CERN ACCELERATOR SCHOOL
Power Converters

Passive components

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Overview

- **Part 1:** Inductors (to be designed)
- **Part 2:** Capacitors (to be selected)
- **Part 3:** A new component: The Supercapacitor, component and applications
Inductors

Overview, typical applications

- AC-applications
- DC applications
- Filtering
- Smoothing (limiting di/dt)
- Components of resonance circuit

References:
Inductors

- 2 main types:
  - Air inductors
  - Inductors with magnetic core

**Solenoid (air)**

\[ L = N^2 \mu_0 A / l \]

- A: area of coil
- L: length of coil
- N: number of turns

**Toroid (core)**

\[ L = N^2 \mu A / \pi d_m \]

- A: section of core
- \( d_m \): mean diameter of tore
- N: number of turns
Toroidal inductor

Permeance of a magnetic circuit is defined as the reciprocal of its reluctance:

$$\mathcal{P} = \frac{1}{\mathcal{R}}$$

$$l = \pi \ast d_m$$

Mean path length $l$

Cross-sectional area $A$

Permeability $\mu$

$$\mathcal{R} = \frac{l}{\mu A} \quad \phi = \frac{Ni}{\mathcal{R}}$$

Figure 3-14 Magnetic reluctance.
Inductors

- Main parameters of inductors
  - Inductance
  - Quality factor
  - Capacity
  - Rated current

- Equivalent scheme

\[ \begin{align*}
L & : \text{Inductance} \\
R_a & : \text{Losses related to AC current component} \\
R_c & : \text{Resistance of winding} \\
C & : \text{Capacity of winding}
\end{align*} \]
Inductors

Relations

For: \( \omega^2 LC \ll 1 \)

\[ Z \approx R' + j\omega L' \]

with

\[ R' = R_c + R_a / (1 + Q_a^2) \]

\[ Q_a = R_a / \omega L \]

if \( Q_a^2 \gg 1 \)

\[ L' \approx L \]

\[ R' \approx R_c + \omega^2 L^2 / R_a \]
Inductors

Factor of losses and quality factor

for \( Q_a^2 \gg 1 \)

\[
\tan \delta = R' / \omega L' \cong R_c / \omega L + \omega L / R_a
\]

\[
\tan \delta = \tan \delta_c + \tan \delta_a
\]

\[
Q = 1 / \tan \delta = \omega L' / R'
\]

Important factor for resonant circuits
Inductors

• Magnetic materials and cores

  – 2 main classes of materials
  1) Iron based
     • Alloys of iron with chrome and silicon (small amounts)
       ⇒ Electrical conductivity
       ⇒ Large value of saturation limit
     • Powdered iron cores (small iron particles isolated from each other)
       ⇒ Greater resistivity, smaller eddy current losses
       ⇒ Suited for higher frequencies
     • Amorphous alloys of iron with other transition metals (METGLAS)
Inductors

- Magnetic materials and cores

2) Ferrites

Oxide mixtures of iron and other magnetic elements

⇒ Large electrical resistivity
⇒ Low saturation flux density (0.3T)
⇒ Have only hysteresis losses
⇒ No significant eddy current losses
Inductors

- Hysteresis losses

\[
P_{m,sp} = kf^a (B_{ac})^d
\]

(specific loss)

k, a, d, constants depending from the material

Loss increase with f and with B\textsubscript{ac}

\[
B_{ac} = \hat{B}
\]

If no time average

\[
B_{ac} = \hat{B} - B_{avg}
\]

If time average
Figure 30-1  Magnetic flux density waveforms having (a) no time average and (b) with a time average.
Inductors

Example of ferrite material (3F3)

\[
P_{m,sp} = 1.5 \times 10^{-6} f^{1.3} (B_{ac})^{2.5}
\]

\[P_{m,sp}\] in mW/cm\(^3\) when \(f\) in kHz and \(B_{ac}\) in mT

For METGLAS:

\[
P_{m,sp} = 3.2 \times 10^{-6} f^{1.8} (B_{ac})^{2}
\]

For 100 kHz and 100 mT:
\[P_{m,sp} = 127\text{mW/cm}^3\]
Inductors

- **Empirical performance factor** \( \text{PF} = f \cdot B_{ac} \)

*Figure 30-3* Empirical performance factor \( \text{PF} = fB_{ac} \) versus frequency for various ferrite core materials. Measurements are made at a power density \( P_{\text{core}} = 100 \text{ mW/cm}^3 \).
Inductors

- $P_{m,sp}$ depends finally on how efficiently the heat dissipated is removed.

- $P_{m,sp}$ is even smaller because of presence of eddy current loss.
Inductors

- **Skin effect limitations (in core)**
  - If conducting material is used: circulation of currents when the magnetic field is time-varying (eddy currents)
  - The magnetic field in the core decays exponentially with distance into the core

\[
B(y) = B_0 e^{-y/\delta}
\]

Skin depth: \(\delta\)
\[
\delta = \sqrt{2/\omega \mu \sigma}
\]

\(\omega = 2\pi f\)
\(\mu : \text{permeability}\)
\(\sigma : \text{conductivity}\)
Inductors

Typical value of skin depth

(Material with large permeability)

1 mm at 60 Hz!

> Thin laminations with each isolated from the other

> Stacking factor (0.9…0.95)

Materials with increased resistivity: increase of skin depth but reduces the magnetic properties

Reasonable compromise for transformers (50/60 Hz): Iron alloy, 97% iron, 3% silicon) and a lamination thickness of 0.3 mm
Inductors

- Example of stacking steel laminations

Figure 30-5  Magnetic core for a transformer or inductor made from a stack of magnetic steel laminations separated by insulators.
Inductors

- Eddy current loss in laminated cores

Specific eddy current loss (estimated optimistic minimum)

\[ P_{ec,sp} = \frac{d^2 \omega^2 B^2}{24 \rho_{core}} \]

- \( d \): thickness of the lamination
- \( d < \delta \) (skin depth)
- \( B(t) = B \sin(\omega t) \)
Inductors

- Core shapes and optimum dimensions

Cross-sectional area of the bobbin: \[ A_w = h_w \cdot b_w \]

Widely used core: Double-E core
\[ b_a = a, \quad d = 1.5a, \quad h_a = 2.5a, \quad b_w = 0.7a, \quad h_w = 2a \]

Figure 30-6  Dimensioned diagram of (a) a double-E core (b) bobbin, and (c) assembled core with winding.
Inductors

- Geometric characteristics of a near optimum core for inductors / transformer

Table 30-1 Geometric Characteristics of a Near Optimum Core for Inductor/Transformer Design

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Relative Size</th>
<th>Absolute Size for ( a = 1 \text{ cm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core area ( A_{\text{core}} )</td>
<td>( 1.5a^2 )</td>
<td>1.5 \text{ cm}^2</td>
</tr>
<tr>
<td>Winding area ( A_w )</td>
<td>( 1.4a^2 )</td>
<td>1.4 \text{ cm}^2</td>
</tr>
<tr>
<td>Area product ( AP = A_w A_c )</td>
<td>( 2.1a^4 )</td>
<td>2.1 \text{ cm}^4</td>
</tr>
<tr>
<td>Core volume ( V_{\text{core}} )</td>
<td>( 13.5a^3 )</td>
<td>13.5 \text{ cm}^3</td>
</tr>
<tr>
<td>Winding volume ( V_w )</td>
<td>( 12.3a^3 )</td>
<td>12.3 \text{ cm}^3</td>
</tr>
<tr>
<td>Total surface area of assembled</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inductor/transformer (^b)</td>
<td>( 59.6a^2 )</td>
<td>59.6 \text{ cm}^2</td>
</tr>
</tbody>
</table>
**Inductors**

- **Copper windings**
  
  Advantages of copper: high conductivity, easy to bend
  - single round wire
  - Litz-wire diameter of each strand: a few hundred of microns
    (skin effect in copper)

  Copper fill factor

  \[ k_{cu} = \frac{NA_{Cu}}{A_w} \]

  from 0.3 (Litz) to 0.5..0.6 for round conductors
Inductors

- Power dissipated in the winding (specific)

\[ P_{Cu,sp} = \rho_{Cu} (J_{rms})^2 \]

or

\[ P_{w,sp} = \kappa_{Cu} \rho_{Cu} (J_{rms})^2 \]

\[ J_{rms} = \frac{I_{rms}}{A_{Cu}} \]
Inductors

- **Skin effect in copper windings**

  Circulating winding current > magnetic field > eddy currents
  ➢ The eddy currents « shield » the interior of the conductor from the applied current

*Figure 30-8*  Isolated copper conductor carrying (a) a current $i(t)$, (b) eddy currents generated by the resulting magnetic field, and (c) the consequences of the skin effect on the current distribution.
Inductors

- Skin depth: $\delta$

<table>
<thead>
<tr>
<th>Frequency</th>
<th>50Hz</th>
<th>5kHz</th>
<th>20kHz</th>
<th>500kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>10.6 mm</td>
<td>1.06 mm</td>
<td>0.53 mm</td>
<td>0.106 mm</td>
</tr>
</tbody>
</table>

Skin depth in Copper at 100°C for several different frequencies
Inductors

- **Thermal considerations**

  Temperature increase of core and winding:
  - degrades the performance of the materials
  - The resistivity of the copper winding increases and so the loss increases
  - The value of the saturation flux density decreases

  It is important to keep the core and winding temperature under a maximum value

  In practice 100-125°C
Inductors

- **Design of the thermal parameters**

\[
R_{\theta sa}, \, R_{\theta rad}, \, R_{\theta conv} \quad \quad R_{\theta sa} = \frac{k_1}{a^2} \quad k_1: \text{constant}
\]

\[
\Delta T = R_{\theta sa}(P_{\text{core}} + P_w)
\]

\[
P_{\text{core}} = P_{c,sp}V_c \quad \quad P_w = P_{w,sp}V_w
\]

\[
P_{c,sp} \approx P_{w,sp} = P_{sp} \quad \text{for an optimal design}
\]

\[
V \ (\text{volume}) \sim a^3 \quad \text{so with} \quad P_{\text{core}} + P_w = k_2a^2 \quad : \quad P_{sp} = \frac{k_3}{a}
\]
Inductors

Maximum current density $J$ and specific power dissipation $P_{sp}$ as functions of the double-E core scaling parameter $a$

$$P_{sp} = \frac{k_3}{a}$$

$$J_{rms} = \frac{k_5}{\sqrt{k_{Cu}} a}$$

![Graph showing $J$ and $P_{sp}$ as functions of core scaling parameter $a$.]