Linear Accelerators

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LINAC APPLICATIONS

- Injectors for synchrotrons
- Medical applications: radiotherapy
- Industrial applications
- Nuclear waste treatment and controlled fission for energy production (ADS)
- Material testing for fusion nuclear reactors
- Spallation sources for neutron production
- Material/food sterilization
- Ion implantation
- National security
- Material treatment

~10^4 LINACs operating around the world
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

1st PART OF THE LECTURE
- Particle source
- Accelerating structures
- Focusing elements: quadrupoles and solenoids

2nd PART OF THE LECTURE
- Longitudinal dynamics of accelerated particles
- Transverse dynamics of accelerated particles
- Accelerated beam
LINAC TECHNOLOGY COMPLEXITY

The overall LINAC has to be designed to obtain the desired beam parameters in terms of:
- output energy/energy spread
- beam current (charge)
- long and transverse beam dimensions/divergence (emittance)

Having, in general, constraints in terms of:
- space
- cost
- power consumption
- available power sources

Particle source

Low Level RF & synchronization

RF Power sources

Accelerating structures (RF cavities)
Magnets (quadrupoles, solenoids)

Diagnostics systems
Vacuum pumps and gauges
Cryostat (superconducting linac)

Magnet power supplies
Vacuum pumps power supplies
Cooling and cryogenics
Diagnostic acquisition system

Control system

Accelerated beam
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

Particle source

Accelerating structures

Longitudinal dynamics of accelerated particles

Transverse dynamics of accelerated particles

Accelerated beam

Focusing elements: quadrupoles and solenoids
The basic equation that describes the acceleration/bending/focusing processes is the Lorentz Force. Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

\[
\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

- \(\vec{p}\) = momentum
- \(m\) = mass
- \(\vec{v}\) = velocity
- \(q\) = charge

**ACCELERATION**
To accelerate, we need a force in the direction of motion

**BENDING AND FOCUSING**
2\textsuperscript{nd} term always perpendicular to motion => no energy gain

**Longitudinal Dynamics**

**Transverse Dynamics**
The first historical linear particle accelerator was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. Electrons emitted by the heated cathode were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced X-rays.

The energy gained by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

\[
\frac{d\vec{p}}{dt} = q \vec{E} \quad \Rightarrow \quad \Delta E = q \Delta V
\]

\( \vec{p} = \text{momentum} \)
\( q = \text{charge} \)
\( E = \text{energy} \)

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt: 1 eV=1.6x10^{-19} \text{ J}
PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

- **Single particle**
  - rest mass $m_0$
  - rest energy $E_0 = m_0 c^2$
  - total energy $E$
  - relativistic mass $m$
  - velocity $v$
  - momentum $p = mv$
  - Kinetic energy $W = E - E_0$

- **Relativistic factors**
  - Relativistic factor
    - $\beta = \frac{v}{c} < 1$
    - $\gamma = \frac{E}{E_0} \geq 1$

- **Formulas**
  - $E^2 = E_0^2 + p^2 c^2$
  - $W = (\gamma - 1) m_0 c^2 \approx \frac{1}{2} m_0 v^2$ if $\beta << 1$
  - $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} = \sqrt{1 - \left(\frac{E_0}{E_0 + W}\right)^2}$

- **Light particles** (as electrons) are practically fully relativistic ($\beta \approx 1$, $\gamma \gg 1$) at relatively low energy and reach a constant velocity ($\approx c$). The acceleration process occurs at constant particle velocity.

- **Heavy particles** (protons and ions) are typically weakly relativistic and reach a constant velocity only at very high energy. The velocity changes a lot during acceleration process.

- **This implies important differences** in the technical characteristics of the accelerating structures. In particular for protons and ions we need different types of accelerating structures, optimized for different velocities and/or the accelerating structure has to vary its geometry to take into account the velocity variation.
ELECTROSTATIC ACCELERATORS

To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a dielectric belt transporting positive charges to an isolated electrode hosting an ion source. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.

LIMITS OF ELECTROSTATIC ACCELERATORS

DC voltage as large as \(~10\) MV can be obtained \((E \sim 10\) MeV). The main limit in the achievable voltage is the breakdown due to insulation problems.

APPLICATIONS OF DC ACCELERATORS

DC particle accelerators are in operation worldwide, typically at \(V < 15\) MV \((E_{\text{max}} = 15\) MeV\), \(I < 100\) mA. They are used for:

\(\Rightarrow\) material analysis
\(\Rightarrow\) X-ray production,
\(\Rightarrow\) ion implantation for semiconductors
\(\Rightarrow\) first stage of acceleration (particle sources)

750 kV Cockcroft-Walton Linac2 injector at CERN from 1978 to 1992
If the length of the tubes increases with the particle velocity during the acceleration such that the time of flight is kept constant and equal to half of the RF period, the particles are subject to a synchronous accelerating voltage and experience an energy gain of $\Delta E = q\Delta V$ at each gap crossing.

In principle a single RF generator can be used to indefinitely accelerate a beam, avoiding the breakdown limitation affecting the electrostatic accelerators.

The Wideroe LINAC is the first RF LINAC.
ACCELERATION: ENERGY GAIN

We consider the acceleration between two electrodes in DC.

\[ E^2 = E_0^2 + p^2c^2 \Rightarrow 2EdE = 2pdp c^2 \Rightarrow dE = v \frac{mc^2}{E} dp \Rightarrow dE = vdp \]

\[ \frac{dp}{dt} = qE_z \Rightarrow v \frac{dp}{dz} = qE_z \Rightarrow \frac{dE}{dz} = qE_z \quad \left( \text{and also} \quad \frac{dW}{dz} = qE_z \right) \]

\[ W = E - E_0 \]

rate of energy gain per unit length

energy gain per electrode

\[ \Rightarrow \Delta E = \int_{gap} \frac{dE}{dz} dz = \int_{gap} qE_z dz \Rightarrow \Delta E = q\Delta V \]
RF ACCELERATION: BUNCHING BEAM

We consider now the acceleration between two electrodes fed by an RF generator.

\[ E_z(z,t) = E_{RF}(z) \cos(\omega_{RF}t) \]

\[ V = V_{RF} \cos(\omega_{RF}t) \]

\[ \Delta V = V_{RF} \cos(\omega_{RF}t) \]

\[ \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}} \]

\[ \Rightarrow \Delta E = q \hat{V}_{acc} \cos(\omega_{RF}t_{inj}) \]

Only these particles are accelerated.

These particles are not accelerated and basically are lost during the acceleration process.

**DC acceleration**

**RF acceleration**

**Bunched beam** (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)
RF ACCELERATION: ACCELERATING FIELD CALCULATION

We consider now the acceleration between two electrodes fed by an RF generator

\[ \Delta V = V_{RF} \cos(\omega_{RF} t) \quad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}} \]

\[ V_{RF} = \int_{\text{gap}} E_{RF}(z) \, dz \quad E_z(z, t) = E_{RF}(z) \cos(\omega_{RF} t) \]

\[ E_z(z, t)_{\text{seen by particle}} = E_{RF}(z) \cos[\omega_{RF}(t + t_{\text{inj}})] = E_{RF}(z) \cos(\omega_{RF} t + \phi_{\text{inj}}) \]

\[ \phi_{\text{inj}} = \omega_{RF} t_{\text{inj}} \]

Hyp. of symmetric accelerating field

\[ \Delta E = q \int_{\text{gap}} E_z(z, t) \, dz = q \int_{-L/2}^{+L/2} E_{RF}(z) \cos\left(\omega_{RF} \frac{z}{v} + \phi_{\text{inj}}\right) \, dz = q \int_{ \text{gap} } E_{RF}(z) \, dz 
\]

\[ \frac{\int_{ \text{gap} } E_{RF}(z) \cos\left(\omega_{RF} \frac{z}{v}\right) \, dz}{\int_{ \text{gap} } E_{RF}(z) \, dz} \]

\[ \int_{\text{gap}} E_{RF}(z) \, dz \]

\[ \cos(\phi_{\text{inj}}) \]

Peak gap voltage

Injection phase

Transit time factor

\[ T < 1 \]

\[ V_{RF} \]

Average accelerating field in the gap

\[ \hat{E}_{\text{acc}} = \frac{\hat{V}_{\text{acc}}}{L} \]

Average accelerating field seen by the particle

\[ E_{\text{acc}} = \frac{V_{\text{acc}}}{L} \]
**DRIFT TUBE LENGTH AND FIELD SYNCHRONIZATION**  
(protons and ions or electrons at extremely low energy)

If now we consider a DTL structure with an injected particle at an energy $E_{in}$, we have that at each gap the maximum energy gain is $\Delta E_n = qV_{acc}$ and the particle increase its velocity accordingly to the previous relativistic formulae.

$$E_n = E_{in} + nqV_{acc}$$

$$v_n = c\beta_n = c\sqrt{1 - \frac{1}{\gamma_n^2}} = c\sqrt{1 - \left(\frac{E_o}{E_n}\right)^2}$$

$$t_n = \frac{L_n}{v_n} = \frac{T_{RF}}{2} \Rightarrow \frac{L_n}{T_{RF}} = \frac{1}{2}v_nT_{RF} = \frac{1}{2}\beta_n cT_{RF}$$

$$L_n = \frac{1}{2}\beta_n \lambda_{RF}$$

$\Rightarrow$ In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** has to be $L_n$:

$$\Delta E = \frac{qV_{acc}}{L_n + L_{gap}} \approx \frac{2qV_{acc}}{\lambda_{RF}\beta_n} [\text{eV/m}]$$
ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important consequences of the previous obtained formulae:

\[ L_n = \frac{1}{2} \beta_n \lambda_{RF} \]

The condition \( L_n \ll \lambda_{RF} \) (necessary to model the tube as an equipotential region) requires \( \beta \ll 1 \). \( \Rightarrow \) The Wideröe technique cannot be applied to relativistic particles.

\[ \frac{\Delta E}{\Delta L} = qE_{acc} \approx \frac{2qV_{acc}}{\lambda_{RF} \beta_n} \]

Moreover when particles get high velocities the drift spaces get longer and one loses on the efficiency. The average accelerating gradient \((E_{acc} \text{[V/m]})\) increase pushes towards small \( \lambda_{RF} \) (high frequencies).

High frequency high power sources became available after the 2\(^{nd}\) world war pushed by military technology needs (such as radar). However, the concept of equipotential DT can not be applied at small \( \lambda_{RF} \) and the power lost by radiation is proportional to the RF frequency.

As a consequence we must consider accelerating structures different from drift tubes.

\( \Rightarrow \) The solution consists of enclosing the system in a cavity which resonant frequency matches the RF generator frequency.

\( \Rightarrow \) Each cavity can be independently powered from the RF generator.
High frequency RF accelerating fields are confined in cavities.

The cavities are metallic closed volumes where the e.m fields have a particular spatial configuration (resonant modes) whose components, including the accelerating field $E_z$, oscillate at some specific frequencies $f_{RF}$ (resonant frequency) characteristic of the mode.

The modes are excited by RF generators that are coupled to the cavities through waveguides, coaxial cables, etc...

The resonant modes are called Standing Wave (SW) modes (spatial fixed configuration, oscillating in time).

The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) by solving the Maxwell equations with the proper boundary conditions.

Courtesy E. Jensen
Alvarez's structure can be described as a special DTL in which the electrodes are part of a resonant macrostructure.

- The DTL operates in 0 mode for protons and ions in the range $\beta = 0.05-0.5$ ($f_{RF} = 50-400$ MHz) 1-100 MeV;
- The beam is inside the “drift tubes” when the electric field is decelerating. The electric field is concentrated between gaps;
- The drift tubes are suspended by stems;
- Quadrupole (for transverse focusing) can fit inside the drift tubes.
- In order to be synchronous with the accelerating field at each gap the length of the $n$-th drift tube $L_n$ has to be:

$$L_n = \beta_n \lambda_{RF}$$
CERN LINAC 2 tank 1: 200 MHz 7 m x 3 tanks, 1 m diameter, final energy 50 MeV.

CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 50 MeV, $\beta=0.08$ to $0.31 \rightarrow$ cell length from 68mm to 264mm.
When the $\beta$ of the particles increases (>0.5) one has to use higher RF frequencies (>400-500 MHz) to increase the accelerating gradient per unit length.

The DTL structures became less efficient (effective accelerating voltage per unit length for a given RF power);

Cylindrical single or multiple cavities working on the $\text{TM}_{010}$-like mode are used.

For a pure cylindrical structure (also called pillbox cavity) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the $\text{TM}_{010}$ mode. It has a well known analytical solution from Maxwell equation.

Real cylindrical cavity
(TM$_{010}$-like mode because of the shape and presence of beam tubes)

\[ f_{\text{res}} = \frac{c}{2.405a} \]

\[ E_z = AJ_0 \left( \frac{2.405r}{a} \right) \cos(\omega_{RF}t) \]

\[ H_\theta = A \frac{1}{Z_0} J_1 \left( \frac{2.405r}{a} \right) \sin(\omega_{RF}t) \]
The shunt impedance is the parameter that qualifies the efficiency of an accelerating mode. The higher is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to maximize the accelerating field for a given dissipated power:

\[ R = \frac{V_{acc}^2}{P_{diss}} \] [Ω]

**SHUNT IMPEDANCE PER UNIT LENGTH**

\[ r = \frac{(V_{acc}/L)^2}{P_{diss}/L} = \frac{E_{acc}^2}{P_{diss}} \] [Ω/m]

NC cavity R~1MΩ
SC cavity R~1TΩ

**Example:**
R~1MΩ
\[ P_{diss} = 1 \text{ MW} \]
\[ V_{acc} = 1 \text{ MV} \]
For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m
- In a multi-cell structure there is one RF input coupler. As a consequence the total number of RF sources is reduced, with a simplification of the layout and reduction of the costs;

- The shunt impedance is $n$ time the impedance of a single cavity

- They are more complicated to fabricate than single cell cavities;

- The fields of adjacent cells couple through the cell irises and/or through properly designed coupling slots.
The N-cell structure behaves like a system composed by \( N \) coupled oscillators with \( N \) coupled multi-cell resonant modes.

The modes are characterized by a cell-to-cell phase advance given by:

\[
\Delta \phi_n = \frac{n\pi}{N-1} \quad n = 0, 1, ..., N - 1
\]

The multi cell mode generally used for acceleration is the \( \pi \), \( \pi/2 \) and 0 mode (DTL as example operate in the 0 mode).

In this case as done for the DTL structures the cell length has to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity.

For ions and protons the cell length has to be increased and the linac will be made of a sequence of different accelerating structures matched to the ion/proton velocity.

\[d = \frac{\beta c}{2f_{RF}} = \frac{\beta \lambda_{RF}}{2}\]

For electron, \( \beta=1 \), \( d=\lambda_{RF}/2 \) and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.
π MODE STRUCTURES: EXAMPLES

LINAC 4 (CERN) PIMS (PI Mode Structure) for protons: \( f_{RF} = 352 \text{ MHz}, \beta > 0.4 \)

- Each module has 7 identical cells
- 12 tanks
- 100 MeV
- 1.28 m
- 1.55 m
- 160 MeV

European XFEL (Desy): electrons

- 800 accelerating cavities
- 1.3 GHz / 23.6 MV/m
- All identical \( \beta = 1 \)
- Superconducting cavities
It is possible to demonstrate that over a certain number of cavities (>10) working on the \( \pi \) mode, the overlap between adjacent modes can be a problem (as example the field uniformity due to machining errors is difficult to tune).

The criticality of a working mode depend on the frequency separation between the working mode and the adjacent mode.

The \( \pi/2 \) mode from this point of view is the most stable mode. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

This allow to increase the number of cells to >20-30 without problems.

\( f_{\text{RF}} \approx 800 - 3000 \, \text{MHz} \) for proton (\( \beta = 0.5-1 \)) and electrons.
SCC STRUCTURES: EXAMPLES

Spallation Neutron Source Coupled Cavity Linac (protons)

4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at **805 MHz** that accelerates the beam **from 87 to 186 MeV** and has a physical installed length of slightly over **55 meters**.
TRAVELLING WAVE (TW) STRUCTURES
(electrons)

⇒ To accelerate charged particles, the electromagnetic field must have an electric field along the direction of propagation of the particle.

⇒ The field has to be synchronous with the particle velocity.

⇒ Up to now we have analyzed the cases standing standing wave (SW) structures in which the field has basically a given profile and oscillate in time (as example in DTL or resonant cavities operating on the TM$_{010}$-like).

$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF}t)$

⇒ There is another possibility to accelerate particles: using a travelling wave (TW) structure in which the RF wave is co-propagating with the beam with a phase velocity equal to the beam velocity.

⇒ Typically these structures are used for electrons because in this case the phase velocity can be constant all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low \(\beta\) particle that changes its velocity during acceleration.
In TW structures an e.m. wave with $E_z \neq 0$ travel together with the beam in a special guide in which the phase velocity of the wave matches the particle velocity ($v$). In this case the beam absorbs energy from the wave and it is continuously accelerated.

As example if we consider a simple circular waveguide the first propagating mode with $E_z \neq 0$ is the TM$_{01}$ mode. Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this constant cross section waveguide will never be synchronous with a particle beam since the phase velocity is always larger than the speed of light $c$.

$$E_z \mid_{TM_{01}} = E_0(r) \cos(\omega_{RF}t - k^*z)$$

$$v_{ph} = \frac{\omega_{RF}}{k^*} > c$$
In order to slow-down the wave phase velocity, iris-loaded periodic structure have to be used.

**CIRCULAR WAVEGUIDE**

\[ E_z|_{TM_{01}} = E_0(r) \cos(\omega_{RF}t - k^*z) \]

**IRIS LOADED STRUCTURE**

\[ E_z|_{TM_{01\text{-like}}} = \hat{E}_{acc}(r,z) \cos(\omega_{RF}t - k^*z) \]

⇒ The field in this type of structures is that of a special wave travelling within a spatial periodic profile.

⇒ The structure can be designed to have the phase velocity equal to the speed of the particles.

⇒ This allows acceleration over large distances (few meters, hundred of cells) with just an input coupler and a relatively simple geometry.

⇒ They are used especially for electrons (constant particle velocity → constant phase velocity, same distance between irises, easy realization).
LINAC TECHNOLOGY
The cavities (and the related LINAC technology) can be of different material:

- **copper** for normal conducting (NC, both SW than TW) cavities;
- **Niobium** for superconducting cavities (SC, SW);

We can choose between NC or the SC technology depending on the required performances in term of:

- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle (see next slide)**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.
- ...

Dissipated power into the cavity walls is related to the surface currents

\[
P_{\text{diss}} = \int_{\text{cavity wall}} \frac{1}{2} R_s H^2_{\tan} \, dS
\]

Between copper and Niobium there is a factor \(10^5-10^6\)
The “beam structure” in a LINAC is directly related to the “RF structure”. There are two possible types of operations:

- **CW** (Continuous Wave) operation ⇒ allow, in principle, to operate with a continuous (bunched) beam
- **PULSED** operation ⇒ there are RF pulses at a certain repetition rate (Duty Cycle (DC)=pulsed width/period)

⇒**SC** structures allow operation at very high Duty Cycle (>1%) up to a CW operation (DC=100%) (because of the extremely low dissipated power) with relatively high gradient (>20 MV/m). This means that a continuous (bunched) beam can be accelerated.

⇒**NC** structures can operate in pulsed mode at very low DC (10^{-2}-10^{-1} %) (because of the higher dissipated power) with, in principle, larger peak accelerating gradient (>30 MV/m). This means that one or few tens of bunches can be, in general, accelerated. NB: NC structures can also operate in CW but at very low gradient because of the dissipated power.
EXAMPLES: EUROPEAN XFEL

**Nominal Energy** | GeV | 17.5
---|---|---
**Beam pulse length** | ms | 0.60
**Repetition rate** | Hz | 10
**Max. # of bunches per pulse** | 2700
**Min. bunch spacing** | ns | 220
**Bunch charge** | nC | 1
**Bunch length, \( \sigma_z \)** | \( \mu m \) | < 20
**Emittance (slice) at undulator** | \( \mu rad \) | < 1.4
**Energy spread (slice) at undulator** | MeV | 1

**101 cryomodules in total**

- RF system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)
- Cryomodule housing: 8 cavities, quadrupole and BPM
- 25 RF stations 5.2 MW each

**800 accelerating cavities**

- 1.3 GHz / 23.6 MV/m

- Undulator

- Input coupler
EXAMPLE: SWISSFEL LINAC (PSI)

Courtesy T. Garvey
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Longitudinal dynamics of accelerated particles

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Accelerating structures

Accelerated beam

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Focusing elements: quadrupoles and solenoids

Linac Beam Dynamics

Linac Components and Technology
RF guns are used in the first stage of electron beam generation in FEL and acceleration.

- Multi cell: typically 2-3 cells
- SW $\pi$ mode cavities
- Operate in the range of 60-120 MV/m cathode peak accelerating field with up to 10 MW input power.
- Typically in L-band- S-band (1-3 GHz) at 10-100 Hz.
- Single or multi bunch (L-band)
- Different type of cathodes (copper,...)

The electrons are emitted on the **cathode** through a laser that hit the surface. They are then accelerated through the electric field that has a longitudinal component on axis TM$_{010}$. 
RF PHOTO-GUNS: EXAMPLES

**LCLS**
- Frequency = 2,856 MHz
- Gradient = 120 MV/m
- Exit energy = 6 MeV
- Copper photocathode
- RF pulse length ~2 µs
- Bunch repetition rate = 120 Hz
- Norm. rms emittance
  - 0.4 mm-mrad at 250 pC

**PITZ L-band Gun**
- Frequency = 1,300 MHz
- Gradient = up to 60 MV/m
- Exit energy = 6.5 MeV
- Rep. rate 10 Hz
- Cs₂Te photocathode
- RF pulse length ~1 ms
- 800 bunches per macropulse
- Normalized rms emittance
  - 1 nC 0.70 mm-mrad
  - 0.1 nC 0.21 mm-mrad

Solenoids field are used to compensate the space charge effects in low energy guns. The configuration is shown in the picture.
Basic principle: create a plasma and optimize its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.
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**Longitudinal dynamics of accelerated particles**

**Transverse dynamics of accelerated particles**

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**Focusing elements: quadrupoles and solenoids**
Let us consider a SW linac structure made by accelerating gaps (like in DTL) or cavities.

In each gap we have an accelerating field oscillating in time and an integrated accelerating voltage \( V_{\text{acc}} \) still oscillating in time than can be expressed as:

\[
V_{\text{acc}} = \hat{V}_{\text{acc}} \cos(\omega_{RF} t)
\]

Let’s assume that the “perfect” synchronism condition is fulfilled for a phase \( \phi_s \) (called synchronous phase). This means that a particle (called synchronous particle) entering in a gap with a phase \( \phi_s \) \((\phi_s = \omega_{RF} t)\) with respect to the RF voltage receive an energy gain (and a consequent change in velocity) that allow entering in the subsequent gap with the same phase \( \phi_s \) and so on.

For this particle the energy gain in each gap is:

\[
\Delta E = q\hat{V}_{\text{acc}} \cos(\phi_s) = qV_{\text{acc} - s}
\]

Obviously both \( \phi_s \) and \( \phi_s^* \) are synchronous phases.

\[
\omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}
\]
Let us consider now the first synchronous phase $\phi_s$ (on the positive slope of the RF voltage). If we consider another particle “near” to the synchronous one that arrives later in the gap ($t_1 > t_s$, $\phi_1 > \phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially compensating its initial delay.

Similarly if we consider another particle “near” to the synchronous one that arrives before in the gap ($t_1 < t_s$, $\phi_1 < \phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

On the contrary if we consider now the synchronous particle at phase $\phi_s^*$ and another particle “near” to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one.

The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: phase stability principle.

The synchronous phase on the negative slope of the RF voltage is, on the contrary, unstable.

Relying on particle velocity variations, longitudinal focusing does not work for fully relativistic beams (electrons). In this case acceleration “on crest” is more convenient.
In order to study the longitudinal dynamics in a LINAC, the following variables are used, which describe the generic particle phase (time of arrival) and energy with respect to the synchronous particle:

Arrival time (phase) of a generic particle at a certain gap (or cavity)

\[ \varphi = \phi - \phi_s = \omega_{RF}(t - t_s) \]

Arrival time (phase) of the synchronous particle at a certain gap (or cavity)

\[ w = E - E_s \]

Energy of the synchronous particle at a certain position along the linac

\[ \Delta L \] (accelerating cell length)

:\[ V_{acc} = \hat{V}_{acc} \cos(\omega_{RF}t) \]

\[ z \]

The energy gain per cell (one gap + tube in case of a DTL) of a generic particle and of a synchronous particle are (we put \( \theta = 0 \) in the generic expression of the accelerating voltage just for simplicity):

\[ \Delta E_s = q\hat{V}_{acc} \cos \phi_s \]

\[ \Delta E = q\hat{V}_{acc} \cos \phi = q\hat{V}_{acc} \cos(\phi_s + \varphi) \]

subtracting

\[ \Delta w = \Delta E - \Delta E_s = q\hat{V}_{acc} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] \]

Dividing by the accelerating cell length \( \Delta L \) and assuming that:

\[ \frac{\hat{V}_{acc}}{\Delta L} = \hat{E}_{acc} \]

\[ \frac{\Delta w}{\Delta L} = q\hat{E}_{acc} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] \]

Approximating

\[ \frac{\Delta w}{\Delta L} \approx \frac{dw}{dz} \]
ENERGY-PHASE EQUATIONS (2/2)
(protons and ions or electrons at extremely low energy)

On the other hand we have that the phase variation per cell of a generic particle and of a synchronous particle are:

\[
\begin{align*}
\Delta \phi_s &= \omega_{RF} \Delta t_s \\
\Delta \phi &= \omega_{RF} \Delta t
\end{align*}
\]

\(\Delta t\) is basically the time of flight between two accelerating cells. \(v, v_s\) are the average particles velocities.

Subtracting

\[
\Delta \phi = \omega_{RF} \left( \Delta t - \Delta t_s \right)
\]

Dividing by the accelerating cell length \(\Delta L\)

\[
\frac{\Delta \phi}{\Delta L} = \frac{\omega_{RF} \left( \frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L} \right)}{\omega_{RF} \left( \frac{1}{v} - \frac{1}{v_s} \right)} = \frac{c E_0 \beta_s^3 \gamma_s^3}{E_{RF}} \approx \frac{d \phi}{d \Delta L} = \frac{d \phi}{d z}
\]

This system of coupled (non linear) differential equations describe the motion of a non synchronous particles in the longitudinal plane with respect to the synchronous one.

\[
\frac{d \phi}{d z} = -\frac{\omega_{RF}}{c E_0 \beta_s^3 \gamma_s^3} w
\]

\[
\frac{d w}{d z} = q \hat{E}_{acc} \left[ \cos(\phi_s + \phi) - \cos \phi_s \right]
\]

**MAT**

\[
\omega_{RF} \left( \frac{1}{v} - \frac{1}{v_s} \right) = \omega_{RF} \left( \frac{v_s - v}{v v_s} \right) \approx -\frac{\omega_{RF}}{c} \frac{\Delta v}{\Delta \phi} \Delta v = -\frac{\omega_{RF}}{c} \frac{\Delta \beta}{\beta_s^2} \quad \text{remembering that} \quad \beta = \frac{\sqrt{1-1/\gamma^2}}{\gamma} \Rightarrow \beta d \beta = d \gamma / \gamma^3 \Rightarrow -\frac{\omega_{RF}}{c} \frac{\Delta \beta}{\beta_s^2} \approx -\frac{\omega_{RF}}{c} \frac{\Delta \gamma}{\beta_s^3 \gamma_s^3} = -\frac{\omega_{RF}}{c} \frac{\Delta E}{E_0 \beta_s^3 \gamma_s^3}
\]
SMALL AMPLITUDE ENERGY-PHASE OSCILLATIONS
(protons and ions or electrons at extremely low energy)

\[
\frac{dw}{dz} = q\hat{E}_{\text{acc}}[\cos(\phi_s + \varphi) - \cos \phi_s]
\]

Assuming small oscillations around the synchronous particle that allow to approximate \( \cos(\phi_s + \varphi) - \cos \phi_s \approx \varphi \sin \phi_s \).

Deriving both terms with respect to \( z \) and assuming an adiabatic acceleration process i.e. a particle energy and speed variations that allow to consider

\[
\frac{d^2 \varphi}{dz^2} = -\frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} \frac{d\varphi}{dz} + q \frac{\omega_{RF} \hat{E}_{\text{acc}} \sin(-\phi_s)}{cE_0 \beta_s^3 \gamma_s^3} \varphi = 0
\]

\( \text{harmonic oscillator equation} \)

The condition to have stable longitudinal oscillations and acceleration at the same time is:

\[
\Omega_s^2 > 0 \Rightarrow \sin(-\phi_s) > 0 \quad \text{and} \quad V_{\text{acc}} > 0 \Rightarrow \cos \phi_s > 0 \quad \Rightarrow \quad -\frac{\pi}{2} < \phi_s < 0
\]

if we accelerate on the rising part of the positive RF wave we have a longitudinal force keeping the beam bunched around the synchronous phase.

\[
\begin{align*}
\varphi &= \hat{\varphi} \cos(\Omega_s z) \\
w &= \hat{w} \sin(\Omega_s z)
\end{align*}
\]
ENERGY-PHASE OSCILLATIONS IN PHASE SPACE
(protons and ions or electrons at extremely low energy)

The energy-phase oscillations can be drawn in the **longitudinal phase space**:

\[
\begin{align*}
\varphi &= \hat{\varphi} \cos(\Omega_s z) \\
\mathcal{W} &= \hat{\mathcal{W}} \sin(\Omega_s z)
\end{align*}
\]

⇒ The trajectory of a generic particle in the longitudinal phase space is an **ellipse**.

⇒ The **maximum energy deviation** is reached at \( \varphi=0 \) while the **maximum phase excursion** corresponds to \( \mathcal{W}=0 \).

⇒ The bunch occupies an area in the longitudinal phase space called **longitudinal emittance** and the projections of the bunch in the energy and phase planes give the **energy spread** and the **bunch length**.

Gain energy with respect to the synchronous one

Loose energy

Longitudinal emittance is the phase space area including all the particles

Energy spread

Number of particles

Bunch length

Loose energy
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS

From previous formulae it is clear that there is no motion in the longitudinal phase plane for ultrarelativistic particles ($\gamma >> 1$).

It is interesting to analyze what happen if we inject an electron beam produced by a cathode (at low energy) directly in a TW structure (with $v_{ph} = c$) and the conditions that allow to capture the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at $v = c$).

Particles enter the structure with velocity $v < c$ and, initially, they are not synchronous with the accelerating field and there is a so called slippage.

After a certain distance they can reach enough energy (and velocity) to become synchronous with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

⇒ This is the case of electrons whose velocity is always close to speed of light $c$ even at low energies.

⇒ Accelerating structures are designed to provide an accelerating field synchronous with particles moving at $v = c$, like TW structures with phase velocity equal to $c$.

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost.
The accelerating field of a TW structure can be expressed by
\[ E_{acc} = \hat{E}_{acc} \cos(\omega_{RF}t - kz) \phi(z,t) \]

\[ \sin\phi_{\text{fin}} = \sin\phi_{\text{in}} + \frac{2\pi E_0}{\lambda_{RF} q \hat{E}_{acc}} \left( \sqrt{\frac{1 - \beta_{\text{in}}}{1 + \beta_{\text{in}}}} - \sqrt{\frac{1 - \beta_{\text{fin}}}{1 + \beta_{\text{fin}}}} \right) \]

Suppose that the particle reach asymptotically the value \( \beta_{\text{fin}} = 1 \) we have:

\[ \sin\phi_{\text{fin}} = \sin\phi_{\text{in}} + \frac{2\pi m_0 c^2}{\lambda_{RF} q \hat{E}_{acc}} \frac{1 - \beta_{\text{in}}}{1 + \beta_{\text{in}}} \]

We always have that
\[ \sin\phi_{\text{fin}} > \sin\phi_{\text{in}} \Rightarrow \phi_{\text{fin}} > \phi_{\text{in}} \]

The equation of motion of a particle with a position \( z \) at time \( t \) accelerated by the TW is then
\[ \frac{d}{dt}(mv) = q\hat{E}_{acc} \cos\phi(z,t) \Rightarrow m_0 \frac{d}{dt}(\gamma\beta) = m_0 \gamma^3 \frac{d\beta}{dt} = q\hat{E}_{acc} \cos\phi \]

It is useful to find which is the relation between \( \beta \) and \( \phi \) from an initial condition (in) to a final one (fin)

This limits the possible injection phases (i.e. the phase of the electrons that is possible to capture)

Should be in the interval \([-1,1]\) to have a solution for \( \phi_{\text{fin}} \)

This quantity is \( >0 \)
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE ACCELERATING FIELD

⇒ For a given injection energy ($\beta_{in}$) and phase ($\phi_{in}$) we can find which is the accelerating field ($E_{acc}$) that is necessary to have the completely relativistic beam at phase $\phi_{fin}$ (that is necessary to capture the beam at phase $\phi_{fin}$)

\[
\hat{E}_{acc} = \frac{2\pi E_0}{\lambda_{RF} q (\sin \phi_{fin} - \sin \phi_{in})} \sqrt{1 - \beta_{in}} / (1 + \beta_{in})
\]

Example:

$E_{in} = 50 \text{ keV}$, (kinetic energy), $\phi_{in} = -\pi/2$

$\phi_{fin} = 0 \Rightarrow \gamma_{in} \approx 1.1; \beta_{in} \approx 0.41$

$f_{RF} = 2856 \text{ MHz} \Rightarrow \lambda_{RF} \approx 10.5 \text{ cm}$

We obtain $E_{acc} \approx 20 \text{ MV/m}$;
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE EFFICIENCY AND BUNCH COMPRESSION

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by velocity modulation (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by velocity modulation (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).
In order to increase the capture efficiency of a traveling wave section, pre-bunchers are often used. They are SW cavities aimed at pre-forming particle bunches gathering particles continuously emitted by a source.

Once the capture condition $E_{RF} > E_{RF,\text{MIN}}$ is fulfilled, the fundamental equation of previous slide sets the ranges of the injection phases $\phi_i$ actually accepted. Particles whose injection phases are within this range can be captured, the other are lost.

$\Rightarrow$ Bunching is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal E-field of a SW cavity. After a certain drift space the velocity modulation is converted in a density charge modulation. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process.

$\Rightarrow$ A TW accelerating structure (capture section) is placed at an optimal distance from the pre-buncher, to capture a large fraction of the charge and accelerate it till relativistic energies. The amount of charge lost is drastically reduced, while the capture section provide also further beam bunching.
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

Longitudinal dynamics of accelerated particles

Transverse dynamics of accelerated particles

Particle source

Accelerating structures

Accelerated beam

Focusing elements: quadrupoles and solenoids

LINAC: BASIC DEFINITION AND MAIN COMPONENTS
RF TRANSVERSE FORCES

The RF fields act on the transverse beam dynamics because of the transverse components of the E and B field.

According to Maxwell equations the divergence of the field is zero and this implies that in traversing one accelerating gap there is a focusing/defocusing term.

\[ \nabla \cdot \vec{E} = 0 \]
\[ \nabla \times \vec{B} = \frac{1}{c^2} \vec{E} \Rightarrow \begin{cases} 
E_r = -\frac{r}{2} \frac{\partial E_z}{\partial z} \\
B_\theta = \frac{r}{2c^2} \frac{\partial E_z}{\partial t}
\end{cases} \]

\[ F_r = q(E_r - vB_\theta) = -q \frac{r}{2} \left( \frac{\partial E_z}{\partial z} - \frac{\beta E_z}{c} \right) \]

\[ F_r \big|_E = -q \frac{r}{2} \frac{\partial E_{RF}(z)}{\partial z} \cos \left( \omega_{RF} \frac{z}{\beta c} + \phi_{inj} \right) \]

\[ F_r \big|_B = q \frac{r}{2} \omega_{RF} \frac{\beta}{c} E_{RF}(z) \sin \left( \omega_{RF} \frac{z}{\beta c} + \phi_{inj} \right) \]

\[ f_{RF} = 350 \text{ MHz} \]
\[ \beta = 0.1 \]
\[ L = 3 \text{ cm} \]
RF DEFOCUSING/FOCUSING

From previous formulae it is possible to calculate the transverse momentum increase due to the RF transverse forces. Assuming that the velocity and position changes over the gap are small we obtain to the first order:

\[ \Delta p_r = \int_{-L/2}^{+L/2} F_r \frac{dz}{\beta c} = -\frac{\pi q\hat{E}_{acc} L \sin \phi}{c \gamma^2 \beta^2 \lambda_{RF}} \]

⇒ transverse defocusing scales as \( \sim 1/\gamma^2 \) and disappears at relativistic regime (electrons)

⇒ At relativistic regime (electrons), moreover, we have, in general, \( \phi=0 \) for maximum acceleration and this completely cancel the defocusing effect

⇒ Also in the non relativistic regime for a correct evaluation of the defocusing effect we have to:

⇒ take into account the velocity change across the accelerating gap

⇒ the transverse beam dimensions changes across the gap (with a general reduction of the transverse beam dimensions due to the focusing in the first part)

Both effects give a reduction of the defocusing force
RF FOCUSING IN ELECTRON LINACS

- RF defocusing is negligible in electron linacs.
- There is a second order effect due to the non-synchronous harmonics of the accelerating field that give a focusing effect. These harmonics generate a ponderomotive force i.e. a force in an inhomogeneous oscillating electromagnetic field.

MECHANISM

The Lorentz force is linear with the particle displacement:

\[ F_r = -q r \left( \frac{\partial E_z}{\partial z} + \frac{\beta}{c} \frac{\partial E_z}{\partial t} \right) \]

Transverse equation of motion:

\[ \ddot{r} = \frac{1}{\gamma m_0} r \sum_a \cos(n \omega t) \]

RF harmonics

This generates a global focusing force.

NON-SYNCHRONOUS RF HARMONICS: SIMPLE CASE OF SW STRUCTURE

Is equivalent to the superposition of two counterpropagating TW waves.

The forward wave only contributes to the acceleration (and does not give transverse effect).

The backward wave does not contribute to the acceleration but generates an oscillating transverse force (ponderomotive force).

Average focusing force:

\[ \bar{F}_r = -r q \frac{\hat{E}_{acc}^2}{8 \gamma m_0 c^2 / e} \eta(\phi) \]

With accelerating gradients of few tens of MV/m can easily reach the level of MV/m².
COLLECTIVE EFFECTS: SPACE CHARGE AND WAKEFIELDS

Collective effects are all effects related to the number of particles and they can play a crucial role in the longitudinal and transverse beam dynamics.

⇒ Effect of Coulomb repulsion between particles (space charge).

⇒ These effects cannot be neglected especially at low energy and at high current because the space charge forces scales as $1/\gamma^2$ and with the current $I$.

**SPACE CHARGE**

EXAMPLE: Uniform and infinite cylinder of charge moving along $z$

\[ \vec{F}_{sc} = q \frac{I}{2\pi\varepsilon_0 R_b^2 \beta c^2 \gamma^2} r \hat{r} \]

In this particular case it is linear but in general it is a non-linear force.

**WAKEFIELDS**

The other effects are due to the wakefield. The passage of bunches through accelerating structures excites electromagnetic field. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), can affect the longitudinal and the transverse beam dynamics. In particular the transverse wakefields, can drive an instability along the train called multibunch beam break up (BBU).

Several approaches are used to absorb these field from the structures like loops couplers, waveguides, Beam pipe absorbers.
MAGNETIC FOCUSING AND CONTROL OF THE TRANSVERSE DYNAMICS

⇒ Defocusing RF forces, space charge or the natural divergence (emittance) of the beam need to be compensated and controlled by focusing forces.

⇒ Quadrupoles are focusing in one plane and defocusing on the other. A global focalization is provided by alternating quadrupoles with opposite signs.

⇒ In a linac one alternates accelerating structures with focusing sections.

⇒ The type of magnetic configuration and magnets type/distance depend on the type of particles/energies/beam parameters we want to achieve.

This is provided by quadrupoles along the beam line. At low energies also solenoids can be used.
Due to the alternating quadrupole focusing system each particle perform transverse oscillations along the LINAC. The equation of motion in the transverse plane is of the type:

\[
\frac{d^2 x}{ds^2} + \left[ K^2(s) + k_{RF}(s) \right] x - F_{SC} = 0
\]

Term depending on the magnetic configuration

RF defocusing/focusing term

Space charge term

The single particle trajectory is a pseudo-sinusoid described by the equation:

\[
x(s) = \sqrt{\varepsilon(s)} \beta(s) \cos \left[ \int_s^s \frac{ds}{\beta(s)} + \phi_0 \right]
\]

Characteristic function (Twiss \( \beta \)-function [m]) that depend on the magnetic and RF configuration

Depend on the initial conditions of the particle

The final transverse beam dimensions \((\sigma_{x,y}(s))\) vary along the linac and are contained within an envelope

\[
\sigma = \int_{L_p}^L \frac{ds}{\beta(s)} \approx \frac{L_p}{\langle \beta \rangle}
\]

Transverse phase advance per period \(L_p\). For stability should be \(0 < \sigma < \pi\)
In case of “smooth approximation” of the LINAC (we consider an average effect of the quadrupoles and RF) we obtain a simple harmonic motion along s of the type ($\beta$ is constant):

$$x(s) = \sqrt{\varepsilon_o} \sqrt{1/K_0} \cos(K_0s + \phi_0)$$

Phase advance per unit length ($\sigma/L_p$)

$$K_0 = \sqrt{\left(\frac{qGl}{2m_0c\gamma\beta}\right)^2 - \frac{\pi q\hat{E}_{acc} \sin(-\phi)}{m_0c^2\lambda_{RF}(\gamma\beta)^3}} - \frac{3Z_0qI\lambda_{RF}(1-f)}{8\pi m_0c^2\beta^2\gamma^3r_xr_yr_z}$$

Magnetic focusing elements (for a FODO)

**NF:** the RF defocusing term $\propto f$ sets a higher limit to the working frequency.

If we consider also the **Space Charge contribution** in the simple case of an **ellipsoidal beam** (linear space charges) we obtain:

Space charge term

For ultrarelativistic electrons RF defocusing and space charge disappear and the external focusing is required to control the emittance and to stabilize the beam against instabilities.
GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (1/2)

⇒ Beam dynamics dominated by space charge and RF defocusing forces
⇒ Focusing is usually provided by quadrupoles
⇒ Phase advance per period (σ) should be, in general, in the range 30-80 deg, this means that, at low energy, we need a strong focusing term (short quadrupole distance and high quadrupole gradient) to compensate for the rf defocusing, but the limited space (βλ) limits the achievable G and beam current.
⇒ As β increases, the distance between focusing elements can increase (βλ in the DTL goes from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV), and can be increased to 4-10βλ at higher energy (>40 MeV).
⇒ A linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy. As β increases, longitudinal phase error between cells of identical length becomes small and we can have short sequences of identical cells (lower construction costs).
⇒ Keep sufficient safety margin between beam radius and aperture

Transverse (x) r.m.s. beam envelope along Linac4

![Graph showing transverse beam envelope](image)

Courtesy A. Lombardi

PROTONS AND IONS
Space charge only at low energy and/or high peak current: below 10-20 MeV (injector) the beam dynamics optimization has to include emittance compensation schemes with, typically solenoids;

At higher energies no space charge and no RF defocusing effects occur but we have RF focusing due to the ponderomotive force: focusing periods up to several meters

Optics design has to take into account longitudinal and transverse wakefields (due to the higher frequencies used for acceleration) that can cause energy spread increase, head-tail oscillations, multibunch instabilities,...

Longitudinal bunch compressors schemes based on magnets and chicanes have to take into account, for short bunches, the interaction between the beam and the emitted synchrotron radiation (Coherent Synchrotron Radiation effects)

All these effects are important especially in LINACs for FEL that requires extremely good beam qualities

![Graph](image1.jpg)

**GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (2/2)**

**ELECTRONS**

**ELECTRONS**

**ELECTRONS**

**ELECTRONS**
RADIO FREQUENCY QUADRUPOLES (RFQ)

At low proton (or ion) energies ($\beta \sim 0.01$), space charge defocusing is high and quadrupole focusing is not very effective. Moreover, cell length becomes small and conventional accelerating structures (DTL) are very inefficient. At this energy it is used a (relatively) new structure, the Radio Frequency Quadrupole (1970).

These structures allow to simultaneously provide:

- Transverse Focusing
- Acceleration
- Bunching of the beam
RFQ: PROPERTIES

1-Focusing
The resonating mode of the cavity (between the four electrodes) is a focusing mode: Quadrupole mode (TE_{210}). The alternating voltage on the electrodes produces an alternating focusing channel with the period of the RF (electric focusing does not depend on the velocity and is ideal at low $\beta$).

2-Acceleration
The vanes have a longitudinal modulation with period = $\beta \lambda_{RF}$. This creates a longitudinal component of the electric field that accelerates the beam (the modulation corresponds exactly to a series of RF gaps).

3-Bunching
The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

The RFQ is the only linear accelerator that can accept a low energy continuous beam.

Courtesy A. Lombardi

Courtesy M. Vretenar and A. Lombardi
RFQ: EXAMPLES

The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA, 425 MHz

The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle

TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA
THE CHOICE OF THE ACCELERATING STRUCTURE

In general **the choice of the accelerating structure depends on:**

- **Particle type**: mass, charge, energy
- **Beam current**
- **Duty cycle** (pulsed, CW)
- **Frequency**
- **Cost** of fabrication and of operation

Moreover a **given accelerating structure has also a curve of efficiency** (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

As example a very general scheme is given in the Table (absolutely not exhaustive).

<table>
<thead>
<tr>
<th>Cavity Type</th>
<th>β Range</th>
<th>Frequency</th>
<th>Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFQ</td>
<td>0.01– 0.1</td>
<td>40-500 MHz</td>
<td>Protons, Ions</td>
</tr>
<tr>
<td>DTL</td>
<td>0.05 – 0.5</td>
<td>100-400 MHz</td>
<td>Protons, Ions</td>
</tr>
<tr>
<td>SCL</td>
<td>0.5 – 1</td>
<td>600 MHz-3 GHz</td>
<td>Protons, Electrons</td>
</tr>
<tr>
<td>SC Elliptical</td>
<td>&gt; 0.5-0.7</td>
<td>350 MHz-3 GHz</td>
<td>Protons, Electrons</td>
</tr>
<tr>
<td>TW</td>
<td>1</td>
<td>3-12 GHz</td>
<td>Electrons</td>
</tr>
</tbody>
</table>
Medical applications

Neutron spallation sources

Security: Cargo scans

Injectors for colliders and synchrotron light sources

Industrial applications:
- Ion implantation for semiconductors
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