CAS: Acceleration of electrons in plasma II

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Topics covered

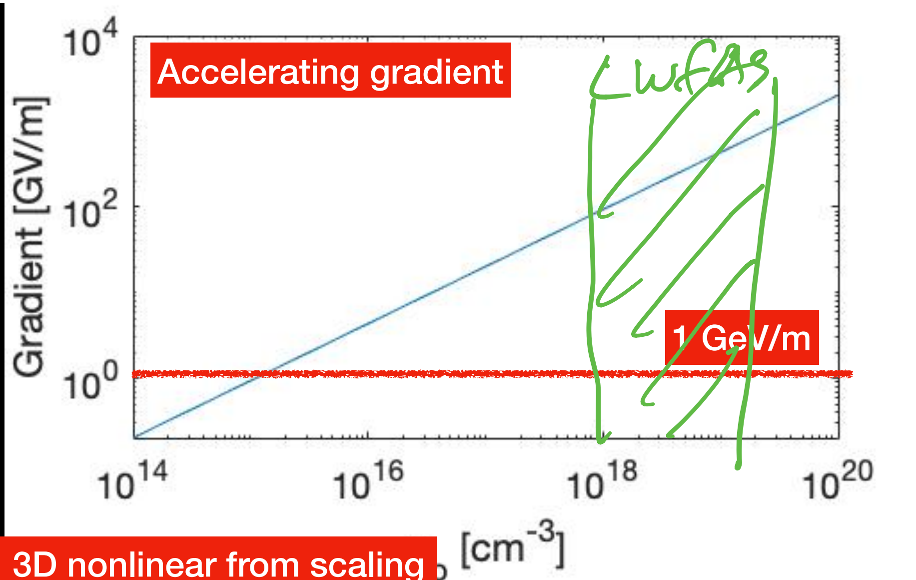
- Parameters for wakefield drivers
- Linear and nonlinear wakes
- Limits on Maximum energy gain of an electron beam
  - Dephasing
  - Depletion
- Wake Hamiltonian
- Wake phase space
- Trapped/untrapped orbits
- Phase space rotation
- Transformer ratio
- Beamloading and energy spread considerations
- Transverse emittance
- Betatron oscillations in wakefields

Recap:

Summary of scalings (LWFA)

Lu PRZ 2007

	a_0	$k_p w_0$	ϵ_{LW}	$k_p L_d$	$k_p L_{pd}$	λ_W	γ_ϕ	$\Delta W/(mc^2)$
Linear:	<1	2π	a_0^2	$\frac{\omega_0^2}{\omega_p^2}$	$\frac{\omega_0^2}{\omega_p^2}\frac{\omega_p\tau}{a_0^2}$	$\frac{2\pi}{k_p}$	$\frac{\omega_0}{\omega_p}$	$a_0^2\frac{\omega_0^2}{\omega_p^2}$
1D Nonlinear:	>1	2π	a_0	$4a_0^2\frac{\omega_0^2}{\omega_p^2}$	$\frac{1}{3}\frac{\omega_0^2}{\omega_p^2}\omega_p\tau$	$\frac{4a_0}{k_p}$	$\sqrt{a_0}\frac{\omega_0}{\omega_p}$	$4a_0^2\frac{\omega_0^2}{\omega_p^2}$
3D Nonlinear:	>2	$2\sqrt{a_0}$	$\frac{1}{2}\sqrt{a_0}$	$\frac{4}{3}\frac{\omega_0^2}{\omega_p^2}\sqrt{a_0}$	$\frac{\omega_0^2}{\omega_p^2}\omega_p\tau$	$\sqrt{a_0}\frac{2\pi}{k_p}$	$\frac{1}{\sqrt{3}}\frac{\omega_0}{\omega_p}$	$\frac{2}{3}\frac{\omega_0^2}{\omega_p^2}a_0$
Ref. [24]:	>20	$\sqrt{a_0}$	$\sqrt{a_0}$		$a_0\frac{\omega_0^2}{\omega_p^2}\omega_p\tau$			$\frac{\omega_0^2}{\omega_p^2}a_0^{3/2}\omega_p\tau$



3D nonlinear from scaling simulations

$$L_d \propto 2\gamma_p^2 \frac{c}{\omega_p} \sim \frac{1}{n^{3/2}}$$

$$\Delta\gamma mc^2 \propto 2\gamma_p^2 \frac{e\Delta\phi}{mc^2} \sim \frac{1}{n}$$

$$\gamma_p(\text{laser}) \sim \frac{\omega_0}{\omega_p} \sim \sqrt{\frac{n_c}{n_e}}$$

Plasma-based accelerators for future colliders

- PWFA-linear collider:
- two-beam accelerator geometry
- 25 GeV drive beams
- 19 plasma stages (1 TeV)
- $n=10^{17}$ cm^{-3} (set by 30 um driver bunch length)

Seryi et al., Proc. PAC (2009)

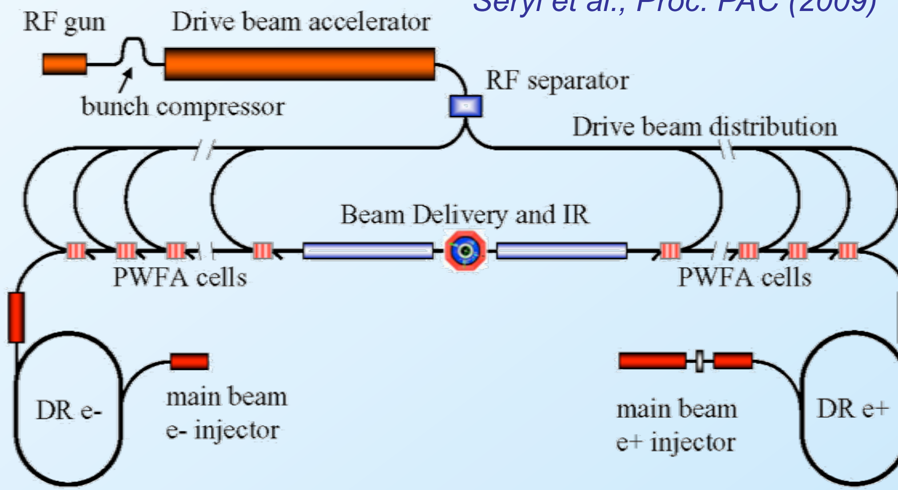
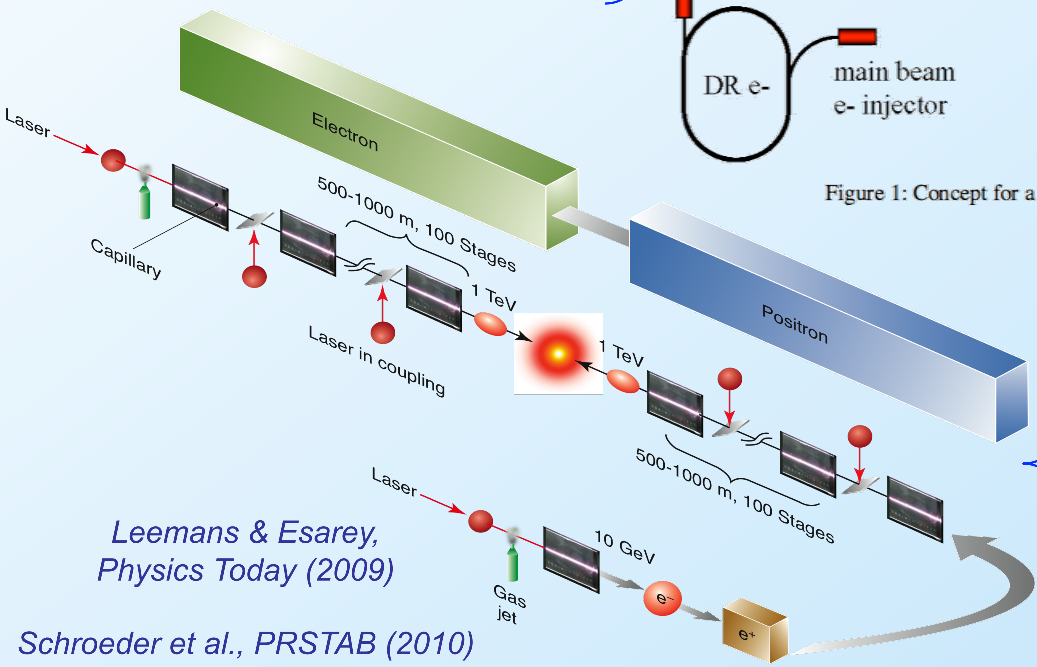


Figure 1: Concept for a multi-stage PWFA-based Linear Collider.

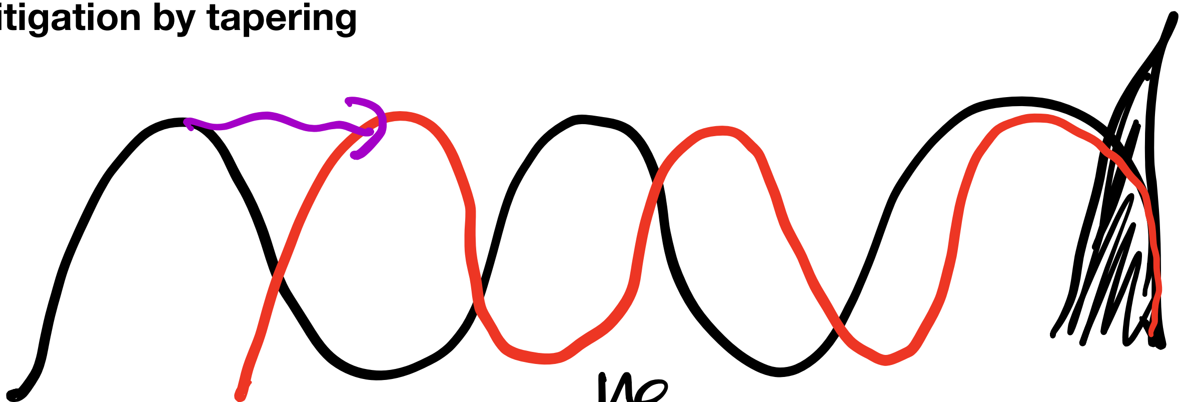


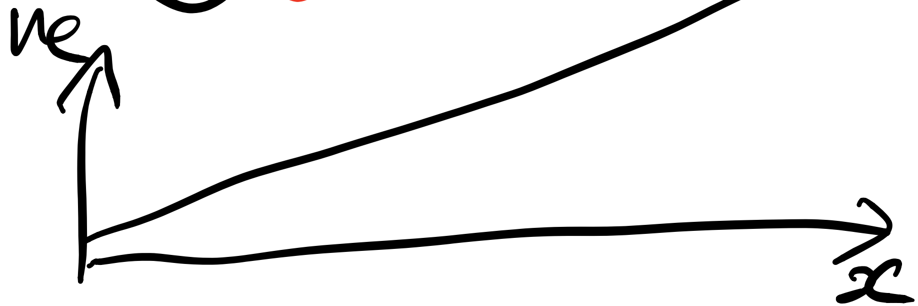
Leemans & Esarey, Physics Today (2009)

Schroeder et al., PRSTAB (2010)

- LPA-linear collider:
- 50 stages (1 TeV collider)
- 10 GeV/stage
- requires ~10 J laser (at tens of kHz, hundreds of kW)
- $n=10^{17}$ cm^{-3} (set by laser depletion)

Mitigation by tapering





$$\lambda_p \sim \frac{2\pi c}{\omega_p}$$

$$\lambda_p \downarrow$$

Recap:

Hamiltonian for wakefield

- Start with Hamiltonian for a system that depends on the wake phase coordinate $\xi = x - v_p t$

$$\boxed{H} = \sqrt{(mc^2)^2 + p_x^2 c^2 + (p_\perp - eA_\perp(\xi))^2\, c^2} - e\phi(\xi) - p_x v_p$$

- We can show that this is the correct Hamiltonian for the coordinates

$$\xi, y, z, p_x, p_y, p_z$$

- using Hamilton's equations…

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i} \qquad \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$$

Wake phase space

- Consider the possible particle trajectories described by the Hamiltonian by solving for p_x

$$p_x c = \gamma_p^2 (H + \phi) \left(\beta_p \pm \sqrt{1 - \frac{m^2c^4 + e^2A^2c^2}{\gamma_p^2 (H + \phi)^2}} \right)$$

- Note we have assumed conservation of transverse canonical momentum

$$p_\perp = p_{\perp 0} = 0 \qquad (p_\perp - eA) = 0$$

kinetic momentum

Wake phase space

- To illustrate, we calculate numerical solutions of the nonlinear Poisson equation (other lectures for derivation):

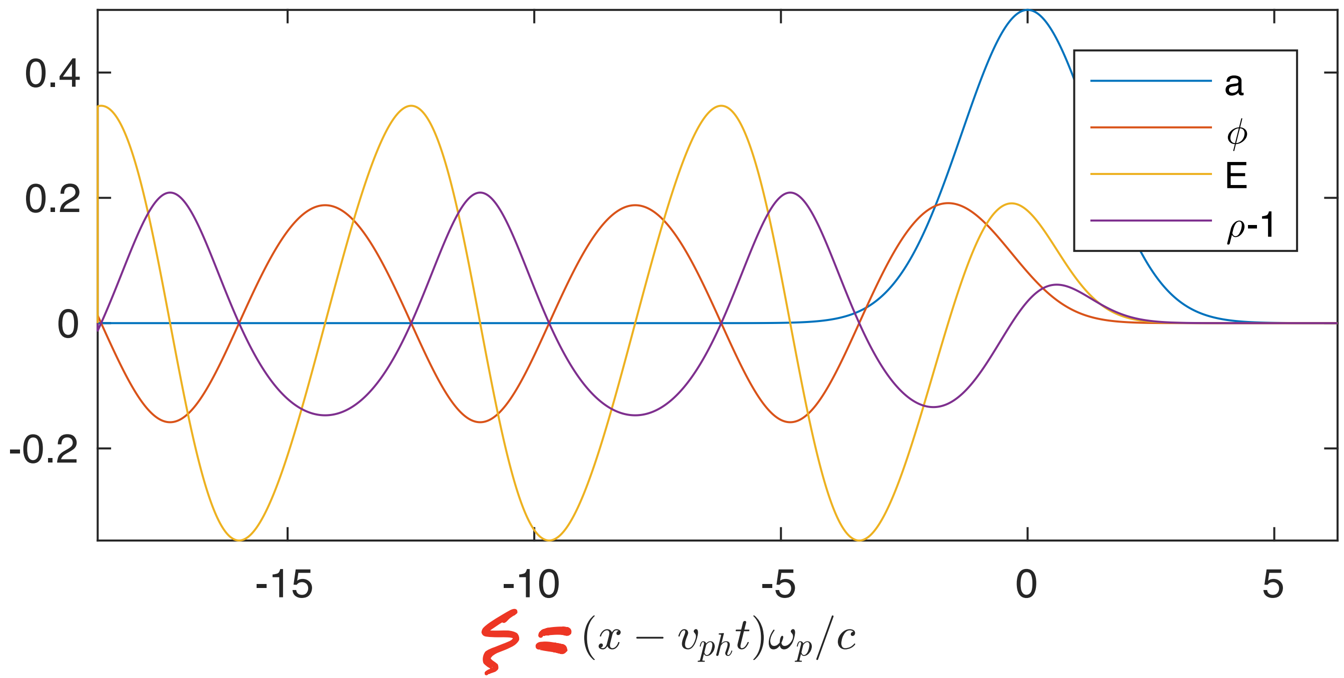
$$\frac{\partial^2 \phi}{\partial \xi^2} = \gamma_p^2 k_p^2 \left(\beta_p \left(1 - \frac{1 + a^2}{\gamma_p^2 \left(1 + \phi\right)^2} \right)^{-1/2} - 1 \right)$$

- with gaussian driver field

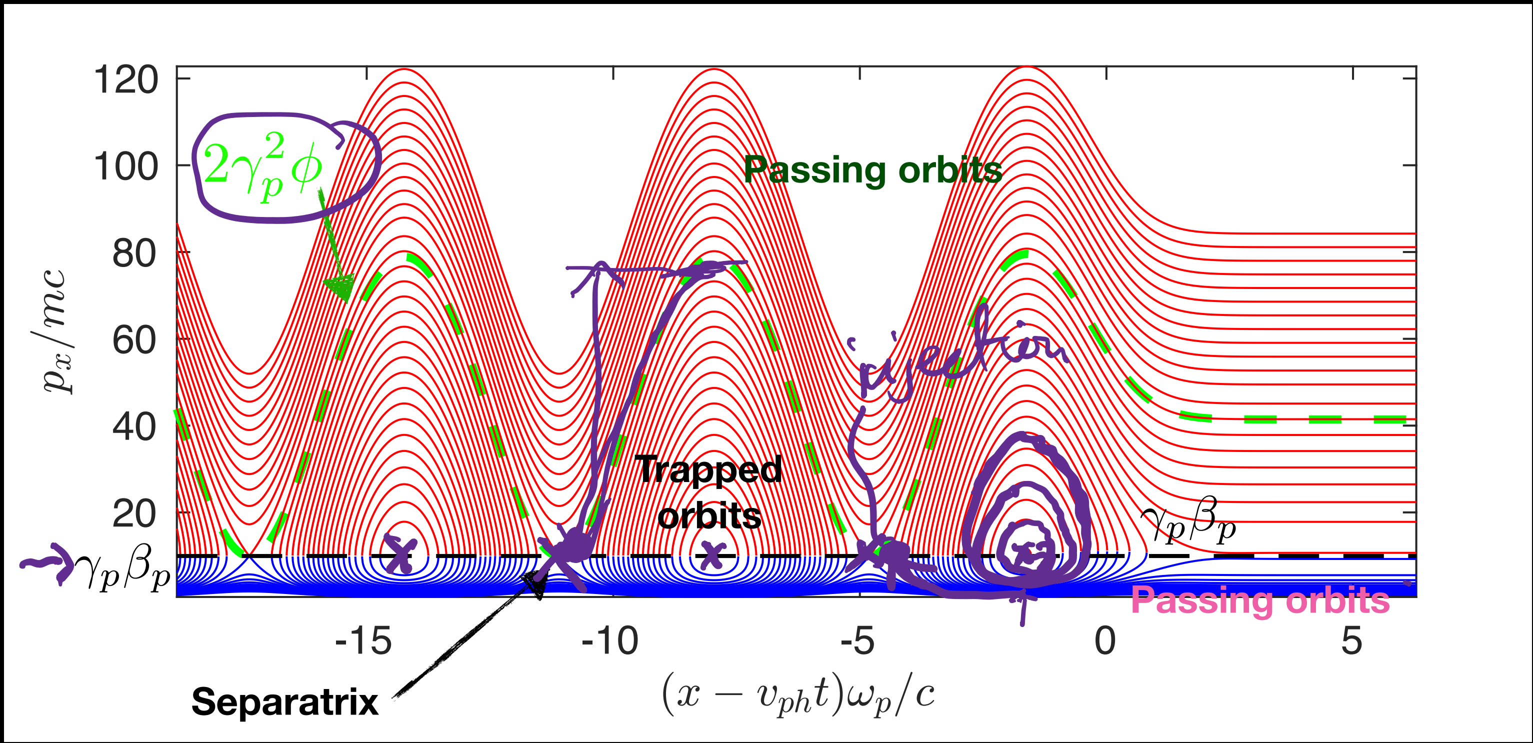
$$a = a_0 \exp \left(-4 \ln 2 \frac{\omega_p^2 \xi^2}{\pi^2 c^2} \right)$$

Wake potential, field, density

$a = 0.5$

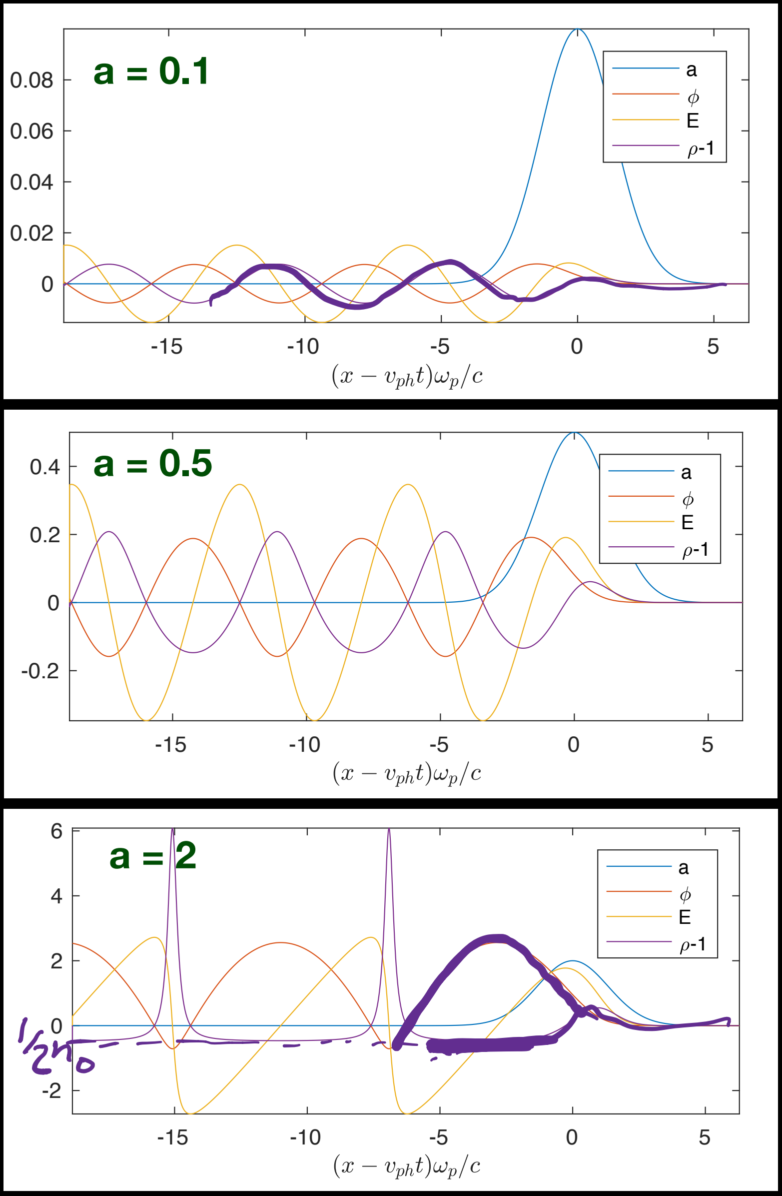


Wake phase space



Wake phase space for different driver amplitudes

- As the driver amplitude increases the potential changes
  - from sinusoidal (with near circular orbits)
  - to inverted parabola (with parabolic forward orbit)



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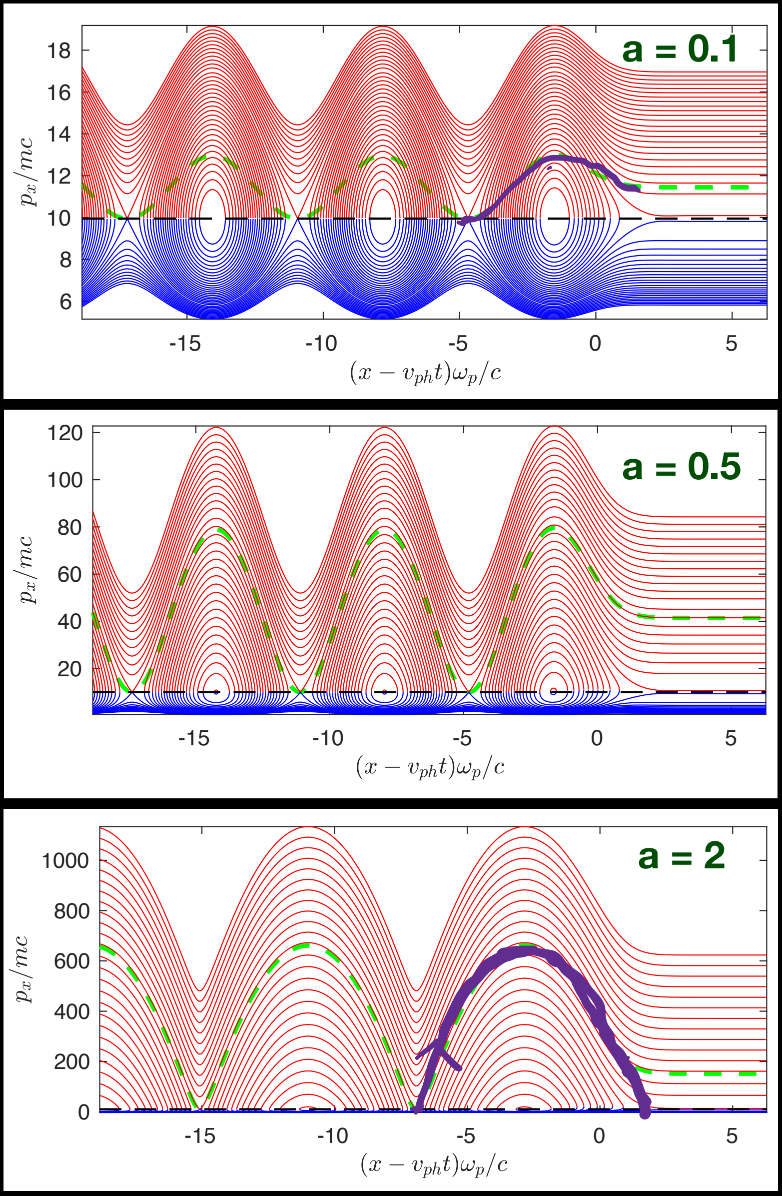
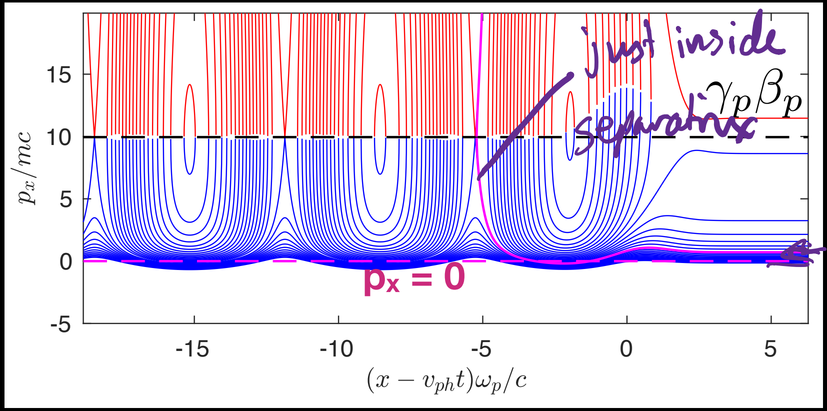
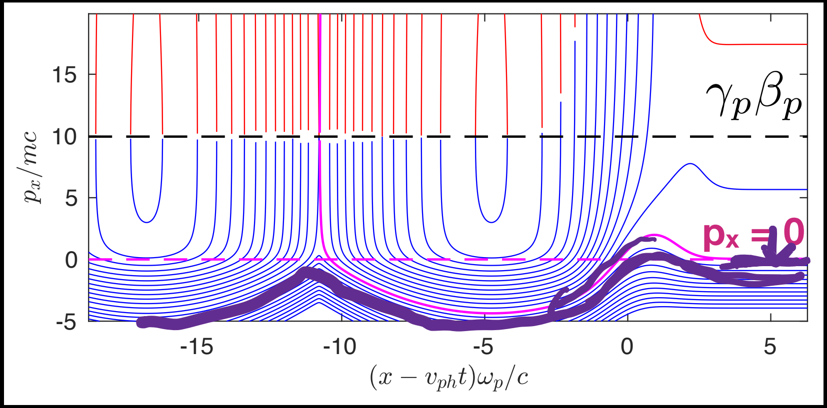


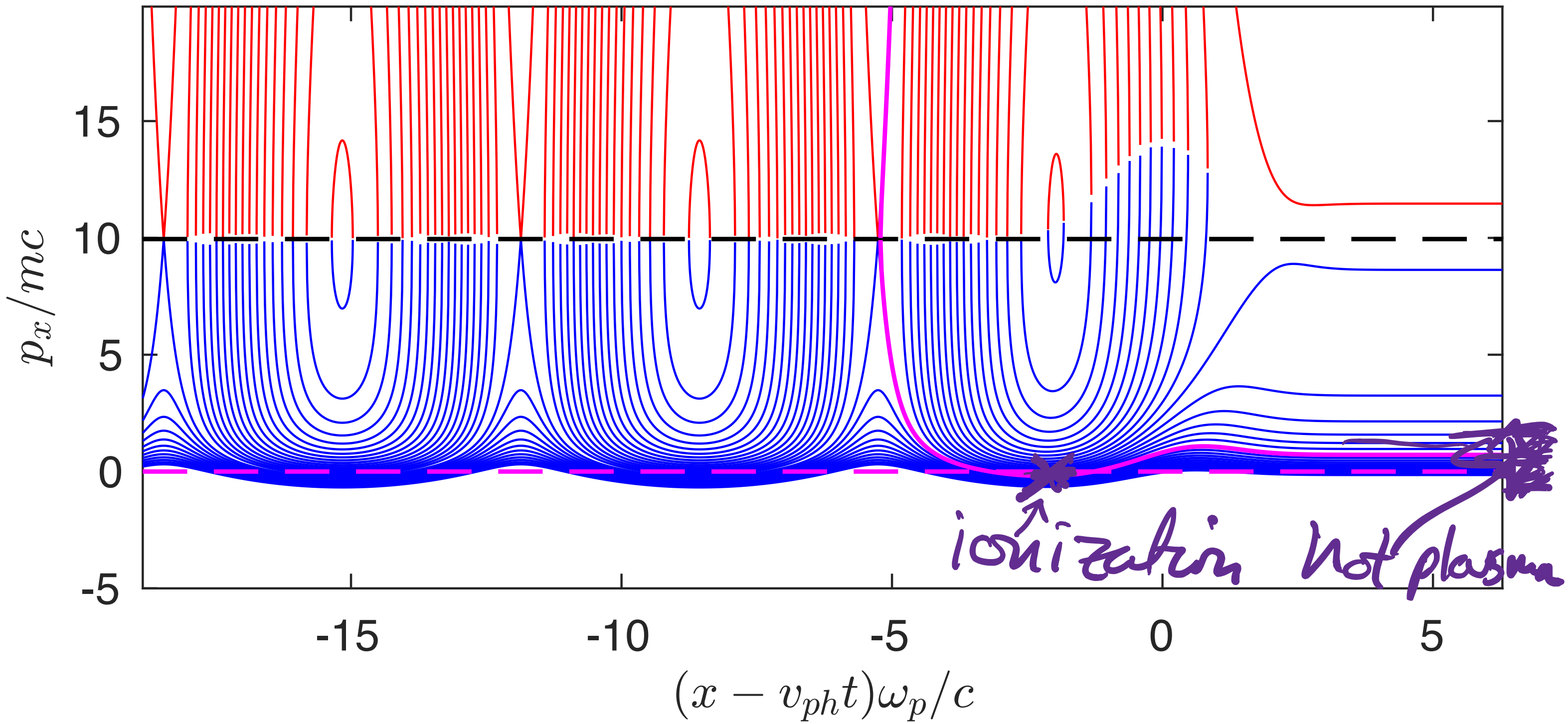
Wake phase space: Injection / trapping



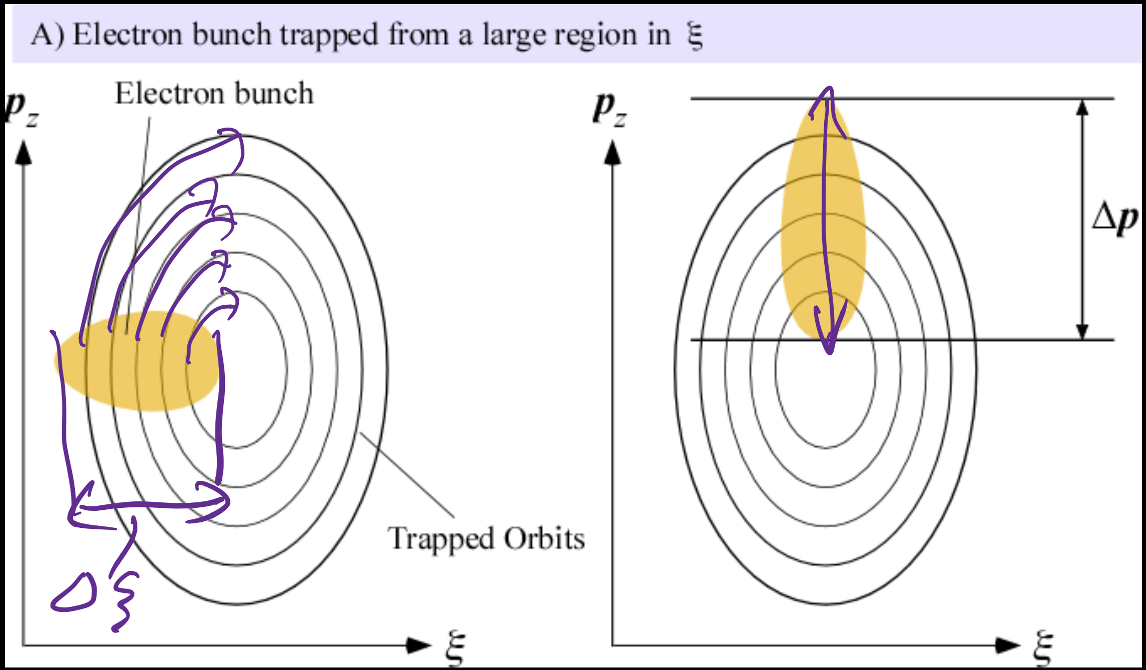


$a_0 = 4$

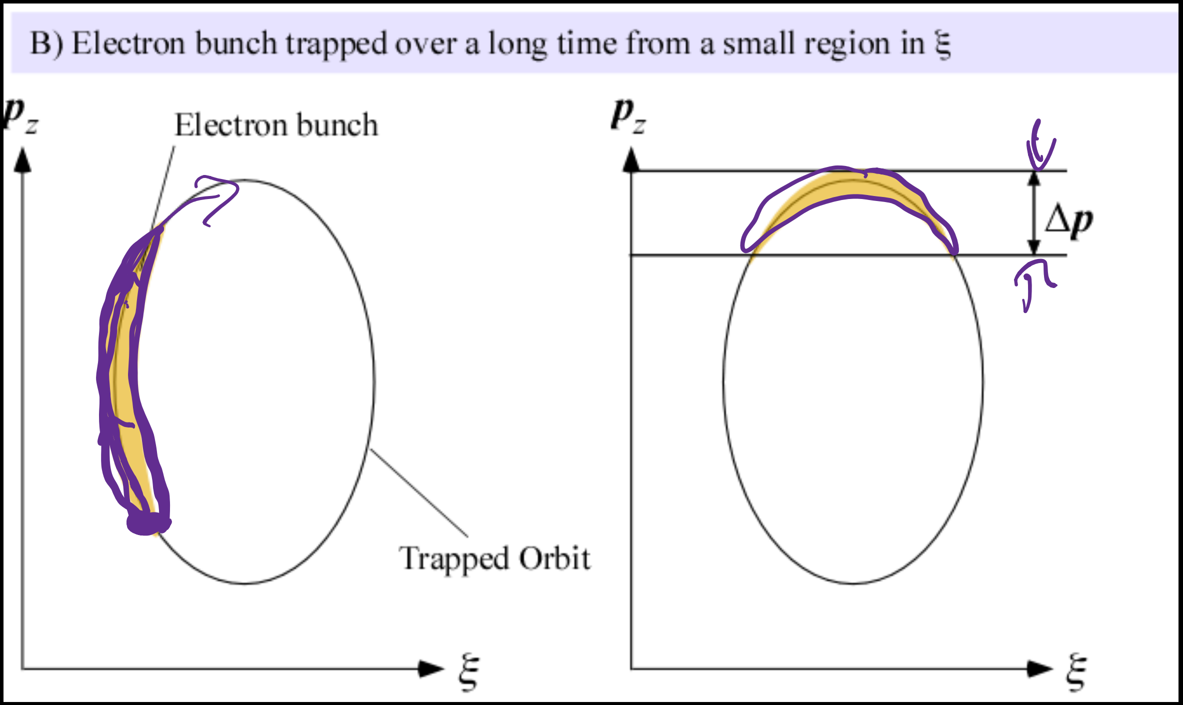
Thermal / ionization induced trapping



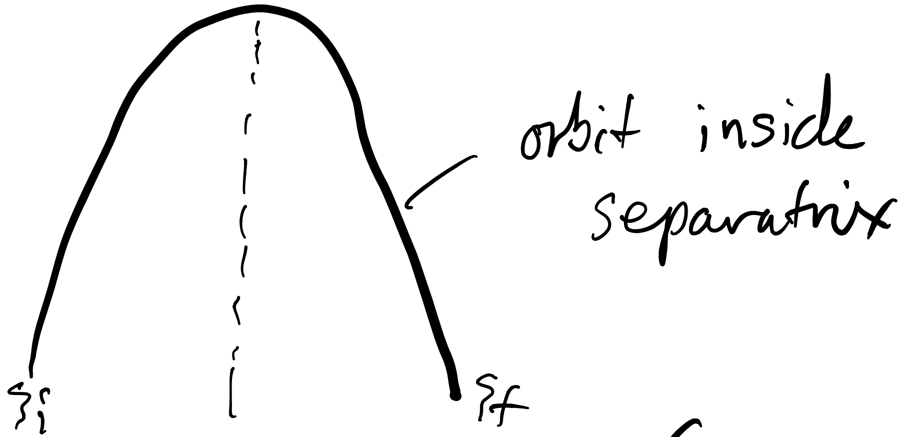
Energy spread compression by phase rotation



Energy spread compression by phase rotation



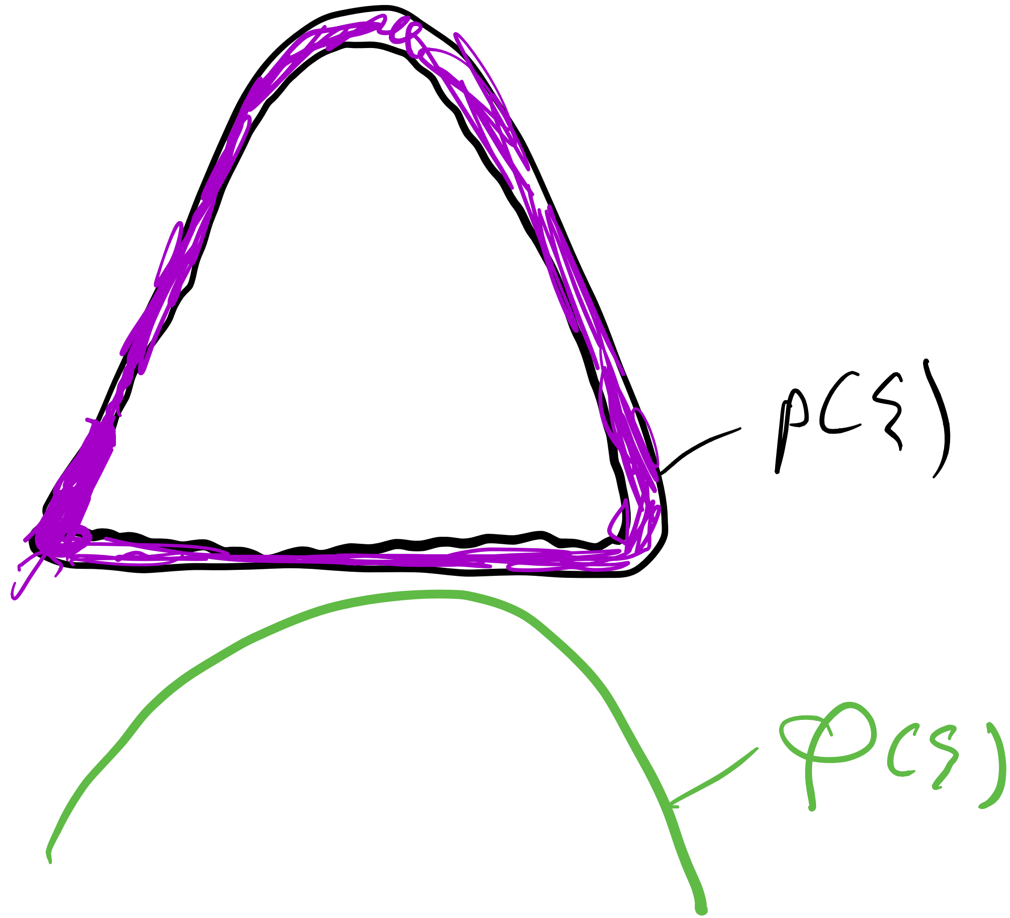
Show spectrum calculated from Hamiltonian



each orbit is described by $\delta(p - p(\xi, h))$

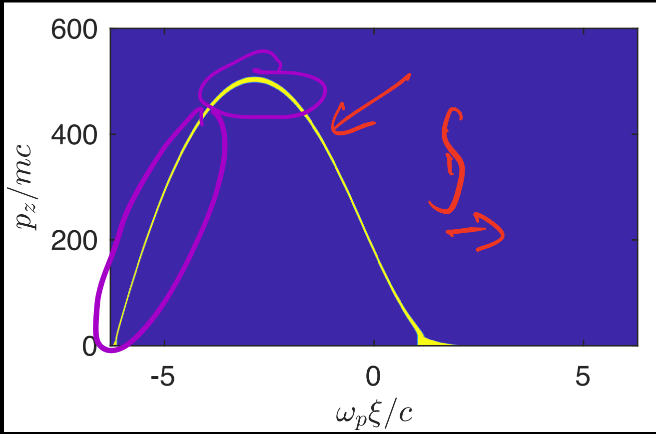
$$f(h)$$

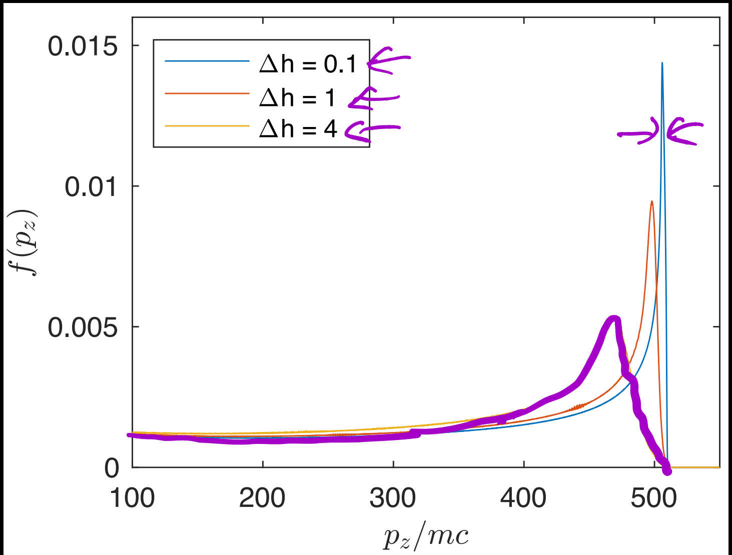
$$\boxed{f(p) = \int_{\xi_i}^{\xi_f} \int_{-\infty}^{H_{upper}} f(h)\, \delta(p - p(\xi, h))\, d\xi}$$

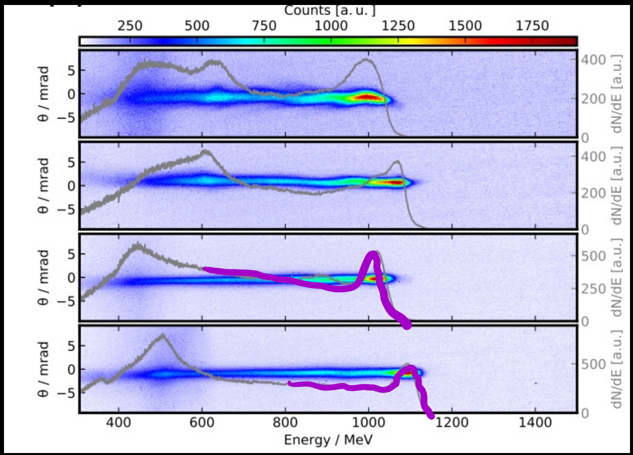

p(ξ)
φ(ξ)

Analytic spectrum for continuous injection

$$f(h) = f_0 \exp(-h^2/\Delta h^2)$$







Hussein Sci. Rep. 2019

Depletion limited energy gain and beamloading

Depletion length

- **The length over which the driver energy is deposited in the plasma**

- Inherently complicated as depletion affects the beam intensity / erodes the head / changes the driver velocity / wakefield amplitude

- We can estimate it in the following way:

pulse energy

$$\underline{\gamma_b mc^2} \sim e\underline{E_x}\,\underline{L_{pd}} \qquad \frac{U_L}{\varepsilon_0 E_x^2 L_{pd} w_0^2} \sim 1$$

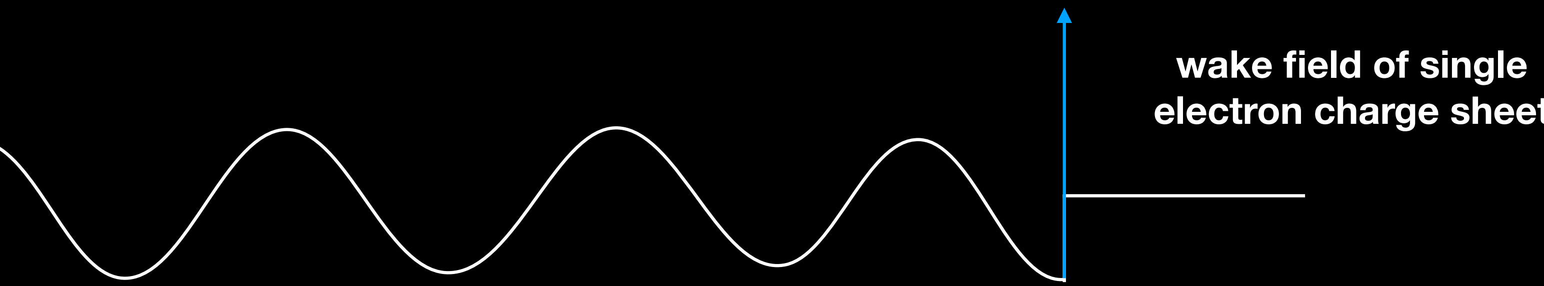
energy density of wake

- For a beam estimate from average decelerating field experienced by the bunch

- For a laser it is more complicated - can estimate using average field energy density of wake

Transformer ratio

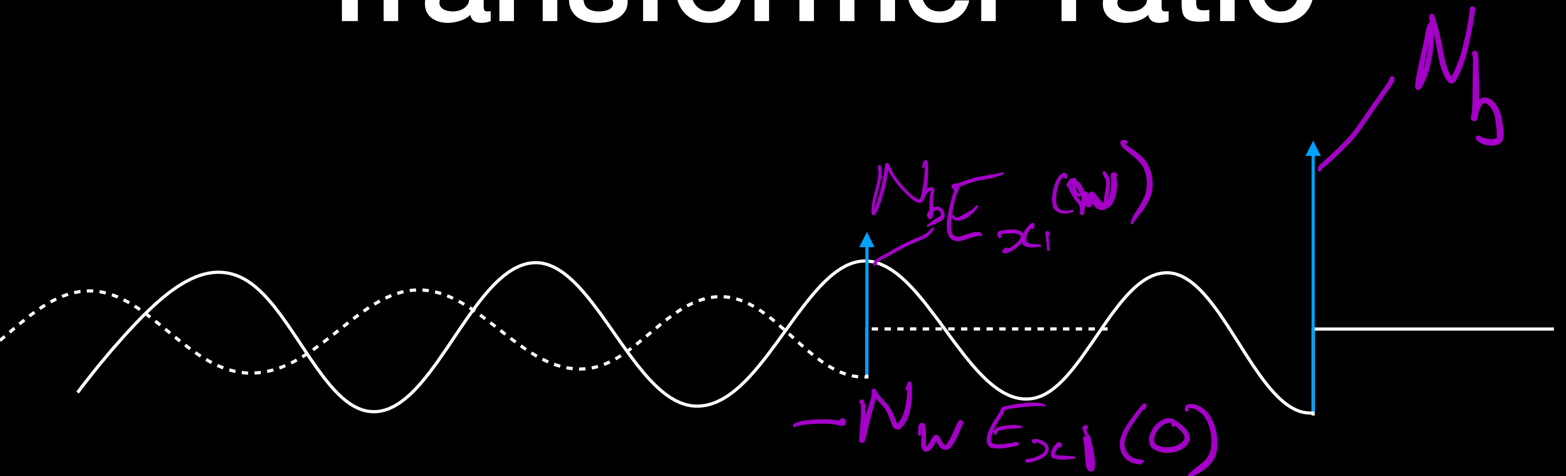
- In PWFA one important definition is the **transformer ratio**



- We accelerate a “witness” bunch in the wake of the first - but this has its own wakefield

- Total wake is linear superposition of wake contributions from drive bunch and witness bunch

Transformer ratio



- Assume two short bunches at $x-v_pt = 0$ and $x-v_pt = w$
- Each ***individual*** electron induces a wakefield $E_{x1}(\xi)$
- Total wakefield is linear superposition of $E_{x1}(\xi)$

$$\frac{d}{dt}\gamma_b mc^2 = -N_b e E_{z1}(0)\, c$$

$$\frac{d\gamma_w mc^2}{dt} = -N_b e E_{z1}(w) - N_w e E_{z1}(0)$$

$$\frac{d}{dt}\left(N_b \gamma_b mc^2 + N_w \gamma_w mc^2\right) = \frac{d}{dt}\left(\text{Total energy}\right) \leq 0$$

$$N_b^2 e E_{z1}(0) + N_b N_w e \underline{E_{z1}(w)} + N_w^2 e E_{z1}(0) \geq 0$$

$E_{z1}(w)$ must negative for acceleration

$$(N_b^2 + N_w^2) e E_{z1}(0) \geq N_b N_w e |E_{z1}(w)|$$

must be true for any N_b, N_w

$$\Rightarrow \boxed{|E_{z1}(w)| \leq 2 E_{z1}(0)}$$

$$E_{z_{accel}} \leq \left(2N_b - N_w\right) E_{z1}(0)$$

$$L_{dp} = \frac{\gamma_b mc^2}{eE_{z1}(0)N_b}$$

$$\Delta\gamma_w mc^2 = eE_z L_{dp} \leq \frac{\left(2N_b - N_w\right)}{N_b}\gamma_b mc^2$$

$$R_T = \frac{\Delta\gamma_w}{\gamma_b} = 2 - \frac{N_w}{N_b}$$

$$R_T = \frac{|E_z(\omega)|}{|E_z(0)|}$$

Transformer ratio

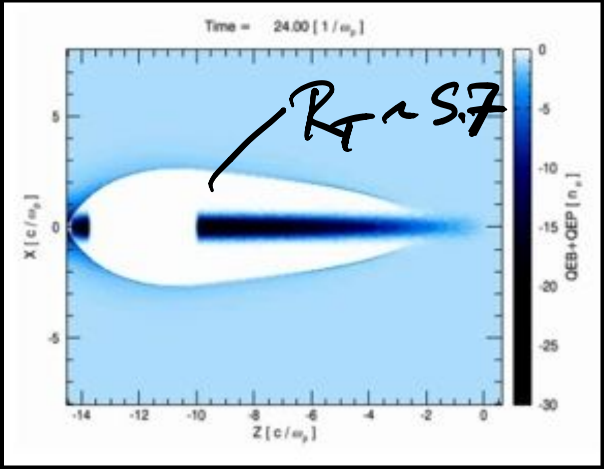
- The ***transformer ratio*** is

$$R_T = \frac{\Delta\gamma_w}{\Delta\gamma_b} = 2 - \frac{N_w}{N_b} \qquad R_T = \frac{|E_x(w)|}{|E_x(b)|}$$

- **This upper limit can be overcome by asymmetric bunches**

- For a wedge bunch of length L

$$R_T \sim \frac{L}{\Lambda_0}$$



W Lu PAC Proceedings 2009

Energy transfer efficiency

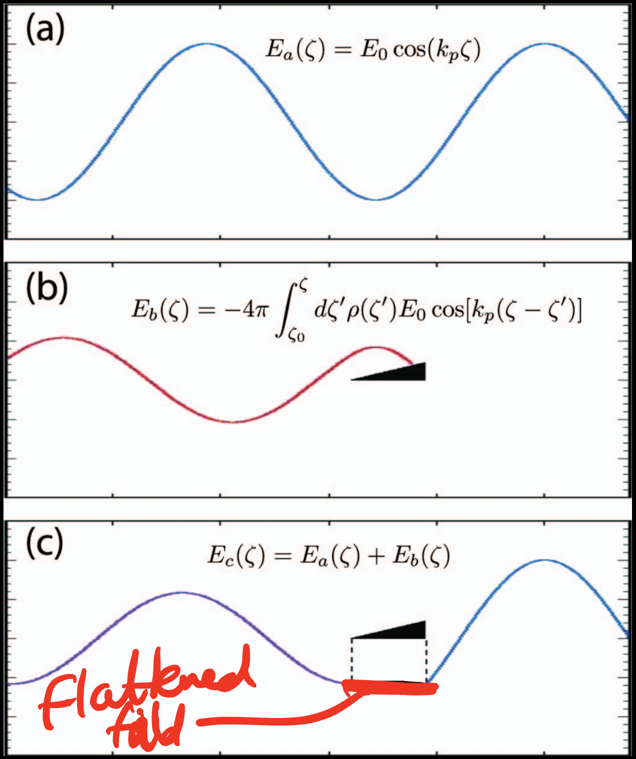
- The overall energy transfer efficiency is the ratio weighted by the bunch charges

$$\eta = \frac{N_w \Delta\gamma_w}{N_b \Delta\gamma_b} \leq \frac{N_w}{N_b}\left(2 - \frac{N_w}{N_b}\right)$$

- Which has maximum efficiency approaching 1 for $N_w = N_b$ (but this is pointless!)

Beamloading

- We have ignored the effect of the fields of the witness bunch so far
- This is ok if the witness bunch charge is small, but if the bunch charge per unit length approaches that of the plasma density perturbation, its effect will be significant
- It is possible to **flatten the field completely** with an appropriately shaped bunch - beamloading
- This will result in the electron beam being accelerated with equal field strength - **preserves beam energy spread**



Tzoufras Phys. Plasmas 2009

Summary

- Having an accelerating field structure move at v_p results in upshift in maximum energy gain and accelerator length by dephasing by

$$2\gamma_p^2$$

- For laser driven case, since $\gamma_p \propto \omega_0/\omega_p$, accelerator length scales as $1/n_0^{3/2}$ and maximum energy as $1/n_0$

- This means the accelerating gradient scales as $\cancel{1/}\sqrt{n_0}$ which implies staging for high energy gain

Summary

- For the beam driven case, the maximum energy gain is limited by driver energy loss under realistic conditions

- The acceleration length scales as the beam energy and inversely to the decelerating field strength

$$L_{pd} \sim \frac{\gamma_b mc^2}{e|E_x(\cancel{w})|}$$

beam

- And the maximum energy gain is limited by the transformer ratio

$$R_T = \frac{|E_z(w)|}{|E_z(b)|}$$

Summary

- The energy spread of the accelerated beam is affected by
  - Phase space rotation
  - The fields of the bunch (Beam loading for flat field)
- And can result in low energy spread for correct extraction timing / accelerated charge profile / localisation of trapping in wake phase

Thank you!