ACCELERATOR MAGNETS

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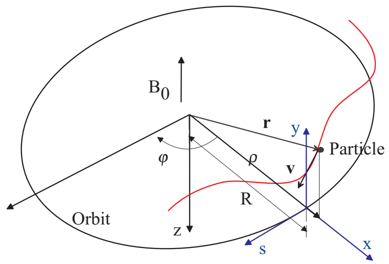
Transverse Motion and the Normalized Gradient

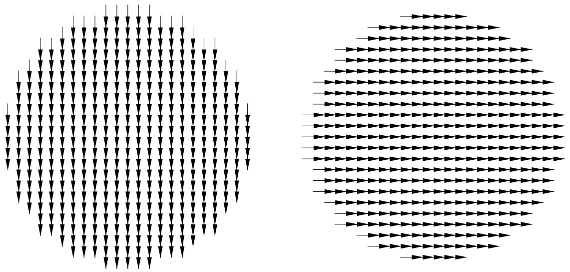


Displaced particle at $\rho = R + x$ with field $B_z = B_0 + \frac{\partial B_z}{\partial x}\Big|_{x=0} x$

$$\frac{1}{p_0}\frac{\mathrm{d}}{\mathrm{d}s}\left(p_0\frac{\mathrm{d}x}{\mathrm{d}s}\right) + \left(\frac{1}{R^2} - k\right)x = 0$$

For $x \ll R$, $e/p_0 = -1/(B_0 R)$ and $k = -\frac{1}{B_0 R}\frac{\partial B_z}{\partial x}\Big|_{x=0}$

Bending Field (Dipole) and Magnetic Rigidity

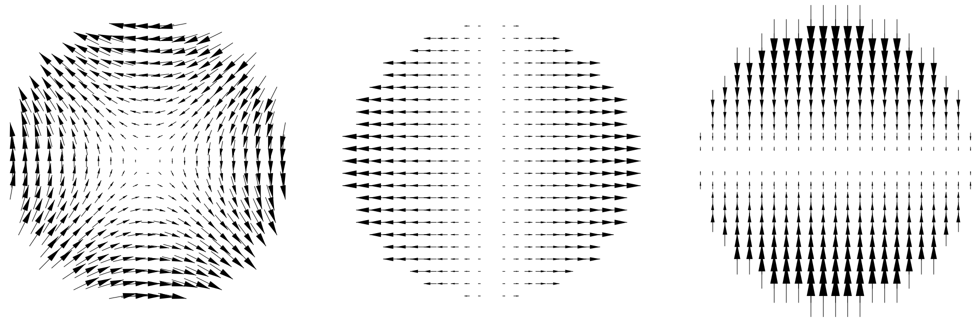


$\vec{B}$ $\quad\quad$ $\vec{v}_z \times \vec{B}$

$$3.3356 p[\mathrm{GeV/c}] = R[\mathrm{m}]\, B_0[\mathrm{T}]$$

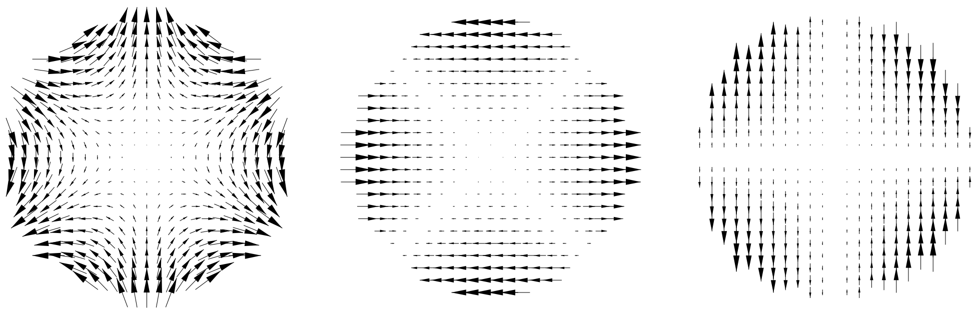
Remark 1: Consider the “filling factor” in accelerators between 0.6 and 0.7.

Quadrupole



$\vec{B}$ $\quad\quad$ $(\vec{v}_z \times \vec{B})_x$ $\quad\quad$ $(\vec{v}_z \times \vec{B})_y$

Higher Order Multipole Fields: Sextupole

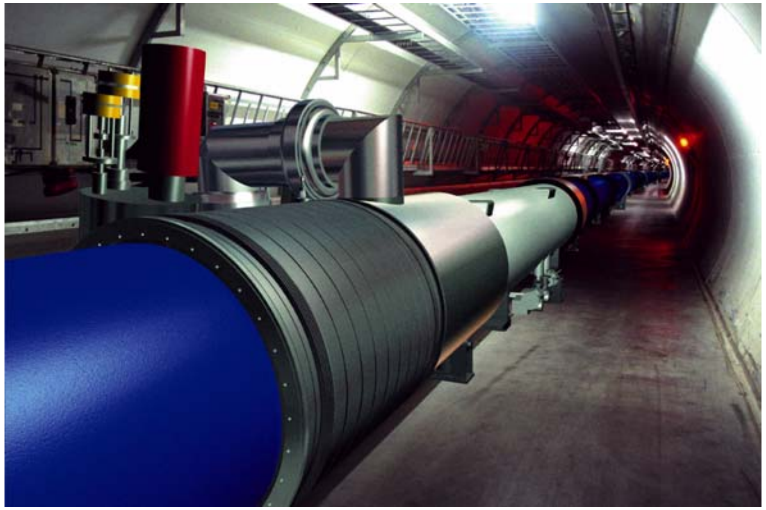


$\vec{B}$ $\quad$ $(\vec{v}_z \times \vec{B})_x$ $\quad$ $(\vec{v}_z \times \vec{B})_y$

Multipole Field Errors and Beam Parameters

b_1, a_1	Closed orbit distortions
b_2	β-beating
a_2	Vertical dispersion, linear coupling
b_3	Off-momentum β beating
a_3	Chromatic coupling inducing detuning
b_4	Dynamic aperture and detuning
a_4	Dynamic aperture at injection
b_5	Dynamic aperture and detuning
a_5	Off-momentum dynamic aperture
b_7	Dynamic aperture at injection
$b_6, a_6, a_7, >$	Dynamic aperture at injection

LHC in the LEP Tunnel (Virtual Reality)

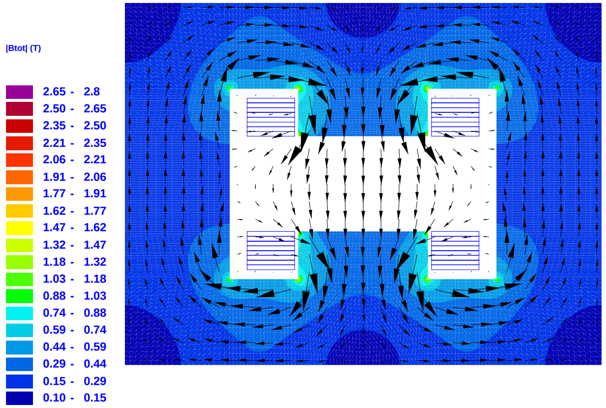


Remark 2: Magnet systems are the most costly item in an accelerator

The 26 GeV Proton Synchrotron (PS) in its Tunnel

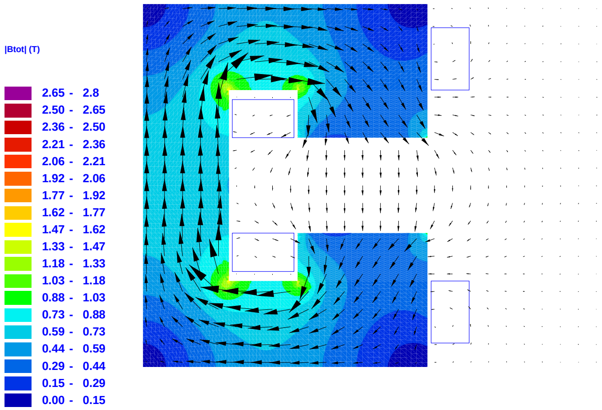
High Energy Ring Accelerator (HERA) at DESY

Magnet Metamorphosis
(H-Magnet)



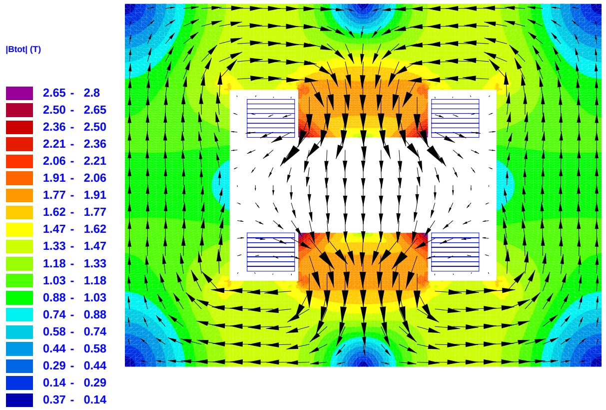
$N \cdot I = 24000$ A $B_1 = 0.3$ T $B_s = 0.065$ T Fill.fac. 0.98

Magnet Metamorphosis
(C- Core, LEP Dipole)



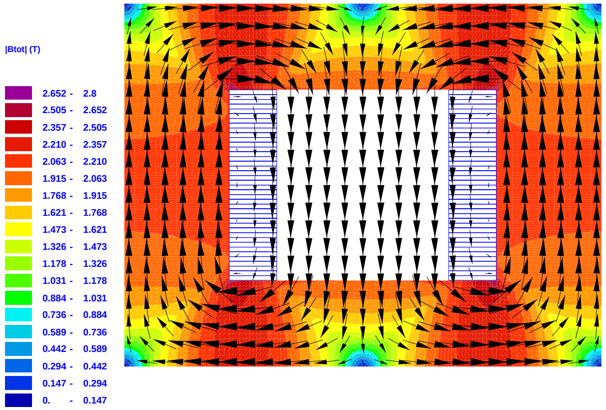
$N \cdot I = 4480$ A $B_1 = 0.13$ T $B_s = 0.042$ T Fill.fac. 0.27

Magnet Metamorphosis
(Cryogenic or Super-Ferric Magnet)



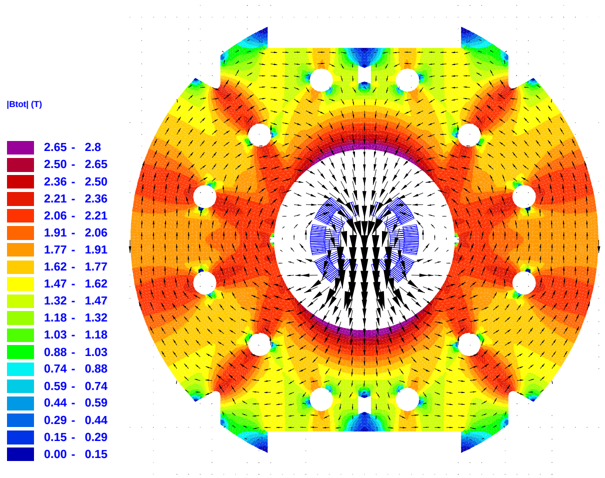
$N \cdot I = 96000$ **A** $B_1 = 1.18$ **T** $B_s = 0.26$ **T**

Magnet Metamorphosis
(Window Frame Dipole)



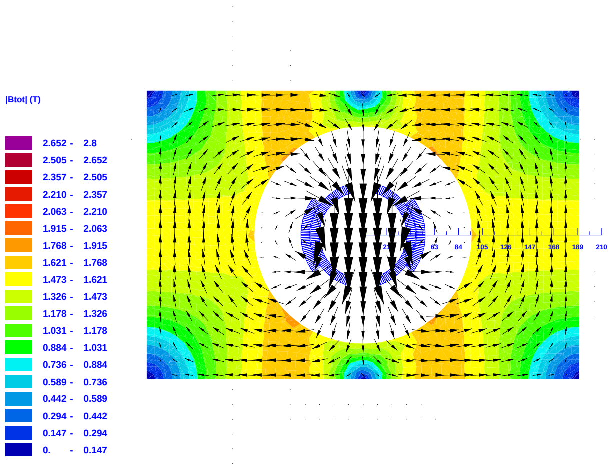
$N \cdot I = 360000$ A $\quad B_1 = 2.08$ T $\quad B_s = 1.04$ T

Magnet Metamorphosis
(LHC Coil-Test Facility



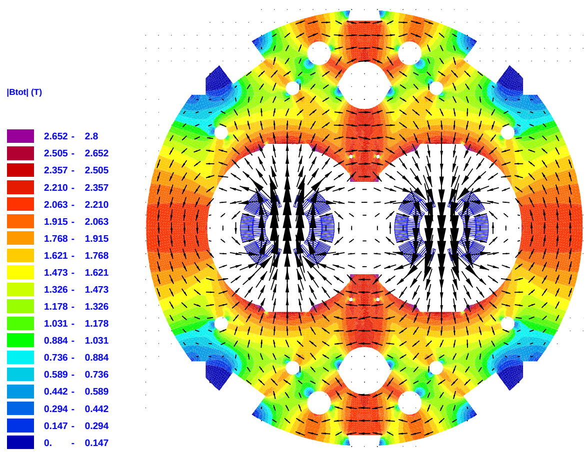
$N \cdot I = 960000$ A $\quad B_1 = 8.33$ T $\quad B_s = 7.77$ T

Magnet Metamorphosis
(Tevatron Dipole)



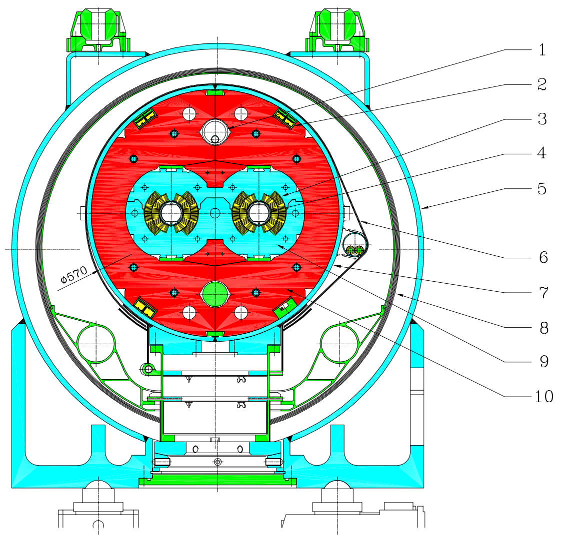
$N \cdot I = 471000$ A $\quad B_1 = 4.16$ T $\quad B_s = 3.39$ T

Magnet Metamorphosis
(LHC Main Dipole)



$N \cdot I = 2x944000$ A $\quad B_1 = 8.32$ T $\quad B_s = 7.44$ T

LHC Main Dipol Cross-Section



1. Heat exchanger, 2. Bus bar, 3. Superconducting coil, 4. Vacuum Pipe und Beam-screen, 5. Cryostat, 6. Thermal shield (55- 75 K), 7. Helium-Tank, 8. Super-Insulation, 9. Collars, 10. Yoke

Overview

Maxwell's equations in integral form
One-dimensional field calculation for conventional magnets

Maxwell's equations in differential form
The solution of Laplace's equation
Harmonic fields
Complex Potentials
Ideal pole shapes of conventional magnets
Feed down

Solution of Poisson's equation
The field of line currents
Generation of pure multipole fields
Sensitivity to manufacturing errors

Numerical field computation
Saturation of the iron yoke
Superconducting filament magnetization

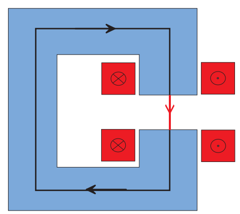
Maxwell's Equations in Integral Form

$$\oint \vec{H} \cdot d\vec{s} = \int_A (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{A}$$
$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \int_A \vec{B} \cdot d\vec{A}$$
$$\int_A \vec{B} \cdot d\vec{A} = 0$$
$$\int_A \vec{D} \cdot d\vec{A} = \int_V \rho dV$$

Constitutive Equations

$$\vec{B} = \mu \vec{H} = \mu_0 (\vec{H} + \vec{M})$$
$$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 (\vec{E} + \vec{P})$$
$$\vec{J} = \kappa \vec{E} + J_{\text{imp.}}$$

1D Field Calculation for a Conventional Dipole



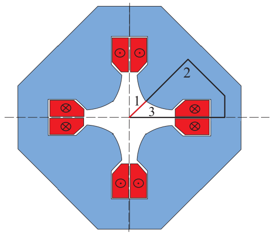
$$\oint \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{A}$$

$$H_{\text{iron}}\, s_{\text{iron}} + H_{\text{gap}}\, s_{\text{gap}} = \frac{1}{\mu_0 \mu_r} B_{\text{iron}}\, s_{\text{iron}} + \frac{1}{\mu_0} B_{\text{gap}}\, s_{\text{gap}} = N\,I$$

$$\mu_r \gg 1 \qquad B_{\text{gap}} = \frac{\mu_0 N\,I}{s_{\text{gap}}}$$

Warning 1: Check that the magnetic circuit contains no flux concentration which increases the magnetic flux density above 1 T, as in this case fringe fields can no longer be neglected.

1D Field Calculation for a Conventional Quadrupole

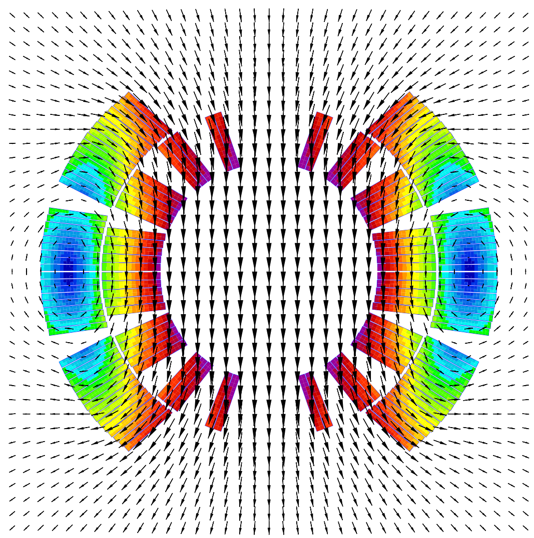


$$\oint \vec{H}\cdot d\vec{s} = \int_1 \vec{H}_1\cdot d\vec{s} + \int_2 \vec{H}_2\cdot d\vec{s} + \int_3 \vec{H}_3\cdot d\vec{s} = N\,I$$

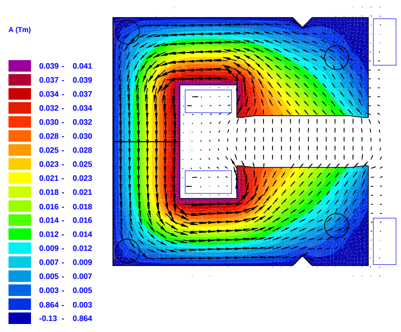
$$B_x = gy \qquad B_y = gx \qquad \Rightarrow \qquad H = \frac{g}{\mu_0}\sqrt{x^2+y^2} = \frac{g}{\mu_0}r$$

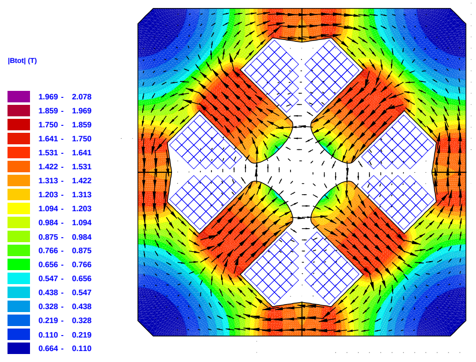
$$\int_0^{r_0} H dr = \frac{g}{\mu_0}\int_0^{r_0} r dr = \frac{g}{\mu_0}\frac{r_0^2}{2} = N\,I \qquad \Rightarrow \qquad g = \frac{2\mu_0 NI}{r_0^2}$$

Field Quality in the Dipole Aperture



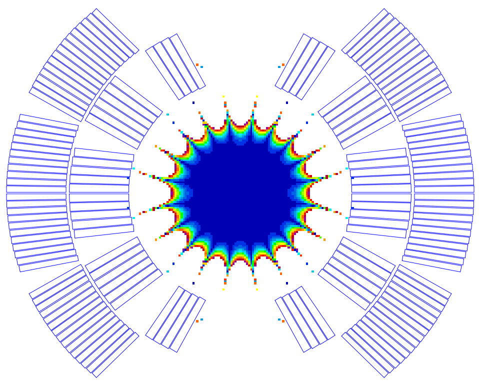
LEP Dipole and Quadrupole





Field Quality in the Dipole Aperture

$$|1 - \frac{B_y}{B_y^{\mathrm{nom}}}|$$



Definition of Field Quality in Accelerator Magnets

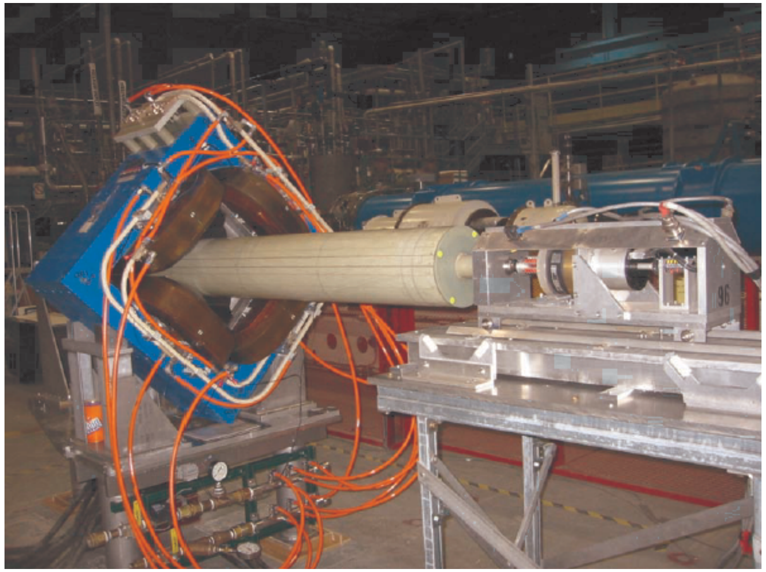
Fourier-series expansion of the radial component of the magnetic flux density on a reference radius inside the aperture

$$
\begin{aligned}
B_r(r_0,\varphi) &= \sum_{n=1}^{\infty}(B_n(r_0)\sin n\varphi + A_n(r_0)\cos n\varphi)\\
&= B_N(r_0)\sum_{n=1}^{\infty}(b_n(r_0)\sin n\varphi + a_n(r_0)\cos n\varphi)
\end{aligned}
$$

$$
A_n(r_0) \approx \frac{1}{P}\sum_{k=0}^{2P-1} B_r(r_0,\varphi_k)\cos n\varphi_k, \qquad B_n(r_0) \approx \frac{1}{P}\sum_{k=0}^{2P-1} B_r(r_0,\varphi_k)\sin n\varphi_k.
$$

Remark 3: This definition is perfectly in line with magnetic field measurements using “harmonic coils” where the periodic flux variation in tangential rotating coils is analyzed with a FFT.

Rotating Coil Measurement Setup at BNL (Brookhaven)



Maxwell’s Equations in Differential Form

$$
\begin{aligned}
\mathrm{curl}\vec{H} &= \vec{J} + \frac{\partial\vec{D}}{\partial t}\\
\mathrm{curl}\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\
\mathrm{div}\,\vec{B} &= 0\\
\mathrm{div}\,\vec{D} &= \rho
\end{aligned}
$$

Constitutive Equations

$$
\begin{aligned}
\vec{B} &= \mu\vec{H} = \mu_0(\vec{H}+\vec{M})\\
\vec{D} &= \varepsilon\vec{E} = \varepsilon_0(\vec{E}+\vec{P})\\
\vec{J} &= \kappa\vec{E} + J_{\mathrm{imp.}}
\end{aligned}
$$

Lemmata of Poincaré

The curl of an arbitrary vector field is source free $\mathrm{div}\,\mathrm{curl}\vec{g} = 0$
An arbitrary gradient field is curl free $\mathrm{curl}\,\mathrm{grad}\phi = 0$

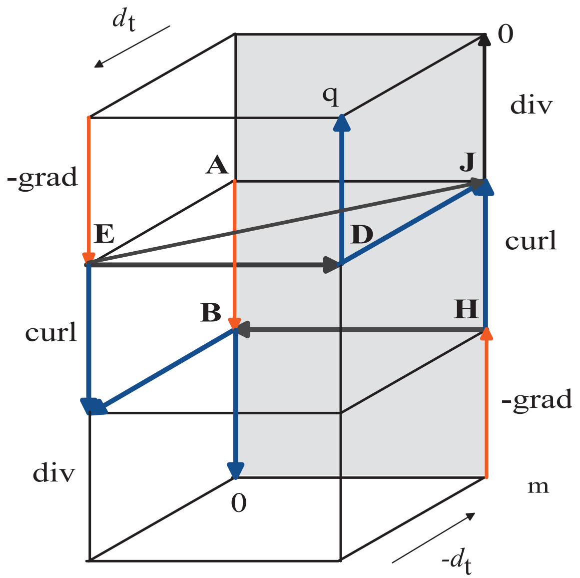
A source free field $\vec{B}$, $(\mathrm{div}\,\vec{B} = 0)$
can be expressed through a vector potential $\vec{A}$ in the form
$\vec{B} = \mathrm{curl}\vec{A}$.

A curl free field $\vec{H}$, $(\mathrm{curl}\vec{H} = 0)$
can be expressed through a scalar potential ϕ in the form $\vec{H} = -\mathrm{grad}\Phi_m$.

Remark 4: The field in the aperture of accelerator magnets is curl and source free

Warning 2: The Lemmata of Poincaré hold only for simply connected domains with connected boundaries.

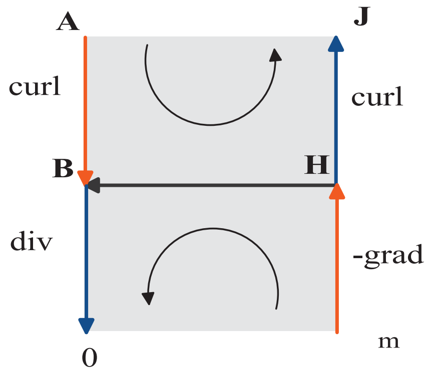
Maxwell's "House"



Warning 3: Only for sub-domains with continuous material parameters.

Maxwell's "Facade"

$$\mathrm{curl}\,\frac{1}{\mu}\mathrm{curl}\vec{A} = \vec{J} \qquad \frac{1}{\mu_0}\mathrm{curl}\,\mathrm{curl}\vec{A} = 0 \qquad \nabla^2\vec{A} - \mathrm{grad}\ \mathrm{div}\,\vec{A} = 0 \qquad \nabla^2 A_z = 0$$



$$\mathrm{div}\,\mu\,\mathrm{grad}\Phi_{\mathrm{m}} = 0 \qquad \mu_0\,\mathrm{div}\,\mathrm{grad}\Phi_{\mathrm{m}} = 0 \qquad \nabla^2\Phi_{\mathrm{m}} = 0$$

$$r^2\frac{\partial^2 A_z}{\partial r^2} + r\frac{\partial A_z}{\partial r} + \frac{\partial^2 A_z}{\partial \varphi^2} = 0$$

General Solution of the Laplace Equation

$$A_z(r,\varphi) = \sum_{n=1}^{\infty}(E_n r^n + F_n r^{-n})(G_n \sin n\varphi + H_n \cos n\varphi)$$

$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1}(C_n \sin n\varphi + D_n \cos n\varphi)$$

$$n r^{n-1} C_n = B_n \qquad n r^{n-1} D_n = A_n$$

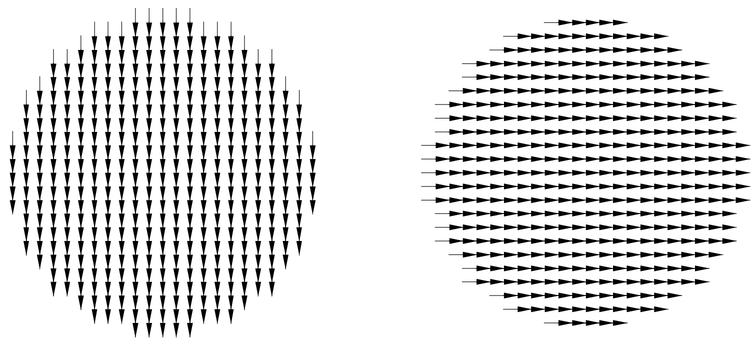
$$B_\varphi(r,\varphi) = -\frac{\partial A_z}{\partial r} = -\sum_{n=1}^{\infty} n r^{n-1}(D_n \sin n\varphi - C_n \cos n\varphi)$$

Normal (Skew) **Dipole** (n=1)

$$B_r = C_1 \cos\varphi + D_1 \sin\varphi \qquad B_\varphi = -C_1 \sin\varphi + D_1 \cos\varphi$$

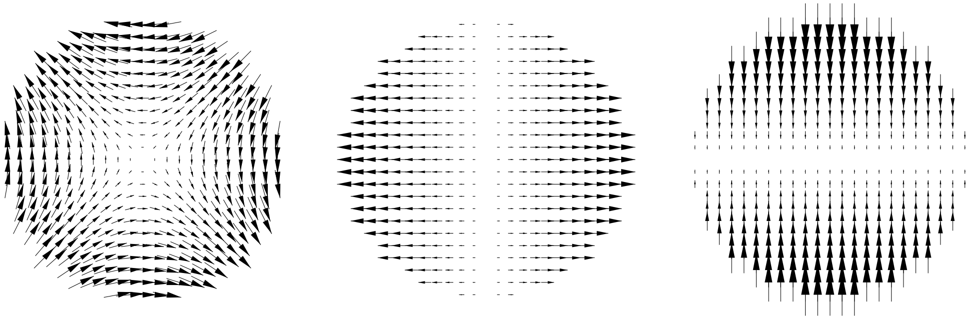
$$B_x = B_r \cos\varphi - B_\varphi \sin\varphi \qquad B_y = B_r \sin\varphi + B_\varphi \cos\varphi$$

$$B_x = C_1 \qquad B_y = D_1$$



Normal (Skew) **Quadrupole** (n=2)

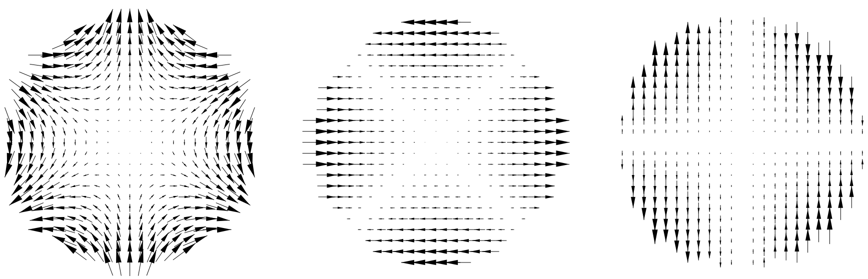
$$\begin{aligned} B_r &= 2C_2 r \cos 2\varphi + 2D_2 r \sin 2\varphi \\ B_\varphi &= -2C_2 r \sin 2\varphi + 2D_2 r \cos 2\varphi \\ B_x &= 2C_2 x + 2D_2 y \\ B_y &= -2C_2 y + 2D_2 x \end{aligned}$$



Remark 5: Notice that we have not yet addressed the problem of how to create such field distributions.

Normal (Skew) **Sextupole** (n=3)

$$\begin{aligned} B_r &= 3C_3 r^2 \cos 3\varphi + 3D_3 r^2 \sin 3\varphi \\ B_\varphi &= -3C_3 r^2 \sin 3\varphi + 3D_3 r^2 \cos 3\varphi \\ B_x &= 3C_3(x^2 - y^2) + 6D_3 xy \\ B_y &= -6C_3 xy + 3D_3(x^2 - y^2) \end{aligned}$$



Remark 6: The treatment of each harmonic separately is a mathematical abstraction. In practical situations many harmonics will be present (to be minimized).

Analytical Field Computation for Superconducting Magnets

Special solution of the inhomogeneous (Poisson) differential equation

$$\nabla^2 \vec{A} = \vec{J}$$

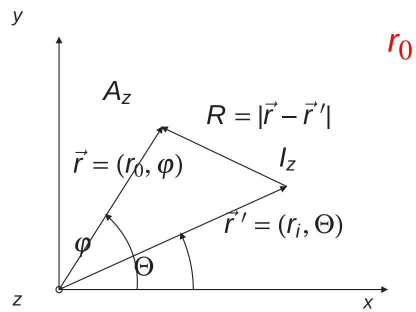
Fundamental solution of the Laplace Operator ∇^2 (Green's Function)

$$\begin{aligned} (2D) \qquad G &= \frac{1}{2\pi} ln\,|\vec{r} - \vec{r}\,'| \\ (3D) \qquad G &= \frac{1}{4\pi} \frac{1}{|\vec{r} - \vec{r}\,'|} \end{aligned}$$

Vector potential of a line current (2D)

$$A_z = -\frac{\mu_0 I}{2\pi} ln\left(\frac{|\vec{r} - \vec{r}\,'|}{a}\right)$$

The Field of Line Currents (2D)



$r_0 < r_i$

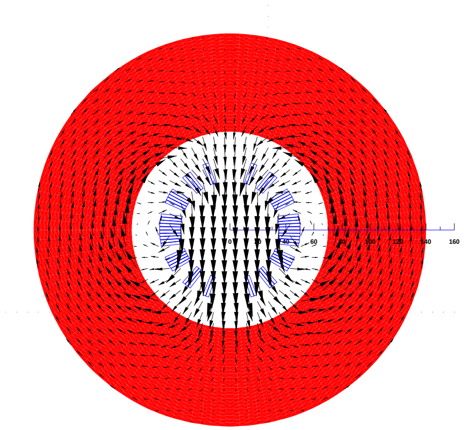
$$A_z = -\frac{\mu_0 I}{2\pi} ln\left(\frac{|\vec{r} - \vec{r}\,'|}{a}\right)$$

$$B_r == -\frac{\mu_0 I}{2\pi} \sum_{n=1}^{\infty} \left(\frac{r_0^{n-1}}{r_i^n}\right)(\sin n\varphi\, \cos n\Theta - \cos n\varphi\, \sin n\Theta)$$

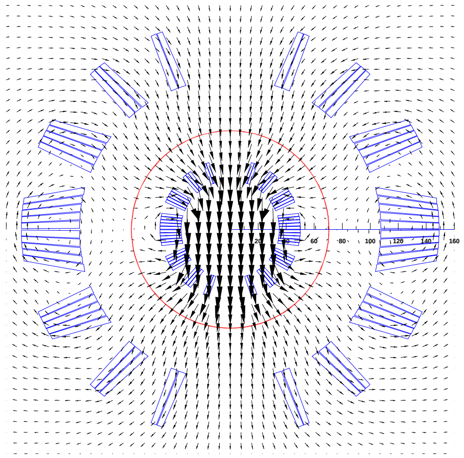
$$B_n(r_0) = -\frac{\mu_0 I}{2\pi} \frac{r_0^{n-1}}{r_i^n} \cos n\Theta \qquad A_n(r_0) = \frac{\mu_0 I}{2\pi} \frac{r_0^{n-1}}{r_i^n} \sin n\Theta$$

The Imaging Method

Coil in non-saturated yoke



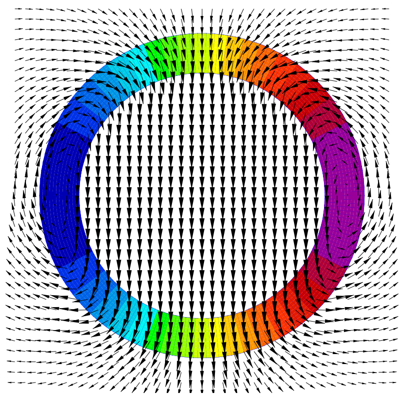
Imaged coil



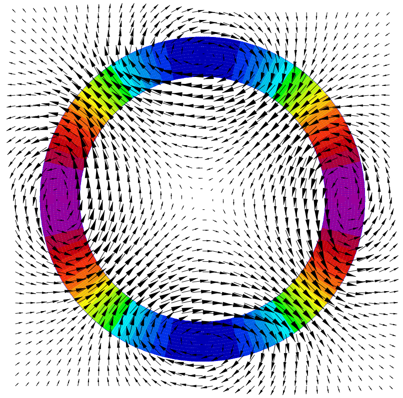
Ideal (cos n θ) Current Distribution

$$B_n(r_0) = -\sum_{i=1}^{n_s}\int_{r_i}^{r_o}\int_0^{2\pi}\frac{\mu_0 J_0 \cos m\Theta}{2\pi}\frac{r_0^{n-1}}{r_i^n}\left(1+\frac{\mu_r-1}{\mu_r+1}\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\cos n\Theta_i\, r\, d\Theta dr$$

Dipole



Quadrupole



Field Harmonics in SC Magnet

$$B_n(r_0) = -\sum_{i=1}^{n_s}\frac{\mu_0 I_i}{2\pi}\frac{r_0^{n-1}}{r_i^n}\left(1+\frac{\mu_r-1}{\mu_r+1}\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$

Scaling laws

$$B_n(r_1) = (r_1/r_0)^{n-1}B_n(r_0)$$

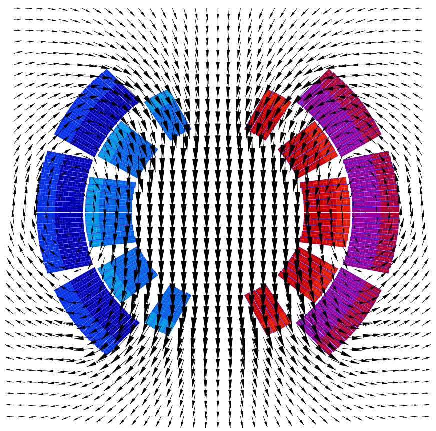
Influence of the iron yoke (non-saturated)

$r_i = 43.5$ mm, $R_{\text{Yoke}} = 89$ mm: B_1 - 19% , B_5 - 0.07%

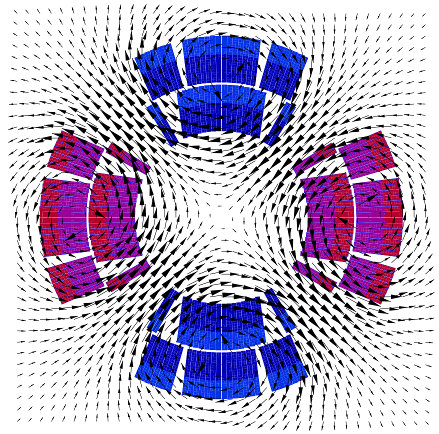
Remark 7: Analytical field optimization can be used for higher order harmonics, finite element calculations (for the estimation of saturation effects in the iron yoke) only needed for the lower order harmonics.

Ideal Current Distribution Approximated with Coil-Blocks

Dipole

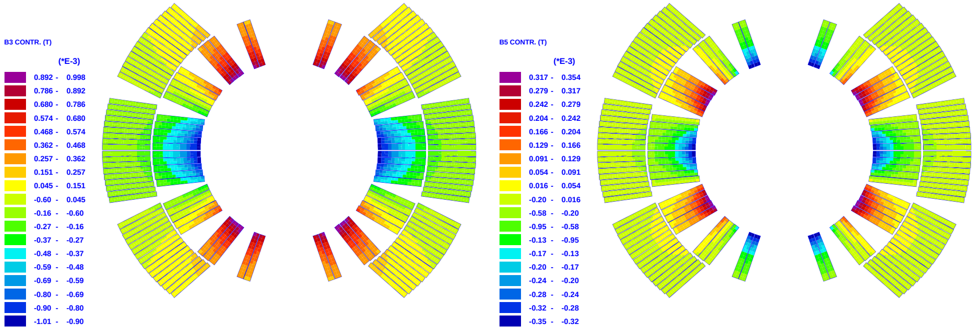


Quadrupole

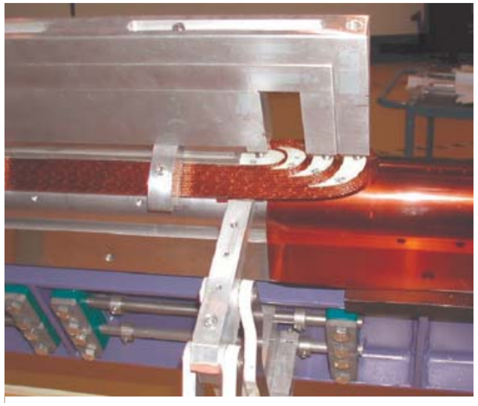


B_3 and B_5 Contribution of Strand Current

$$B_n(r_0) = -\frac{\mu_0 I_i}{2\pi}\frac{r_0^{n-1}}{r_i^n}\left(1+\frac{\mu_r - 1}{\mu_r + 1}\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$



Coil Winding



Sensitivity to Coil Block Displacements

$$B_n(r_0) = -\frac{\mu_0 I_i}{2\pi}\frac{r_0^{n-1}}{r_i^n}\left(1+\frac{\mu_r - 1}{\mu_r + 1}\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$

$$\frac{\partial B_n(r_0)}{\partial \Theta_i} = -\frac{\mu_0 I_i}{2\pi}\frac{n r_0^{n-1}}{r_i^n}\left(1+\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\sin n\Theta_i$$

$$\frac{\partial B_n(r_0)}{\partial r_i} = \frac{\mu_0 I_i}{2\pi}\frac{n r_0^{n-1}}{r_i^{n+1}}\left(1-\left(\frac{r_i}{R_{\text{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$

Increase of the azimuthal coil size by 0.1 mm produces (in units of 10^{-4}):

$b_1 = -14. \quad b_3 = 1.2 \quad b_5 = 0.03$

Specified tolerances on coils: $\pm\, 0.025$ mm

Curing Press and Curing Mold

Complex Potentials

$$\vec{H} = -\mathrm{grad}\Phi_m = -\frac{\partial \Phi_m}{\partial x}\vec{e}_x - \frac{\partial \Phi_m}{\partial y}\vec{e}_y$$

$$\vec{B} = \mathrm{curl} A_z = \frac{\partial A_z}{\partial y}\vec{e}_x - \frac{\partial A_z}{\partial x}\vec{e}_y$$

Cauchy-Riemann DEQ

$$\frac{\partial A_z}{\partial y} = -\mu_0 \frac{\partial \Phi_m}{\partial x} \qquad \frac{\partial A_z}{\partial x} = \mu_0 \frac{\partial \Phi_m}{\partial y}$$

$$W(z) = A_z(x,y) + i\mu_0 \Phi_m(x,y) \qquad z = x + iy$$

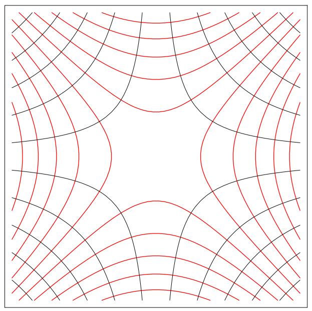
$$-\frac{\mathrm{d}W}{\mathrm{d}z} = -\frac{\partial A_z}{\partial x} - i\mu_0 \frac{\partial \Phi_m}{\partial x} = B_y(x,y) + iB_x(x,y)$$

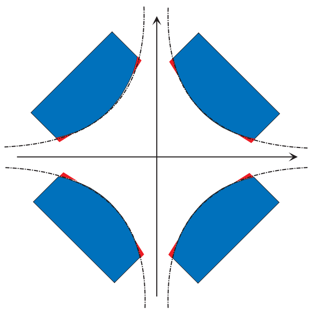
Cartesian Field Components

$$W(z) = A_z(x,y) + i\mu_0 \Phi_m(x,y) \qquad z = x + iy$$

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n)\left(\frac{z_0}{r_0}\right)^{n-1} = -\frac{\mu_0 I}{2\pi} \sum_{n=1}^{\infty} \frac{z_0^{n-1}}{z^n} \qquad |z_0| < |z|$$

Ideal (Iron) Pole Shapes





Feed down

$$\sum_{n=1}^{\infty} c_n \left(\frac{z}{r_0}\right)^{n-1} = \sum_{n=1}^{\infty} c'_n \left(\frac{z'}{r_0}\right)^{n-1} = B_y + \mathrm{i} B_x$$

$$z = z' + \Delta z \qquad z = x + \mathrm{i}y$$

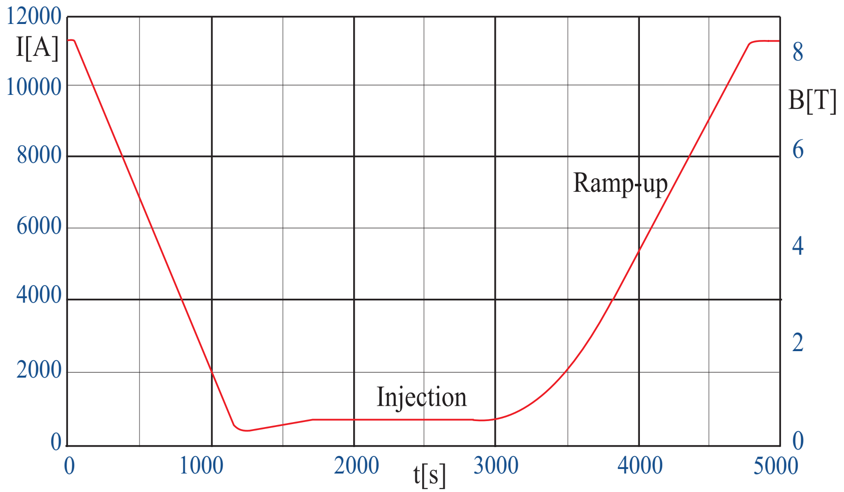
$$c'_n = \sum_{k=n}^{\infty} c_k \frac{(k-1)!}{(k-n)!\,(n-1)!} \left(\frac{\Delta z}{r_0}\right)^{k-n}$$

$$c'_2 = c_2 + 2\,c_3 \left(\frac{\Delta z}{r_0}\right) + 3\,c_4 \left(\frac{\Delta z}{r_0}\right)^2 + \ldots$$

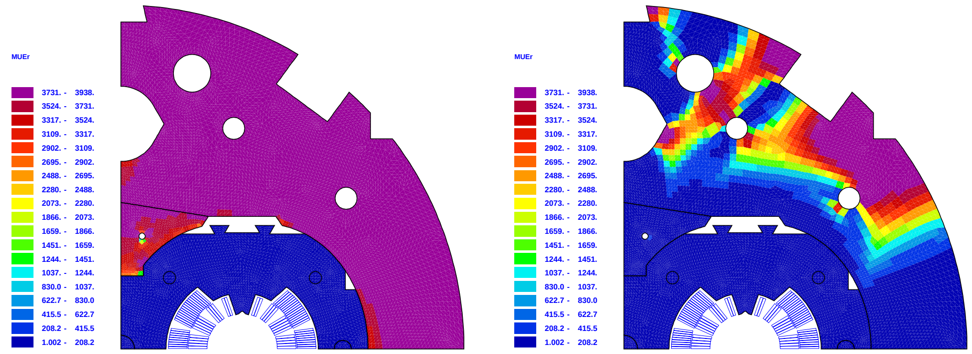
- Measurement of magnetic axis in dipole by powering the coil as a quadrupole.
- The feed down effect can be used to center the measurements coil by minimization of the b_{10} which can only occur as feed down from b_{11}.
- The dipole magnetic axis has to be well aligned with respect to the closed orbit: $\pm\,0.1$ mm for both Δx and Δy systematic and 0.5 mm for both σ_x and σ_y random (r.m.s.).
- Alignment tolerances of MCS and MCDO correctors w.r.t. MB: 0.3 mm radially

Mechanical Axis Measurement for Spool Piece Alignment

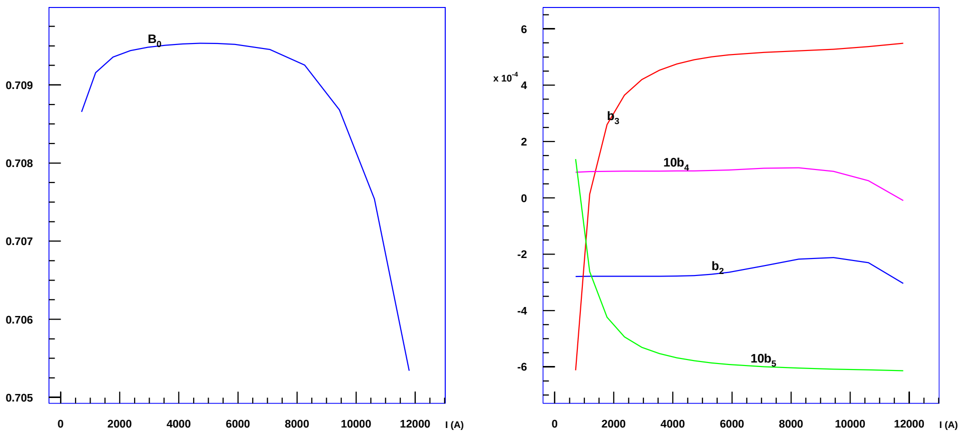
Excitation Cycle of the LHC Dipoles



Saturation Effect in LHC Dipoles



Field Variation with Excitation



Objectives for the ROXIE Development

Automatic generation of coil geometry

Field computation specially suited for magnet design (BEM-FEM)

No meshing of the coil

No artificial boundary conditions

Confine numerical errors to the iron magnetization

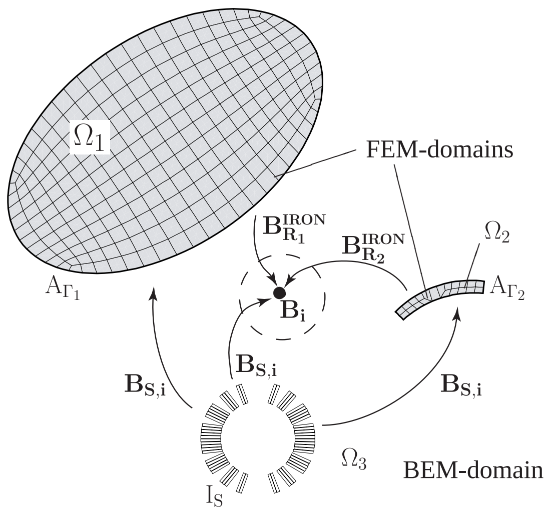
Higher-order quadrilateral finite-elements

Parametric mesh generator

Mathematical optimization

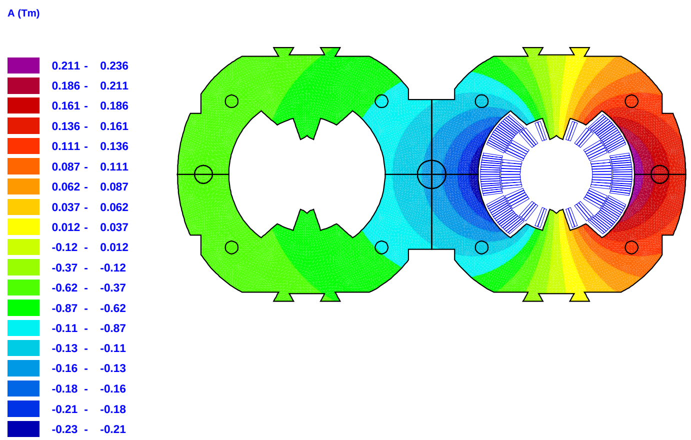
CAD/CAM interfaces

Field Computation with the BEM-FEM Coupling Method



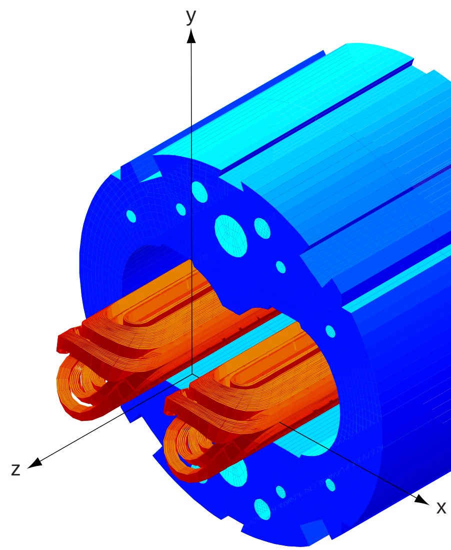
Preparation and Measurement of Collared Coils at BNN, Germany

Field Quality Calculations for Collared Coils

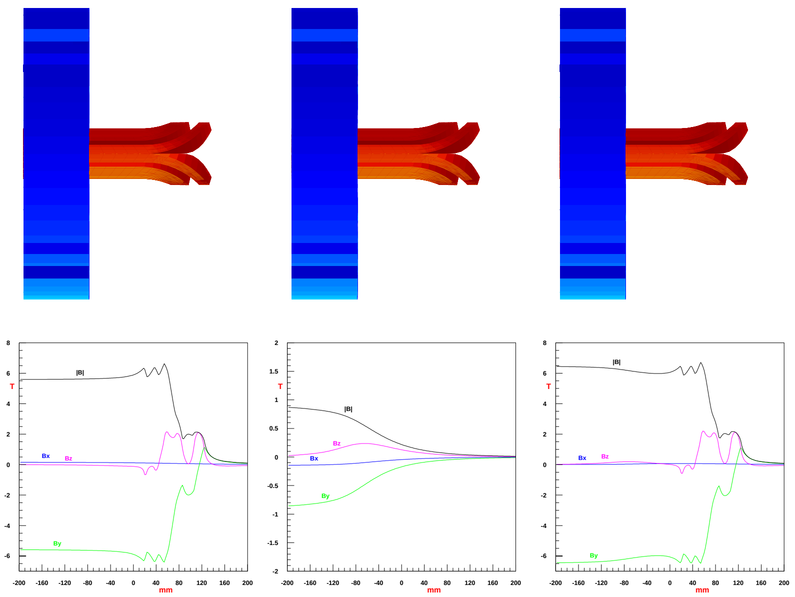


$b_2 = -0.239$ $b_3 = 2.742$ $b_4 = -0.012$ $b_5 = -0.733$

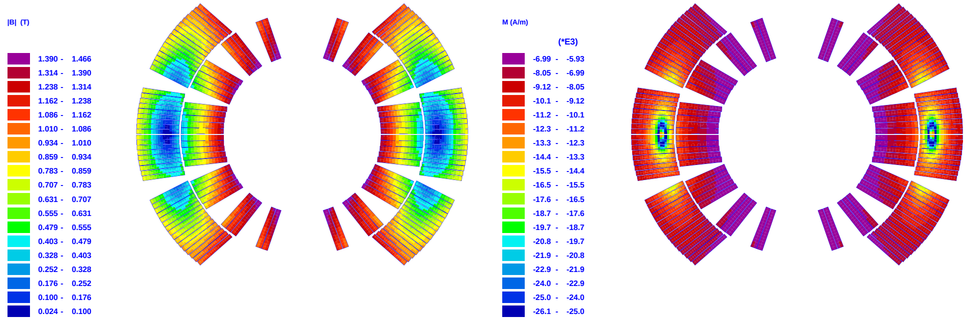
Calculation of Fringe Fields in a LHC Dipole Model



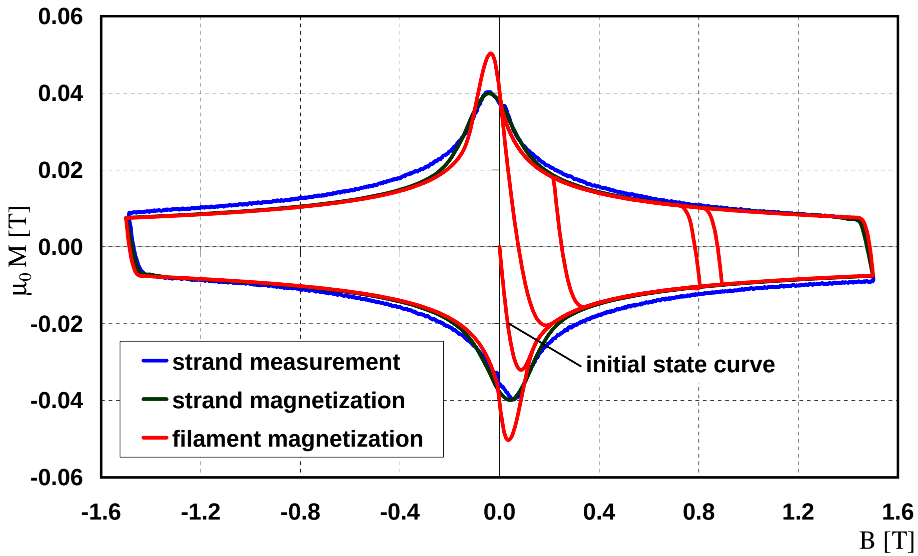
Source field **Reduced field** **Total field**



Field and Magnetization in the LHC Dipole Coil



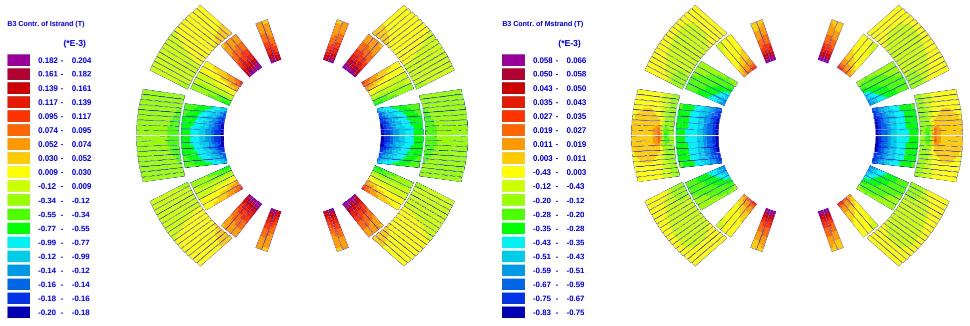
Hysteresis in Superconductor Magnetization



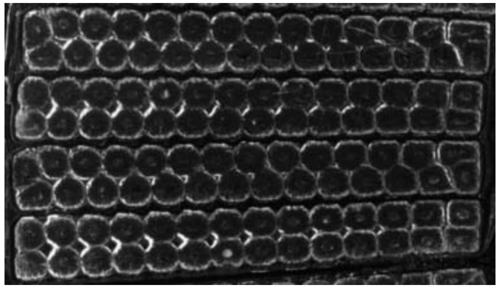
Generated B_3 Field Errors

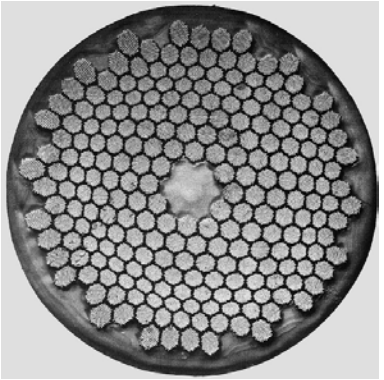
$$B_n(r_0) = -\frac{\mu_0 I_i}{2\pi}\frac{r_0^{n-1}}{r_i^n}\cos n\Theta$$

$$B_n(r_0) = \frac{\mu_0}{2\pi}\frac{r_0^{n-1}}{r_i^{n+1}}n(m_{r'}\sin n\theta + m_\theta\cos n\theta)$$



Macro-Photography of Cable and Strand





Inner (outer) layer LHC dipole coil:
Filament diameter 7 (6) μm, Strand diameter 1.065 (0.825) mm

Scaling Laws Hold for Integrated Multipoles

$$\nabla^2 \Phi_{\mathrm{m}}(x,y,z) = \frac{\partial^2 \Phi_{\mathrm{m}}(x,y,z)}{\partial x^2} + \frac{\partial^2 \Phi_{\mathrm{m}}(x,y,z)}{\partial y^2} + \frac{\partial^2 \Phi_{\mathrm{m}}(x,y,z)}{\partial z^2} = 0$$

$$\overline{\Phi}_{\mathrm{m}}(x,y) = \int_{-z_0}^{z_0} \Phi_{\mathrm{m}}(x,y,z)\mathrm{d}z$$

$$\nabla^2 \overline{\Phi}_{\mathrm{m}}(x,y) = \frac{\partial^2 \overline{\Phi}_{\mathrm{m}}(x,y)}{\partial x^2} + \frac{\partial^2 \overline{\Phi}_{\mathrm{m}}(x,y)}{\partial y^2} = 0$$

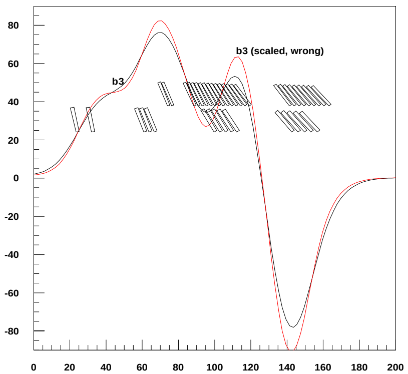
Sufficient condition: Integration path is extended far enough away from (into) the magnet so that the axial component of the field has dropped to zero.

$$\frac{\partial^2 \overline{\Phi}_{\mathrm{m}}}{\partial x^2} + \frac{\partial^2 \overline{\Phi}_{\mathrm{m}}}{\partial y^2} = \int_{-z_0}^{z_0} \left(\frac{\partial^2 \Phi_{\mathrm{m}}}{\partial x^2} + \frac{\partial^2 \Phi_{\mathrm{m}}}{\partial y^2} \right) \mathrm{d}z = \int_{-z_0}^{z_0} \left(- \frac{\partial^2 \Phi_{\mathrm{m}}}{\partial z^2} \right) \mathrm{d}z = - \left. \frac{\partial \Phi_{\mathrm{m}}}{\partial z} \right|_{-z_0}^{z_0} = H_z|_{-z_0} - H_z|_{z_0}$$

Warning 4:

Scaling Laws for Multipoles are Wrong in 3 Dimensions

$$B_n(r_1) = (\frac{r_1}{r_0})^{n-1} B_n(r_0) \qquad b_n(r_1) = (\frac{r_1}{r_0})^{n-N} b_n(r_0)$$



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