Concept of Luminosity in particle colliders

(or: explaining the jargon...)

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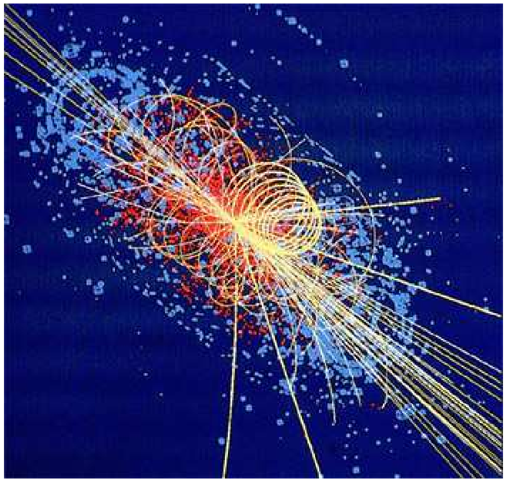
http://cern.ch/Werner.Herr/CAS2010/lectures/Varna_luminosity.pdf

Concept of Luminosity in particle colliders

(or: explaining the jargon*) ...)

) (beta, squeeze, inverse femtobarn, lumi scan, crossing angle, filling schemes, hour glass effect, crab crossing ...)

Particle colliders ?



- Used in particle physics
- Look for rare interactions
- Want highest energies
- Many interactions (events)

Figures of merit for a collider:

- energy
- number of collisions

Why colliding beams ?

- **Two beams:** $E_1, \vec{p_1}, E_2, \vec{p_2}, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2}$
- **Collider versus fixed target:**

  Fixed target: $\vec{p_2} = \mathbf{0}$ ➡ $E_{cm} = \sqrt{2m^2 + 2E_1 m}$

  Collider: $\vec{p_1} = -\vec{p_2}$ ➡ $E_{cm} = E_1 + E_2$
- **LHC (pp): 14000 GeV versus $\approx$ 115 GeV**
- **LEP ($\mathbf{e^+e^-}$): 210 GeV versus ?**

Collider performance issues

- Available energy
- Number of interactions per second (useful collisions)
- Total number of interactions
- Secondary issues:
  - Time structure of interactions (how often and how many at the same time)
  - Space structure of interactions (size of interaction region)
  - Quality of interactions (background, dead time etc.)

Luminosity:

We want:

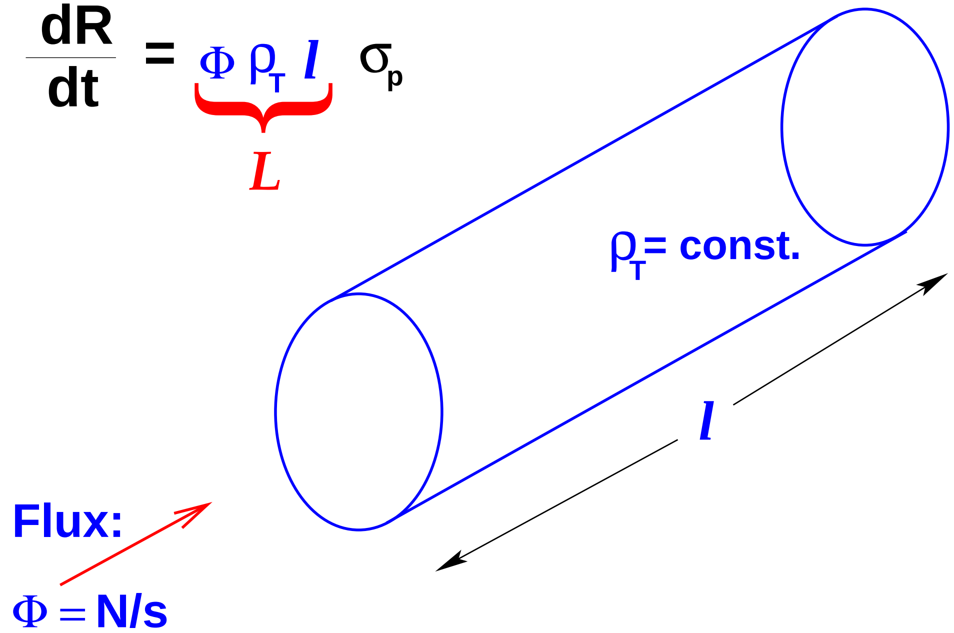
- **Proportionality factor between cross section σ_p and number of interactions per second $\frac{dR}{dt}$**

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p \qquad (\rightarrow \quad \mathrm{units:} \ \ \mathrm{cm}^{-2}\mathrm{s}^{-1})$$

- **Relativistic invariant**
- **Independent of the physical reaction**
- **Reliable procedures to compute and measure**

Fixed target luminosity

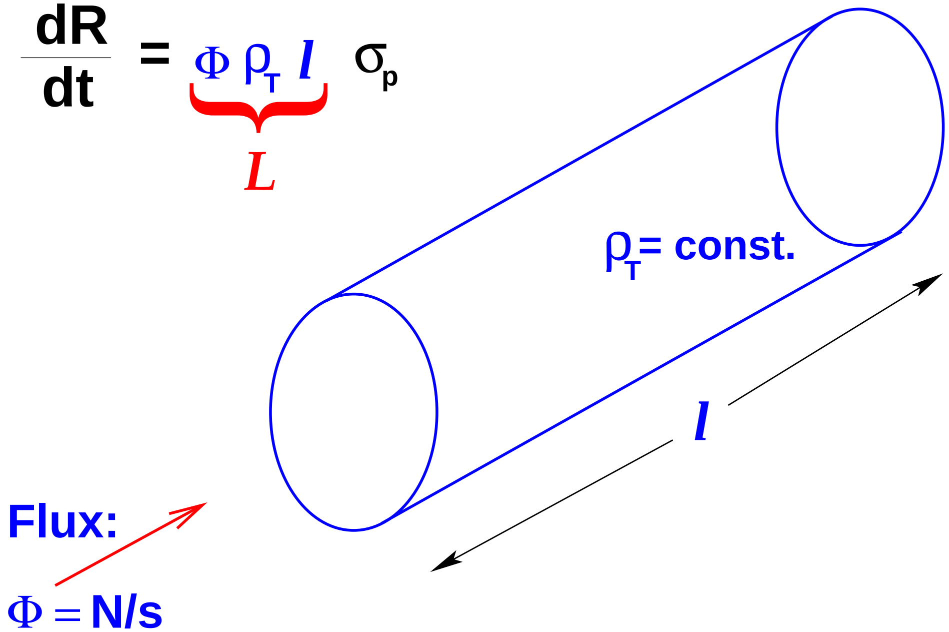
$$\frac{dR}{dt} = \underbrace{\Phi\, \rho_T\, l}_{L}\ \sigma_p$$



- Interaction rate from flux and target density and size

Fixed target luminosity

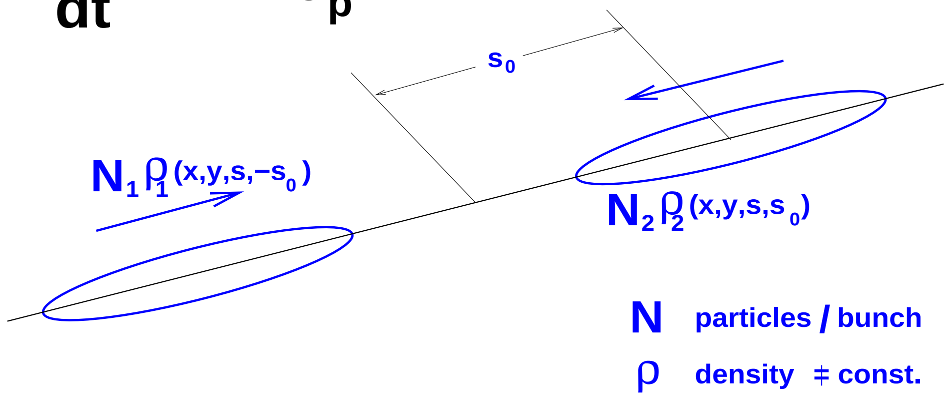
$$\frac{dR}{dt} = \underbrace{\Phi\, \rho_T\, l}_{L}\; \sigma_p$$



- In a collider: target is the other beam ... (and it is moving !)

Collider luminosity (per bunch crossing)

$$\frac{dR}{dt} = L\,\sigma_p$$



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

s_0 is "time"-variable: $s_0 = c\cdot t$

Kinematic factor: $K = \sqrt{(\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2/c^2}$

Collider luminosity (per beam)

- **Assume uncorrelated densities in all planes**

→ **factorize:** $\rho(x, y, s, s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$

- **For head-on collisions ($\vec{v_1} = -\vec{v_2}$) we get:**

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$

$$\rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s + s_0)$$

- **In principle: should know all distributions**

→ **Mostly use Gaussian ρ for analytic calculation (in general: it is a good approximation)**

Gaussian distribution functions

- $$\rho_z(u) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \qquad u = x, y$$

- $$\rho_s(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

- **For non-Gaussian profiles not always possible to find analytic form, need a numerical integration**

Luminosity for two beams (1 and 2)

- **Simplest case : equal beams**
  - $\sigma_{1x} = \sigma_{2x}, \qquad \sigma_{1y} = \sigma_{2y}, \qquad \sigma_{1s} = \sigma_{2s}$
  - **but:** $\qquad \sigma_{1x} \neq \sigma_{1y}, \quad \sigma_{2x} \neq \sigma_{2y}$ **is allowed**
- **Further: no dispersion at collision point**

Integration (head-on)

for beams of equal size: $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1 \rho_2 = \rho^2$ **:**

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx dy ds ds_0$$

integrating over s **and** s_0**, using:**

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y: $\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$

Luminosity for two (equal) beams (1 and 2)

- **Simplest case :** $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

  or: $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y}, \quad but : \sigma_{1s} \approx \sigma_{2s}$

$$\Longrightarrow \quad \boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}\left(\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}\right)}$$

- **Here comes β^*:** $\qquad \sigma_{x,y} = \sqrt{\epsilon \cdot \beta^*_{x,y}}$

Luminosity for two (equal) beams (1 and 2)

- **Simplest case :** $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

  or: $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y},\quad but : \sigma_{1s} \approx \sigma_{2s}$

$$\Longrightarrow \quad \boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \left(\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \right)}$$

- **Would you increase n_b or N_1, N_2 ?**
- **Special case LHC:** $n_b \rightarrow n_{coll}$ **(filling schemes)**

Examples

	Energy (GeV)	$\mathcal{L}_{max}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$	rate s^{-1}	σ_x/σ_y μm/μm	Particles per bunch
SPS (p$\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron (p$\bar{p}$)	1000x1000	$100\ 10^{30}$	$7\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA (e^+p)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$\approx 11\ 10^{10}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 50\ 10^{10}$
PEP (e^+e^-)	9x3	$8000\ 10^{30}$	NA	150/5	$\approx 2/6\ 10^{10}$

What else ?

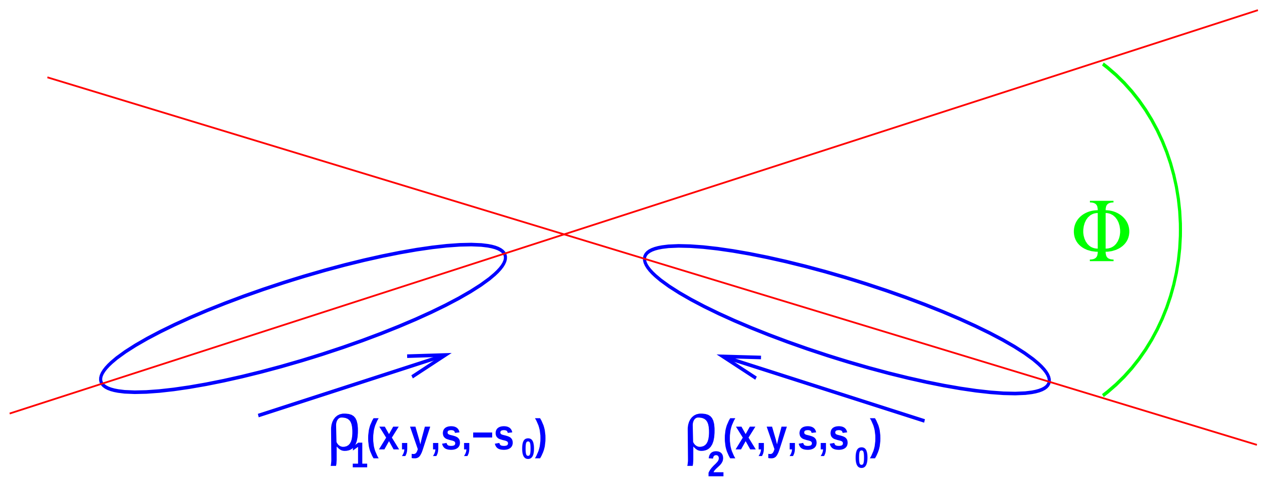
- **What about linear colliders ?**

➡ **See later ...**

Complications

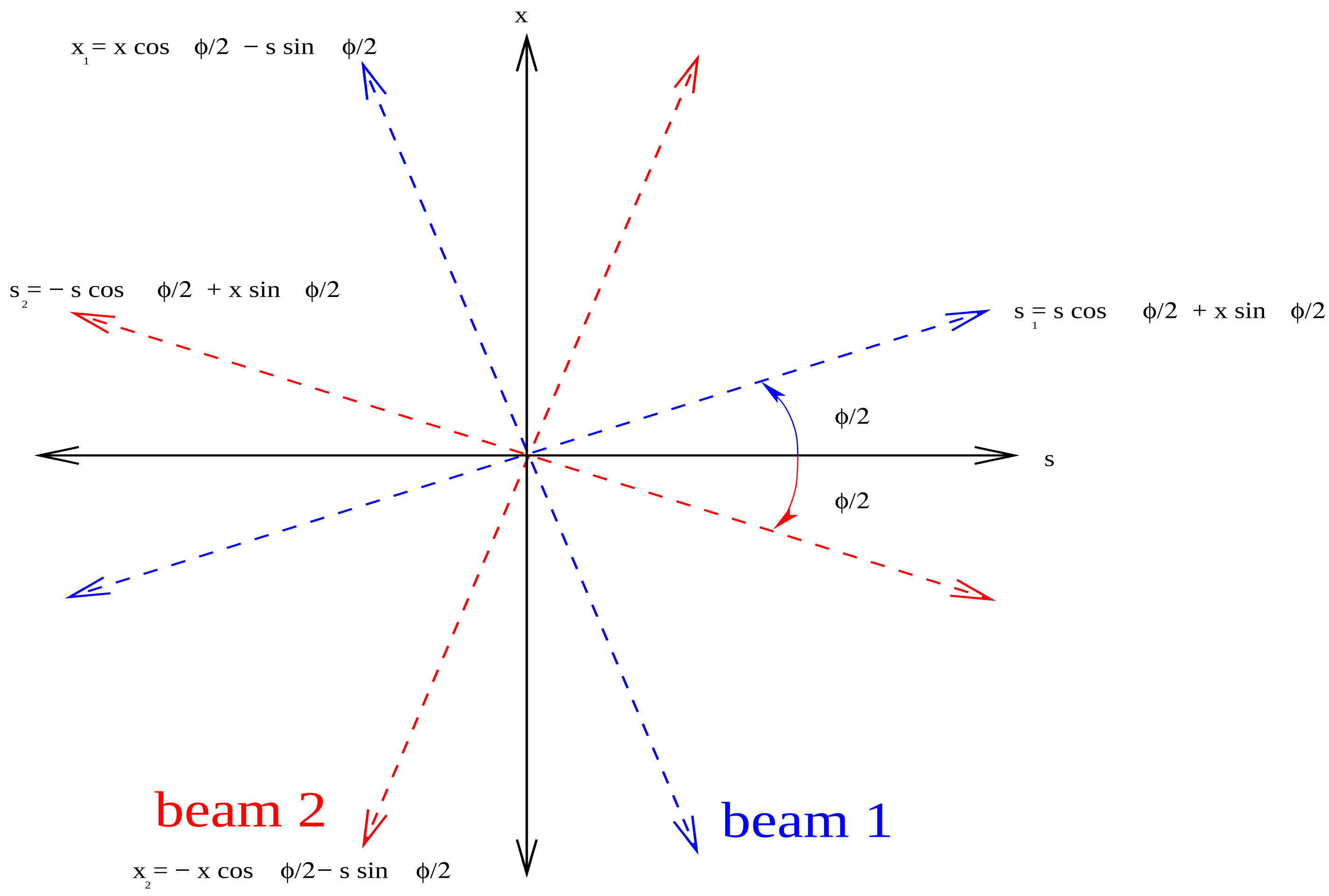
- **Crossing angle**
- **Hour glass effect**
- **Collision offset (wanted or unwanted)**
- **Non-Gaussian profiles**
- **Dispersion at collision point**
- $\delta\beta^*/\delta s \ = \ \alpha^* \ \neq \ 0$
- **Strong coupling**
- **etc.**

Collisions at crossing angle



- Needed to avoid unwanted collisions
  - For colliders with many bunches: LHC, CESR, KEKB
  - For colliders with coasting beams

Collisions angle geometry (horizontal plane)



Crossing angle

Assume crossing in horizontal (x, s)- plane.
Transform to new coordinates:

$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, \quad & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, \quad & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\boxed{\begin{aligned} \mathcal{L} = 2\cos^2\frac{\phi}{2} N_1 N_2 f n_b \int\int\int\int_{-\infty}^{+\infty} dx\,dy\,ds\,ds_0 \\ \rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0) \end{aligned}}$$

Integration (crossing angle)

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt \quad = \quad \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

Further:

- **Since σ_x, x and sin($\phi/2$) are small:**
  - **drop all terms $\sigma^k{}_x sin^l(\phi/2)$ or $x^k sin^l(\phi/2)$ for all:** **k+l** $\geq$ 4
  - **approximate: sin($\phi/2$) $\approx$ tan($\phi/2$)$\approx$ $\phi/2$**

Crossing angle

- **Crossing Angle** $\Rightarrow$ $\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S}$

- S **is the geometric factor**

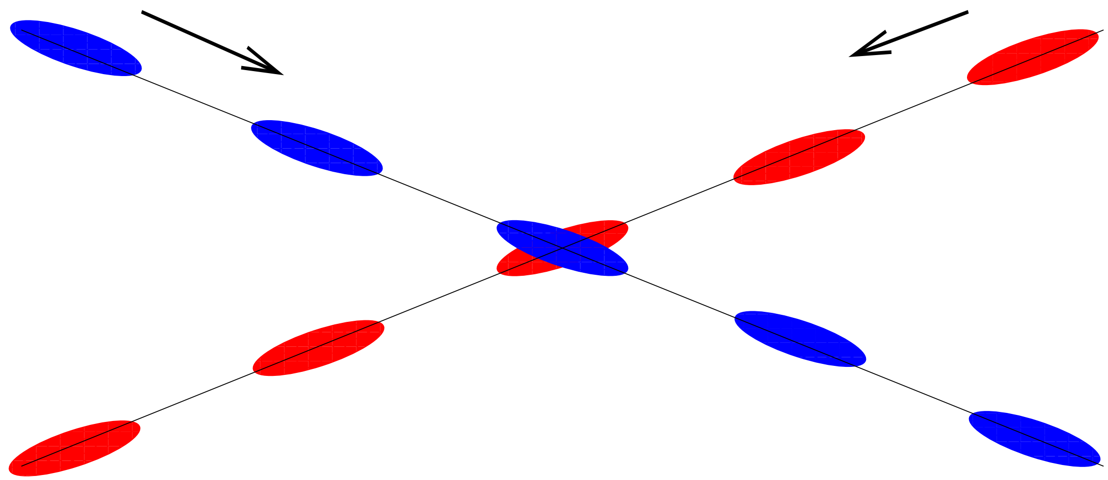
- **For small crossing angles and** $\sigma_s \gg \sigma_{x,y}$

$$\Rightarrow S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

Example LHC (at 7 TeV):

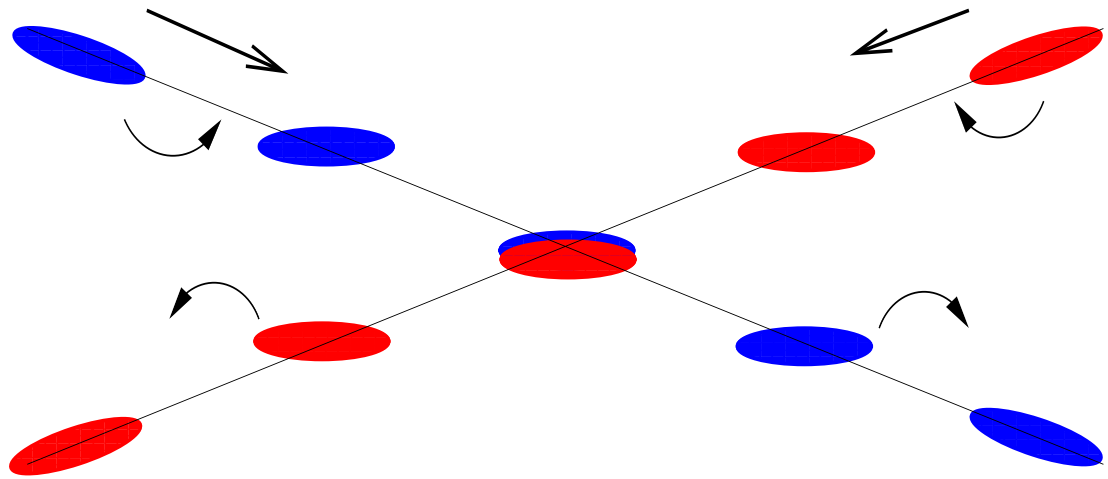
Φ **= 285** μ**rad,** $\sigma_x \approx$ **17** μ**m,** σ_s **= 7.5 cm,** **S = 0.84**

Large crossing angle



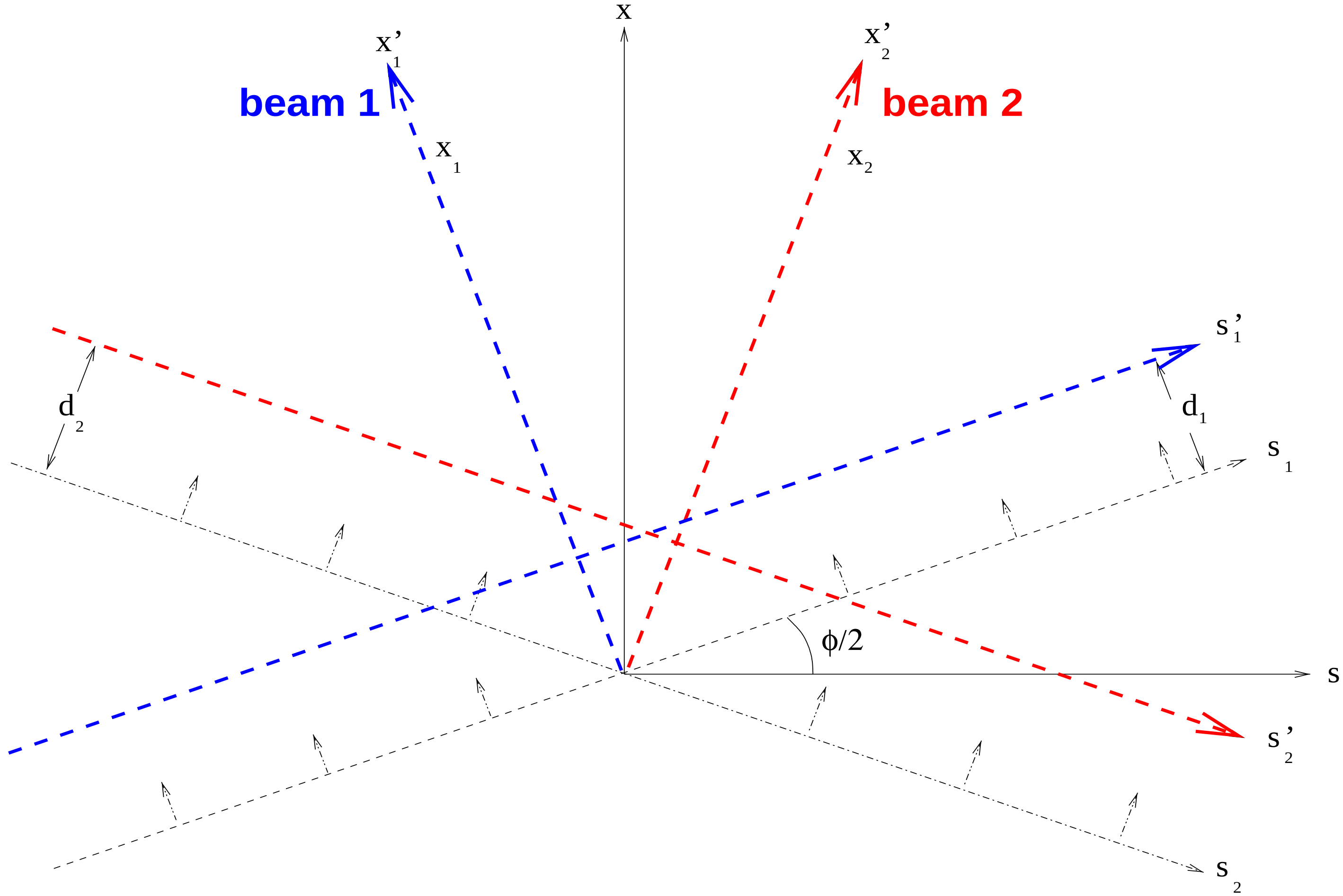
- Large crossing angle: large loss of luminosity
- "crab" crossing can recover geometric factor

"crab" crossing scheme



- Done with transversely deflecting cavities (if you wondered what they can be used for)
- Feasibility needs to be demonstrated

Offset and crossing angle



Offset and crossing angle

- **Transformations with offsets in crossing plane:**

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

- **Gives after integration over y and s_0:**

$$\mathcal{L} = \frac{\mathcal{L}_0}{2\pi\sigma_s\sigma_x} 2\cos^2\frac{\phi}{2} \int\int e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times \quad e^{-\frac{d_1^2+d_2^2+2(d_1+d_2)x\cos(\phi/2)-2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}} dxds.$$

Offset and crossing angle

After integration over x:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}}\sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int_{-\infty}^{+\infty} W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} \qquad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2}$$

and $\quad W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$

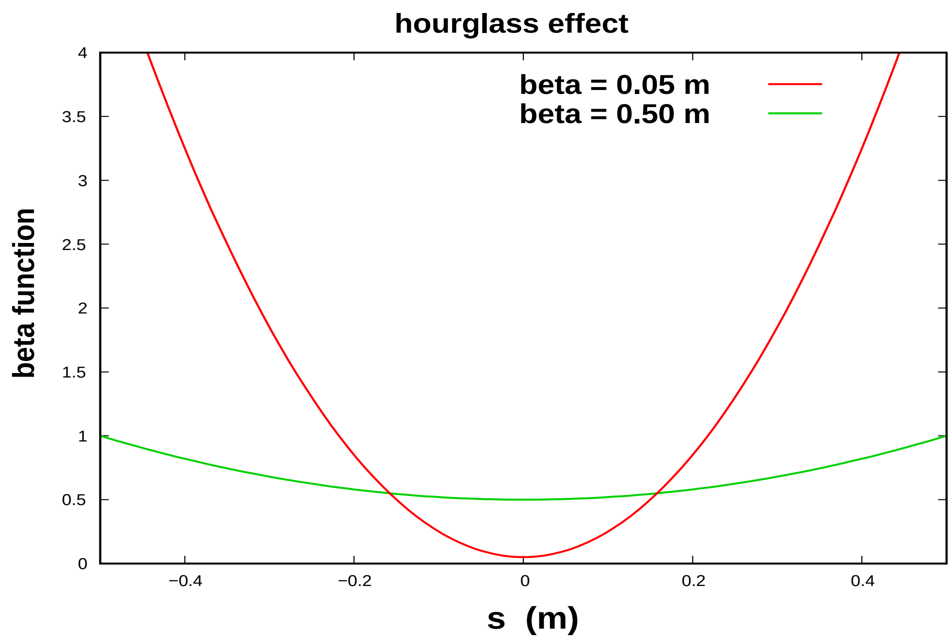
$\Longrightarrow$ **After integration: Luminosity with correction factors**

Luminosity with correction factors

$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S}$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**

Hour glass effect

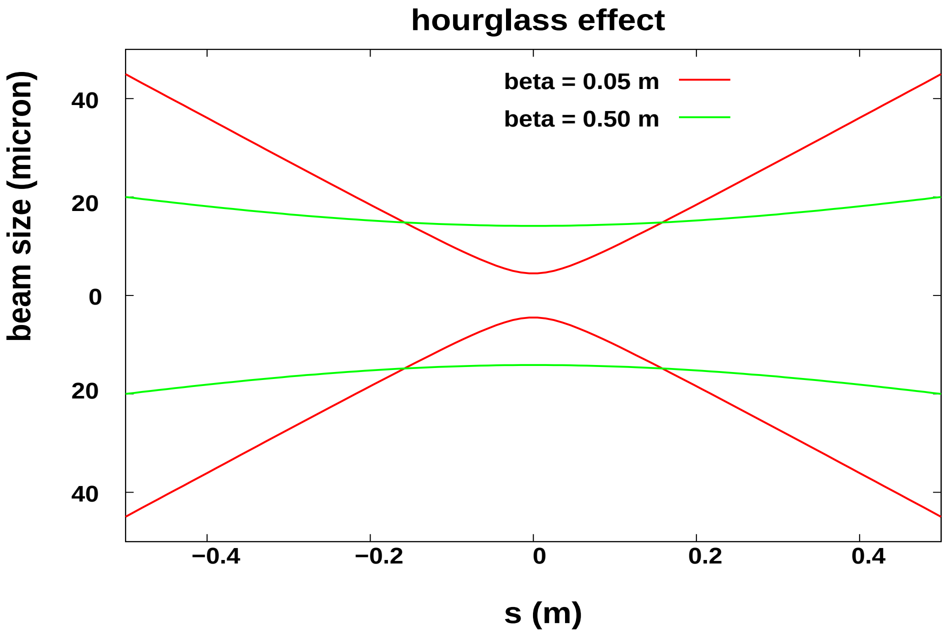


- **β-functions depends on position s**
- $\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$

Hour glass effect

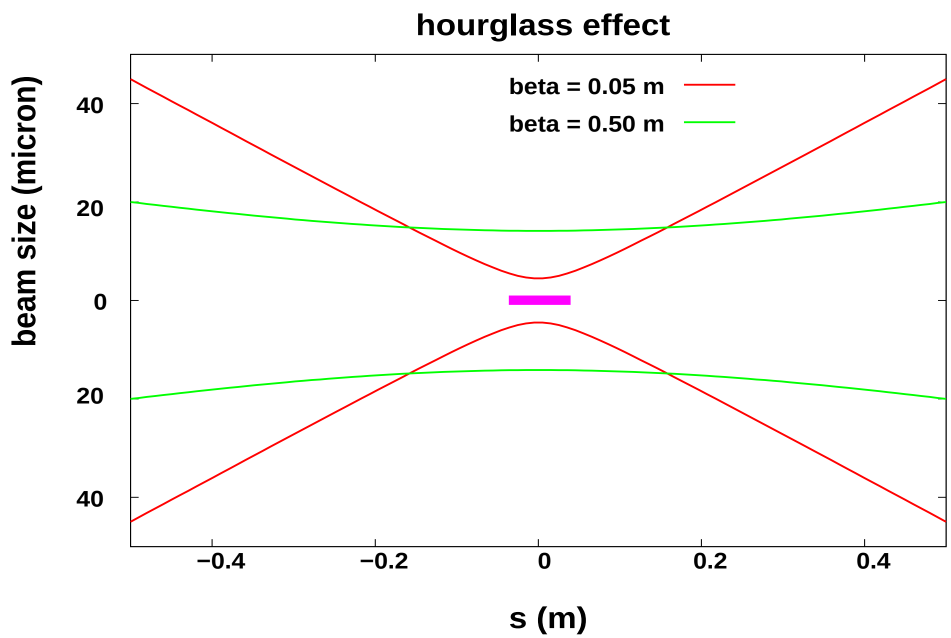
- β-functions depends on position s

Hour glass effect



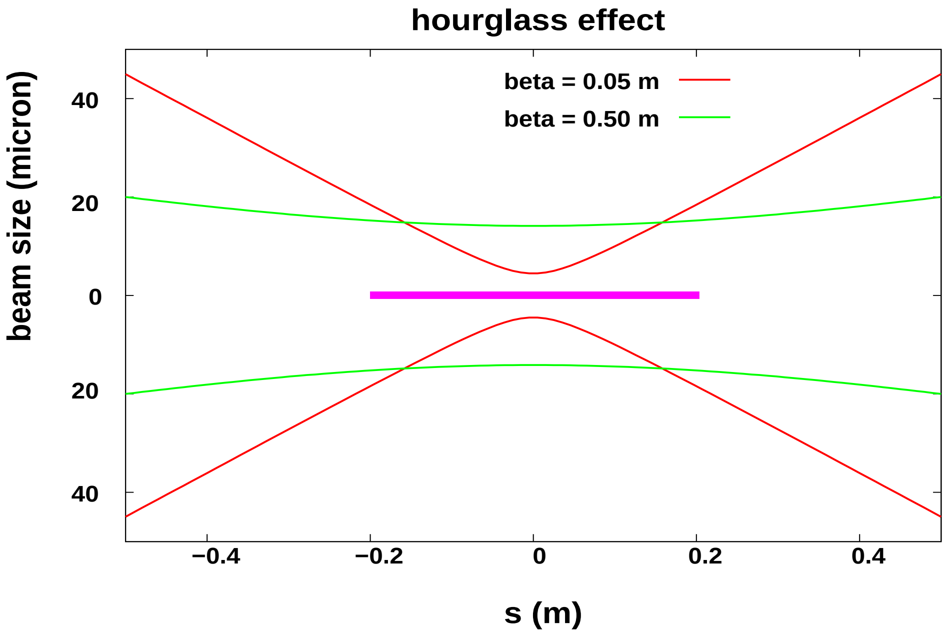
- **Beam size** σ $\quad (\propto \sqrt{\beta^*(s)})$ **depends on position s**

Hour glass effect - short bunches



- Small variation of beam size along bunch

Hour glass effect - long bunches



- Significant effect for long bunches and small β^*

Hour glass effect

- **β-functions depends on position s**
- **Usually:** $\beta(s) = \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$
  - → **i.e.** $\sigma \implies \sigma(s) \neq$ **const.**
  - → $\sigma(s) = \sigma^* \sqrt{(1 + \left(\frac{s}{\beta^*}\right)^2)}$
- **Important when β^* comparable to the r.m.s. bunch length σ_s (or smaller !)**

Hour glass effect

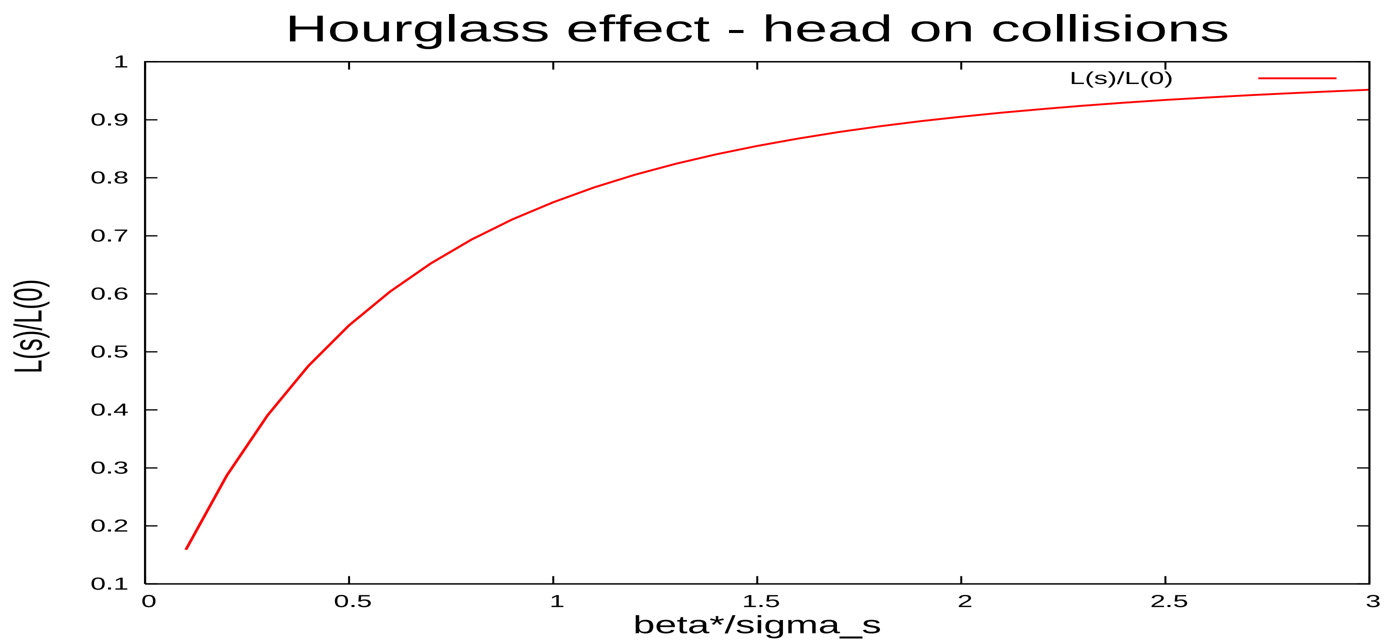
- **Without crossing angle and for symmetric, round Gaussian beams we get the relative luminosity reduction as:**

$$\frac{\mathcal{L}(\sigma_s)}{\mathcal{L}(0)} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-u^2}}{[1 + (\frac{u}{u_x})^2]} du = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

Using the expression: $u_x = \beta^* / \sigma_s$

$$\mathcal{L}(\sigma_s) = \mathcal{L}(0) \cdot H \quad \text{with :} \quad H = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

Hour glass effect



➡ **Hourglass reduction factor as function of ratio β^*/σ_s.**

Luminosity with (more) correction factors

$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**
- H: **correction for hour glass effect**

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**
- $n_b = 2808$ **bunches/beam**
- $f = 11.2455$ kHz, $\quad \phi = 285\ \mu$**rad**
- $\beta_x^* = \beta_y^* = 0.55$ m
- $\sigma_x^* = \sigma_y^* = 16.6\ \mu$m, $\quad \sigma_s = 7.7$ cm

- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.200 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle:**

$$\mathcal{L} = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

If the beams are not Gaussian ??

Exercise:

- **Assume flat distributions (normalized to 1)**

  $\rho_1 = \rho_2 = \frac{1}{2a}$, **for** $[-a \leq z \leq a]$**,** $z = x, y$

  Calculate r.m.s. in x and y:

$$< (x,y)^2 > = \int_{-\infty}^{+\infty} (x,y)^2 \cdot \rho(x,y) dxdy$$

and

$$\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x,y)\rho_2(x,y)\ dxdy$$

- **Compute:** $\mathcal{L} \cdot \sqrt{< x^2 > \cdot < y^2 >}$
- **Repeat for various distributions and compare**

Integrated luminosity

- $$\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L}(t)dt$$
- **The figure of merit:**
  $\mathcal{L}_{\text{int}} \cdot \sigma_p =$ **number of events**
- **Unit is:** cm^{-2}**, i.e. inverse cross-section**
- **Often expressed in inverse** $barn$ $(10^{-24}\ cm^2)$
- **1 fbarn**$^{-1}$ **is** $10^{39} cm^{-2}$
- **for 1 fbarn**$^{-1}$**: requires** 10^7**s at L** $= 10^{32} cm^{-2} s^{-1}$

Integrated luminosity

- **Experiments: continuous recording of $\mathcal{L}$**
- **For studies: assume some life time behaviour.**
  E.g. $\mathcal{L}(t) \longrightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- **Contributions to life time from: intensity decay, emittance growth etc.**

Integrated luminosity

- **Knowledge of preparation time allows optimization of $\mathcal{L}_{\text{int}}$**

Integrated luminosity

- **Typical run times LEP:**
  t_r $\approx$ **8 - 10 hours**
- **For LHC long preparation time t_p expected**

→ **Optimum combination of t_r and t_p gives maximum luminosity**

→ t_r **is usually a "free" parameter, i.e. can be chosen**

Maximising Integrated Luminosity

- **Assume exponential decay of luminosity**
  $\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{t/\tau}$

- **Average (integrated) luminosity** $< \mathcal{L} >$
  $< \mathcal{L} >= \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$

- **(Theoretical) maximum for:**
  $t_r \approx \tau \cdot \ln(1 + \sqrt{2t_p/\tau} + t_p/\tau)$

- **Example LHC:** $t_p \approx$ **10h,** $\tau \approx$ **15h,** $\Rightarrow t_r \approx$ **15h**

- **Exercise: Would you improve** τ **(long** t_r**) or** t_p **?**

Interactions per crossing

- **Luminosity/$fn_b \propto$ $\mathbf{N_1 N_2}$**
- **In LHC: crossing every 25 ns**
- **Per crossing approximately 20 interactions**
- **May be undesirable (pile up in detector)**
- $\Longrightarrow$ **more bunches n_b, or smaller N ??**

Beware: maximum (peak) luminosity $\mathcal{L}_{max}$

is not the whole story ... !

Luminosity measurement

- One needs to get a signal proportional to interaction rate → Beam diagnostics
- Large dynamic range:
  10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}
- Very fast, if possible for individual bunches
- Used for optimization
- For absolute luminosity need calibration

Luminosity calibration

- **Remember the basic definition:**

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$

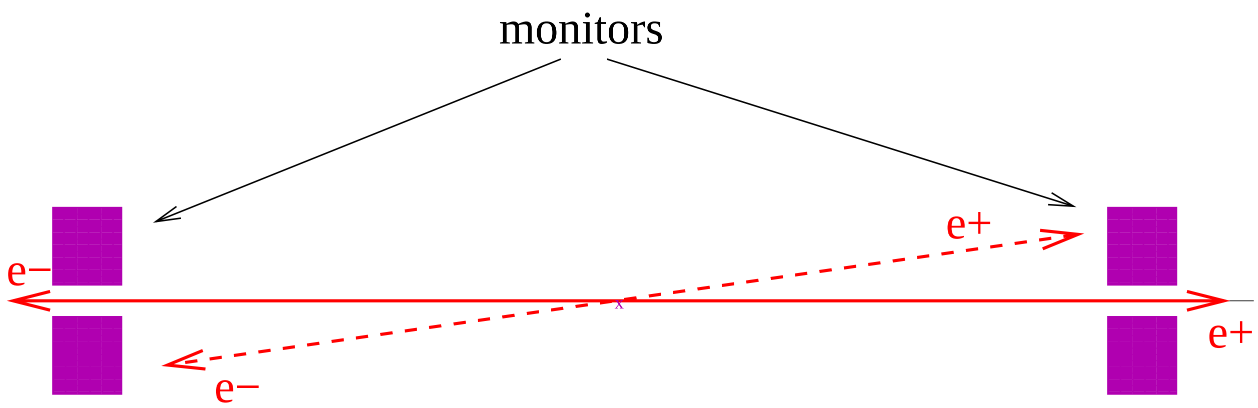
- **For a well known and calculable process we know** σ_p
- **The experiments measure the counting rate** $\frac{dR}{dt}$ **for this process**
- **Get the absolute, calibrated luminosity**

Luminosity calibration

(e^+e^-)

- **Use well known and calculable process**
- $e^+e^- \rightarrow e^+e^-$ **elastic scattering (Bhabha scattering)**
- **Have to go to small angles** $(\sigma_{el} \propto \Theta^{-3})$
- **Small rates at high energy** $(\sigma_{el} \propto \frac{1}{E^2})$

Luminosity calibration



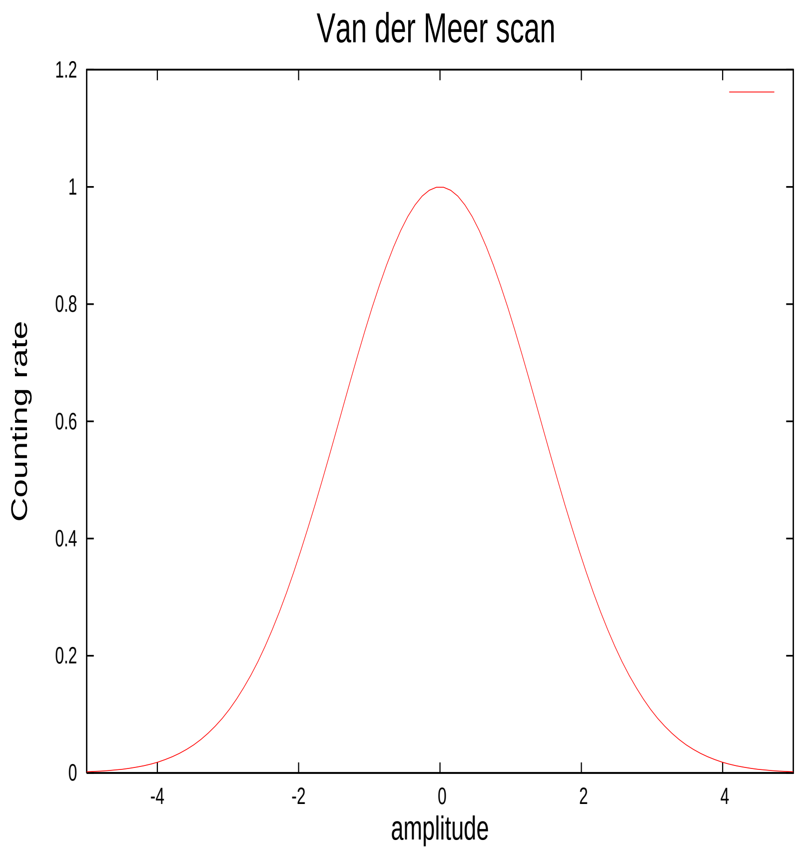
- Measure coincidence at small angles
- Low counting rates, in particular for high energy !
- Background may be problematic

Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- **Must measure beam current and beam sizes**
- **Beam size measurement:**
  - **Wire scanner or synchrotron light monitors**
  - **Measurement with beam ... $\rightarrow$ remember luminosity with offset**
  - **Move the two beams against each other in transverse planes (van der Meer scan, ISR 1973 - LHC 2010)**

Luminosity optimization



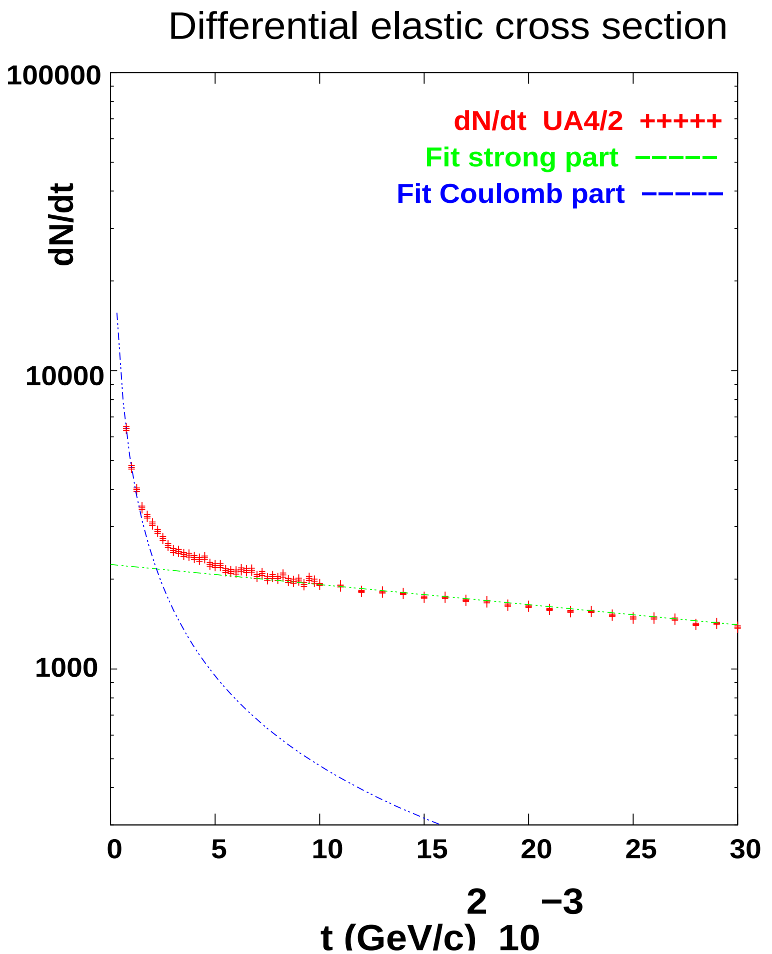
- Record counting rates ***R(d)*** as function of movement ***d***
- Since ***R(d)*** is proportional to luminosity ***L(d)***
- Get ratio of luminosity ***L(d)/L(0)***

Luminosity optimization

- From ratio of luminosity $\mathcal{L}(\mathrm{d})/\mathcal{L}_0$
- Remember: $W = e^{-\frac{1}{4\sigma^2}(d_2-d_1)^2}$
- Determines σ
- ... and centres the beams !
- Others:
  - Beam-beam deflection scans **LEP**
  - Beam-beam excitation

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By Coulomb normalization:**
  - **Coulomb amplitude exactly calculable:**
  - $$\lim_{t\to 0}\frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}\Big|_{t=0} = \pi \mid f_C + f_N \mid^2$$
    $$\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i)e^{b\frac{t}{2}} \mid^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \Big|_{|t|\to 0}$$
  - **Fit gives:** σ_{tot}, ρ, b **and** $\mathcal{L}$
- **Can be done measuring elastic scattering at small angles**



- Measure dN/dt at small t (0.01 < (GeV/c)**2) and extrapolate to t = 0.0

- Needs special optics to allow measurement at very small t

- Measure total counting rate N_{el} + N_{inel}
  Needs good detector coverage

- Often use slightly modified method, precision 1 − 2 %

Luminosity in linear colliders

- Mainly (only) $e+e-$ colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Special issues:
  - Particles collide only once (dynamics) !
  - Particles collide only once (beam power) !

→ Must be taken into account

Luminosity in linear colliders

- **Basic formula:**

$$\text{From :}\quad \mathcal{L} = \frac{N^2\ f\ n_b}{4\pi\sigma_x\sigma_y} \quad \text{to :}\quad \mathcal{L} = \frac{N^2\ f_{rep}\ n_b}{4\pi\sigma_x\sigma_y}$$

- **Replace frequency f by repetition rate f_{rep}.**
- **And introduce effective beam sizes $\overline{\sigma_x}, \overline{\sigma_y}$:**

$$\mathcal{L} = \frac{N^2\ f_{rep}\ n_b}{4\pi\overline{\sigma_x}\ \overline{\sigma_y}}$$

Luminosity in linear colliders

- **Using the enhancement factor H_D:**

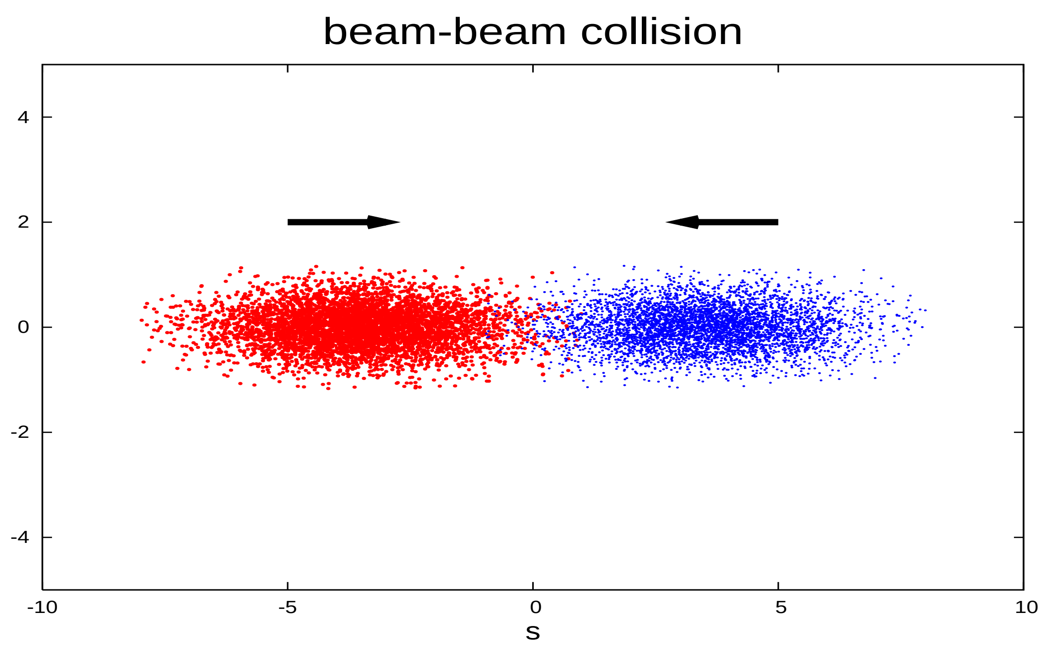
$$\mathcal{L} = \frac{N^2 \; f_{rep} \; n_b}{4\pi \overline{\sigma_x} \; \overline{\sigma_y}} \quad \Rightarrow \quad \mathcal{L} = \frac{H_D \cdot N^2 \; f_{rep} \; n_b}{4\pi \sigma_x \; \sigma_y}$$

- **Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (pinch effect)**

- **Related to disruption parameter $\mathcal{D}$:**

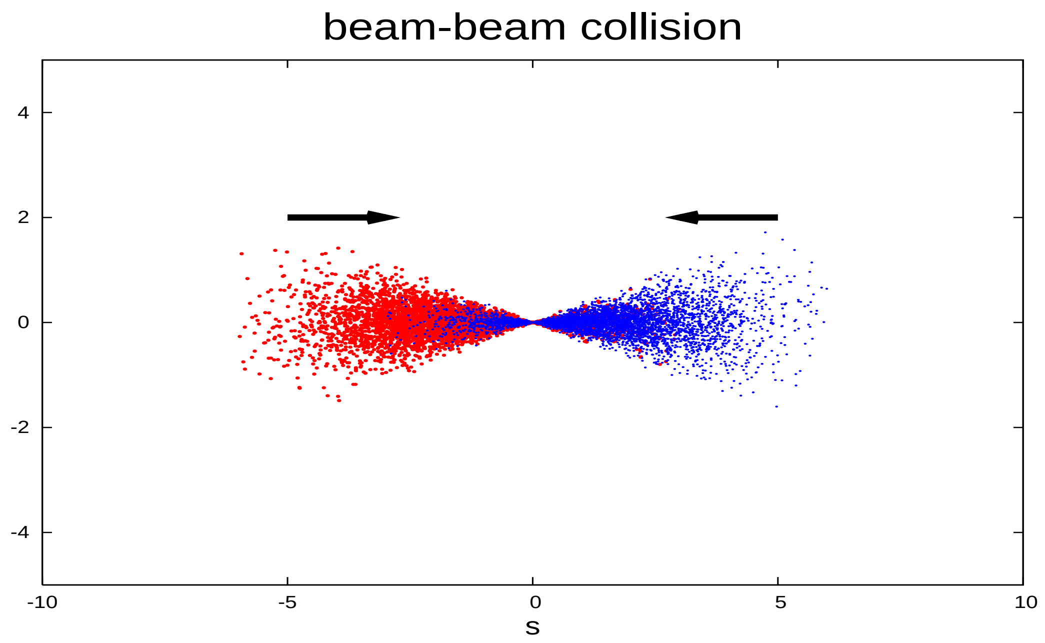
$$\mathcal{D}_{x,y} = \frac{2 r_e N \sigma_z}{\gamma \sigma_{x,y}(\sigma_x + \sigma_y)}$$

Pinch effect - disruption



- **Additional focusing by opposing beams**

Pinch effect - disruption



- **Additional focusing by opposing beams**

Luminosity in linear colliders

- **For weak disruption $\mathcal{D} \ll 1$ and round beams:**

$$H_D = 1 \; + \; \frac{2}{3\sqrt{\pi}}\mathcal{D} \; + \; \mathcal{O}(\mathcal{D}^2)$$

- **For strong disruption and flat beams: computer simulation necessary, maybe can get some scaling**

Beamstrahlung

- Disruption at interaction point is basically a strong "bending"
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
  - Spread of centre-of-mass energy
  - Pair creation and detector background
- Again: luminosity is not the only important parameter

Beamstrahlung Parameter Y

- **Measure of the mean field strength in the rest frame normalized to critical field B_c:**

$$Y = \frac{< E + B >}{B_c} \approx \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

with:

$$B_c = \frac{m^2 c^3}{e\hbar} \approx 4.4 \times 10^{13} G$$

Energy loss and power consumption

- **Average fractional energy loss δ_E:**

$$\delta_E = 1.24 \frac{\alpha \sigma_z m_e}{\lambda_C E} \frac{Y}{(1 + (1.5Y)^{2/3})^{1/2}}$$

where E is beam energy at interaction point and λ_C the Compton wavelength.

Luminosity in linear colliders

- **Using the beam power P_b and beam energy E in the luminosity:**

$$\mathcal{L} = \frac{H_D \cdot N^2 \ f_{rep} \ n_b}{4\pi\sigma_x \ \sigma_y} \quad \rightarrow \quad \mathcal{L} = \frac{H_D \cdot N \cdot P_b}{eE \cdot 4\pi\sigma_x \ \sigma_y}$$

- **Beam power P_b related to AC power consumption P_{AC} via efficiency η_b^{AC}**

$$P_b \quad = \quad \eta_b^{AC} \quad \cdot \quad P_{AC}$$

Figure of merit in linear colliders

- **Luminosity at given energy normalized to power consumption and momentum spread due to beamstrahlung:**

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b}P_{AC}}$$

- **With previous definition (and reasonably small beamstrahlung) this becomes:**

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b}P_{AC}} \quad \propto \quad \frac{\eta_b^{AC}}{\sqrt{\epsilon_y^*}}$$

- **These are optimized in the linear collider design**

Not treated :

- Coasting beams (e.g. ISR)
- Asymmetric colliders (e.g. PEP, HERA, LHeC)

How to cook high Luminosity ?

- **Get high intensity**
- **Get small beam sizes (small ϵ and β^*)**
- **Get many bunches**
- **Get small crossing angle (if any)**
- **Get exact head-on collisions**
- **Get short bunches**

Luminosity in a nutshell

$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

Luminosity in a nutshell

$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

- **Are there limits to what we can do ?**

Luminosity in a nutshell

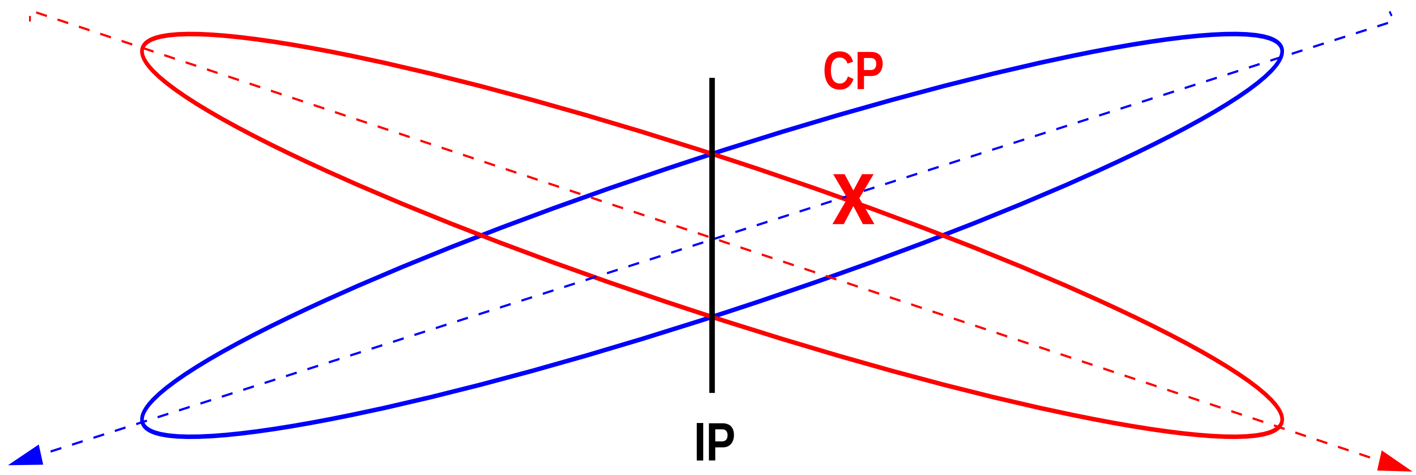
$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

- **Are there limits to what we can do ?**
- **Yes, there are beam-beam effects**

Luminosity in a nutshell

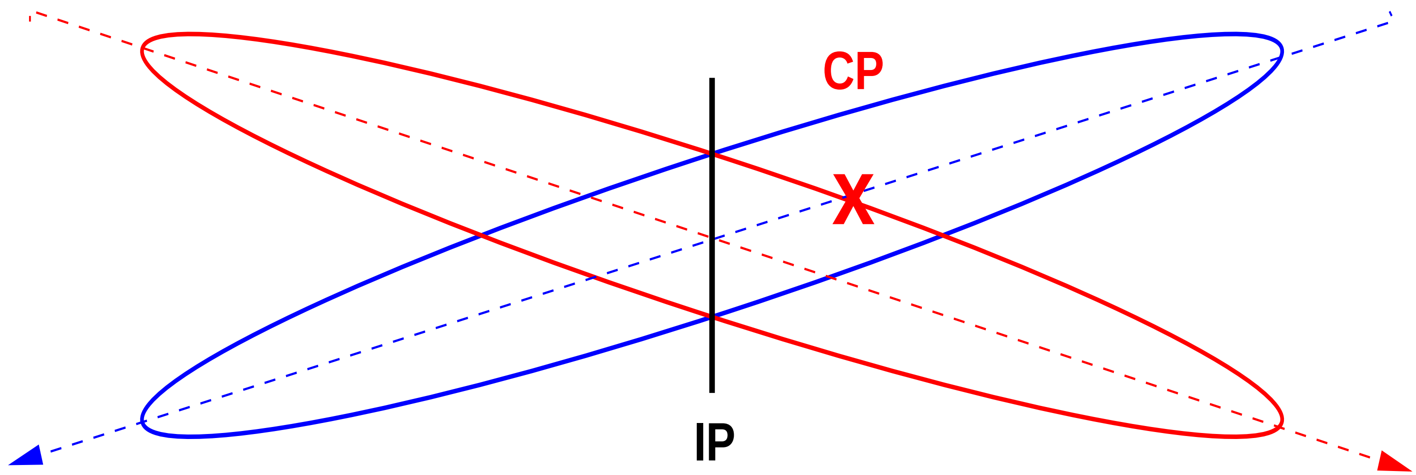
$$\boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

Large crossing angle



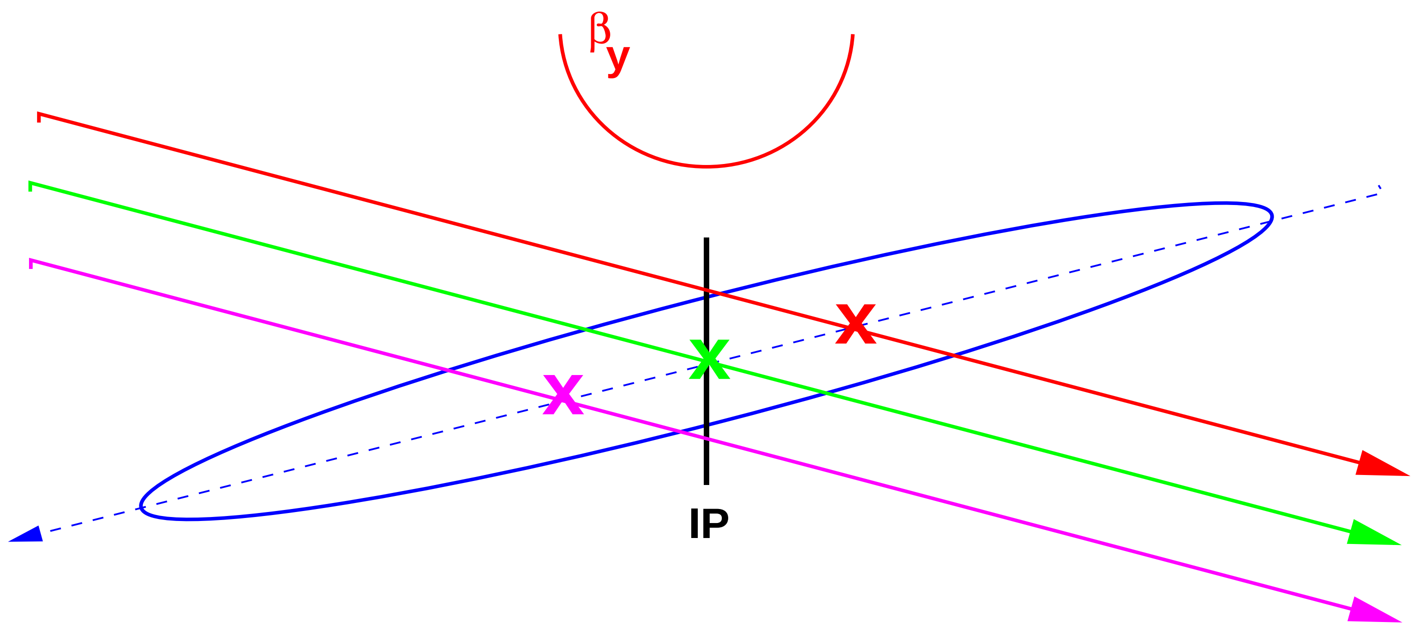
For large amplitude particles: collision point longitudinally displaced

Large crossing angle



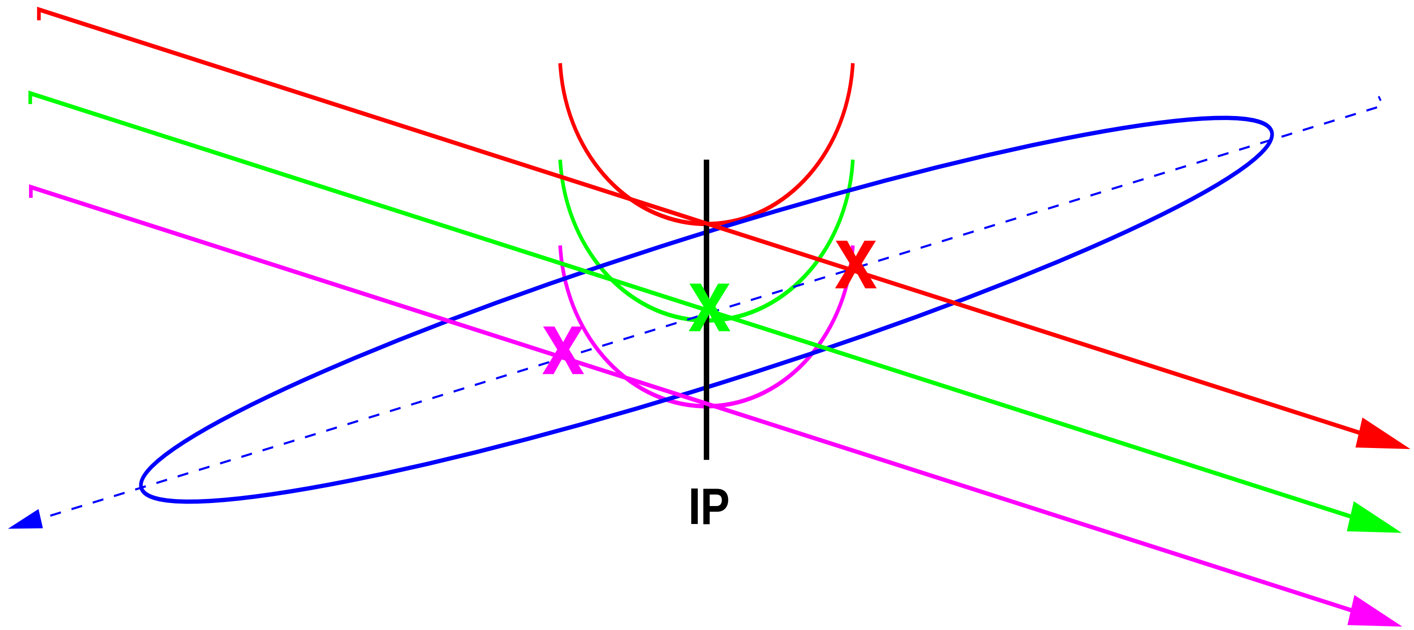
- For large amplitude particles: collision point longitudinally displaced
- Can introduce coupling (transverse and synchro betatron, bad for flat beams)

Large crossing angle



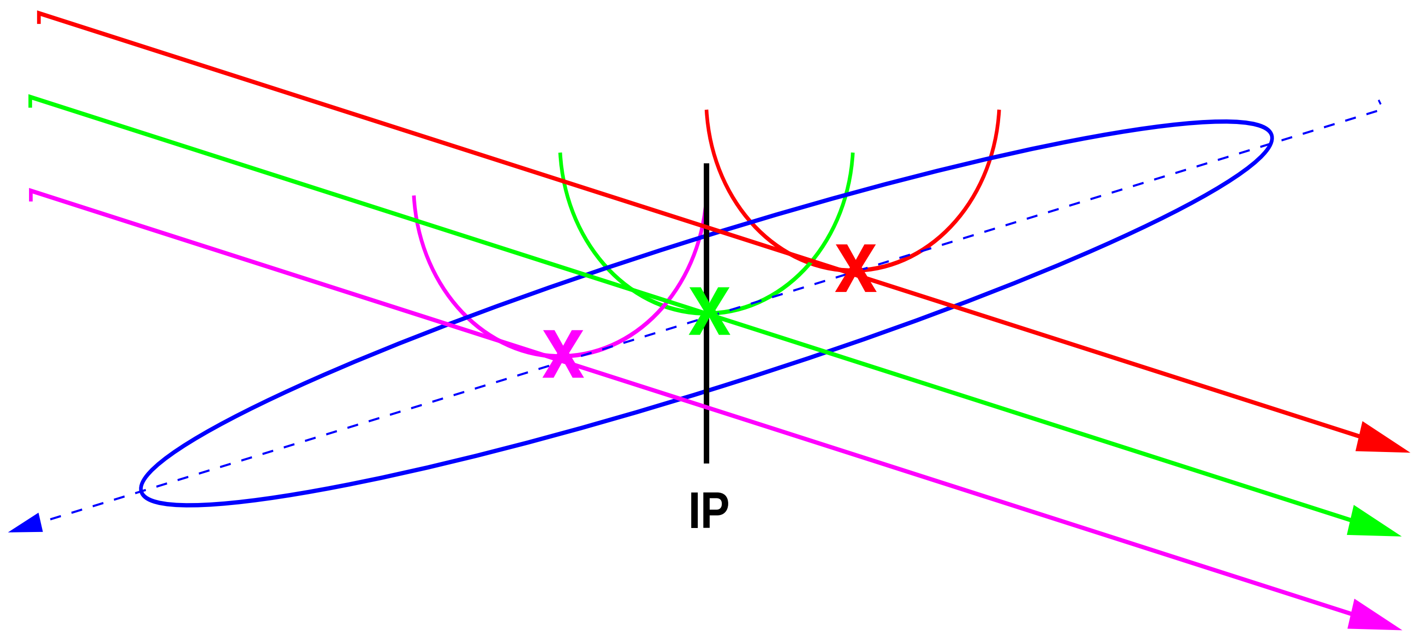
- A particle's collision point amplitude dependent
- Different (vertical) β functions at collision points

Large crossing angle



- A particle's collision point amplitude dependent
- Different β functions at collision points (hour glass !)

"crab waist" scheme



- **Make vertical waist (β_y^{min}) amplitude (x) dependent**
- **All particles in both beams collide in minimum β_y region**

"crab waist" scheme

- **Make vertical waist (minimum of β) amplitude (x) dependent**
- **Without details: can be done with two sextupoles**
- **First tried at DAPHNE (Frascati) in 2008**
- **Geometrical gain small**
- **Smaller vertical tune shift as function of horizontal coordinate**
  - **Less betatron and synchrotron coupling**
  - **Good remedy for flat (i.e. lepton) beams with large crossing angle**