Introduction

In the particle-mesh method the calculation of the space charge fields from spatially distributed charges requires the solution of the Poisson equation. The solution can be strongly influenced by the applied boundary conditions (b.c.). Conducting b.c. are often applied on the rectangular boundaries of the calculation domain. However, the rectangular cross section is not the best approximation to the real geometry of the beam pipe. We present an algorithm applying conducting b.c. on an arbitrary elliptic cross section of the beam pipe. Further we present results from our tracking routine, which employs the new space charge solver.

CONDUCTING B.C. ON BEAM PIPES WITH ELLIPTICAL CROSS SECTION

Considering beam pipes with elliptical cross section, we solve the Poisson equation on the cylindrical domain $\Omega$:

$$\Delta \phi = \frac{\rho}{\varepsilon} \quad \text{in} \ 0 \in \mathbb{R}^3,$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on} \ \partial \Omega_1,$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on} \ \partial \Omega_2,$$

$\partial \Omega_1$ is the surface of the cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad -c < z < c,$$

$\partial \Omega_2$ are the two elliptical bases of the cylinder satisfying

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad z = \pm c.$$

RESULTS

Space charge simulation

Example to compare the calculated $E$ fields with open and conducting b.c. on a rectangular and elliptic beam pipe

- spherical bunch ($r << a,b$)
- uniformly distributed charge of -1nC

Electric field $E$ along $y = b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right), computed with open (w/o.b.c.) and conducting b.c. (w.b.c.) on a rectangular and elliptic (elliptic b.c.) pipe.

Conclusions

Field differences between simulations with different b.c. and different geometries get significant as we move towards the boundary of the pipe. Consequently a bunch tracked with different b.c. has a different transversal expansion, as shown with our first tracking results.

REFERENCES