Outline

• Overview

• Insertion Device e-Beam Interaction

• Radiation Properties of Insertion Devices

• Insertion Device Technology
History of Insertion Devices

1947  First discussion of undulator radiation by Ginzburg

1951 / 1953  First production of undulator light in the mm and visible regime by Motz et al.

1976  FEL radiation from a superconducting helical undulator at Stanford: Madey et al.

1979 / 1980  first operation of insertion devices in storage rings (SSRL, LURE, VEPP3)

1980...  first operation of wavelength shifters in storage rings (VEPP3, SRS, VEPP2M)

today  about a dozen of 3rd generation synchrotron radiation light sources; SASE FELs in the visible and UV (80nm) regime

future  SASE-FELs for the energy regime up to 10 keV
Synchrotron Radiation Integrals

\[I_1 = \int \frac{\eta(s)}{\rho} ds\]
\[I_2 = \int \frac{1}{\rho^2} ds\]
\[I_3 = \int \frac{1}{|\rho|^3} ds\]
\[I_{3a} = \int \frac{1}{\rho^3} ds\]
\[I_4 = \int \frac{(1 - 2n(s))\eta(s)}{\rho^3} ds\]
\[H(s) = \frac{1}{\beta} [\eta^2 + <\beta\eta^\prime - 0.5\beta^\prime \eta>]\]
\[n(s) = \rho^2 \frac{\partial}{\partial x} (1/\rho)\]

Beam Parameter Dependence on SR-Integrals

\[\Delta E = \frac{2}{3} r_e \frac{E^4}{3(mc^2)^3} I_2\]

energy loss per revolution

\[\left(\frac{\sigma E}{E}\right)^2 = C_q \gamma^2 \frac{I_3}{2I_2 + I_4}\]

energy spread

\[\epsilon = C_q \gamma^2 \frac{I_5}{I_2 - I_4}\]

emittance

\[\tau_i = 3T_0 / r_0 \gamma^3 J_i I_2\]

damping times

\[i = x, z, \epsilon\]

\[J_x = 1 - I_4 / I_2\]

\[J_z = 1\]

\[J_\epsilon = 2 + I_4 / I_2\]

damping partition numbers

\[1/\tau_p = \frac{5\sqrt{3}}{8} \frac{\hbar r_e (\frac{E_0}{mc^2})^5}{m} \frac{I_3}{2\pi R}\]

polarization time

\[P_{max} = \frac{8}{5\sqrt{3}} \frac{I_{3a}}{I_3}\]

degree of polarization
Fokussing of Insertion Devices

Equation of motions in linear optics

\[
x''(s) + \left(\frac{1}{\rho^2(s)} - \kappa(s)\right) \cdot x(s) = 0
\]

\[
y''(s) + \kappa(s) \cdot y(s) = 0
\]

averaged focussing terms

\[
< K_x(s) > = < (1/\rho^2(s) - \kappa(s)) >
\]

\[
< K_y(s) > = < \kappa(s) >
\]

evaluate &\zeta along curved trajectory

\[
\kappa = \frac{e}{\gamma mc} \frac{\partial B_y}{\partial \xi}
\]

\[
= \frac{e}{\gamma mc} \frac{\partial B_y}{\partial y}
\]

\[
= \frac{e}{\gamma mc} \left[ \frac{\partial B_y}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial B_y}{\partial s} \cdot \frac{\partial s}{\partial \xi} \right]
\]

\[
\approx \frac{e}{\gamma mc} \left[ \frac{\partial B_y}{\partial x} - \frac{\partial B_y}{\partial s} \cdot x' \right]
\]

using Halbach magnetic fields

\[
B_x = \frac{k_x}{k_y} B_0 \cdot sinh(k_x x) \cdot sinh(k_y y) \cdot cos(ks)
\]

\[
B_y = B_0 \cdot cosh(k_x x) \cdot cosh(k_y y) \cdot cos(ks)
\]

\[
B_s = -\frac{k}{k_y} B_0 \cdot cosh(k_x x) \cdot sinh(k_y y) \cdot sin(ks)
\]

with \( k_x^2 + k_y^2 = k^2 \) (Maxwell)

focussing strength:

\[
< K_x > = \frac{k_x^2}{2 \rho_0^2 k^2}
\]

\[
< K_y > = \frac{k_y^2}{2 \rho_0^2 k^2}
\]
Tracking of Particles in Undulator Fields

1: Expand \( x(s) \), \( y(s) \) and \( B \) with respect to initial coordinates and \( 1/\Box \) (\( =x3 \))

\[
\begin{align*}
  x(s) &= x_i + s \cdot x'_i + \sum_{k,l,m} a_{klm} (x_i, y_i, s) \cdot x'_i^k \cdot y'_i^l \cdot x_3^m \\
  y(s) &= y_i + s \cdot y'_i + \sum_{k,l,m} b_{klm} (x_i, y_i, s) \cdot x'_i^k \cdot y'_i^l \cdot x_3^m
\end{align*}
\]

\[B(x(s), y(s), s) = B(x_i, y_i, s) + \sum_{kl} \frac{1}{k!l!} \frac{\partial^2 B(x_i, y_i, s)}{\partial x^k \partial y^l} \cdot \Delta x^k \cdot \Delta y^l\]

2: Insert \( x(s) \), \( y(s) \) and \( B \) into equations of motion and determine \( a_{klm} \) and \( b_{klm} \)

3: change to canonical coordinates and modify transformation

\[
\begin{align*}
  q_x &= x \\
  p_x &= A^x/(B\rho) + x'\sqrt{1 + x'^2 + y'^2} \\
  q_y &= y \\
  p_y &= A^y/(B\rho) + y'\sqrt{1 + x'^2 + y'^2}
\end{align*}
\]

\[
(\begin{align*}
  q_x, p_x, q_y, p_y \\
  \text{old}
\end{align*}) \quad \Rightarrow \quad (\begin{align*}
  q_x, p_x, q_y, p_y \\
  \text{new}
\end{align*})
\]

4: set up generating function and get canonical transformation from derivatives

\[
F = F_{00} + F_{10} \cdot px_f + F_{01} \cdot py_f + F_{20} \cdot px_f^2 + F_{11} \cdot px_f \cdot py_f + F_{02} \cdot py_f^2
\]

\[
\begin{align*}
  qx_f &= \frac{\partial F}{\partial px_f} \\
  px_i &= \frac{\partial F}{\partial qx_i}
\end{align*}
\]
Radiation Emitted by Accelerated Charged Particles

acceleration fields as derived from the Lienard Wiechert potentials:

\[
E(t) = \frac{e}{4\pi \epsilon_0 c} \cdot \frac{1}{R} \cdot \left[ \hat{n} \times [(\hat{n} - \hat{\beta}) \times \hat{\beta}] \right]_{\text{ret}} \frac{1}{(1 - \hat{\beta} \cdot \hat{n})^3}
\]

retarded time \( t' = t - R(t')/c \)

\[
\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \left[ \hat{n} \times [(\hat{n} - \hat{\beta}) \times \hat{\beta}] \right]_{\text{ret}} e^{i\omega t} dt \right|^2
\]

far field approximation:

\[
R(t') \approx R_0(t') - \hat{n}_0 \cdot \vec{r}(t') \\
\hat{n} = \hat{n}_0
\]

\[
\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \left[ \hat{n} \times [(\hat{n} - \hat{\beta}) \times \hat{\beta}] \right] e^{i\omega (t-\vec{n}\vec{r})} dt \right|^2
\]
Bending Magnets

\[
\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{3e^2}{16\pi^3 c \epsilon_0 y^2 \gamma^2 (1 + X^2)^2} \left| K_{2/3}(\xi), i \sqrt{\frac{X^2}{1 + X^2}} K_{1/3}(\xi) \right|^2
\]

with

\[
\xi = \frac{y}{2} \left(1 + (\gamma \theta_y)^2\right)^{3/2}
\]

\[
y = \frac{\omega}{\omega_c}
\]

\[
X = \gamma \theta_y
\]

\[
\omega_c = \frac{3\gamma^3 c}{2\rho}
\]

Polarization

\[
\Delta E_x \sim -\dot{\beta}_x \cdot (n_y^2 + n_s^2) + \dot{\beta}_s \cdot n_x n_s
\]

\[
\Delta E_y \sim \dot{\beta}_x \cdot n_x n_y + \dot{\beta}_s \cdot n_y n_s
\]

on axis flux density

\[
\frac{\partial^2 \tilde{F}}{\partial (\Delta \omega/\omega) \partial \Omega} = 1.327 \cdot 10^{13} \cdot E^2 (GeV) \cdot I(A) \cdot H_2
\]

integration over vertical angle

\[
\frac{\partial^2 \tilde{F}}{\partial (\Delta \omega/\omega) \partial \theta_x} = 2.457 \cdot 10^{13} E (GeV) \cdot I(A) \cdot G_1(y)
\]
Undulator Radiation

resonance condition

$$\lambda = \frac{\lambda_0}{2\gamma^2} (1 + K^2/2 + \gamma^2\theta^2)$$

$$K = 93.4 \cdot \lambda_0 \cdot B_0$$

figure 8 motion in moving frame produces higher harmonics

$$x(t) = \frac{Kc}{\gamma\omega_u} \sin(\omega_u t)$$

$$s(t) = \beta ct - \frac{K^2 c}{8\gamma^2\omega_u} \sin(2\omega_u t)$$
Analytical Approach for Undulator Radiation

\[ \frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{e^2 \gamma^2 N^2}{4\pi \epsilon_0 c} \cdot F_n(K_x, K_y, \gamma \theta, \phi) \cdot \frac{\sin^2(N\pi \frac{\Delta \omega}{\omega_1(\theta)})}{N^2 \sin^2(\pi \frac{\Delta \omega}{\omega_1(\theta)})} \]

Fn represents an infinite sum over BESSEL functions. The last term is called the line shape function and describes the interference effects.

The angular divergence and the spectral width can be derived from the line shape function

**divergence** \[ \sigma_{\gamma'} = \sqrt{\frac{\lambda}{2L}} \]

**spectral width** \[ \frac{\Delta \omega}{\omega_n} = \frac{1}{nN} \]
Useful Equations in Practical Units

on axis flux density

\[ \frac{\partial^2 \tilde{F}}{\partial (\Delta \omega/\omega) \partial \Omega} = 1.744 \cdot 10^{14} \cdot N^2 \cdot E^2 (GeV) \cdot I(A) \cdot F_n(K) \]

flux over the central cone

\[ \frac{\partial \tilde{F}'}{\partial (\Delta \omega/\omega)} = 1.431 \cdot 10^{14} \cdot N \cdot Q_n \cdot I(A) \]

\[ Q_n = (1 + K^2/2) \cdot F_n/n \]
Brightness (Wigner, K.-J. Kim)

\[
B_0(\vec{x}, \vec{\Phi}) = c \cdot \int d^2\xi \cdot A(\vec{x}, \vec{\xi}) \cdot \exp\left(\frac{2\pi i}{\lambda} \cdot \vec{\Phi} \cdot \vec{\xi}\right),
\]

\[
A(\vec{x}, \vec{\xi}) = E_y^*(\vec{x} + \vec{\xi}/2) \cdot E_y(\vec{x} - \vec{\xi}/2) + E_z^*(\vec{x} + \vec{\xi}/2) \cdot E_z(\vec{x} - \vec{\xi}/2).
\]

The brightness is not positive definite.
Physical quantities are the angular or spatial flux density, which are derived via integration of the brightness in space or solid angle.
The electron beam emittance can be convoluted with the 4D-brightness.

Assuming a angular and spatial Gaussian distribution of the photon beam the brightness can be evaluated from:

\[
\frac{\partial^3 \tilde{F}_e}{\partial (\Delta\omega/\omega) \partial x \partial \Omega} = \frac{\partial^2 \tilde{F}_e}{\partial (\Delta\omega/\omega) \partial \Omega} \cdot \frac{\sigma_r^2}{\sqrt{\sigma_r^2 + \sigma_x^2} \sqrt{\sigma_r^2 + \sigma_y^2} \sqrt{\sigma_r^2 + \sigma_x^2} \sqrt{\sigma_r^2 + \sigma_y^2}}
\]

The beam size can be approximated with:

\[
\sigma_r = \frac{1}{\pi \sqrt{2}} \frac{\gamma L}{\pi \sqrt{2}}
\]
Polarization

\[ F_n(K_x, K_y, \gamma \theta, \gamma \phi) = \frac{n^2}{(K_x^2/2 + K_y^2/2 + (\gamma \theta^2))^2} |A_x, A_y|^2 = a \cdot (A_x^2 + A_y^2) \]

Planar Undulator, K=1

Definition of Stokes Parameters

\[ S_0 = E_x^2 + E_y^2 \]
\[ S_1 = E_x^2 - E_y^2 \]
\[ S_2 = 2 \cdot E_x E_y \cos(\delta) \]
\[ S_3 = 2 \cdot E_x E_y \sin(\delta) \]

\[ S_1^2 + S_2^2 + S_3^2 \leq S_0^2 \]
Polarization Properties

Planar Devices

Helical Devices

rel. polarized flux  degree of polarization

rel. polarized flux  degree of pol.
Sources of Brightness Degradation

Undulator errors

Beam parameters

Beam emittance: 6.e-9 m rad
\( \delta x = 0.94 \) m
\( \delta y = 2.1 \) m

Energy spread: 1.e-3

\[ R = \frac{1 - e^{-\sigma^2 \phi N^2}}{N^2} \]

\[ \Delta \Phi = \frac{2\pi}{\beta \lambda (B\rho)^2} \cdot \int_0^z \left[ \int_0^{z'} B_y^{\text{int}} dz'' \cdot \int_0^{z'} B_y^{\text{res}} dz'' \right] dz' \]

\[ + \frac{1}{\beta \lambda (B\rho)^2} \cdot \frac{0.5}{\lambda} \cdot \int_0^z \left[ \int_0^{z'} B_y^{\text{res}} dz'' \cdot \int_0^{z'} B_y^{\text{res}} dz'' \right] dz' \]

black: without emittance, energy spread
red: emittance included
blue: energy spread included
magenta: emittance and energy spread incl.
Angular Flux Density of Insertion Devices

\[ \frac{\partial P}{\partial \Omega} (W/mrad^2) = 0.01344 \cdot E(GeV)^2 \cdot I(A) \cdot N \cdot \int_{-\lambda_0/2}^{\lambda_0/2} \left[ \frac{v_x'^2 + v_y'^2}{D^3} - \frac{(v_x'^2 + v_y'^2)}{D^5} \right] ds \]

\[ D = 1 + v_x^2 + v_y^2 \]

\[ v_{x/y} = \gamma(\beta_{x/y} - \theta_{x/y}) \]

energy=1.7GeV, current=0.1A, N=100, \( \bullet = 50\)mm

Devices with low on axis power:

helical device, \( K_{eff}=4 \)

figure-8 undulator

angular flux density

\( K_x/K_y=0, 0.25, 0.5, 0.75, 1.0 \)
Asymmetric Wiggler

Elliptical Wiggler


Elliptical / Helical Undulators

Permanent magnet devices

Various types of planar helical devices

ESRF

SPRING-8

ELETTRA

APPLE II provides highest fields

electromagnetic devices

+ fast helicity switching
+ mechanically simple
- limited to long periods
- weak fields

Advanced Photon Source
Fast Helicity Switching with Double Undulators

**SPRING-8**
- dynamic electron orbit bump,
- angular separation

**SLS**
- static displacement,
- separation in focal plane

**BESSY**
- static angle,
- angular separation
BESSY UE56 Double Undulator for fast Helicity Switching
IDs with different states of polarization at different harmonics
+ polarization switching without complicated mechanics
+ suitable for in vacuum applications
- less flux
- slow switching frequency

period length of vert. field half the value of hor. field
relative phase = 0 deg. \(\theta\) parabolic undulator
relative phase = 90 deg. \(\theta\) figure-8 undulator
asymmetric figure-8 und.


figure-8 undulator
alternatively horizontal and vertical polarization at successive harmonics

asymmetric figure-8 undulator
up to 80% circular polarization in certain harmonics

parabolic undulator
off axis circularly polarized light
Quasiperiodic Undulators

generate 1D-quasiperiodic lattice

ESRF / ELETTRA design

BESSY design

spectra derived from measured magn. fields

\[ x_m = \frac{d}{r \cdot \tan(\alpha)} \cdot (m + (r \cdot \tan(\alpha) - 1) \left[ \frac{\tan(\alpha)}{r + \tan(\alpha)} m + 1 \right]) \]

\[ r = \frac{b}{a} \]
Small Period Devices

in vacuum undulators

Complicated but mature technique

- Coating of magnets to reduce outgassing
  Ti+TiN ion plating of NdFeB magnets (SPRING8)
- high coercive magnetic material (bakeout at 125°)
- thin metal sheet to reduce image current heating
  (50Ωm Ni + 10Ωm Cu)
- water cooled RF-fingers
- special shimming techniques

in vacuum revolver (SPRING 8)

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superconducting undulators
under development

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NUS-ID

\[ B = 1.3 \text{ Tesla} \]
\[ \bullet = 14\text{mm} \]
\[ \text{gap} = 5\text{mm} \]
\[ 50 \text{ periods} \]
\[ \varphi = 5.7^\circ \]

(nach Dipolkorr.)
High Field Devices

**non superconducting**

**Hybrid wiggler**
- B = 2 Tesla
  - (many SR-facilities)

**Asymmetric wiggler**
- 3.1 Tesla
  - 11m gap, 378mm
  - (ESRF)

**5 T WLS**
- Perm. magn. + coils
  - (Budker Institute)

**superconducting**

**HMI Multipole wiggler (BESSY)**

**3.5 Tesla wiggler**
- 46 poles, 61mm
  - gap=10.2mm
  - (MAX-Lab, ELETTRA)

**Superbends**
- ALS,
  - 2 years of operation
Long Undulators

spontaneous emission

SPRING-8
25 m ID
in vacuum

stimulated emission (SASE)

saturation demonstrated:
VISA 800 nm
LEUTL 300 nm
TTF 80 nm

projects:
TESLA 0.1 nm
LCLC 0.15 nm
SCSS SPRING8 3.6 nm
BESSY 1.2 nm