Electron dynamics with Synchrotron Radiation

Lenny Rivkin

*Paul Scherrer Institute (PSI)*

*and*

*Swiss Federal Institute of Technology Lausanne (EPFL)*
Curved orbit of electrons in magnet field

Accelerated charge ➔ Electromagnetic radiation
Electromagnetic waves
Crab Nebula
6000 light years away

First light observed
1054 AD

GE Synchrotron
New York State

First light observed
1947
Synchrotron radiation: some dates

- 1873  Maxwell’s equations
- 1887  Hertz: electromagnetic waves
- 1898  Liénard: retarded potentials
- 1900  Wiechert: retarded potentials
- 1908  Schott: Adams Prize Essay

... waiting for accelerators ...
1940: 2.3 MeV betatron, Kerst, Serber
Maxwell equations (poetry)

Was it a God whose inspiration
Led him to write these fine equations
Nature’s fields to me he shows
And so my heart with pleasure glows.

translated by John P. Blewett
Synchrotron radiation: some dates

- 1873  Maxwell’s equations
- 1887  Hertz: electromagnetic waves
- 1898  Liénard: retarded potentials
- 1900  Wiechert: retarded potentials
- 1908  Schott: Adams Prize Essay

... waiting for accelerators ...
1940: 2.3 MeV betatron, Kerst, Serber
THEORETICAL UNDERSTANDING

1873 Maxwell’s equations
→ made evident that changing charge densities would result in electric fields that would radiate outward

1887 Heinrich Hertz demonstrated such waves:

It's of no use whatsoever[...] this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.
Electron Beam Dynamics, L. Rivkin, Intro to Accelerator Physics, Vysoke Tatry
Synchrotron radiation: some dates

- 1873: Maxwell’s equations
- 1887: Hertz: electromagnetic waves
- 1898: Liénard: retarded potentials
- 1900: Wiechert: retarded potentials
- 1908: Schott: Adams Prize Essay

... waiting for accelerators ...
1940: 2.3 MeV betatron, Kerst, Serber
Donald Kerst: first betatron (1940)

"Ausserordentlich hochgeschwindigkeitelektronenentwicklung undenschwerarbeitsbeigollitron"
Synchrotron radiation: some dates

- **1946**: Blewett observes energy loss due to synchrotron radiation from a 100 MeV betatron.
- **1947**: First visual observation of synchrotron radiation from a 70 MeV synchrotron at GE Lab.
- **1949**: Schwinger's PhysRev paper.
- **1976**: Madey's demonstration of the first free electron laser.
THE ELECTROMAGNETIC SPECTRUM

Wavelength continuously tunable!
60,000 SR users world-wide

Electron Beam Dynamics, L. Rivkin, Intro to Accelerator Physics, Vysoke-Tatry
A larger view
Crab Nebula sent a 450 TeV photon
Why do they radiate?
Synchrotron Radiation is not as simple as it seems

... I will try to show that it is much simpler
Charge at rest
Coulomb field, no radiation
Uniformly moving charge does not radiate

\[ v = \text{constant} \]

But! Cerenkov!
Free isolated electron cannot emit a photon

Easy proof using 4–vectors and relativity

- momentum conservation if a photon is emitted

\[ P_i = P_f + P_\gamma \]

- square both sides

\[ m^2 = m^2 + 2P_f \cdot P_\gamma + 0 \Rightarrow P_f \cdot P_\gamma = 0 \]

- in the rest frame of the electron

\[ P_f = (m, 0) \quad P_\gamma = (E_\gamma, p_\gamma) \]

this means that the photon energy must be zero.
We need to separate the field from charge
Bremsstrahlung
or
“braking” radiation
Transition Radiation

\[ c_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_1}} \quad \quad c_2 = \frac{1}{\sqrt{\varepsilon_2 \mu_2}} \]
Liénard–Wiechert potentials

\[ \varphi(t) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r(1 - \mathbf{n} \cdot \mathbf{\beta})} \]

\[ \vec{A}(t) = \frac{q}{4\pi\varepsilon_0 c^2} \left[ \frac{\vec{v}}{r(1 - \mathbf{n} \cdot \mathbf{\beta})} \right] \]

and the electromagnetic fields:

\[ \nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0 \quad \text{(Lorentz gauge)} \]

\[ \vec{B} = \nabla \times \vec{A} \]

\[ \vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \]
Fields of a moving charge

\[ \vec{E}(t) = \frac{q}{4\pi\varepsilon_0} \left[ \frac{\vec{n} - \vec{\beta}}{(1 - \vec{n} \cdot \vec{\beta})^3 \gamma^2} \cdot \frac{1}{r^2} \right]_{ret} + \text{“near field”} \]

\[ \frac{q}{4\pi\varepsilon_0 c} \left[ \vec{n} \times \left( \frac{(\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{n} \cdot \vec{\beta})^3 \gamma^2} \right) \cdot \frac{1}{r} \right]_{ret} \text{“far field”} \]

\[ \vec{B}(t) = \frac{1}{c} [\vec{n} \times \vec{E}] \]
Energy flow integrated over a sphere

*Power* \( \sim E^2 \cdot \text{Area} \)

\[ A = 4\pi r^2 \]

Near field

\[ P \propto \frac{1}{r^4} r^2 \propto \frac{1}{r^2} \]

Far field

\[ P \propto \frac{1}{r^2} r^2 \propto \text{const} \]

*Radiation = constant flow of energy to infinity*
Transverse acceleration

Radiation field quickly separates itself from the Coulomb field
Radiation field cannot separate itself from the Coulomb field

Longitudinal acceleration

\[ \mathbf{a} \]
Synchrotron Radiation
Basic Properties
Beams of ultra-relativistic particles: e.g. a race to the Moon

An electron with energy of a few GeV emits a photon... a race to the Moon!

\[ \Delta t = \frac{L}{\beta c} - \frac{L}{c} = \frac{L}{\beta c} (1 - \beta) \approx \frac{L}{\beta c} \cdot \frac{1}{2\gamma^2} \]

Electron will lose
- by only 8 meters
- the race will last only 1.3 seconds

\[ \Delta L = L(1 - \beta) \approx \frac{L}{2\gamma^2} \]
Moving Source of Waves: Doppler effect

“redshift”

“blueshift”

Cape Hatteras, 1999
Time compression

Electron with velocity $\beta$ emits a wave with period $T_{\text{emit}}$ while the observer sees a different period $T_{\text{obs}}$ because the electron was moving towards the observer.

The wavelength is shortened by the same factor

$$\lambda_{\text{obs}} = (1 - \beta \cos \theta) \lambda_{\text{emit}}$$

in ultra-relativistic case, looking along a tangent to the trajectory.

Since

$$1 - \beta = \frac{1 - \beta^2}{1 + \beta} \approx \frac{1}{2\gamma^2}$$
Radiation is emitted into a narrow cone

\[ \theta = \frac{1}{\gamma} \cdot \theta_e \]

\[ \theta_e \sim c \]

\[ v \sim c \]

\[ V \ll C \]

\[ V \approx C \]
Sound waves (non-relativistic)

Angular collimation

\[ \theta = \frac{v_{s\perp}}{v_{s\parallel}} + v = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{v}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{v}{v_s}} \]

Doppler effect (moving source of sound)

\[ \lambda_{\text{heard}} = \lambda_{\text{emitted}} \left(1 - \frac{v}{v_s}\right) \]
Synchrotron radiation power

Power emitted is proportional to:

\[ P \propto E^2 B^2 \]

\[ P = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2} \]

\[ C_\gamma = \frac{4\pi re}{3 (m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[ \frac{m}{\text{GeV}^3} \right] \]

\( E = \text{Energy!} \)
The power is all too real!

Fig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.
Synchrotron radiation power

Power emitted is proportional to:

\[ P \propto E^2 B^2 \]

\[ P_\gamma = \frac{c C_\gamma \cdot E^4}{2\pi \rho^2} \]

\[ C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[ \frac{m}{\text{GeV}^3} \right] \]

Energy loss per turn:

\[ U_0 = C_\gamma \cdot \frac{E^4}{\rho} \]

\[ U_0 = \frac{4\pi}{3} \alpha \hbar c \gamma^4 \frac{\gamma^4}{\rho} \]

α = \frac{1}{137}

\[ \hbar c = 197 \text{ Mev} \cdot \text{ fm} \]
Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory (a few mm)

\[ l \sim \frac{2\rho}{\gamma} \]

Pulse length: difference in times it takes an electron and a photon to cover this distance

\[ \Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c} (1 - \beta) \]

\[ \omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0 \]

\[ \Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2} \]
Short magnet: higher energy photons

When Lorentz factor is not very high (e.g. protons)...

\[ l \ll \frac{2\rho}{\gamma} \]

\[ \Delta t \sim \frac{l}{c} \frac{1}{2\gamma^2} \]

Pulse length: difference in times it takes an electron and a photon to cover this distance

\[ \Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1 - \beta) \]
Spectrum of synchrotron radiation

• Synchrotron light comes in a series of flashes every $T_0$ (revolution period)

• the spectrum consists of harmonics of

\[ \omega_0 = \frac{1}{T_0} \]

• flashes are extremely short: harmonics reach up to very high frequencies

\[ \omega_{typ} \approx \gamma^3 \omega_0 \]

• At high frequencies the individual harmonics overlap

\[
\omega_0 \sim 1 \text{ MHz} \\
\gamma \sim 4000 \\
\omega_{typ} \sim 10^{16} \text{ Hz!}
\]

continuous spectrum!
Wavelength continuously tunable!
\[
\frac{dP}{d\omega} = \frac{P_{\text{tot}}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)
\]

\[
S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')dx'
\]

\[
\int_0^\infty S(x')dx' = 1
\]

\[
P_{\text{tot}} = \frac{2}{3} \hbar c^2 \alpha \gamma^4 \rho^2
\]

\[
\omega_c = \frac{3 c \gamma^3}{2 \rho}
\]

\[
\varepsilon_c [\text{eV}] = 665 E^2 [\text{GeV}] B [\text{T}]
\]

\[
G_1(x) = x \int_x^\infty K_{5/3}(x')dx'
\]

\[
\sim 2.1 x^{1/3}
\]

\[
\sim 1.3 \sqrt{x} e^{-x}
\]

\[
50\%
\]
Synchrotron radiation flux for different electron energies

 Flux [photons/s/mrad/0.1%BW] vs. Photon energy [eV]

- LEP Dipole Flux
  - I = 1 mA

- 20 GeV
- 50 GeV
- 100 GeV

*EPFL*

Electron Beam Dynamics, L. Rivkin, Intro to Accelerator Physics, Vysoke-Tatry
Angular divergence of radiation

The rms opening angle R’

- at the critical frequency: $\omega = \omega_c \quad R' \approx \frac{0.54}{\gamma}$

- well below $\omega \ll \omega_c \quad R' \approx \frac{1}{\gamma} \left( \frac{\omega_c}{\omega} \right)^{\frac{1}{3}} \approx 0.4 \left( \frac{\lambda}{\rho} \right)^{\frac{1}{3}}$

- well above $\omega \gg \omega_c \quad R' \approx \frac{0.6}{\gamma} \left( \frac{\omega_c}{\omega} \right)^{\frac{1}{2}}$

independent of $\gamma$!
Synchrotron light polarization
An electron in a storage ring

Polarization:
Linear in the plane of the ring
the electric field vector

elliptical out of
the plane
Angular distribution of SR
Synchrotron light based electron beam diagnostics
Seeing the electron beam (SLS)

X rays

visible light, vertically polarised

$\sigma_x \sim 55 \mu m$
Seeing the electron beam (SLS)

Making an image of the electron beam using the vertically polarised synchrotron light
High resolution measurement

Wavelength used: 364 nm

For point-like source the intensity on axis is zero

Peak-to-valley intensity ratio is determined by the beam height

Present resolution: 3.5 μm
Useful books and references

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

H. Wiedemann, *Particle Accelerator Physics I and II*

A. Hofmann, *The Physics of Synchrotron Radiation*
Cambridge University Press 2004

Synchrotron Radiation and Free Electron Lasers

Grenoble, France, 22 - 27 April 1996
(A. Hofmann’s lectures on synchrotron radiation)
CERN Yellow Report 98-04

Brunnen, Switzerland, 2 – 9 July 2003
CERN Yellow Report 2005-012

Previous CAS Schools Proceedings