Mini-CAS on Mechanical and Materials Engineering for Accelerators

Numerical Tools I

Federico Carra

CERN Engineering Department

Mechanical and Materials Engineering Group

November 6th, 2020

Outline

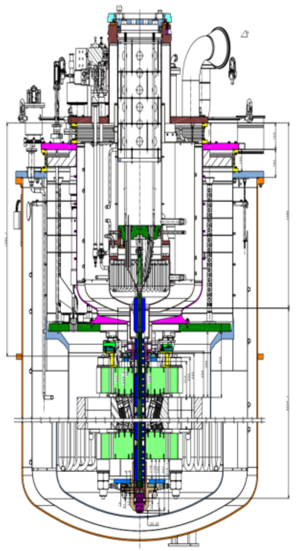
Computer-Aided Technologies (CAx)

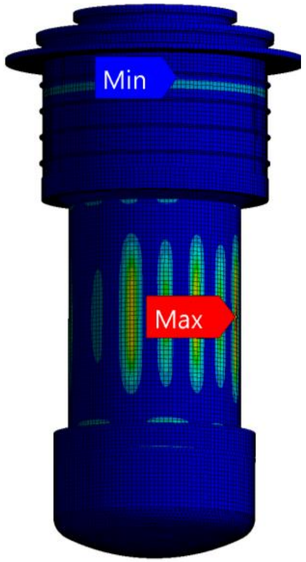
CAx

Computer-Aided Design
(CAD)

Computer-Aided Engineering
(CAE – e.g. **FEA**, CFD)

Computer-Aided
Manufacturing
(CAM)





Computer-Aided Design and Engineering (CAD/CAE)

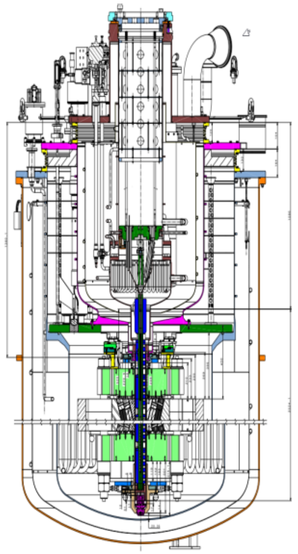
CAD/CAE

Design and Analysis* → Virtual Prototyping → Physical Prototyping → Final Product Testing

$ Relatively cheap

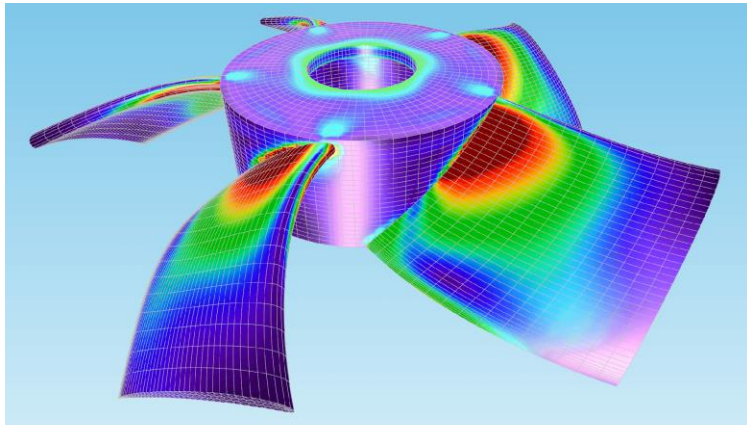
$$ Not so cheap

$$$ Really expensive



***The more time spent here, the less money and time spent later**

CAE Fields

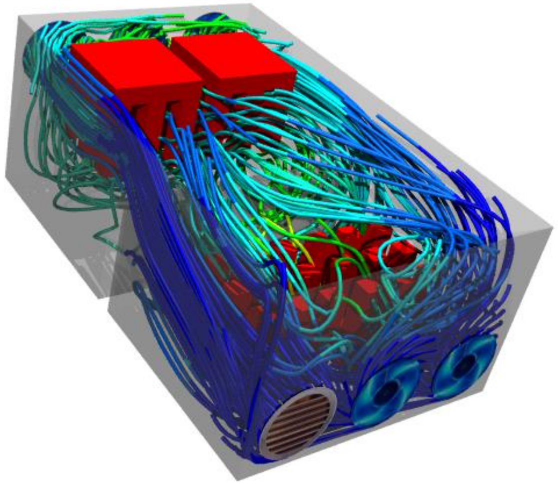


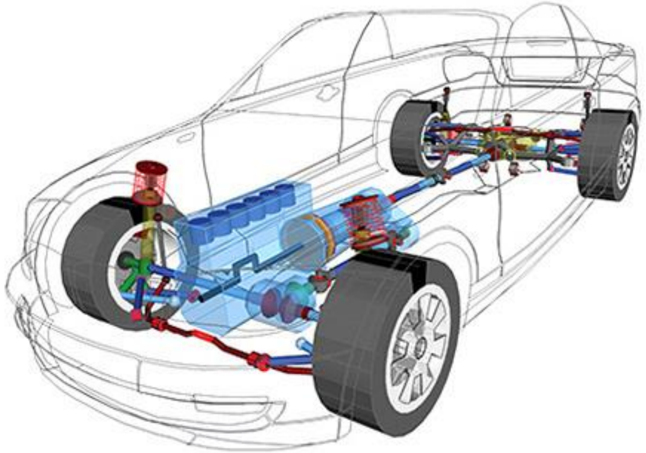
Finite-Element Analysis

(FEA)

Computational Fluid Dynamics

(CFD)



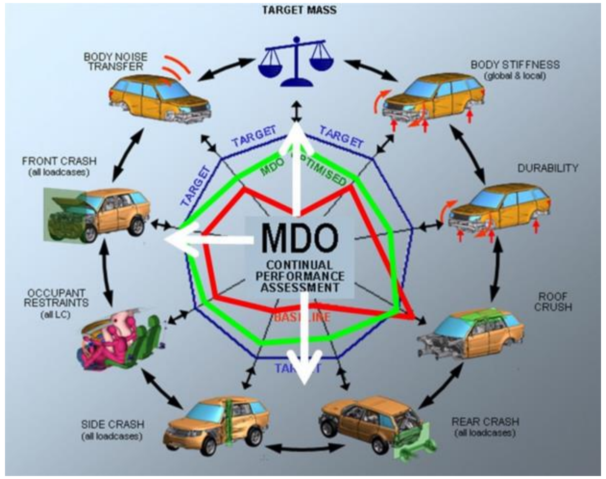


Multibody Dynamics

(MBD)

Multidisciplinary design optimization

(MDO)



FEM Theory in a Nutshell

Physical phenomenon → Model → Governing equations & BCs → Approximation → Finite element equations

- Solid Mechanics
  e.g. Axially loaded elastic bar
  $$\frac{d}{dx}\left(AE\frac{du}{dx}\right)+b=0$$
- Fluid Mechanics
  e.g. Poiseuille flow in pipe
  $$\frac{d}{dx}\left(A\frac{D^2}{32\mu}\frac{dp}{dx}\right)+Q=0$$
- Thermal Conduction
  e.g. 1-D heat flow
  $$\frac{d}{dx}\left(Ak\frac{dT}{dx}\right)+Q=0$$
- Diffusion
  e.g. 1-D diffusion
  $$\frac{d}{dx}\left(AD\frac{dC}{dx}\right)+Q=0$$
- Electrical Conduction
  e.g. 1-D electric current flow
  $$\frac{d}{dx}\left(A\sigma\frac{dV}{dx}\right)+Q=0$$

+ BCs (Boundary Conditions) →

$[K]\{u\} = \{F\}$ Static problems

$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$ Dynamic problems

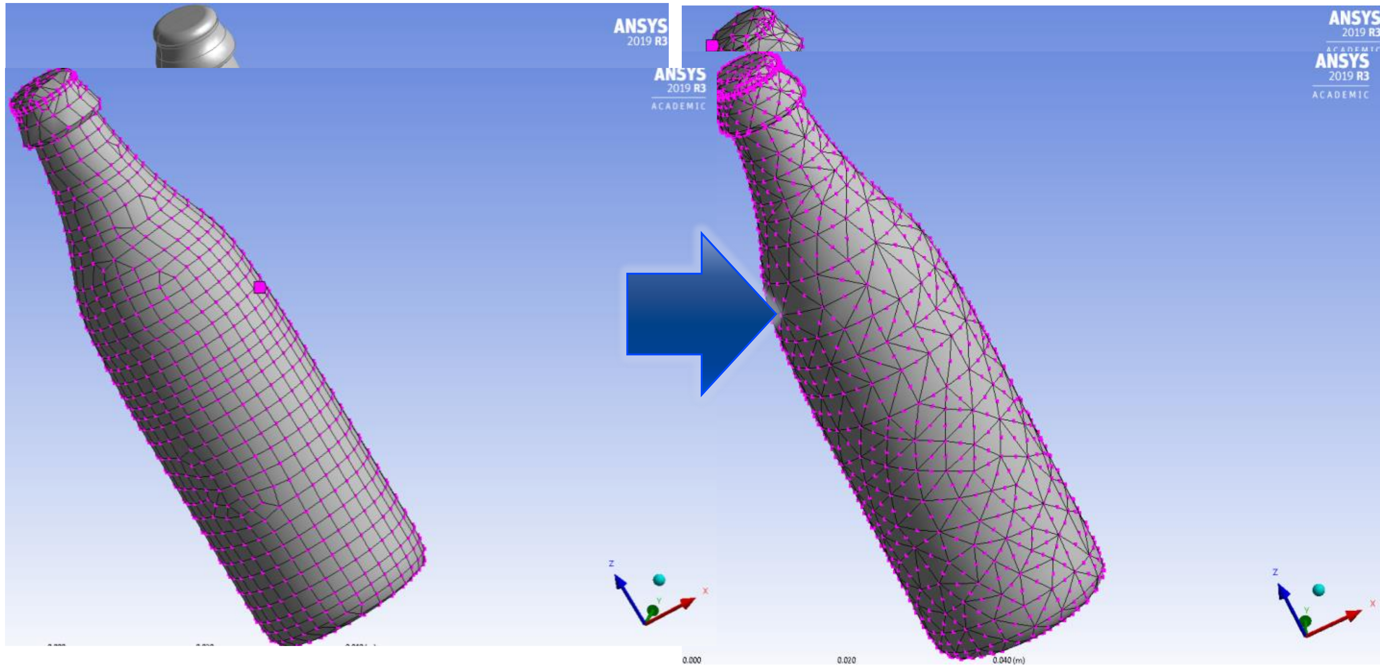
Linear problems: $[K], [C], [M] = [K_0], [C_0], [M_0]$

Nonlinear problems: $[K], [C], [M] = f(u, \dot{u}, \ddot{u})$

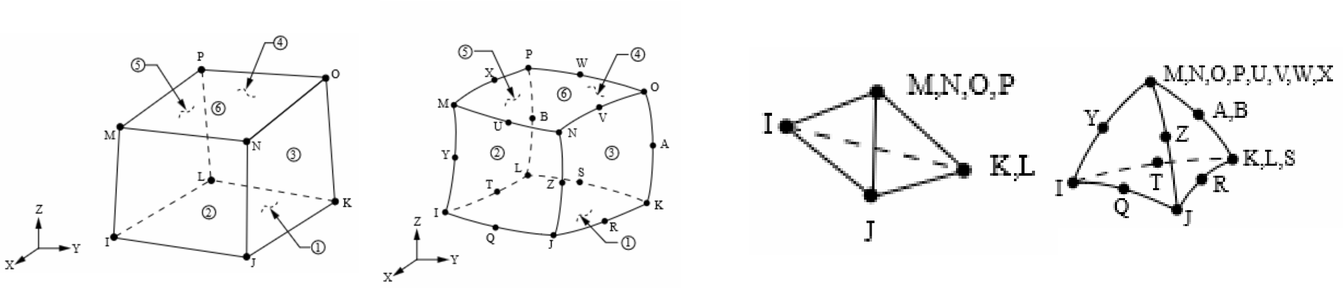
FEM Theory in a Nutshell

The displacements of all the points in a continuum under the action of external forces depends on the displacements of discrete points known as **nodes**.

This dependence is regulated by interpolating functions known as **shape functions.**



To study a body with FEM, we must thus **discretize the continuum in a finite number of elements**, each one featuring a number of nodes which depends on the type of element chosen.



FEM Theory in a Nutshell

FEM: solving for the nodal displacements $\{s\}$

$$\{s\} = [K]^{-1}\{F\}$$

After calculation of $\{s\}$:

$$[\partial] = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix}$$

$$\{u\} = [N]\{s\}$$

N — Shape functions

$$\{\varepsilon\} = [\partial]\{u\} = [\partial][N]\{s\}$$

C — Compatibility Equations

$$\{\sigma\} = [D]\{\varepsilon\} = [D][\partial][N]\{s\}$$

M — Material Constitutive Law (e.g. Hooke's law)

N + C + M → **Solution known in all points of the structure (not only at the nodes!)**

Properties of shape functions

1. It **must** be a continuous function, and must possess a derivative at least until to the *n-1* order required by the problem under study (*e.g.* n = 1 for a truss element, n = 2 for a beam or plane element, etc.)
2. It **must** reproduce rigid motion of the element with a null deformation energy (*i.e.* in an eigenvalue problem, the rigid motion d.o.f. gave a null eigenvalue → in a 3D space, for an unconstrained body there will be 6 null eigenvalues)
3. It **must** guarantee a constant deformation along the element (minimal condition when element size tends to zero)

Complete (1–3)

4. It **must** guarantee continuity among elements (*i.e.* identical displacement field on a segment belonging to two adjacent elements)

Compatible (4)

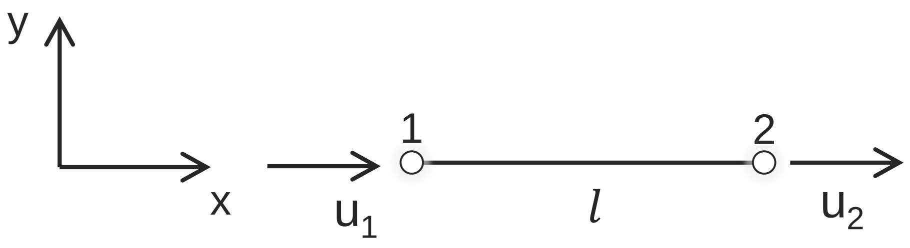
Conform (1–4)

5. It **should** be geometrically isotropic (*i.e.* displacement field is invariant wrt the reference system, not presenting preferential directions)

↓

Polynomials

Shape function of a truss element



$$u = a_1 + a_2 x = [1 \quad x]\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

a_1, a_2 are coefficients that can be calculated imposing the b.c. $x_1 = 0, x_2 = l$

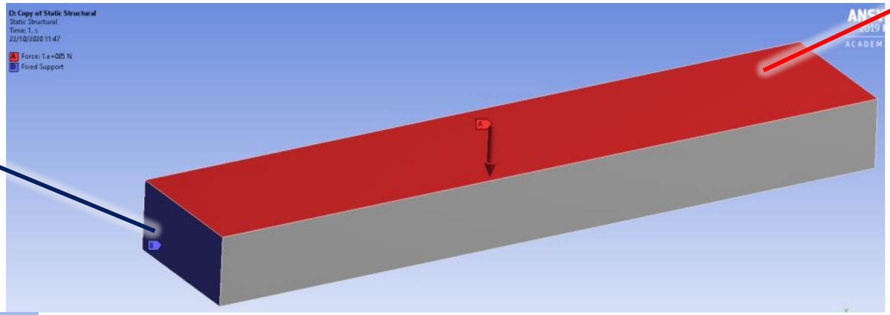
$$[N] = \left[\left(1 - \frac{x}{l}\right) \quad \frac{x}{l}\right]$$

- Displacements will be varying linearly over the length of the element, while **strains and stresses will be constant**
- This shape function respects the properties discussed earlier, in particular the ***n* order** of the problem under study
- Choose the right element for the right problem! In case of bending and shear, use a **beam element instead**

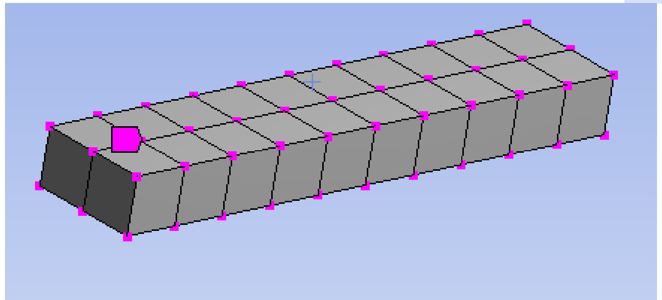
Linear vs quadratic elements

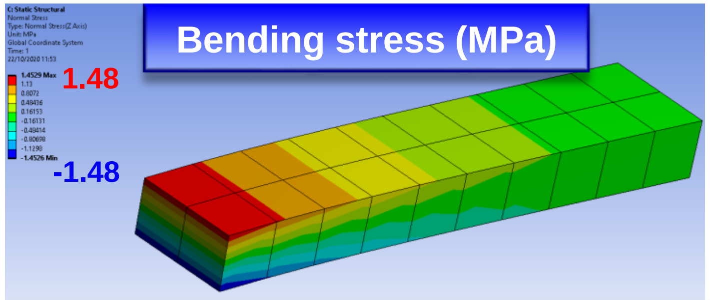
Distributed load

Fixed support

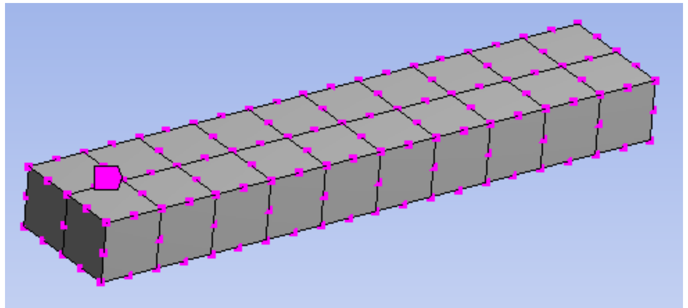


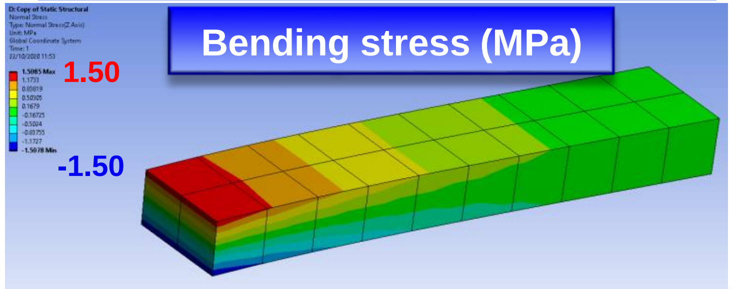
Linear elements





Quadratic elements





Linear elements: computationally more efficient, but when a nonlinear stress state is expected, use quadratic elements or more linear elements over the thickness

FEM Solvers

- **Explicit solvers**: derive the unknowns (displaceme… …) at a time instant…
- **Explicit solvers**: suggested for fast transient …
- **Implicit solvers**: suggested for slow tr… equilibrium equations at the time t…

ANSYS

COM…

N NA…

SIMU… ABAQU…

Implici…

- …
- …
- …
- …roblem

- …onally stable
- Small time steps
- «Lumped» matrix multiplication
- Uncoupled equations
- «Keep going»

AD

LS-DYNA

WAVE FEA

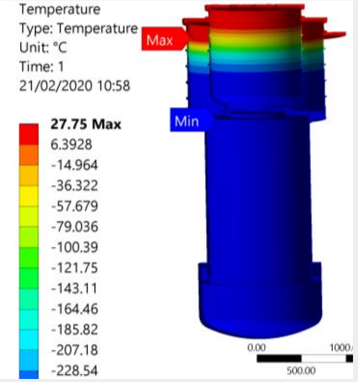
- O. C. Zienkiewicz, “The Finite Element Method: Its Basis and Fundamentals”, ISBN 978-1-85617-633-0.
- O. C. Zienkiewicz, “The Finite Element Method for Solid and Structural Mechanics”, ISBN 978-1-85617-634-7.
- D. Braess, “Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics”, ISBN 978-0-52170-518-9.

FEM tips: from reality to model

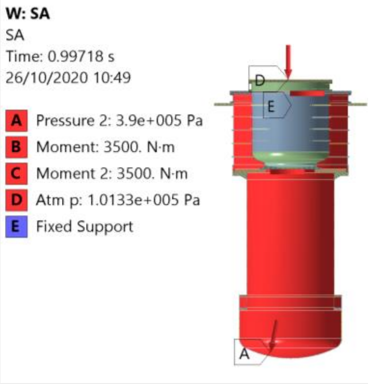
- **Simplification of the model**: removal of details not contributing to the solution of the problem under study
  - **Screws, welds** typically defeatured in the FEA, and calculated “by hand” extracting internal loads from FEA
  - **Chamfers, radii** can be verified via submodels
- **Loads and boundaries:**
  - As accurate as possible representation of the real working conditions
  - Compromise sometimes to be made to simplify the problem (*e.g.* nonlinear contacts, etc.)
  - Most critical step of the process
- **Safety factors!** (*i.e. factor of ignorance*)
- When approximating, always be on the **conservative side**
- **Start simple, complexify later**

Particle accelerator components: typical loads

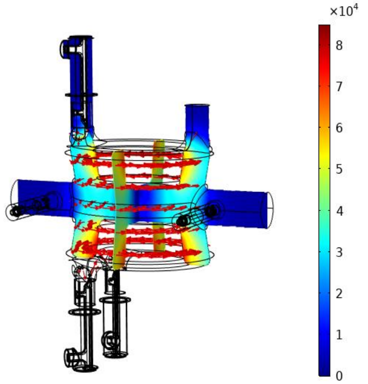
Temperature gradient



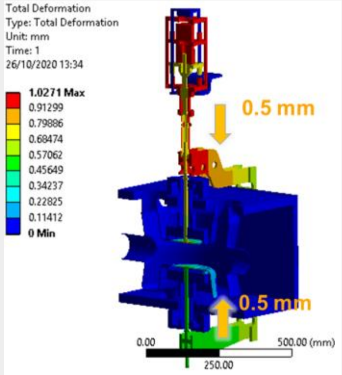
Mechanical forces



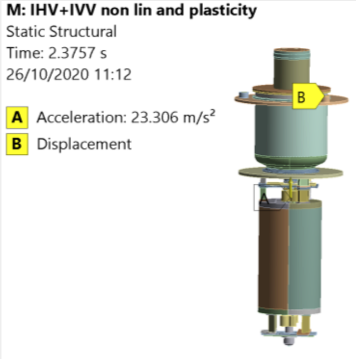
Electromagnetic forces



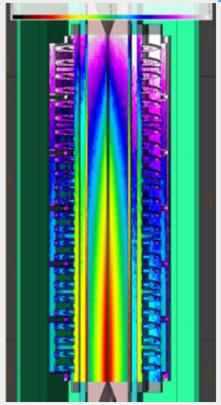
Displacements



Accelerations



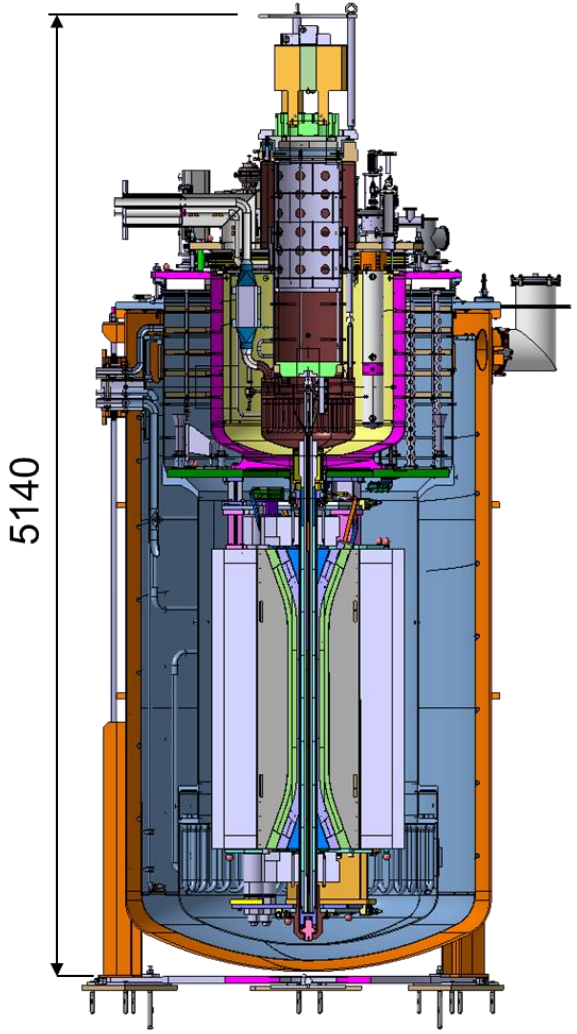
Beam loads (RF & particle losses)



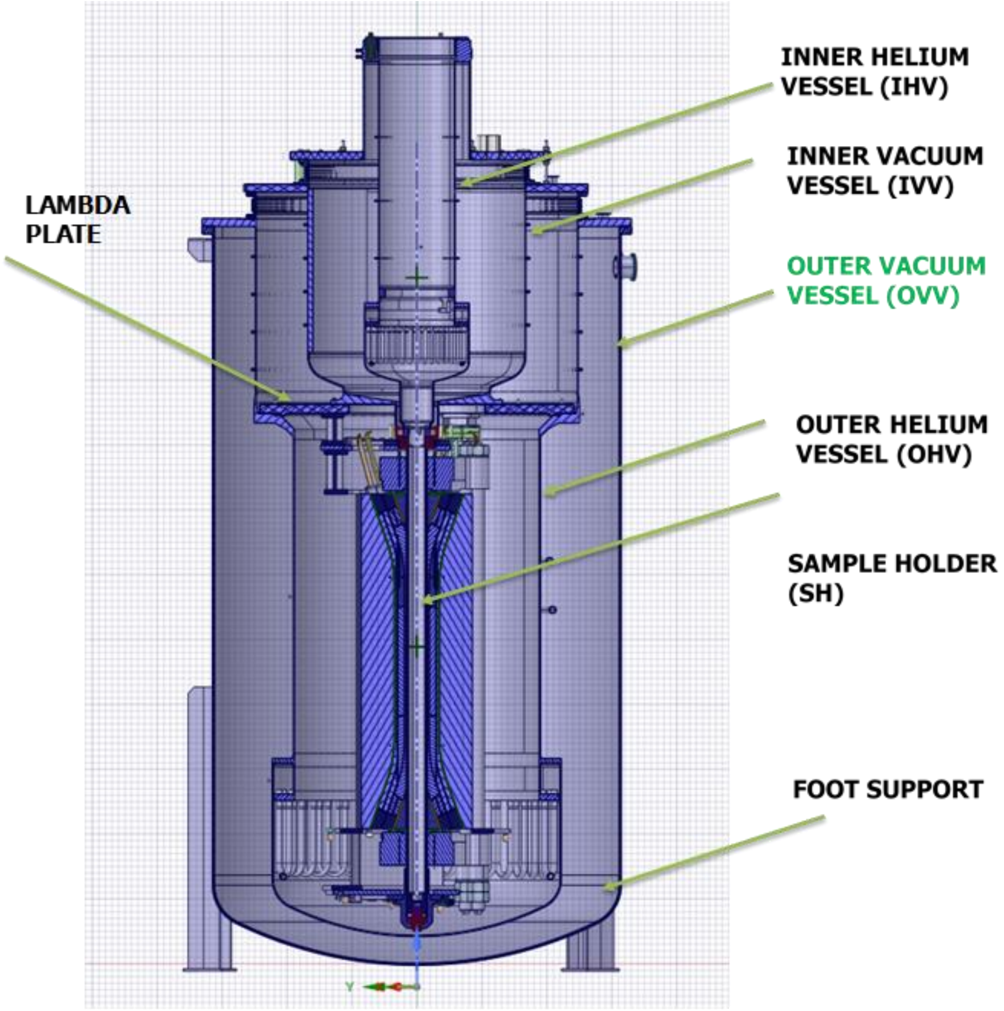
Can be typically studied with **implicit FE codes**

Examples

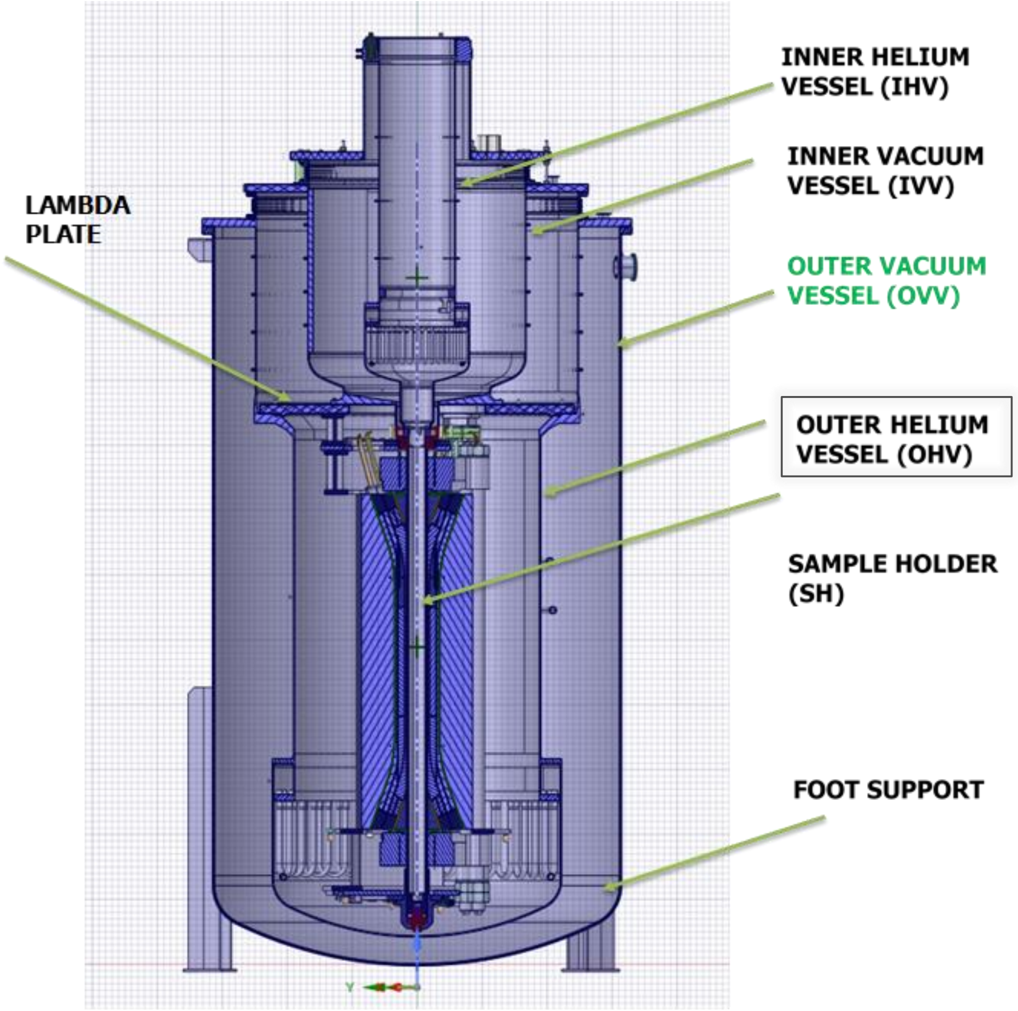
FRESCA2: a facility for testing SC samples



Simplified model for computations



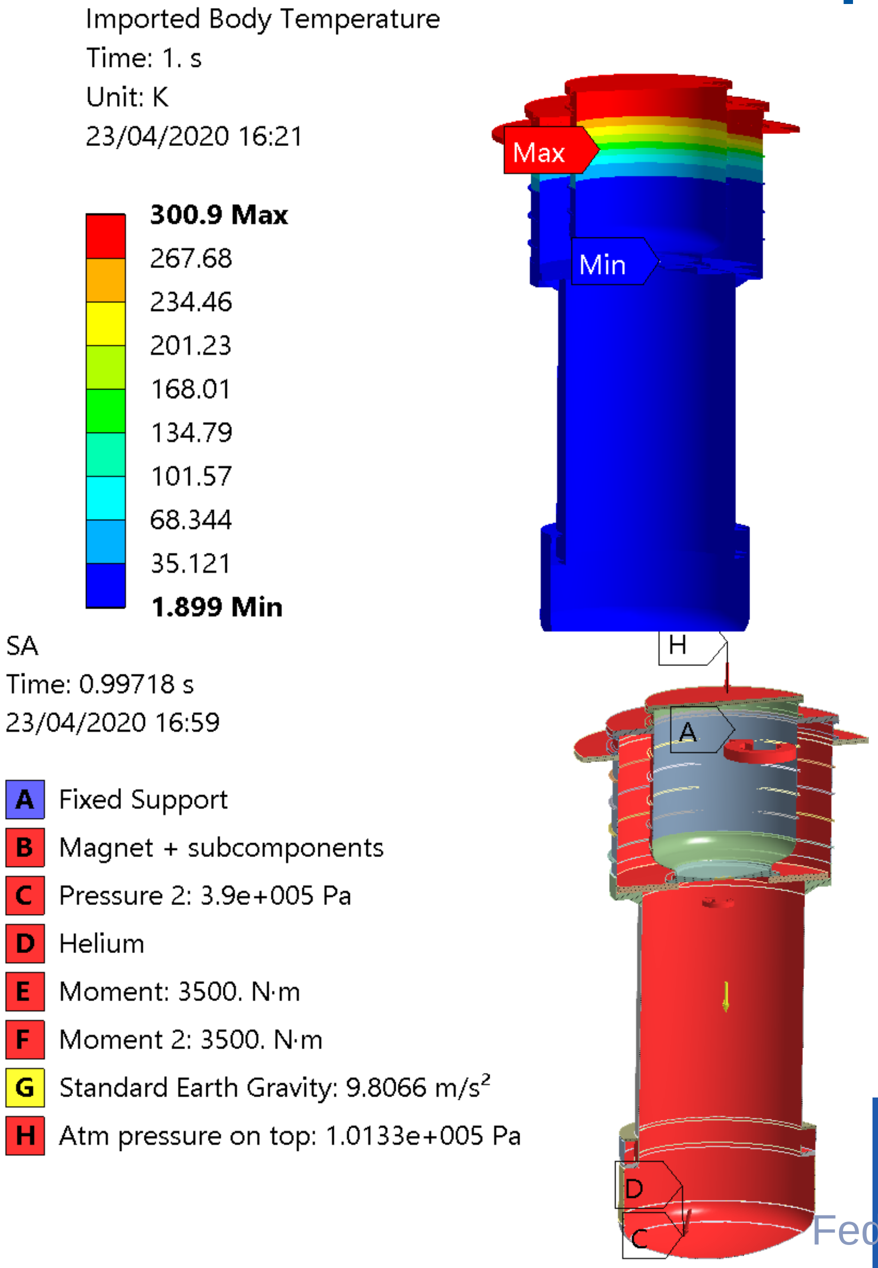
FRESCA2: design of the OHV

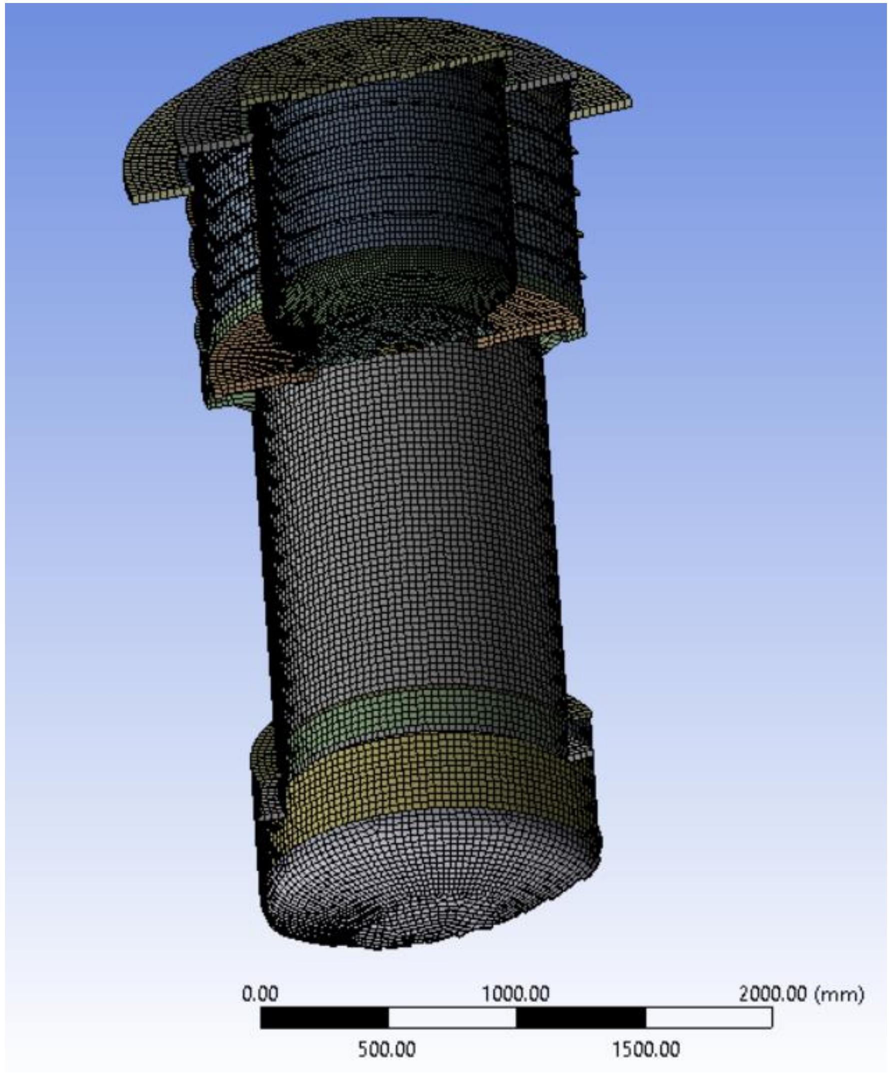


Two main design cases:

1. **Operation**:
   - **Internal pressure** in the OHV 3.9 bara
   - **Thermal gradient** 2-300 K
   - **EM torque** 3500 Nm
   - Most likely failure scenario is by **plastic deformation**
2. **Vacuum loss during OHV purging**:
   - **External pressure** on the OHV 1.5 bara
   - Most likely failure scenario is by **buckling**

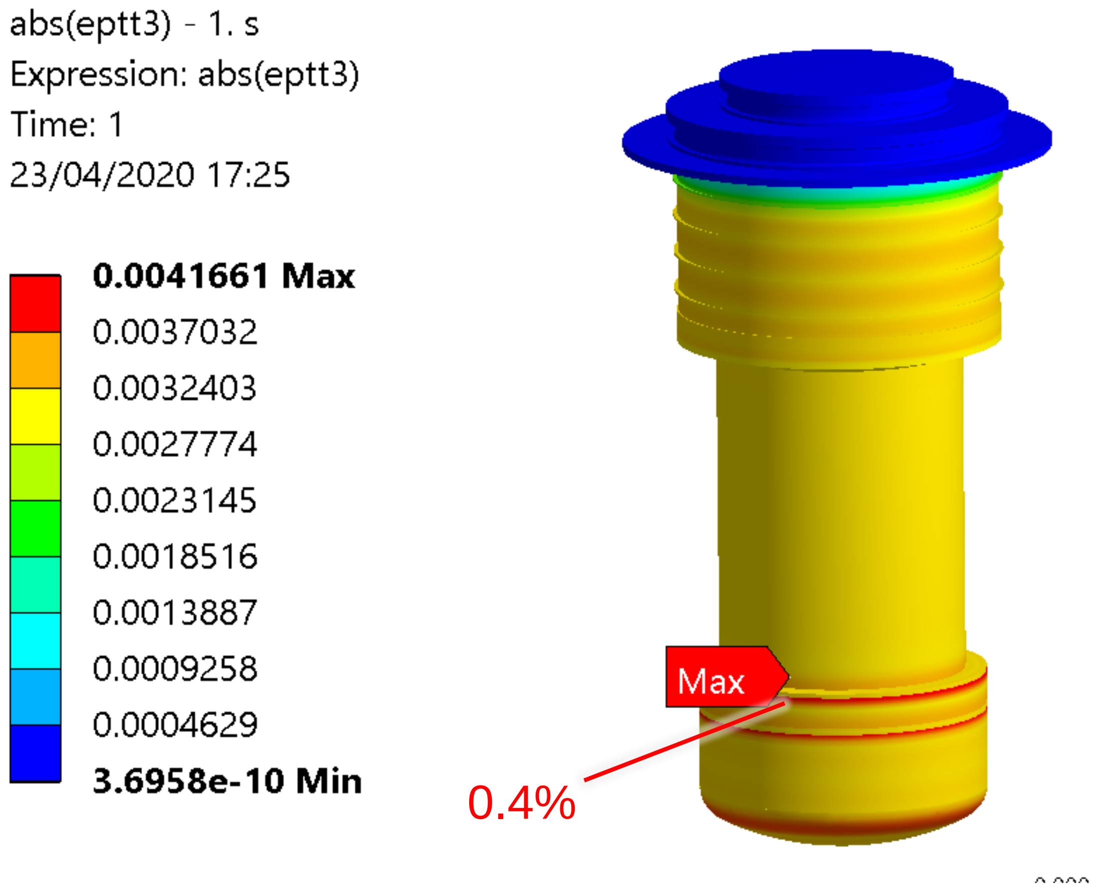
FRESCA2 OHV: operational scenario



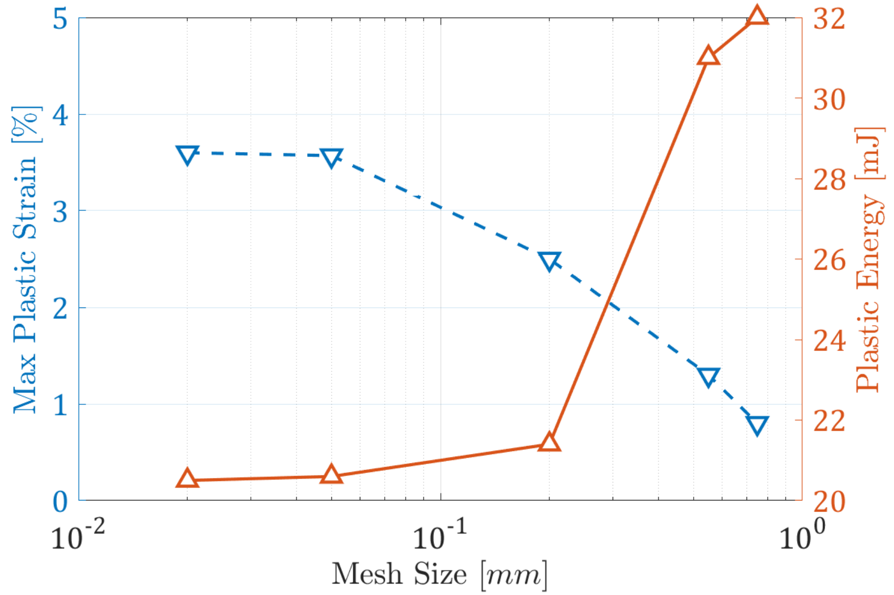


- Use **shell elements** wherever possible wrt solids
- **Nonlinearity of materials** (temperature, strain, …)
- Structure verified against EN-13445 Direct Route: **plastic strain must be less than 5%**
- T field can be calculated in a separated **thermal analysis**, then imported into structural (thermomechanical coupling)

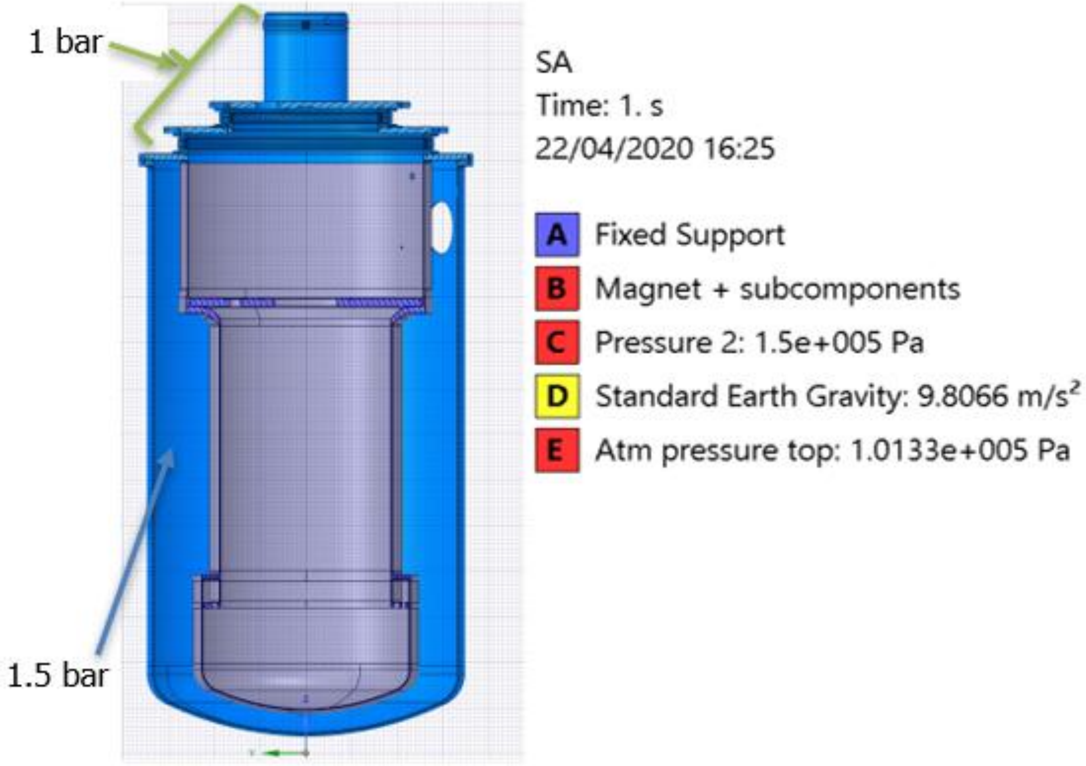
FRESCA2 OHV: operational scenario

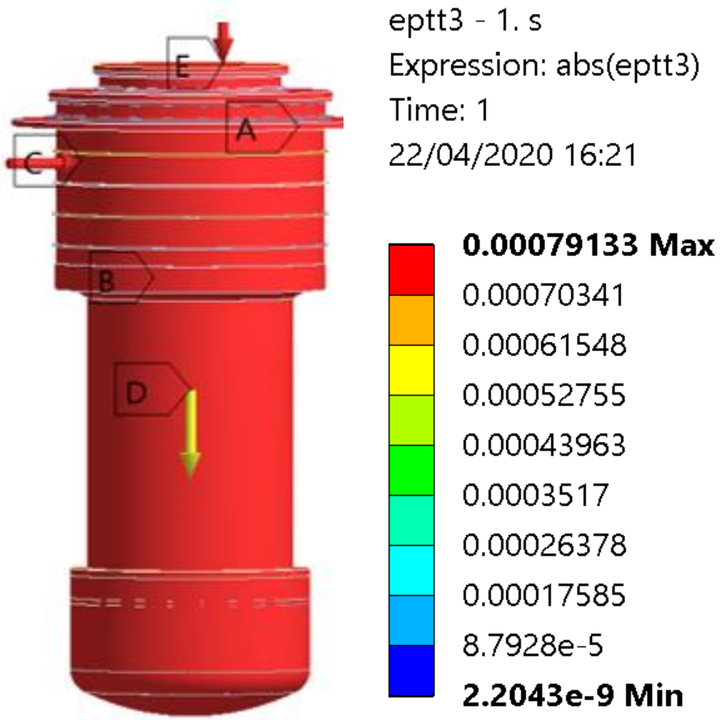


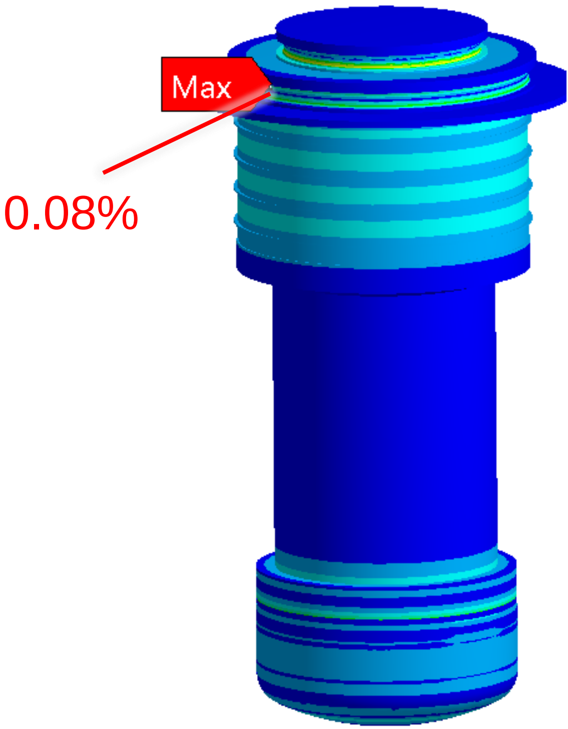
- Direct route requires $max(|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|) < 5\%$
- How to make sure of accuracy of the results?
  - Convergence study
  - Submodeling



FRESCA2 OHV: vacuum loss during purging

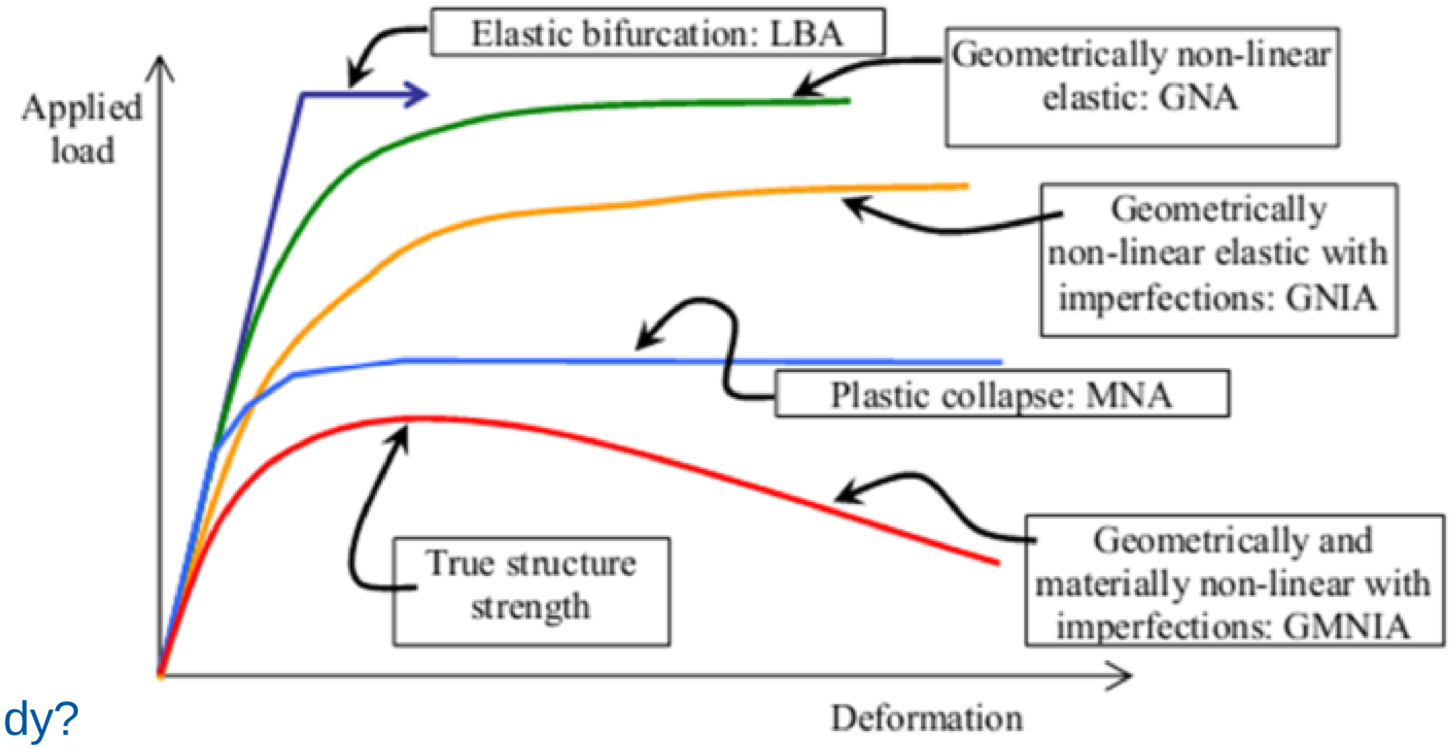






- Direct route check: $0.08\% < 5\%$ → **not enough!**
- (Especially) with external pressure, important to **verify buckling**

FRESCA2 OHV: vacuum loss during purging – Buckling



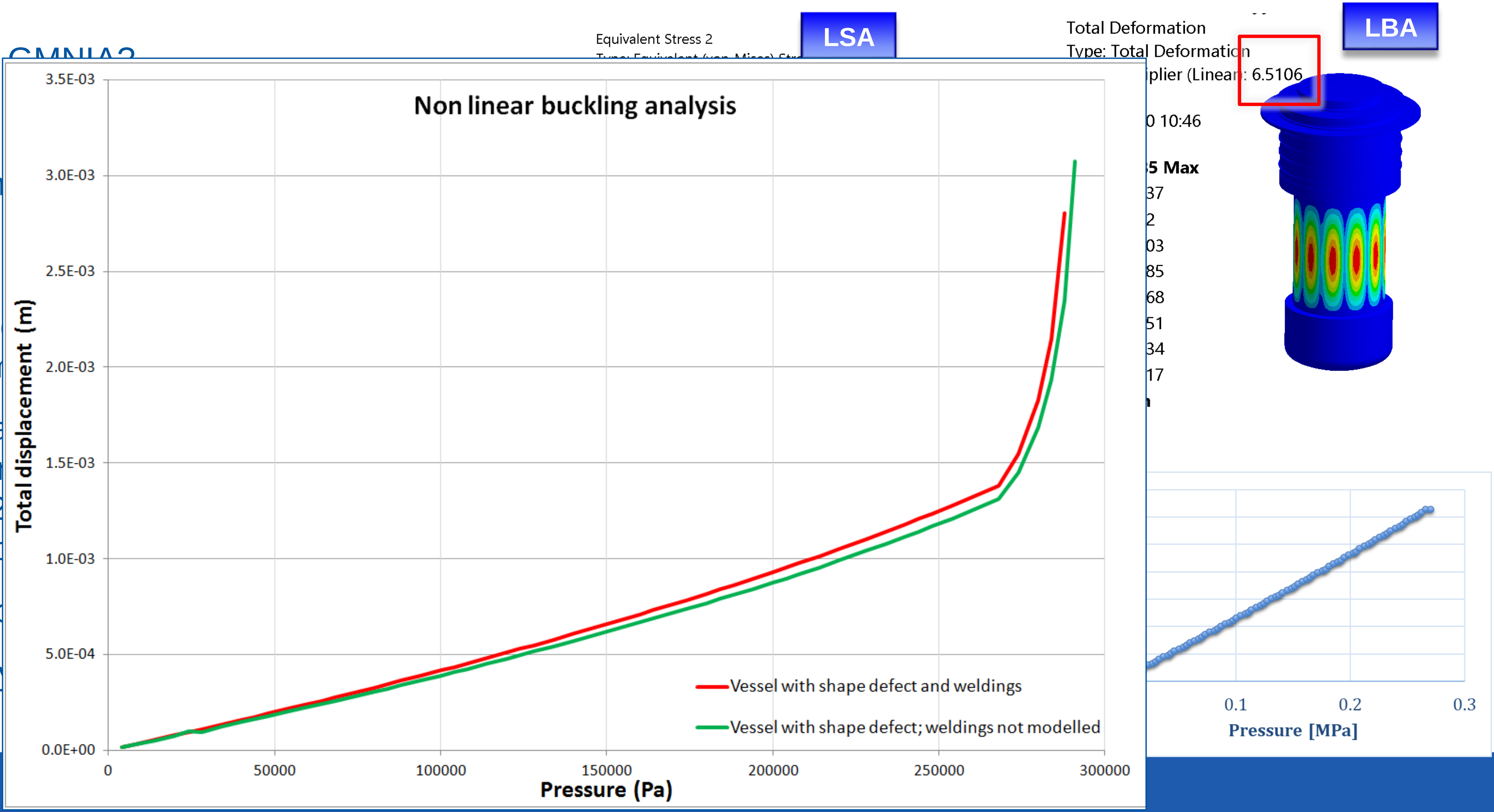
- Which kind of buckling study?
- **When going with FEM, better to directly take the most accurate one (GMNIA)**
- Also required by direct route. It accounts for large deformation theory, material nonlinearities, and initial geometry imperfections (e.g. shape errors, etc.)

FRESCA2 OHV: vacuum loss during purging – Buckling

How to perform a GMNIA?

Steps:

1. LSA: Run a lin… imperfections)
2. LBA: Perform … buckling) and … and the load m…
3. GMNIA: Run a … deformation a… geometry of (2… Increase the lo…
   - Buckling …
   - The safety … reached



Conclusions

- Nowadays, Computer-Aided Engineering (CAE) is of paramount importance in the design phase of components, to decrease cost, time, risk for the project
- CAE require a number of iterations with CAD, with the goal of optimizing the component
- What cannot be calculated, must be validated through tests / prototypes (calculation cannot replace everything!)
- Thanks to the increase in the computational power, the Finite-Element Method (FEM) has been, in the last years, the most adopted tool for CAE
- When engineering particle accelerator components, implicit codes are typically adopted over explicit ones
- Explicit codes become necessary when dealing with short transient simulations (*e.g.* beam impact on dumps, windows, etc.) and with strongly nonlinear problems (e.g. fabrication technologies: cutting, welding, brazing, forming, etc.)
- **Graphical interfaces of FEM tools are becoming simpler: easier for the work, riskier if we do not well master the method!**

Symbols

- $[M]$: mass matrix $[kg]$
- $[C]$: damping matrix $[N/({}^{m}/_{s})]$
- $[K]$: stiffness matrix $[N/m]$
- $\{\ddot{u}\}$: acceleration vector $[m/s^2]$
- $\{\dot{u}\}$: velocity vector $[m/s]$
- $\{u\}$: displacement vector $[m]$
- $\{F\}$: external force vector $[N]$
- $\{s\}$: nodal displacements vector $[m]$
- $[N]$: shape functions matrix $[-]$
- $\{\varepsilon\}$: strain vector $[-]$
- $[\partial]$: strain-displacement matrix $[m^{-1}]$
- $\{\sigma\}$: stress vector $[Pa]$
- $[D]$: material constitutive matrix $[Pa]$
- $\{a\}$: polynomial coefficients vector $[-]$
- $[P]$: position matrix $[m]$
- $\{a\}$: nodal position matrix $[m]$
- ε_1: maximum principal strain $[-]$
- ε_2: middle principal strain $[-]$
- ε_3: minimum principal strain $[-]$

Thank you!
Questions?

Backup slides

FEM Theory (backup)

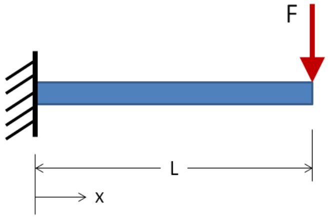
E EQUILIBRIUM

C COMPATIBILITY
Displacements & strains

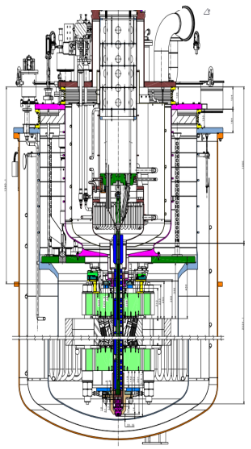
M MATERIAL LAWS
Stress & strain

IF E C M are satisfied everywhere inside and on the surface of an elastic body

strand strain fields are **exactly correct**



BUT, impossible for complex structures !



FE usually guarantees two of the laws are satisfied exactly: C M

E is **approximated**, by:
- Principle of Virtual Work
- Principle of Minimum Potential

Solution will converge to exact answer as number of elements tends to infinity.

Manual FE in 10 steps

I .Displacement $u=[N]\{s\}$.

II. Strain $\{\varepsilon\}=[B]\{s\}$.

III. Define material property matrix [D].

IV. Stress $\{\sigma\}=[D][B]\{s\}$.

V. Determine element stiffness matrix and force vectors.

VI. Transform nodal variables to global.

VII. Assemble the global stiffness matrix and load vector.

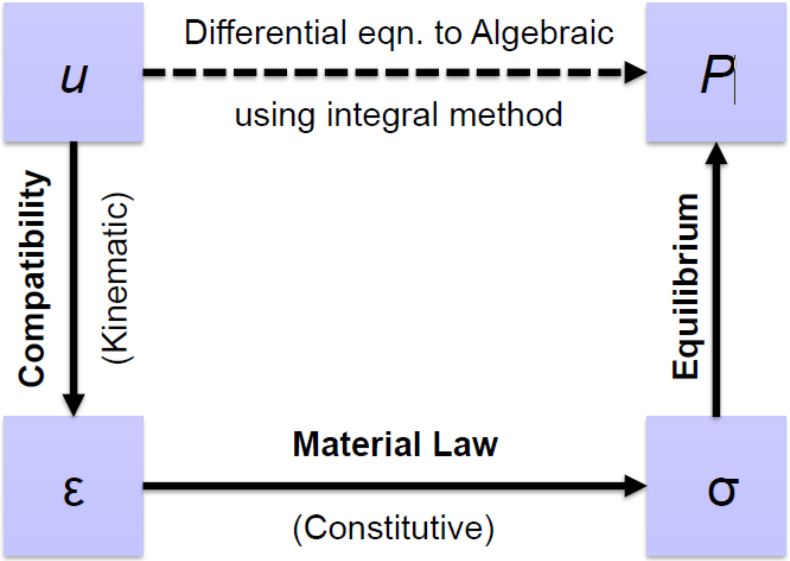
VIII. Boundary conditions.

IX. Get unknown displacements and reactions.

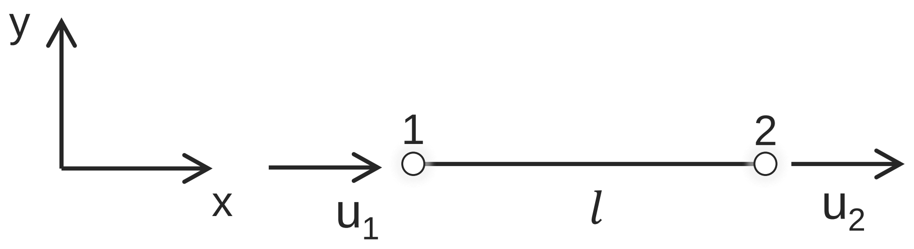
X. $\{\sigma\}=[D][B]\{u\}$.

$$u \leftrightarrow \varepsilon \leftrightarrow \sigma \leftrightarrow P$$

$$[k]\{u\}=\{P\}$$



Derivation of shape function for a truss element



$$u = a_1 + a_2 x = [1 \quad x]\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = [P]\{a\} \rightarrow \quad u_1 = [1 \quad x_1]\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}; \quad u_2 = [1 \quad x_2]\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

(1)

$$b.c.: \quad x_1 = 0; \quad x_2 = x_1 + l = l$$

$$\{s\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = [A]\{a\} \rightarrow \quad [A]^{-1} = \frac{1}{l}\begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \rightarrow \quad \{a\} = [A]^{-1}\{s\} = \frac{1}{l}\begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

(2)

$$(1) + (2) \rightarrow u = [P][A]^{-1}\{s\} = [1 \quad x]\frac{1}{l}\begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \left[\left(1 - \frac{x}{l}\right) \quad \frac{x}{l}\right]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$[N] = [P][A]^{-1} = \left[\left(1 - \frac{x}{l}\right) \quad \frac{x}{l}\right]$$

Calculation of the element stiffness matrix

Deformation matrix $[B] = [\partial][N]$

Applying principle of virtual works: $[K] = \int_V [B]^T [D] [B] dV$

Implicit and Explicit Problems

Implicit analysis

- aim to solve the unknowns {x} through matrix inversion.
- in nonlinear problems - the solution is obtained in a number of steps and the solution for the current step is based on the solution from the previous step.
- for large models - inverting the matrix is highly expensive and will require advanced iterative solvers
  - these solutions are unconditionally stable and facilitate larger time steps.

Despite this advantage, the implicit methods can be extremely time-consuming when solving dynamic and nonlinear problems.

Explicit analysis

- aim to solve for acceleration {x''}
- in most cases, the mass matrix is considered as lumped → a diagonal matrix → inversion is straightforward
  - once the accelerations are calculated in n^{th} step, the velocity at n+1/2 step and displacement at n+1 step are calculated
- in these calculations, the scheme is not unconditionally stable and thus smaller time steps are required.

To be more precise, the time step in an explicit finite element analysis must be less than the Courant time step (*the time taken by a sound wave to travel across an element*).

Explicit vs Implicit

Explicit

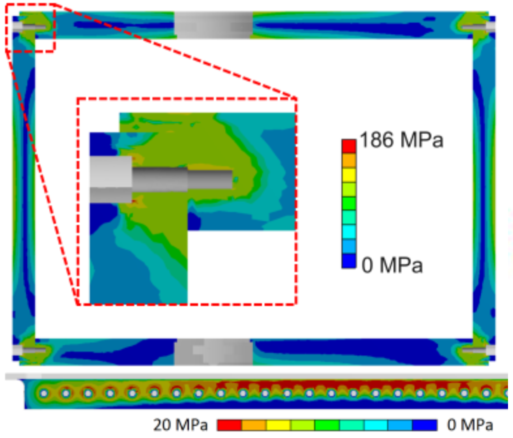
- robust, even for strongly non linear models
- low memory requirements
- expensive to conduct long term simulations
- conditionally stable
- equations are uncoupled
- only matrix multiplication

Implicit

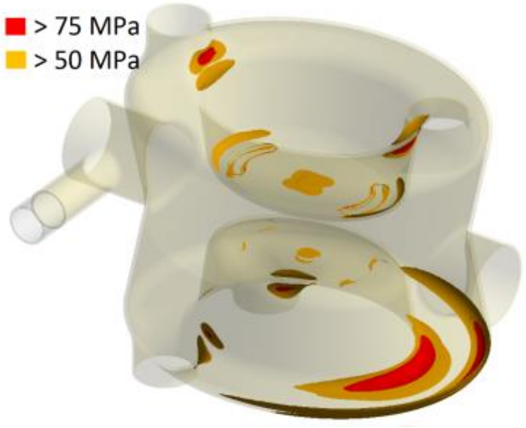
- can be unconditionally stable
- eventually problematic for strongly non-linear models
- high memory requirements(inverting stiffnes matrix)
- relatively inexpensive for long duration analysis
- equations are coupled
- matrix inversion
- convergence problem

Examples of studies performed (2)

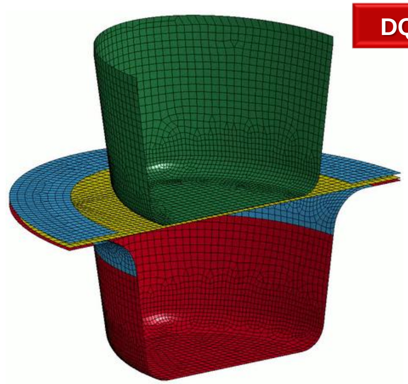
- **Crab cavities (BE-RF)**
  - Studies for the **design of dressed cavities and cryomodule**, optimization of the **thermal loss balance**

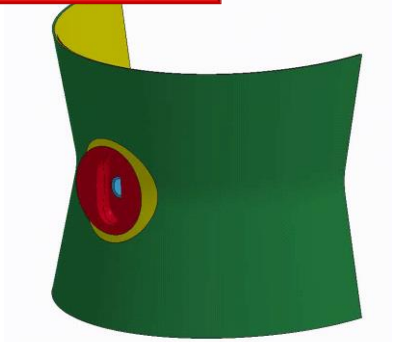


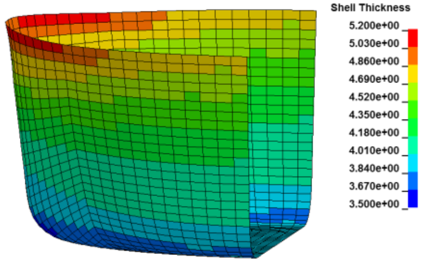
Stress on He vessel and DQW cavity

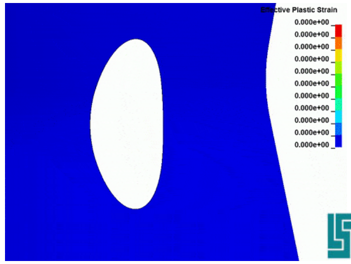


DQW bowl and diabolo









LS-DYNA

- Explicit analyses for the **sheet metal forming** of cavity components (bowl, diabolo, etc.), to explore **different tool shapes and forming steps** as well as predict the stress-strain field and thickness distribution

Job example: TIDVG5 hipping

- **User**: EN-STI
- **Problem**: TIDVG5 cooling circuit bursting during hipping
- **Method**: FEA (implicit + explicit)
- **Proposed solution**: reduce pipe/housing gap, adapted shape of housing
- **Future developments**: standardize bending process