A Transverse Feedback System Using Multiple Pickups for Noise Minimization

Mouhammad Alhumaidi and Abdelhak M. Zoubir Signal Processing Group Email: {malhumai,zoubir}@spg.tu-darmstadt.de

TECHNISCHE UNIVERSITÄT DARMSTADT

Motivation

Transversal beam oscillations can happen:

- directly after injection
- due to higher beam intensity, which excites instabilities, when natural damping becomes not enough for attenuation

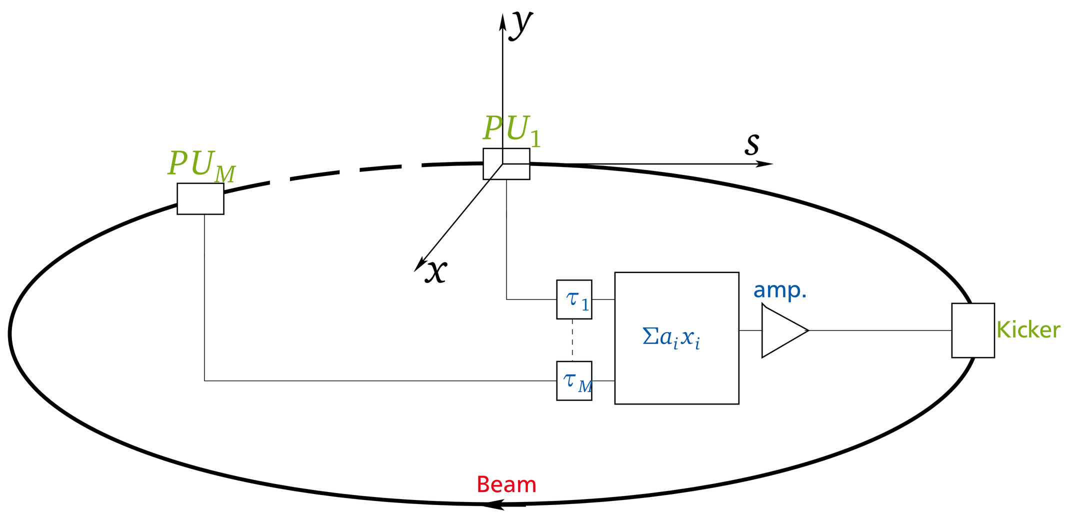
$\Longrightarrow$ Transversal Feedback System (TFS) must be used for actively damping beam transversal instabilities, however:

- at the Pickups (PUs) only beam displacement is measured
- noise generated by the PUs deteriorates feedback

$\Longrightarrow$ multiple PUs for estimating beam direction at Kicker position with minimized noise

System model

Block diagram of proposed technique and coordinates for circular accelerator:



- Further notation will use $x(s)$ as beam horizontal displacement at position s, $\tilde{x}_i$ at position s_i of PU_i and $x'(s)$ as beam direction in horizontal plane at position s
- $\tilde{\mathbf{x}} = [\tilde{x}_1, \tilde{x}_2 \cdots, \tilde{x}_M]^T$ denotes vector of beam horizontal displacements at the M PUs positions
- $x_i = \tilde{x}_i + z_i$ denotes output signal of PU_i, where z_i denotes noise perturbation
- $\mathbf{x} = [x_1, x_2, \cdots, x_M]^T$ denotes vector of PUs output signals and $\mathbf{z} = [z_1, z_2 \cdots, z_M]^T$ denotes the noise vector at the PUs so:
  $\mathbf{x} = \tilde{\mathbf{x}} + \mathbf{z}$
- For beam vertical displacement every thing holds by using y instead of x

Beam displacement-direction transmission from position s_0 to position s:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \mathbf{M} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

where

$$\mathbf{M} = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos(\Psi(s)) + \alpha_0 \sin(\Psi(s))) & \sqrt{\beta\beta_0}\sin(\Psi(s)) \\ \frac{1}{\sqrt{\beta\beta_0}}[(\alpha_0 - \alpha)\cos(\Psi(s)) - (1 + \alpha\alpha_0)\sin(\Psi(s))] & \sqrt{\frac{\beta_0}{\beta}}(\cos(\Psi(s)) - \alpha\sin(\Psi(s))) \end{pmatrix}$$

$$\Psi(s) = \int_{s_0}^{s} \frac{1}{\beta(s)} \mathrm{d}s$$

knowing beam dispacements $\tilde{x}_{i_1}$ and $\tilde{x}_{i_2}$ and by solving transmission equations one can calculate beam direction at Kicker position as:

$$x' = \alpha_{i_1}\tilde{x}_{i_1} + \alpha_{i_2}\tilde{x}_{i_2}$$

using output signals of the M PUs:

$$\begin{aligned} \bar{x} &= \alpha_{i_1}x_{i_1} + \alpha_{i_2}x_{i_2} \\ &= \alpha_{i_1}\tilde{x}_{i_1} + \alpha_{i_2}\tilde{x}_{i_2} + \alpha_{i_1}z_{i_1} + \alpha_{i_2}z_{i_2} \\ &= x' + z \end{aligned}$$

Proposed Technique

Basic idea:

- Filter out the noise by using multiple PUs signals

Optimization problem of finding optimal weighted sum of PUs signals can be formulated as:

$$[a_1, \cdots, a_M] = \underset{a_1, \cdots, a_M}{\operatorname{argmin}} \mathrm{E}|\sum_{i=1}^{M} a_i z_i|^2$$

$$s.t. \quad \sum_{i=0}^{M} a_i \tilde{x}_i = x'$$

Solution for this optimization can be solved through equivalent problem as follows:

- take

$$\begin{pmatrix} \tilde{x}_i \\ \vdots \\ \tilde{x}_{M-1} \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 & & \cdots & 0 \\ 0 & \alpha_{22} & \alpha_{23} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 & \cdots \\ \vdots & \vdots & \cdots & 0 \ddots & \ddots & \cdots \\ 0 & \cdots & 0 & \cdots & \alpha_{M-1M-1} & \alpha_{M-1M} \end{pmatrix}}_{\Lambda} \begin{pmatrix} x_1 \\ \vdots \\ x_M \end{pmatrix}$$

$$= \begin{pmatrix} x' \\ \vdots \\ x' \end{pmatrix} + \Lambda \mathbf{z}$$

- define $\mathbf{w} = [w_1, w_2, \cdots, w_{M-2}, 1 - \sum_{i=1}^{M-2} w_i]^T \in \mathcal{R}^{M-1\times 1}$

then we have

$$\mathbf{w}^T \begin{pmatrix} \tilde{x}_i \\ \vdots \\ \tilde{x}_{M-1} \end{pmatrix} = x' + \mathbf{w}^T \Lambda \mathbf{z} \quad \forall w_1, \cdots, w_{M-2} \in \mathcal{R}^{M-2}$$

Solving the optimization problem is equivalent to finding the optimal vector $\mathbf{w}_{opt}$, which minimizes the noise power, where

$$[a_1, \cdots, a_M]_{opt} = \mathbf{w}_{opt}^T \Lambda$$

This equivalence is due to the same number of degrees of freedom

we have

$$\mathbf{w} = \mathbf{D}\hat{\mathbf{w}} + \mathbf{e}_{M-1}$$

where $\mathbf{D} \in \mathcal{R}^{M-1\times M-2}$ with all-ones on the main diagonal, all -1 on the last row and zeros elsewhere

$$\hat{\mathbf{w}} = [w_1, w_2, \cdots, w_{M-2}]^T \text{ and } \mathbf{e}_{M-1} = [0, 0, \cdots, 0, 1]^T \in R^{M-1\times 1}$$

- Setting the derivative of noise power with respect to $\hat{\mathbf{w}}$ to zero and solving, one can find

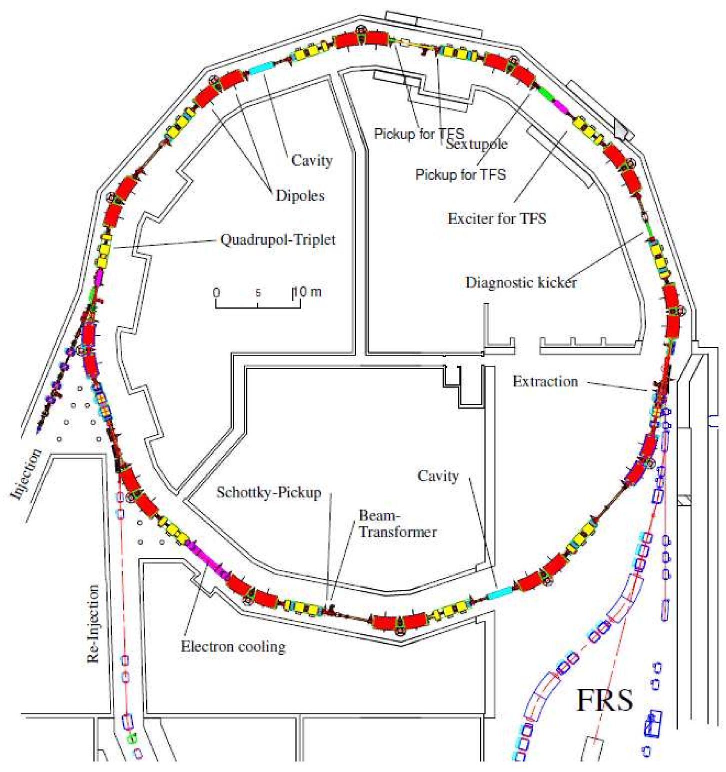
$$\hat{\mathbf{w}}_{opt} = -(\mathbf{D}^T \Lambda \mathbf{R}_{\mathbf{zz}} \Lambda^T \mathbf{D})^{-1} \mathbf{D}^T \Lambda \mathbf{R}_{\mathbf{zz}} \Lambda^T \mathbf{e}_{M-1}$$

Simulations

Simulations of this technique has been done for the Synchrotron SIS 18 at the GSI

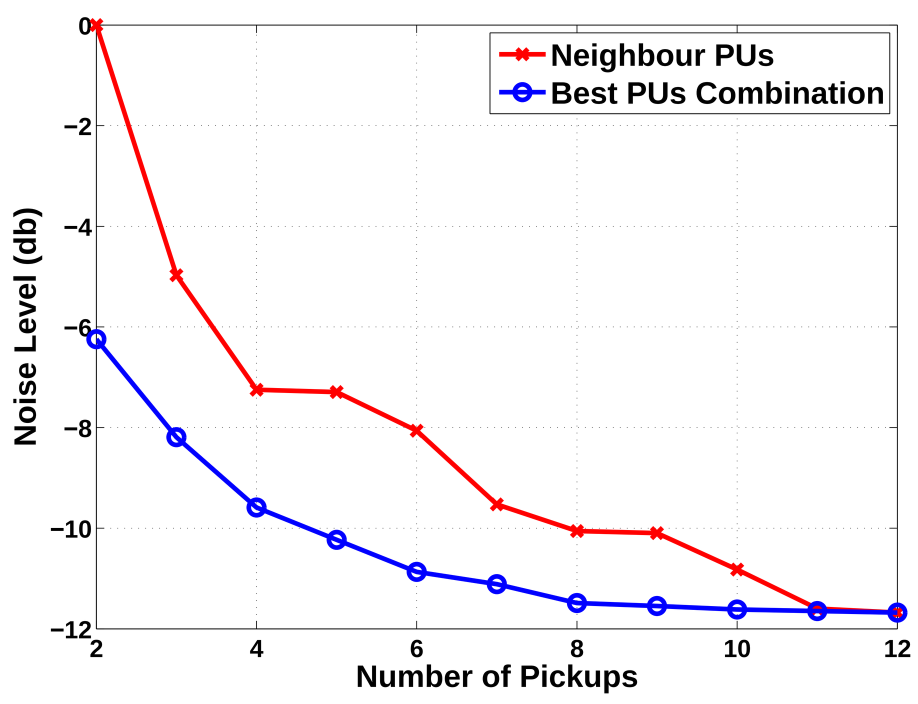
SIS 18 at the GSI

- 12 periods with 12 beam position PUs
- one Kicker
- from doublet mode to triplet mode focusing during acceleration
- tunes: $Q_x = 4.29$, $Q_y = 3.29$



Noise reduction for horizontal direction

- reference: noise power for using the closest two horizontal PUs to the kicker



Noise reduction for vertical direction

- reference: noise power for using the closest two vertical PUs to the kicker