



Longitudinal Beam diagnostics

T. Lefevre
CERN



A big thanks to all the people who
provided materials for this lecture !!

A. Gillepsie, S. Jamison, A. Cianchi,

- Longitudinal beam profile in accelerators
- Invasive and Non-invasive techniques
 - Explain concepts
 - Review performances and limitations



Accelerating charged particles



DC Accelerator



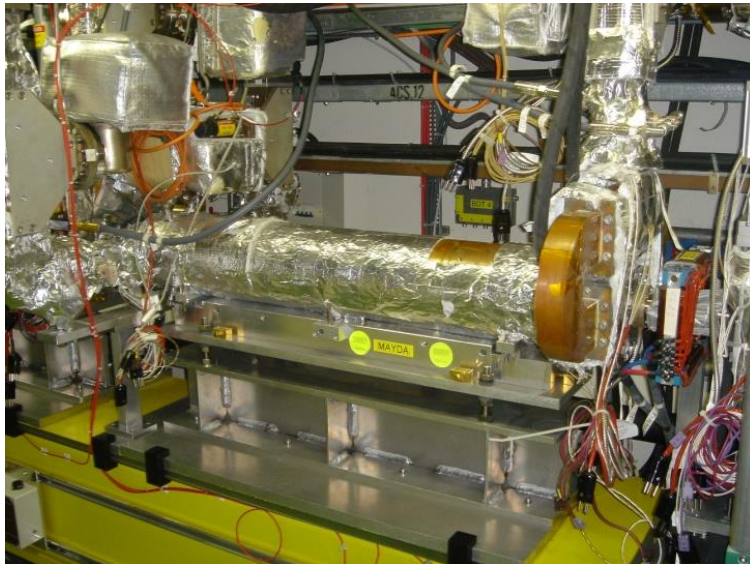
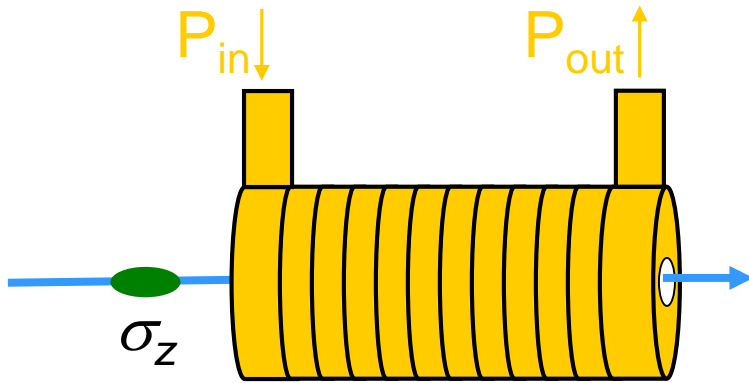
RF Accelerator



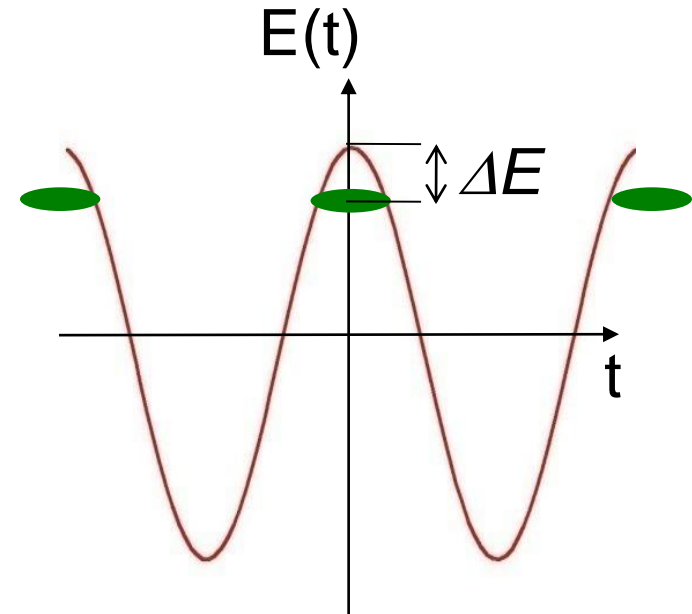
synchronizing particle
with an
electromagnetic wave!

Acceleration techniques

RF Accelerating structures



RF Accelerating Field



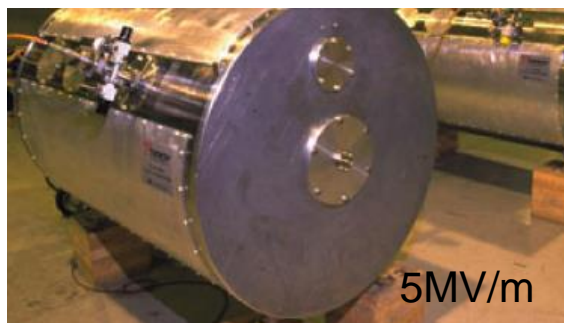
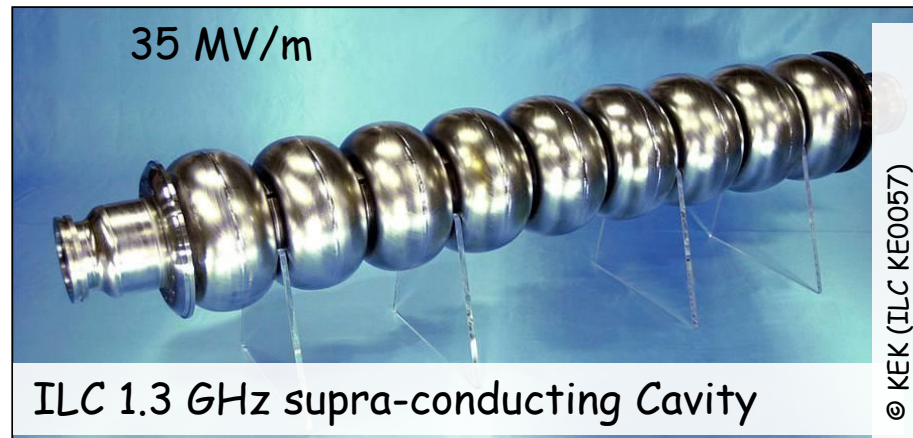
At 3GHz accelerating frequency

1 period = 333ps : Bunch spacing
 Typical bunch length : few deg ~ few ps

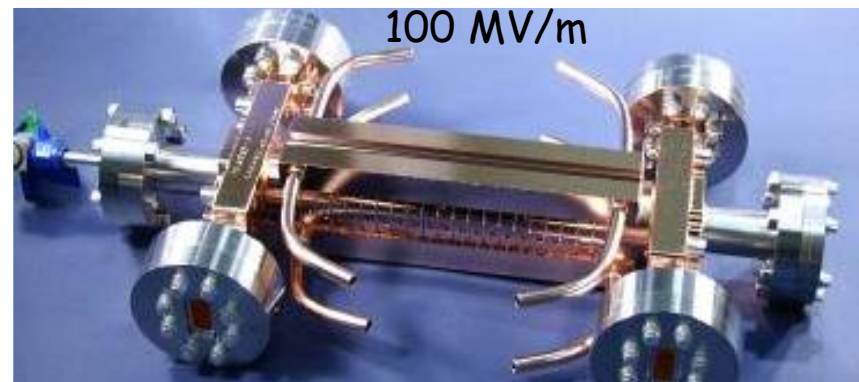
Accelerating cavities

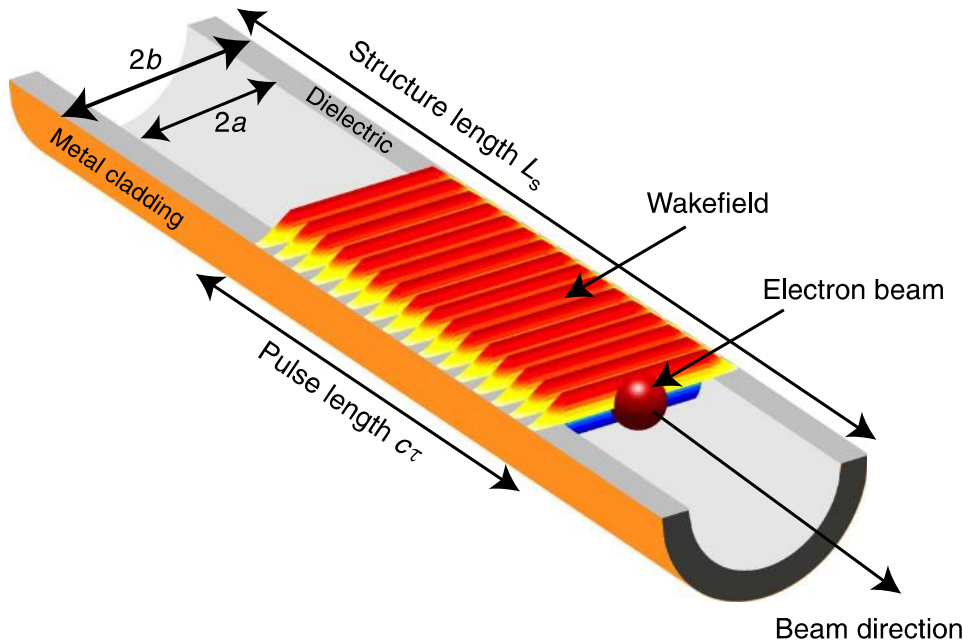


CERN PS 19 MHz Cavity (prototype 1966)



400MHz LHC Cavity in its cryo-module
T. Lefevre





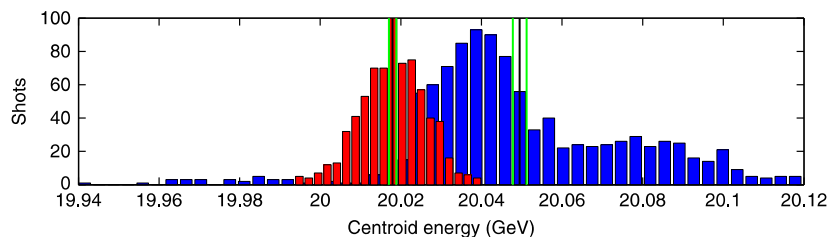
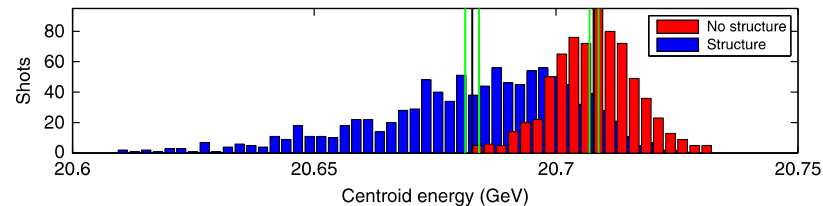
ARTICLE

Received 10 Mar 2016 | Accepted 29 Jul 2016 | Published 14 Sep 2016

DOI: 10.1038/ncomms12763 OPEN

Observation of acceleration and deceleration in giga-electron-volt-per-metre gradient dielectric wakefield accelerators

B.D. O'Shea^{1,2}, G. Andonian¹, S.K. Barber¹, K.L. Fitzmorris¹, S. Hakimi¹, J. Harrison¹, P.D. Hoang¹, M.J. Hogan², B. Naranjo¹, O.B. Williams¹, V. Yakimenko² & J.B. Rosenzweig¹



SiO₂ - 15cm long dielectric

Outer diameter : 2b-400um

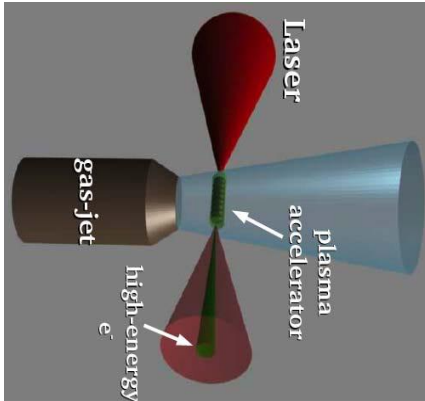
Inner diameter : 2a-300um

Beam size 30um

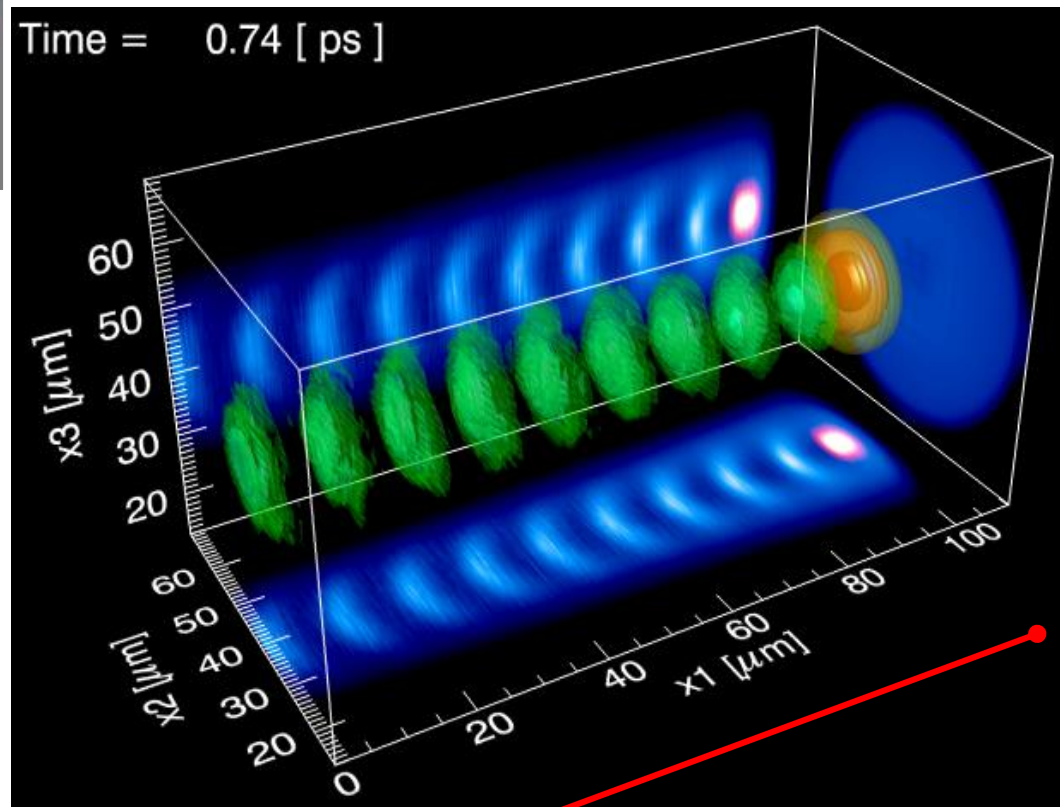
Bunch length 25um (W) and 55um (D)

Δt (D-W) = 250um – 833fs

Laser Plasma Wakefield Acceleration



Courtesy of W. Mori & L. da Silva

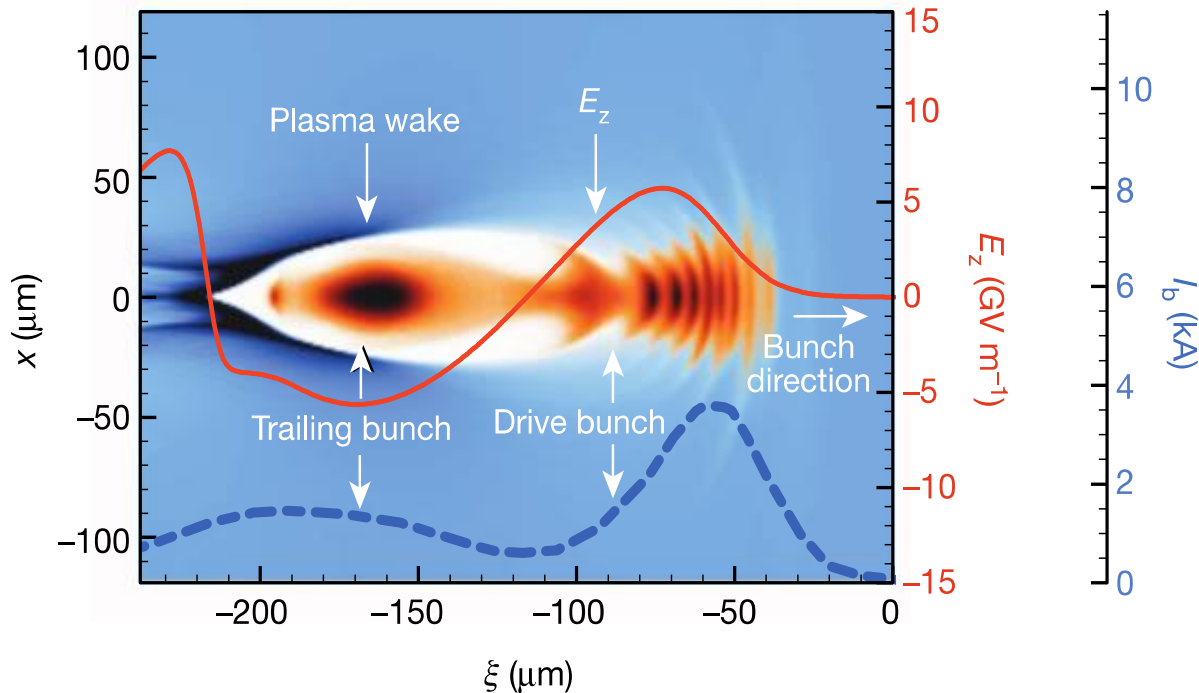


LETTER

doi:10.1038/nature13882

High-efficiency acceleration of an electron beam in a plasma wakefield accelerator

M. Litos¹, E. Adli^{1,2}, W. An³, C. I. Clarke¹, C. E. Clayton⁴, S. Corde¹, J. P. Delahaye¹, R. J. England¹, A. S. Fisher¹, J. Frederico¹, S. Gessner¹, S. Z. Green¹, M. J. Hogan¹, C. Joshi⁴, W. Lu⁵, K. A. Marsh⁴, W. B. Mori³, P. Muggli⁶, N. Vafaei-Najafabadi⁴, D. Walz¹, G. White¹, Z. Wu¹, V. Yakimenko¹ & G. Yocky¹



Typical bunch length

H ⁻ @ SNS	100ps
H ⁺ @ LHC	230ps
e ⁻ @ CLIC	130fs
e ⁻ @ XFEL	10fs
e ⁻ @ DWFA	<60fs
e ⁻ @ PWFA	<30fs
e ⁻ @ LWFA	<10fs



Bunch length measurement techniques

Bunch length measurement techniques

Radiative techniques

Optical Method

1. Produce visible light
2. Analyse the light pulse using dedicated instruments

Bunch Frequency Spectrum

The shorter the bunches, the broader the bunch frequency spectrum

RF manipulation

Use RF techniques to convert time information into spatial information

Laser-based beam diagnostic

Using short laser pulses and sampling techniques

1- Longitudinal Profile



RMS or FWHM values

- *More precise information on the beam characteristic*

2- Single shot measurements



Sampling measurements

- *Do not care about the beam reproducibility*
- *No additional problem due to timing jitter*

3- Non interceptive



Destructive Devices

- *Can be used for beam study and beam control for on-line monitoring*
- *Beam Power density : No risk of damage by the beam itself*

Do not forget about **Simplicity and Reliability**

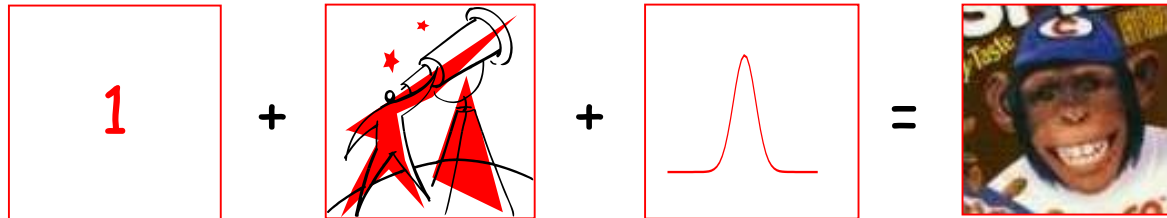
‘Beam diagnostics should help you to understand the beam properties, **it should not be the opposite**’



A detector, what for ?

- Online Beam stability → Non-intercepting and **reliable**
Only have access to a partial information (RMS values)
- Beam characterization and beam physics study → **Full information**
Complexity and time consuming

Can we perform non intercepting, single shot, beam profile measurement in an easy way ?



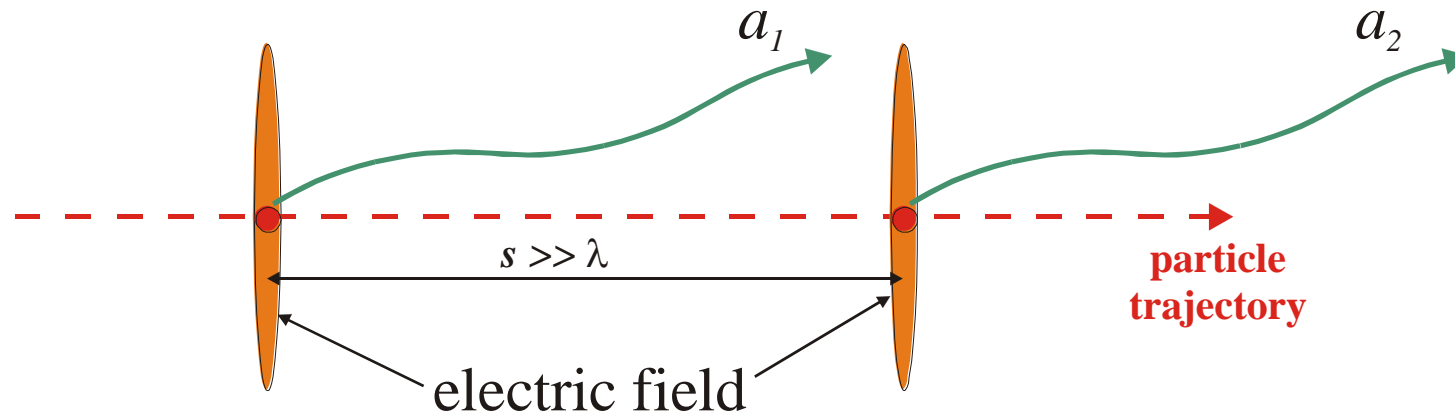


Radiative techniques

‘How to convert particles into photons’

Incoherent versus Coherent Radiation

At wavelength much shorter than the bunch length, the radiation is emitted incoherently because each particle emits photon independently from the others without a defined phase relation

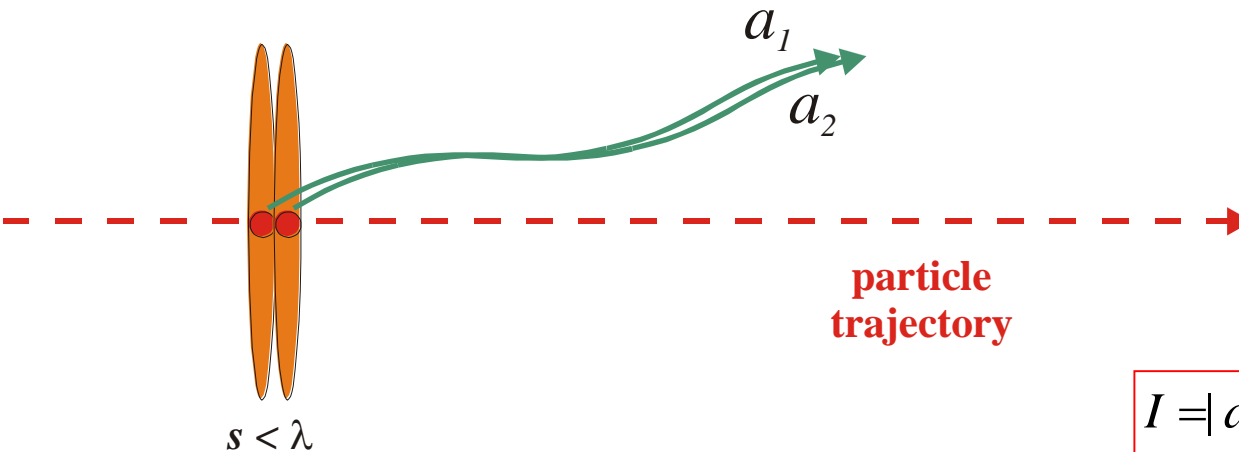


$$I = |a_1|^2 + |a_2|^2 = 2|a|^2 \rightarrow N|a|^2$$

Incoherent radiation

Incoherent versus Coherent Radiation

A coherent enhancement occurs at wavelengths which are equal to or longer than the bunch length, where fixed phase relations are existing, resulting in the temporal coherence of the radiation



$$I = |a_1 + a_2|^2 = |2a|^2 = 4|a|^2 \rightarrow N^2 |a|^2$$

Coherent radiation

Total radiation spectrum

Incoherent term

Coherent term

$$S(\omega) = S_p(\omega) \left[N + N(N-1) F(\omega) \right]$$

$S(\omega)$ – radiation spectrum

$S_p(\omega)$ – single particle spectrum

N – number of electrons in a bunch

$F(\omega)$ – longitudinal bunch form factor

$$F(\omega) = \left| \int_{-\infty}^{\infty} \rho(s) e^{-i\frac{\omega}{c}s} ds \right|^2$$

$\rho(s)$ – Longitudinal particle distribution in a bunch

Total radiation spectrum

Incoherent term

Coherent term

$$S(\omega) = S_p(\omega) \left[N + N(N-1) F(\omega) \right]$$

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$\rho(s)$ – Longitudinal particle distribution in a bunch

Radiative processes



- Transition radiation
- Cherenkov radiation

Better for $\gamma > 100$

$$\beta > 1/n$$



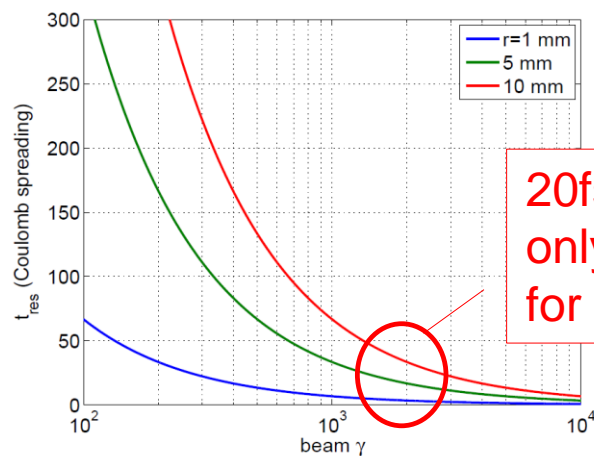
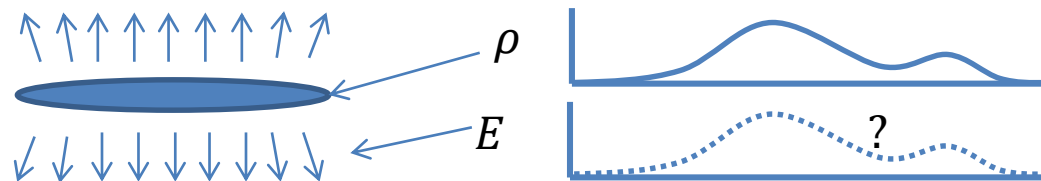
- Diffraction radiation
- Cherenkov Diffraction radiation
- Synchrotron radiation

For incoherent radiation
 $\gamma > 3000$

For coherent radiation
$$\gamma > \frac{0.06}{\sigma_z(m)}$$

Radiative processes

Field radiated or probed is related to **Coulomb field near the electron bunch**



20fs resolution
only obtainable
for >1 GeV beam

High γ is an advantage!

Time response & spectrum of field
is dependent on spatial position, r :

$$\delta t \sim 2r / c\gamma$$

\Rightarrow ultrafast time resolution requires close proximity to bunch



Optical method using Incoherent light



Time correlated single photon counting

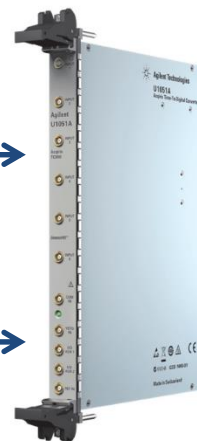


Geiger-mode Avalanche photodiode converts photon to electrical pulse

Visible photon



Precise trigger synchronized with the beam



Time to Digital converter records pulse arrival time

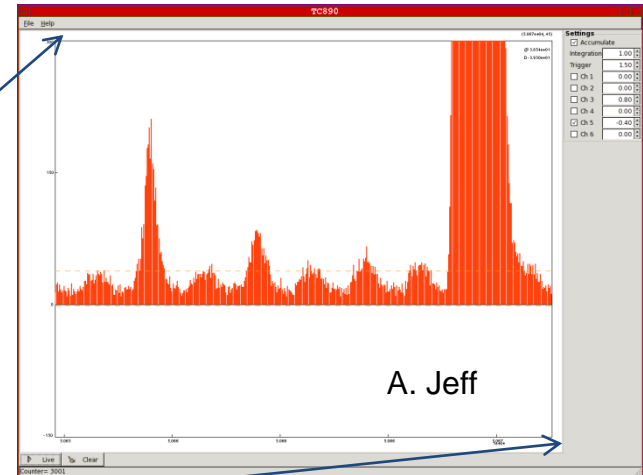
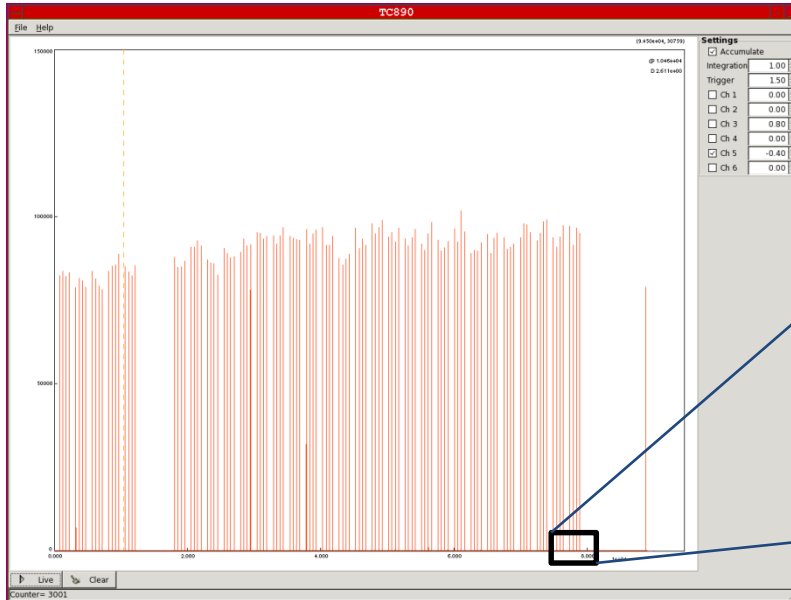
- Sampling Method allowing very high dynamic range if you measure long enough
- Avalanche photodiode have deadtime and are subject to after-pulsing
- State of the art TDC typically limited to 10ps sampling

D.V. O'Connor, D. Phillips, Time-correlated Single Photon Counting, Academic Press, London, 1984
C.A. Thomas et al., Nucl. Instr. and Meth. A566 (2006) p.762

Time correlated single photon counting



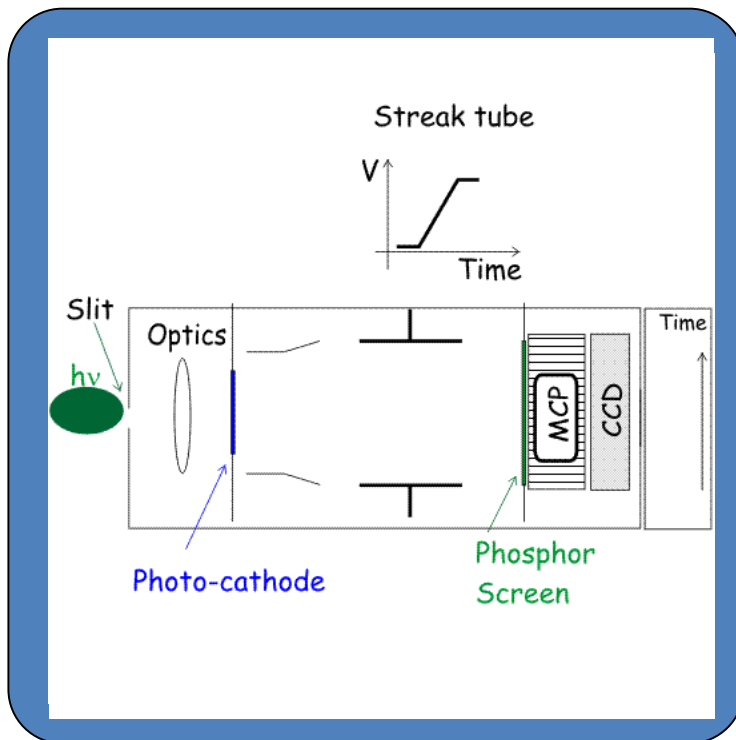
Longitudinal profile of the entire LHC ring (89us) with 50ps resolution using SR light



A very large dynamic range should make it possible to see ghost bunches as small as $5e5$ protons / 50ps with long integration



1



‘Streak cameras uses a time dependent deflecting electric field to convert time information in spatial information on a CCD’

M. Uesaka et al, *NIMA* 406 (1998) 371

200fs time resolution obtained using reflective optics and 12.5nm bandwidth optical filter (800nm) and the Hamamatsu FESCA 200

Limitations : Time resolution of the streak camera :

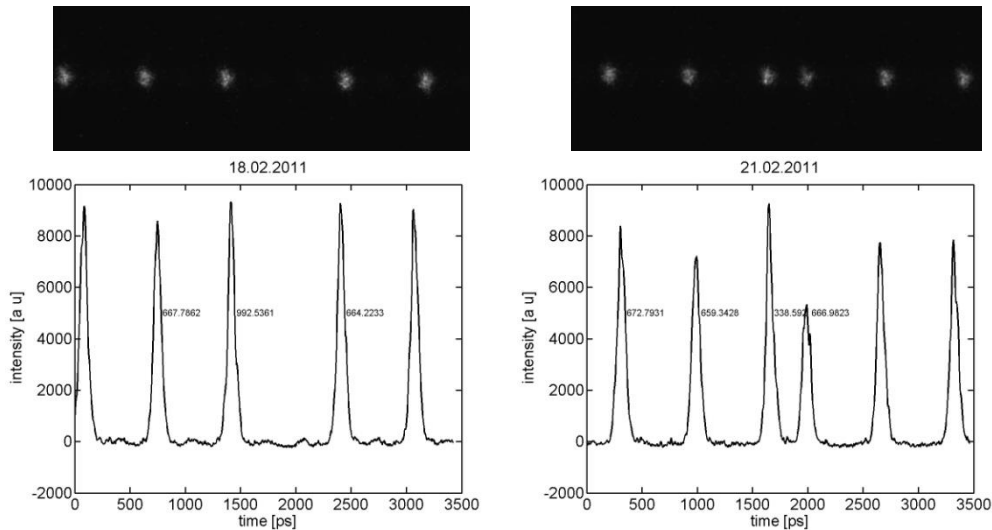
- (i) Initial velocity distribution of photoelectrons : *narrow bandwidth optical filter*
- (ii) Spatial spread of the slit image: *small slit width*
- (iii) Dispersion in the optics

Streak Camera

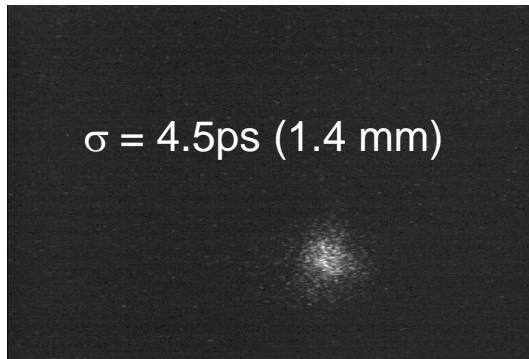
Observation of 5MeV electron bunch train using Cherenkov radiation
Sweep speed of 250ps/mm



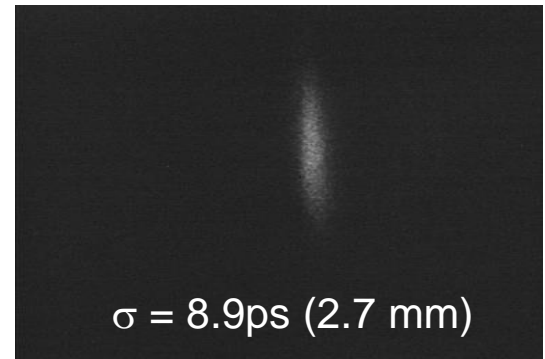
1



Measure of bunch length using OTR and OSR



*Sweep
speed of
10ps/mm*





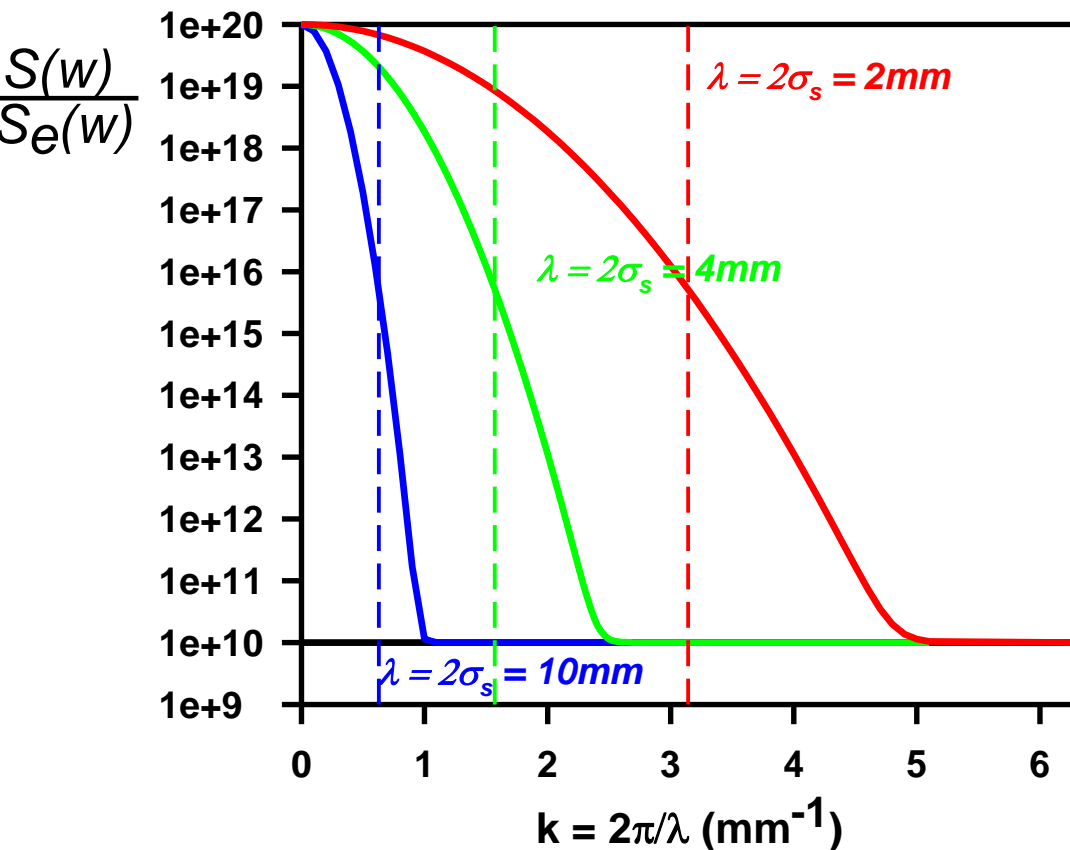
Bunch length measurement using using Coherent light

‘The shorter in time the broader in frequency’

Bunch form factor

$$F(\omega) = \left| \int_{-\infty}^{\infty} dz \rho(z) e^{i(\omega/c)z} \right|^2$$

$$\rho(z) = \frac{1}{\pi c} \int_0^{\infty} d\omega \sqrt{F(\omega)} \cos\left(\frac{\omega z}{c}\right)$$



Coherent radiation appears when the bunch length is comparable to or shorter than the emitted radiation wavelength

$$S(\omega) \gg N^2 S_p(\omega) F(\omega)$$

- ✓ $S(\omega)$ – radiation spectrum (known in the experiment)
- ✓ N – number of particles s / bunch (known from the experiment)
- ✓ $F(\omega)$ – bunch form factor (what you want to find out)
- ✓ $S_p(\omega)$ – single particle spectrum (should be known)



Coherent Transition Radiation (CTR)

P. Kung et al, **Physical review Letters** 73 (1994) 96



Coherent Diffraction (CDR) or Coherent Synchrotron (CSR)

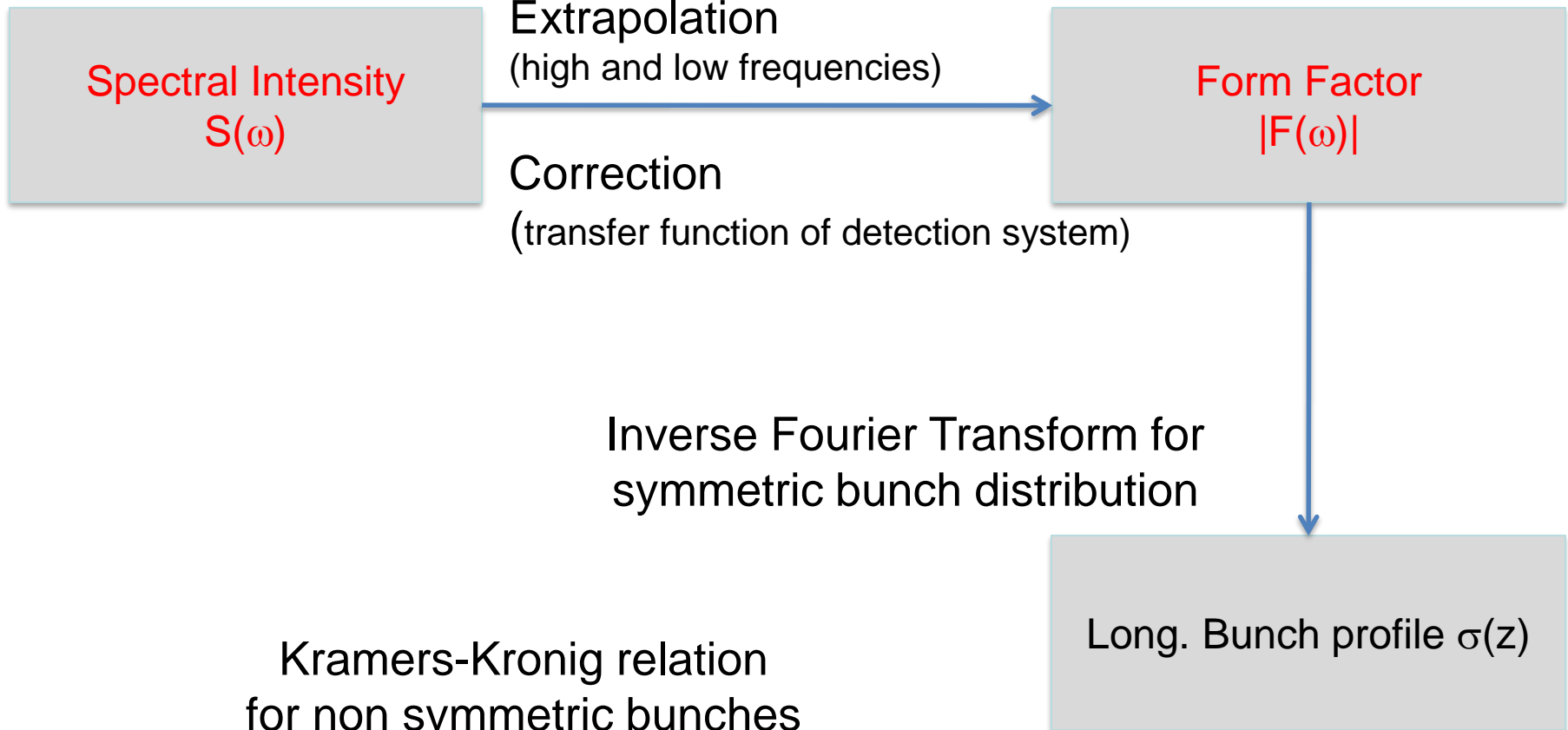
B. Feng et al, **NIM A** 475 (2001) 492–497 ; A.H. Lumpkin et al, **NIM A** 475 (2001) 470–475

C. Castellano et al, **Physical Review E** 63 (2001) 056501

T. Watanabe et al, **NIM A** 437 (1999) 1-11 & **NIM A** 480 (2002) 315–327

Measuring the Radiation Spectrum

Frequency Domain



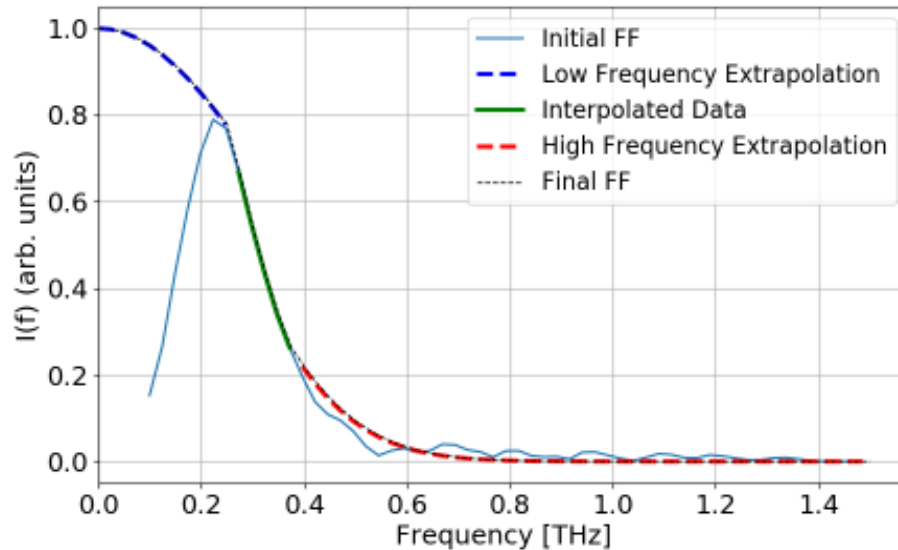
Kramers-Kronig relation
for non symmetric bunches

R. Lai and A.J. Sievers, NIM-A 397 (1997) 221 -231

Time Domain

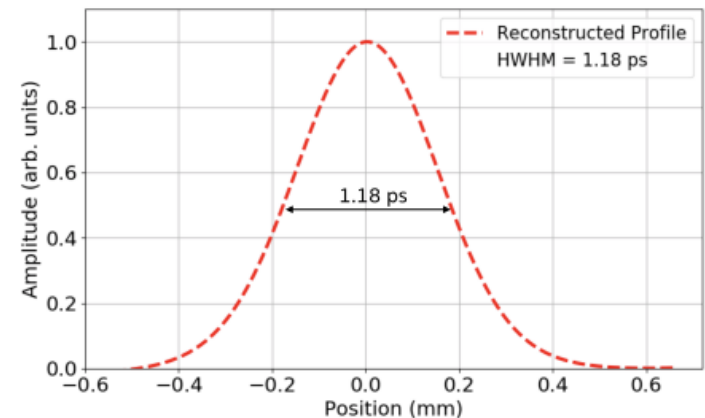
Measuring the Radiation Spectrum

Frequency Domain



- Extrapolation (high and low frequencies)
- Correction (transfer function of detection system)

Inverse Fourier Transform using
Kramer kronig relation



Time Domain

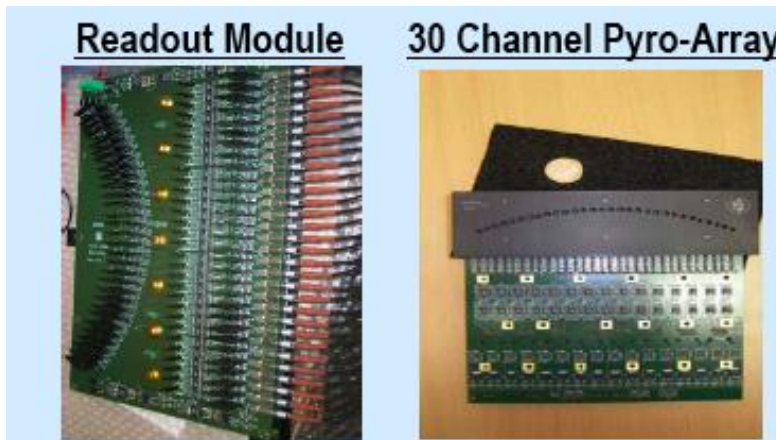
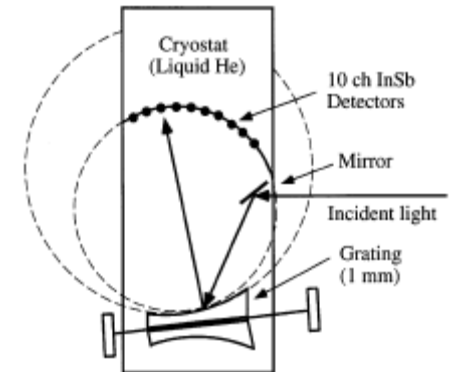
Measuring the Radiation Spectrum

1

*‘The **polychromator** enables to get the spectrum directly by a single shot. The radiation is deflected by a grating and resolved by a multi-channels detector array’*

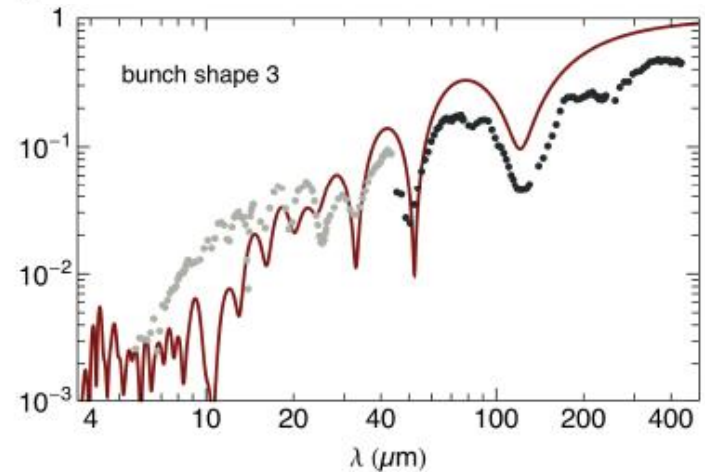
T. Wanatabe et al., NIM-A 480 (2002) 315-327

H. Delsim-Hashemiet al., Proc. FLS, Hamburg 2006, WG512



B. Schmidt, DESY

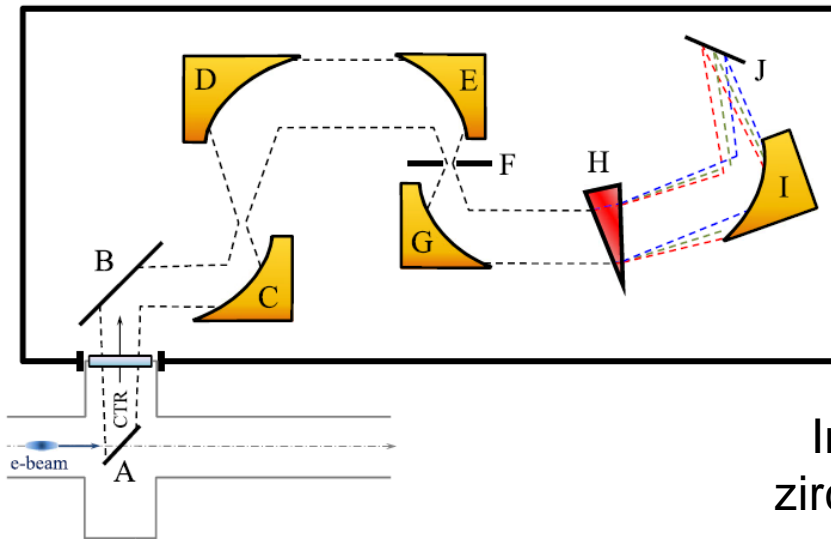
Measured & calculated spectra



(E. Hass et al., Proc. SPIE 8778, May 2013)

Single shot CTR measurements

- T. J. Maxwell et al. "Coherent-radiation spectroscopy of few-femtosecond electron bunches using a middle-infrared prism spectrometer." *Physical review letters* 111.18 (2013)



KRS-5 (thallium bromoiodide) prism based spectrometer

Images CTR from foil onto 128 lead zirconate titanate pyroelectric elements with 100 μm spacing line array

1



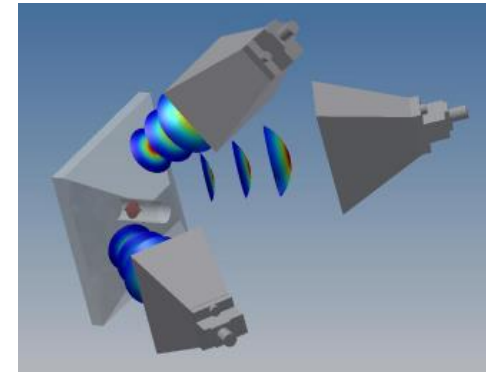
Single-shot Cherenkov diffraction

measurement

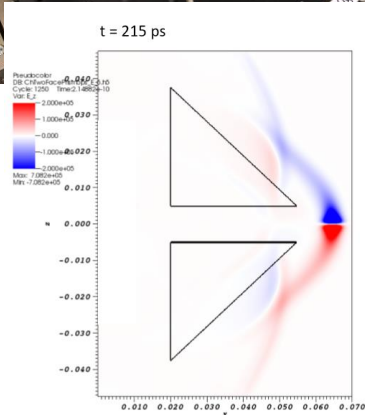
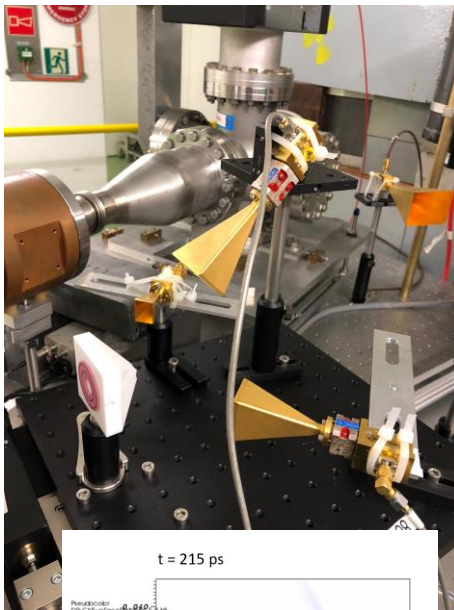
Cherenkov diffraction radiation

Measured in 3 bands (60-90-110GHz)

1



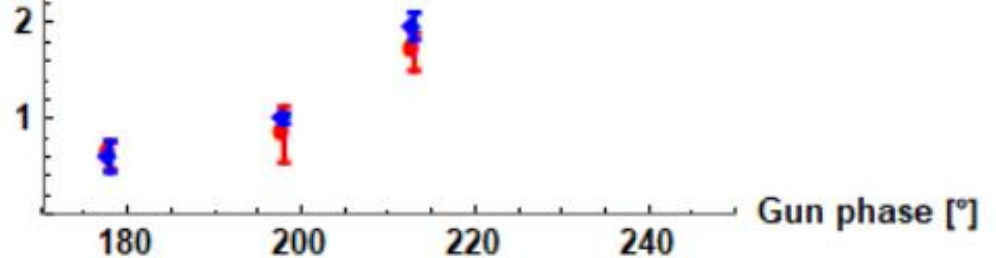
Pyramidal cone
with 1cm aperture



Bunch
Length [ps]

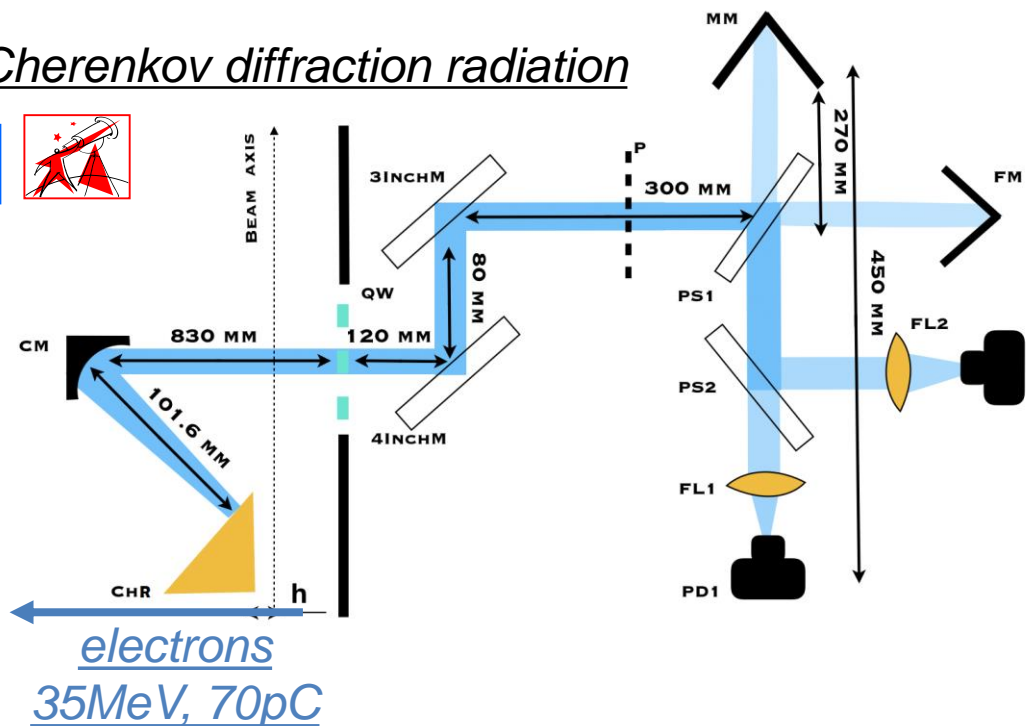
electrons
200MeV, 100pC

● CChDR-spectrometer
◆ RF-deflector

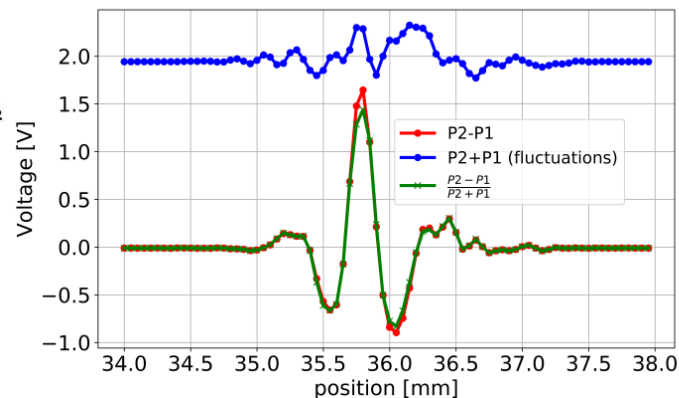


Martin-Puplett Interferometer

Cherenkov diffraction radiation

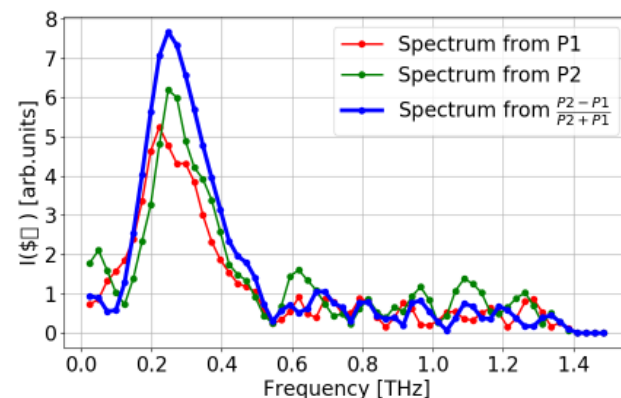


$$I(\delta) \propto \int_{-\infty}^{\infty} |E(t) + E(t + \delta/c)|^2 dt$$



“the Fourier transform of the autocorrelation function is the power spectrum”

$$I(\omega) \propto \int_{-\infty}^{\infty} I(\delta) \cos\left(\frac{\omega\delta}{c}\right) d\delta$$



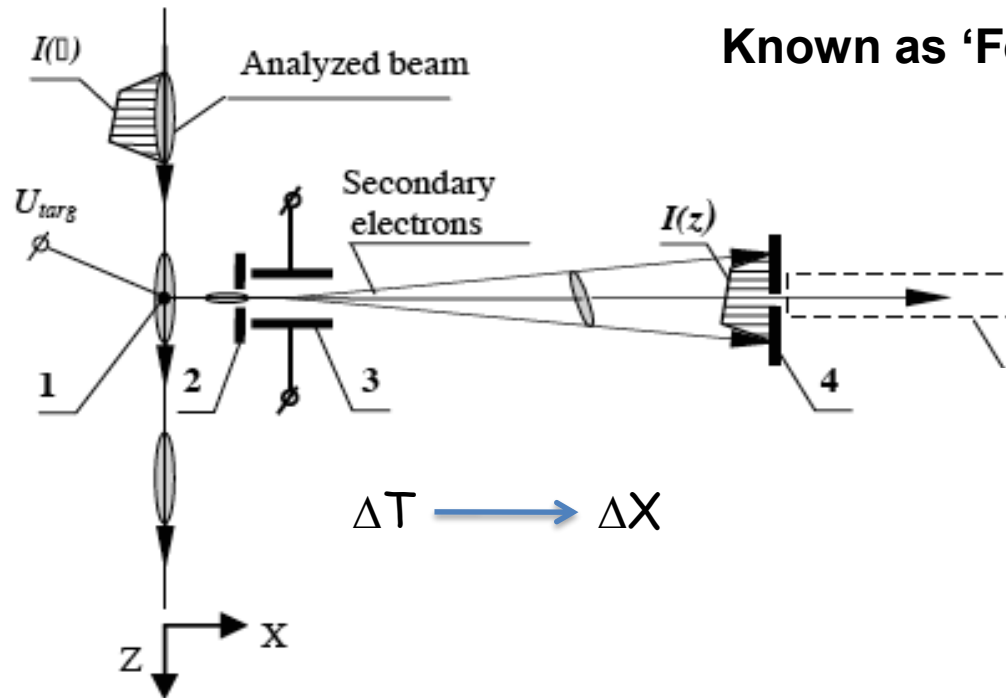
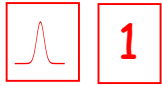


Radiofrequency manipulation

‘How to transform time information into spatial information’

Bunch shape monitor

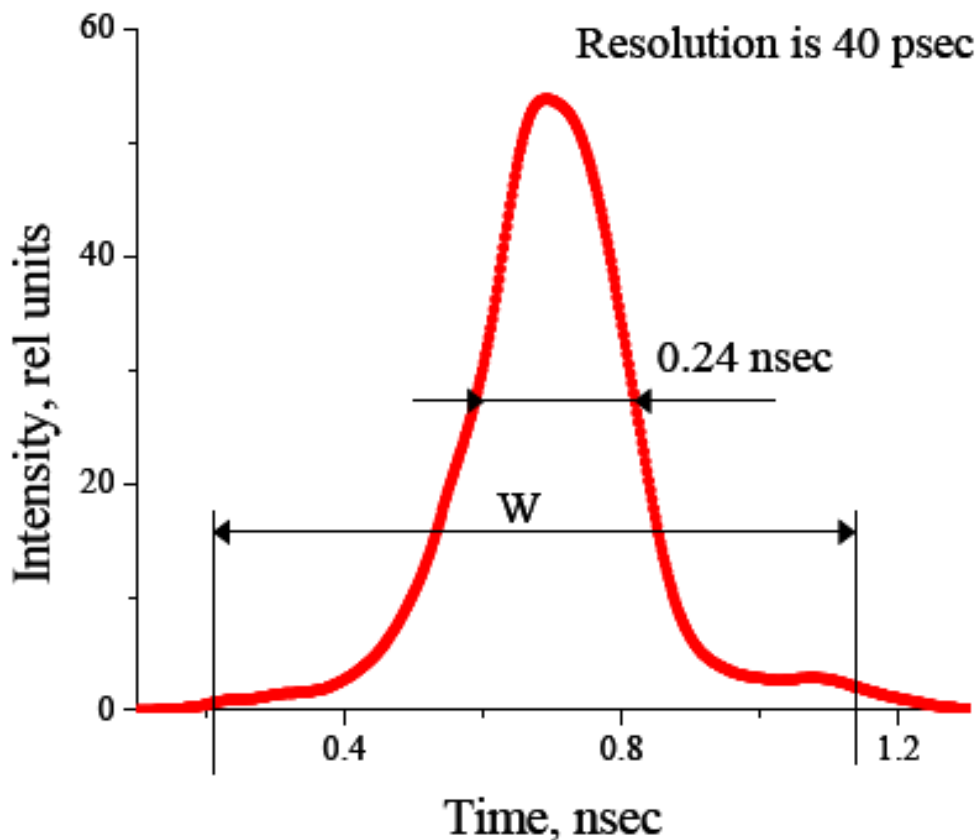
Known as 'Feschenko monitor'



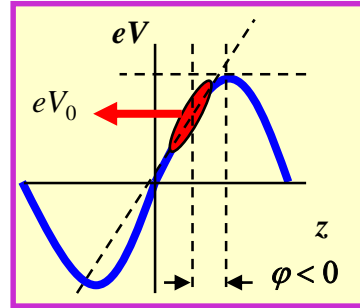
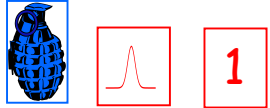
- 1 - Target (wire, screen, laser for H^-) : Source of secondary electrons
- 2 - Input collimator
- 3 - RF deflector (100MHz, 10kV) combined with electrostatic lens
- 4 - Electron Beam detector (electron multiplier, ..)

Bunch shape monitor

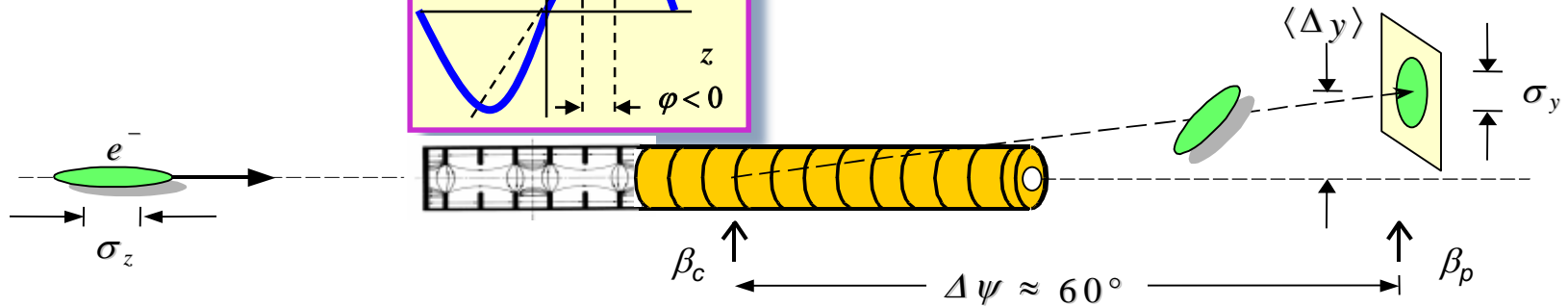
Longitudinal Bunch profile @ SNS



RF Deflecting cavities



- Old idea from the 60's
- RF Deflector ~ relativistic streak tube



Beam profile RF on

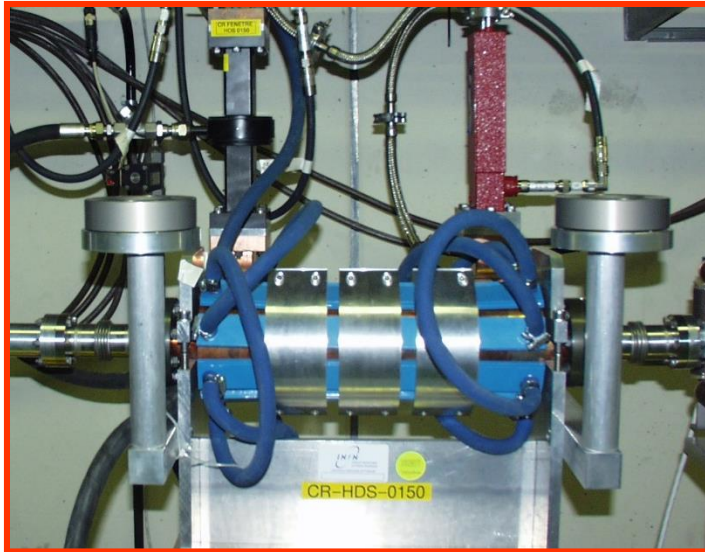
$$\sigma_y = \sqrt{\sigma_{y_0}^2 + \sigma_z^2 \beta_c \beta_p \left(\frac{2\pi}{\lambda} \frac{eV_0}{E_0} \sin(\Delta\Psi) \cos(\varphi) \right)^2}$$

Deflecting Voltage: eV_0
 RF deflector wavelength: λ
 Beam energy: E_0
 Betatron phase advance (cavity-profile monitor): $\Delta\Psi$
 Bunch length: σ_z
 Beta function at cavity and profile monitor: $\beta_c \beta_p$

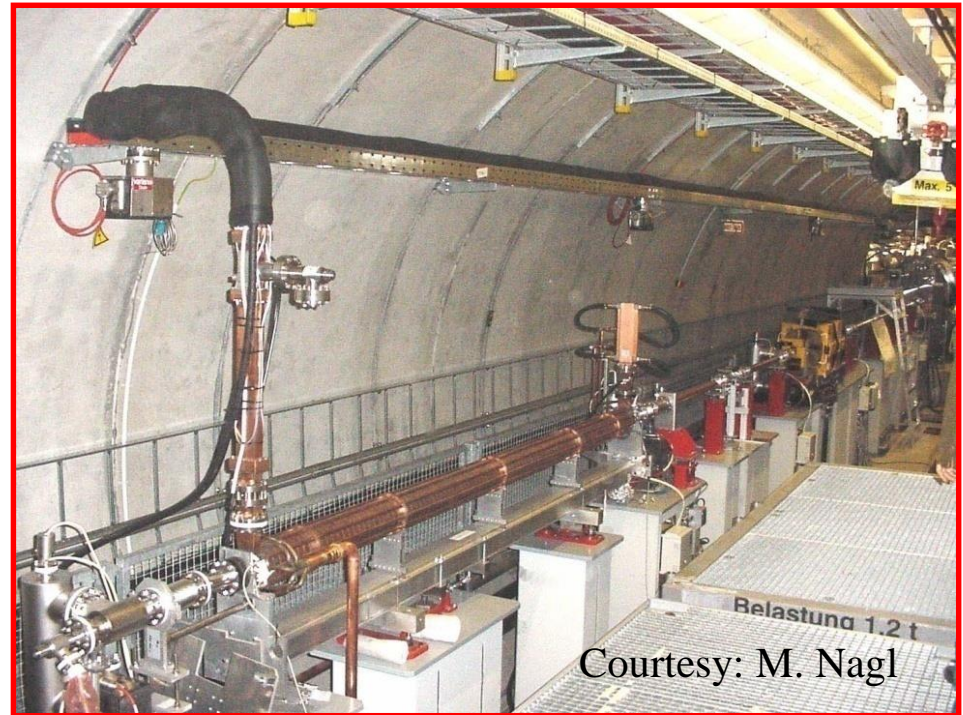
$\sin\Delta\psi = 1$, β_p small
Make β_c large

RF Deflecting cavities

CTF3



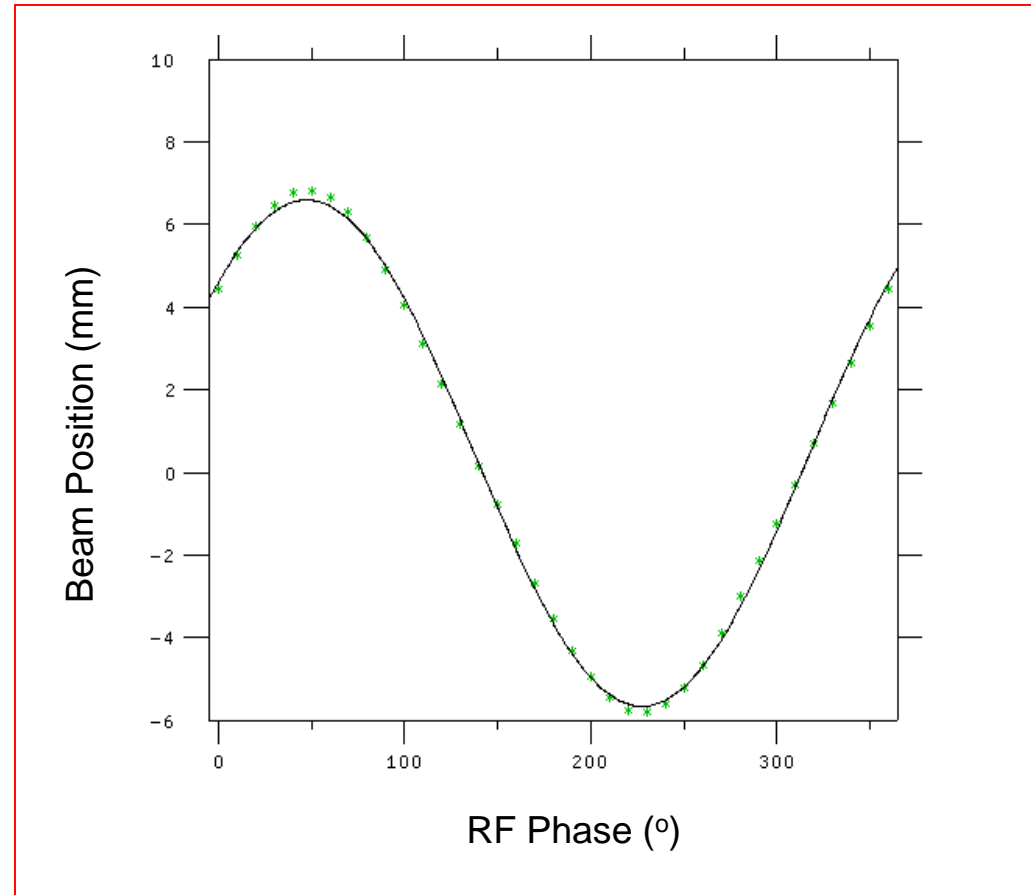
LOLA @ Flash



Calibration of RF Deflector

$\Delta X(\text{mm}) \longrightarrow \Delta\varphi(^{\circ})$
 $\Delta T(\text{ps})$

Monitor the Beam Position on (or close to) the Profile monitor to calibrate the deflection angle

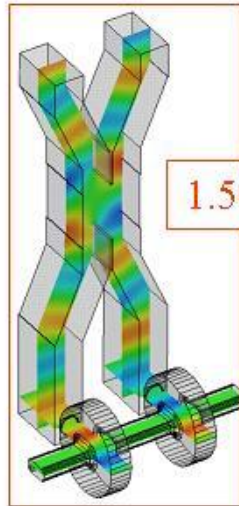


Beam offset on the screen

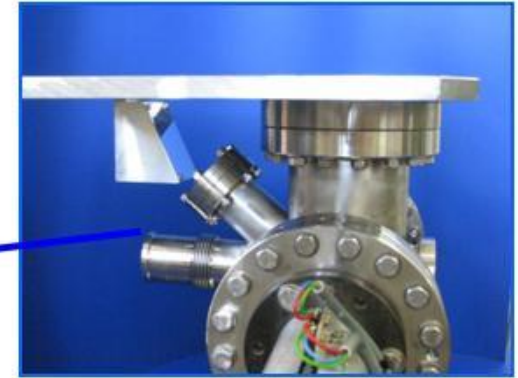
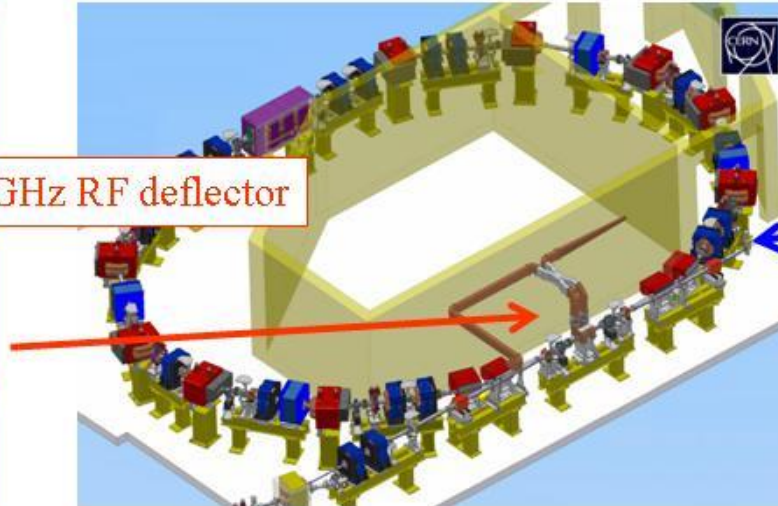
$$\Delta y(z) \approx \frac{eV_0}{E_0} \cdot \sqrt{\beta_c \beta_p} \sin(\Delta\Psi) \left(\frac{2\pi}{\lambda} - z \cos(\varphi) + \sin(\varphi) \right)$$

RF deflector phase

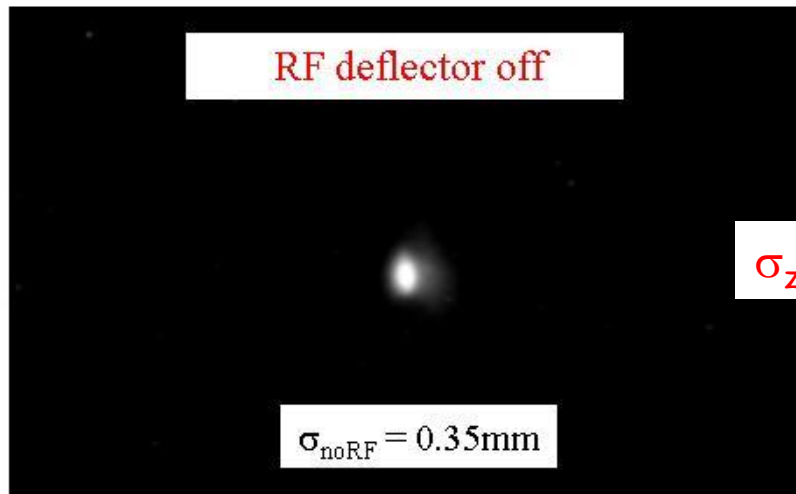
RF Deflecting cavities



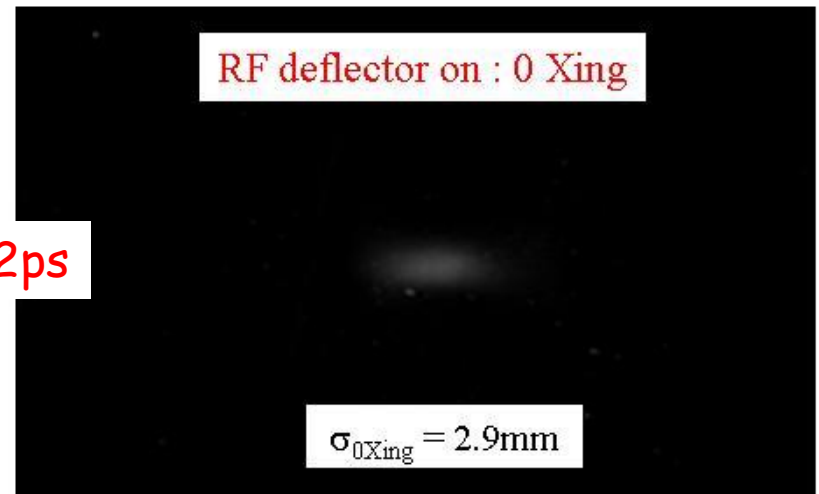
1.5GHz RF deflector



OTR screen

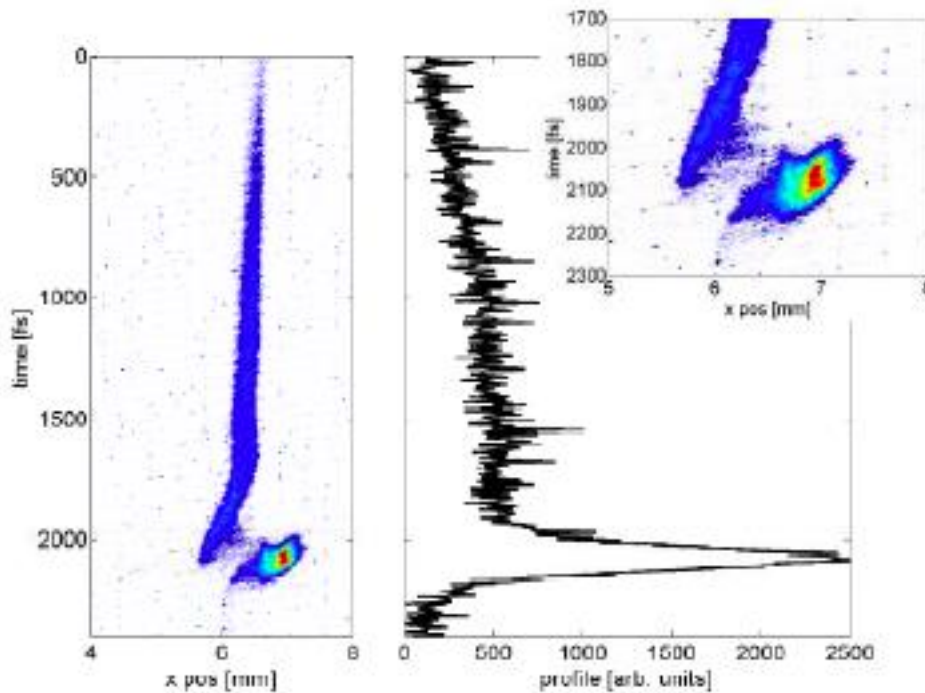


$\sigma_z = 2\text{ps}$



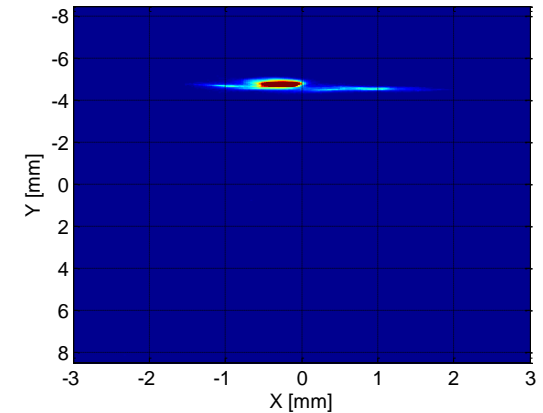
RF Deflecting cavities

Bunch length measurement @ Flash

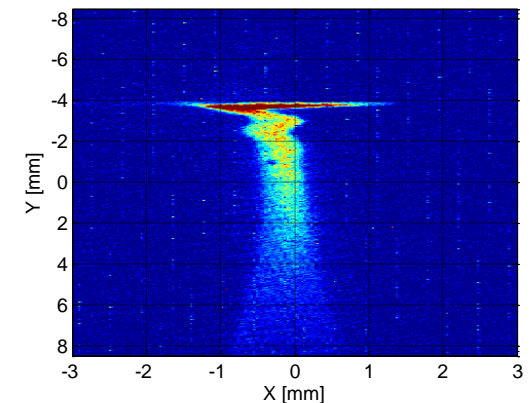


→ Resolution of 4fs/pixels

LOLA off:

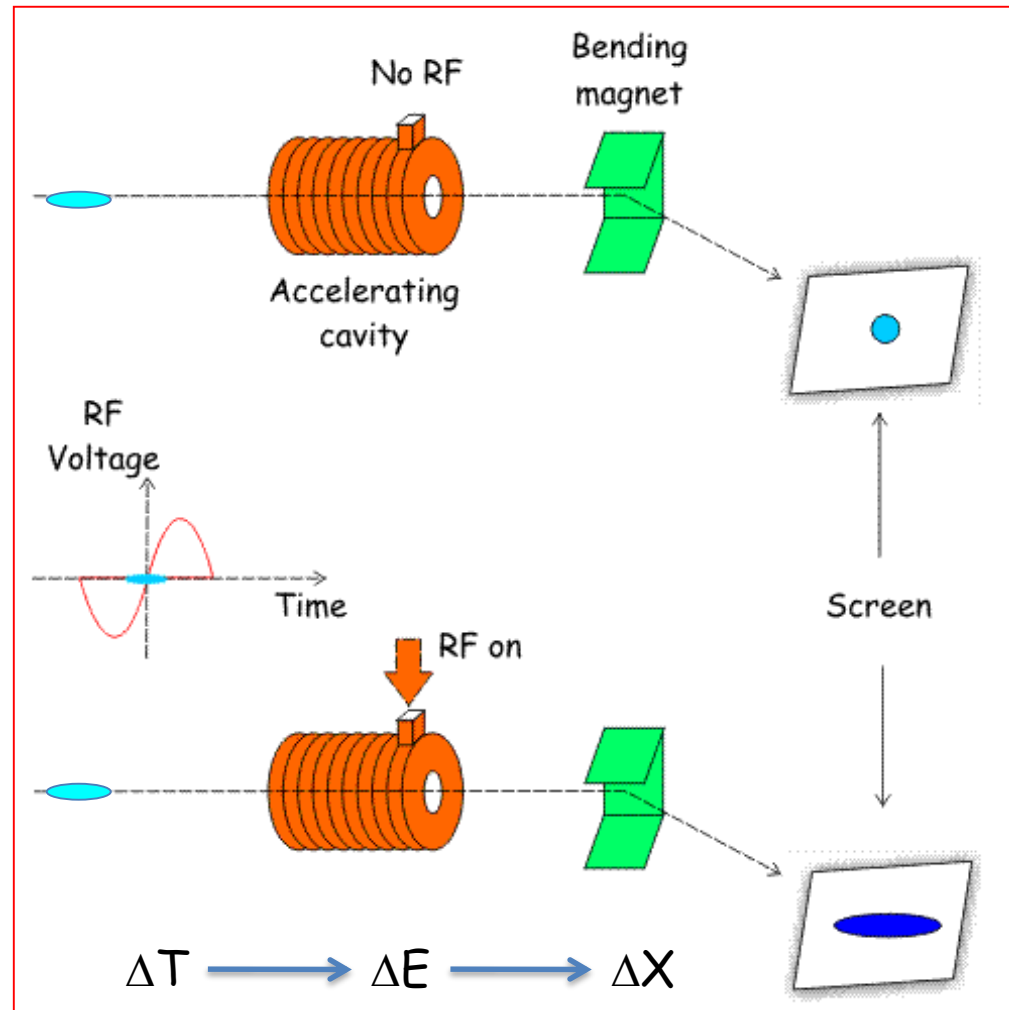
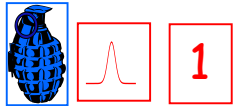


LOLA on:



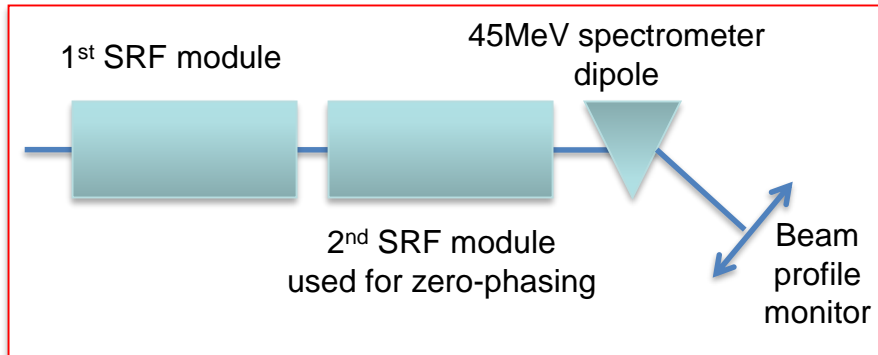
RF Accelerating cavities

'The electron energy is modulated by the **zero-phasing** RF accelerating field and the bunch distribution is deduced from the **energy dispersion** measured downstream using a spectrometer line'



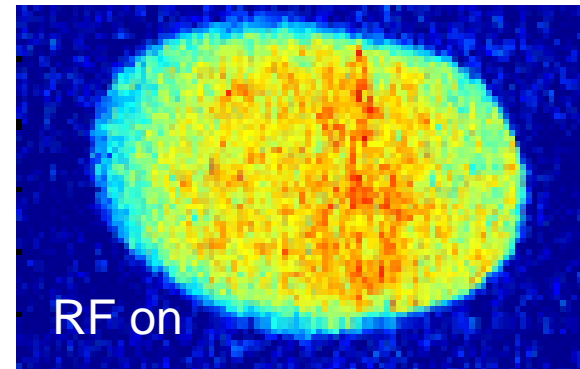
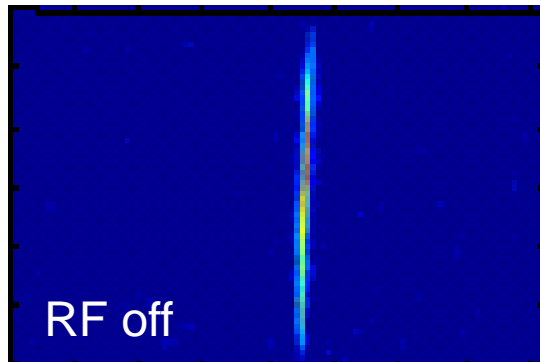
RF Accelerating cavities

CEBAF injector, Newport News



D. X. Wang *et al*, Physical Review E57 (1998) 2283

84fs, 45MeV beam but low charge beam

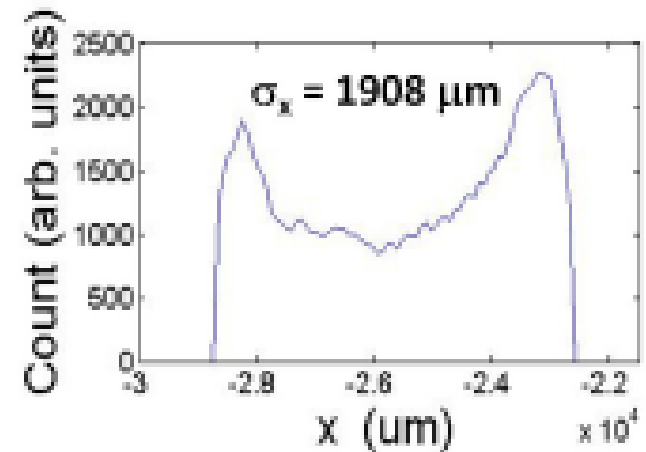
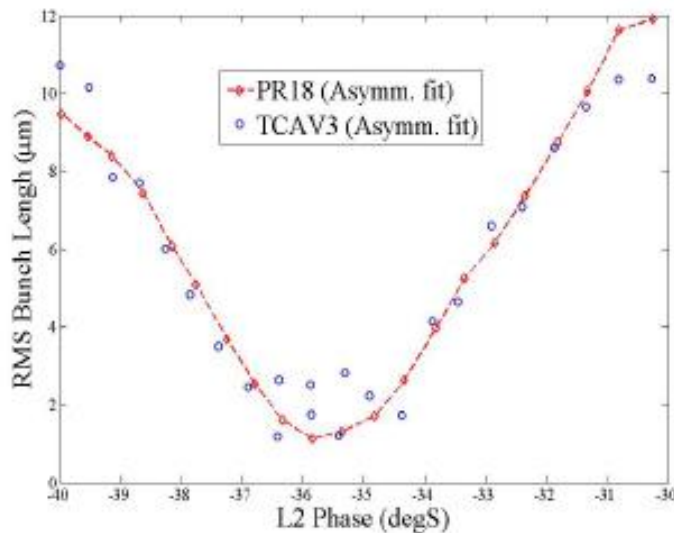
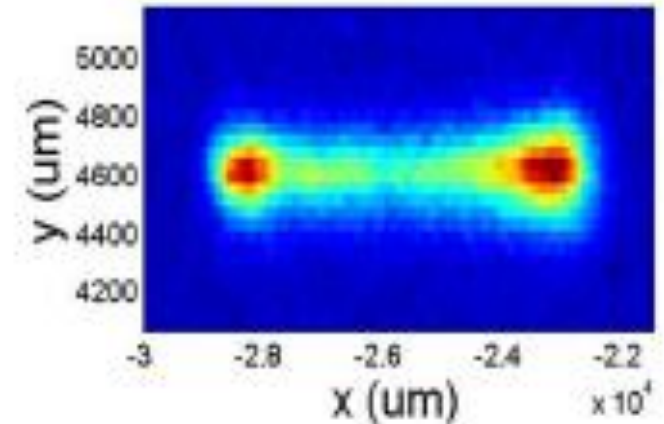


Limitations

RF non linearities
Beam loading and wakefield for high charge beam

SLAC LCLS: at 4.7 GeV

- 550m of linac at RF zero crossing!
- 6m dispersion on A-line spectrometer!



TCAV3

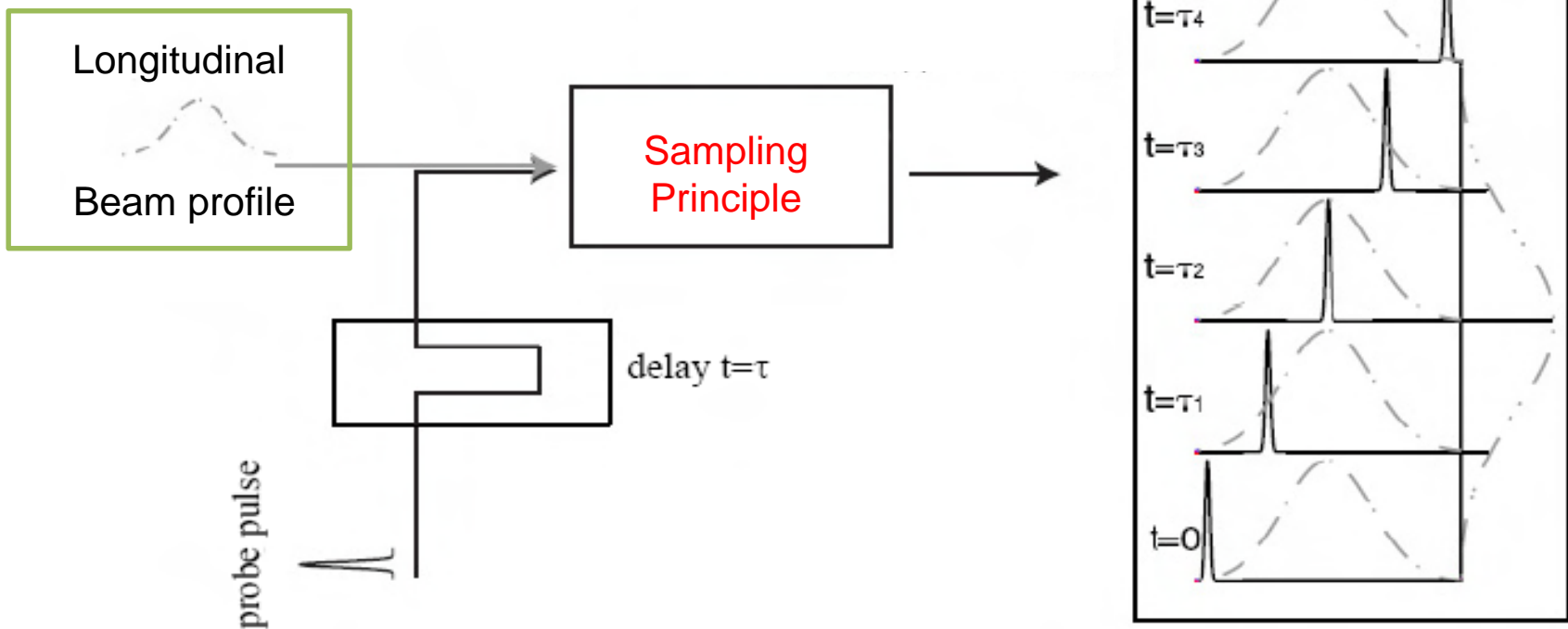
~ 1 fs rms bunch length at 4.7 GeV



Laser-based diagnostics

Sampling techniques

Using a short laser pulse to scan through the beam profile

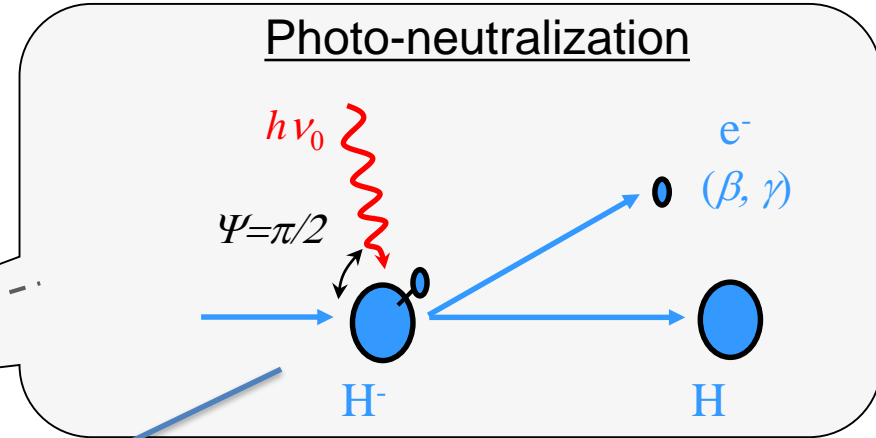
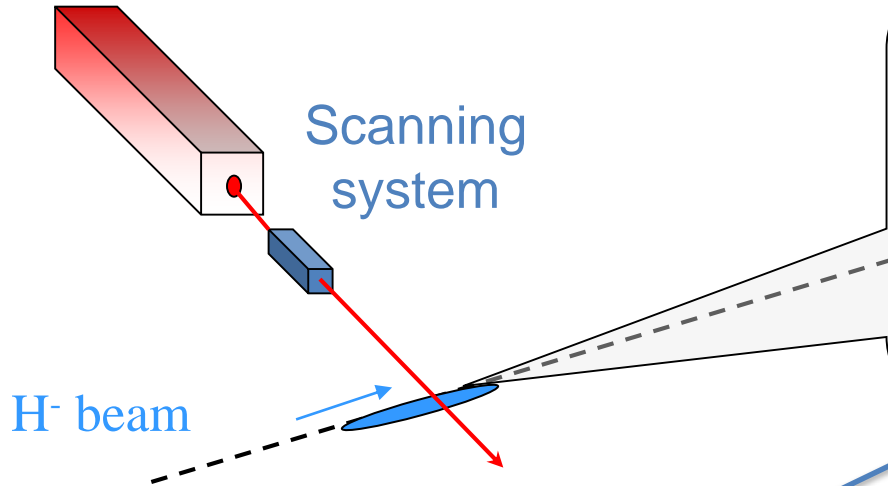


Limitation

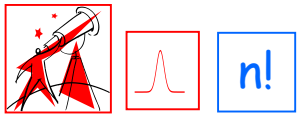
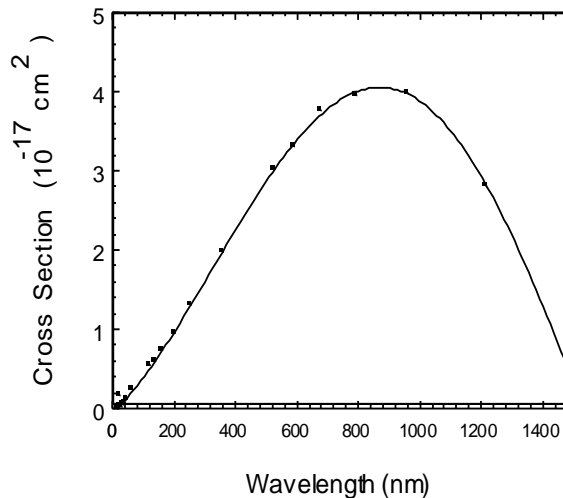
Laser-beam synchronization jitter should be smaller than the bunch length to measure

Laser Wire Scanner

High power laser



- First ionization potential for H⁻ ions is 0.75eV
- Photo-neutralization cross section : $\sigma \sim 4 \cdot 10^{-17} \text{ cm}^2$

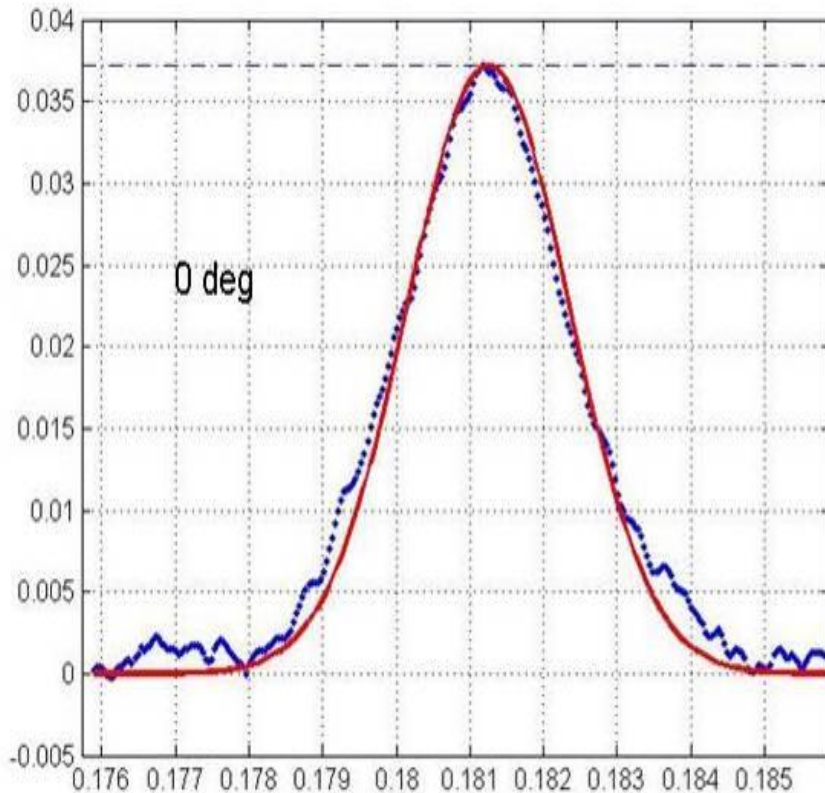


Detection system based on

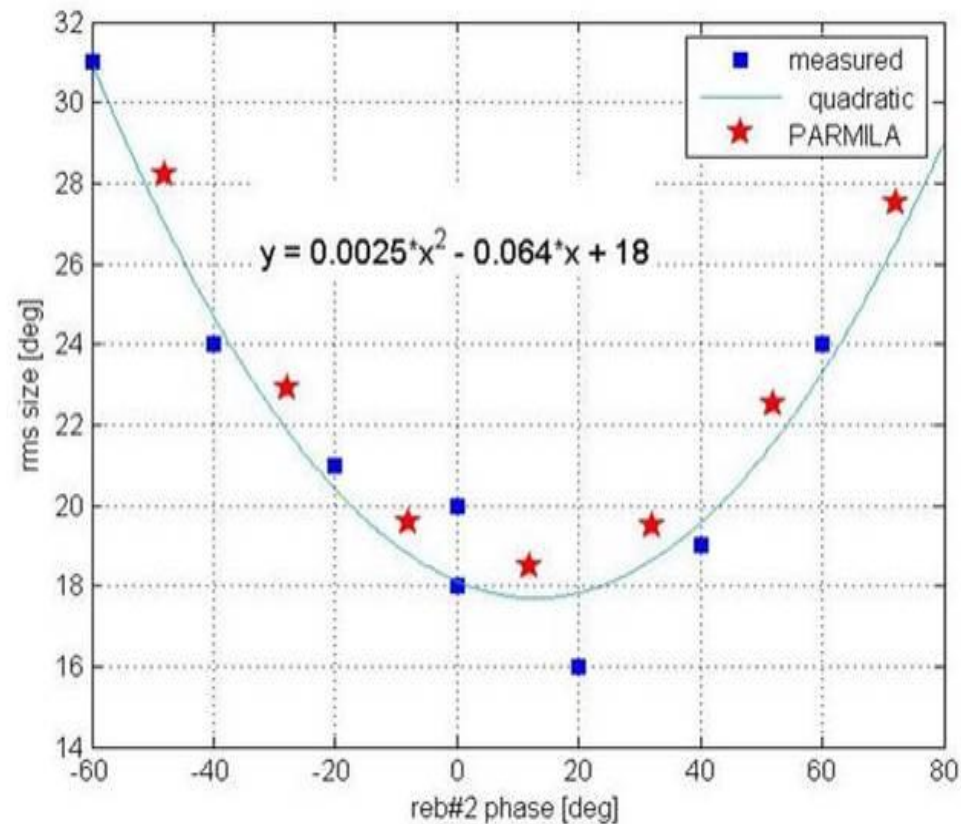
- The measurement of released electrons using a magnet and a collector (faraday cup, MCP,..)
- Measured the conversion of H⁻ into H with a current monitor

Longitudinal Measurements @ SNS

2.5 MeV H^- , 402.5 MHz bunching freq, Ti-Sapphire laser phase-locked @ $1/5^{\text{th}}$ bunching frequency



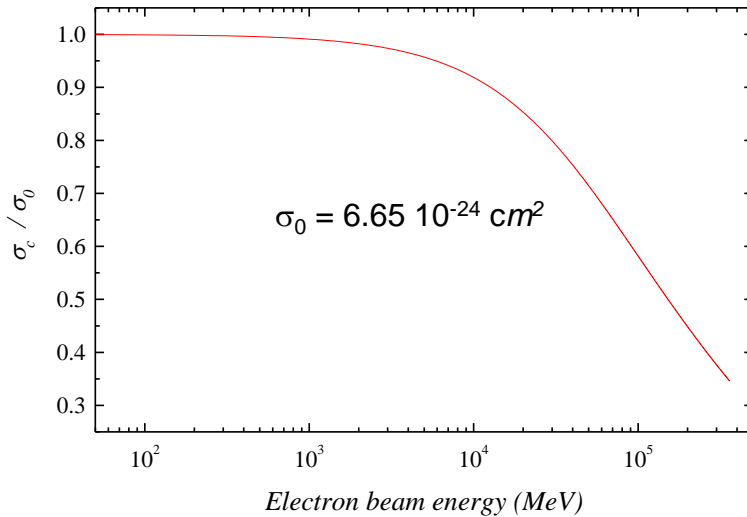
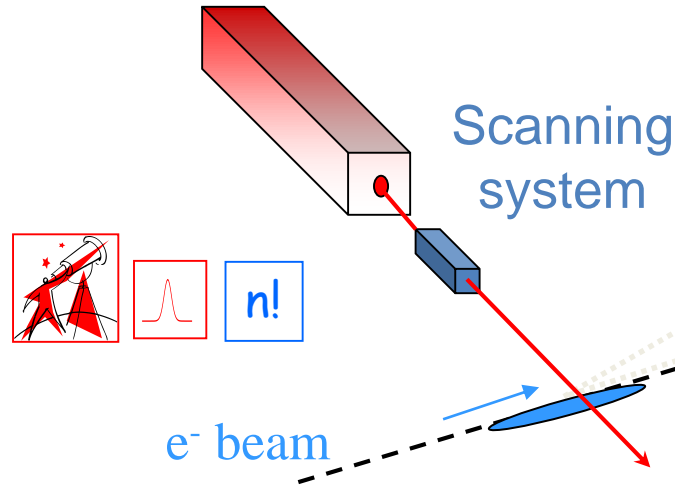
Collected electron signal plotted vs. phase



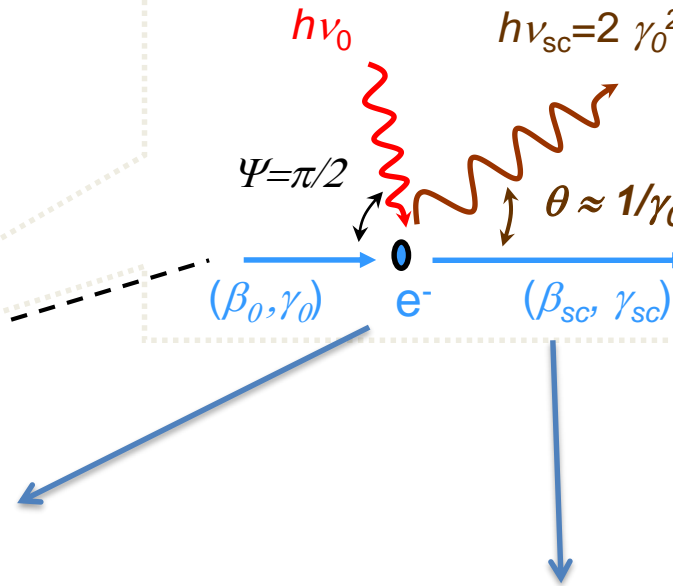
Measured and predicted bunch length vs. cavity phase setting

Laser Wire Scanner

High power laser



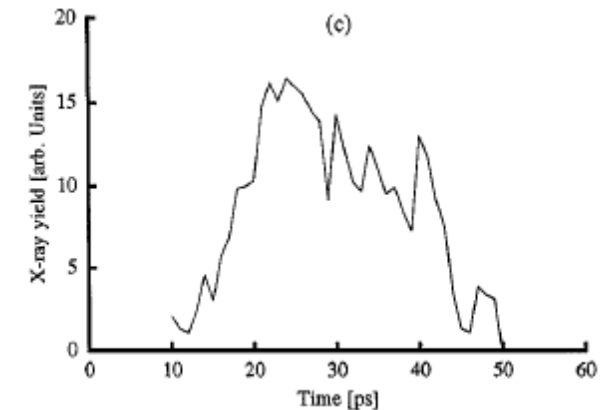
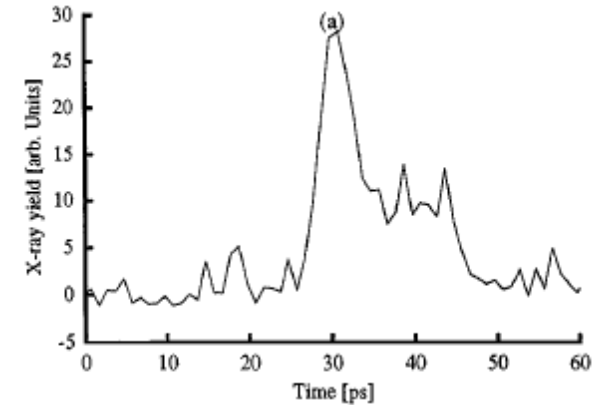
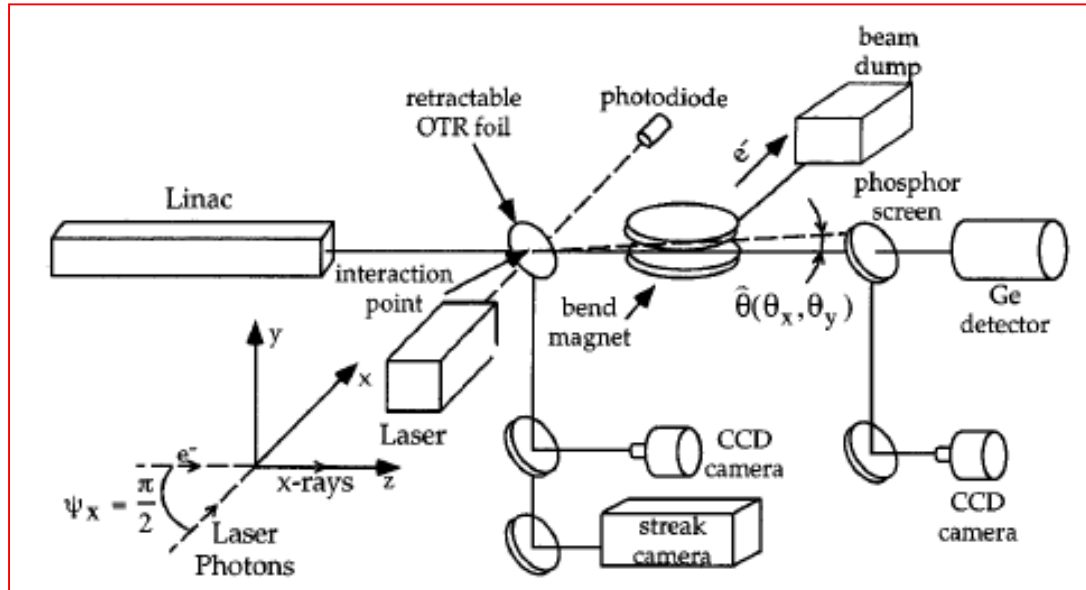
Thomson/Compton scattering



- Detection system based on**
- The measurement of the scattered photons
 - The measurement of degraded electrons

Laser Wire Scanner

ALS @ LBNL

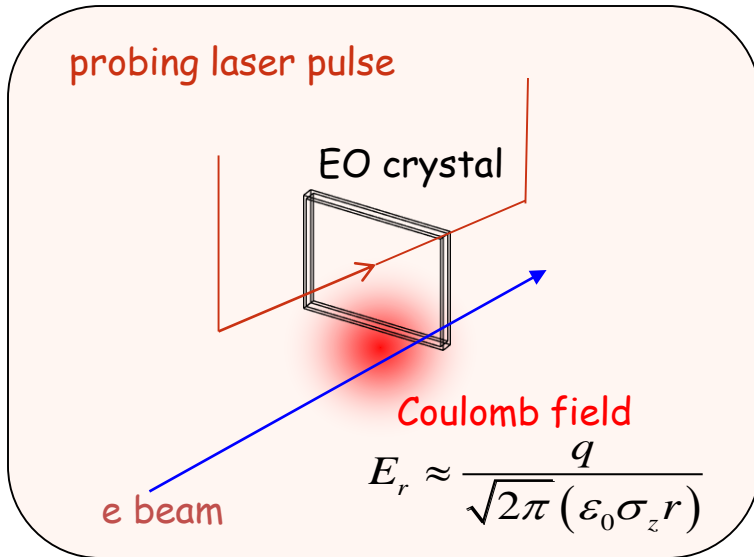


Using a 10TW Ti:Al₂O₃ laser system. Detecting $5 \cdot 10^4$ 10-40 keV X-rays using either an X-ray CCD and Ge detector.

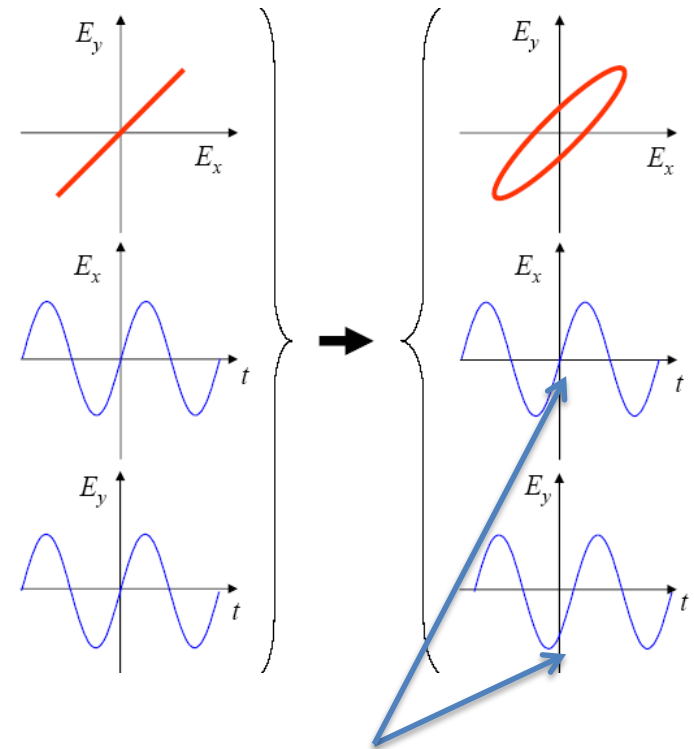
Electro-optical techniques

'This method is based on the polarization change of a laser beam which passes through a crystal itself polarized by the electrons electric field'

E-field induced birefringence in EO-crystal : Pockel/Kerr effect



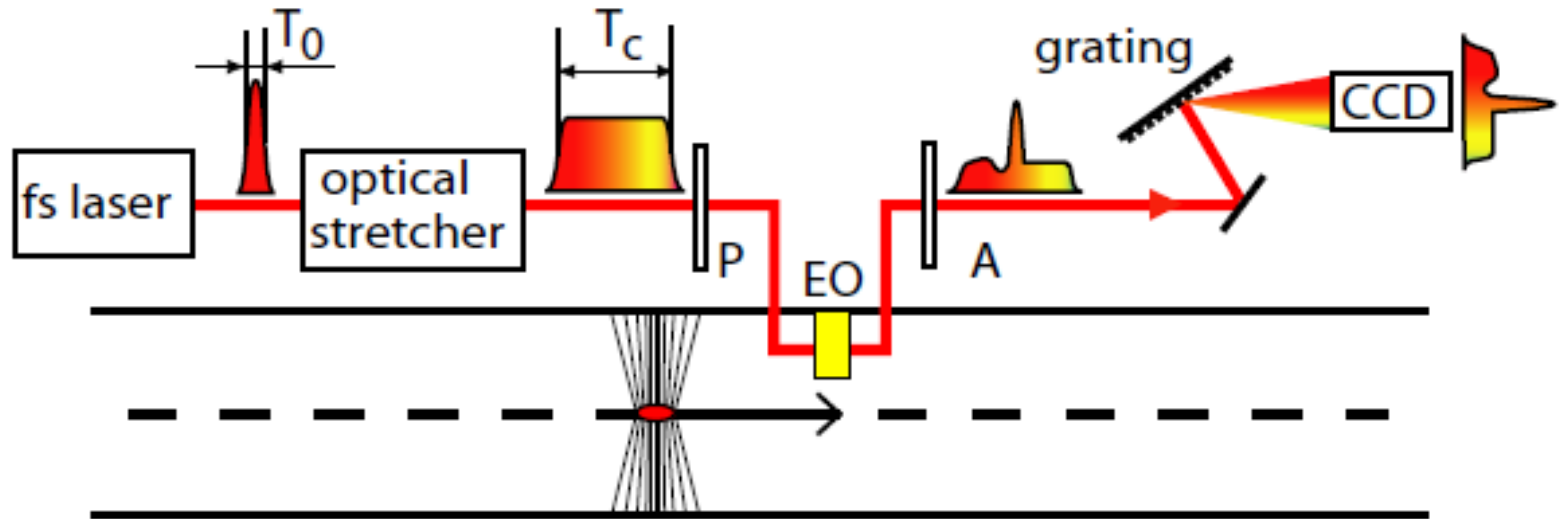
- Polarization diagram
- electric field of the horizontal polarization
- electric field of the vertical polarization



$$G = \frac{2pd}{l_0} (n_x - n_y) = \frac{2pd}{l_0} n_0^3 r_{41} E_r$$

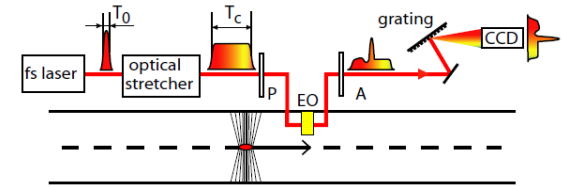
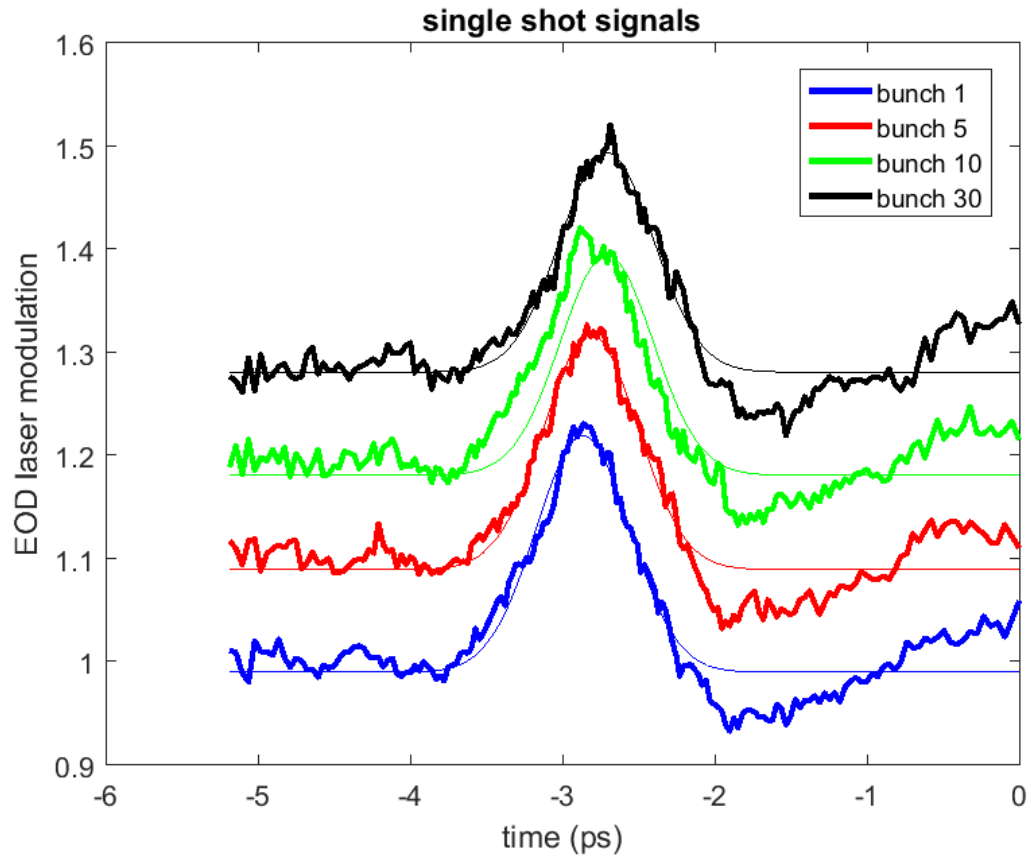
Relative phase shift between polarizations increases with the beam electric field

Spectral decoding

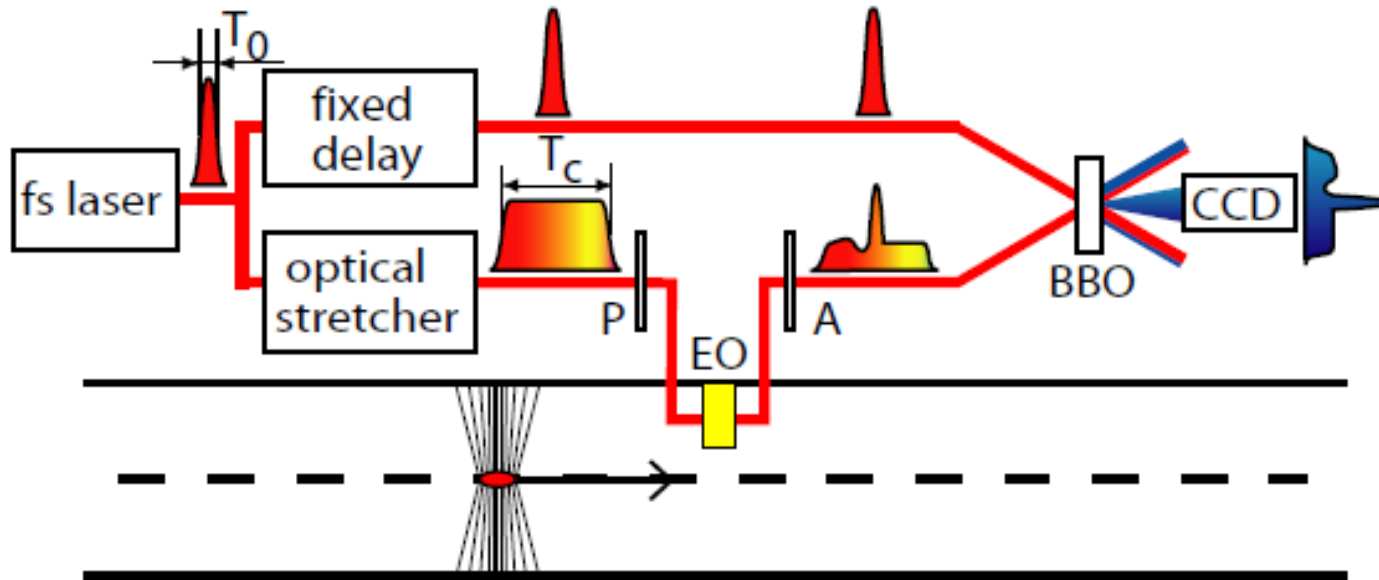


- Artifacts due to frequency mixing
- Minimum resolution in the order $T_{\text{lim}} \approx 2.6 \sqrt{T_0 T_c}$

Single shot measurements at the XFEL bunch compressor 1

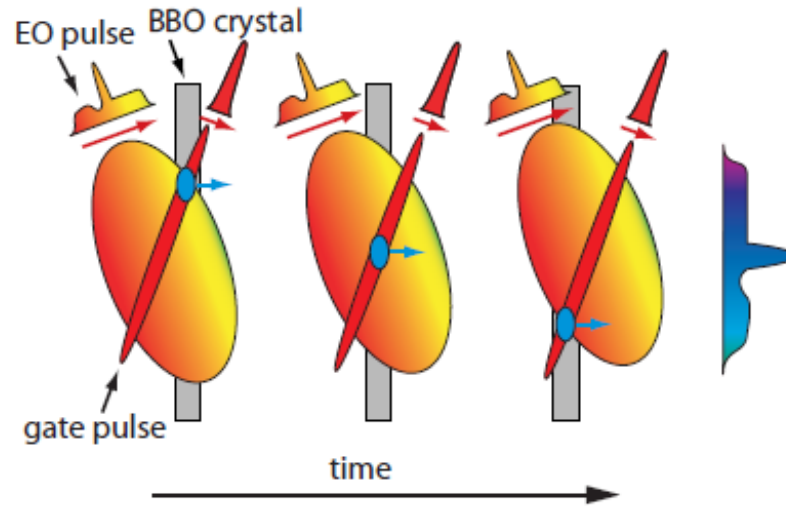


Temporal decoding



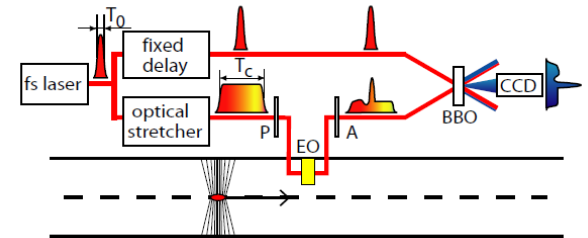
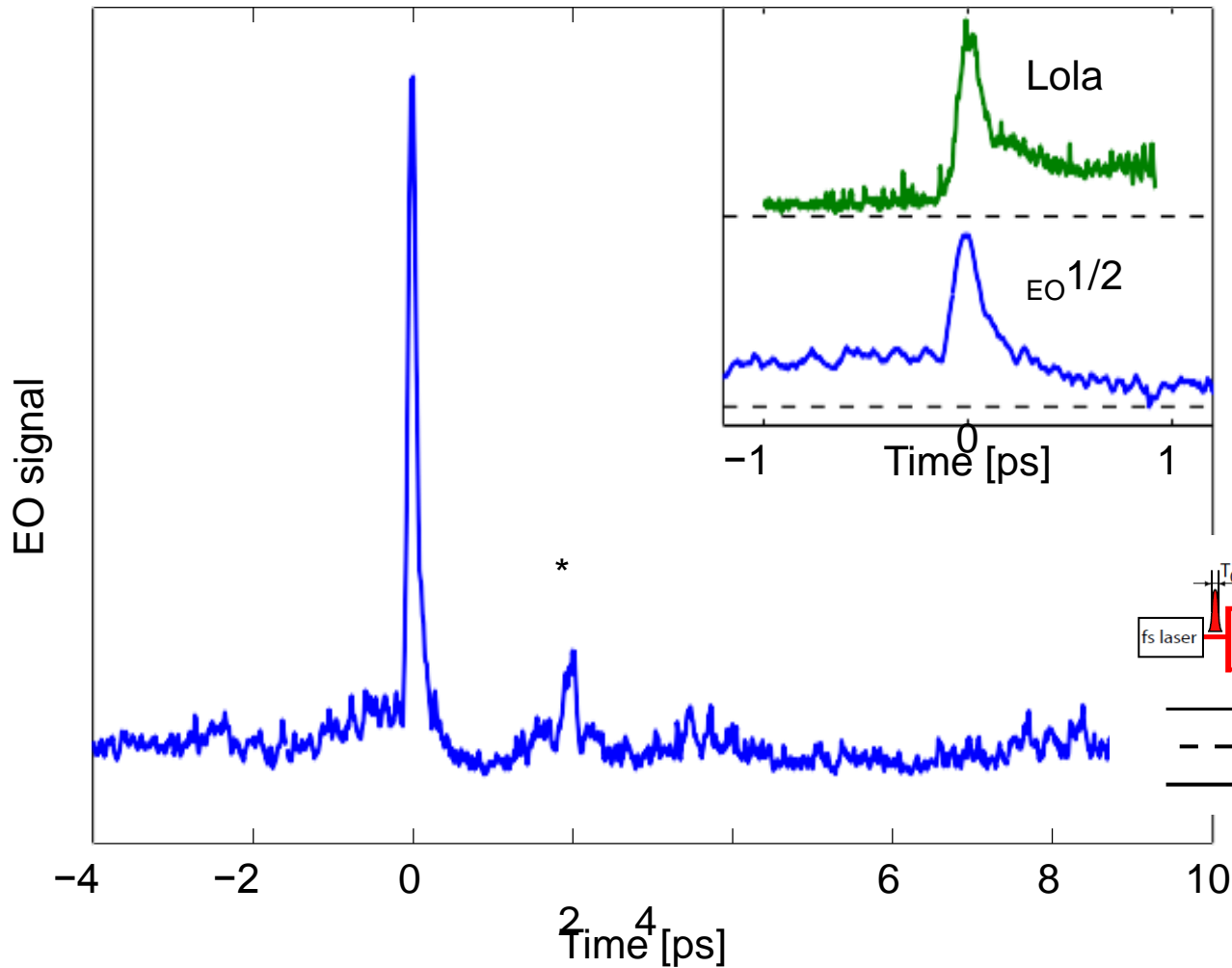
- Resolution : duration of the gate beam, thickness of the SHG crystal
- 50 fs or slightly better
- low efficiency SHG process, approx. 1mJ laser pulse energy necessary

Temporal decoding

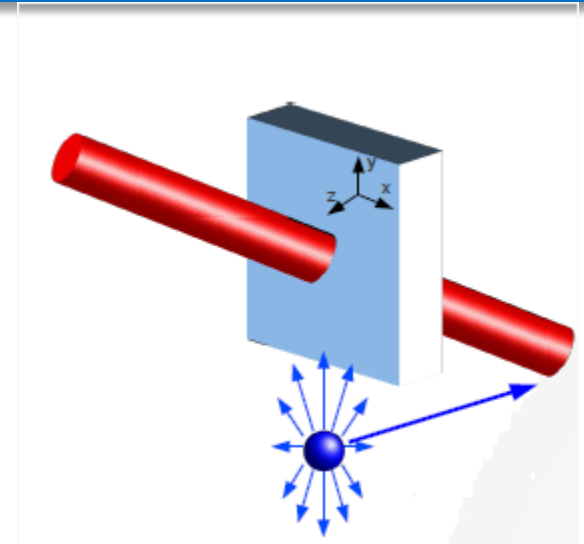
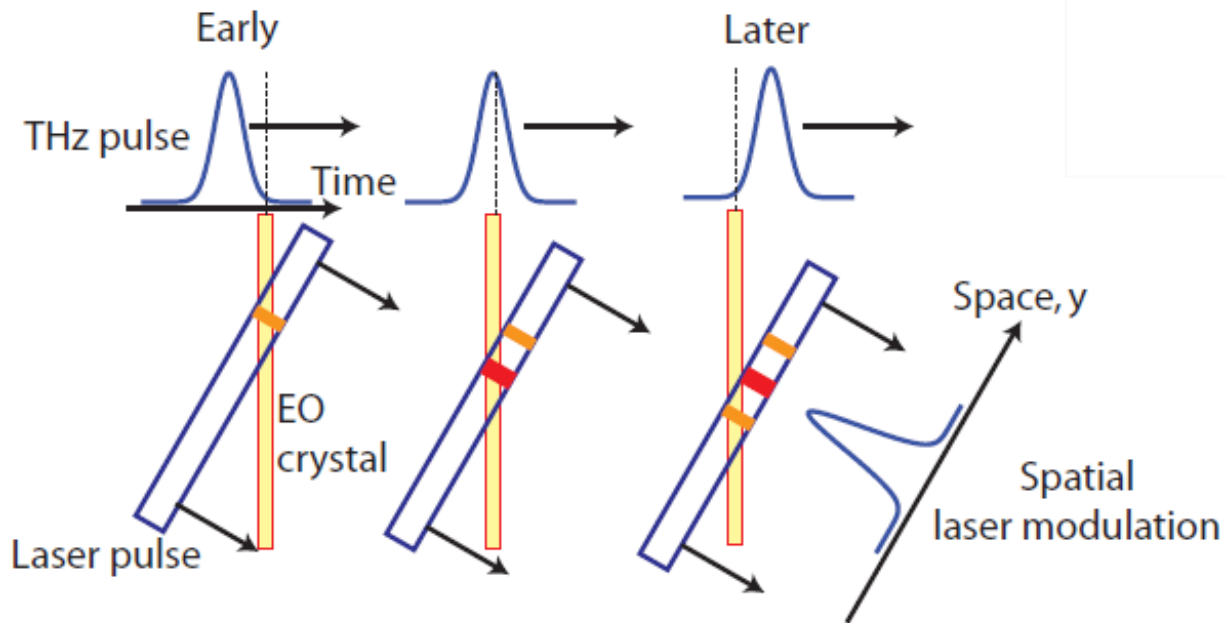


- The short gate pulse overlaps with different temporal slices of the EO pulse at different spatial positions of the BBO crystal. Thus the temporal modulation of the EO pulse is transferred to spatial distribution of the SHG light.

Measurement performed at FLASH/DESY



Spatial decoding



Cavaliere *et. al*, PRL 94 (2005) 114801

Jamison *et. al*, Opt. Lett. 28 (2003) 1710

Van Tilborg *et. al*, Opt. Lett. 32 (2007) 313

- Optical radiation
 - Cherenkov / OTR radiation
 - ODR / OSR Radiation
 - Streak camera
- Coherent radiation : Bunch spectrum
 - Interferometry
 - Polychromator
- RF techniques
 - 'Feschenko' monitor
 - RF Deflector
 - Zero phasing techniques
- Laser based Method
 - Sampling
 - Non linear mixing
 - Thomson/Compton scattering
 - Photo-neutralization
 - Electro-Optic Sampling
 - E-O Spectral decoding
 - E-O Spatial decoding
 - E-O Temporal decoding



Limitations

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

X

200fs

Hadron, 20ps

1fs

10fs

Jitter (10fs)

Electron

H⁺

~ 200fs

~ 50fs

~ 50fs

- Short bunch length measurements are challenging
- Resolution of few fs achieved operationally
- Field in constant move driven by the advances in FELs and novel accelerating technologies
- An exciting field as well !



The CERN Accelerator School

Extra slides



“When you are courting a nice girl an hour seems like a second. When you sit on a red-hot cinder a second seems like an hour. That's relativity. “

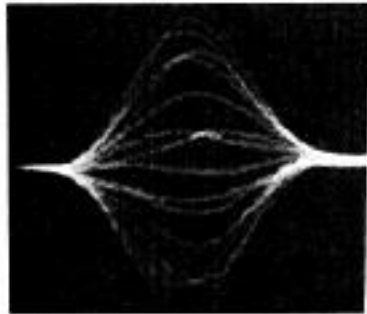
Albert Einstein



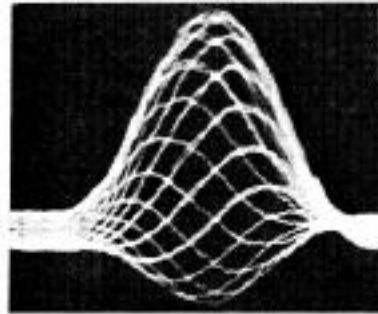


Transverse Diagnostics for measuring instabilities

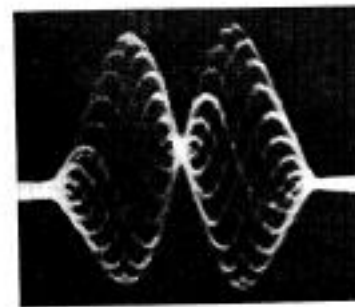
From Booster in 70's



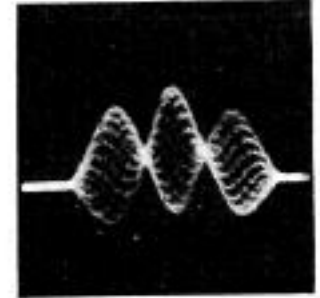
a) mode $m = 0$, $\chi = 0$



b) $m = 0$, $\chi = 2.3$ radians



b) $m = 1$, $\chi = 6.9$ radians



d) $m = 2$, $\chi = 6.9$ radians

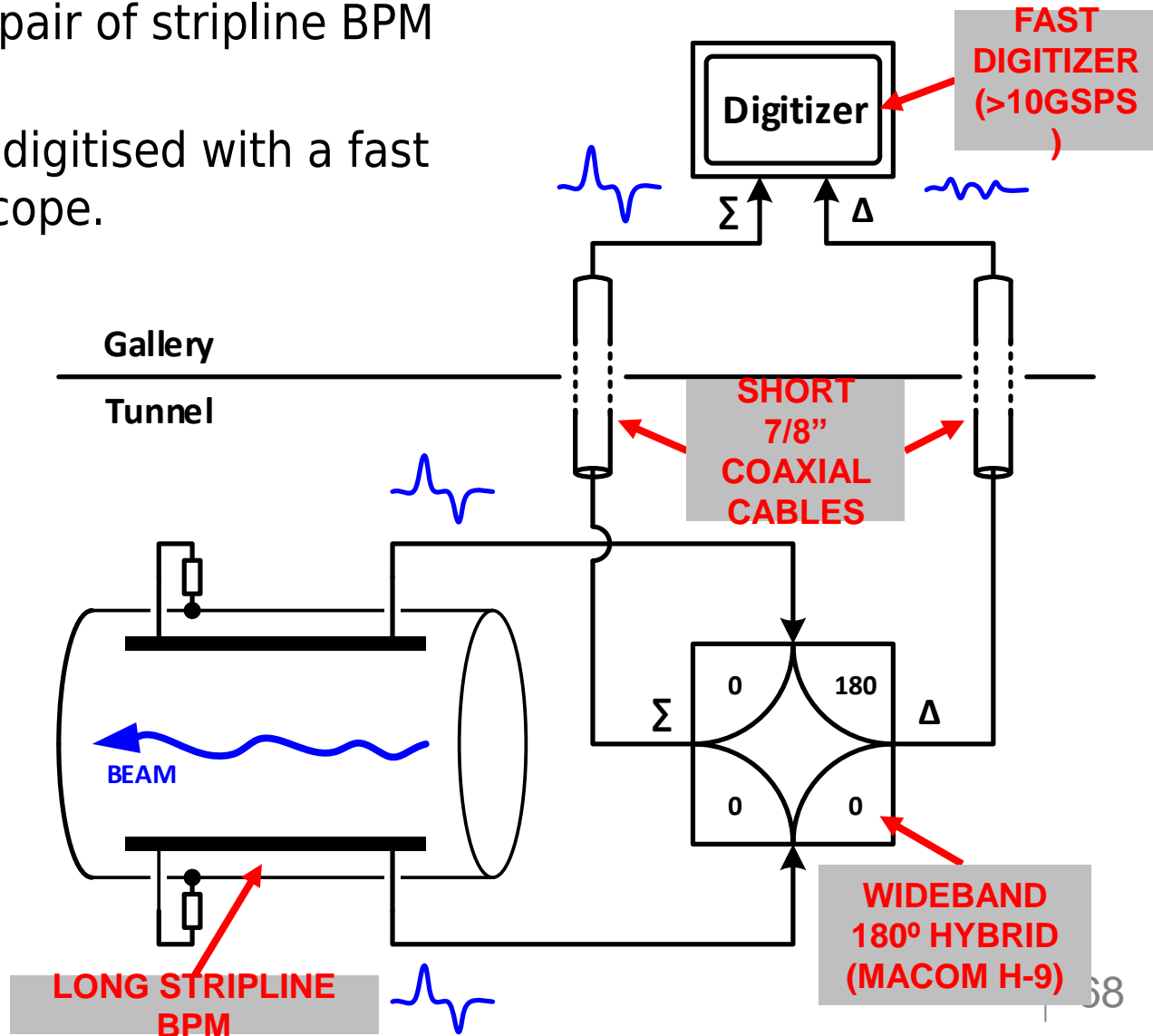
Very long pulses – 100ns

Transverse instability monitoring

A wideband 180° hybrid calculates the sum and difference of a pair of stripline BPM electrodes.

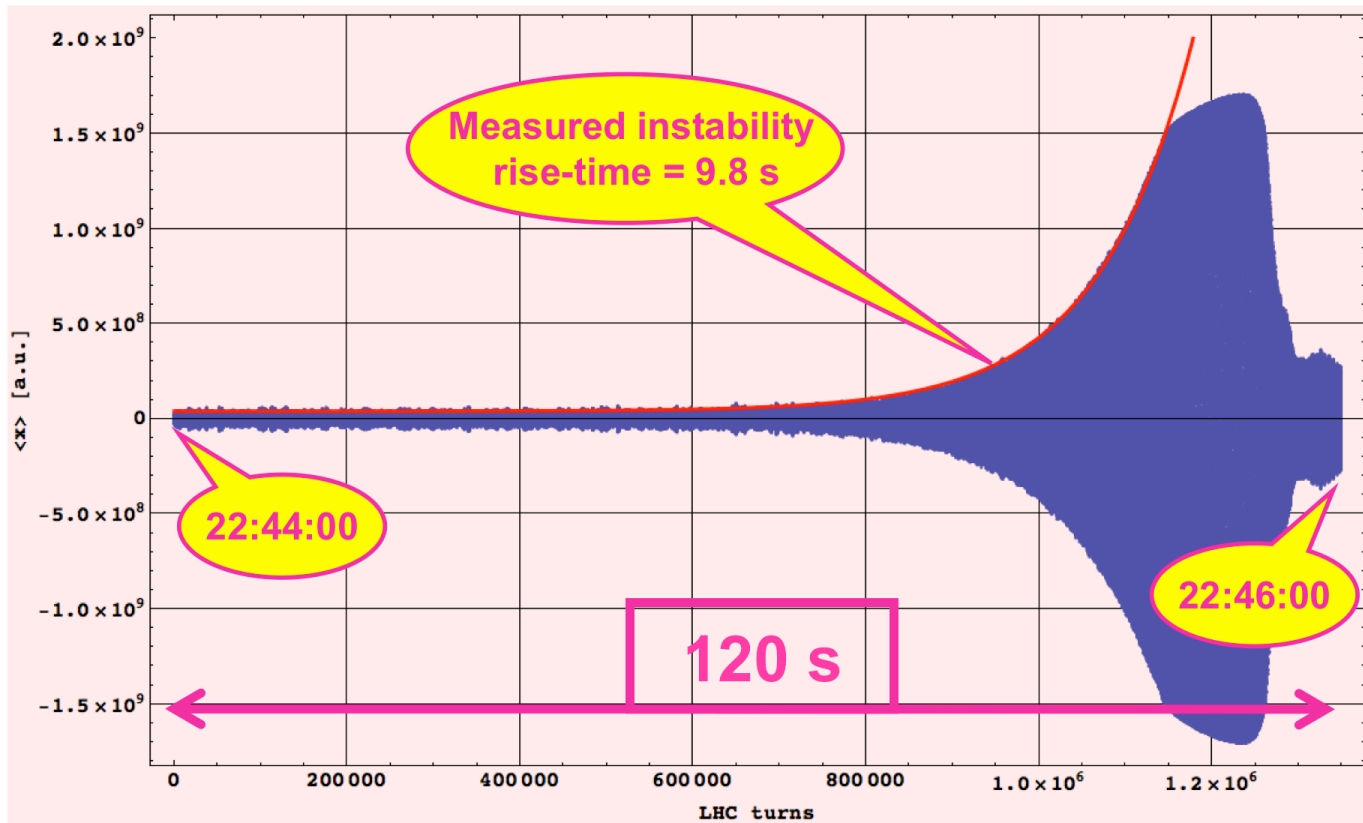
Signals are directly digitised with a fast (>10GSPS) oscilloscope.

Originally planned for chromaticity measurement (H-T phase shift), but excitation amplitude too large for regular operation. Now primarily used for measuring intra-bunch instabilities.



Transverse instability monitoring

Looking at the beginning of an instability on Large Hadron Collider



The rise time is defined as the time taken for the amplitude of the envelope to increase by: $e^1 \approx 2.7$.

Transverse instability monitoring

The LHC BBQ system is most sensitive instrument available for detecting transverse oscillations. Instability detection can be performed by looking at the growth in BBQ amplitude spectrum. Initial developments of algorithm by J. Ellis. Since 2015 the algorithm has been running online in the LHC (FPGA implementation).

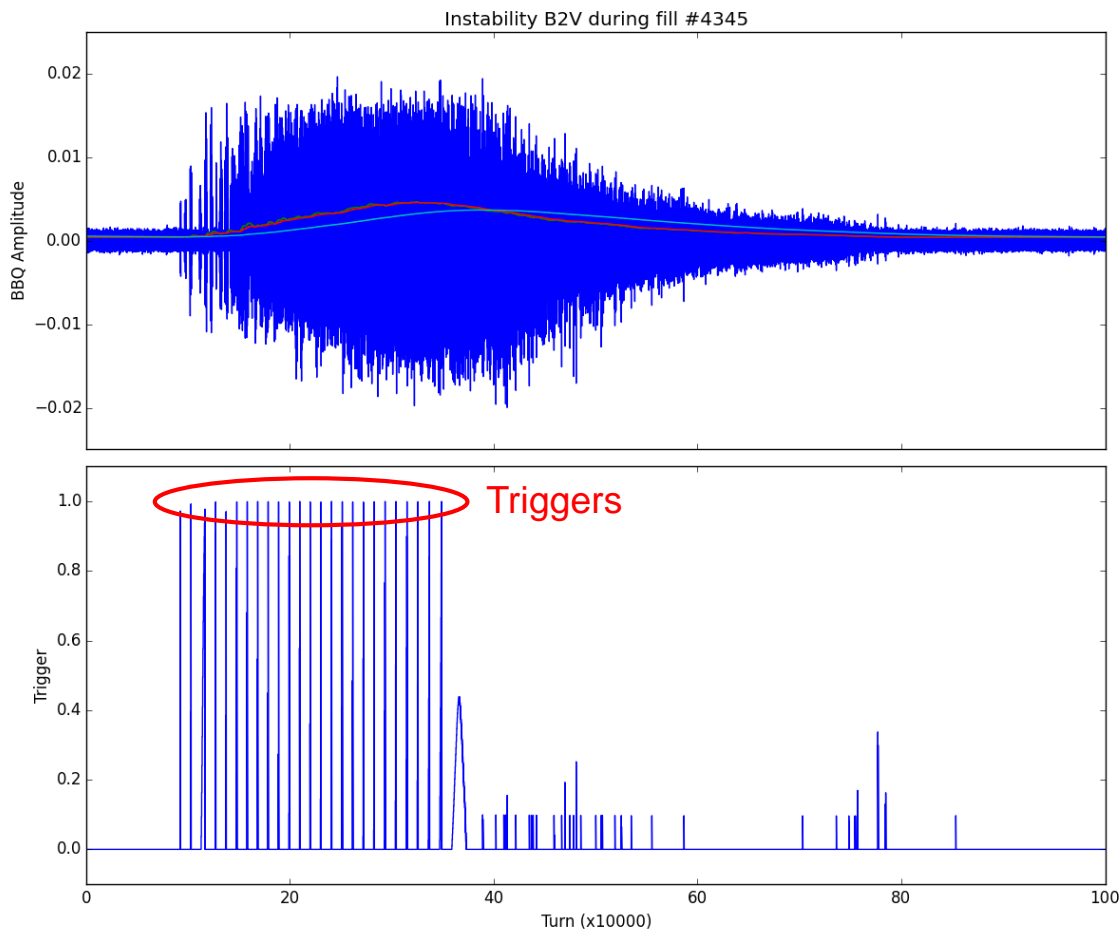
Three moving average filters of different lengths are applied to r.m.s. input signal.

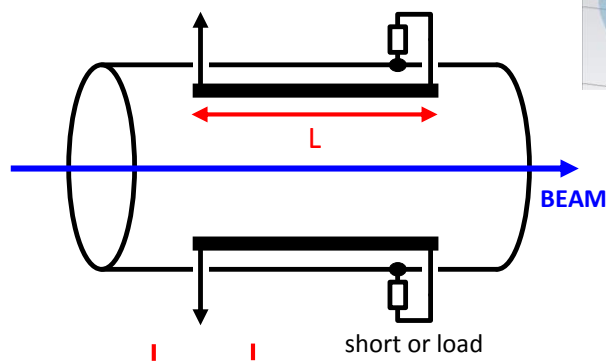
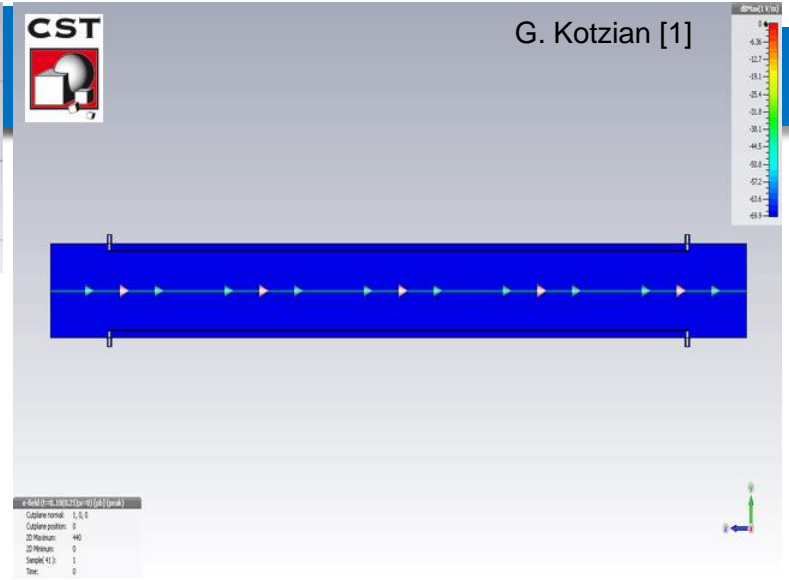
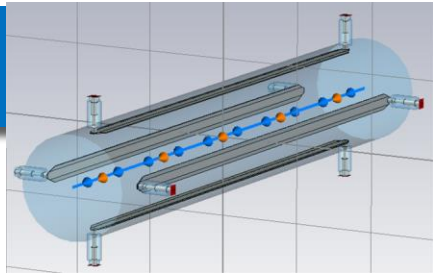
If the condition:

$$\sigma_{\text{short}} > \sigma_{\text{medium}} > \sigma_{\text{long}}$$

is exceeded for a certain number of turns the trigger is fired.

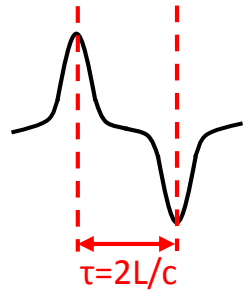
Works reasonably but still being tuned in order to be robust against injection transients, abort gap cleaning excitation, ...



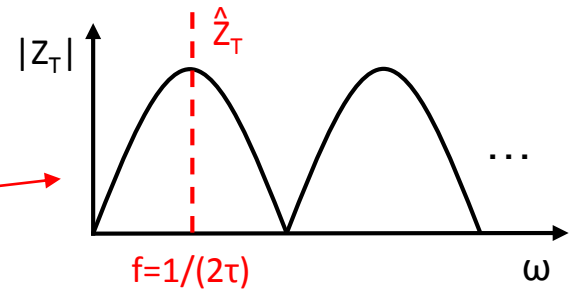


LHC BPLX
L=40cm, $\tau=2.6$ ns

SPS BPCL
L=60cm, $\tau=4.0$ ns

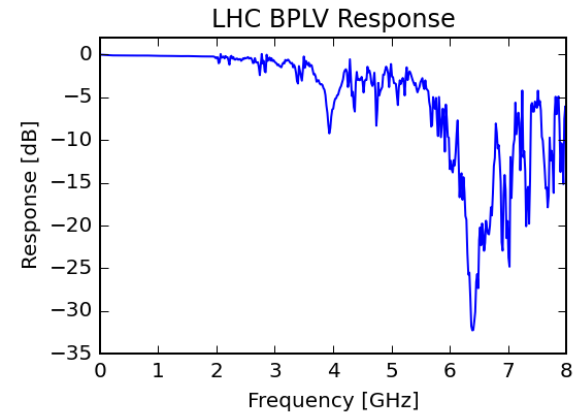


$$V_{PU}(t) = \underbrace{Z_T I_b(t)}_{\text{BUNCH PROFILE}} * \underbrace{[\delta(t) - \delta(t - \tau)]}_{\text{NOTCHES IN FREQ. RESPONSE}}$$



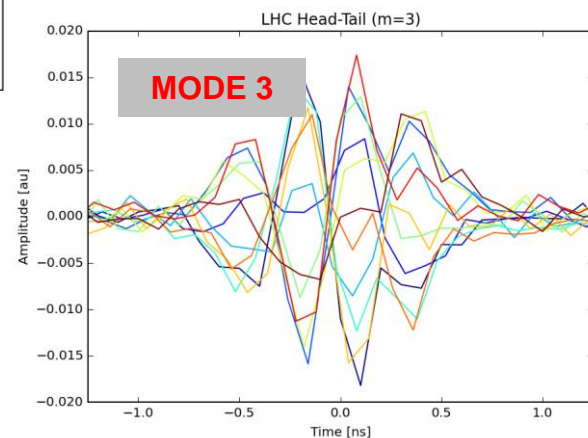
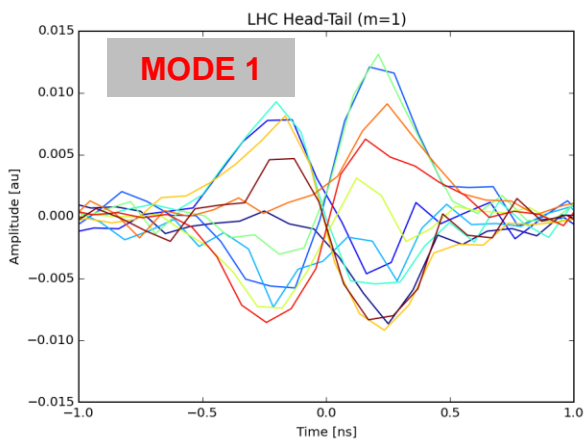
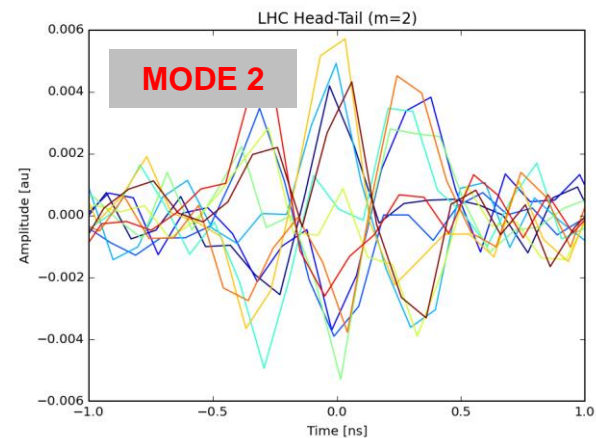
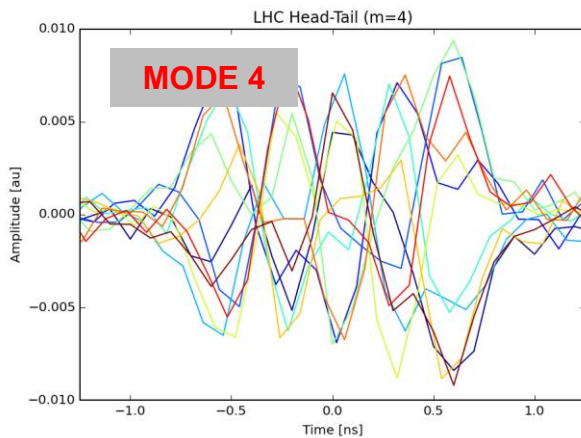
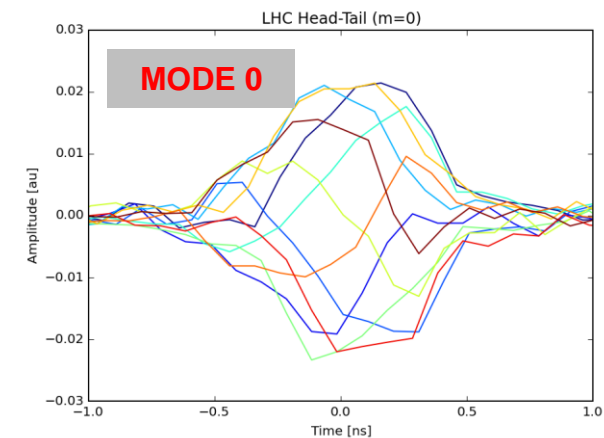
Notches can be removed gating on the initial pulse in the time domain and discarding second pulse. Frequency response is then limited by the BPM structure, feed-throughs, etc.

NB: This requires long BPM and adequate bunch spacing to avoid mixing of the two pulses from the same or subsequent bunches.



Instability triggering

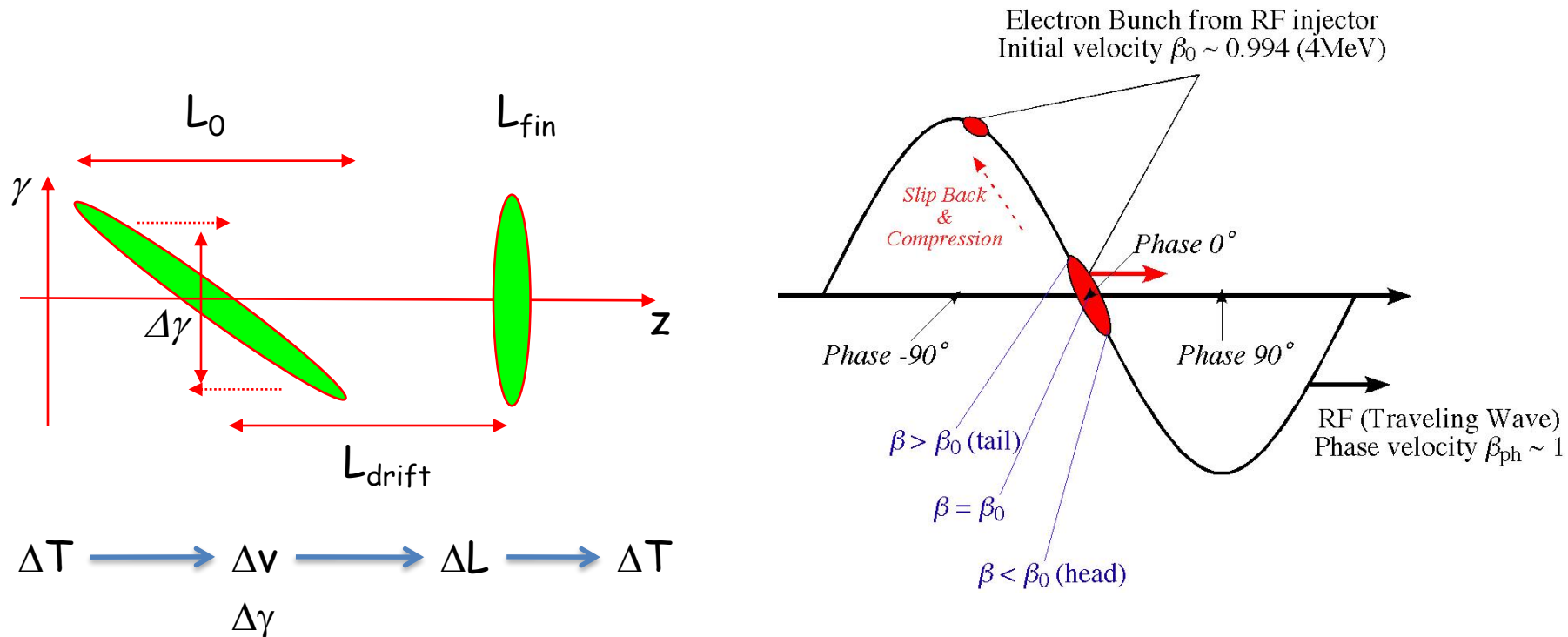
From LHC in 2018



Bunch length manipulation

- Ballistic Compression
- Magnetic Compression

Short bunches by Ballistic/Velocity Compression

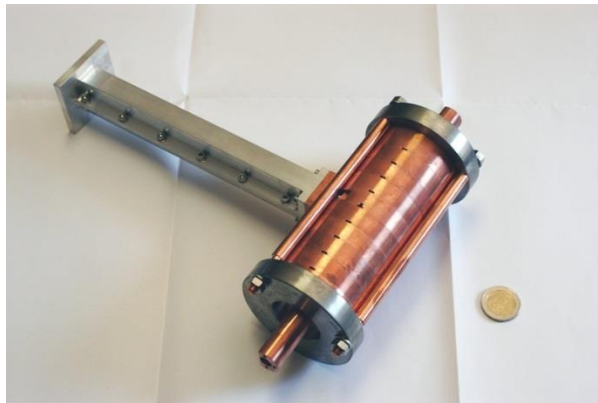
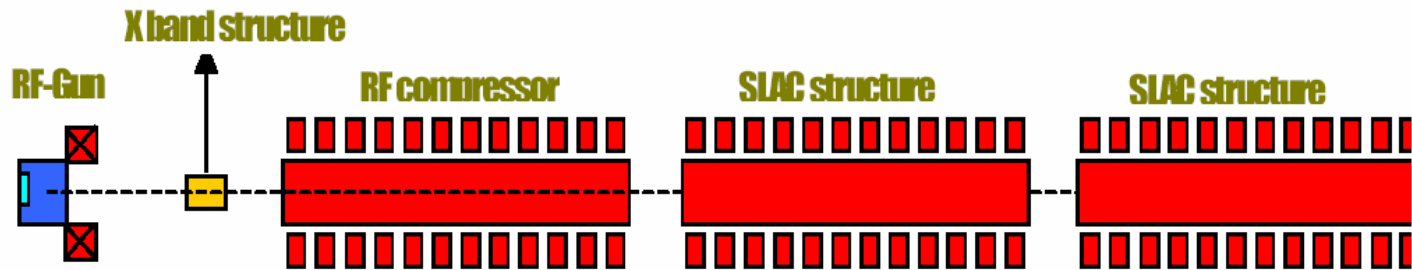


Provide a correlated velocity spread enough to produce, in a drift of length L_{drift} a path difference equal to ΔL

$$DL = \left[\frac{L_{drift}}{g^2} \right] \frac{Dg}{g}$$

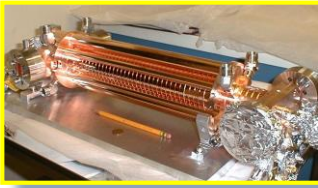
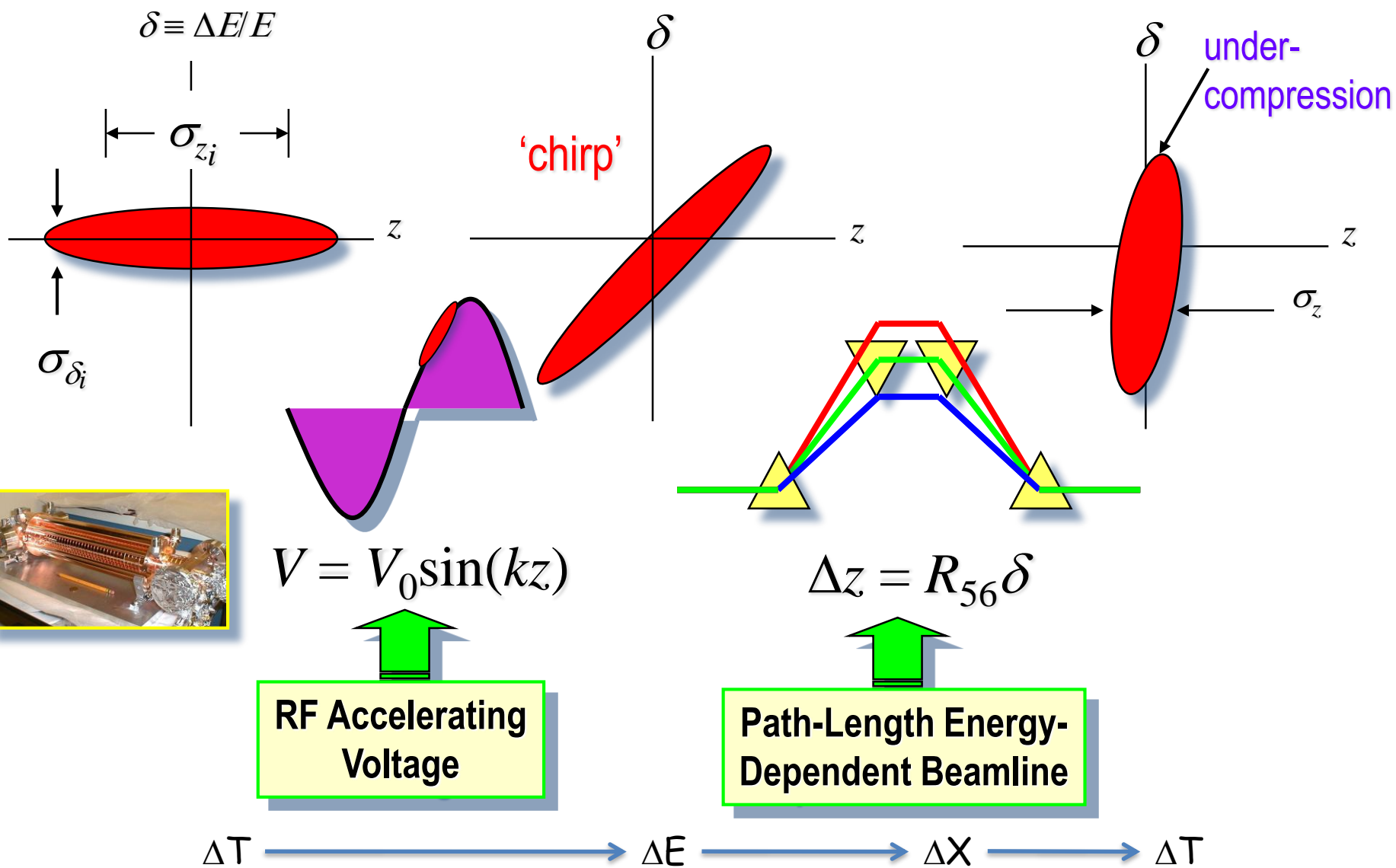
P. Piot *et al*, PRSTAB 6 (2003) 033503
S.G. Anderson *et al*, PRSTAB 8 (2005) 014401

Short bunches by Ballistic Compression

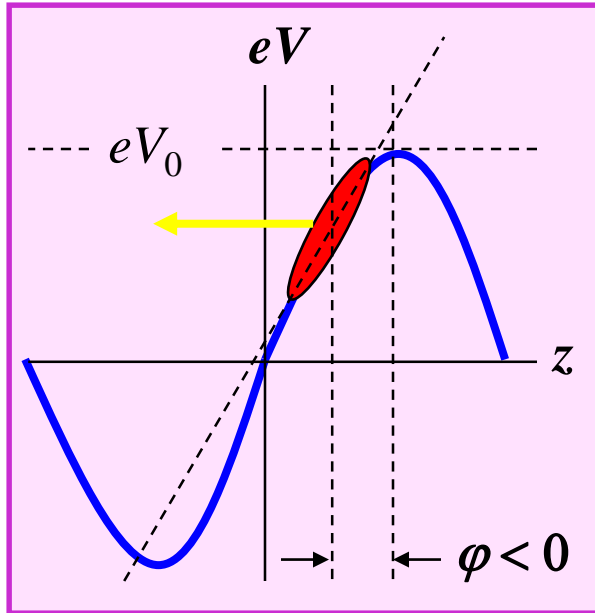


- Works well for non ultra-relativistic beam energies
- no Coherent Synchrotron Radiation effect and bend-plane emittance growth
- Longitudinal emittance growth due to RF non linearities

Short bunches by Magnetic Compression



Short bunches by Magnetic Compression



$$E(z) = E_0 + eV_0 \cos(\varphi + 2\pi z/\lambda)$$

$$\delta \equiv \frac{\Delta E}{E} \approx \dots$$

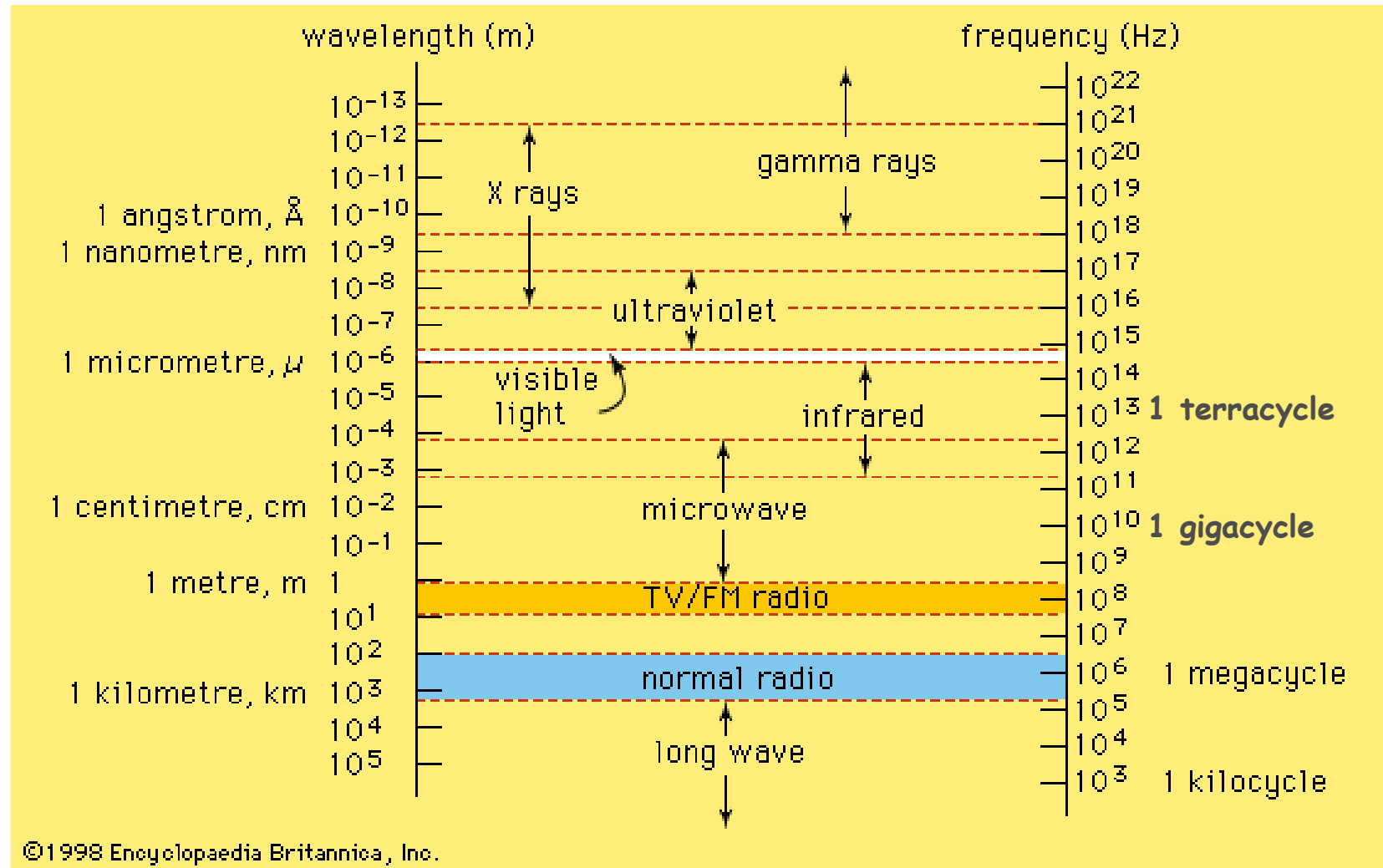
$$\delta_0 \frac{E_0}{E} + \left(1 - \frac{E_0}{E}\right) \left[\frac{\cos(\varphi + \Delta\varphi) - (2\pi z/\lambda) \sin(\varphi + \Delta\varphi)}{\cos(\varphi)} - 1 \right]$$

$$k(\varphi) \equiv \frac{\partial \delta}{\partial z} = -\frac{2\pi}{\lambda} \left(1 - \frac{E_0}{E}\right) \frac{\sin(\varphi + \Delta\varphi)}{\cos(\varphi)} \quad \text{'chirp'}$$

final bunch length and energy spread...

$$\sigma_z = \sqrt{(1 + kR_{56})^2 \sigma_{z0}^2 + R_{56}^2 \sigma_{\delta_0}^2 E_0^2 / E^2} \quad , \quad \sigma_\delta = \sqrt{k^2 \sigma_{z0}^2 + \sigma_{\delta_0}^2 E_0^2 / E^2}$$

Bunch Frequency Spectrum



Coherent Synchrotron Radiation in Magnetic Chicane

- Powerful radiation generates energy spread in bends
- Energy spread breaks achromatic system
- Causes emittance growth (short bunch worse)

