Staging in High Gradient Wakefield Accelerators

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INTRODUCTION

- > Why staging? What is staging?
- > Transverse beam dynamics (refresher)
- > Calculating minimum staging length

Why staging?

> Goal: accelerating particles to **high energies**.

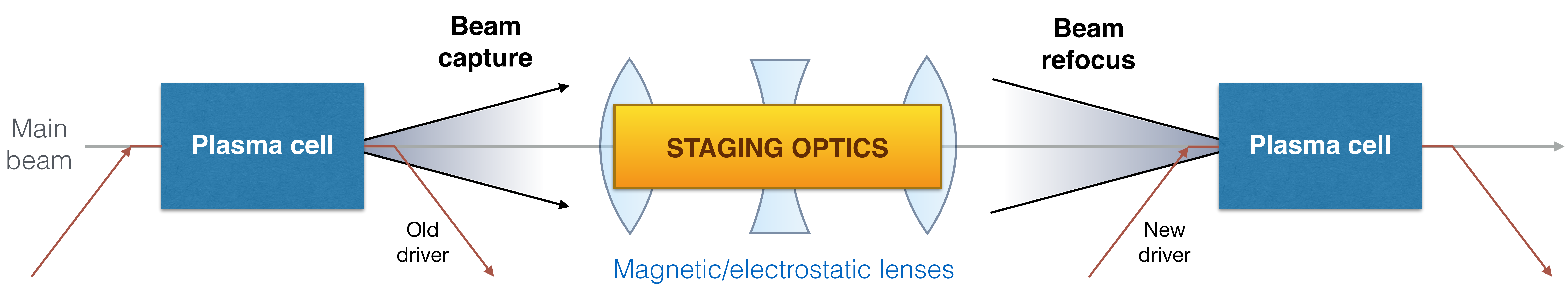
> Challenges:
  > **Depletion**
    > Final energy of the witness bunch is larger than what can be contained in a single driver
  > **Dephasing**
    > The driver moves slower than the witness, such that the witness drifts out of the accelerating phase
    > (mainly a problem for laser drivers)
  > **Diffraction**
    > The drive beam diverges too much, stopping further acceleration
    > (laser-specific term, but also applies to particle beams: known as *head erosion* [1])

> If any of the above problems apply, the solution is to **use several acceleration stages**.
> For TeV-scale acceleration using beam-driven PWFA, depletion is the main challenge.

*[1] Ian Blumenfeld, "Scaling of the longitudinal electric fields and transformer ratio in a non-linear plasma wakefield accelerator," **Ph.D. thesis (Stanford University, 2009)***

Staging in a Nutshell

- Two things need to occur between stages:
  - 1. Out-coupling of the depleted driver, and in-coupling of a fresh driver
  - 2. Capture and refocusing of the accelerated/witness bunch
- Has historically been an under-studied and under-estimated topic.
- We will focus mostly on plasma-based accelerators (because they are the most challenging).



Refresher: Transverse beam dynamics

> Geometric emittance: (trace space area)

$$\epsilon_g^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2$$

> Normalized emittance: (preserved with acceleration)

$$\epsilon_n = \frac{\epsilon_g}{\gamma\beta} \approx \frac{\epsilon_g}{\gamma}$$

> Beam covariance matrix:

$$\Sigma_x = \begin{bmatrix} \langle x^2 \rangle & \langle x x' \rangle \\ \langle x x' \rangle & \langle x'^2 \rangle \end{bmatrix} = \epsilon_g \begin{bmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{bmatrix}$$

> Twiss parameters:

$$\beta_x = \frac{\langle x^2 \rangle}{\epsilon_g} \qquad \alpha_x = -\frac{1}{2}\frac{\partial \beta_x}{\partial s} = -\frac{\langle x x' \rangle}{\epsilon_g} \qquad \gamma_x = \frac{1+\alpha_x^2}{\beta_x} = \frac{\langle x'^2 \rangle}{\epsilon_g}$$

> Transport of beams through a "lattice" of optical elements (quadrupoles, etc):

$$\Sigma = \mathbf{M}\Sigma_0\mathbf{M}^\top$$

> Transfer matrix of a lattice of elements (1–n):

$$\mathbf{M} = \mathbf{M}_n\mathbf{M}_{n-1}\ldots\mathbf{M}_2\mathbf{M}_1$$

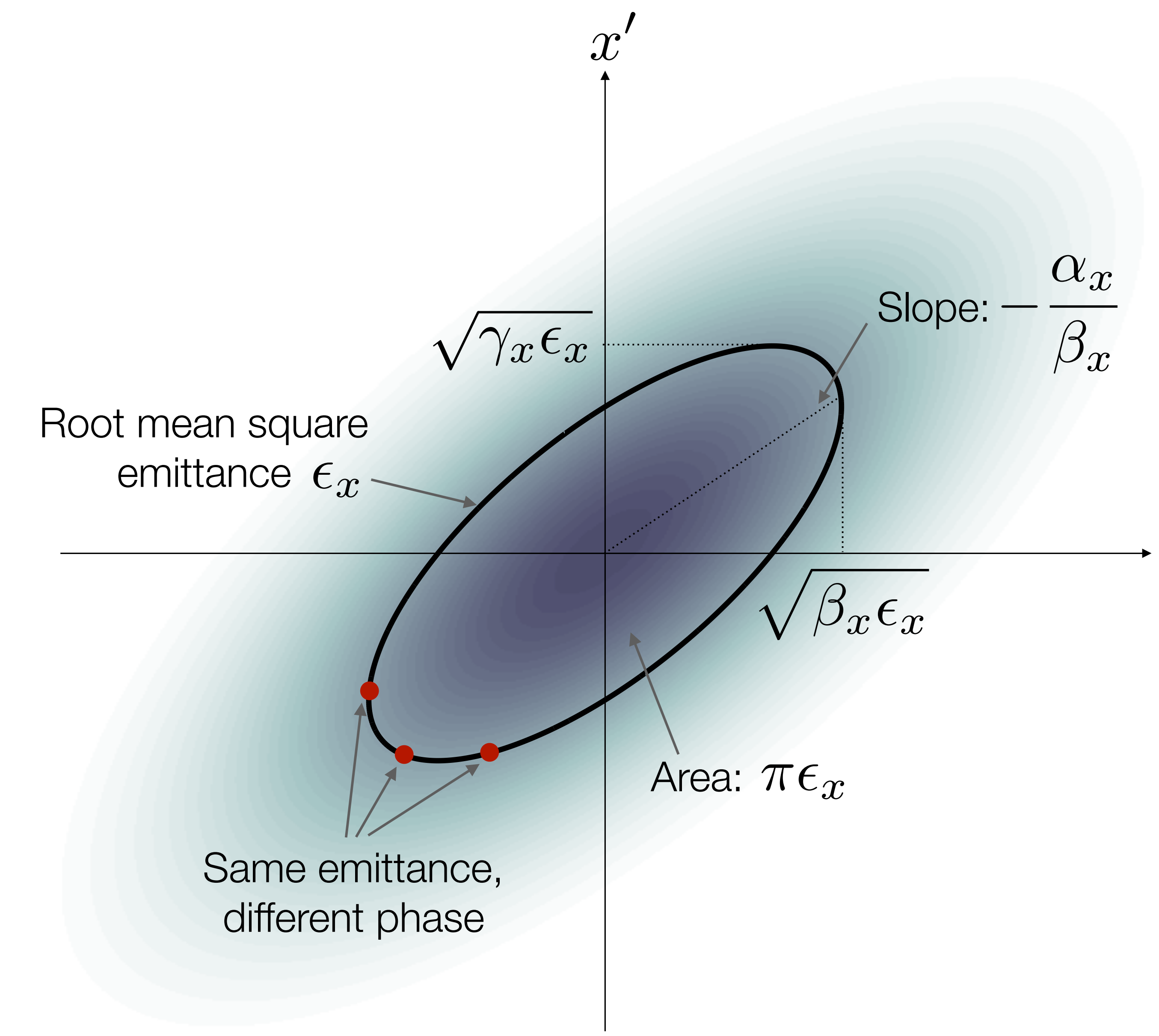


Image source: C. A. Lindstrøm, PhD thesis (University of Oslo, 2019) [2]

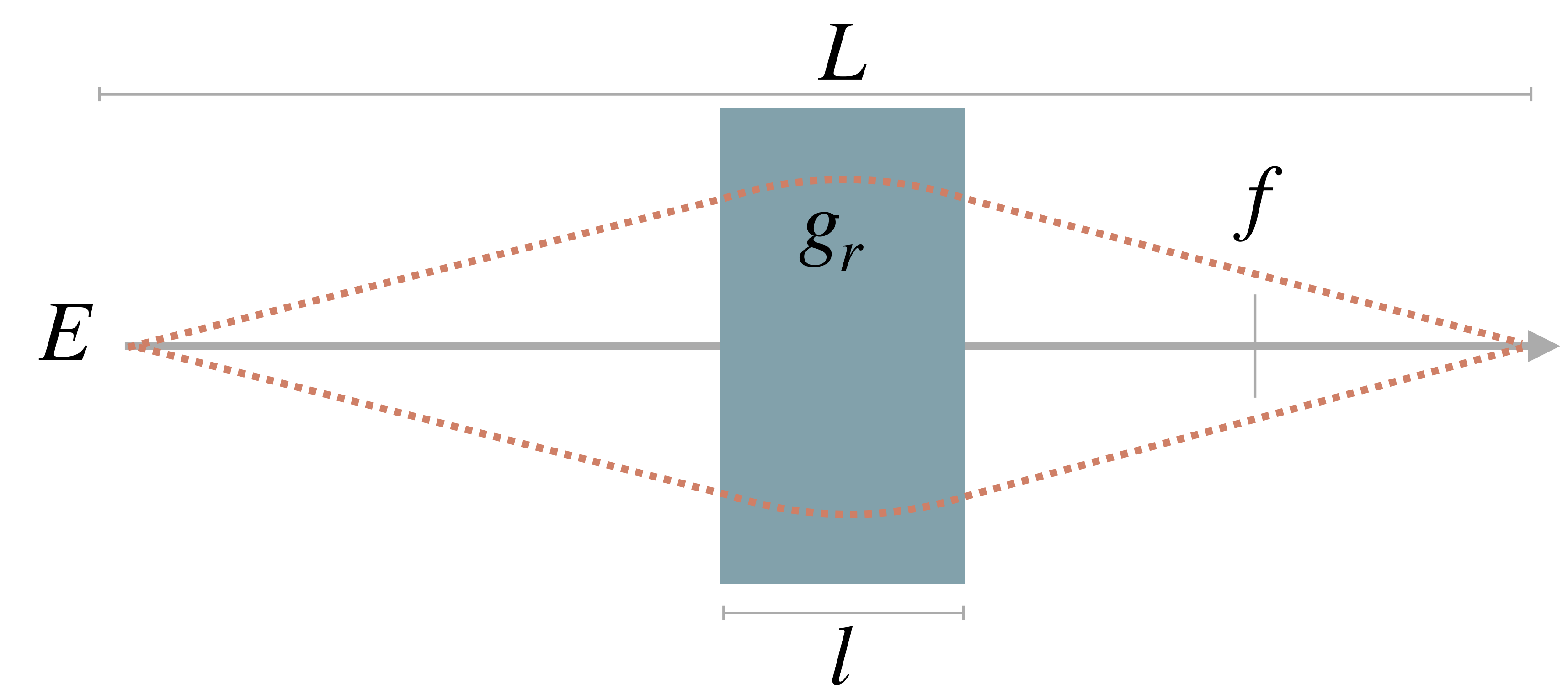
[2] C. A. Lindstrøm, "Emittance growth and preservation in a plasma-based linear collider", **PhD thesis (University of Oslo, 2019)**

Exercise: Minimum length of beam capture and refocusing

> Assume point source beam
(small initial beta function compared to staging length)

> Assume radial focusing

> Assume in- and out coupling sections are short:
limited by the focal length of the focusing optic(s).



> Focusing strength (k), thin lens approximation for focal length (f):

$$k = \frac{g_r e c}{E} \qquad f = \frac{1}{kl} \qquad \Longrightarrow \qquad fl = \frac{E}{g_r e c}$$

> Total staging length (L):

$$L = l + 4f$$

> Optimize length of optic compared to focal length:

$$\frac{dL}{dl} = 1 - \frac{4}{kl^2} = 0 \qquad \Longrightarrow \qquad l = 4f = \frac{2}{\sqrt{k}}$$

> **Minimum length (L_{min}) given by:**

$$L_{\text{min}} = \frac{4}{\sqrt{k}} = 4\sqrt{\frac{E}{g_r e c}}$$

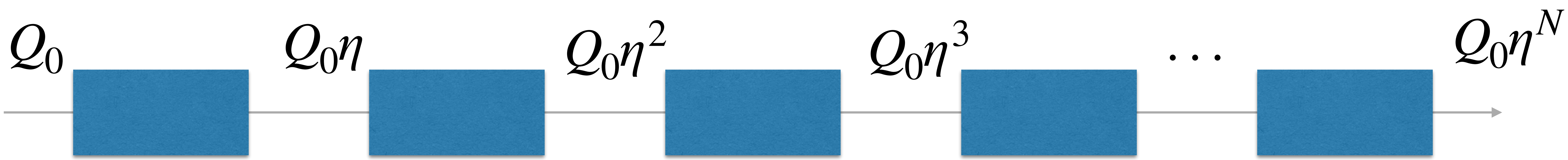
> Example: g_r = 10 T/m, E = 50 GeV ⇒ L_{min} = 16 m

TECHNICAL REQUIREMENTS

- Charge coupling
- Compact in- and out coupling of drive beams
- Emittance preservation
- Isochronicity ($R_{56} = 0$)
- Overall compactness (high effective gradient)

CHARGE COUPLING EFFICIENCY

- Charge coupling decays exponentially with the number of stages (assuming no correlations)
- Therefore the **charge coupling efficiency should be close to 100%.**



- Example: initial charge Q = 180 pC, 1.1×10^9 particles, η = 50% coupling efficiency, N = 30 stages

  $\Rightarrow$ Final charge: 1 electron.

- The only current staging experiment (BELLA at LBNL) had a charge coupling efficiency of 3.5%.
- Needs to improve in the future!

In- and out-coupling of drive beams

> **Beam drivers:**

- > High energy particle beams (multi-GeV) are difficult to bend: need strong electromagnetic fields.
- > Cannot use normal kickers, as these have rise times of several ns (drive–witness separation is sub-ps)
- > Solution: **energy separation with *dipoles***
  - > Requires beams of different energy (or charge) for separation ⇒ may be problematic at low witness beam energy
  - > **Witness energy should be higher than the driver energy**, as driver will develop ~100% energy spread
- > Esoteric possibility: Using transverse wakefields as ultra-fast kicker/deflecting structure.

> **Laser drivers:**

- > 1. Use a *magnetic chicane*
  Challenge: difficult at high witness beam energies
- > 2. Use a *mirror*
  Challenge: intense laser beams lead to burning of mirrors.
  Solution: **Plasma mirrors** (e.g. VHS tape) [3]

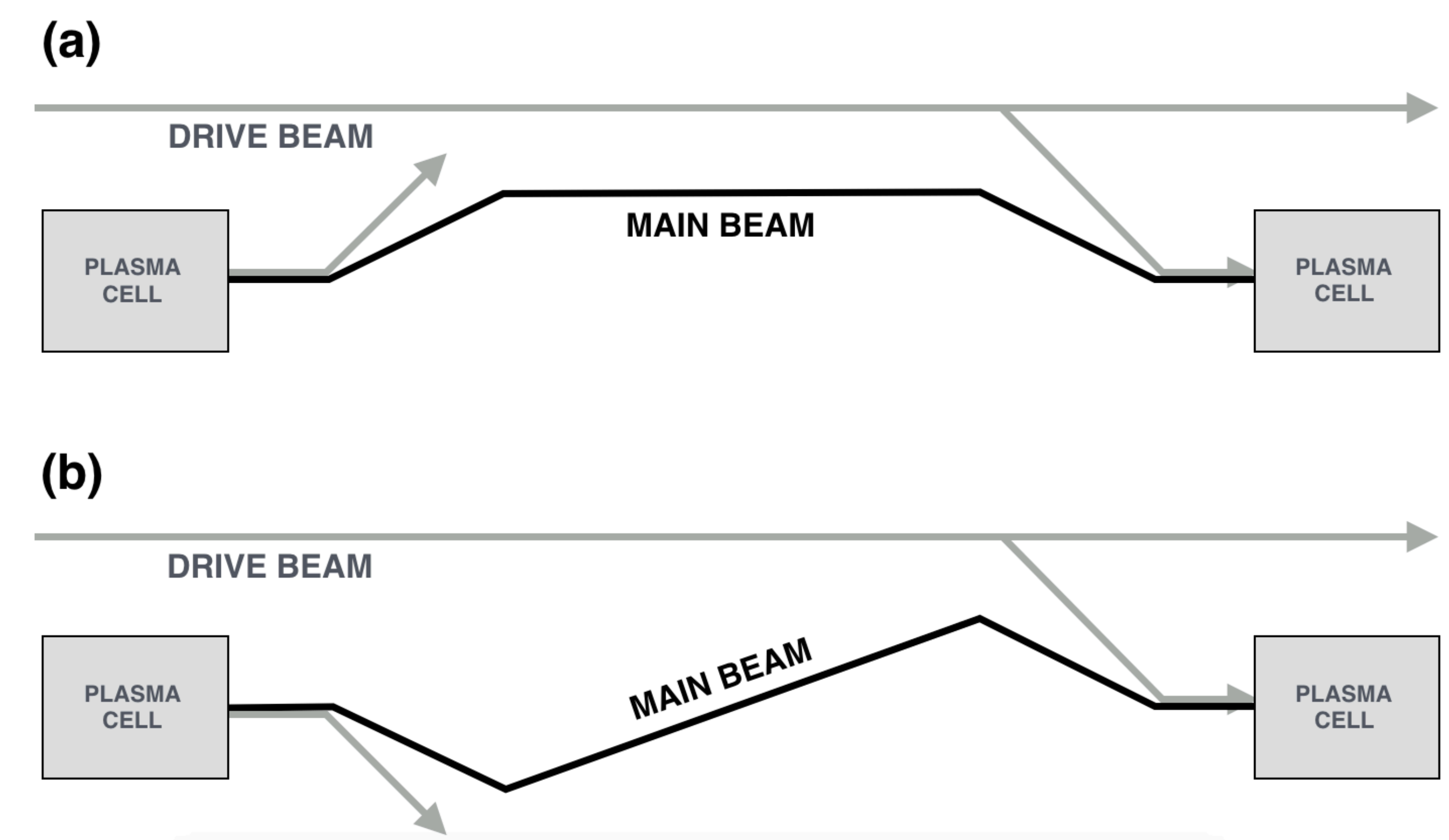


Image source: C. A. Lindstrøm et al.,
Nucl. Instrum. Methods Phys. Res. A 829, 224–228 (2016).

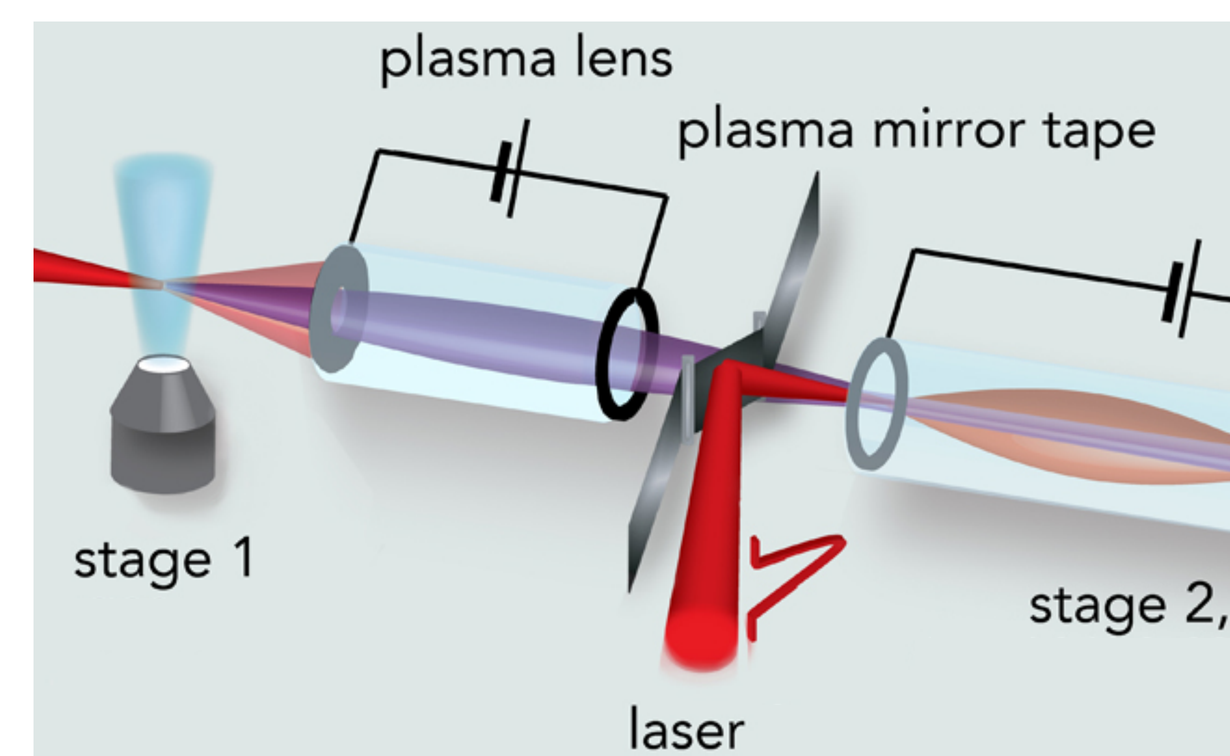


Image source: LBNL

[3] C. Thaury, et al., "Plasma mirrors for ultrahigh-intensity optics", **Nature Phys. 3, 424–429 (2007)**

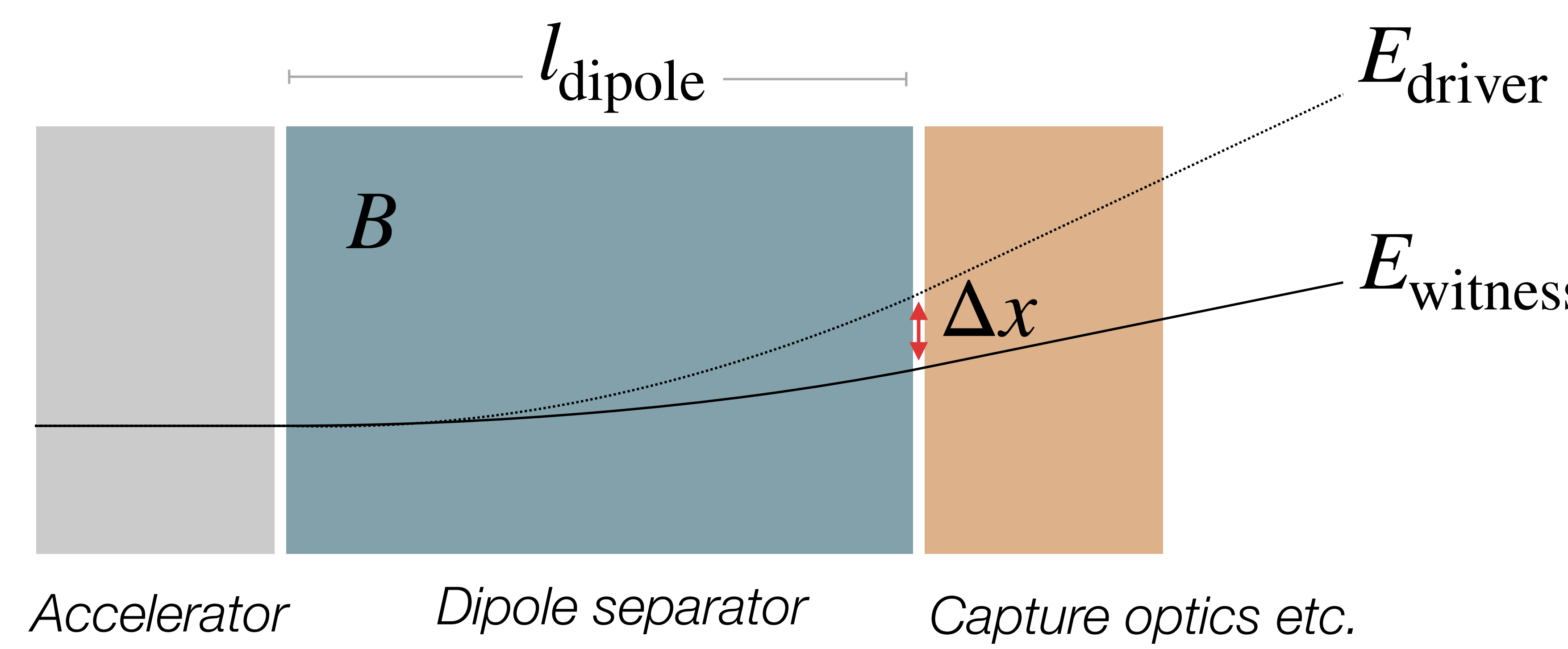
Dipole separation in beam-driven wakefield accelerators

- Horizontal separation in a dipole of two different energy beams:

$$\Delta x = \frac{{l_{\mathrm{dipole}}}^2 Bce}{2}\left(\frac{1}{E_{\mathrm{driver}}} - \frac{1}{E_{\mathrm{witness}}}\right)$$

- Assuming E_{driver} = 20 GeV, E_{witness} = 50+ GeV, B = 0.5 T, Δx > 5 mm

  ⇒ **l_{dipole} > 1.5 m** ⇒ Meter-scale dipole lengths



- Cannot capture the witness bunch before driver–witness separation, as the beam energies are different.
  ⇒ This will destroy the lower-energy drive bunch.

- Result: **Long drift before witness bunch capture.**

- Implies **two regimes of staging design**:
  - 1. Capture length limited by driver–witness separator (low energy)
  - 2. Capture length limited by focal length of optics (high energy)

- Energy loss in dipoles from **synchrotron radiation should be minimized.** Favours longer dipoles and weaker B-fields (for constant offset):

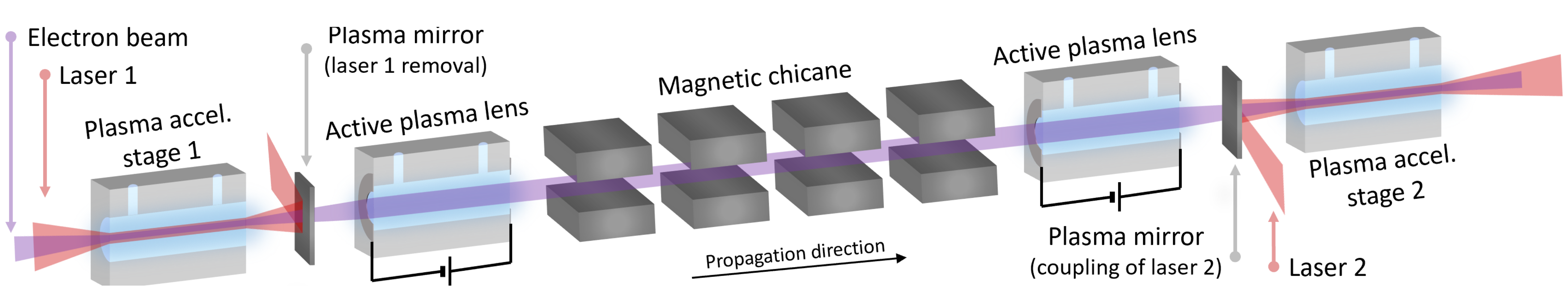
$$\Delta E_{SR} = P_{SR} l_{\mathrm{dipole}} = \left(\frac{e^4 c^3}{6\pi\epsilon_0}\right)\gamma^2 B^2 l_{\mathrm{dipole}}$$

Isochronicity (R_{56}) — Avoiding bunch compression/stretching

- If dipoles are part of the staging, **different energy slices must have the same path length** to avoid bunch lengthening/contraction.
- Governed by transfer matrix element ***R*₅₆**: relation between the longitudinal coordinate and the energy

$$\left| R_{56} \right| \ll \frac{\sigma_z}{\sigma_\delta} \approx \frac{1}{k_p \sigma_\delta}$$

- Example: Witness bunch length σ_z = 10 μm rms, energy spread σ_δ = 1% rms $\Rightarrow$ $R_{56} << 1$ mm
- Quadrupoles also affect R_{56} — can be tweaked to cancel it.
- A suggested option [4] is to use a controlled R_{56} (using a chicane) and two stages to remove the energy chirp from a laser plasma accelerator.



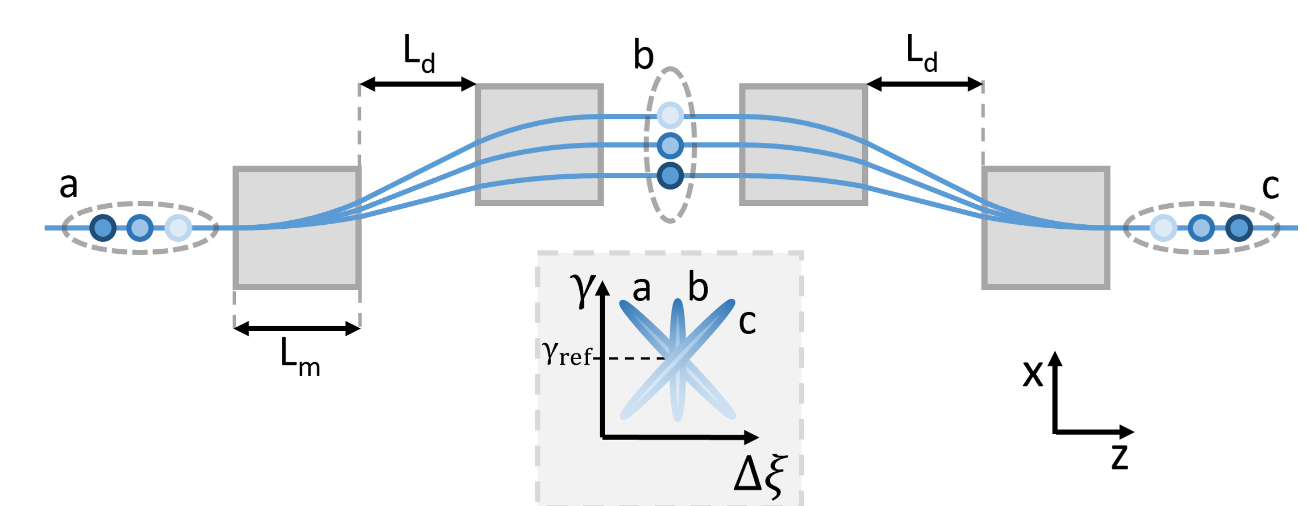


Image source: A. Ferran Pousa et al., arXiv:1811.07757 (2018) [4]

[4] A. Ferran Pousa et al., "Correlated Energy Spread Compensation in Multi-Stage Plasma-Based Accelerators," **arXiv:1811.07757 (2018)**

Emittance preservation

> Low normalized emittance is key to both linear colliders and FELs: must be preserved throughout the acceleration.

> Emittance can only grow (Liouville's theorem)—only exception is radiation damping (damping rings).

> Large accelerators will have an "emittance budget"

> > Typically ~100% growth or less in the entire linac

> > For many stages, this implies a limit of approximately **%-level emittance growth per stage**

Luminosity of a collider:

$$\mathscr{L} = \frac{H_D}{8\pi m_e c^2} \frac{P_{\mathrm{wall}}}{\sqrt{\beta_x \epsilon_{Nx}} N} \frac{\eta}{\sqrt{\beta_y \epsilon_{Ny}}}$$

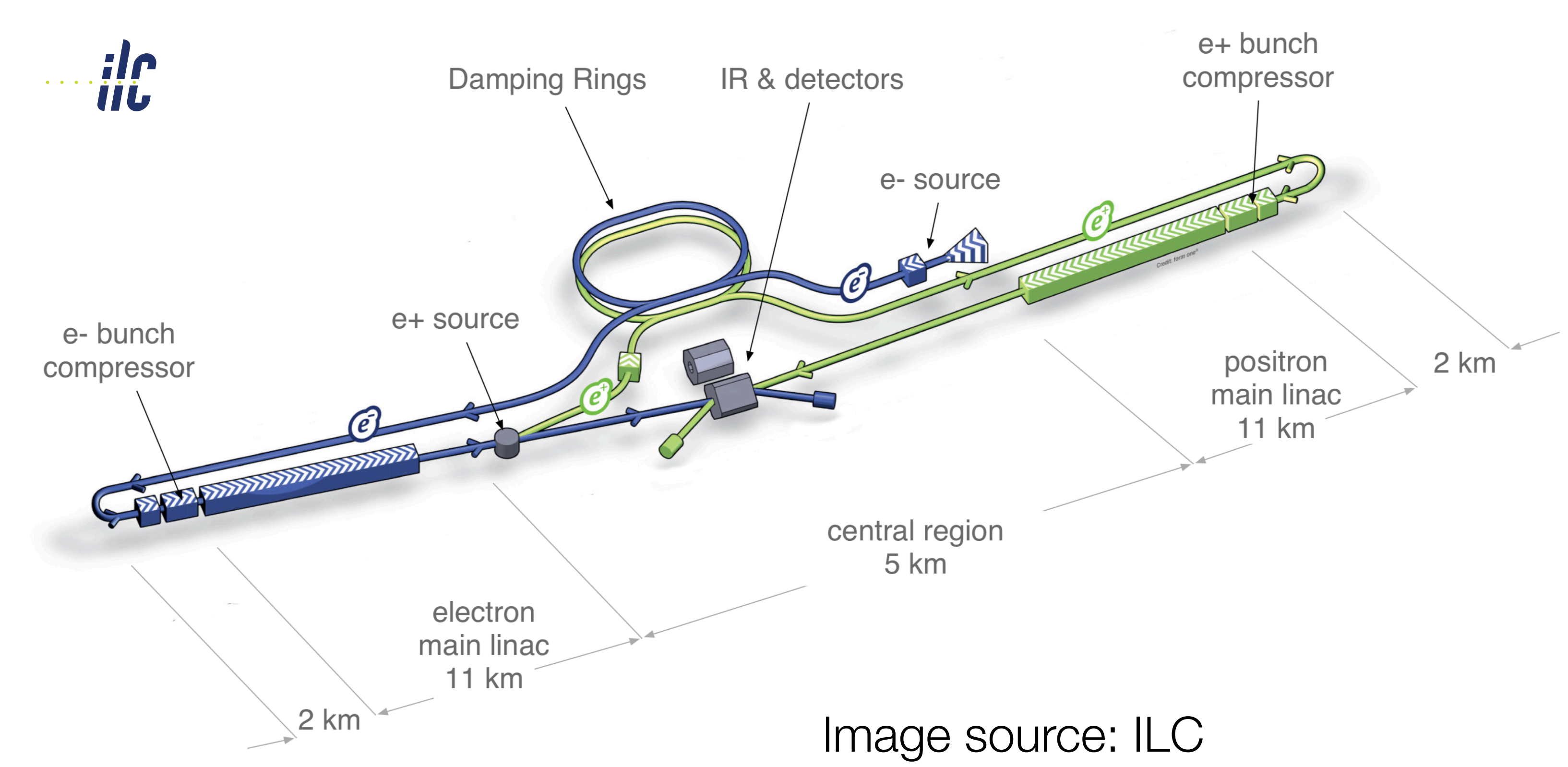


Image source: ILC

> Main challenges:

> > ***Chromaticity* = different focusing for different energies**

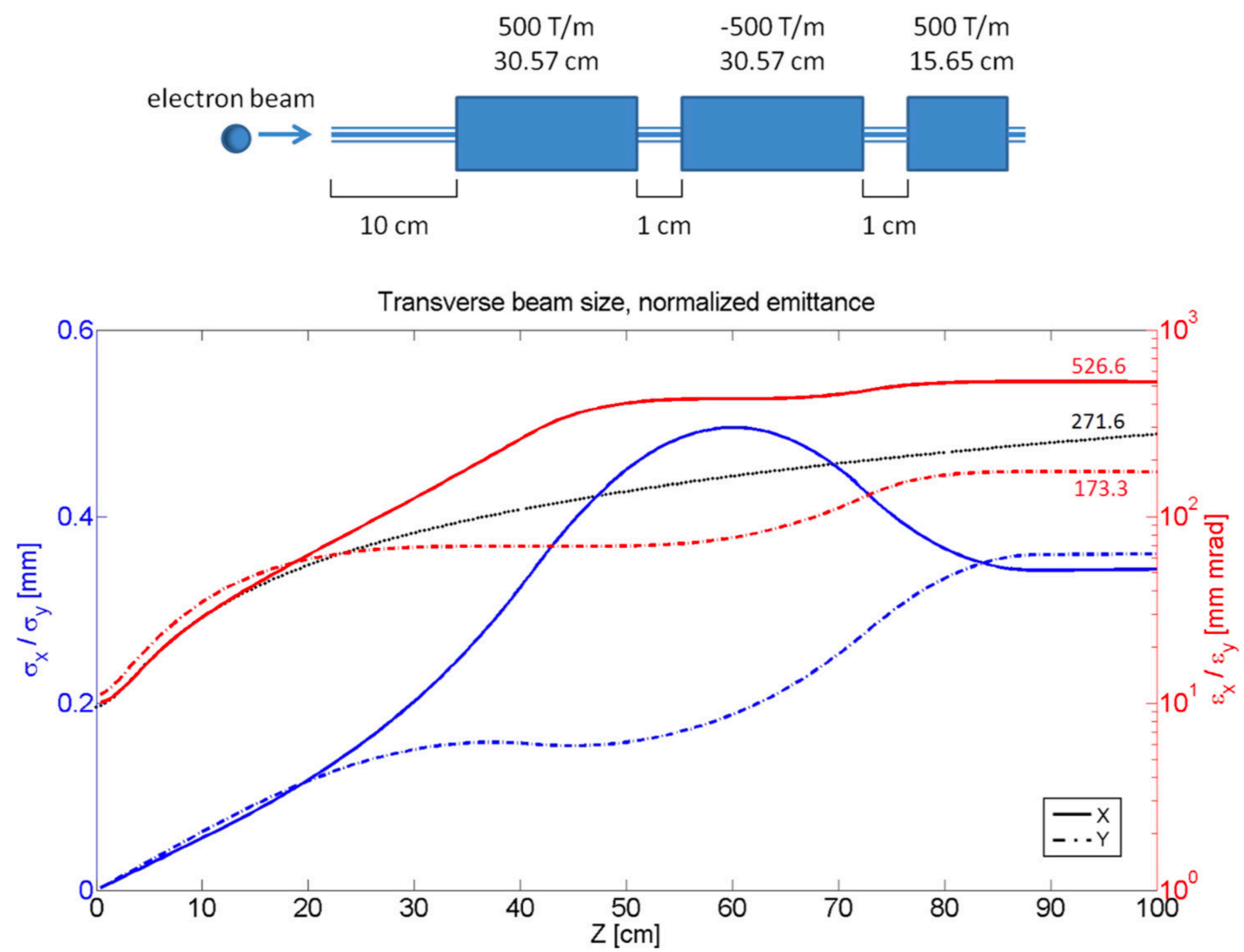
> > *Dispersion* = different centroid offsets for different energies

> > *Geometric terms* = nonlinear focusing effects in sextupoles, etc

> > *Transverse misalignments*

> > *Gas scattering*

Chromaticity: Emittance growth in conventional lattices



Large emittance growth in a quadrupole sextuplet (half shown). Energy spread 6% rms (LWFA).

Image source: M. Scisciò, J. Appl. Phys. 119, 094905 (2016) [5]

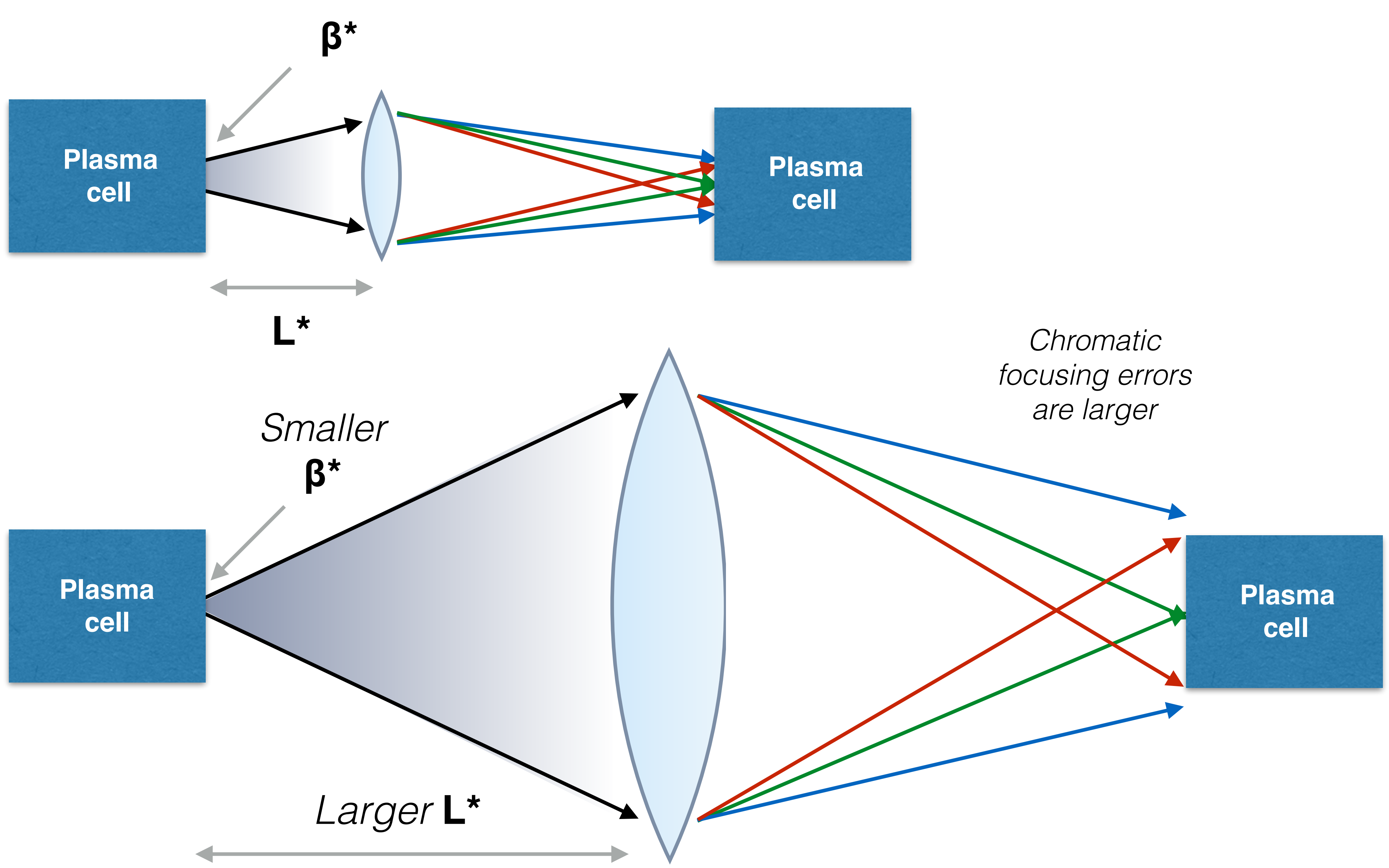
[5] M. Scisciò, "Parametric study of transport beam lines for electron beams accelerated by laser-plasma interaction", ***J. Appl. Phys. 119, 094905 (2016)***

CHROMATICITY — ONE OF THE MAIN CHALLENGES

> Chromaticity = Different energies are focused differently.

> While the emittance of each energy slice IS preserved, the **projected (energy-averaged) emittance IS NOT preserved.**

> For plasma wakefield accelerators, all energy slices must be matched to avoid further emittance growth in the plasma ion column.



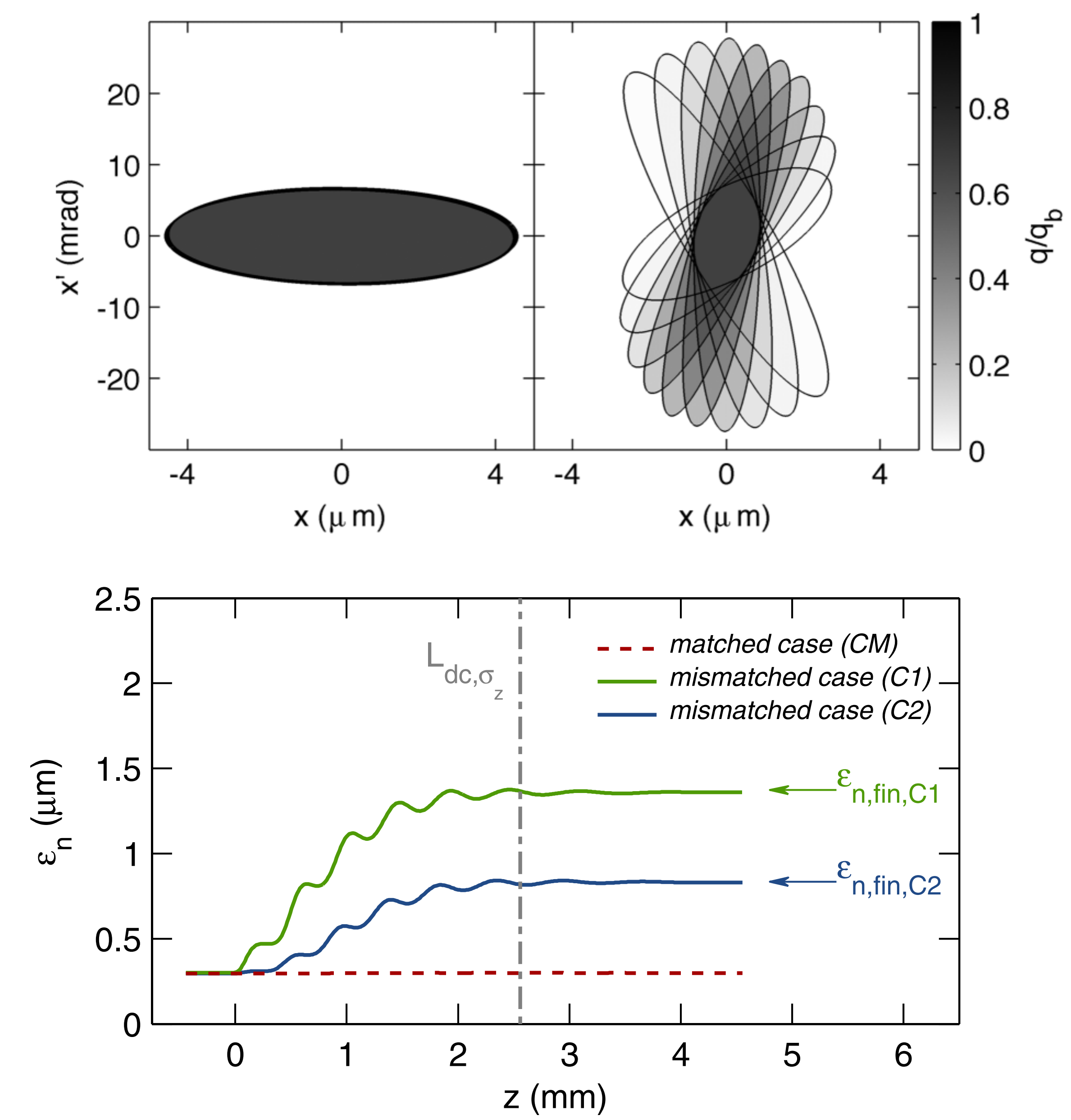


Image source: T. Mehrling et al., Phys. Rev. ST Accel. Beams 15, 111303 (2012) [6]

[6] T. Mehrling et al., "Transverse emittance growth in staged laser-wakefield acceleration", Phys. Rev. ST Accel. Beams 15, 111303 (2012)

Chromaticity – Projected emittance growth

> Chromaticity is inherent in any focusing element:

$$k(\delta) = \frac{k_0}{1+\delta} \qquad \delta = \frac{\Delta p}{p} \approx \frac{\Delta E}{E}$$

> Quantified in by the so-called **"chromatic amplitude" or W-function**, first introduced by B. Montague [7] in 1979.

$$W = \sqrt{\left(\frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta}\right)^2 + \left(\frac{1}{\beta}\frac{\partial \beta}{\partial \delta}\right)^2}$$

> The added chromatic amplitude per optic is approximately: (larger beta in the optic and stronger focusing is bad)

$$\Delta W = \beta k l \approx \frac{\beta}{f}$$

> There is also a corresponding *chromatic phase*, which evolves at twice the rate of the betatron phase, and determines how the chromatic amplitude from each new optic is added.

> Chromaticity is **linked to relative emittance growth to lowest order in σ_δ by** [8]:

$$\frac{\Delta \epsilon_n^2}{\epsilon_n^2} = W^2 \sigma_\delta^2 + \mathcal{O}(\sigma_\delta^4)$$

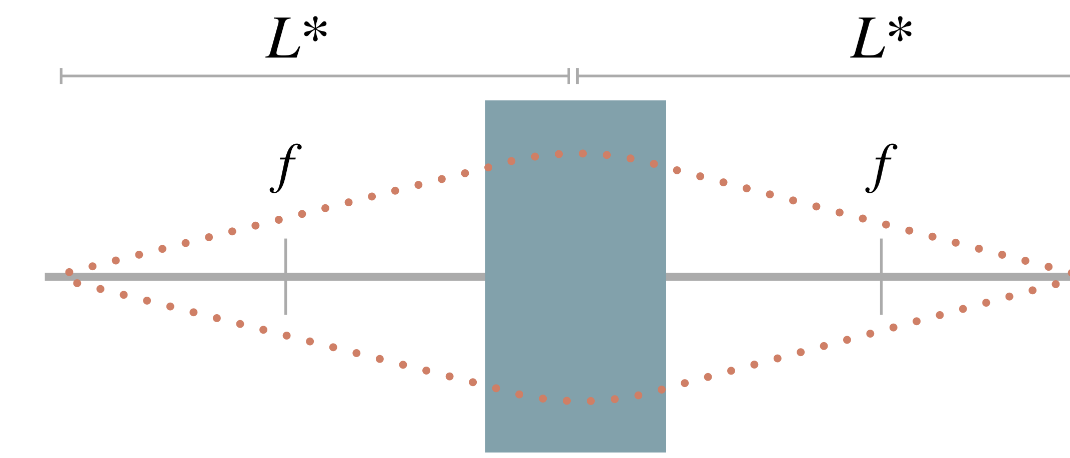
> Mnemonic: To keep emittance growth below 100%, the W-function must be below 1/(rms energy spread).

[7] Brian W. Montague, "Linear optics for improved chromaticity correction," LEP Note 165 (CERN, Geneva, 1979)
[8] C. A. Lindstrøm and E. Adli, "Design of general apochromatic drift-quadrupole beam lines," Phys. Rev. Accel. Beams 19, 071002 (2016)

Chromaticity for in plasma wakefield accelerator staging

> Matched beta function, assumed to be small compared to staging length (if not, we do not require any staging optics).

$$\beta_m = \frac{\sqrt{2\gamma}}{k_p}$$



> Assume radial focusing.

$$k_p^2 = \frac{n_0 e^2}{\epsilon_0 m_e c^2}$$

> Beam expands over a distance L^* before being captured:

$$\beta_{\max} = \beta_m + \frac{L^{*2}}{\beta_m} \approx \frac{L^{*2}}{\beta_m}$$

> The beam is capture and refocused with a lens of focal length:

$$f = \frac{L^*}{2}$$

> The induced chromaticity is approximately:
and the emittance growth is given by:

$$W = \frac{\beta_{\max}}{f} \approx \frac{2L^*}{\beta_m} \qquad \frac{\Delta\epsilon_n^2}{\epsilon_n^2} = \frac{4L^{*2}}{\beta_m^2}\sigma_\delta^2 + \mathcal{O}(\sigma_\delta^4)$$

> Recommendations for reducing emittance growth:
- **Decrease plasma density at entry/exit**
- **Increase focusing gradient (decrease focal length)**
- OR cancel chromaticity in other ways

Assuming minimum focal length staging (result will be wrong by a numerical factor)

$$\frac{\Delta\epsilon_n^2}{\epsilon_n^2} \approx 4\frac{E}{g_r e c}\frac{k_p^2}{2\gamma}\sigma_\delta^2 \approx \frac{2n_0 e \sigma_\delta^2}{g_r \epsilon_0 c}$$

Plasma density ramps

> Plasma density ramps are gradual changes of the plasma density at the entry and exit of the plasma stage.

> Reduces the chromaticity problem outside the ramp (larger matched beta function)

> Different classes of density ramps:

> > **Adiabatic** (α ≈ 0 throughout the ramp): longer, but high energy acceptance [9]

> > **Non-adiabatic** (α ≠ 0): shorter, but low energy acceptance [10]

> Warning: Long ramps will decelerate or induce energy spread.

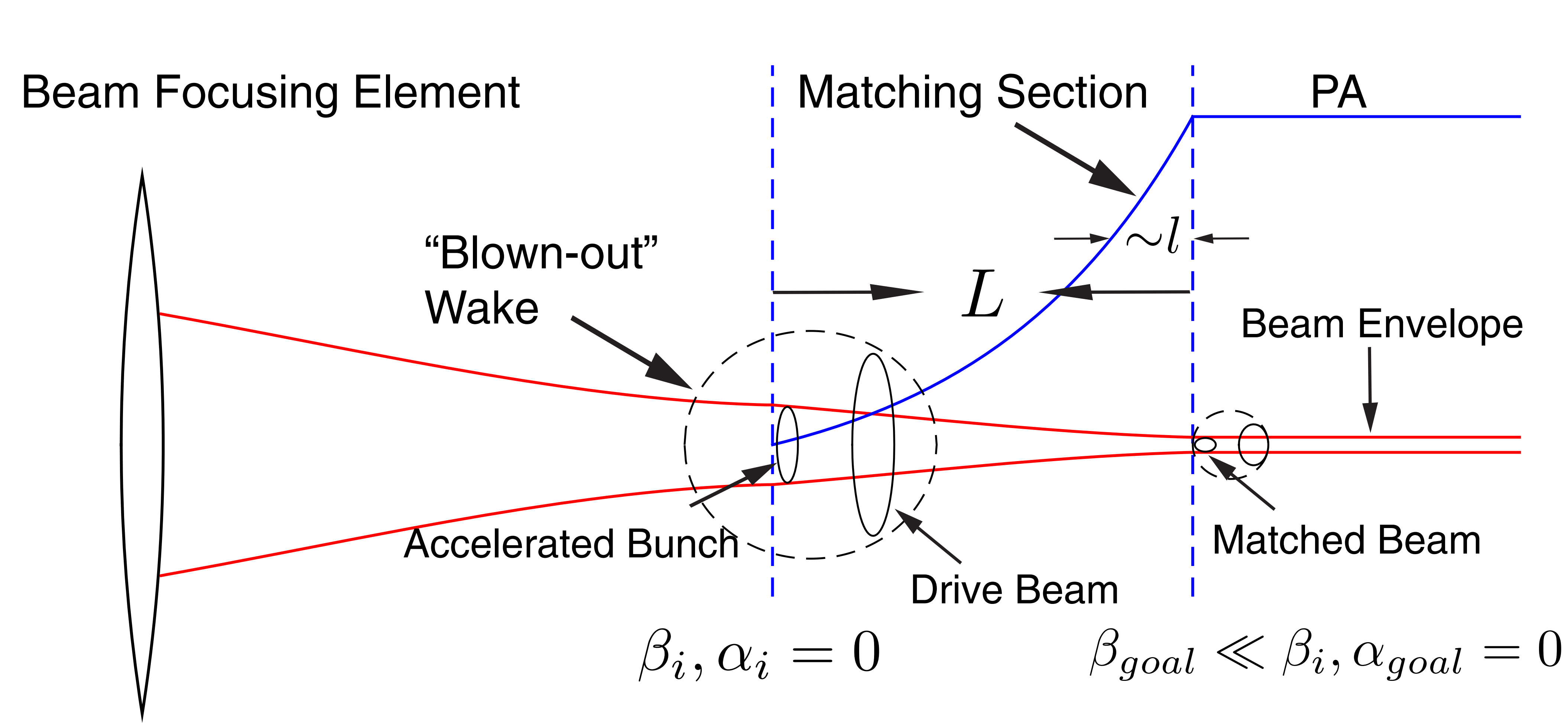
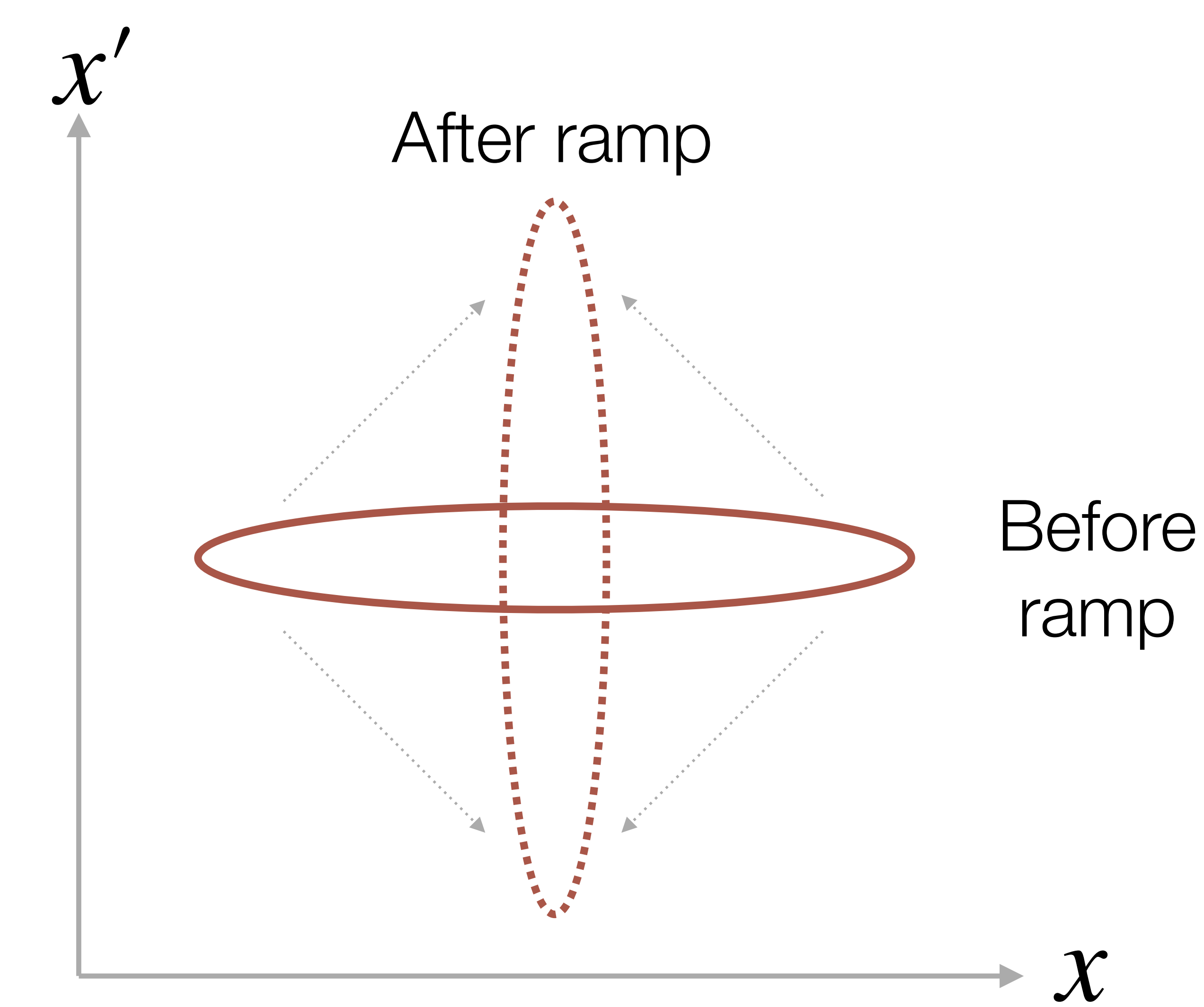


Image source: X. Xu et al., Phys. Rev. Lett. 116, 124801 (2016) [10]



[9] Klaus Floettmann, "Adiabatic matching section for plasma accelerated beams", Phys. Rev. ST Accel. Beams 17, 054402 (2014)
[10] X. L. Xu et al., "Physics of Phase Space Matching for Staging Plasma and Traditional Accelerator Components Using Longitudinally Tailored Plasma Profiles", Phys. Rev. Lett. 116, 124801 (2016)

TRANSVERSE OFFSET TOLERANCES

> Emittance growth from misalignment into a plasma stage:

> (1) Decoherence

> (2) Seed for beam-breakup instability

> Requirement (approximate): Offsets of the witness beam relative to the driver/structure should be smaller than its transverse size.

> For high-energy, low-emittance beams in high-gradient PWFAs, this can be very small: ~**10 nm tolerances** [11, 12]

> (LCLS best: 100–200 nm rms)

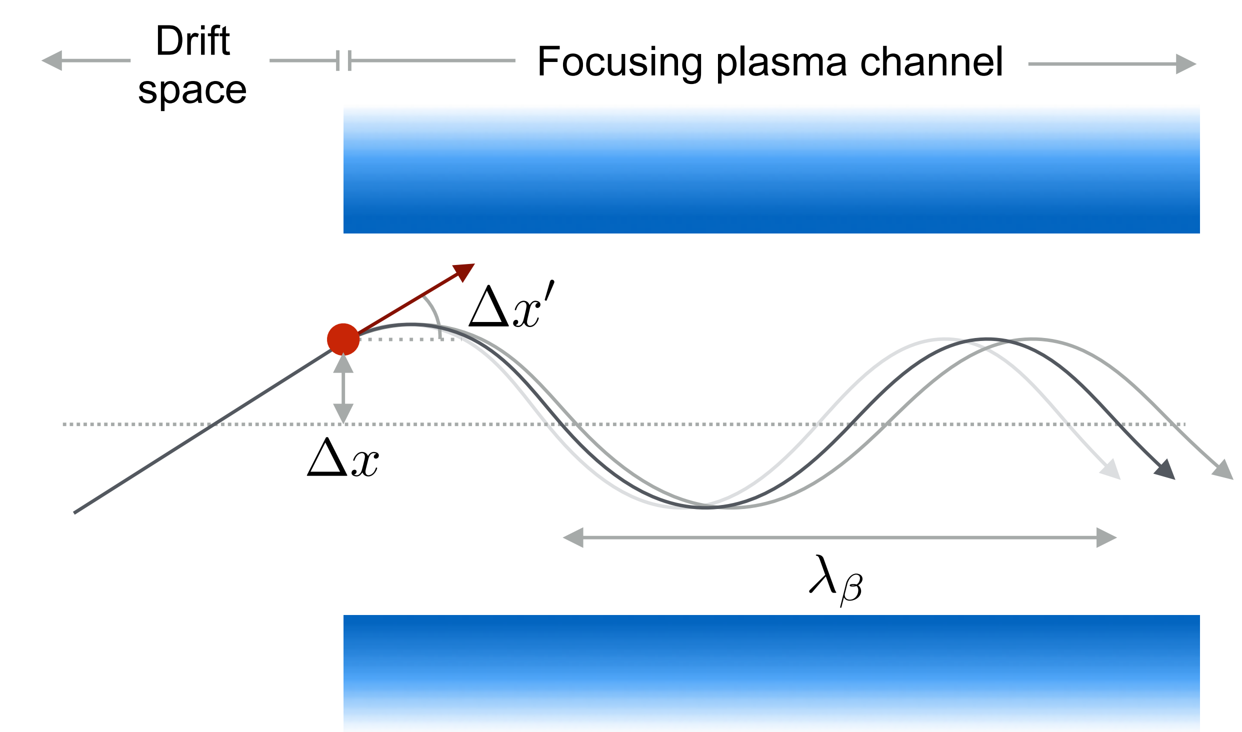
> Linear collider misalignment tolerance at IP is also few nm-level

> **The offset of a beam waist is locked to the offset of the final quads.**

> Similarly, staging optics components just after capture/before refocusing must therefore be aligned to roughly the same level: 1–10 nm

> The main problem is random jitter (static offsets can be removed by tuning).

> Effect of **plasma ramps: position tolerances improve / angular tolerances worsen.**



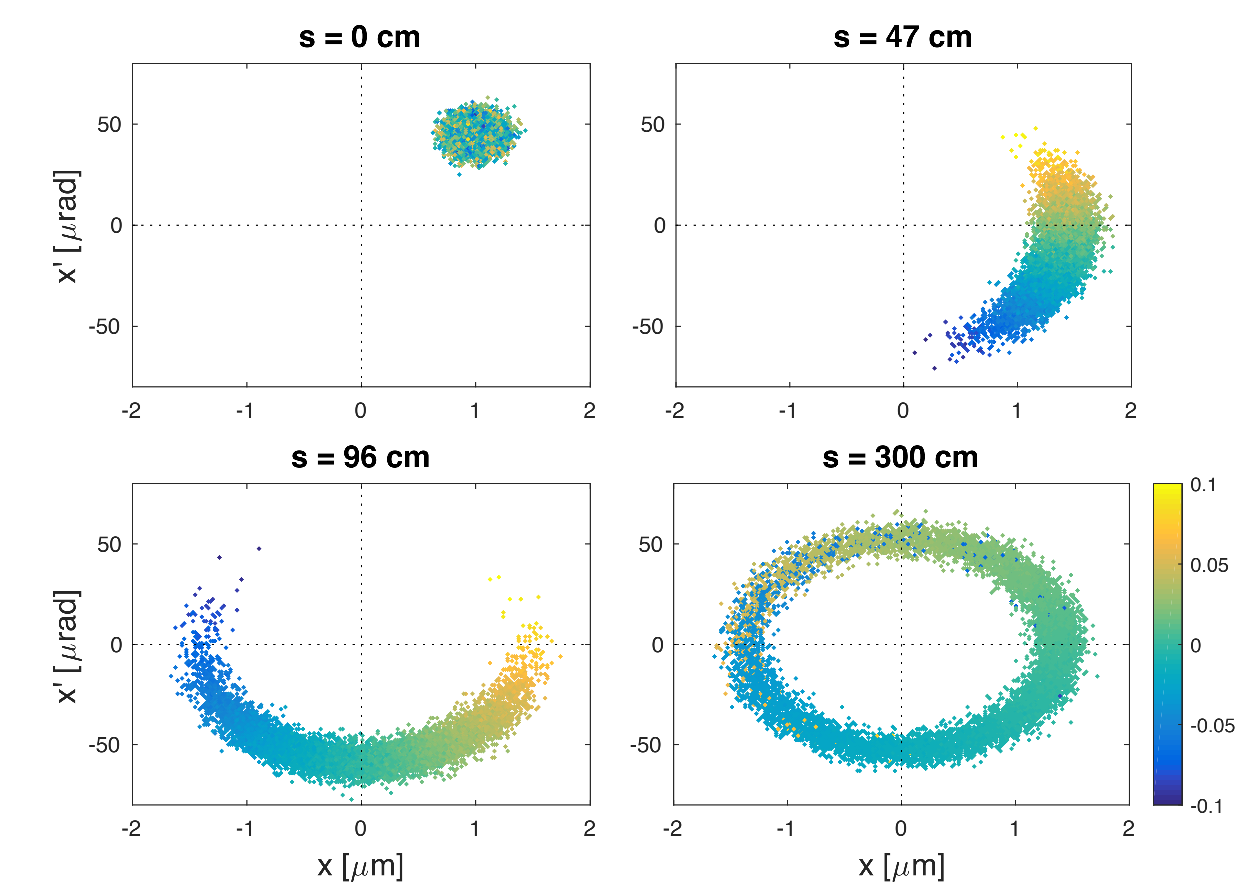


Image source: C. A. Lindstrøm et al., Proc. of IPAC2016, p. 2561 (2016) [12]

[11] Ralph Assmann and Kaoru Yokoya, "Transverse beam dynamics in plasma- based linacs," **Nucl. Instrum. Methods Phys. Res. A 410, 544 (1998)**
[12] C. A. Lindstrøm et al., "Transverse tolerances of a multi-stage plasma wakefield accelerator," **Proc. of IPAC2016, p. 2561 (2016)**

ENERGY SCALING OF STAGING LENGTH

> Minimum staging length scales as $\sqrt{E}$

> Matched beta function scales as $\sqrt{E}$

> **In the high-energy regime (limited by focusing strength), staging length scales as $\sqrt{E}$**

> **All beta functions also scale as $\sqrt{E}$**

⇒ The same optics is valid for any energy (scaled by $\sqrt{E}$)

> The relative emittance growth from chromaticity is the same for each stage (independent of energy).

> *Question:*
Does this mean that wakefield accelerators will eventually be worse than conventional accelerators (scaling as E)?

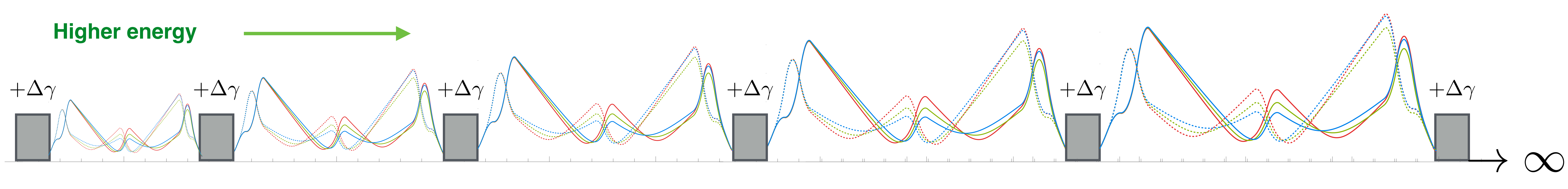
$$L_{\min} = \frac{4}{\sqrt{k}} = 4\sqrt{\frac{E}{g_r e c}}$$

$$\beta_m = \frac{\sqrt{2\gamma}}{k_p}$$

$$\beta_{\max} = \beta_m + \frac{L^{*2}}{\beta_m} \approx \sim \sqrt{E}$$

Emittance growth from chromaticity
(does not depend on energy)

$$\frac{\Delta\epsilon_n^2}{\epsilon_n^2} = \frac{4L^{*2}}{\beta_m^2}\sigma_\delta^2 + \mathcal{O}(\sigma_\delta^4)$$



Alternative staging schemes — Logarithmic/Recursive staging

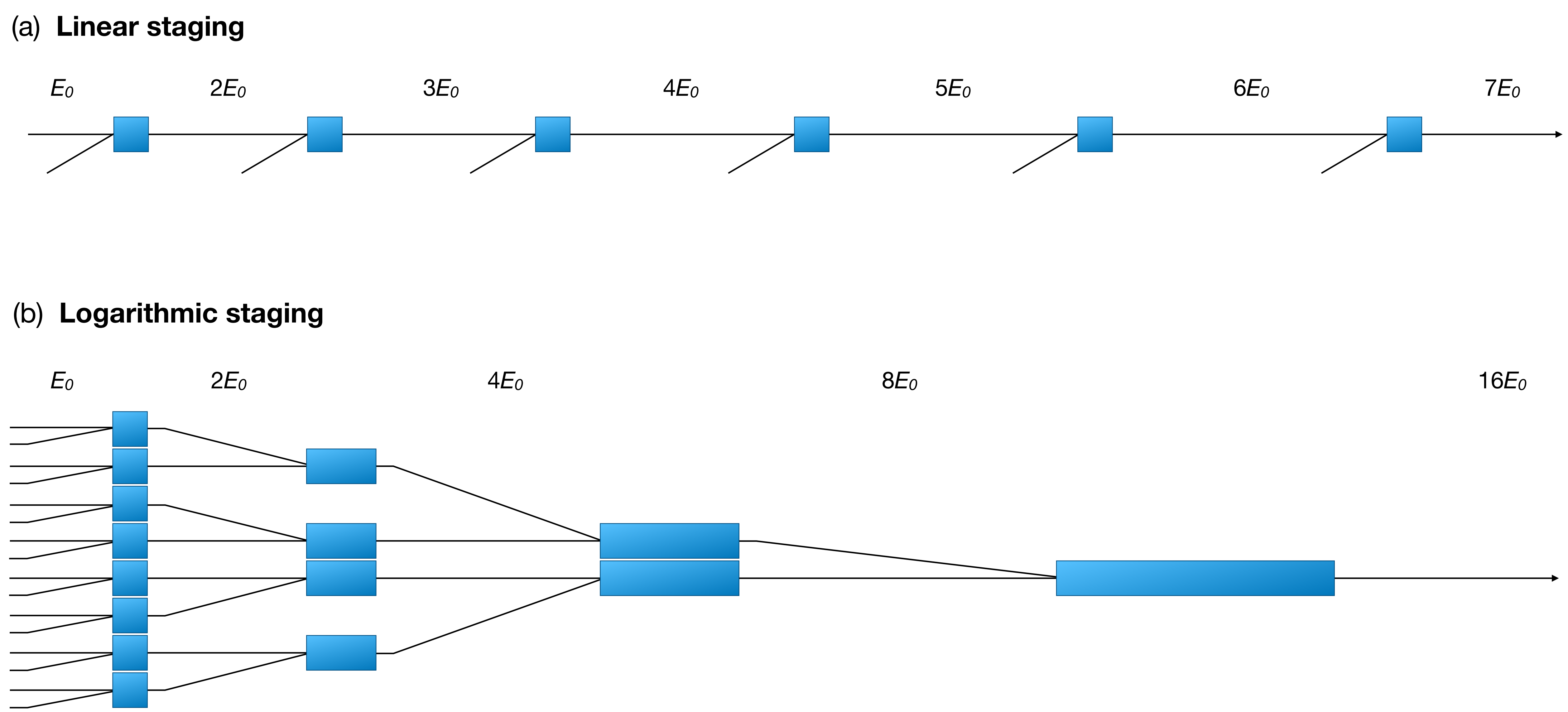
> Can reduce overall staging length by using higher energy drivers.

> For beam-driven PWFA, we can use an witness bunch as a higher-energy drive bunch

> Leads to **exponential (not linear) growth of energy per stage**—but also exponentially longer plasma stages.

⇒ Fewer stages, less staging length

> Requires high driver–witness energy transfer efficiency to be useful.



> *This is included in **Case Study 4** (so we won't reveal all the conclusions)*

SUGGESTED STAGING TECHNIQUES

> Single-stage / focus-free acceleration
> Sextupoles in dispersive sections
> A(po)chromatic optics
> Passive and active plasma lenses
> Bent channels

HIGH-TOTAL-ENERGY DRIVERS (AVOIDING STAGING 1/2)

> If all the energy required is already contained in one driver, **single-stage acceleration** is sufficient.

> Cannot use a single high total energy electron/positron driver, as electron/positron acceleration is then unnecessary.

> However, two options are attractive:

> > **Proton bunches**, which can be accelerated to high energy in synchrotrons to contain several kJ per bunch (laser/electron bunches: ~J).

> > **Bunch trains**, resonantly driving the wakefield. Possible with both particle and laser [13] beams.

> The AWAKE experiment at CERN is using both of these concepts, by employing self-modulation of proton bunches into bunch trains [15].

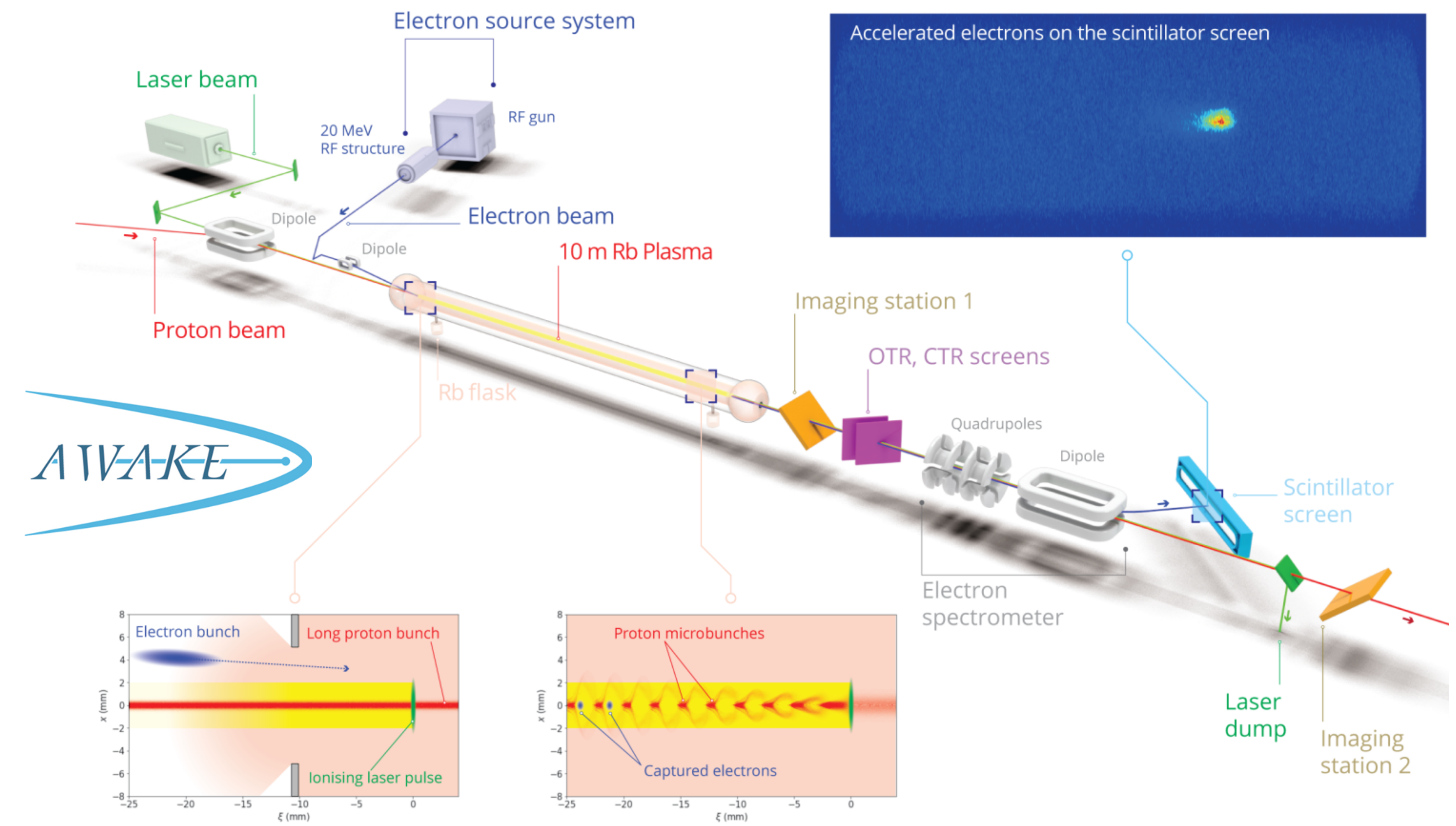


Image source: E. Adli et al. (AWAKE Collaboration), Nature 561, 363–367 (2018) [15]

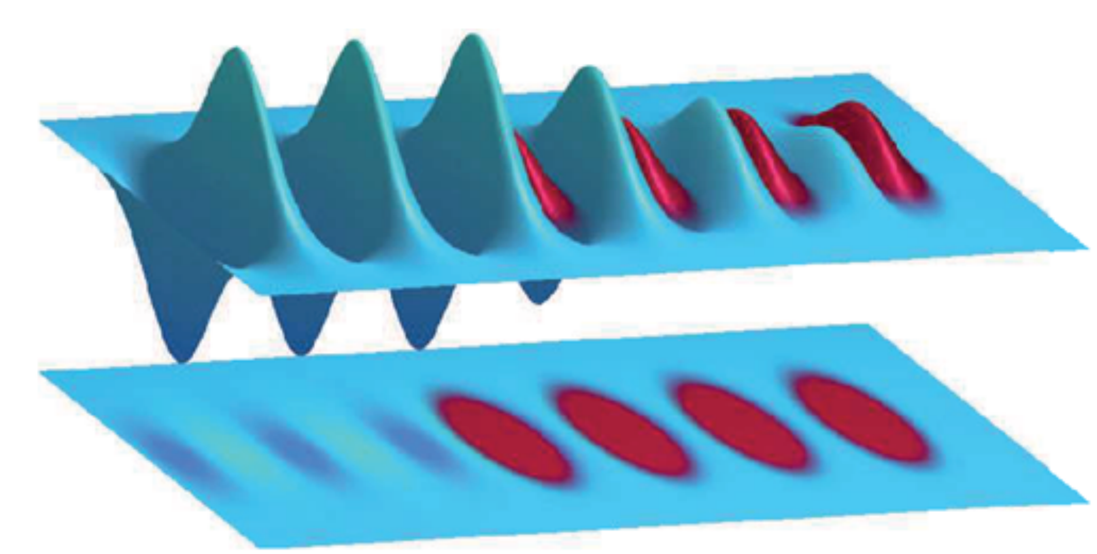


Image source: S. Hooker/University of Oxford

[13] J. Cowley et al., "Excitation and Control of Plasma Wakefields by Multiple Laser Pulses", **Phys. Rev. Lett. 119, 044802 (2017)**
[14] K. Nakajima et al., "Plasma wake-field accelerator experiments at KEK," **Nucl. Instrum. Methods Phys. Res. A 292, 12 (1990)**
[15] E. Adli et al. (AWAKE Collaboration), "Acceleration of electrons in the plasma wakefield of a proton bunch", **Nature 561, 363–367 (2018)**

Dielectric or Hollow Plasma Channels (avoiding staging 2/2)

> Structure-based dielectric [16] or hollow channel plasma [17] wakefield accelerators can provide strong acceleration (GV/m-scale) **without on-axis focusing**.

> No strong focusing removes the chromaticity problem
⇒ Easier staging (less focusing optics)

> However, very strong **dipole-like transverse wakefields** [18, 19] lead to beam breakup and strong witness bunch deflections.

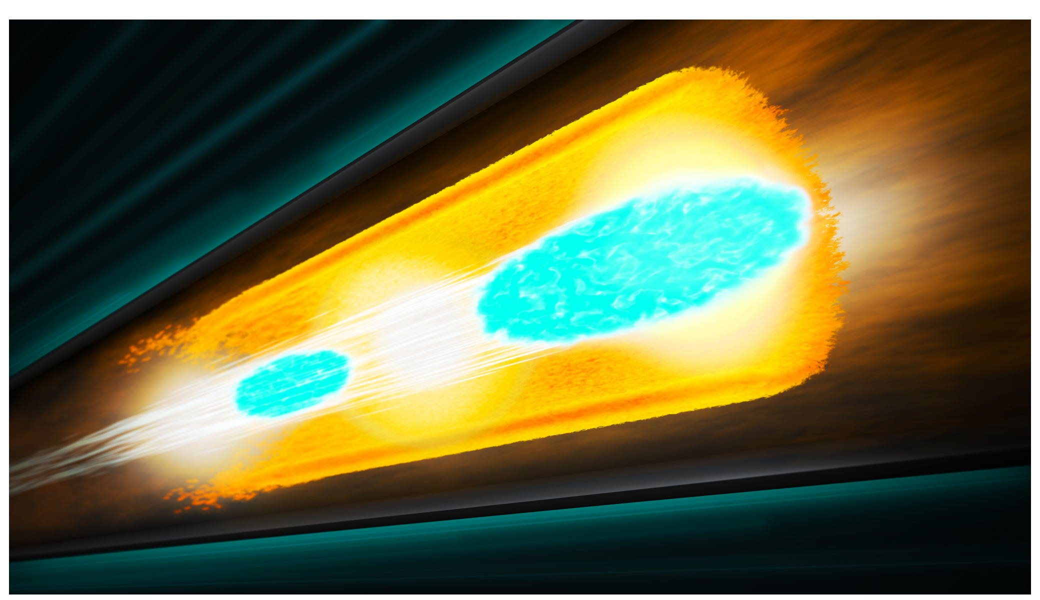


Image source: SLAC (2016)

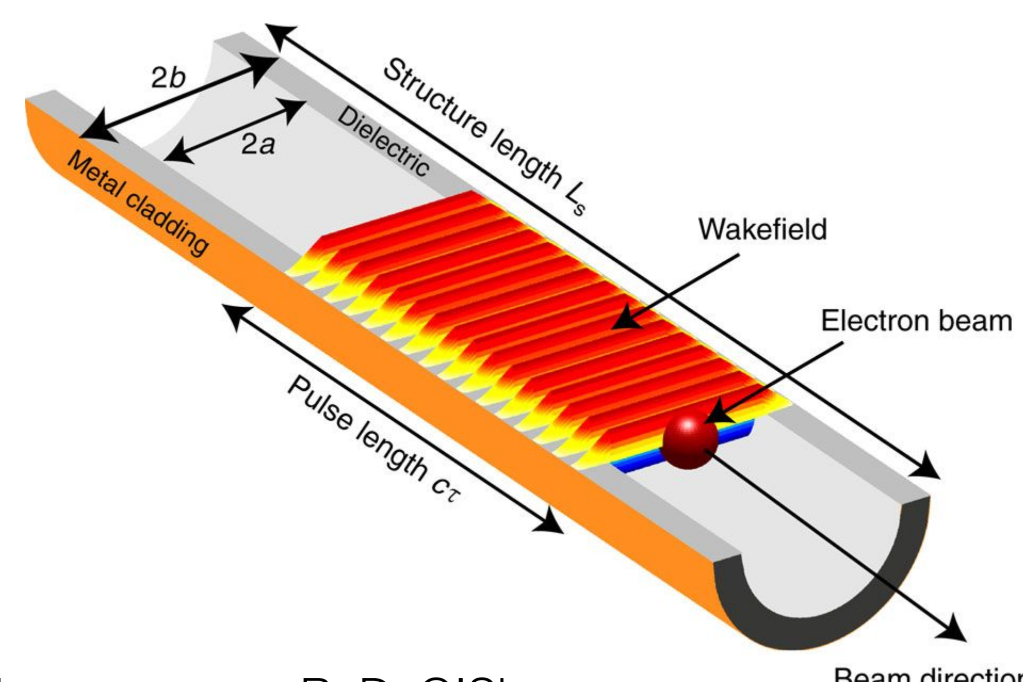


Image source: B. D. O'Shea, Nat. Commun. 7, 12763 (2016) [16]

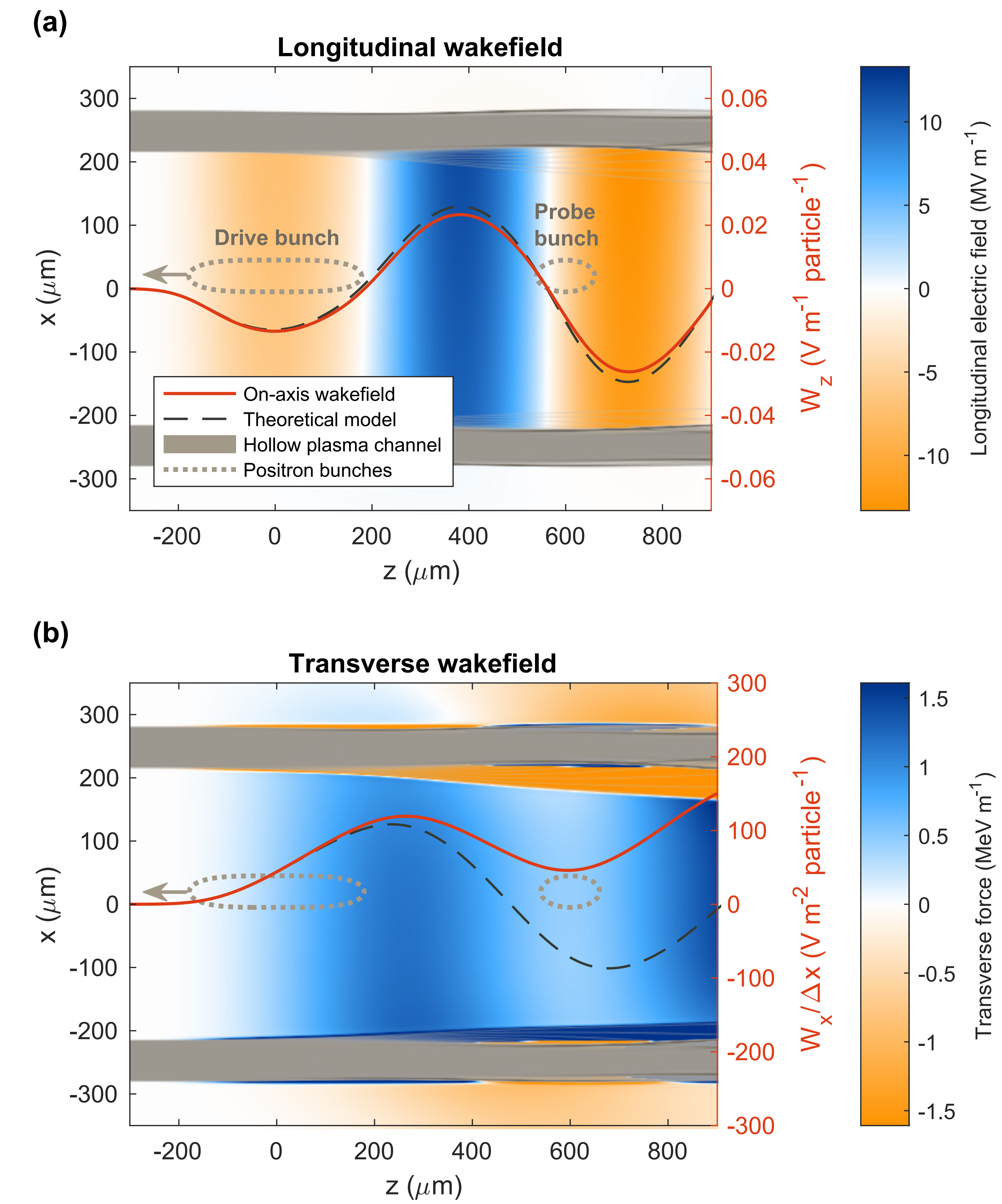


Image source: C. A. Lindstrøm et al., Phys. Rev. Lett. 120, 124802 (2018) [19]

[16] B. D. O'Shea et al., "Observation of acceleration and deceleration in gigaelectron-volt-per-metre gradient dielectric wakefield accelerators", **Nat. Commun. 7, 12763 (2016)**
[17] S. Gessner et al., "Demonstration of a positron beam-driven hollow channel plasma wakefield accelerator", **Nat. Commun. 7, 11785 (2016)**
[18] C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, "Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator", **Phys. Rev. Lett. 82, 1177 (1999)**
[19] C. A. Lindstrøm et al., "Measurement of Transverse Wakefields Induced by a Misaligned Positron Bunch in a Hollow Channel Plasma Accelerator", **Phys. Rev. Lett. 120, 124802 (2018)**

Sextupoles in dispersive sections (the conventional solution)

- > **Sextupoles: Different focal length at each offset** (+ "geometric terms")
  - > Energetically disperse the beam onto sextupole (different energy at each offset)
  - > Adjust sextupole strength to cancel chromaticity
  - > Cancel the geometric terms (*B* ~ *xy*, etc.)
- > Used in final focus systems, where betas are demagnified by 10^6–10^9
- > Two methods of chromaticity cancellation [20]
  - > ***Global* chromaticity correction** (used in Stanford Linear Collider)
  - > ***Local* chromaticity correction** (planned for ILC, CLIC, FCC, CEPC)

Sextupole B-fields:

$$B_x \sim xy + \delta D_x y$$

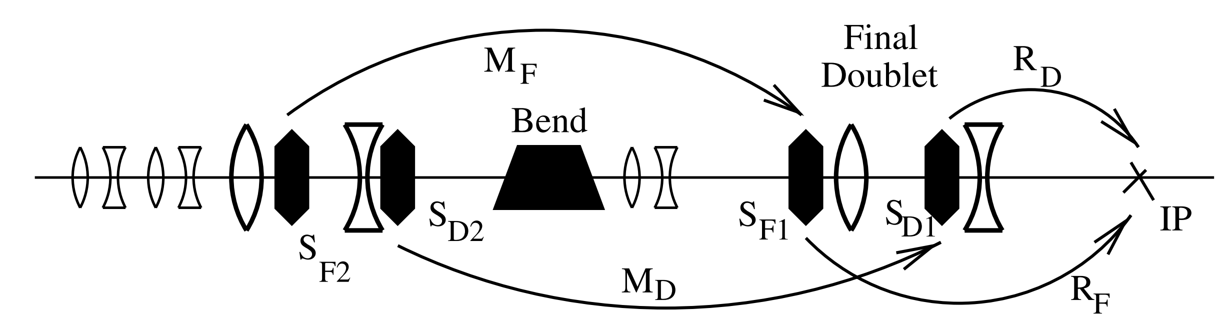
Non-linear geometric terms

Linear chromatic terms
CORRECT CHROMATICITY

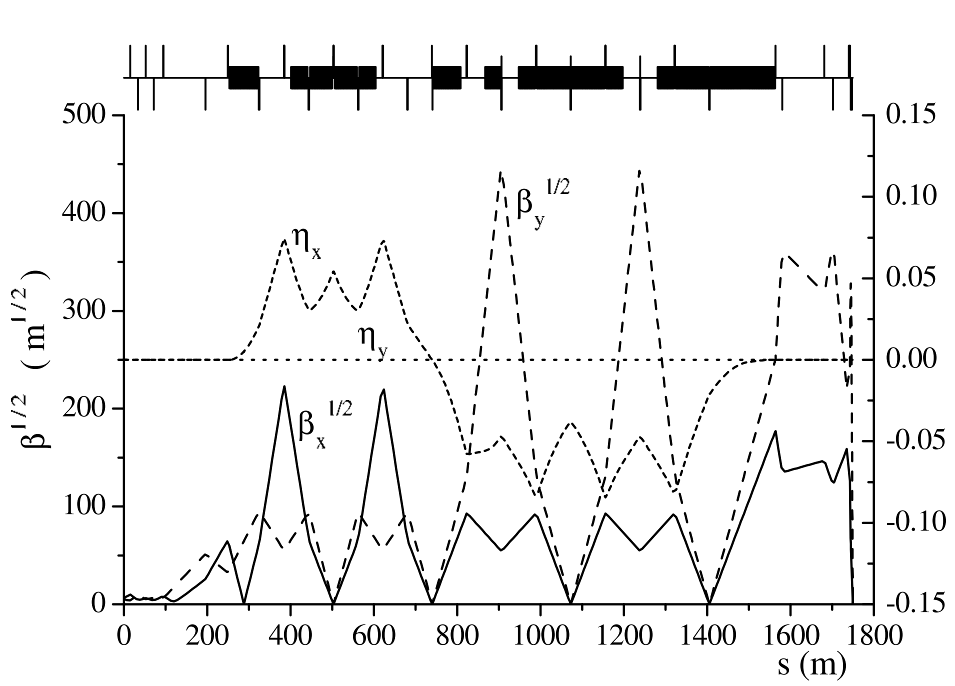
$$B_y \sim \frac{1}{2}(x^2 - y^2) + x\delta D_x + \frac{1}{2}\delta^2 D_x^2$$

Non-linear chromatic term

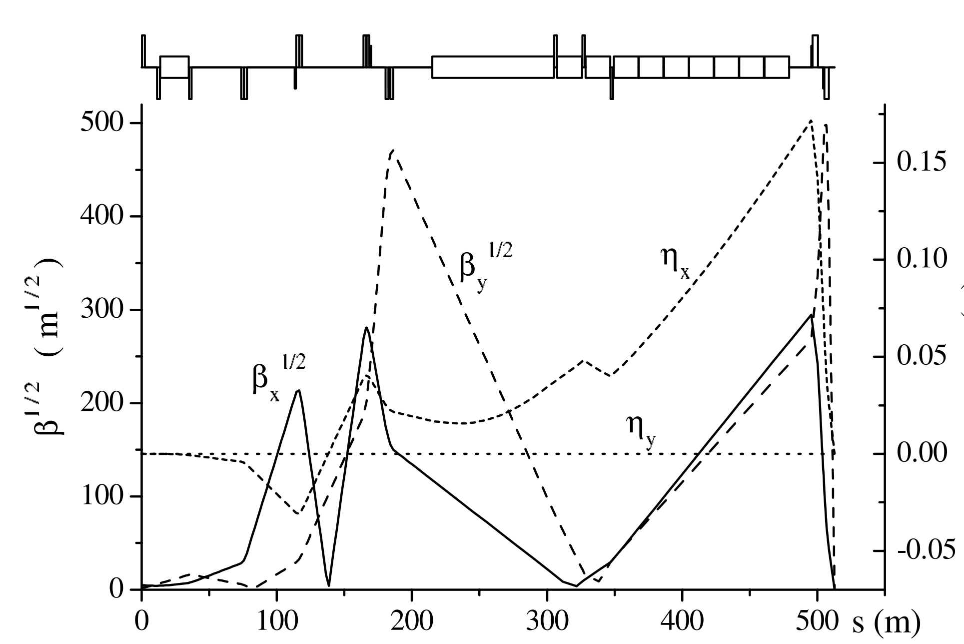
Geometric term cancellation:



Global correction



Local correction (6x shorter)



Comparison of IP beam size

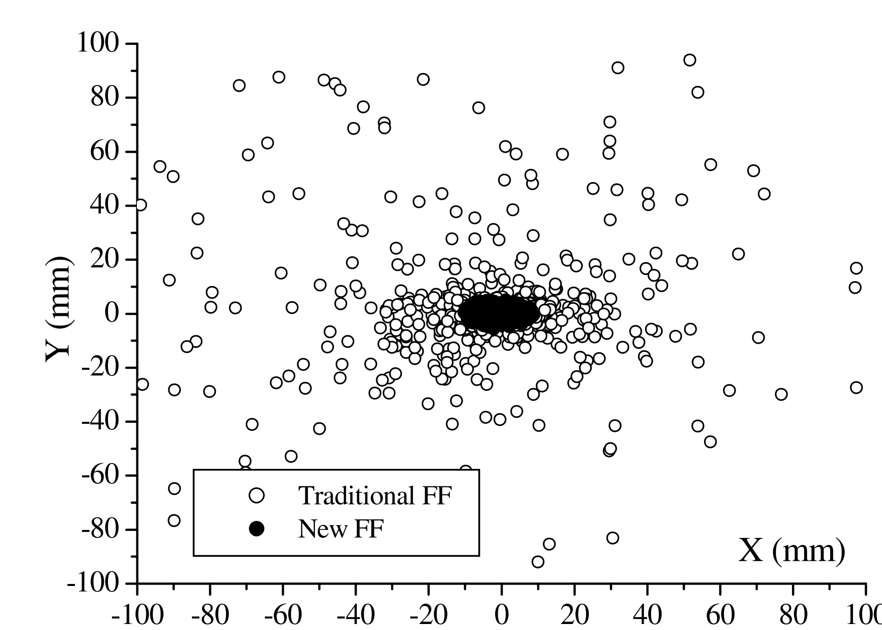


Image source (all): P. Raimondi and A. Seryi, Phys. Rev. Lett. 86, 3779 (2001) [18]

[20] P. Raimondi and A. Seryi, "Novel Final Focus Design for Future Linear Colliders", ***Phys. Rev. Lett. 86, 3779 (2001)***

SEXTUPOLES IN DISPERSIVE SECTIONS (THE CONVENTIONAL SOLUTION)

> Example solution (very complex, large number of magnets) [2] — Effectively two back-to-back final focus systems

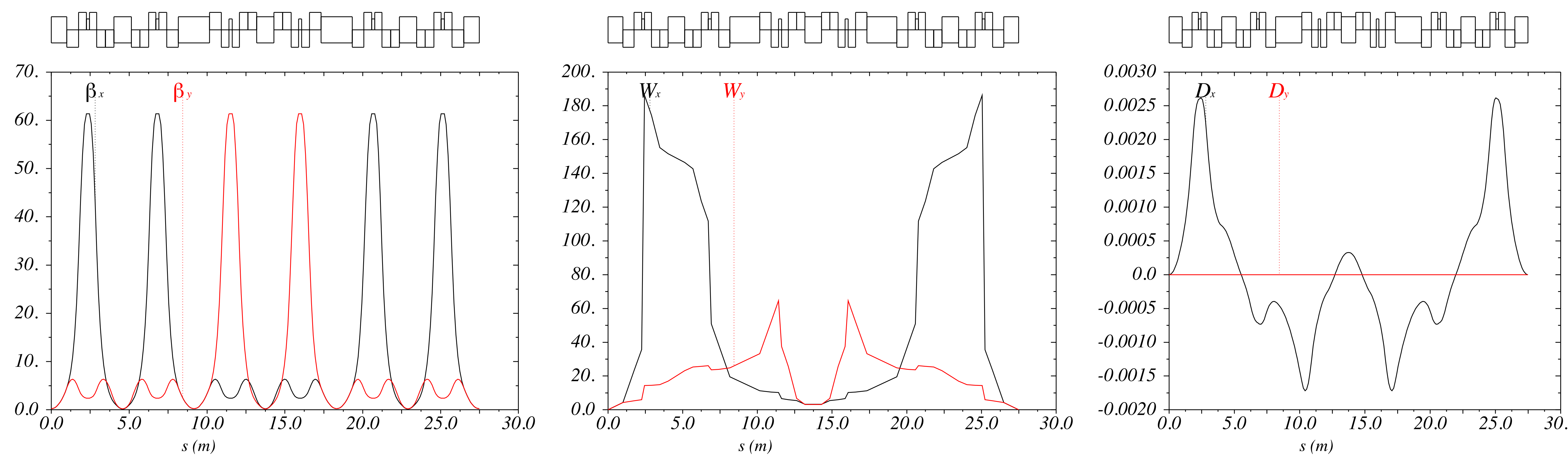


Image source: C. A. Lindstrøm, PhD thesis (2019)

> **Pros**:

> Currently the only successfully demonstrated method (FFTB at SLAC, ATF2 at KEK)

> **Can correct large chromaticity**

> **Cons**:

> Introduces nonlinear terms, requires long sections to cancel

> Requires strong dipoles (large dispersion): **scales unfavourably to high energy** regarding synchrotron radiation.

[2] C. A. Lindstrøm, "Emittance growth and preservation in a plasma-based linear collider", ***PhD thesis (University of Oslo, 2019)***

A(PO)CHROMATIC CORRECTION (1/3)

> Possible to cancel chromaticity at a given location using only linear optics.

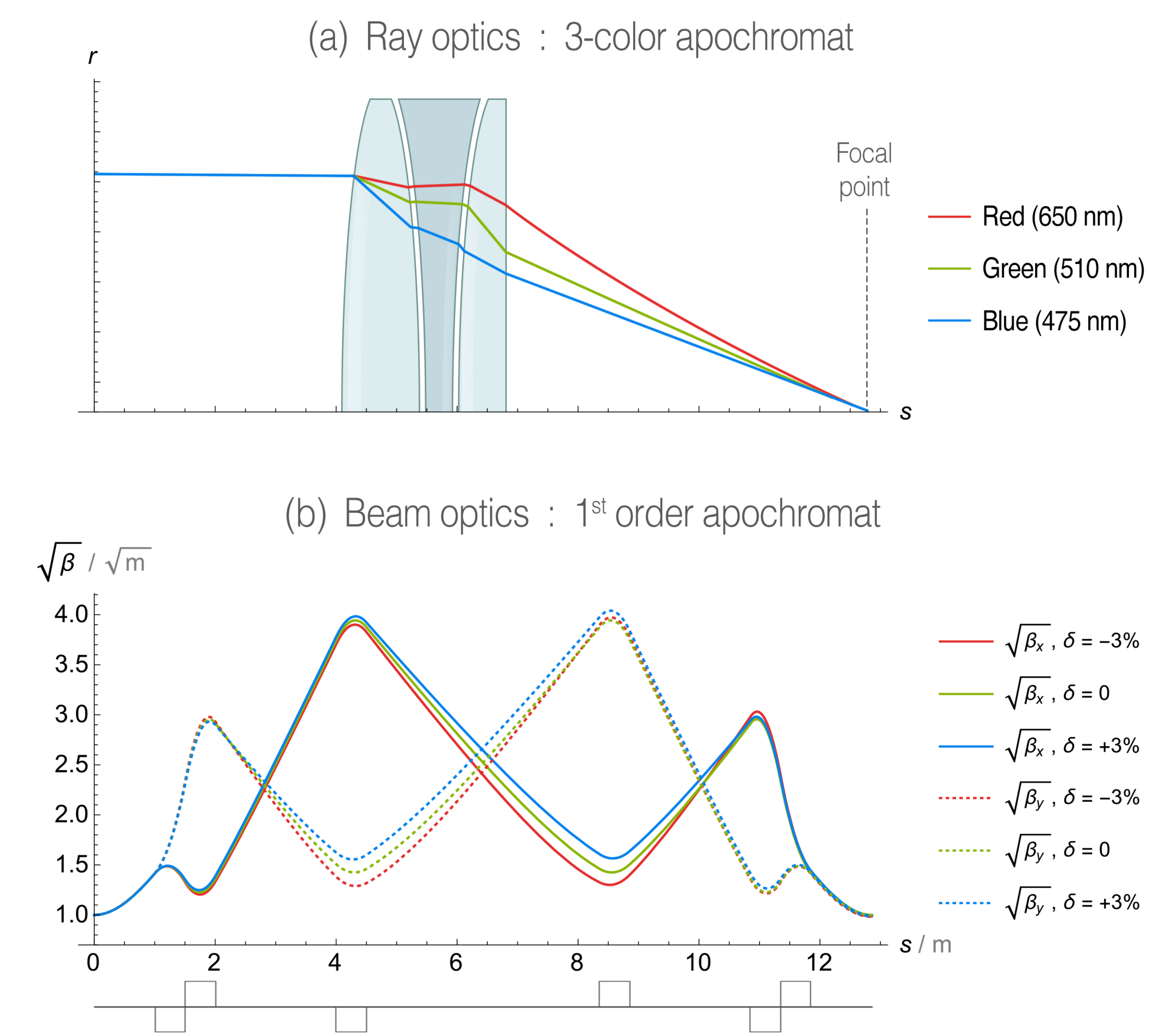
> First suggested by Montague and Ruggiero (CLIC) in 1987 [21]: they called in "apochromatic focusing".

> Working principle:

> > **Each energy slice traverses the lattice differently, but end up with the same Twiss parameters.**

> > The phase advance of particles at different energy still varies with energy (requires sextupoles to cancel)

> **Limited in how much energy spread it can correct for**, must therefore be used in tandem with plasma density ramps or similar [8].



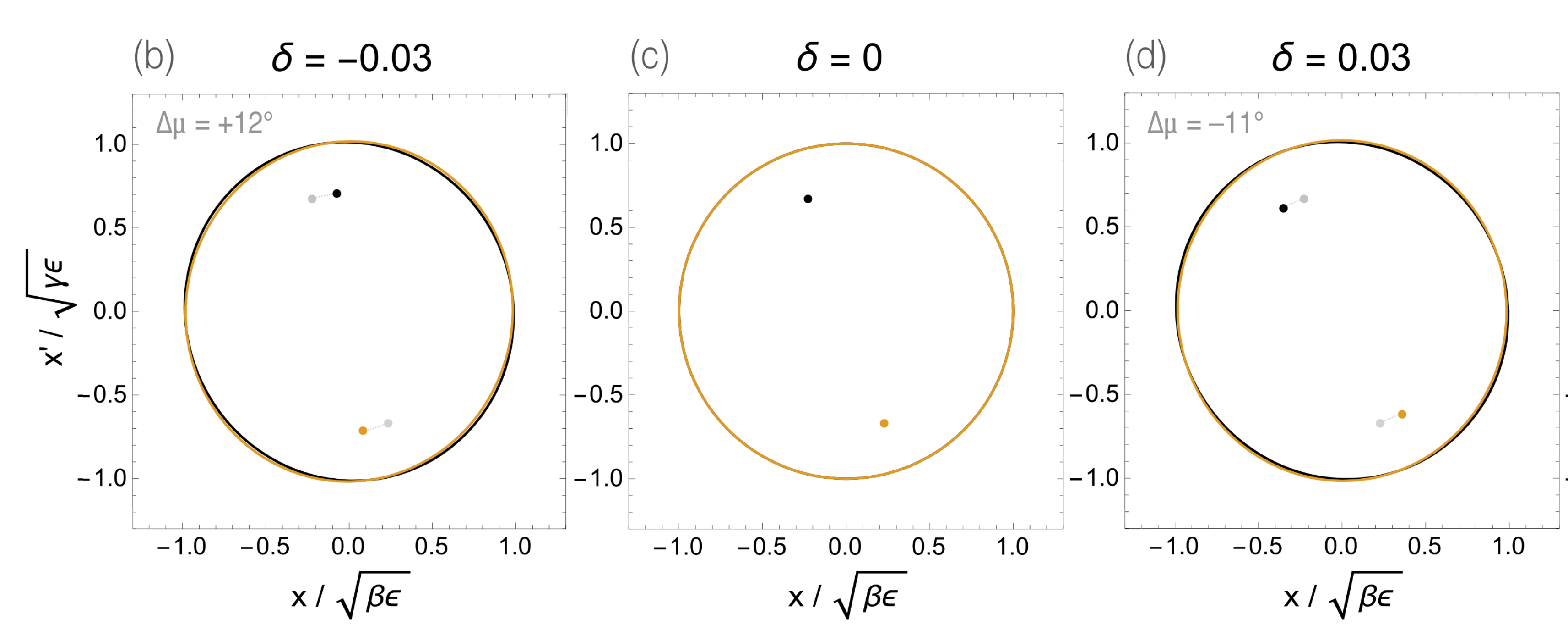


Image source (all): C. A. Lindstrøm and E. Adli, Phys. Rev. Accel. Beams 19, 071002 (2016) [8]

[21] Brian W. Montague and Francesco Ruggiero, "Apochromatic focusing for linear colliders," ***CLIC Note 37 (CERN, Geneva, 1987)***

A(PO)CHROMATIC CORRECTION (2/3)

> Chromaticity can be canceled to any order, at the cost of longer lattices.

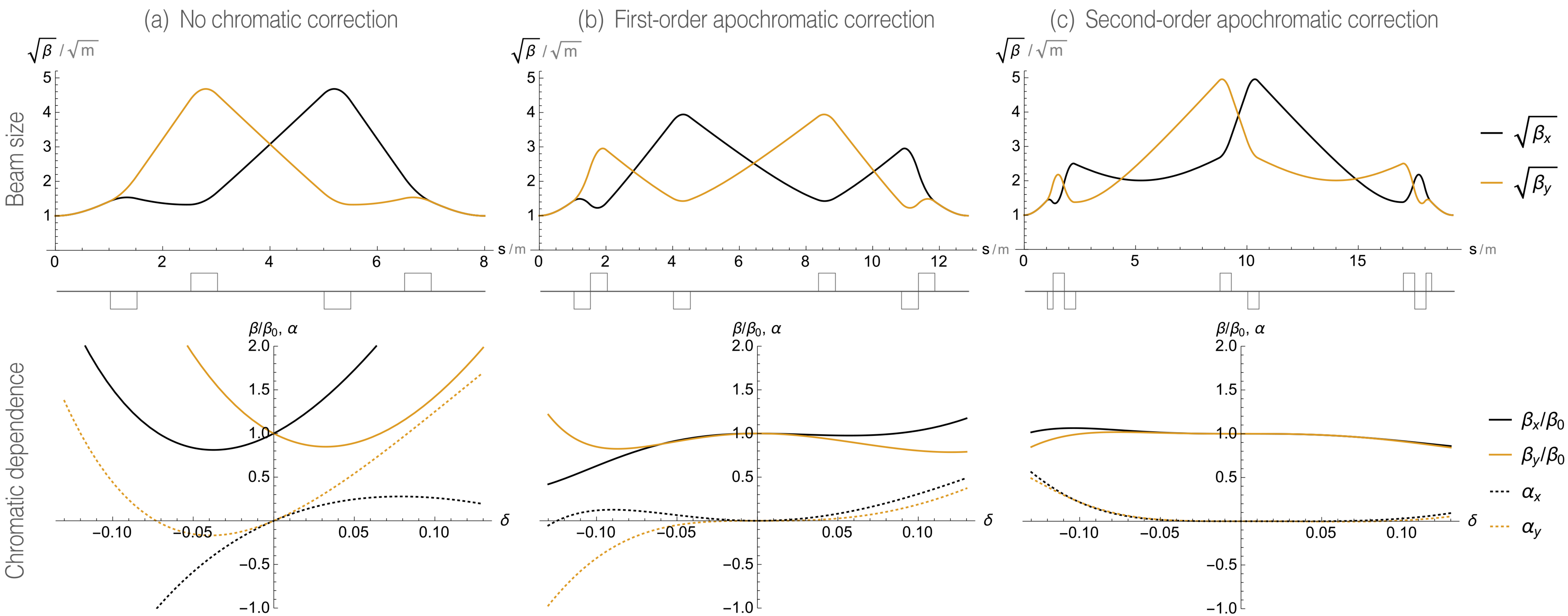
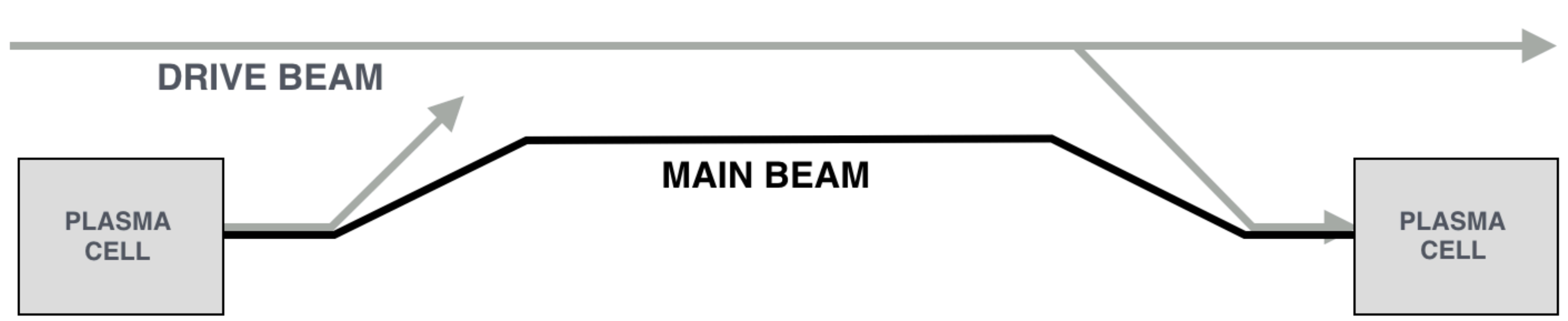


Image source (all): C. A. Lindstrøm and E. Adli, Phys. Rev. Accel. Beams 19, 071002 (2016) [22]

A(PO)CHROMATIC CORRECTION (3/3)

- Example solution [22]:
  - Working staging optics for a 500 GeV, 0.5% rms energy spread, 80 cm matched beta
  - 39 m long, 5 dipoles, 8 quadrupoles
  - Cancels (1st order) chromaticity and (1st order) dispersion.
  - 1% emittance growth (due to 2nd order dispersion)



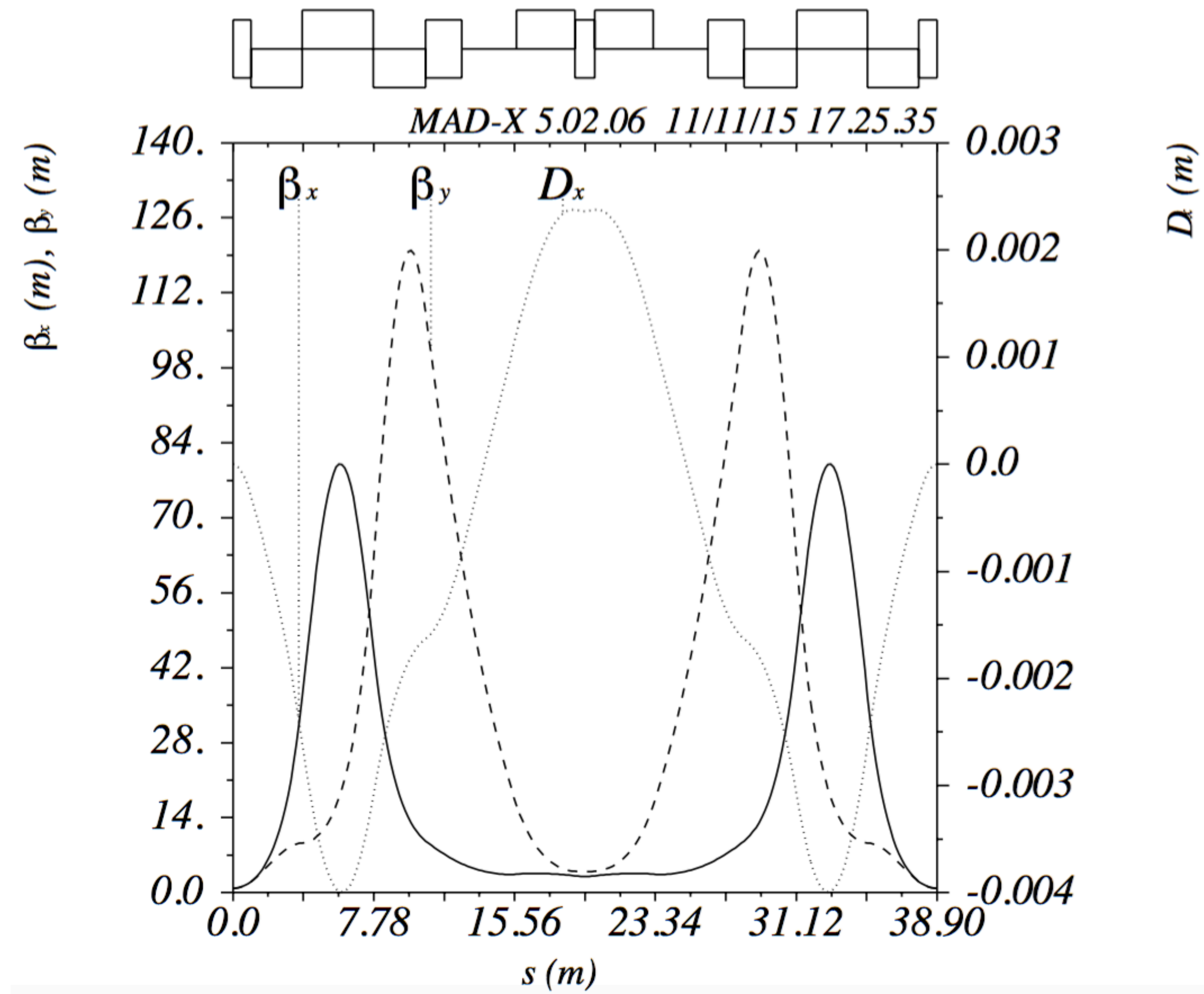


Image source: C. A. Lindstrøm et al., Nucl. Instrum. Methods Phys. Res. A 829, 224–228 (2016) [23]

[22] C. A. Lindstrøm et al., "Staging optics considerations for a plasma wakefield acceleration linear collider", **Nucl. Instrum. Methods Phys. Res. A 829, 224–228 (2016)**.

SIDE NOTE: RADIAL FOCUSING

- *Question: How does the effective focusing strength of a quadrupole channel (focus–defocus-etc) compare to that of a radial focusing channel (focusing in both planes)?*
- To achieve the same matched beta function, **a quadrupole channel has to be ~10 times stronger than a radial focusing channel with the same gradient** [2].

$$g_{r,\text{quads}} \approx \frac{g_r}{10}$$

- In addition, quadrupole channels are not very effective for other energies — is **defocusing for lower energies**!
- Conclusion: it is highly beneficial to use radial focusing optics compared to quadrupole optics.
- Maxwell's equations dictate that **radial focusing is impossible in a vacuum**: on-axis current or charge is required.

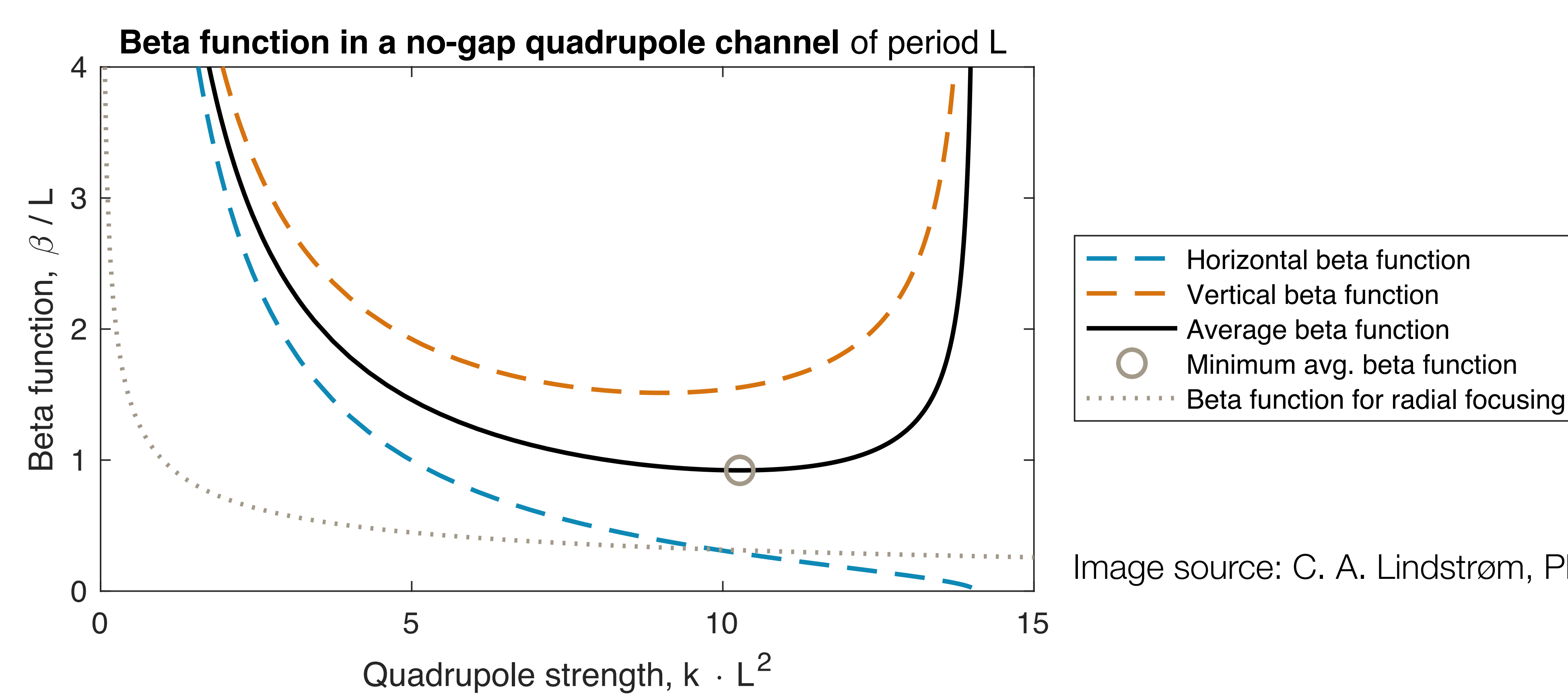


Image source: C. A. Lindstrøm, PhD thesis (2019)

[2] C. A. Lindstrøm, "Emittance growth and preservation in a plasma-based linear collider", ***PhD thesis (University of Oslo, 2019)***

PASSIVE PLASMA LENSES

> (1922) The principle was used for cathode rays (J. B. Johnson) [24].
(1987) Proposed in its current form by P. Chen [23].
(1990) First demonstrated by at Argonne by J. Rosenzweig *et al.* [25].

> Principle: Electrostatic focusing by an ion column (same focusing as in plasma wakefield accelerators)

$$g_r = \frac{en_0}{2c\epsilon_0}$$

> Very **strong focusing gradients**.

> Example: $n_0 = 10^{17}$ cm^{-3} $\Rightarrow$ $g_r = 3$ MT/m

> The plasma density ramp is an example of a passive plasma lens.

> Generally **non-uniform focusing in the longitudinal and transverse**

> Uniform focusing requires a driver (e.g. blowout regime) – only works for electrons, not positrons/protons

> Beam-based: hard to meet transverse tolerance requirements.

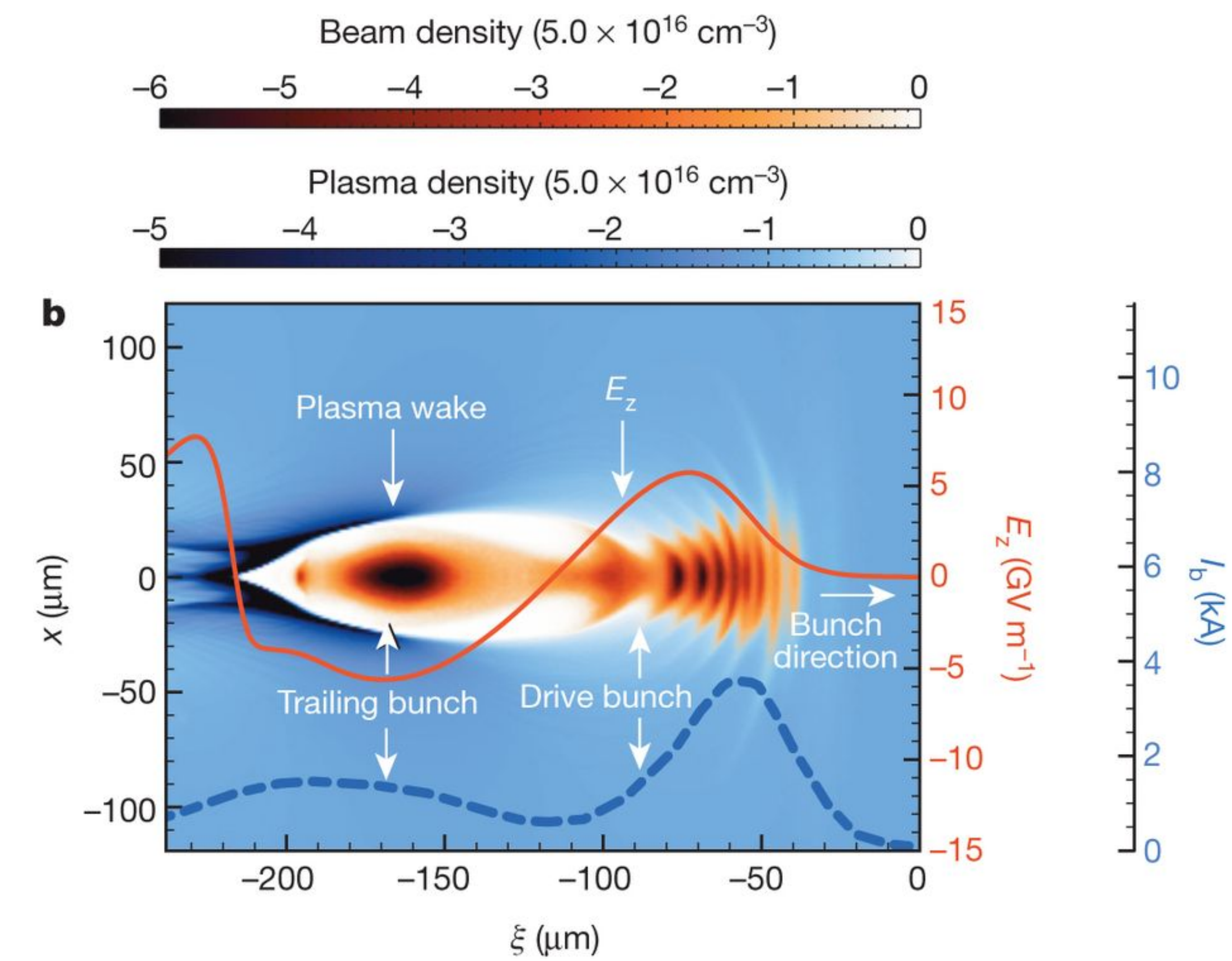


Image source: M. Litos et al., Nature 515, 92–95 (2014) [26]

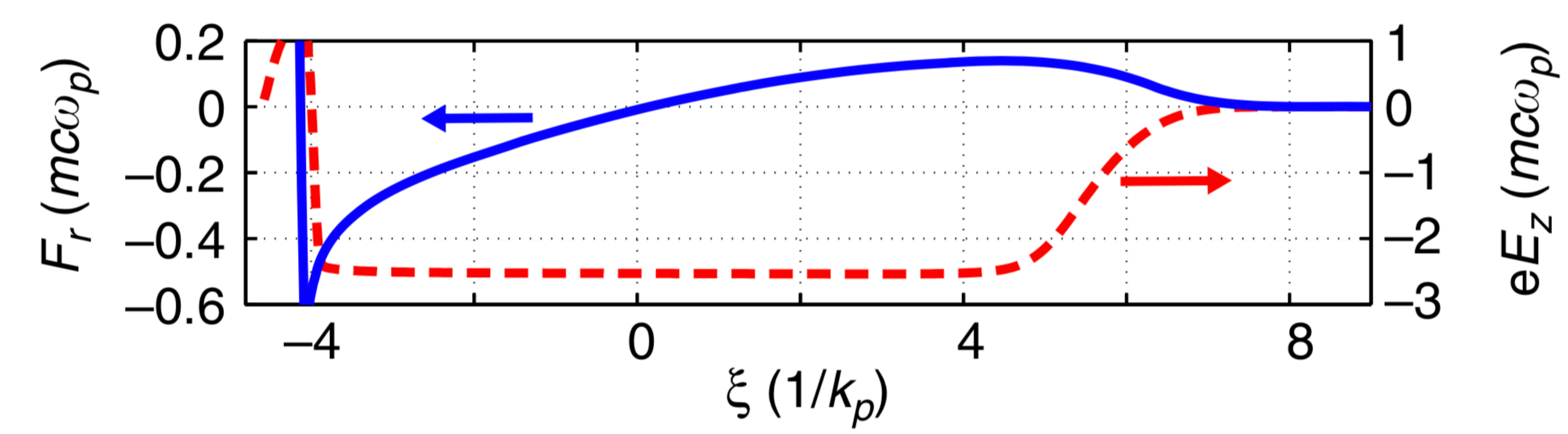


Image source (adapted): C. E. Clayton et al., Nat. Commun. 7, 12483 (2016) [27]

[23] Pisin Chen, "A possible final focusing mechanism for linear colliders," **Part. Accel. 20, 171 (1987)**.
[24] John B. Johnson, "A low voltage cathode ray oscillograph," **J. Opt. Soc. Am. 6, 701 (1922)**.
[25] J. B. Rosenzweig et al., "Demonstration of electron beam selffocusing in plasma wake fields," **Phys. Fluids B 2, 1376 (1990)**
[26] M. Litos et al., "High-efficiency acceleration of an electron beam in a plasma wakefield accelerator", **Nature 515, 92–95 (2014)**
[27] C. E. Clayton et al., "Self-mapping the longitudinal field structure of a nonlinear plasma accelerator cavity", **Nat. Commun. 7, 12483 (2016)**

ACTIVE PLASMA LENSES

- First use by Panofsky and Baker [28] at the Berkeley Rad Lab in 1950.
- Principle of operation:
  - **Uniform longitudinal current density**
  - Sets up **B-field which increases linearly with radius**
- Fields up to 3 kT/m have been demonstrated [29].
- Charge-agnostic: Works for both electrons and positrons/protons.
- Several labs pursuing this technology (LBNL, DESY, INFN, CERN).
- Not beam-based: better transverse stability.
- Nonlinear focusing fields: can be mitigated with high-Z gases (argon) [30]
- Emittance preservation has been demonstrated for low charge (20 pC)
- **Limited by passive plasma lensing (for intense bunches):** may be no possibility for using APLs for linear collider/FEL beams [31]
- Limited focusing strength from z-pinching (self-focusing of the current)
- Gas scattering in the lens (large beta functions + high plasma density)

$$g_r = \frac{\mu_0 I}{2\pi R^2}$$

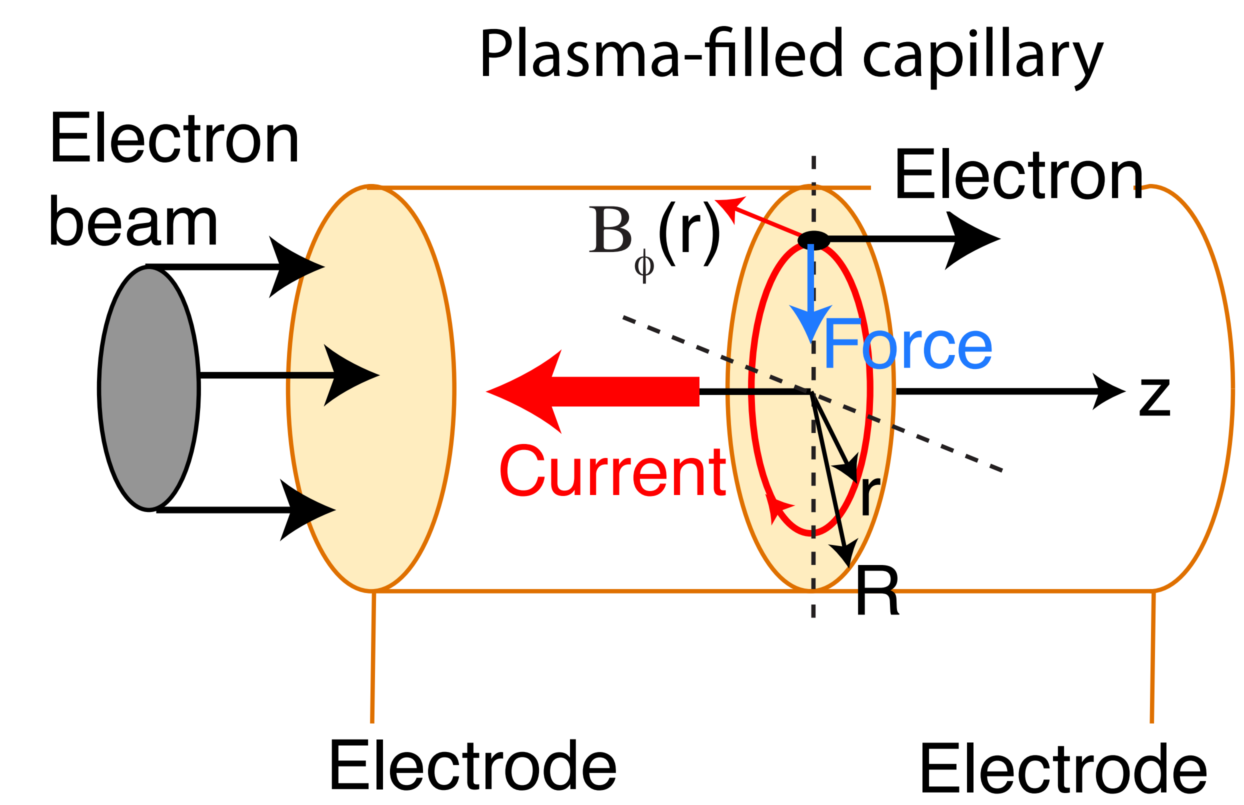


Image source: J. van Tilborg et al., Phys. Rev. Lett. 115, 184802 (2015) [29]

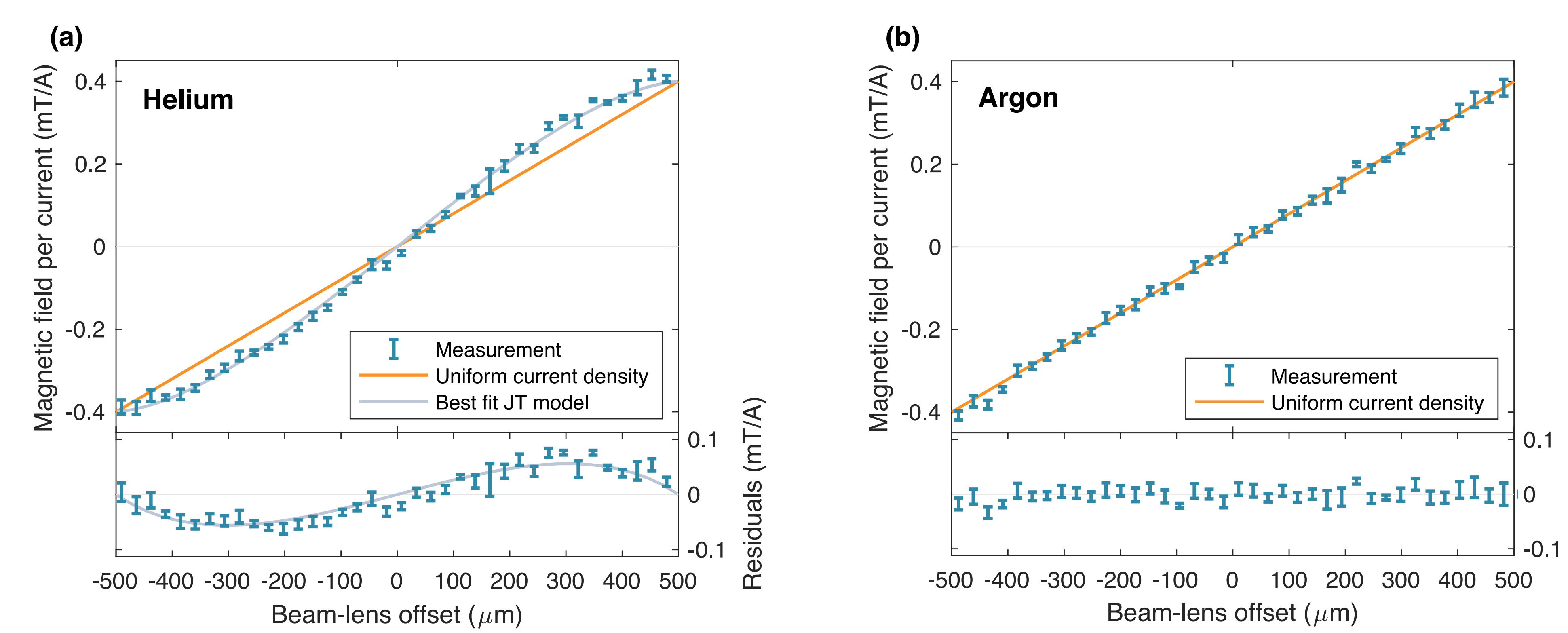


Image source: C. A. Lindstrøm et al., Phys. Rev. Lett. 121, 194801 (2018) [30]

[28] W. K. H. Panofsky and W. R. Baker, "A focusing device for the external 350-Mev proton beam of the 184-inch cyclotron at Berkeley," **Rev. Sci. Instrum. 21, 445 (1950)**
[29] J. van Tilborg et al., "Active plasma lensing for relativistic laser-plasma- accelerated electron beams," **Phys. Rev. Lett. 115, 184802 (2015)**
[30] C. A. Lindstrøm et al., "Emittance preservation in an aberration-free active plasma lens," **Phys. Rev. Lett. 121, 194801 (2018)**
[31] C. A. Lindstrøm and E. Adli, "Analytic plasma wakefield limits for active plasma lenses," **arXiv:1802.02750 (2018)**

APLs: EXPERIMENTAL DEMONSTRATION OF STAGING AT BELLA

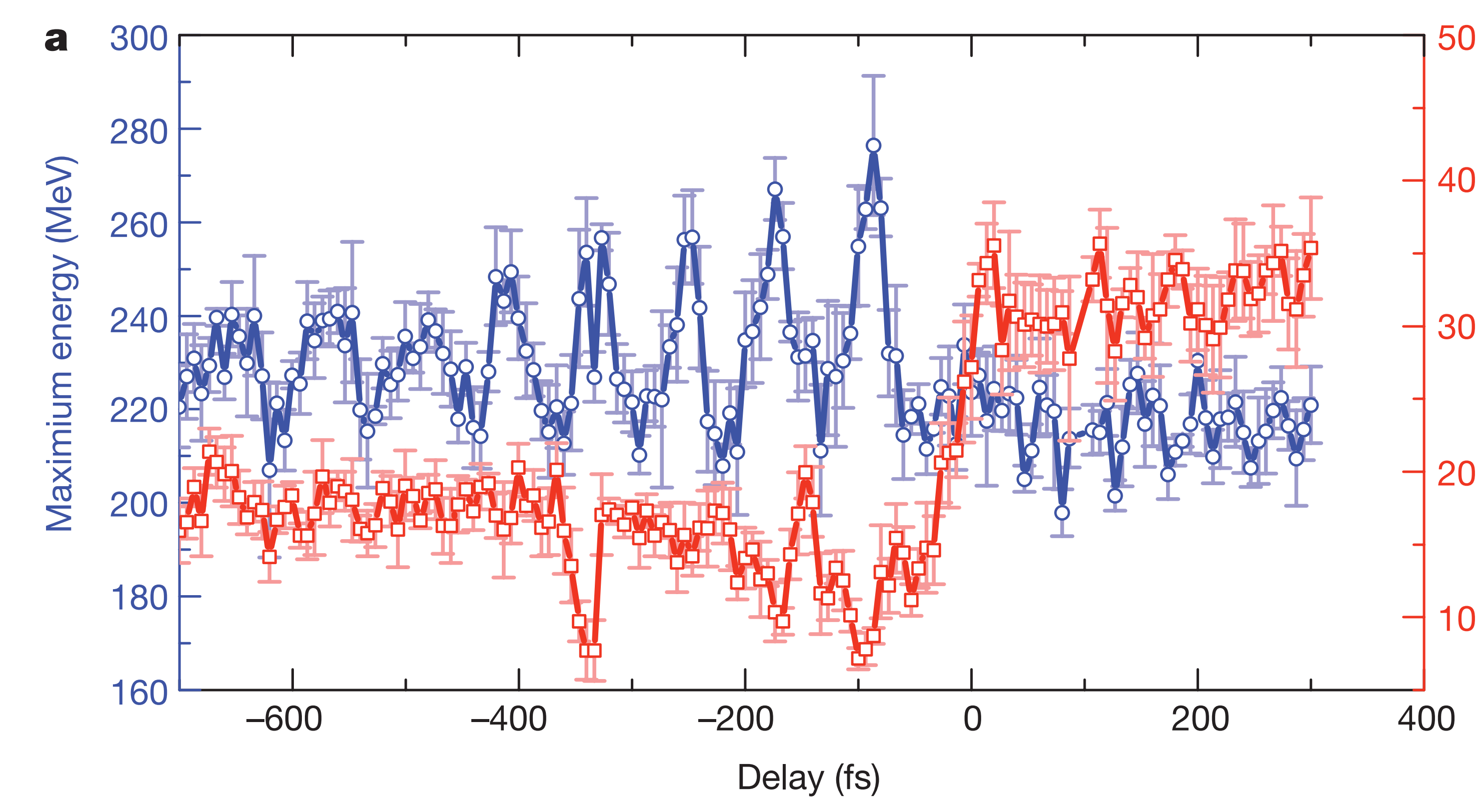
> BELLA laser wakefield accelerator facility at LBNL.

> The only staging experiment so far.

> ~100 MeV beam from Stage 1, large energy spread [32]

> Active plasma lensing used for capture/refocusing: **3.5% charge coupling**

> Further staging experiments are currently underway at BELLA.
Similar experiments soon at APOLLON (quadrupoles) and at RAL (no optics).



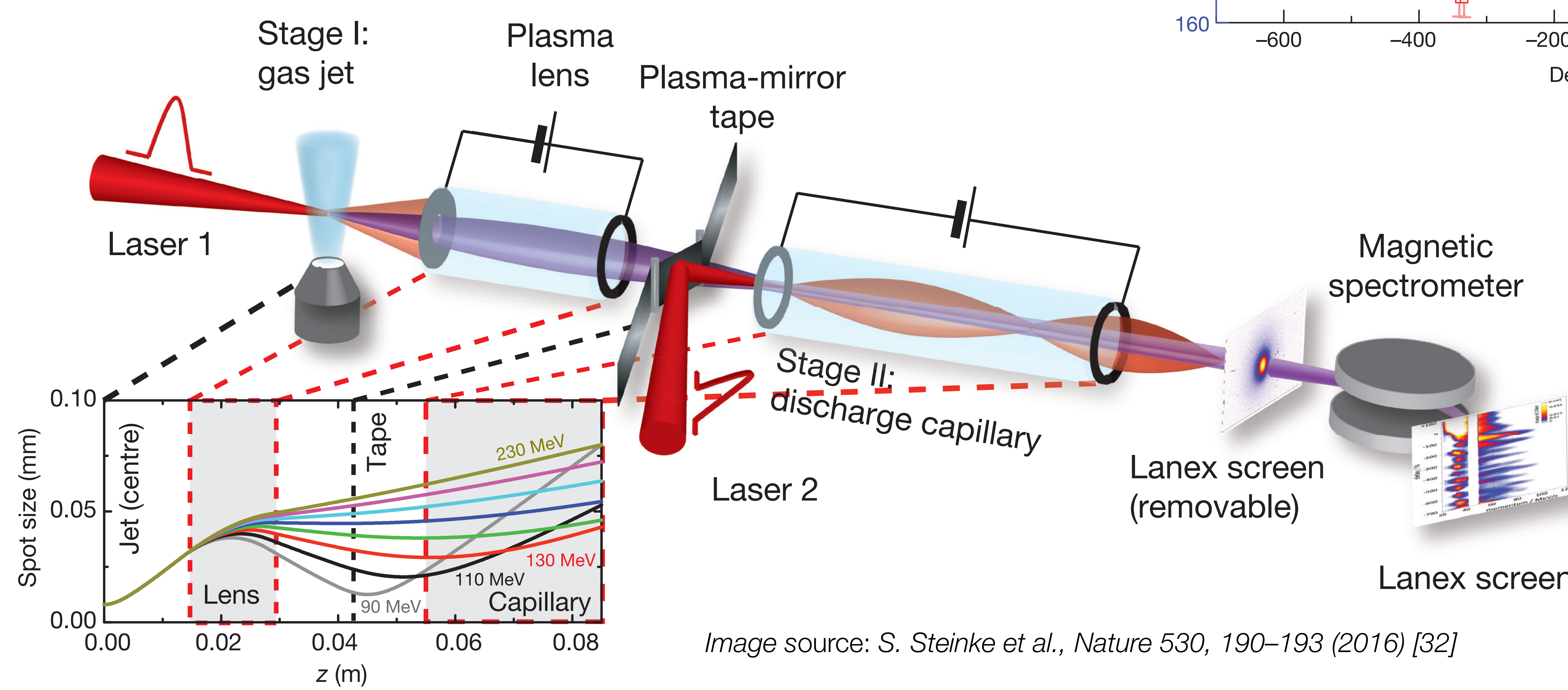


Image source: S. Steinke et al., Nature 530, 190–193 (2016) [32]

*[32] S. Steinke et al., "Multistage coupling of independent laser-plasma accelerators", **Nature 530, 190–193 (2016)***

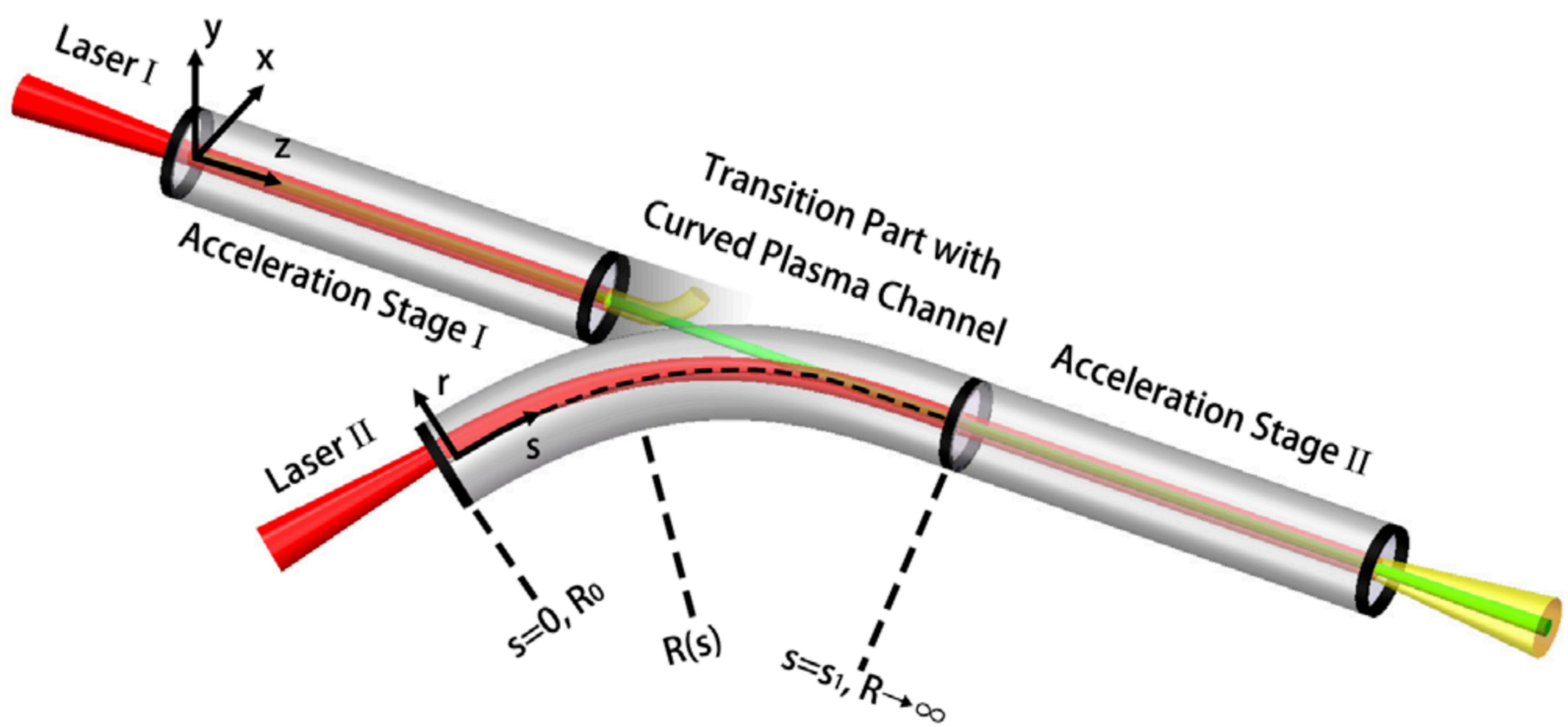
Curved channels for laser in-coupling

> An alternative to plasma mirrors is to use curved channels.

> The **distance between stages can be made very short, reducing chromaticity**

> However, misalignment and dispersion is induced, which will dramatically increase emittance.

> Not yet experimentally demonstrated.



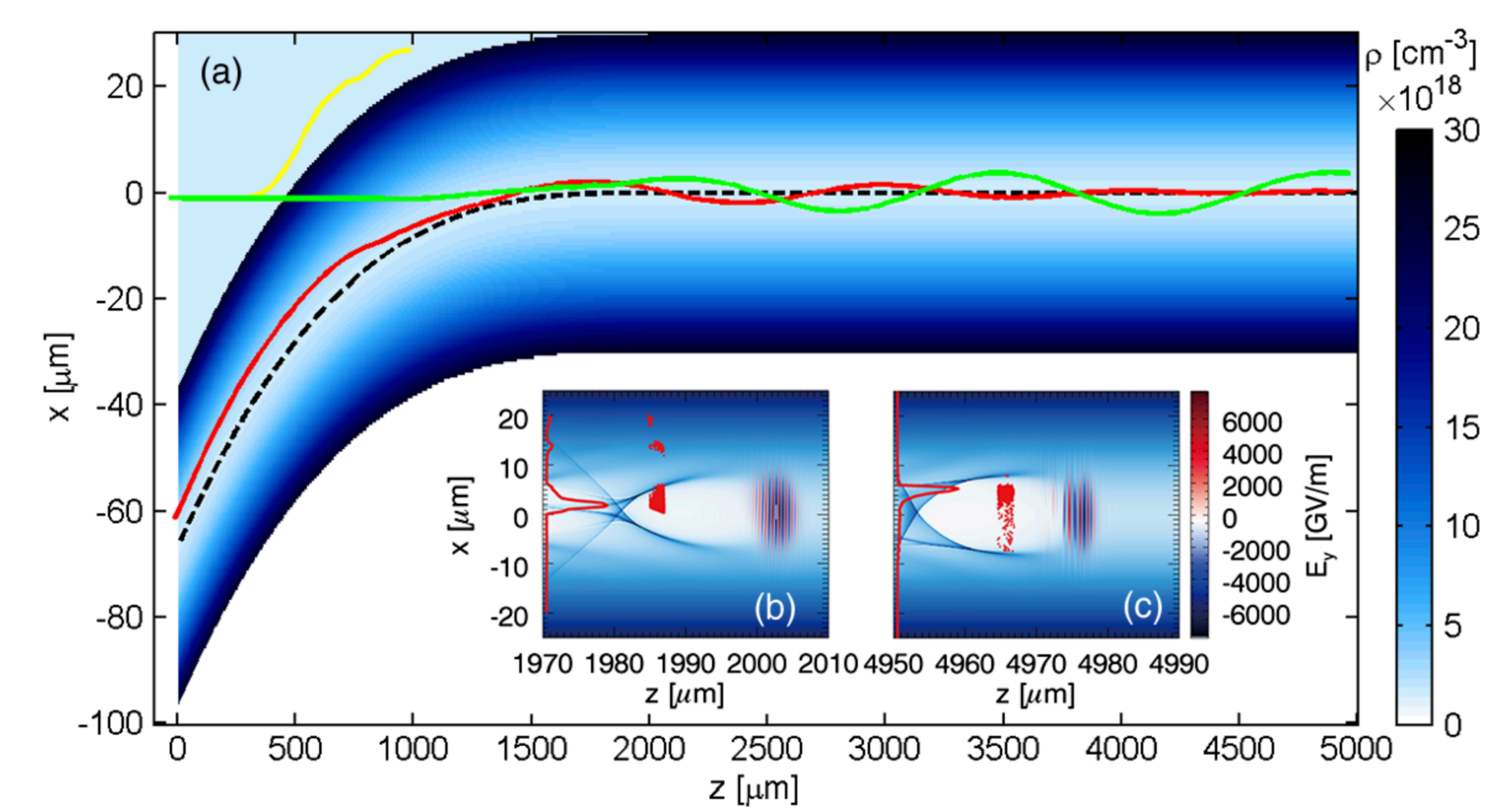


Image source: *J. Luo et al., Phys. Rev. Lett. 120, 154801 (2018)*

*[32] J. Luo et al., "Multistage Coupling of Laser-Wakefield Accelerators with Curved Plasma Channels", **Phys. Rev. Lett. 120, 154801 (2018)***

DRIVE BEAM DISTRIBUTION SCHEMES

- Synchronization/timing tolerances
- Four ways of distributing the drive bunches

Synchronization/Timing Tolerances

> For plasma accelerators, the **timing tolerance is given by the size of the accelerating phase and the energy offset tolerance**:
  > Assume that the energy gain must be accurate to ~1% or better
  > The size/duration of the accelerating phase (a few k_p for plasma wakefields)

> Timing tolerance **scales inversely with gradient**.

> For GV/m fields, the timing tolerance is around **tens of femtoseconds**.

> No current experiments have had to face this tolerance, as witness bunches have either been injected or made from “scraping” a single bunch into two.

$$k_p \Delta\xi \approx \frac{\Delta E_z}{E_z}$$

$$\Delta t \lesssim \left(\frac{\Delta E_z}{E_z}\right)\frac{1}{ck_p} \approx \left(\frac{\Delta E_z}{E_z}\right)\frac{m_e c}{eE_z}$$

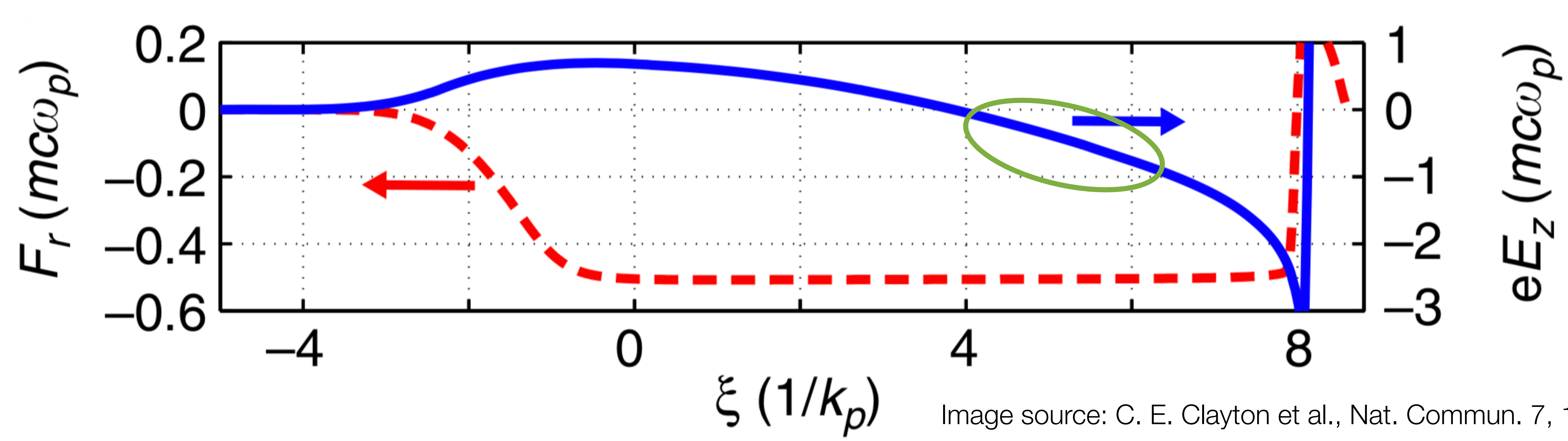


Image source: C. E. Clayton et al., Nat. Commun. 7, 12483 (2016)

U-TURN CHICANES

> Most basic solution: One drive beam complex per stage — Problem: **How can we make it more cost effective?**

> A train of drive bunches is produced in a drive beam accelerator complex (CLIC-like [33])

> **Each driver is coupled in via its own U-turn chicane** [34, 35]

> The first bunch of the train is used in the first cell

> Can be **expensive** to build: requires a large number of long chicanes (worse with higher drive beam energy)

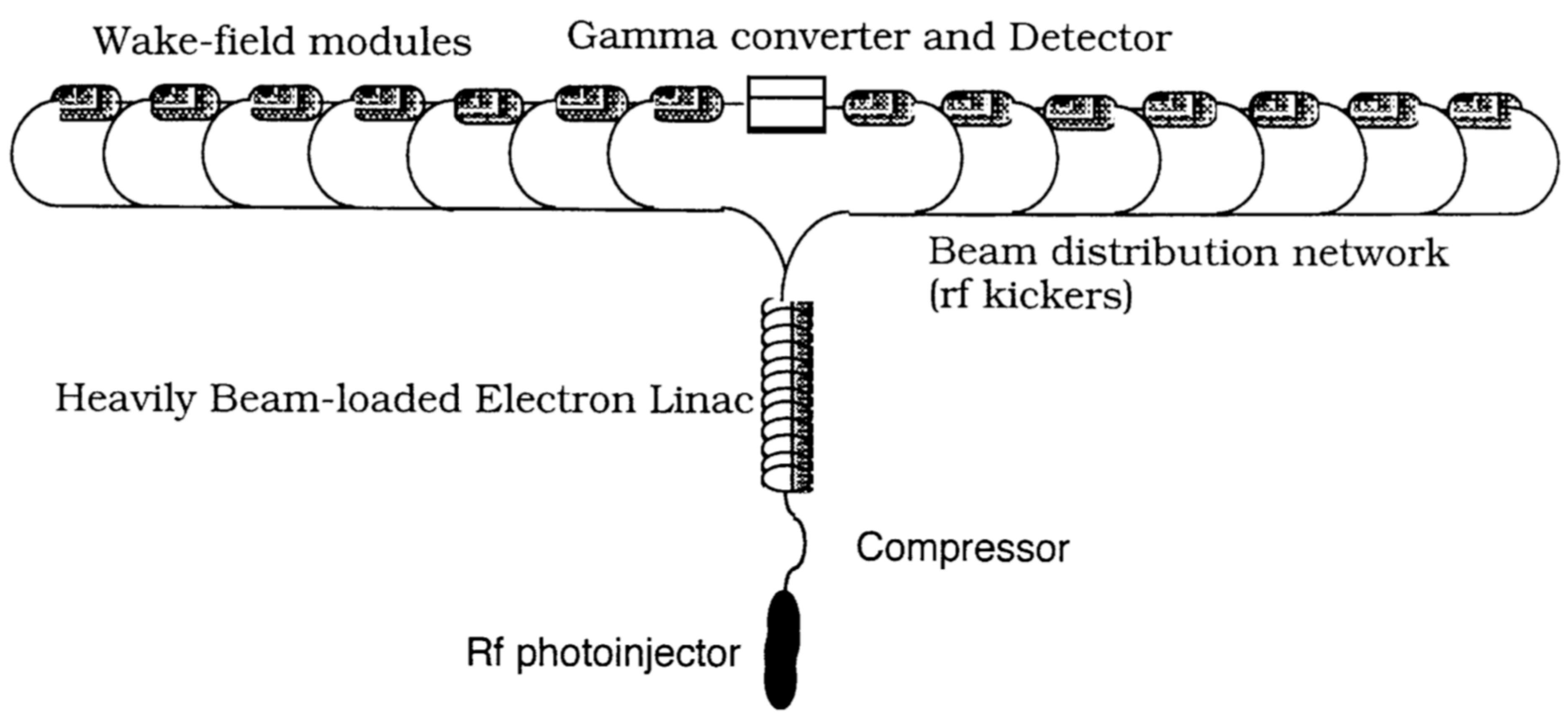


Image source: J. Rosenzweig et al., Nucl. Instrum. Methods Phys. Res. A 410, 532 (1998) [34]

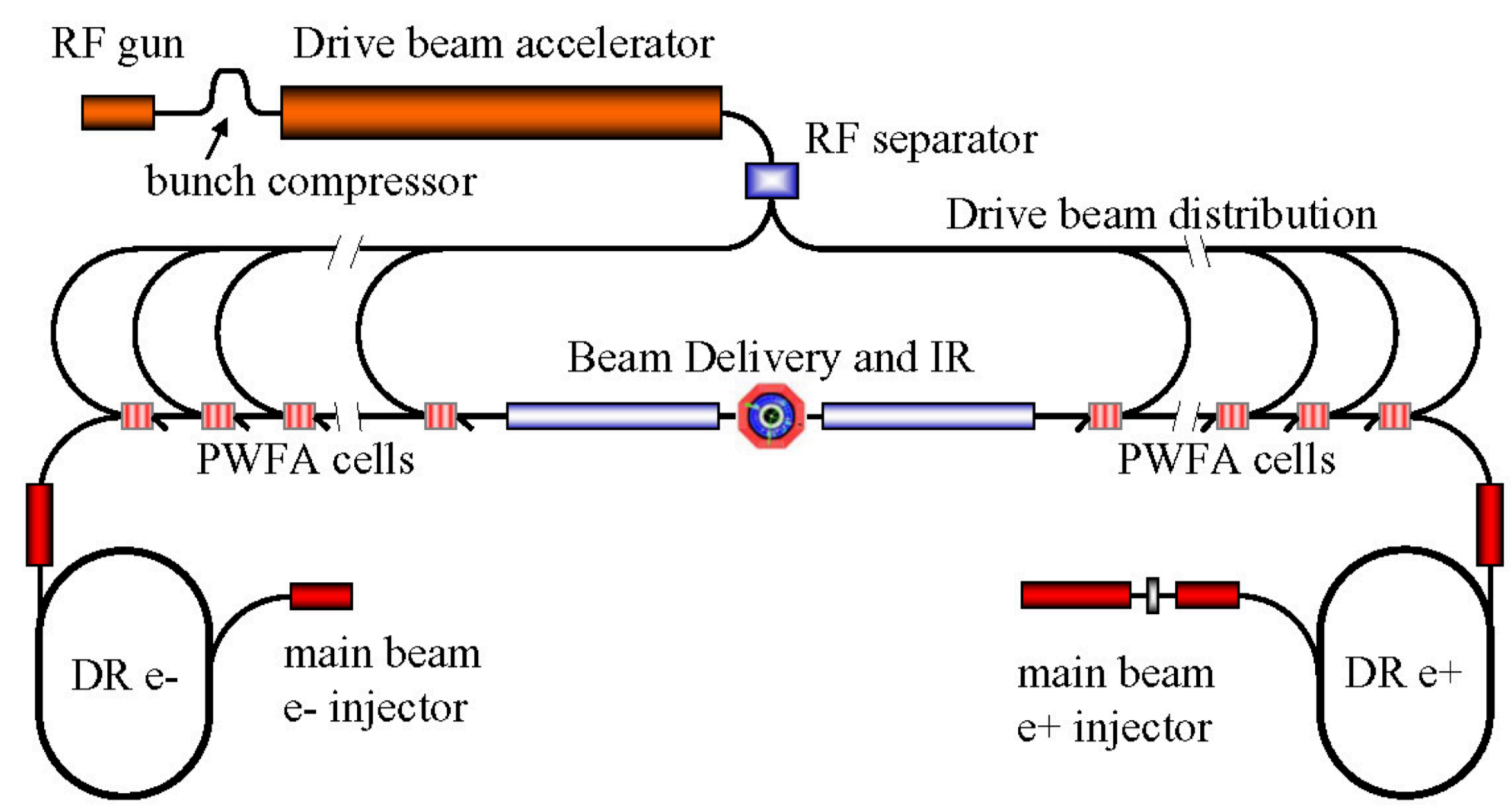


Image source: A. Seryi et al., Proceedings of PAC2009, p. 2688 (2010) [35]

*[33] M. Aicheler (editor) et al., "A Multi-TeV linear collider based on CLIC technology", **CLIC Conceptual Design Report (CERN, Geneva, 2013)***
*[34] J. Rosenzweig et al., "Towards a plasma wake-field acceleration-based linear collider," **Nucl. Instrum. Methods Phys. Res. A 410, 532 (1998)***
*[35] A. Seryi et al., "A concept of plasma wake field acceleration linear collider (PWFA-LC)," **Proceedings of PAC2009, p. 2688 (2010)***

MULTI-BEND (SNAKE-LIKE) DELAY CHICANE

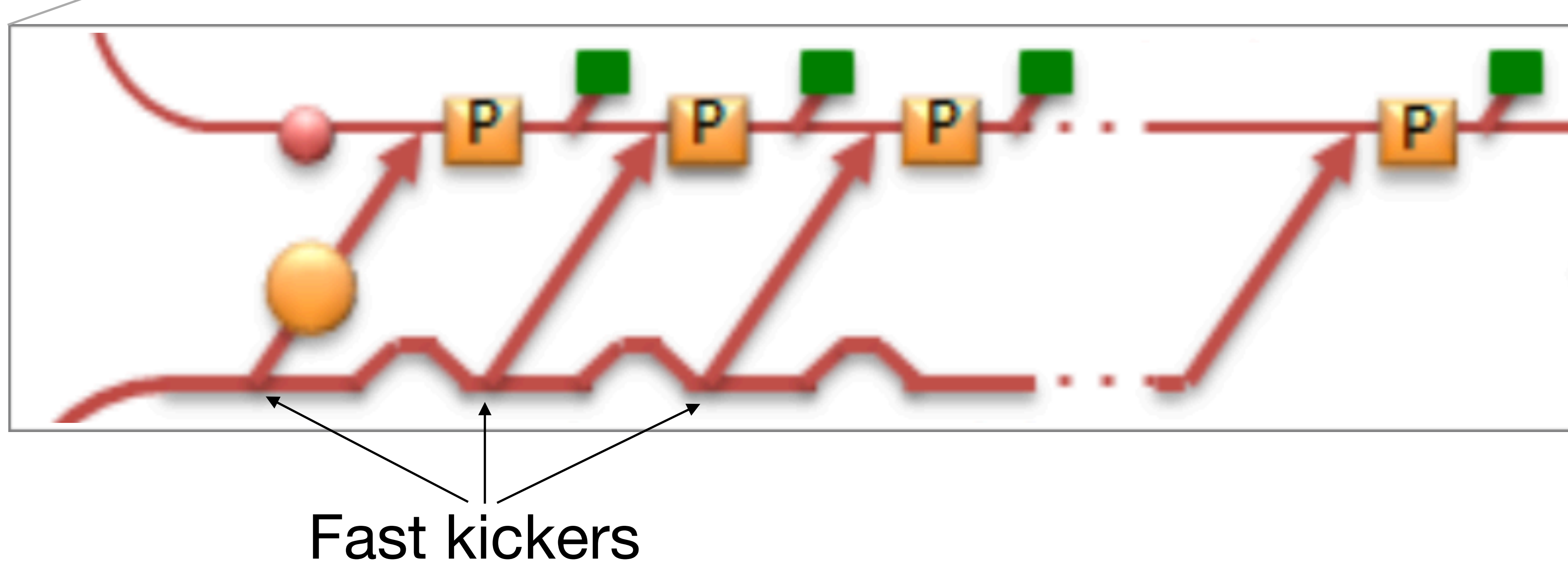
> To save on tunnel and magnet cost, it is possible to make **a single multi-bend delay chicane** [36].

> The last bunch is kicked before each delay chicane.

> **Requires very fast kickers** (ns-level)

> Current fastest kickers have a ~4 ns rise time, but these are large and expensive.

> Compact delay chicanes will require strong bending and **induce coherent synchrotron radiation** (CSR)



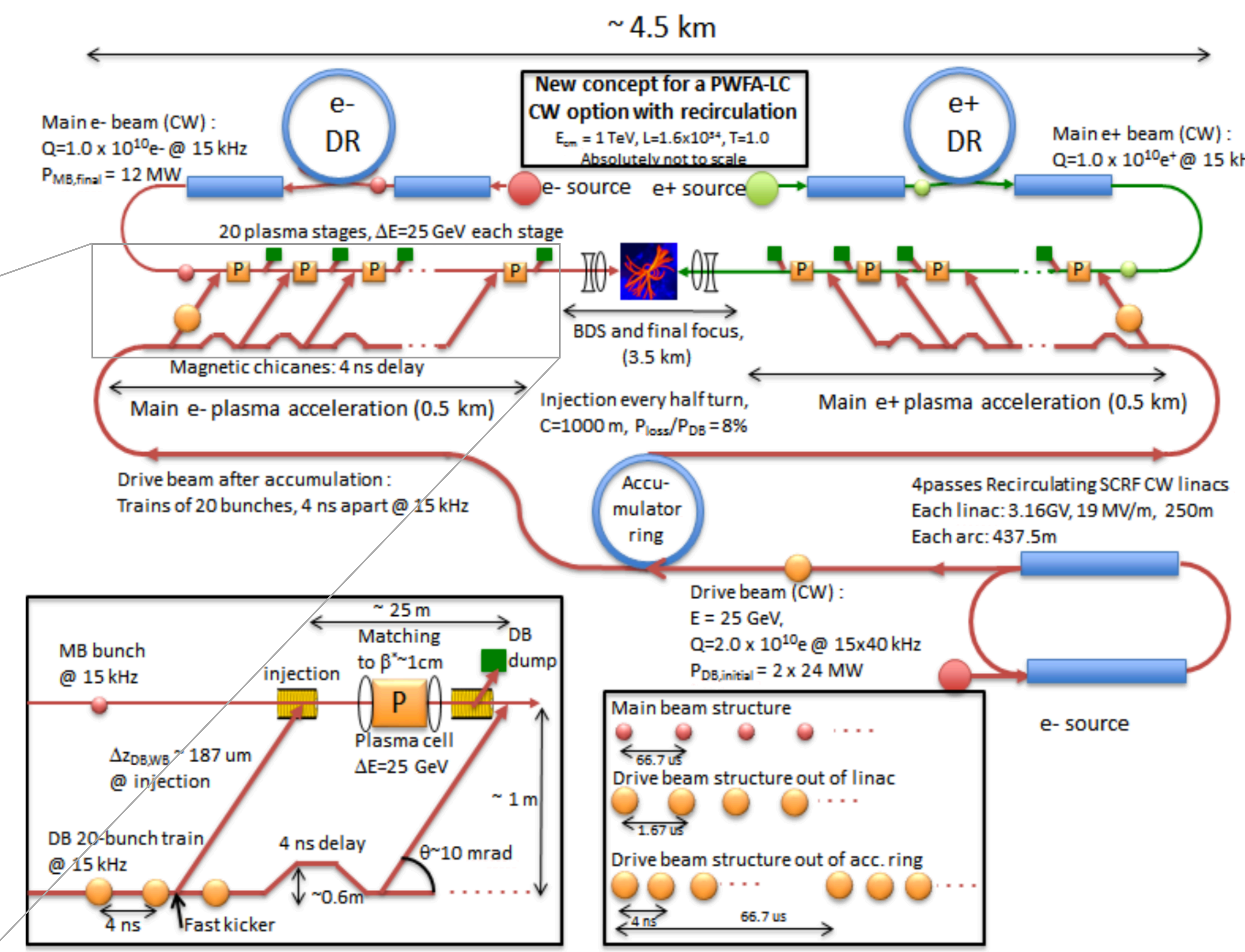


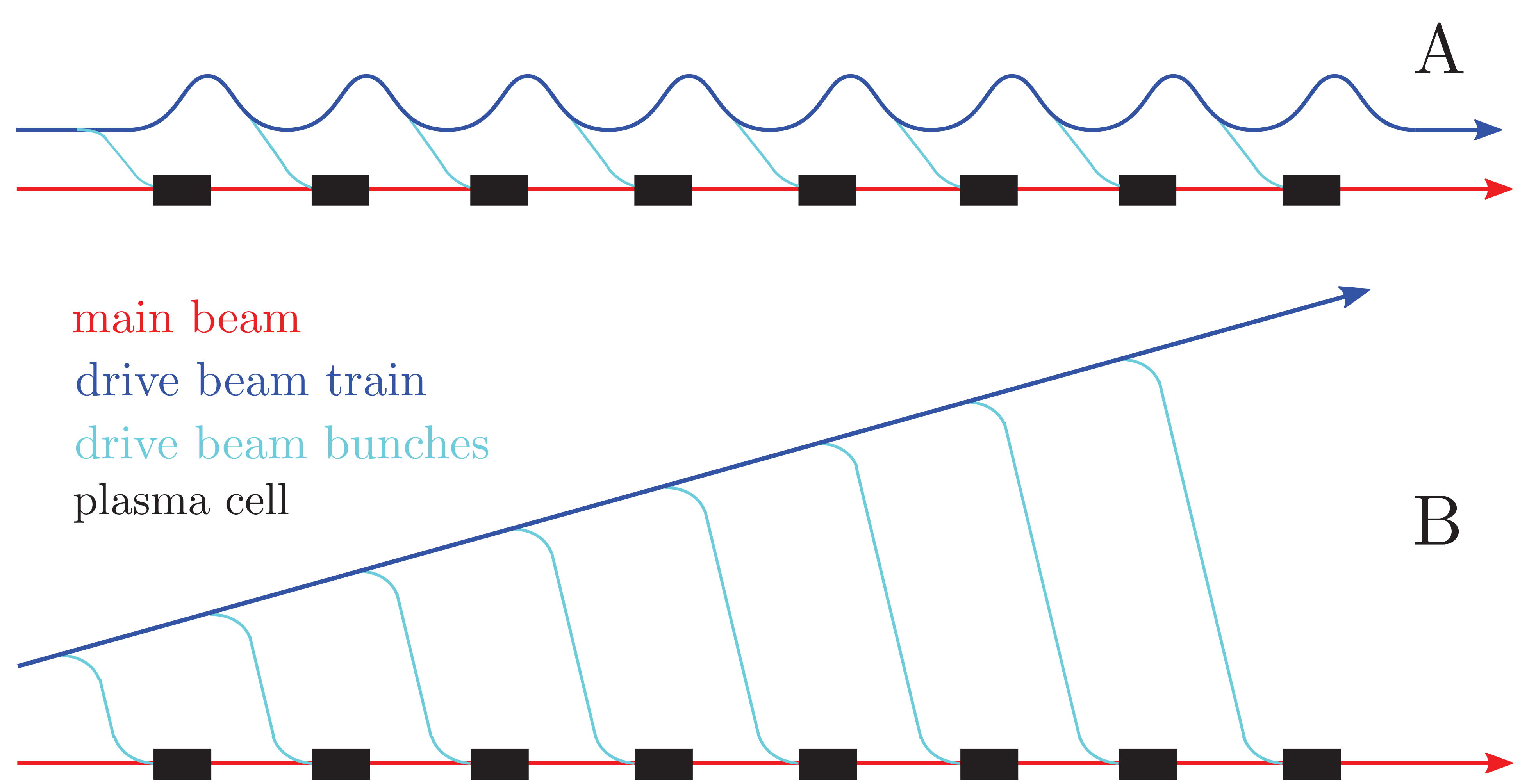
Image source: E. Adli et al., Proc. Snowmass Process CSS2013 (2013); arXiv:1308.1145 [36]

[36] E. Adli et al., "A beam driven plasma-wakefield linear collider: from Higgs factory to multi-TeV," **Proceedings of the Snowmass Process CSS2013 (2013); arXiv:1308.1145**

MULTI-BEND WITH REDUCED SYNCHROTRON RADIATION

> At the expense of longer chicanes, the synchrotron radiation can be greatly reduced using a different geometry.

> A compromise between the two previous schemes.



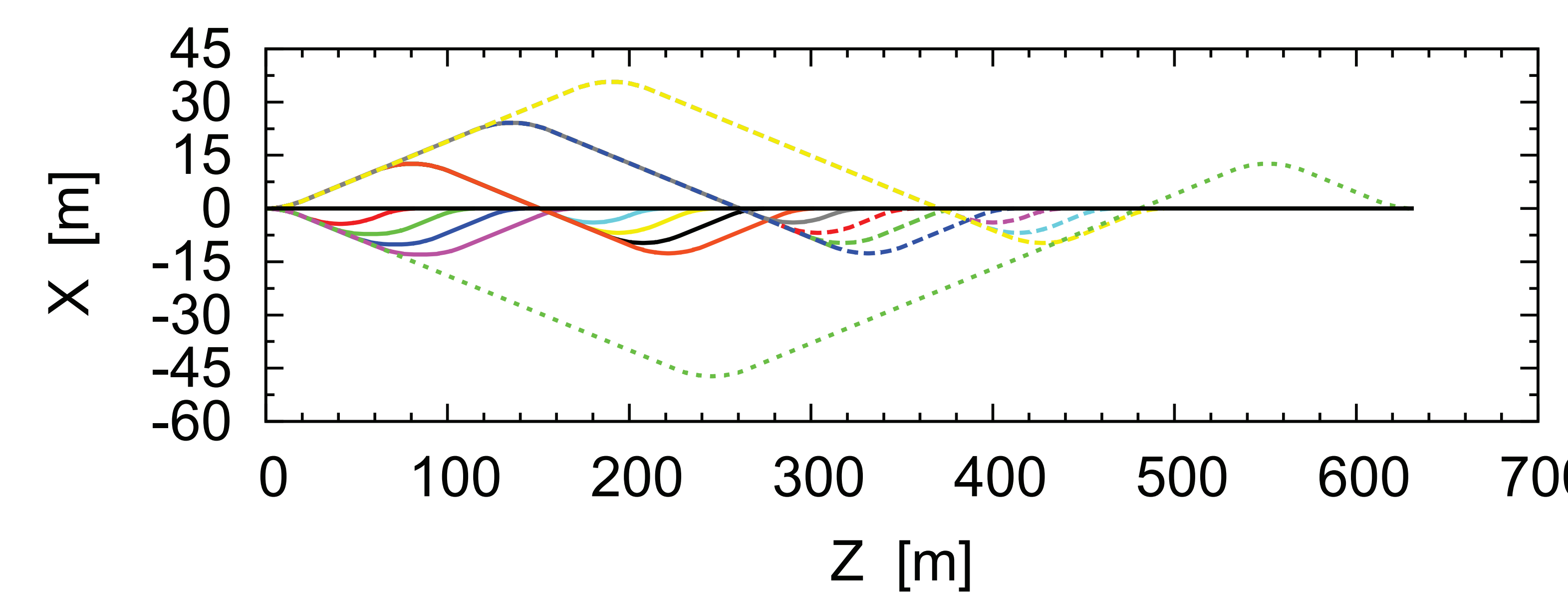


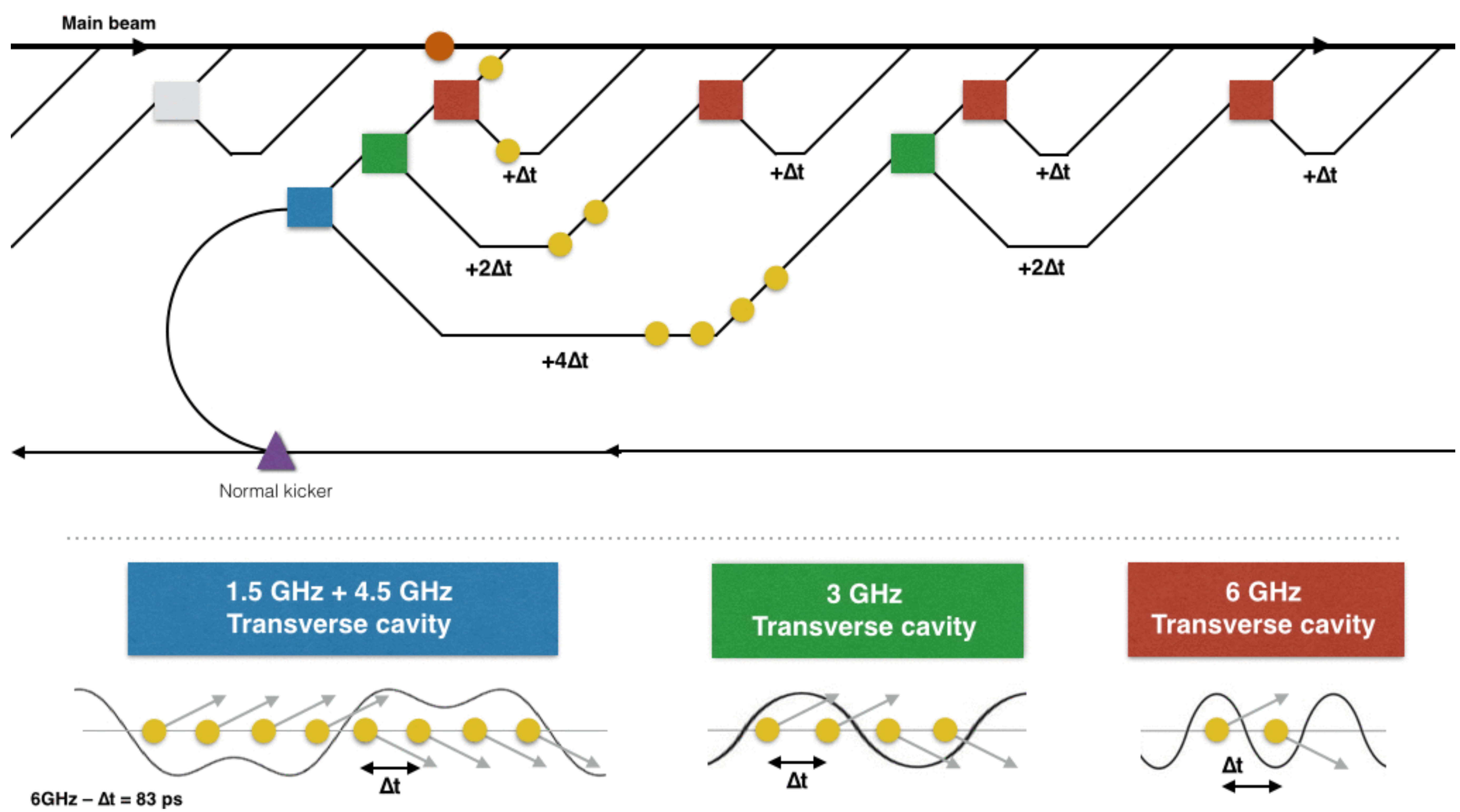
Image source: J. Pfingstner et al., Proc. IPAC2016, Busan, Korea (JACoW, Geneva, 2016), p. 2565 [37]

[37] J. Pfingstner et al., "Considerations for a drive beam scheme for a plasma wakefield linear collider," ***Proc. IPAC2016, Busan, Korea (JACoW, Geneva, 2016), p. 2565***

TREE STRUCTURE DELAY CHICANES WITH TRANSVERSE DEFLECTING CAVITIES

> To reduce the chicane delay (by a factor 10–100), we can **switch from using kickers to transverse deflecting cavities.**

> Can combine different cavity modes to make ~flat top deflecting profiles



> **Overall: Choice of scheme is a cost optimization issue**, dependent on the specific parameters.

SUMMARY AND CONCLUSIONS

Summary and Conclusions

- Staging is a method to reach higher energies than is possible in a single stage.
- May be avoided with high-total-energy drivers (protons, trains) or with curved channels (laser)
- Several challenges, including:
  - Compact in/out coupling (especially for particle drivers)
  - Emittance preservation — chromaticity is a big challenge (!)
  - Isochronicity ($R_{56} = 0$)
  - Energy scaling (optics gets longer with energy)
- Many suggested techniques, all have pros and cons (no clear winner yet)
  - Conventional: Sextupoles in dispersive sections (works, but is long/complex)
  - Apochromatic correction (simple, but limited energy acceptance)
  - Plasma ramps (useful reducing chromaticity, but decelerates and takes up space)
  - Active plasma lenses (uniform, incompatible with high intensity beams?)
  - Passive plasma lenses (non-uniform or needs driver)
- Drive beam distribution schemes include
  - U-turn delay chicanes (with slow kickers)
  - Multi-bend chicanes (with fast kickers)
  - Tree-structure delay chicane (with deflecting cavities)

THANKS FOR LISTENING!

(thanks to Erik Adli for input)