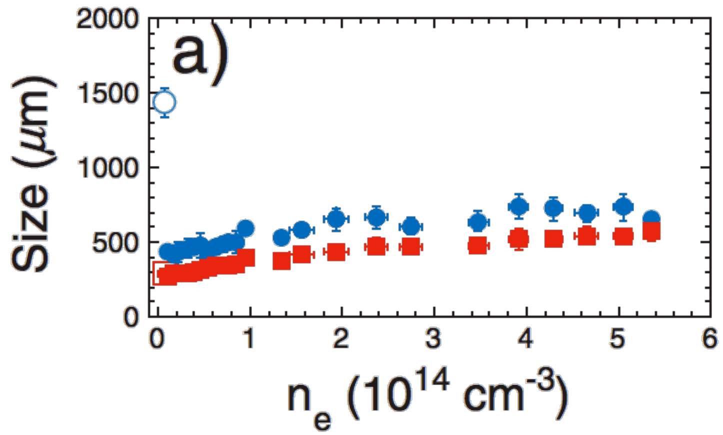
EMiTTANCE PRESERVATiON

e^- Clayton, PRL (2002)

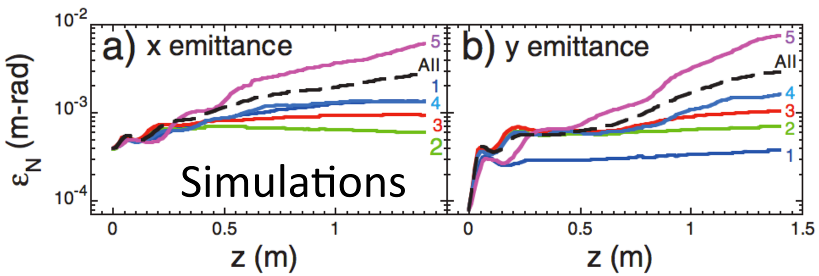




e^+



✧Emittance not preserved with e^+, W_r not ~r



Muggli, PRL 101, 055001 (2008)

✧Focused shape is "Gaussian", as expected (no halo) in blow-out regime, W_r~r

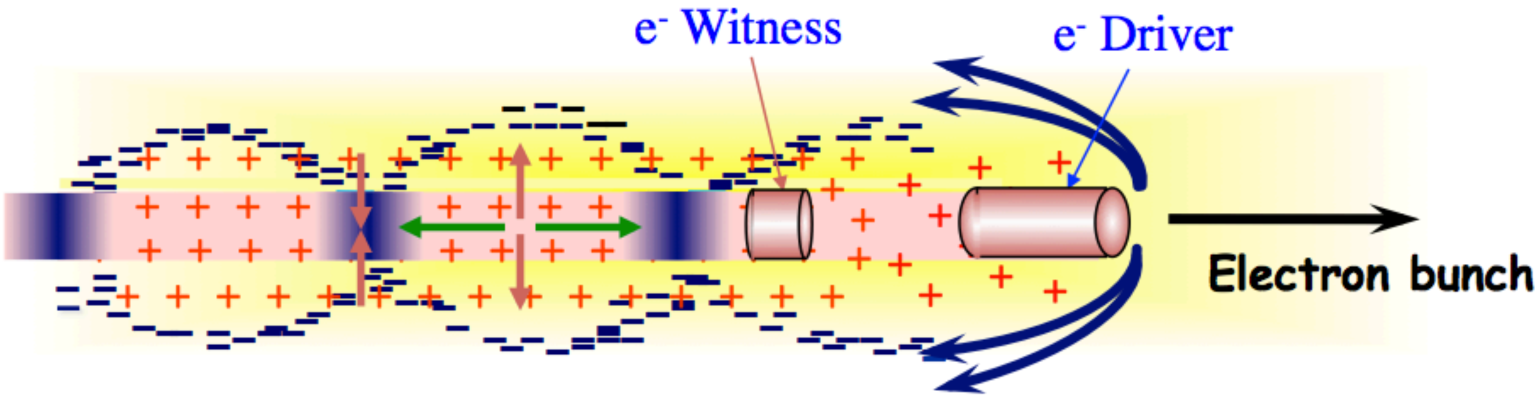
✧Fit of betatron oscillation focusing as a function of plasma density consistent with a single (input) emittance => emittance preservation

✧Also consistent with matching to the plasma pure ion column

✧Focused has a halo

✧Consistent with numerical simulations that show emittance growth

IDEAL PWFA (e^-)



Drive bunch with blow-out regime ($n_b >> n_{e0}$) for: → Demonstrated in focusing experiments Clayton, Phys. Rev. Lett. 88, 154801 (2002)

✧W_r~r: slice emittance preservation (γ-dependent, i.e, ξ)

✧W_z independent of r: narrow slice ΔE/E (γ-dependent, i.e, ξ)

Witness bunch with beam loading for: → Demonstrated in acceleration experiments Litos, Nature 515, 92 (2014).

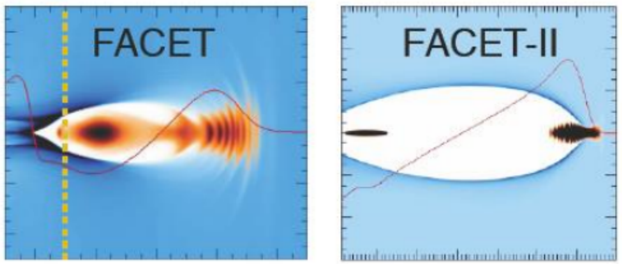
✧Most slices to experience the same gradient

✧High energy transfer efficiency (D->W)

Witness bunch matched to the plasma focusing force for → Demonstrated in acceleration and focusing experiments Muggli, Phys. Rev. Lett. 93, 014802 (2004)

✧Minimum betatron radiation

✧Weak dependency of exit divergence vs plasma density



✧ Narrow energy spread, low emittance accelerated W-bunch!

CONCLUSiONS

The PWFA is a very interesting and active research topic

Barely scratched the surface of the topic

There is a large body of results … (theory, simulations, experiments)

The number of PWFA experiments/facilities is growing …

PWFA applications are to free electron lasers (FELs) and particle physics (HEP)

PWFA works well to accelerate e^-, difficult for e^+ (see S. Corde's lecture)

PWFA applications to fixed target physics or e^-/proton physics is promising

LWFA-PWFA for FELs, brightness transformer is a new topic for FELs

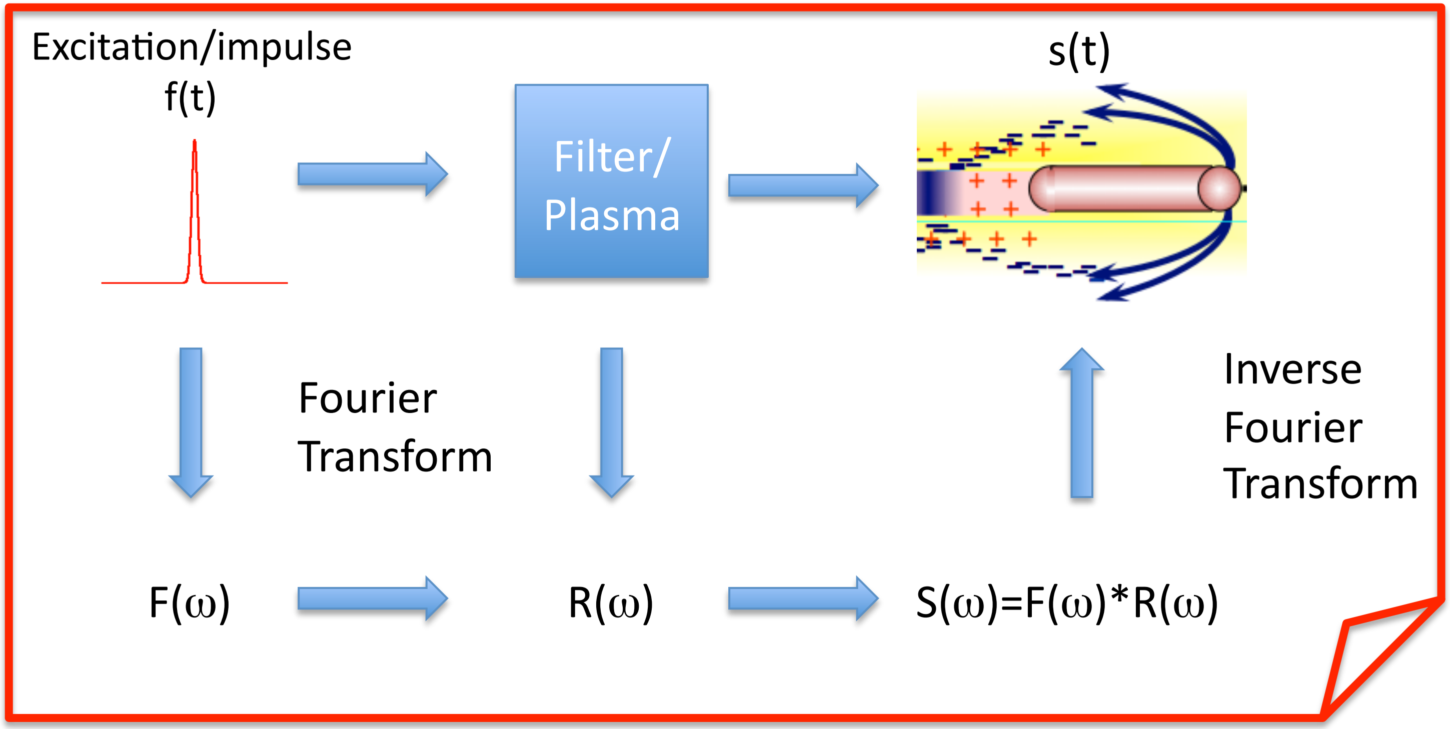
"From Acceleration to Accelerator"

I hope you have a better "feel" for:
- What the PWFA is … how it works …
- Correspondence between theory-simulations-experiments
- Effects that are are play in the PWFA

… and you have a few cues to start YOUR study of the PWFA

Thank you!

Consider the plasma as a harmonic oscillator with an eigen-frequency $f_{pe}=\omega_{pe}/2\pi$ (p^+: f_{pe}~237GHz, ω_{pe}~1.23mm)



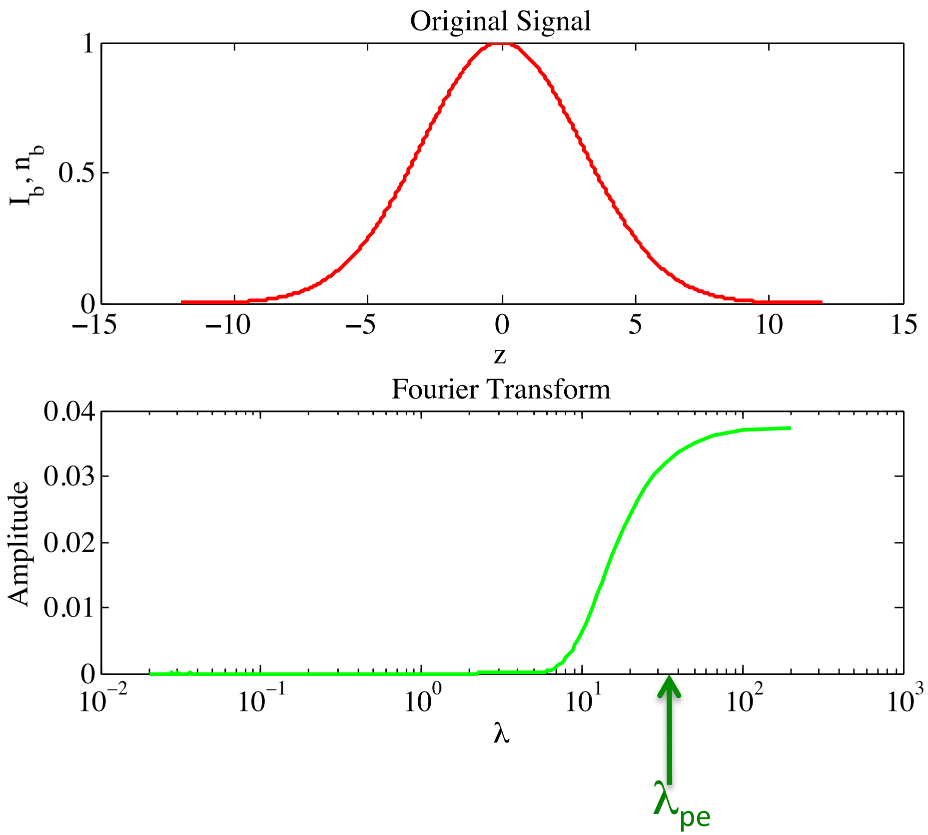
The oscillator is driven by a time signal given by the bunch current profile

Therefore, the effectiveness of the driving is given by the amount of energy at frequency f_{pe} (in a narrow bandwidth around) …

Remember linear equation for the plasma density:

BUNCH LENGTH

Single, short bunch … $k_{pe}\sigma_z$~√2~1

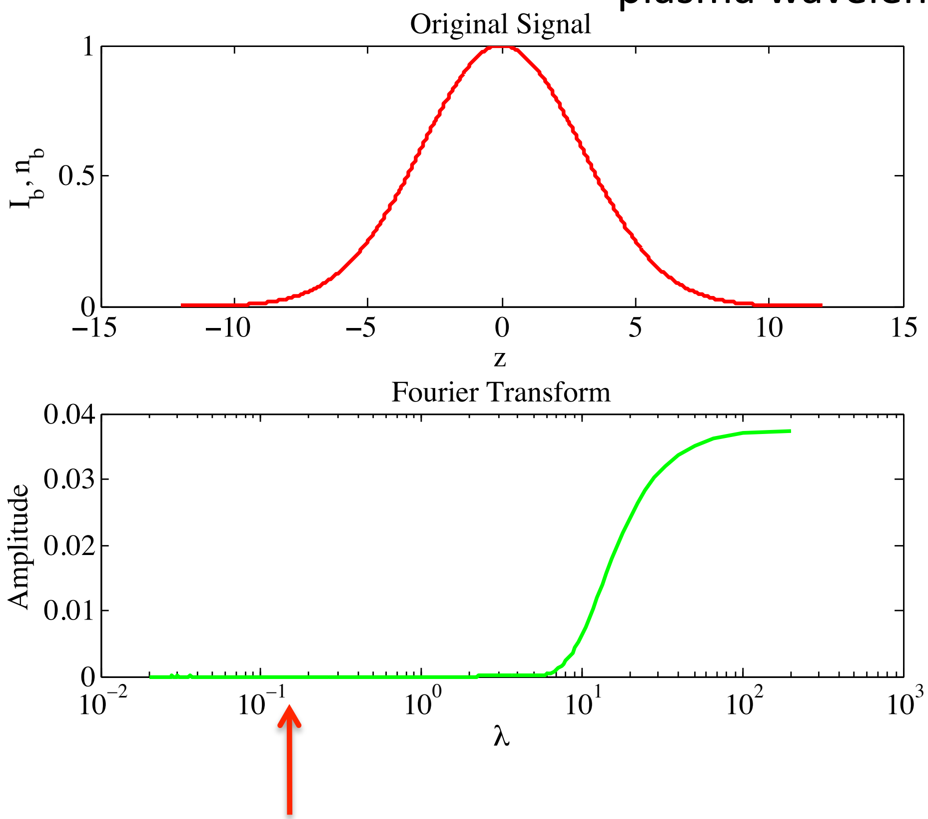


Effective
When $k_{pe}\sigma_z$~√2

BUNCH LENGTH

Long bunch in high plasma density … $k_{pe}\sigma_z$>>1

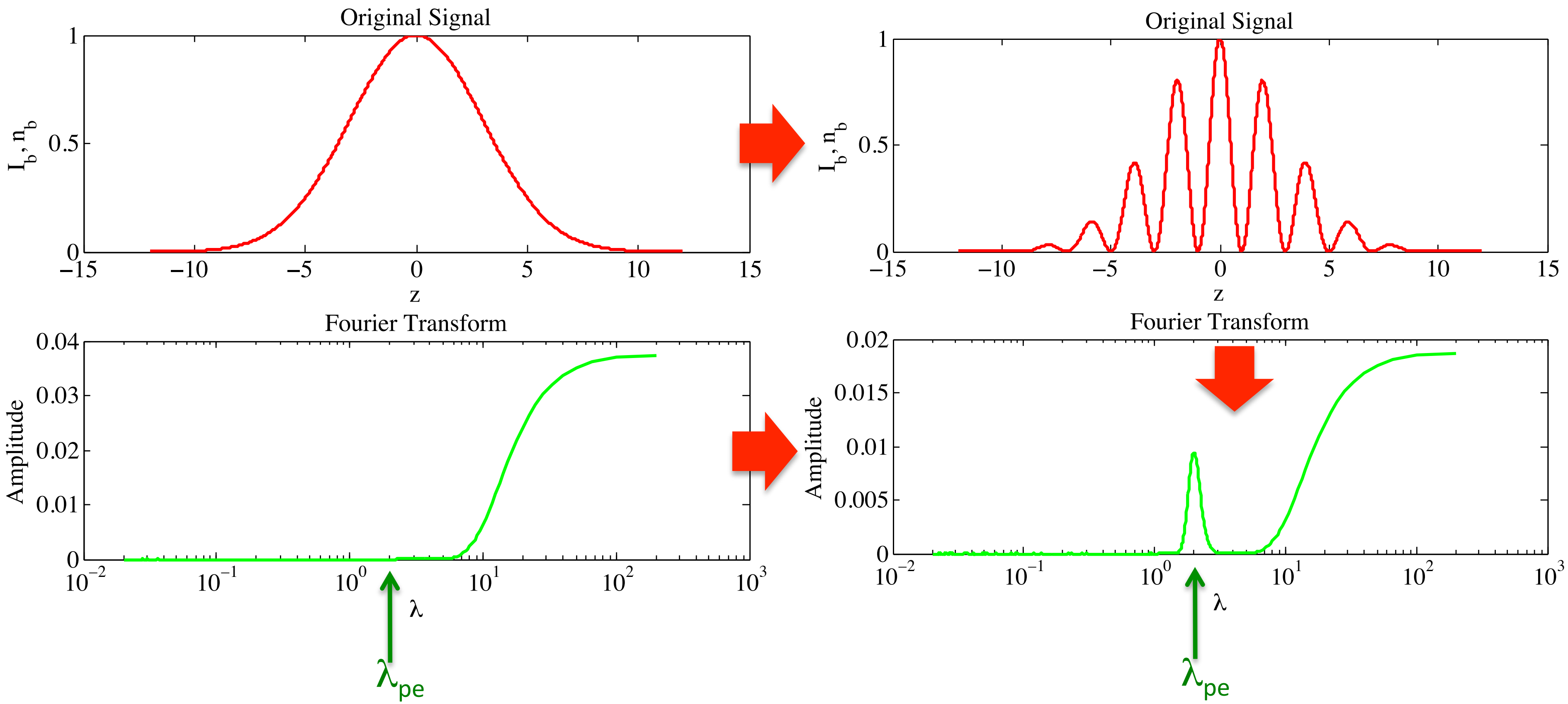
Example: bunch with σ=3 in a plasma with plasma wavelength λ_{pe}=2



Not effective for $k_{pe}\sigma_z$>>1 or λ_{pe}<<σ_z

BUNCH LENGTH

Example: bunch with σ=3, modulated with λ=2 in a plasma with plasma wavelength λ_{pe}=2



Not effective

Effective

=> (self-)Modulating the bunch density (n_b~$1/\sigma_r^2\sigma_z$) at ~λ_{pe} is a very effective way to drive larger (than in the $k_{pe}\sigma_z$~√2 case) wakefields

SELF-MODULATION INSTABILITY (SMI)

✧CERN p^+ bunches (PS, SPS, LHC) ~12cm long

✧$E_{WB} \sim \omega_{pe} \sim n_e^{1/2}$ and $\sigma_z \sim n_e^{-1/2}$

PRL 104, 255003 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

Self-Modulation Instability of a Long Proton Bunch in Plasmas

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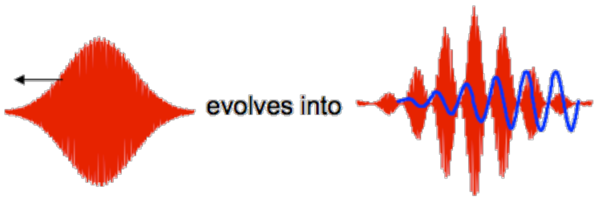
(Received 16 April 2010; published 25 June 2010)

An analytical model for the self-modulation instability of a long relativistic proton bunch propagating in uniform plasmas is developed. The self-modulated proton bunch resonantly excites a large amplitude plasma wave (wakefield), which can be used for acceleration of plasma electrons. Analytical expressions for the linear growth rates and the number of exponentiations are given. We use full three-dimensional particle-in-cell (PIC) simulations to study the beam self-modulation and transition to the nonlinear stage. It is shown that the self-modulation of the proton bunch competes with the hosing instability which tends to destroy the plasma wave. A method is proposed and studied through PIC simulations to circumvent this problem, which relies on the seeding of the self-modulation instability in the bunch.

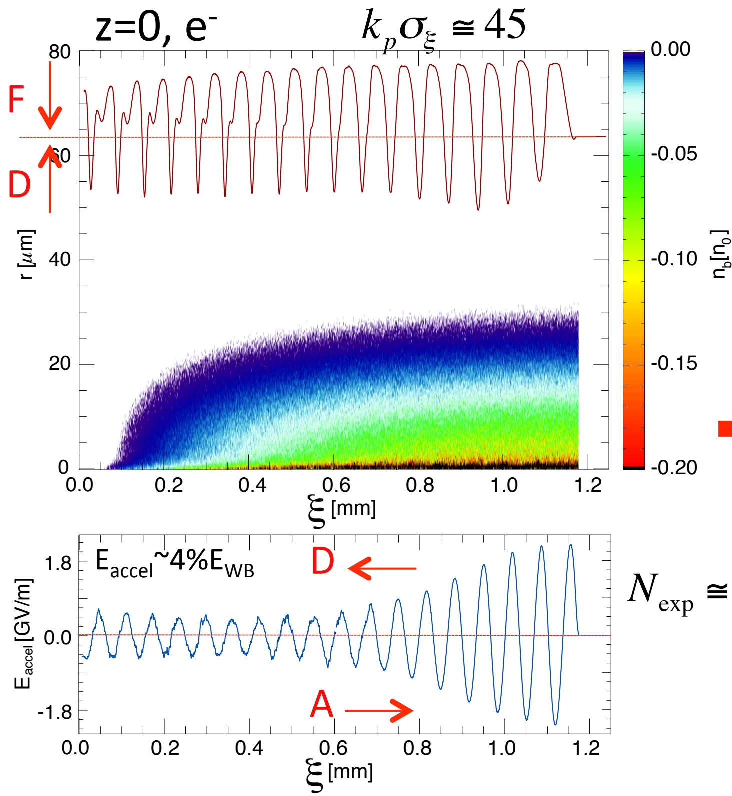
DOI: 10.1103/PhysRevLett.104.255003 PACS numbers: 52.35.−g, 52.40.Mj, 52.65.−y

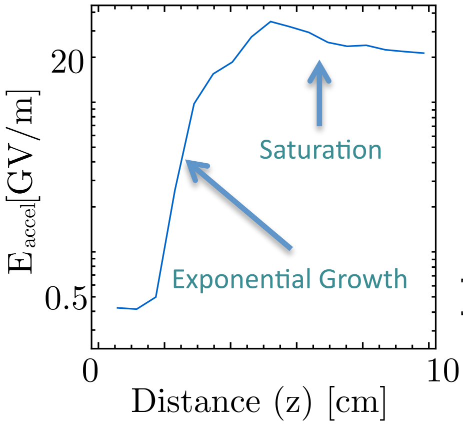
✧Idea developed "thanks" to the non-availability of short p^+ bunches

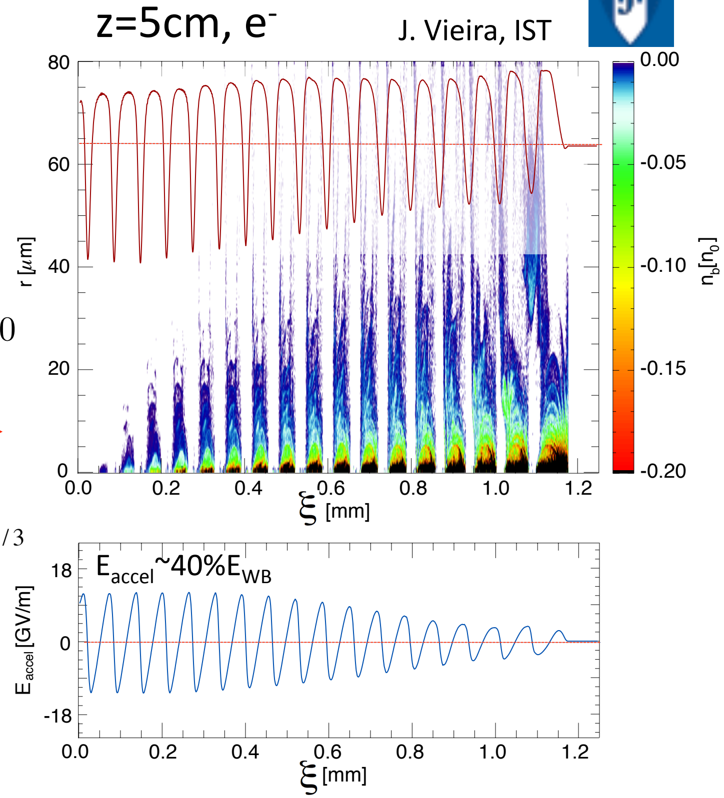
✧Very similar to Raman self-modulation of long laser pulses (LWFA of the 20th century)



SELF-MODULATION INSTABILITY (SMI)







$$N_{exp} \cong \frac{3\sqrt{3}}{4}\left(\frac{n_b}{n_e}\frac{m_e}{\gamma M_b}\left(k_p|\xi|\right)\left(k_p z\right)^2\right)^{1/3}$$

Grows along the bunch & along the plasma

Pukhov et al., PRL 107, 145003 (2011)
Schroeder et al., PRL 107, 145002 (2011)

✧Initial small transverse wakefields modulate the bunch density

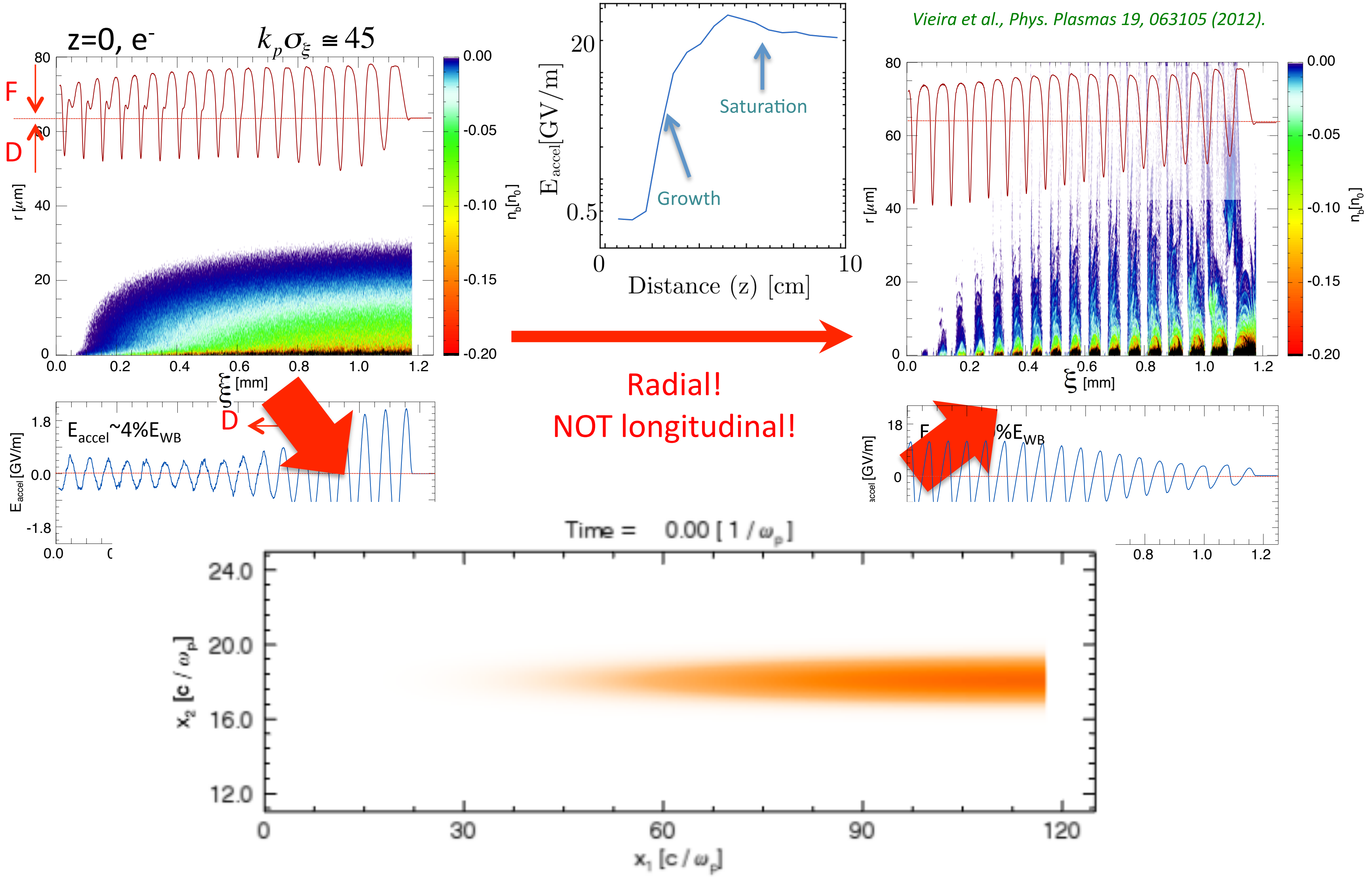
✧Longitudinal, transverse wakefields ~n_b

✧Associated longitudinal wakefields reach large amplitude through resonant excitation

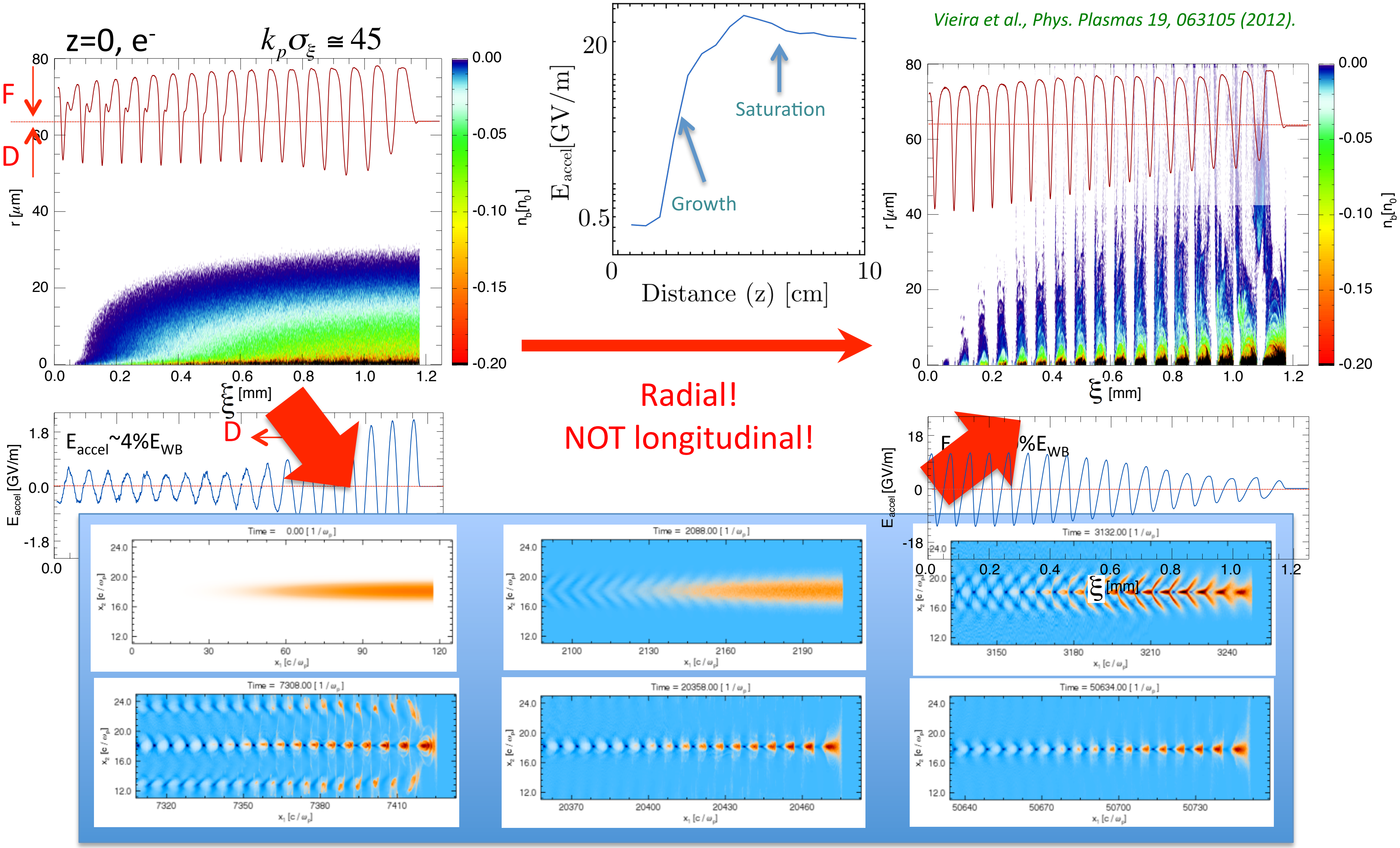
J. Vieira et al., Phys. Plasmas 19, 063105 (2012)

SELF-MODULATION INSTABILITY (SMI)

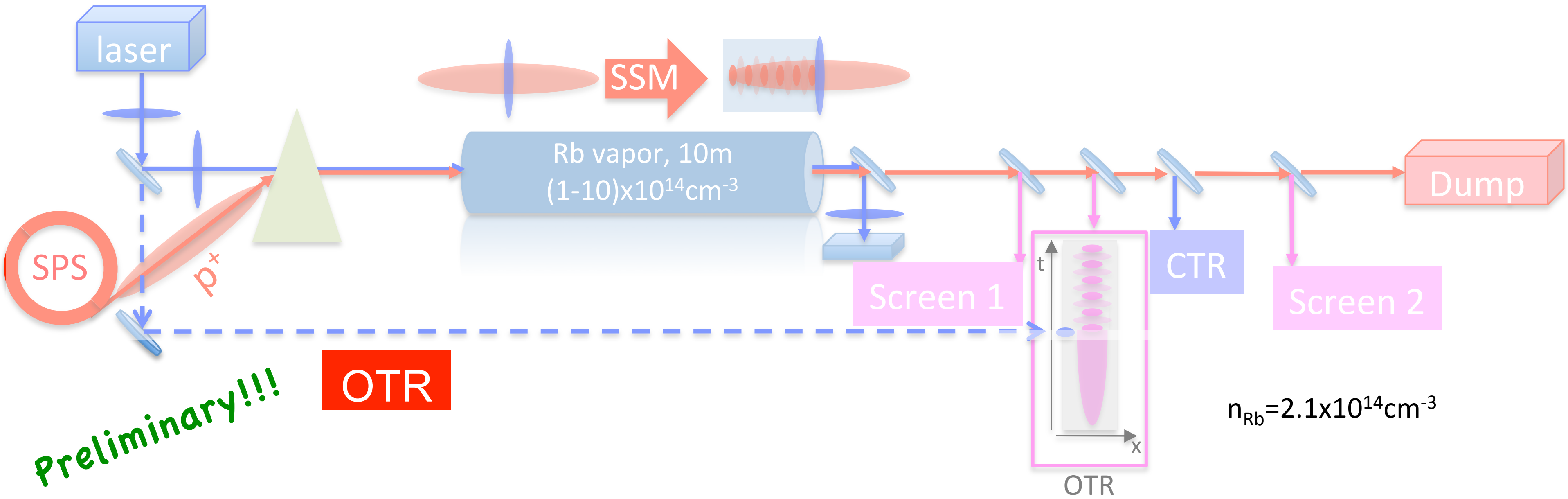
Vieira et al., Phys. Plasmas 19, 063105 (2012).



SELF-MODULATION INSTABILITY (SMI)



μBUNCH TRAIN



Streak camera Images