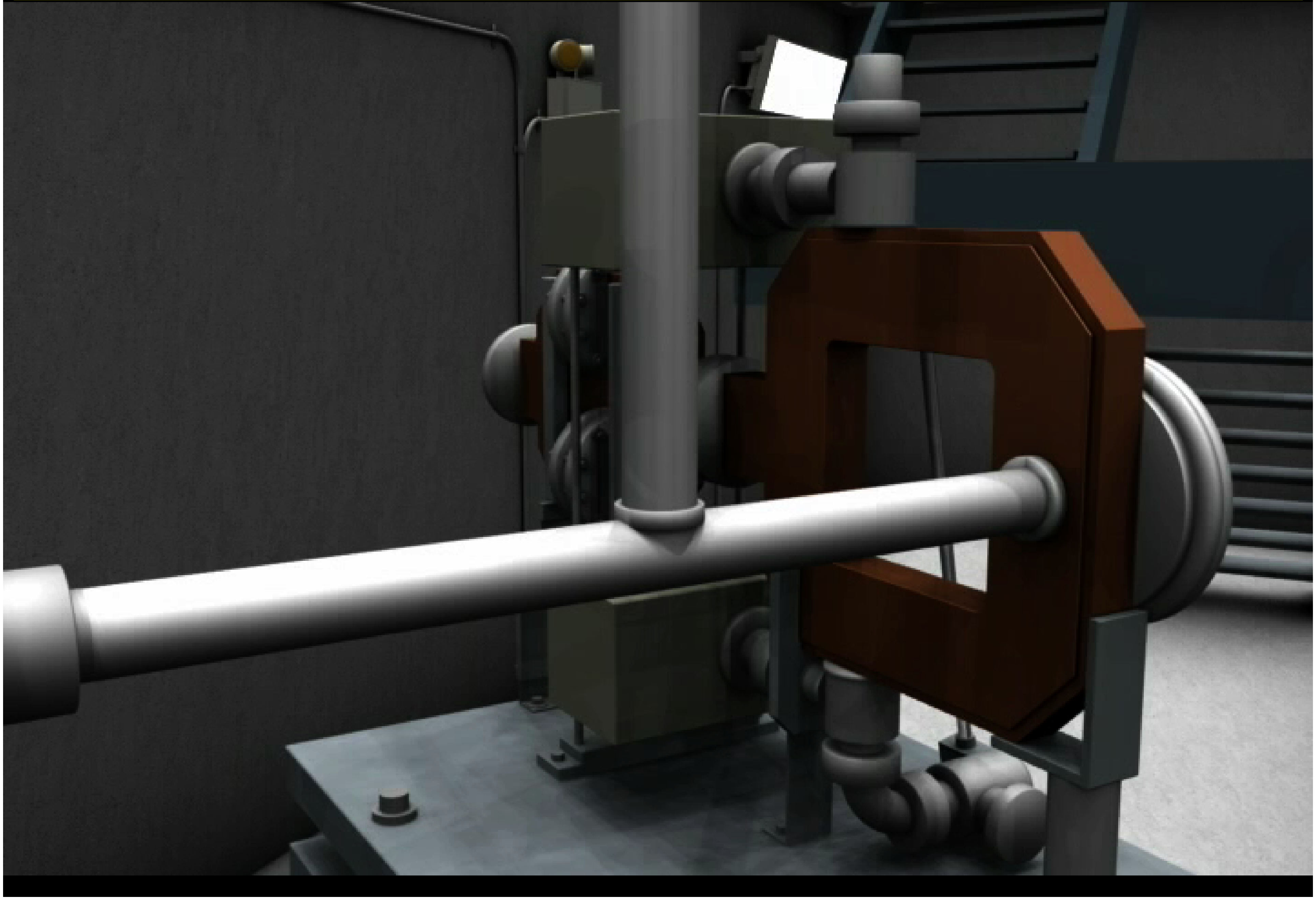
Linac Driven Free Electron Lasers (I)

Massimo.Ferrario@lnf.infn.it

CAS
THE CERN ACCELERATOR SCHOOL

Injector

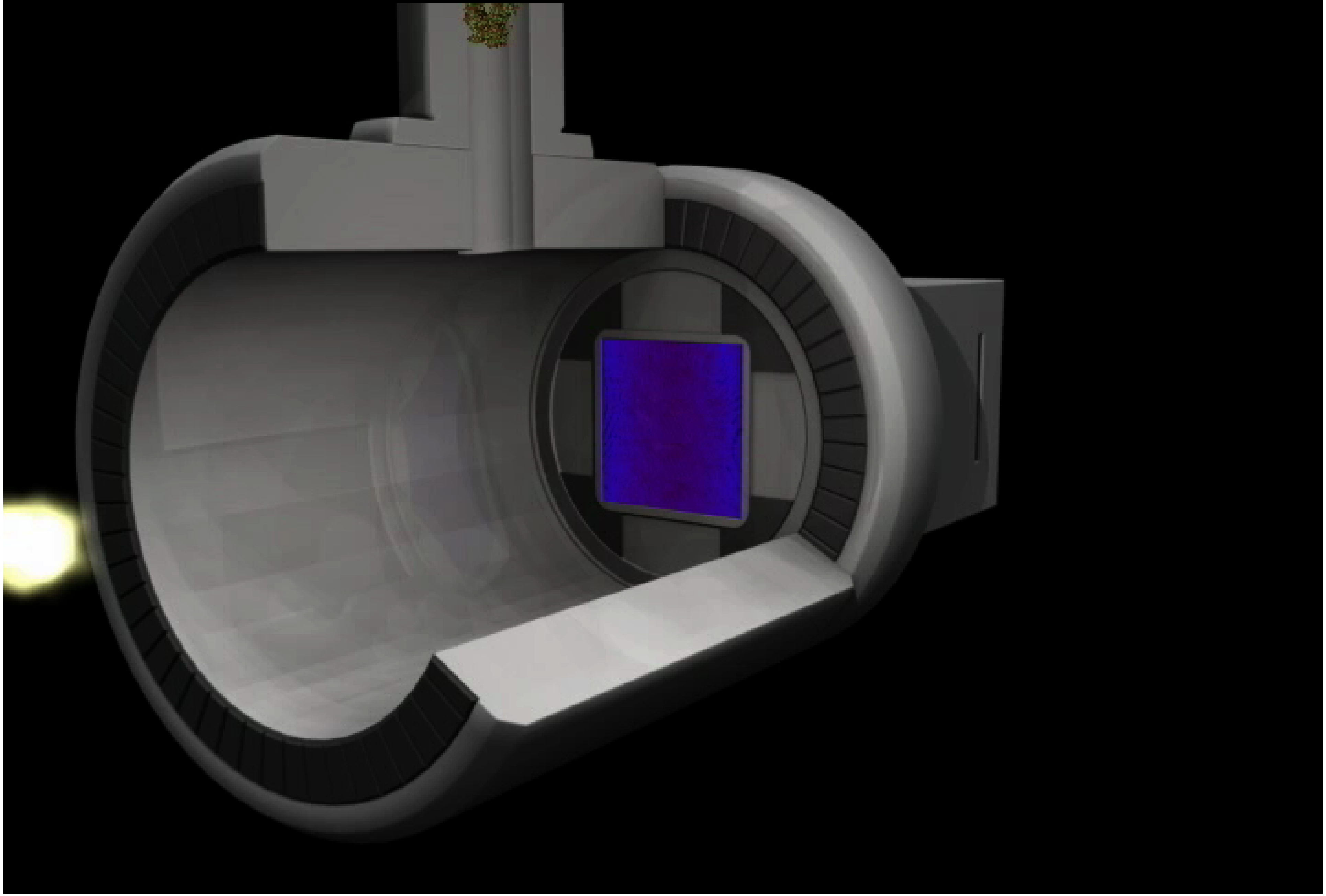


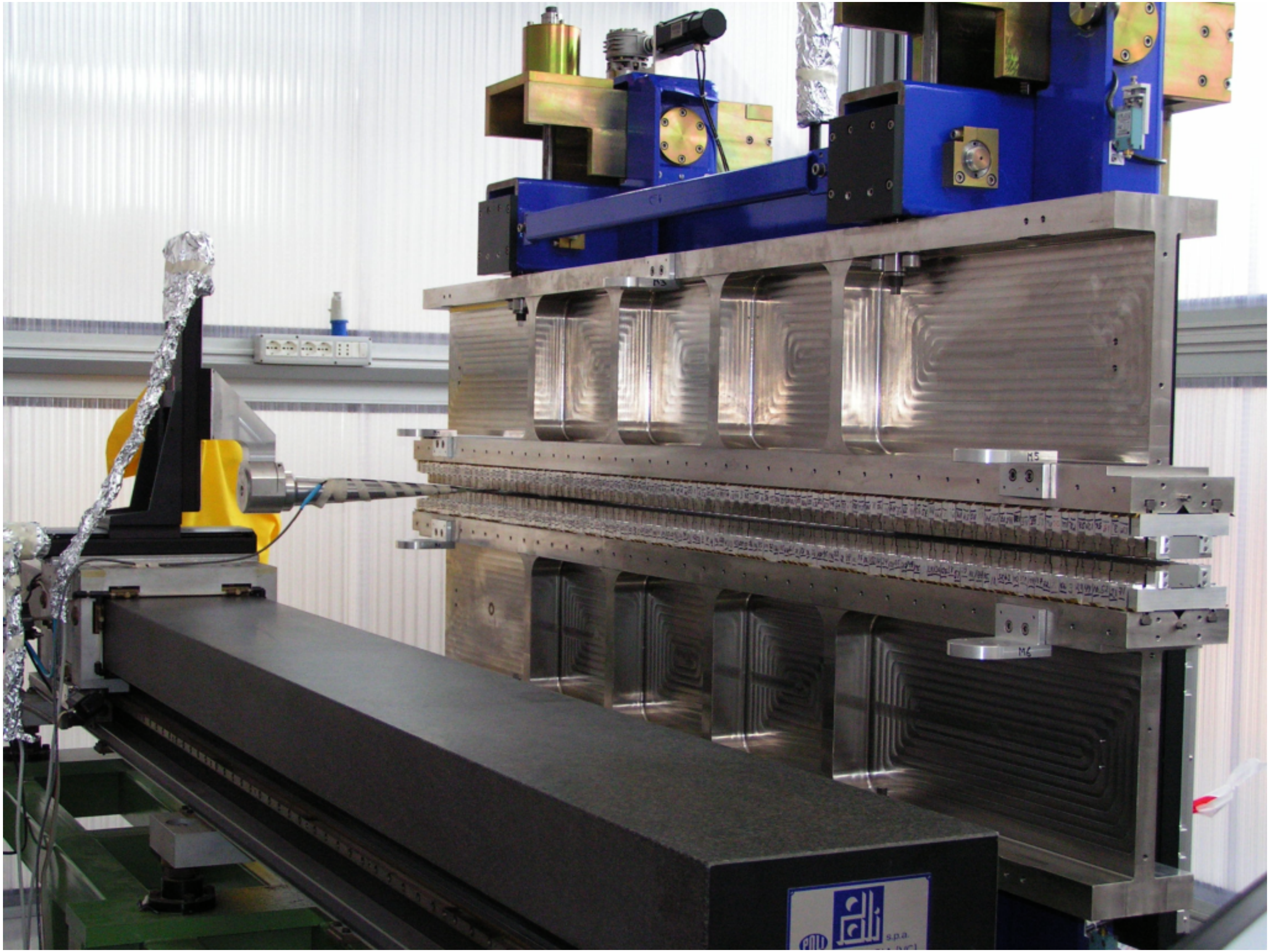
Magnetic bunch compressor (Chicane)

Long undulators chain

Beam separation

Experimental hall (Single Protein Imaging)




M5
M6
s.p.a.

Electron motion in an Undulator:

$$B_y(z) = B_0 \sin(k_u z) \quad \text{with} \quad k_u = 2\pi/\lambda_u,$$

$$m\gamma \frac{d^2x}{dt^2} = e(v_y B_z - v_z B_y) = -eB_0 c \sin(k_u z) \quad v_z \approx c.$$

$$\beta_\perp = \frac{K}{\gamma} cos(k_u z)$$

$$K = eB_0/(mck_u)$$

$$\beta = \frac{v}{c}$$

$$\beta_{//} = \sqrt{\beta^2 - \beta_\perp^2} = \sqrt{1 - \frac{1}{\gamma^2} - \beta_\perp^2} \approx 1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \beta_\perp^2\right)$$

$$\beta_{//} = \overline{\beta}_{//} - \frac{K^2}{4\gamma^2} cos(2k_u z)$$

$$\overline{\beta}_{//} = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

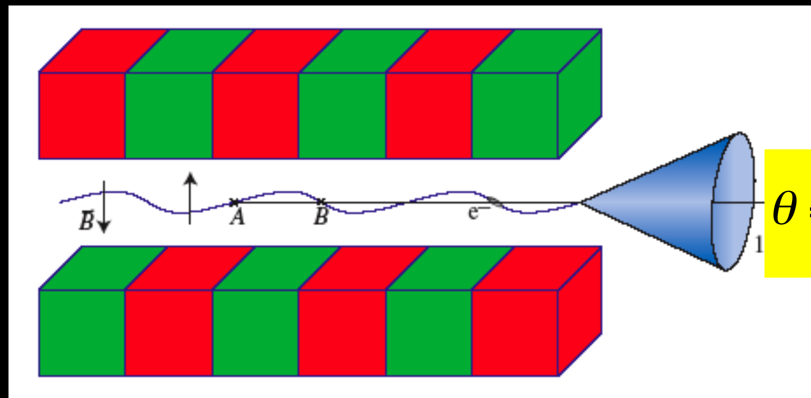
$$\beta_\perp = \frac{K}{\gamma}\cos(k_u z)$$

$$K = eB_0/(mck_u)$$

$$\beta = \frac{v}{c}$$

$$x = \frac{K}{\gamma k_u}\sin(k_u z).$$

The electron trajectory is determined by the undulator field and the electron energy



$$\theta = \frac{1}{\gamma}$$

$$\beta_{x\,Max} = \frac{K}{\gamma}$$

The electron trajectory is inside the radiation cone if $K \leq 1$

Relativistic Mirrors

$$\lambda'_u = \frac{\lambda_u}{\gamma_{//}}$$

Counter propagating pseudo-radiation

$$\lambda'_{rad} = \lambda'_u$$

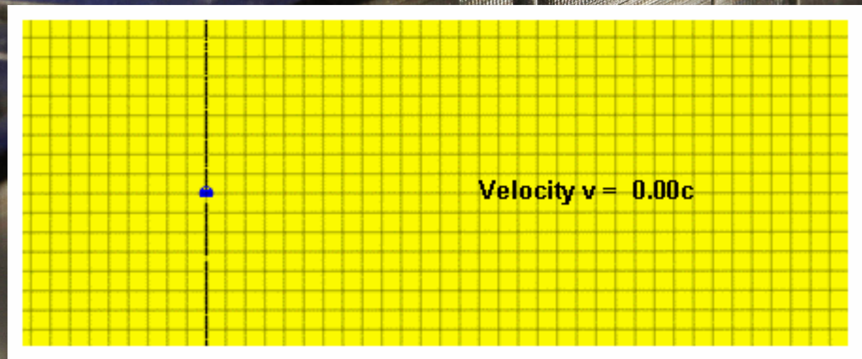
Thompson back-scattered radiation in the mirror moving frame

$$\lambda_{rad} = \gamma \lambda'_{rad}\left(1 - \beta \cos\vartheta\right) \approx \lambda_u\left(1 - \bar{\beta}_{//} \cos\vartheta\right)$$

Doppler effect in the laboratory frame

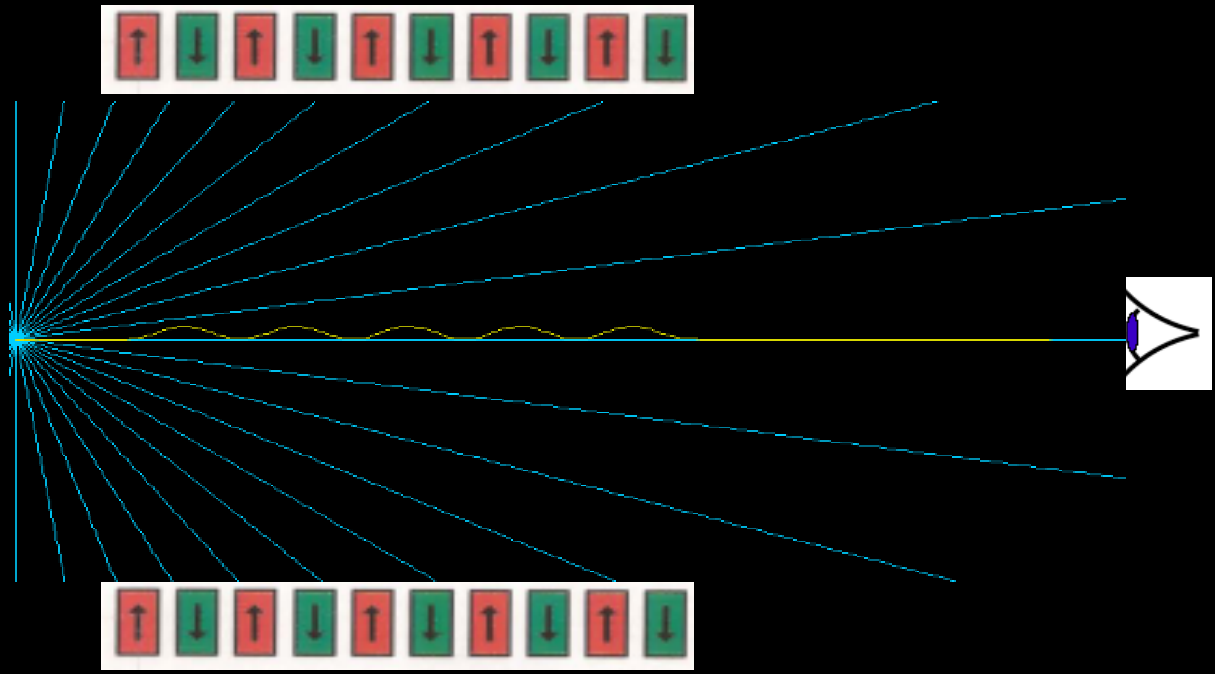
$$\bar{\beta}_{//} = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

$$\cos\vartheta \approx 1 - \frac{\vartheta^2}{2}$$

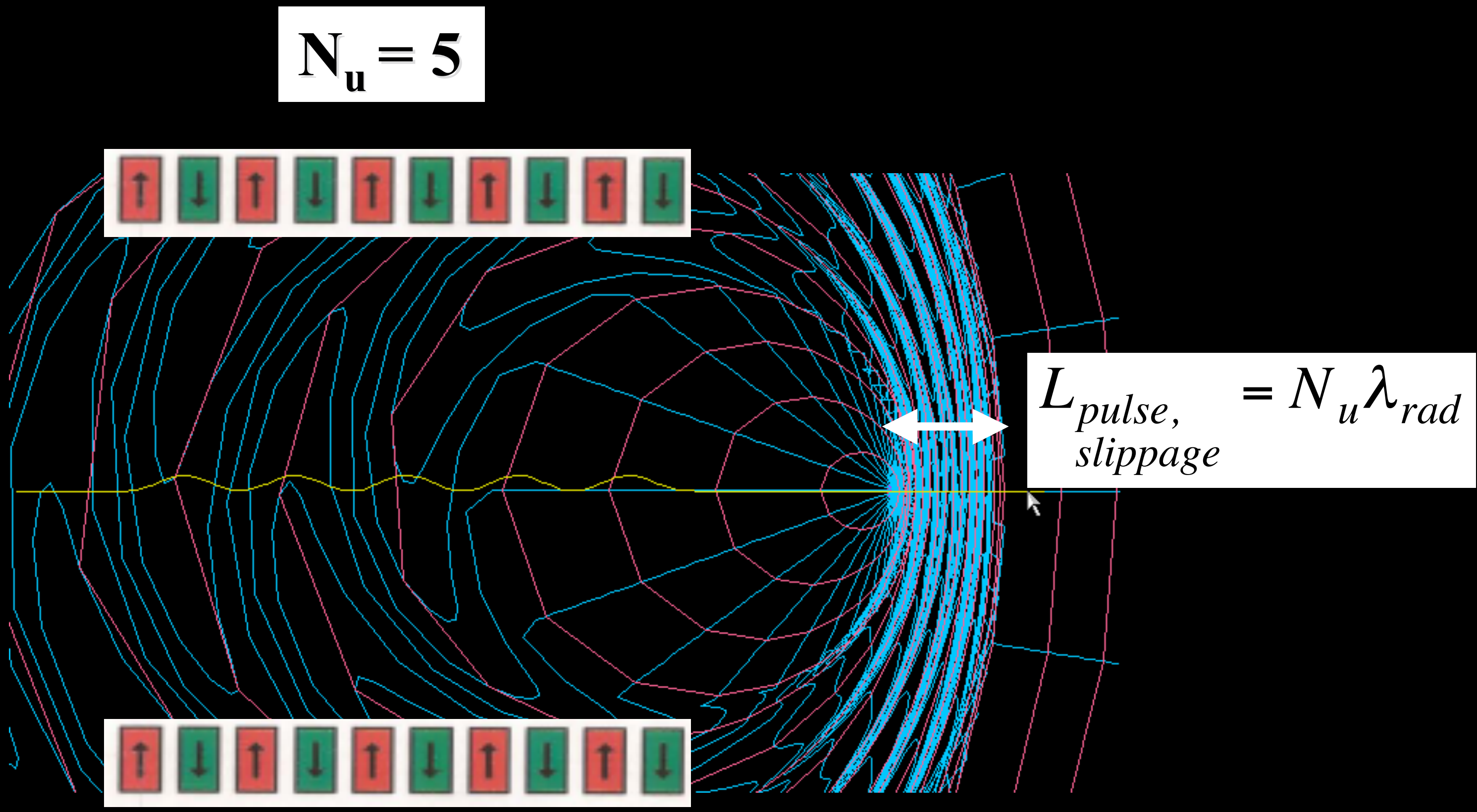


$$\lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\vartheta^2\right)$$

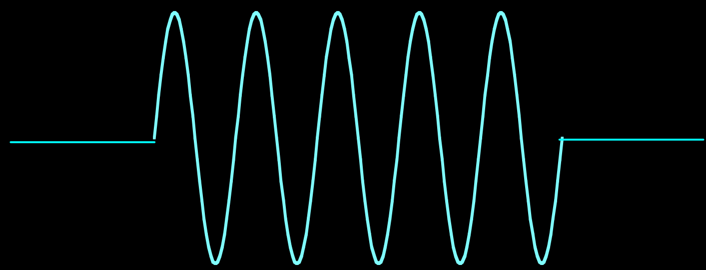
Tunability & Red Shift



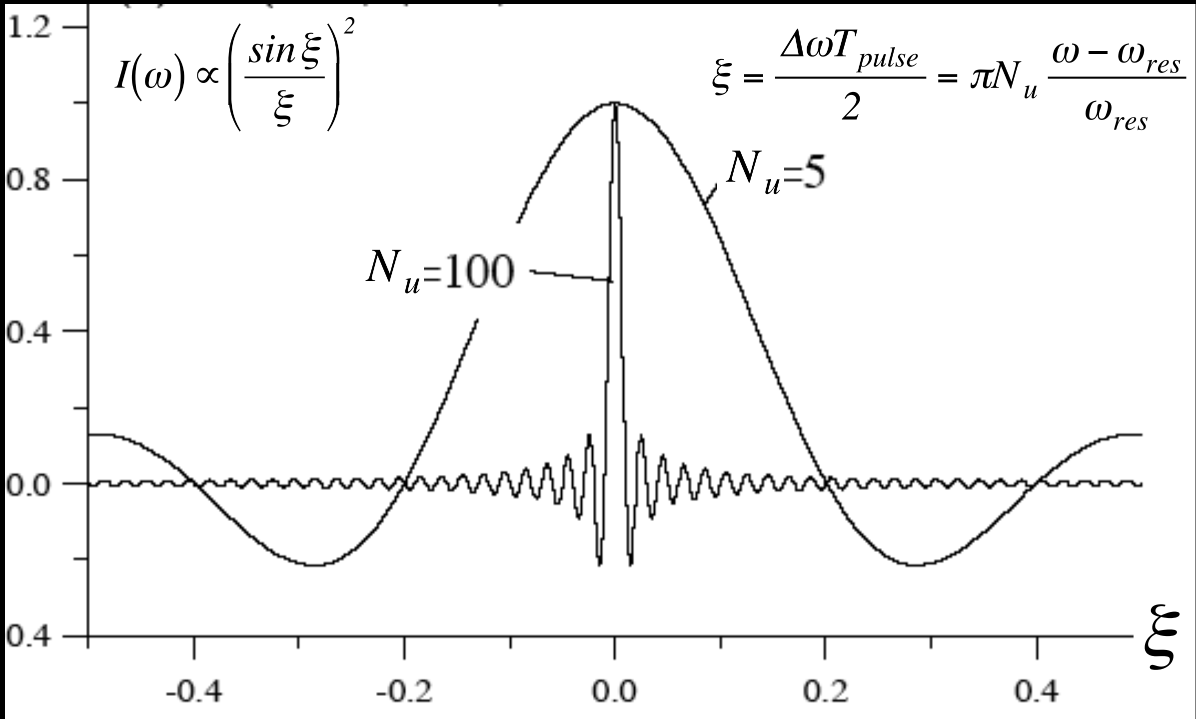
Radiation Simulator – T. Shintake, @ http://www-xfel.spring8.or.jp/Index.htm



Due to the finite duration the radiation is not monochromatic but contains a frequency spectrum which is obtained by Fourier transformation of a truncated plane wave



Spectral Intensity



$$\frac{\Delta\omega}{\omega} \approx \frac{1}{N_u}$$

Line width

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda(\vartheta) - \lambda(0)}{\lambda(0)} = \frac{\gamma^2\vartheta^2}{1 + \frac{K^2}{2}} \approx \frac{1}{N_u}$$

==>

$$\vartheta \approx \sqrt{\frac{1}{\gamma^2 N_u}\left(1 + \frac{K^2}{2}\right)} \approx \frac{1}{\gamma\sqrt{N_u}}$$

Angular width

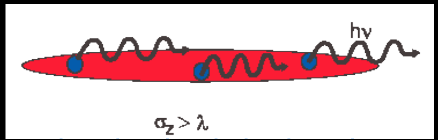
Peak power of one accelerated charge:

$$P_1 = \frac{e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$

Different electrons radiate indepedently hence the total power depends linearly on the number N_e of electrons per bunch:

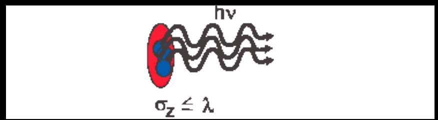
Incoherent Spontaneous Radiation Power:

$$P_T = N_e \frac{e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$



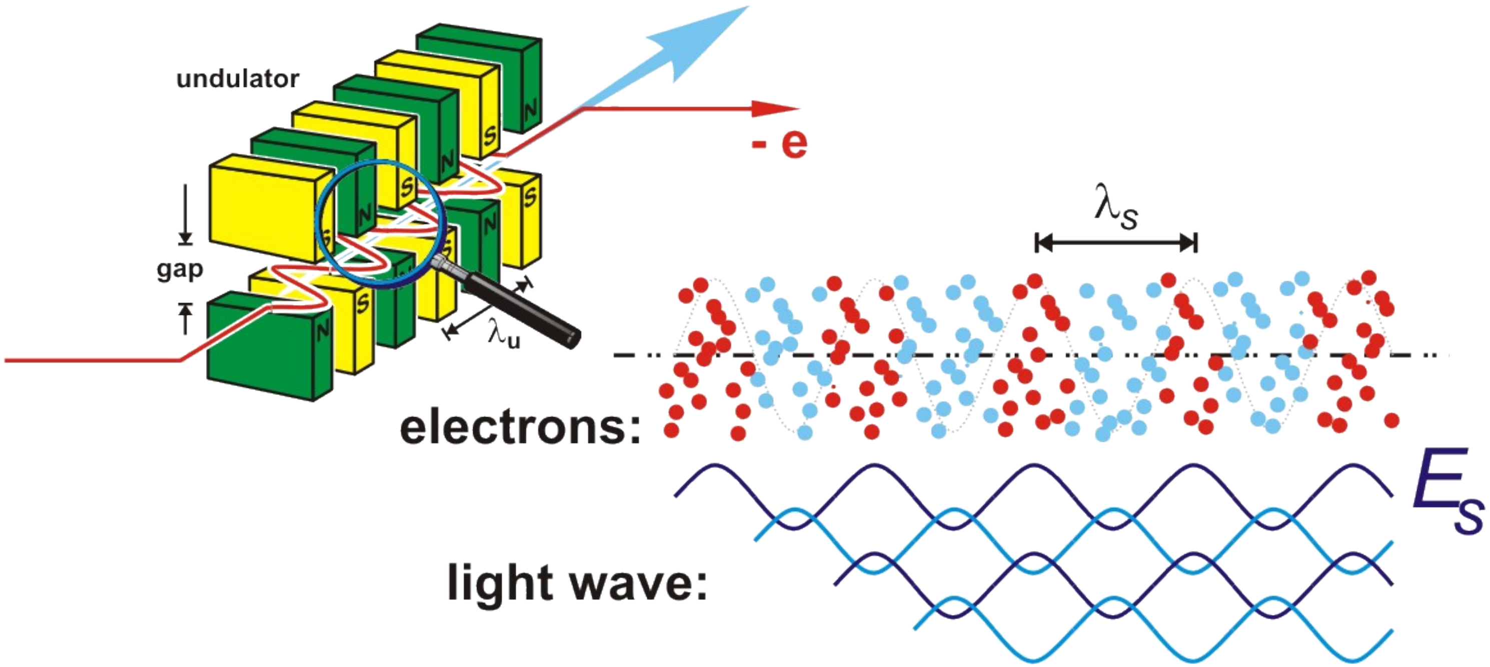
Coherent Stimulated Radiation Power:

$$P_T = \frac{N_e^2 e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$



Bunching on the scale of the wavelength:

Spontaneous Emission ==> Random phases



Radiated Power:

$$P \propto N$$

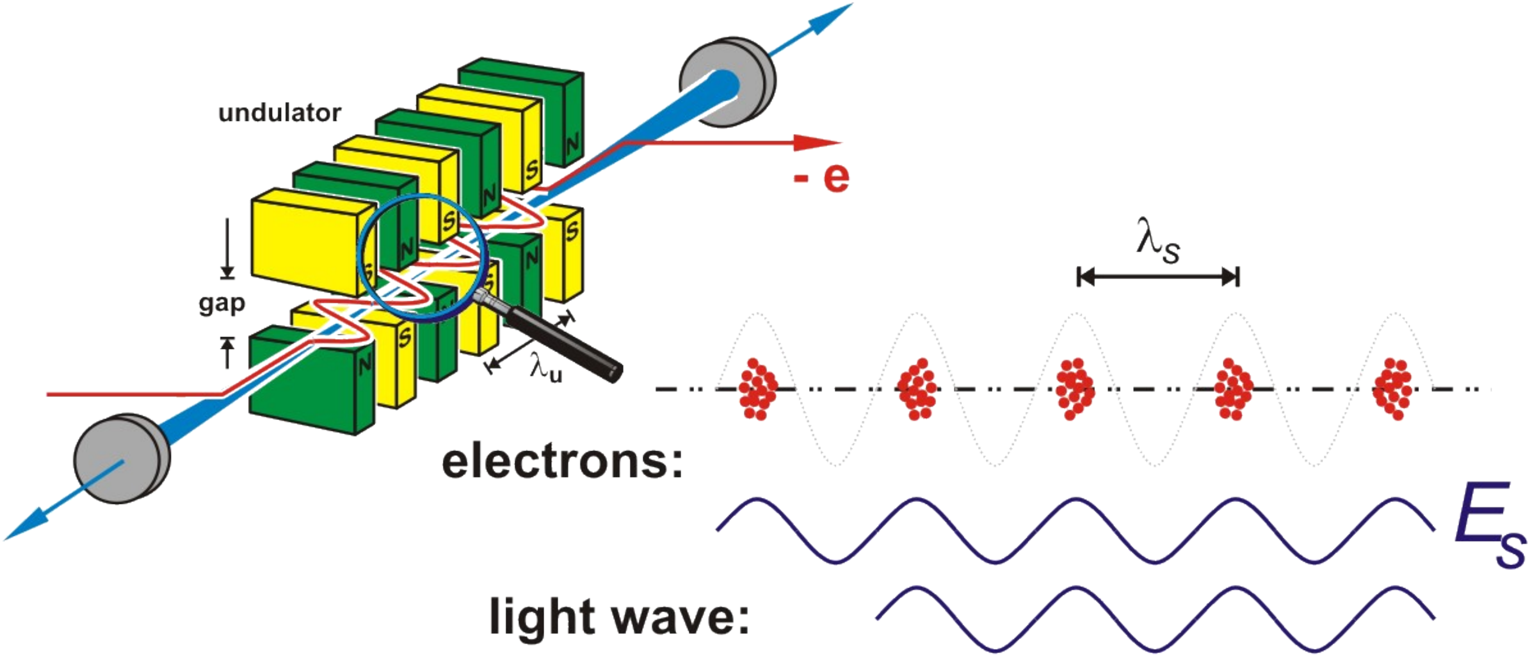
destructive interference

⟶ **shotnoise radiation**

Spontaneous Emission ==> Random Claps

Coherent Claps ==> Stimulated Emission

Coherent Light ==> Stimulated Emission

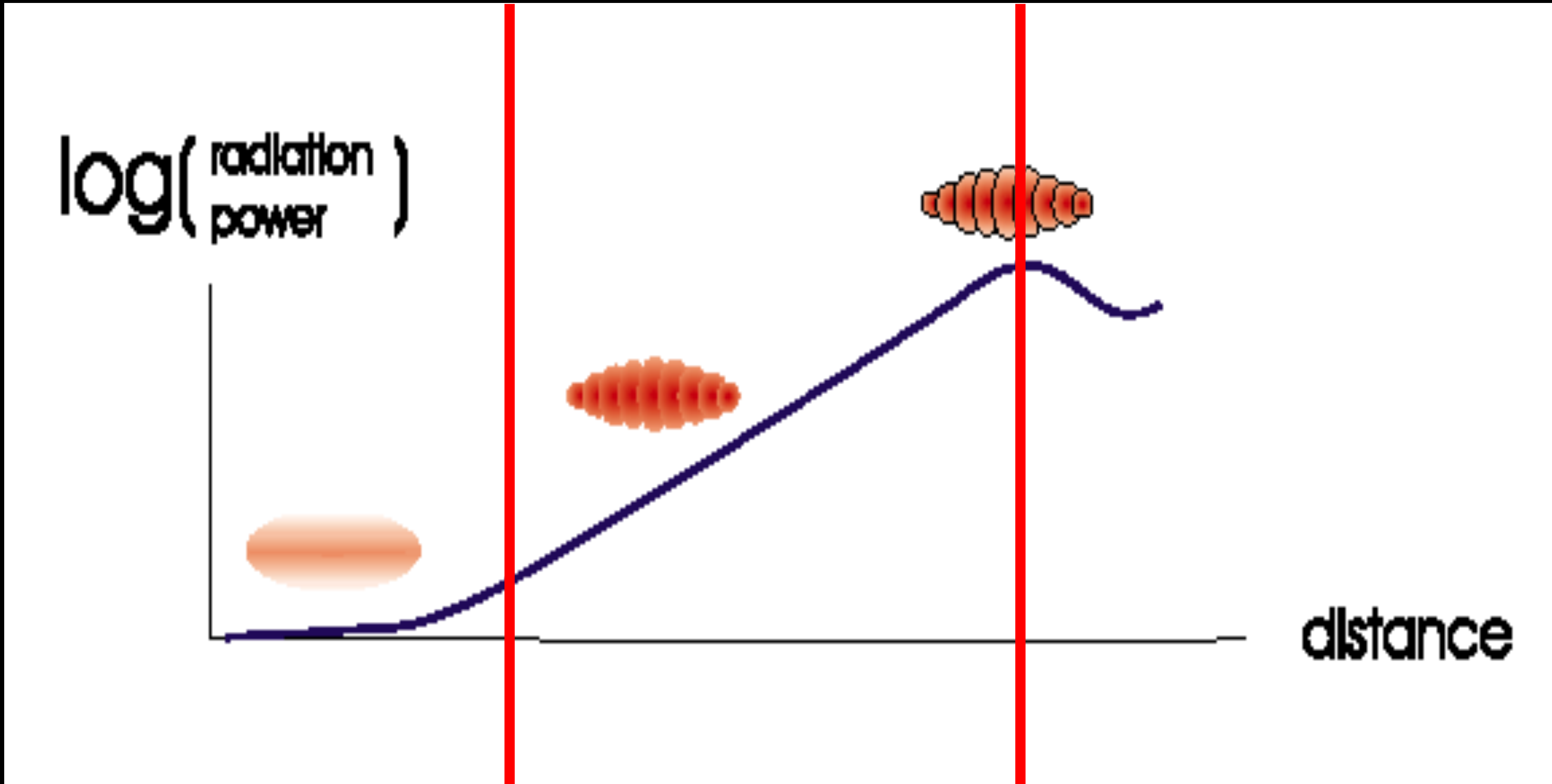


Radiated Power :

$P \propto N^2$

constructive interference

⟶ **enhanced emission**



Letargy

Spontaneous Emission

Low Gain

Slow Bunching

Exponential Growth

Stimulated emission

High Gain

Enhanced Bunching

Saturation

Absorption

No Gain

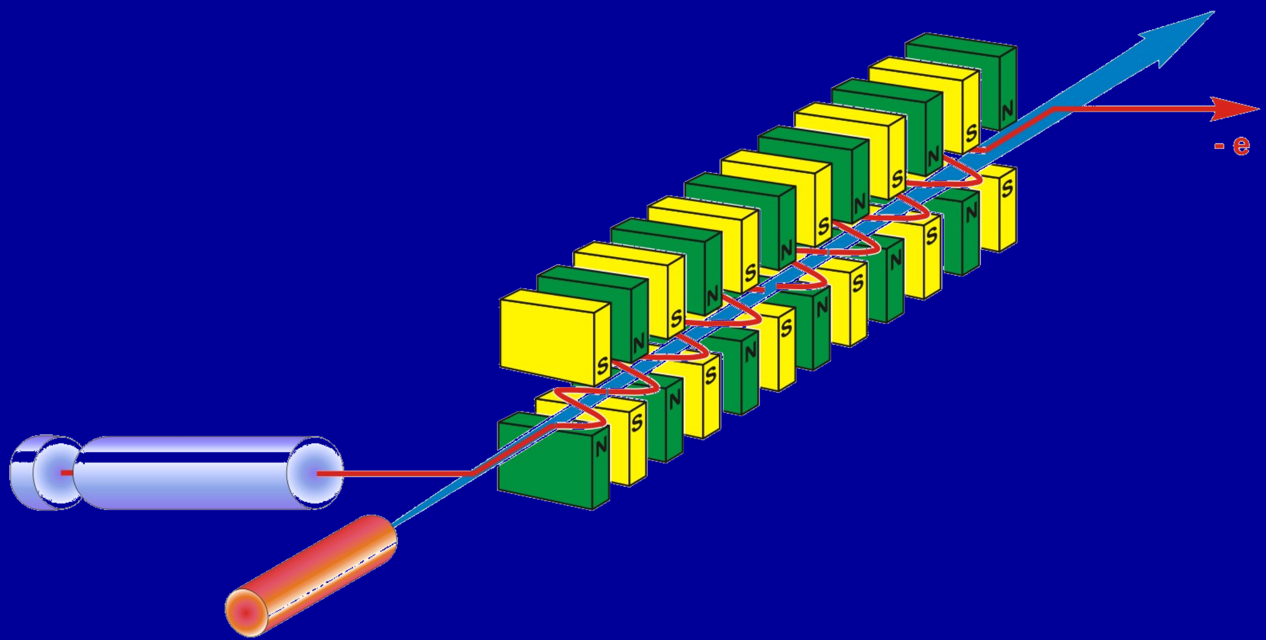
Debunching

Free Electron Laser 1D Self Consistent Model

Consider“seeding”by an external light source with wavelength λ_r
The light wave is co-propagating with the relativistic electron beam

$$\frac{d\gamma}{dt} = -\frac{e}{mc}\vec{E}\cdot\vec{\beta} = -\frac{e}{mc}E_{\perp}\beta_{\perp}$$

Energy exchange occurs only if there is transverse motion



Newton Lorentz Equations

E, B

Problem: electrons are slower than light

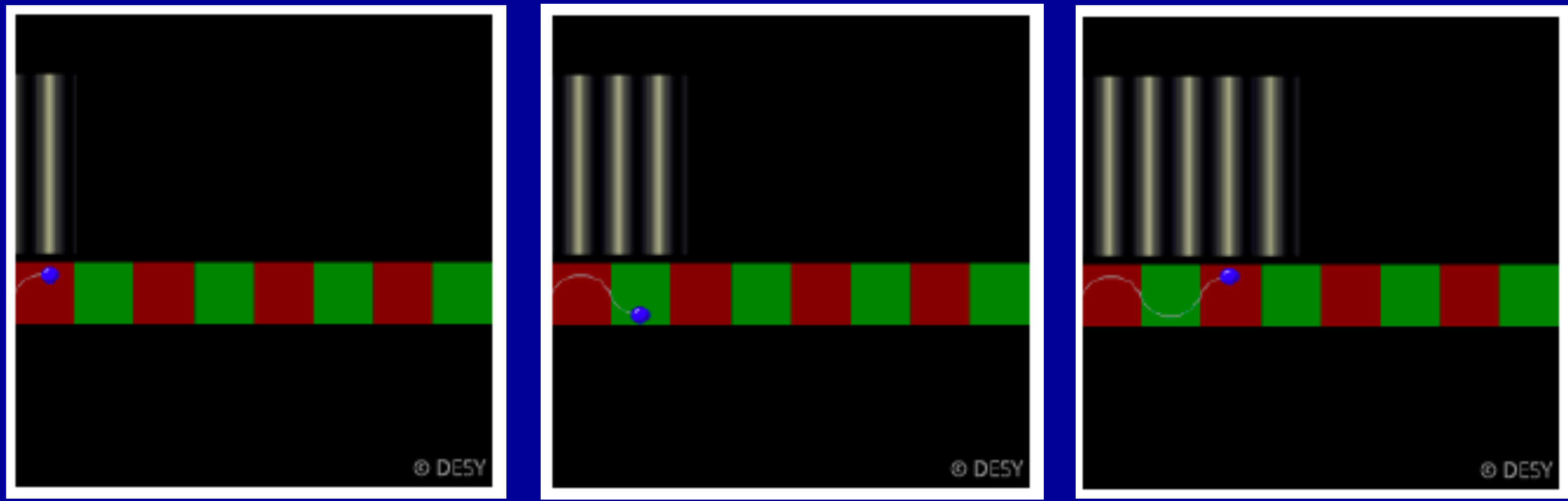
Question: can there be a continuous energy transfer from electron beam to light wave?

Answer: We need a Self Consistent Model

$J_\perp$

Maxwell Equations

(R. Bonifacio, C.Pellegrini, L.Narducci, Opt. Comm., 50, 373 (1984))



After one wiggler period the electron sees the radiation with the same phase if the flight time delay is exactly one radiation period: $\Delta t = t_e - t_{ph} = T_{rad}$

$$\Delta t = \frac{\lambda_u}{c\beta_{//}} - \frac{\lambda_u}{c} = \frac{\lambda_{rad}}{c} \longrightarrow \lambda_{rad} = \frac{1-\bar{\beta}_{//}}{\bar{\beta}_{//}}\lambda_u \xrightarrow{\bar{\beta}_{//}\approx 1} \lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$\gamma_{res} \approx \sqrt{\frac{\lambda_u}{2\lambda_{rad}}\left(1+\frac{K^2}{2}\right)}$$

The relative slippage of the radiation envelope through the electron beam can be neglected, provided that $l_b \gg N_u\lambda_r$ (Steady State Regime)

Plane wave with constant amplitude, co-propagating with the electron beam:

$$E_x(z,t) = E_o \cos(k_l z - \omega_l t + \psi_o)$$

$$k_l = \frac{\omega_l}{c} = \frac{2\pi}{\lambda_l}$$

$$\lambda_l \approx \frac{\lambda_u}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)$$

$$\beta_{\perp j} = \frac{K}{\gamma_j} \cos(k_u z_j)$$

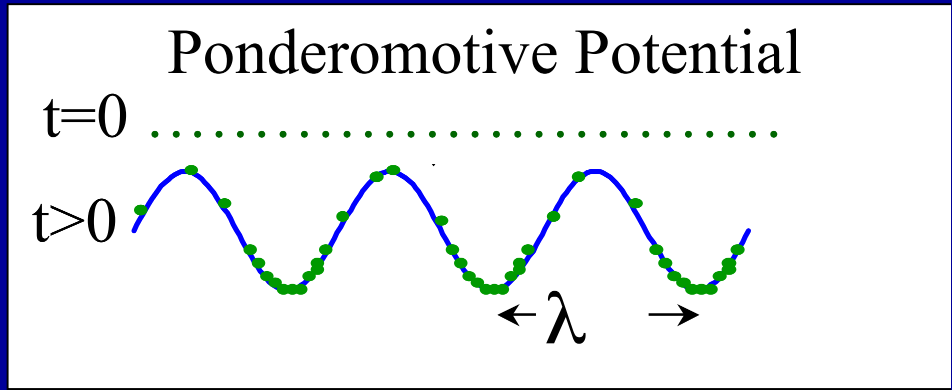
$$\frac{d\gamma_j}{dt} = -\frac{e}{m_e c} E_\perp \beta_{\perp j} = -\frac{eE_o K}{2\gamma_j m_e c}\left[\cos\left((k_l + k_u)z_j - \omega_l t + \psi_{oj}\right) + \ldots\right]$$

Ponderomotive phase:

$$\psi_j(t) = (k_l + k_u)z_j - \omega_l t + \psi_{oj}$$

In a resonant and randomly phased electron beam, nearly one half of the electrons absorbs energy and one half loses energy, with no net energy exchange.

If the undulator is sufficiently long the energy modulation becomes a phase modulation: the electrons self-bunch on the scale of a radiation wavelength.



The phase of the combined “ponderomotive” (radiation + undulator) field, propagates in forward direction with a phase velocity that corresponds to the velocity of the resonant particle:

$$\frac{d\psi}{dt} = (k_l + k_u)\bar{v}_z - k_l c = 0 \longrightarrow \bar{v}_z = \frac{k_l c}{k_l + k_u} = c\left(1 - \frac{1}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)\right)$$

The particles bunch around a phase ψ_r for which there is weak coupling with the radiation:

Bunching Parameter:

$$b(z,t) = \frac{1}{N}\sum_{j=1}^{N} e^{-i\psi_j} = \left\langle e^{-i\psi_j} \right\rangle$$

$|b| \approx 0$ Spontaneous emission

$|b| \to 1$ Stimulated emission

Motion in the potential well: the electron pendulum equations

For particles with off resonance energy $\gamma \neq \gamma_r$, the ponderomotive phase is no longer constant

$$\frac{d\psi}{dt} = (k_l + k_u)\bar{v}_z - k_l c \overset{k_u << k_l}{\approx} k_l c\left(\frac{\lambda_l}{\lambda_u} - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right) = \frac{k_l c}{2}\left(\frac{1}{\gamma_r^2} - \frac{1}{\gamma^2}\right)\left(1 + \frac{K^2}{2}\right)$$

$$\frac{d\psi}{dt} \approx k_u c \frac{\gamma^2 - \gamma_r^2}{\gamma_r^2} \approx 2k_u c \frac{\gamma - \gamma_r}{\gamma_r} = 2k_u c\eta$$

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r} << 1$$

$$\frac{d\eta}{dt} = \frac{1}{\gamma_r}\frac{d\gamma}{dt} = -\frac{eE_oK}{2\gamma_r^2 m_e c}\cos\psi$$

Two coupled first order differential equations

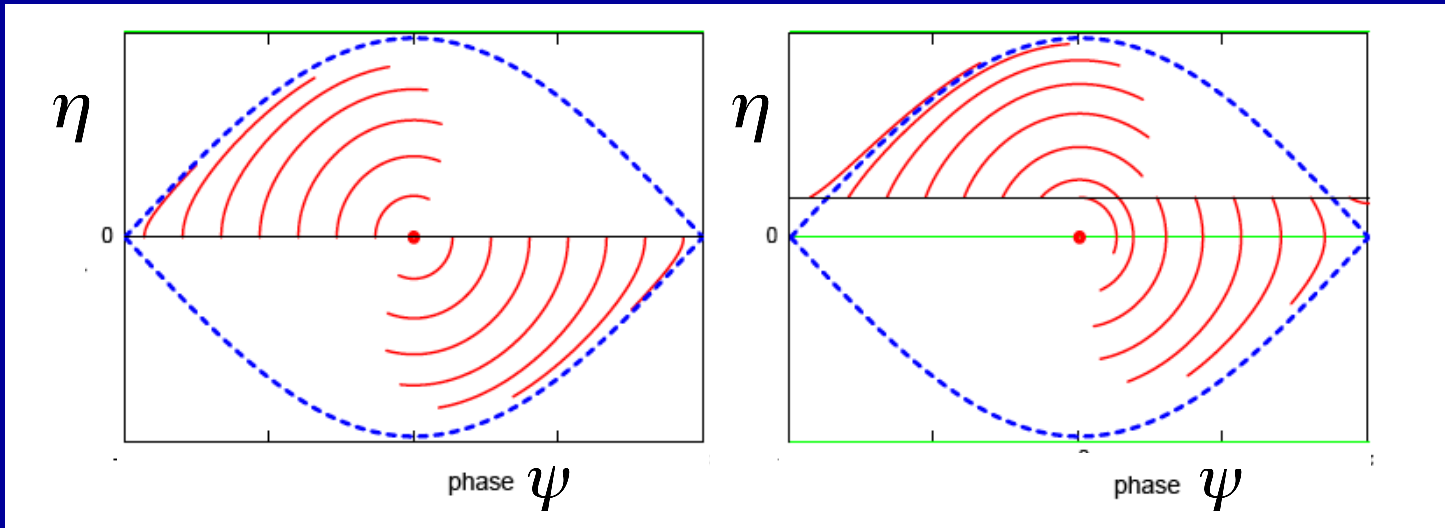
Combining the two coupled first order differential equations:

$$\begin{cases} \dfrac{d\psi}{dt} = 2k_u c\eta \\ \dfrac{d\eta}{dt} = -\dfrac{eE_oK}{2\gamma_r^2 m_e c}\cos\psi \end{cases}$$

$$\frac{d^2\psi}{dt^2} = -\frac{eE_oKk_u}{\gamma_r^2 m_e}\cos\psi$$

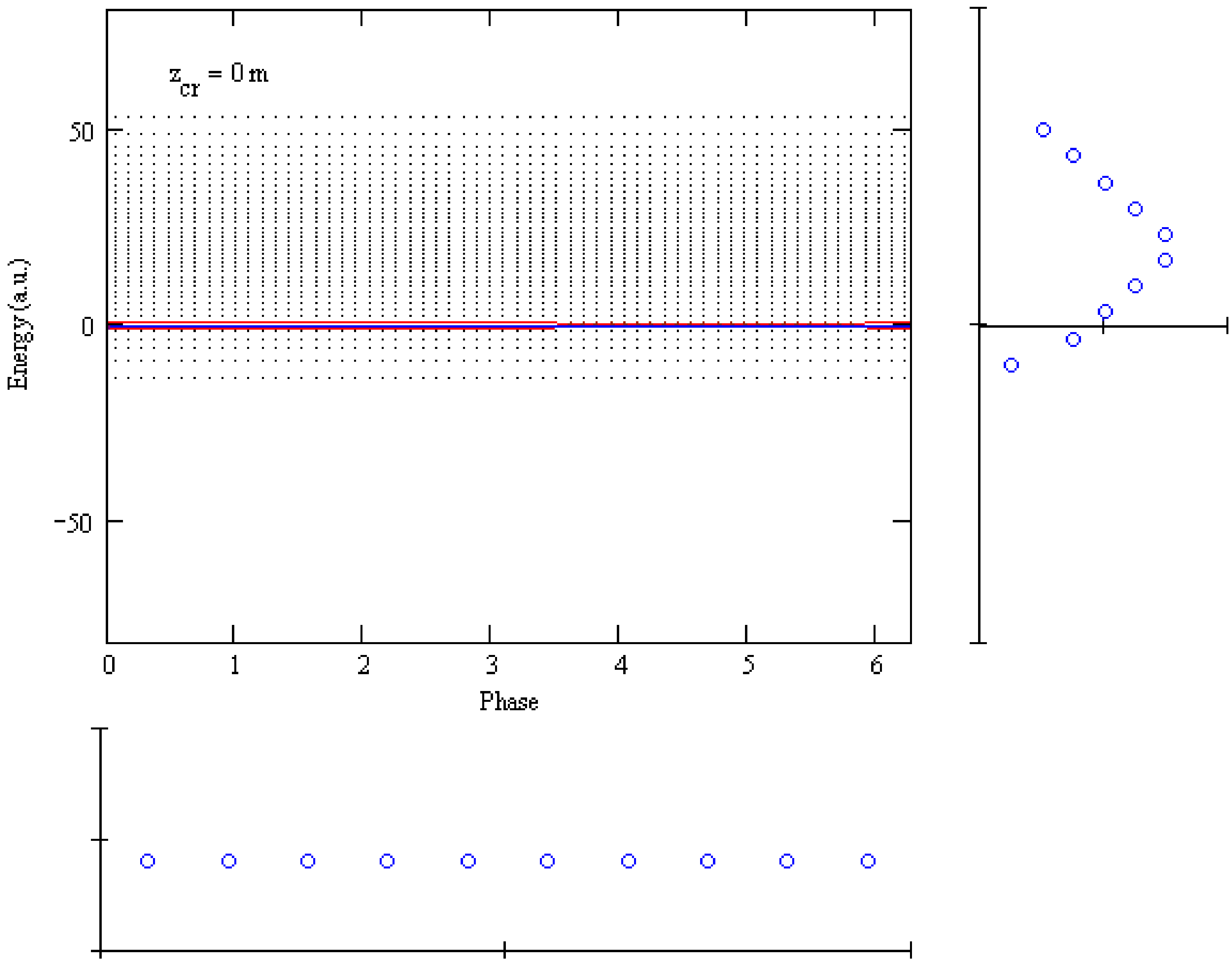
$$\frac{d^2\psi}{dt^2} + \Omega^2\cos\psi = 0$$

$$\Omega^2 = \frac{eE_oKk_u}{\gamma_r^2 m_e}$$

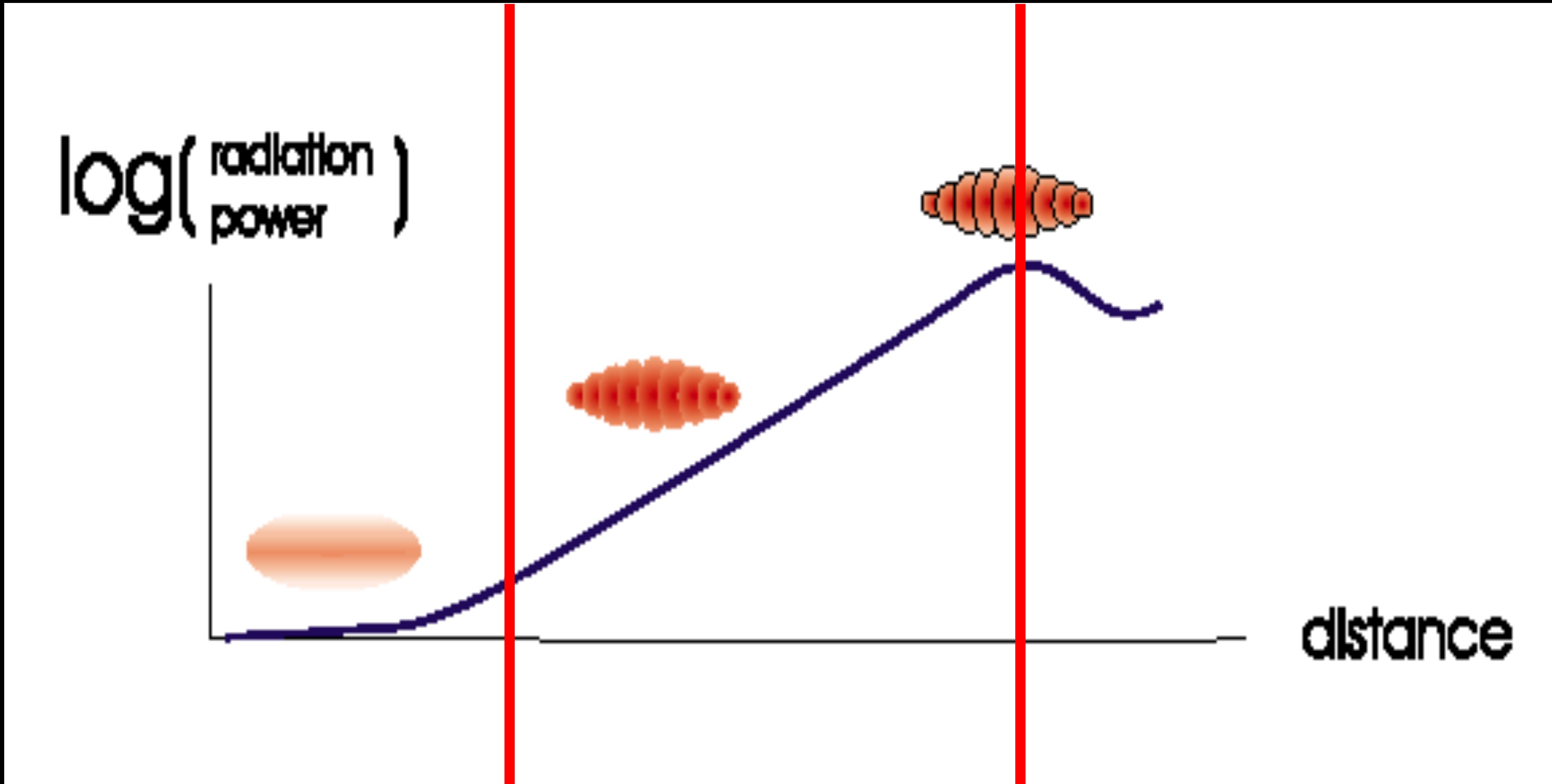


Separatrix

$$\eta_{sep} = \pm\sqrt{\frac{eEK}{k_u m_e c^2\gamma_r^2}}\cos\left(\frac{\psi-\psi_r}{2}\right)$$



Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)



Letargy

Spontaneous Emission

Low Gain

Slow Bunching

Exponential Growth

Stimulated emission

High Gain

Enhanced Bunching

Saturation

Absorption

No Gain

Debunching

High gain FEL regime

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\tilde{E}_x(z,t) = \mu_o \frac{\partial j_x}{\partial t}$$

Test solution

$$\tilde{E}_x(z,t) = \tilde{E}_x(z) e^{i(k_l z - \omega_l t)} = \frac{E_o(z) e^{i\varphi}}{2} e^{i(k_l z - \omega_l t)}$$

$$\left[2ik_l \tilde{E}'_x(z) + \tilde{E}''_x(z)\right] e^{i(k_l z - \omega_l t)} = \mu_o \frac{\partial j_x}{\partial t}$$

Slowly Varying Envelope Approximation (SVEA):

the amplitude variation within one undulator period is very small

$$\tilde{E}'_x(z) << \frac{\tilde{E}_x(z)}{\lambda_u} \quad \Rightarrow \quad \tilde{E}''_x(z) << \frac{\tilde{E}'_x(z)}{\lambda_u}$$

$$\frac{d\tilde{E}_x(z)}{dz} = -\frac{i\mu_o}{2k_l}\frac{\partial j_x}{\partial t} e^{-i(k_l z - \omega_l t)}$$

To be consistent with SVEA we should average also the source term over a time $T \approx n\lambda_l / c$ in which $\tilde{E}_x(z)$ could be considered constant

$$2ik_l \tilde{E}'_x = \mu_o \frac{1}{T}\int_t^{t+T} \frac{\partial \tilde{j}_x}{\partial t} e^{-i(k_l z - \omega_l t)} dt$$

$$\frac{1}{T}\int_t^{t+T}\frac{\partial\tilde{j}_x}{\partial t}e^{-i(k_l z-\omega_l t)}dt = \frac{-i\omega_l}{T}\int_t^{t+T}\tilde{j}_x e^{-i(k_l z-\omega_l t)}dt$$

Integration by parts

$$\tilde{j}_x = \frac{e}{S}\sum_{j=1}^{N}v_{xj}\delta\left(z-z_j(t)\right) = \frac{e}{Sv_z}\sum_{j=1}^{N}v_{xj}\delta\left(t-t_j(z)\right)$$

Beam model

S: transverse beam area

Exercise: verify there are not misprints (~mistakes):

$$\frac{1}{T}\int_t^{t+T}\tilde{j}_x e^{-i(k_l z-\omega_l t)}dt = \frac{e}{Sv_zT}\int_t^{t+T}\sum_{j=1}^{N}v_{xj}\delta\left(t-t_j(z)\right)e^{-i(k_l z-\omega_l t)}dt$$

$$= \frac{e}{V}\sum_{j=1}^{N}v_{xj}e^{-i\left(k_l z-\omega_l t_j\right)} \qquad \textbf{where :}\ V = Sv_zT$$

$$= \frac{e}{V}\sum_{j=1}^{N}\frac{Kc}{\gamma_j}\cos\left(k_u z\right)e^{-i\left(k_l z-\omega_l t_j\right)} \qquad \textbf{using}\ v_{xj} = \ldots\ldots$$

$$= \frac{eKc}{V\gamma_r}\sum_{j=1}^{N}e^{-i\left((k_l+k_u)z-\omega_l t_j\right)} = \frac{eKc}{V\gamma_r}\sum_{j=1}^{N}e^{-i\psi_j} \qquad \textbf{using}\ \gamma_j \approx \gamma_r$$

$$= \frac{eKc}{V\gamma_r}N\left\langle e^{-i\psi_j}\right\rangle = \frac{eKc}{\gamma_r}n_e\left\langle e^{-i\psi_j}\right\rangle \qquad \textbf{where}\ n_e = \frac{N}{V}$$

Three coupled first order differential equations.

They describe a collective instability of the system which leads to electron self-bunching and to exponential growth of the radiation until saturation effects set a limit on the conversion of electron kinetic energy into radiation energy.

$$\begin{cases} \dfrac{d\tilde{E}_x}{dz} = \dfrac{\omega_l \mu_o}{2k_l} \dfrac{eKc}{\gamma_r} n_e \left\langle e^{-i\psi_j} \right\rangle \\ \dfrac{d\psi_j}{dz} = 2k_u \eta_j \\ \dfrac{d\eta_j}{dz} = -\dfrac{eK}{2m_e c^2 \gamma_r^2} \Re e\left(\tilde{E}_x e^{i\psi_j} \right) \end{cases}$$

$$j = 1, N_e$$

$$b = \frac{1}{N} \sum_{j=1}^{N} e^{-i\psi_j} = \left\langle e^{-i\psi_j} \right\rangle$$

Bunching parameter

Saturation effects prevent the beam to radiate as N^2, limting the radiated power scaling to $N^{4/3}$, due to a competition between neighbours slices .

When propagation effects and slippage are relevant, i.e. when the elctron beam is as short as a slippage length, the emitted radiation leaves the bunch before saturation occurs and the power scaling becomes N^2 (Super-radiant or Single Spike regime)

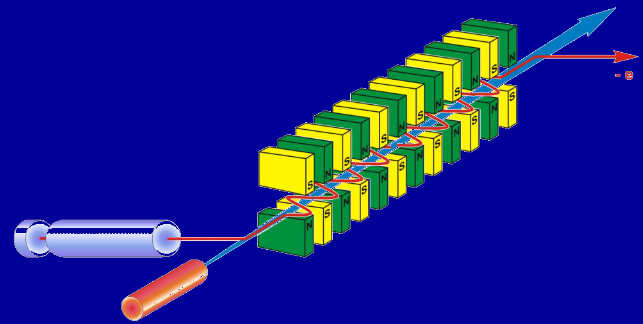
$$\begin{cases} \dfrac{d\tilde{E}_x}{dz} = \dfrac{\omega_l \mu_o}{2k_l} \dfrac{eKc}{\gamma_r} n_e b(z,t) \\ \dfrac{d\psi_j}{dz} = 2k_u c\eta_j \\ \dfrac{d\eta_j}{dz} = -\dfrac{eK}{2m_e c^2 \gamma_r^2} \Re e\left(\tilde{E}_x e^{i\psi_j}\right) \end{cases}$$

- Case 1: not interesting

$$\left.\begin{array}{l} b(z,t=0)=0 \\ \tilde{E}_x(z,t=0)=0 \end{array}\right\} \Rightarrow \tilde{E}_x(z,t)=0$$

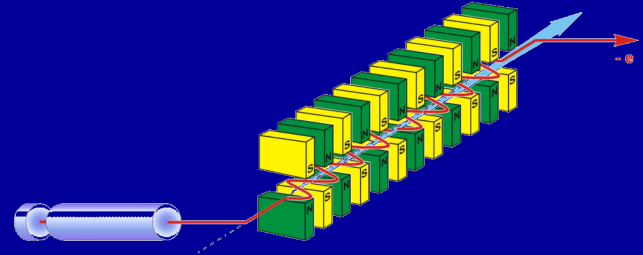
- Case 2: Amplification of input signal (Seeding)

$$\left.\begin{array}{l} b(z,t=0)=0 \\ \tilde{E}_x(z,t=0)\neq 0 \end{array}\right\} \Rightarrow \tilde{E}_x(z,t)\neq 0$$

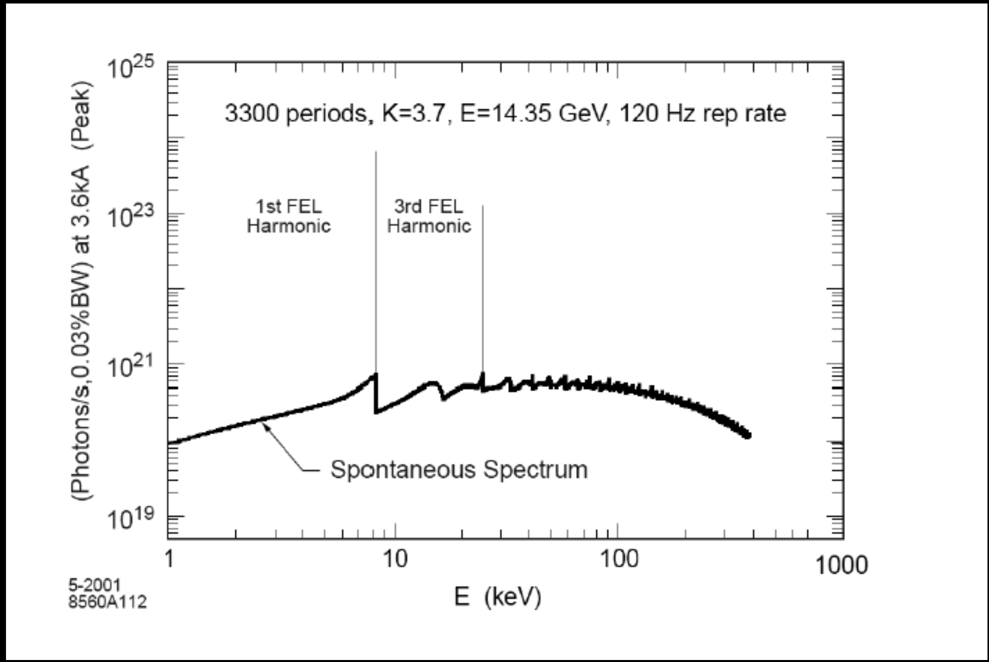


- Case 3: Self Amplification of Spontaneous Emission (SASE)

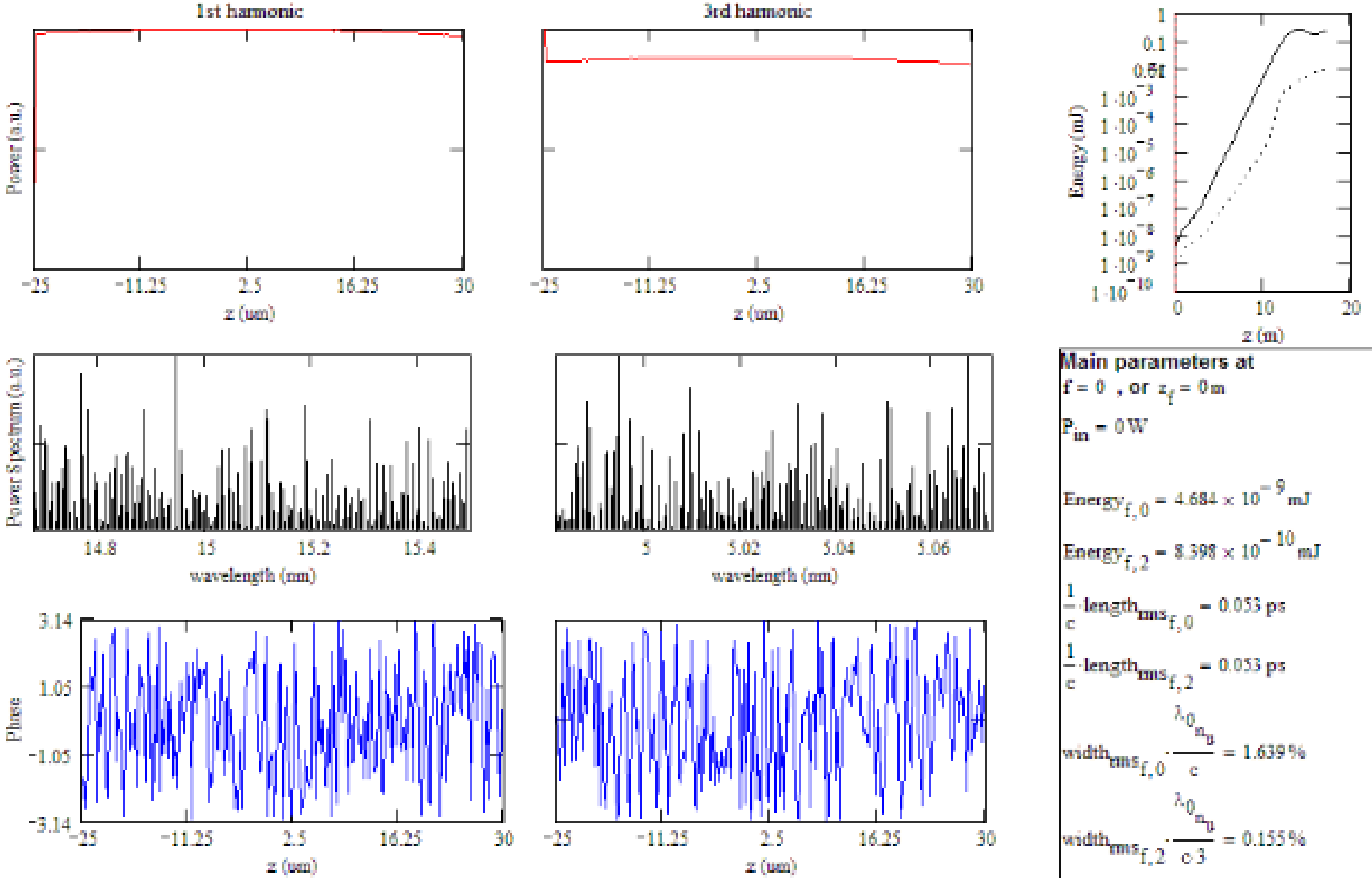
$$\left.\begin{array}{l} b(z,t=0)\neq 0 \\ \tilde{E}_x(z,t=0)=0 \end{array}\right\} \Rightarrow \tilde{E}_x(z,t)\neq 0$$



A Free Electron Laser is a device that converts a fraction of the electron kinetic energy into coherent radiation via a collective instability in a long undulator



SASE



Main parameters at
f = 0 , or z_f = 0 m

P_{in} = 0 W

$Energy_{f,0} = 4.684 \times 10^{-9}$ mJ

$Energy_{f,2} = 8.398 \times 10^{-10}$ mJ

$\frac{1}{c} \cdot length_{rms_{f,0}} = 0.053$ ps

$\frac{1}{c} \cdot length_{rms_{f,2}} = 0.053$ ps

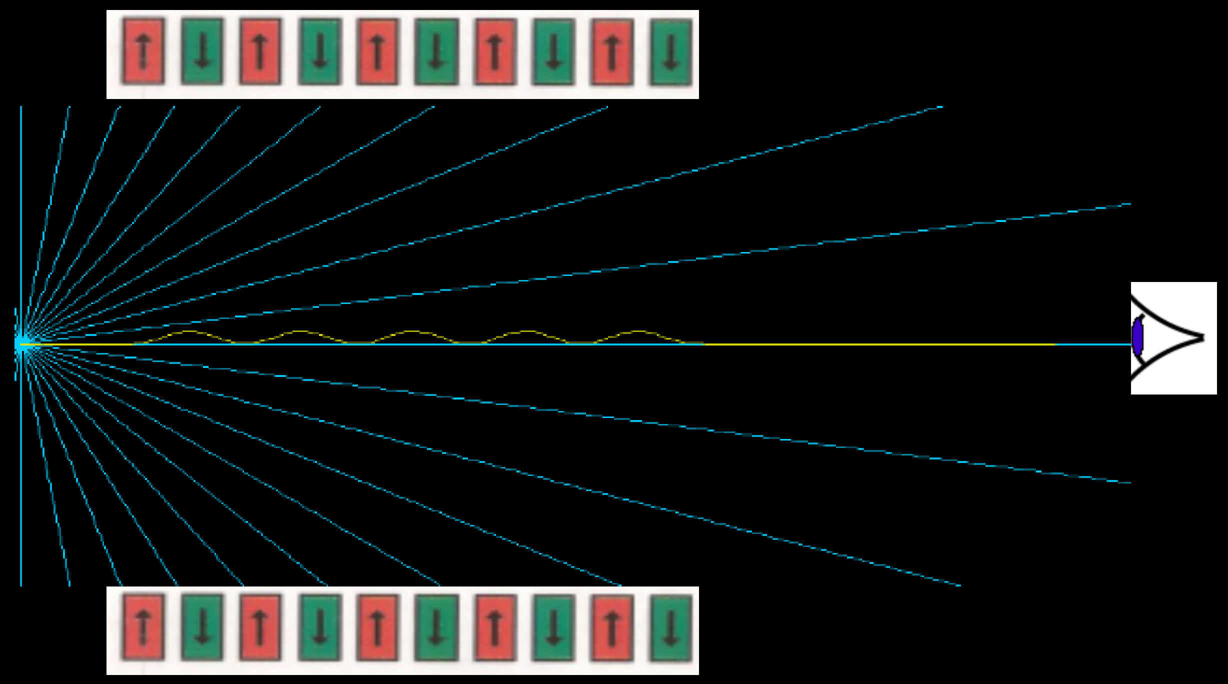
$width_{rms_{f,0}} \cdot \frac{\lambda_{0_{n_u}}}{c} = 1.639\,\%$

$width_{rms_{f,2}} \cdot \frac{\lambda_{0_{n_u}}}{c \cdot 3} = 0.155\,\%$

$dfl_1 = 4.198$

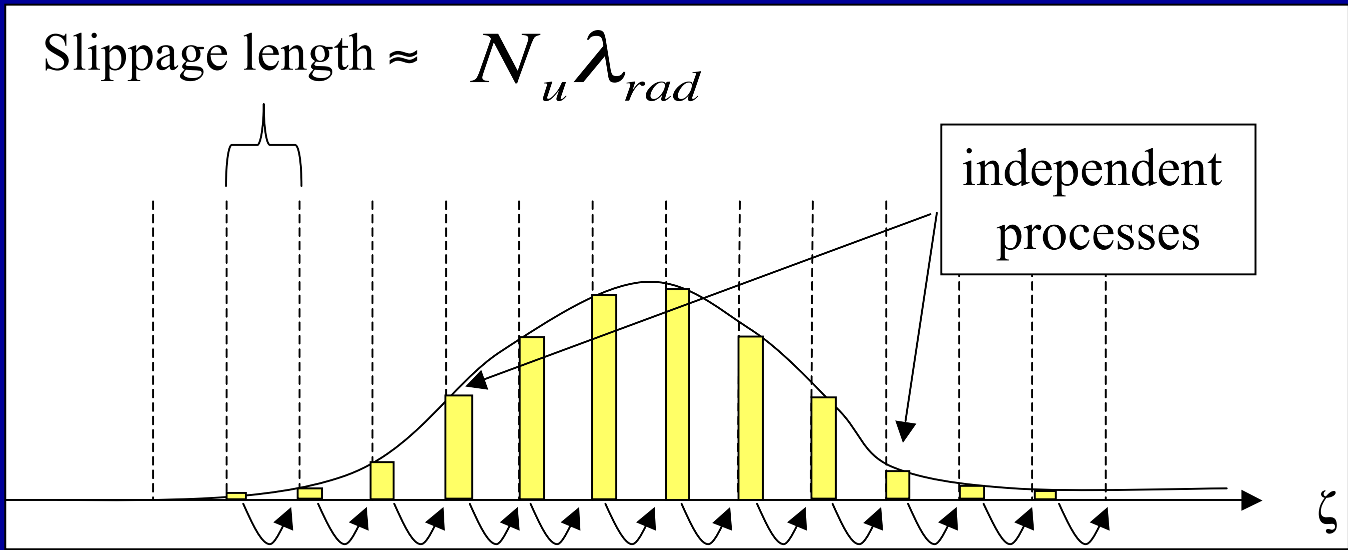
$dfl_3 = 7.55$

Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)



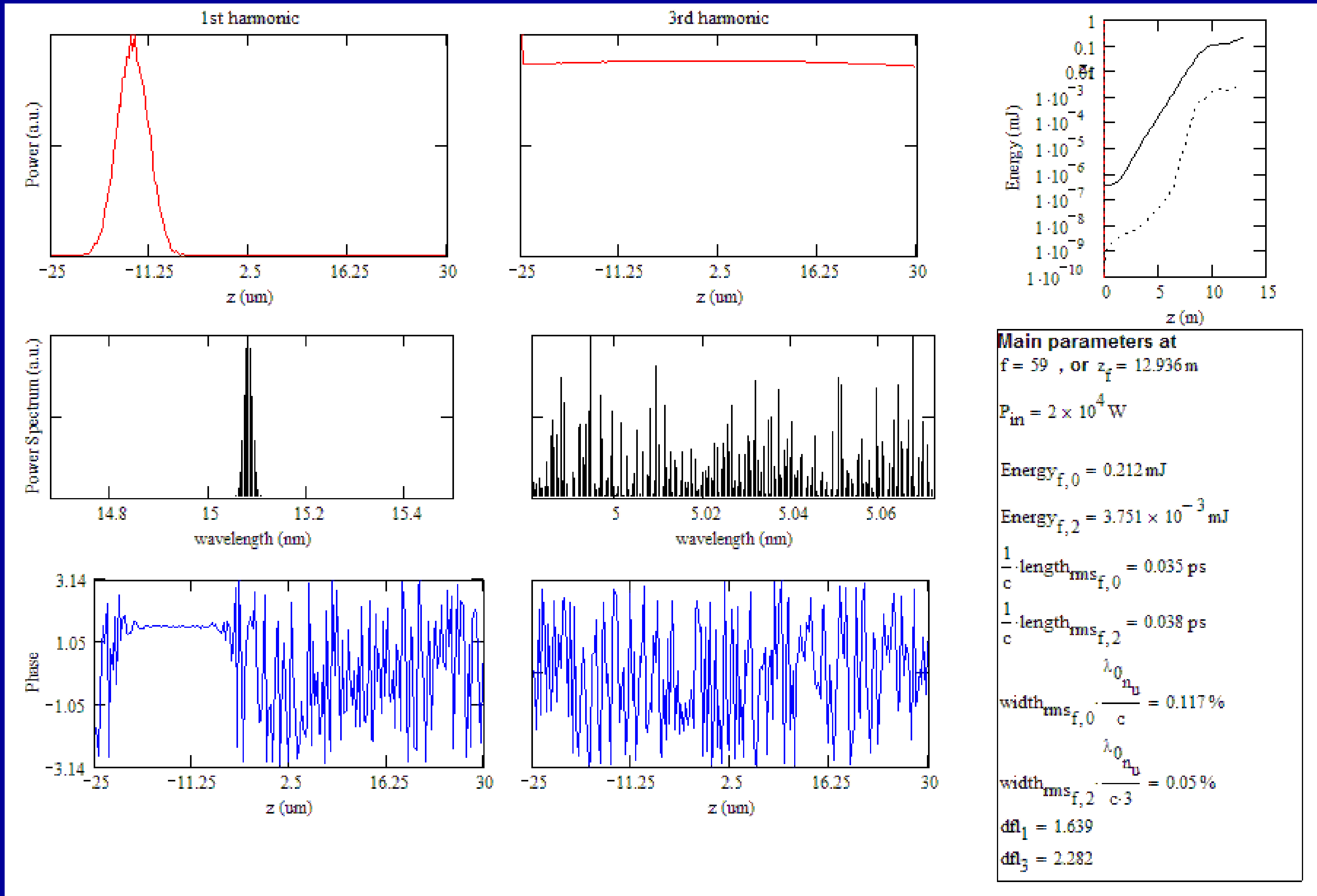
Radiation Simulator – T. Shintake, @ http://www-xfel.spring8.or.jp/Index.htm

SASE Longitudinal coherence



The radiation "slips" over the electrons for a distance $N_u \lambda_{rad}$

SEEDING



Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)

THE END

References:

-R. Bonifacio, F. Casagrande and C. Pellegrini, Hamiltonian model of a freeelectron laser, Opt. Comm. Vol. 61, 55 (1987)

-Laser Handbook Vol. 6, Free Electron Lasers, eds. W.B. Colson, C. Pellegrini and A. Renieri, North Holland, Amsterdam, Oxford, New York, Tokyo 1990

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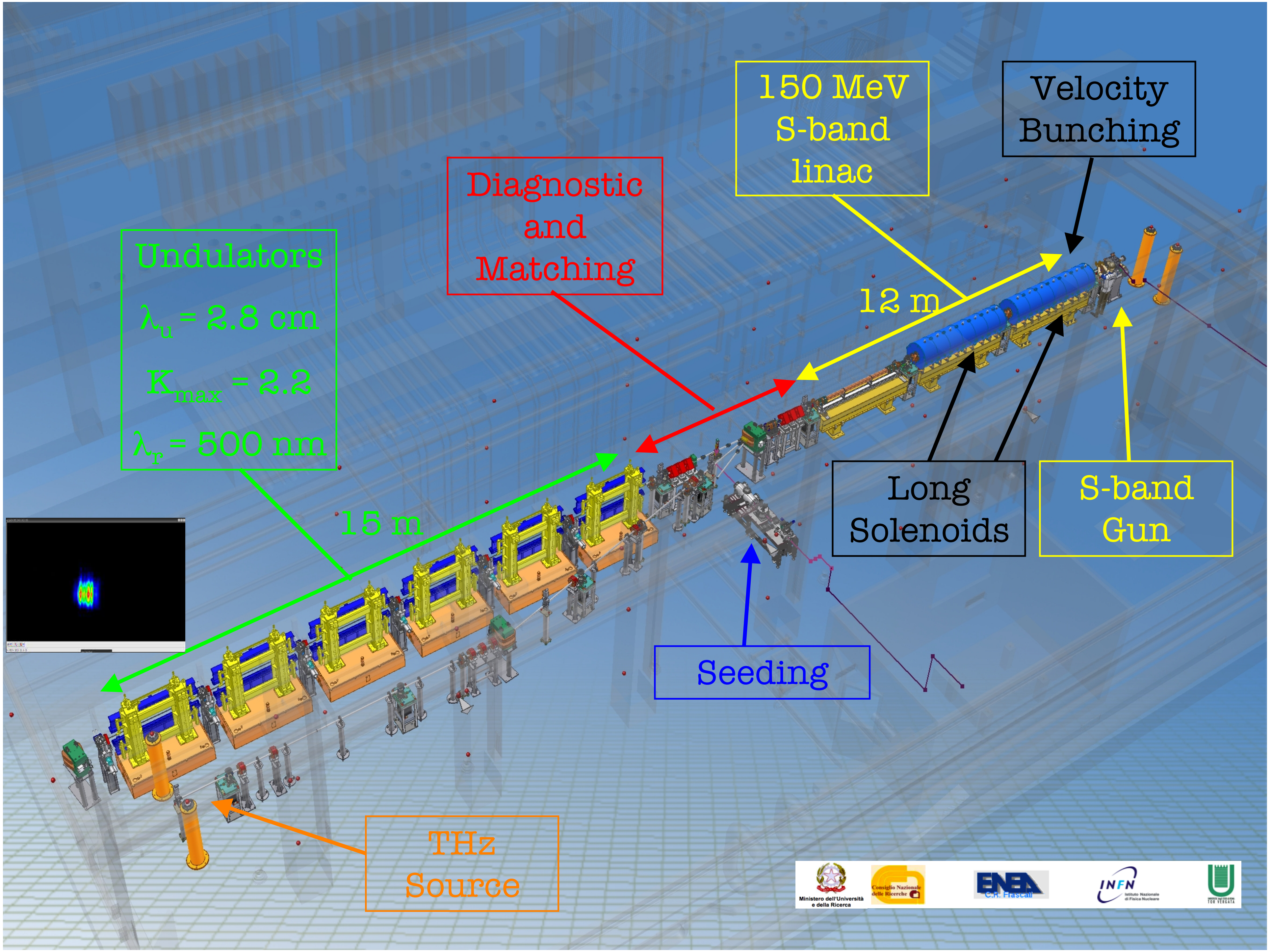
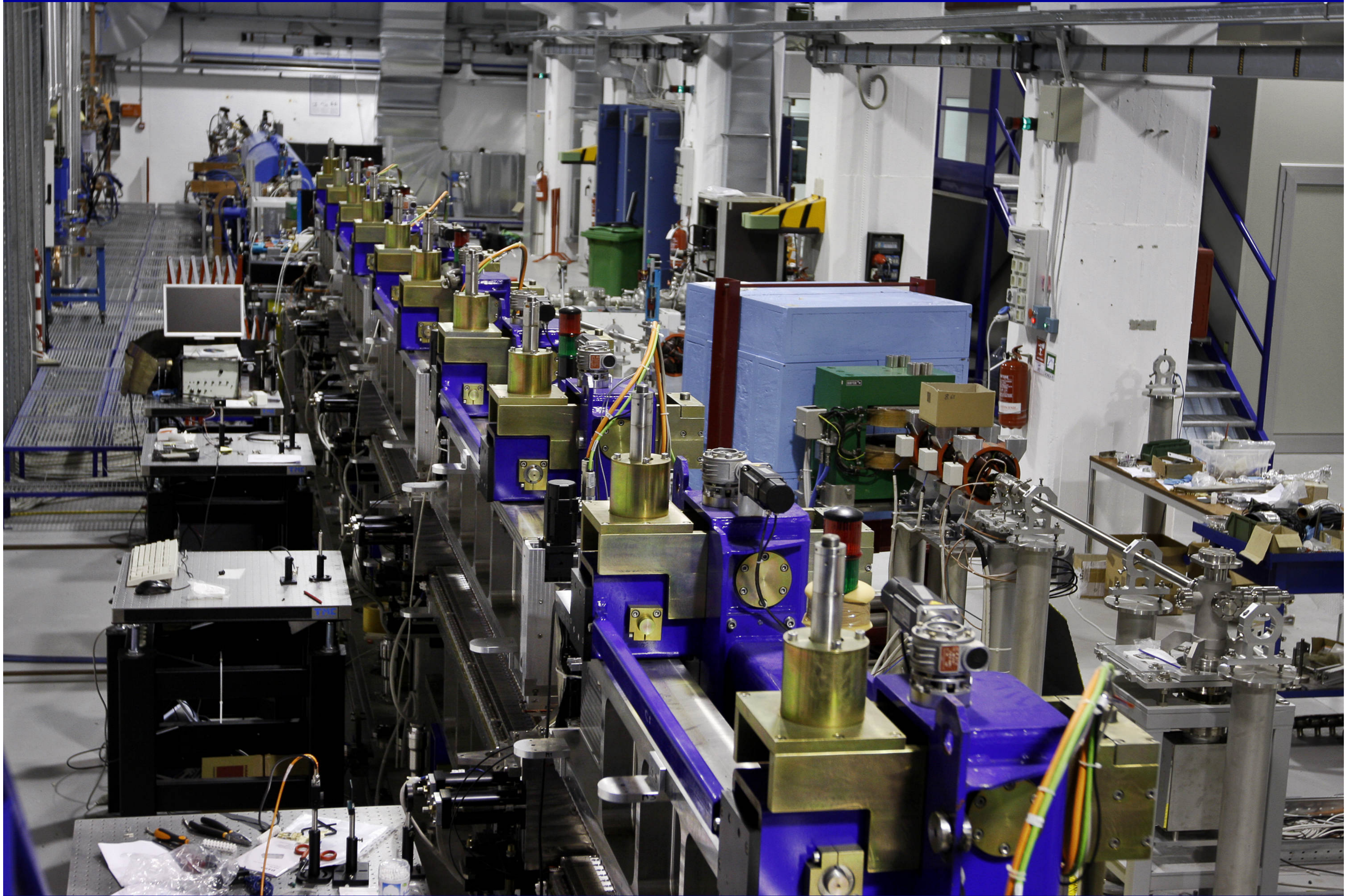
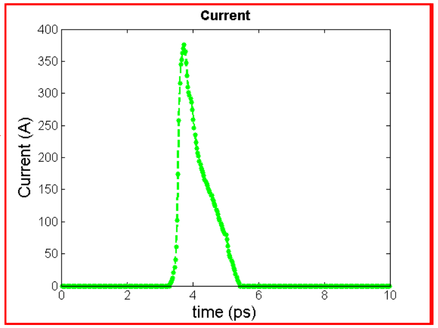


Velocity Bunching
150 MeV S-band linac
Diagnostic and Matching
Undulators
λ_u = 2.8 cm
K_{max} = 2.2
λ_r = 500 nm
12 m
15 m
Long Solenoids
S-band Gun
Seeding
THz Source

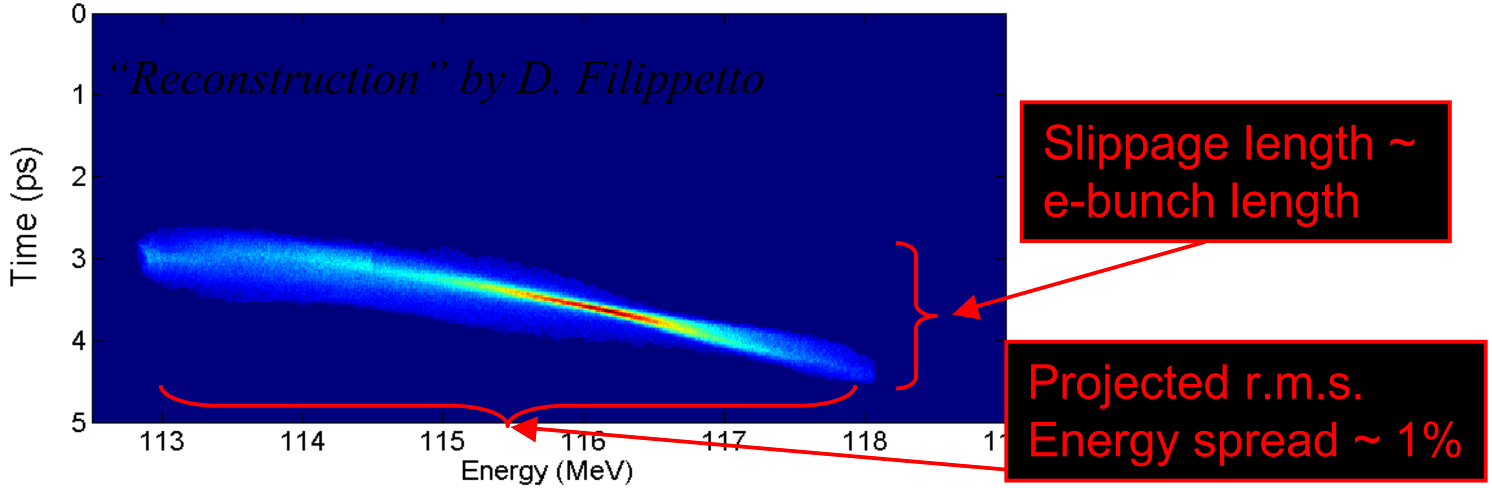


SASE with chirped & compressed beam

- Compression with "Velocity Bunching"
  - High peak current (up to 380A)

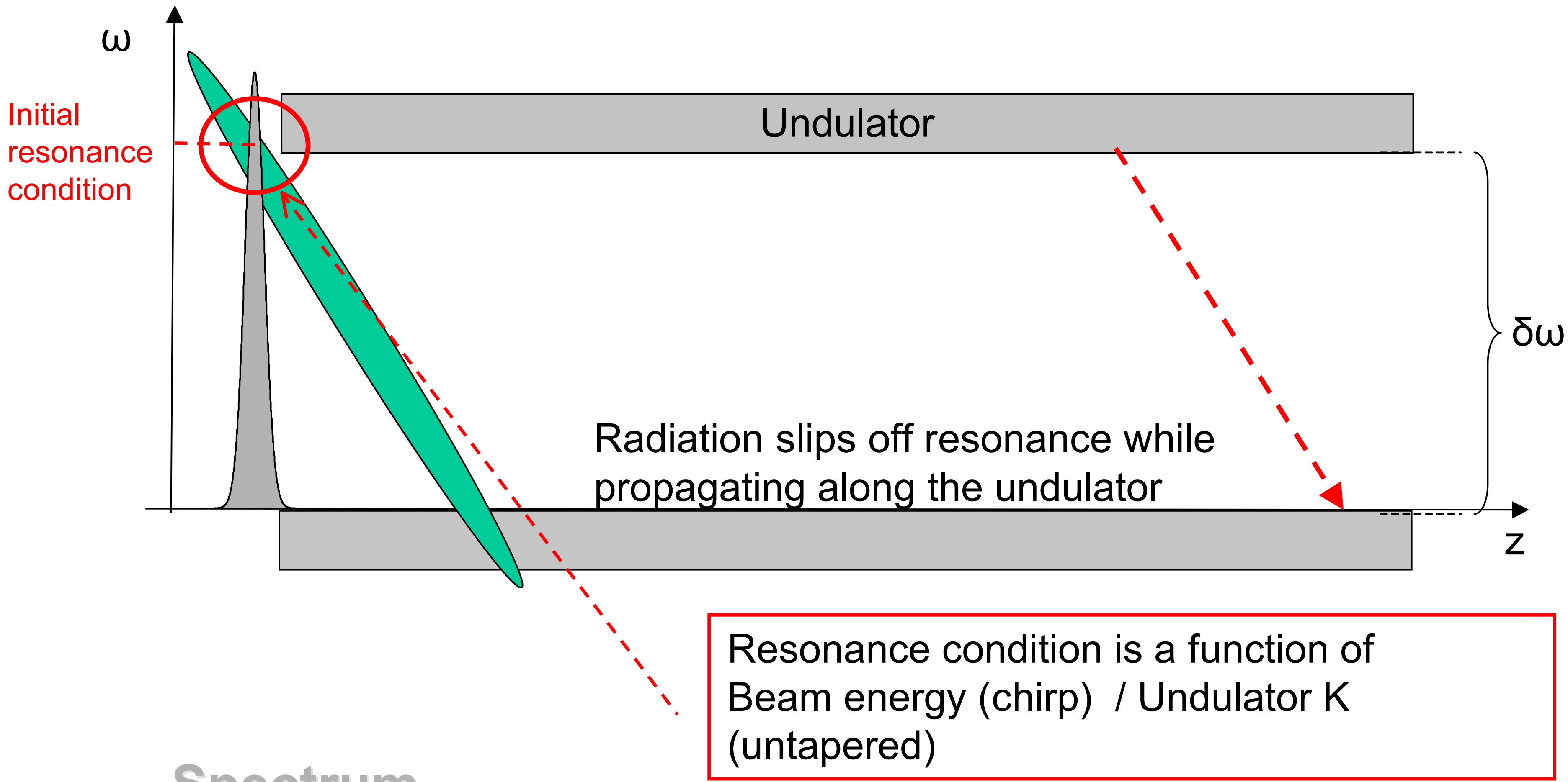


Strong chirp / energy spread in the longitudinal phase space

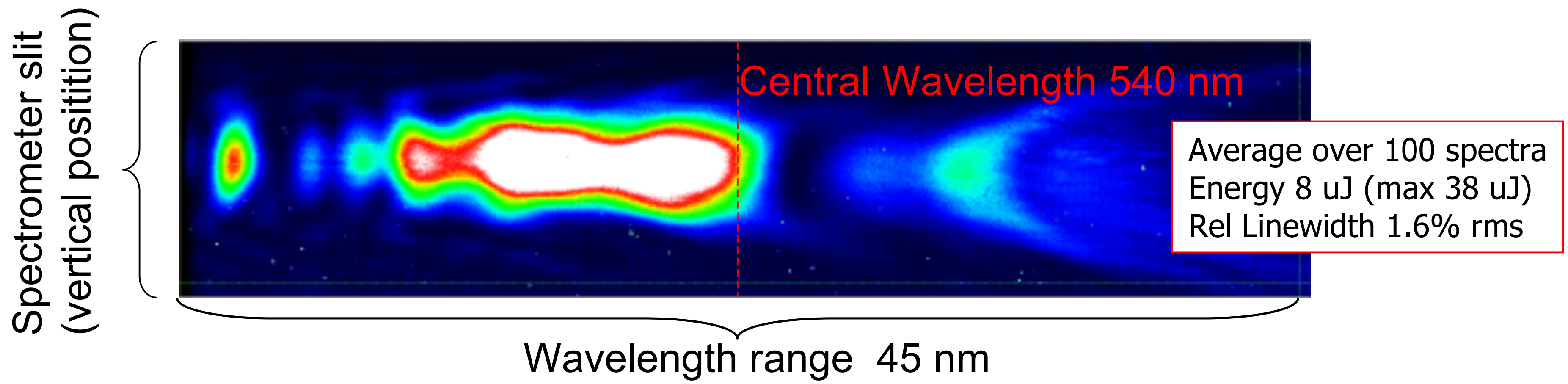


Compensation of the chirp with UM Taper

E. L. Saldin, E. A. Schneidmiller, and M.V. Yurkov, Self-amplified spontaneous emission FEL with energy-chirped electron beam and its application for generation of attosecond x-ray pulses, PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 9, 050702 (2006)


ω
Initial
resonance
condition
Undulator
δω
Radiation slips off resonance while
propagating along the undulator
z
Resonance condition is a function of
Beam energy (chirp) / Undulator K
(untapered)

Spectrum


Spectrometer slit
(vertical position)
Central Wavelength 540 nm
Average over 100 spectra
Energy 8 uJ (max 38 uJ)
Rel Linewidth 1.6% rms
Wavelength range 45 nm

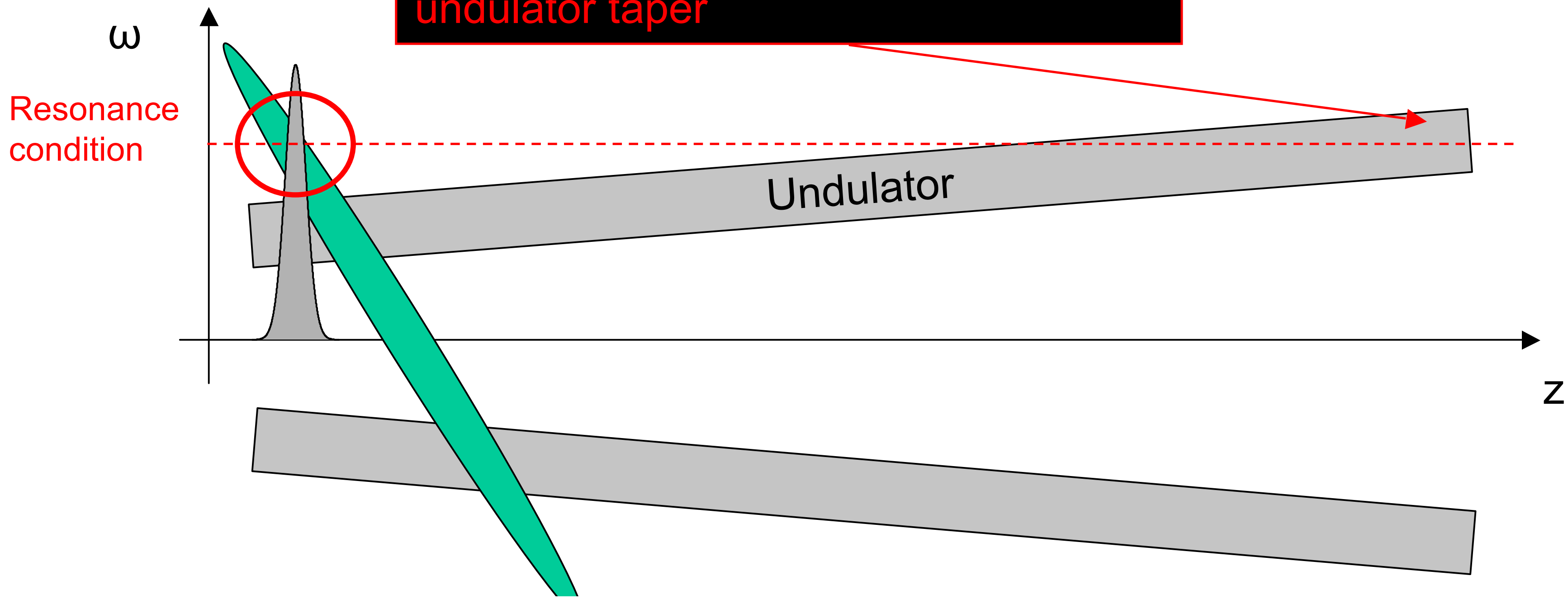
Compensation with Undulator taper

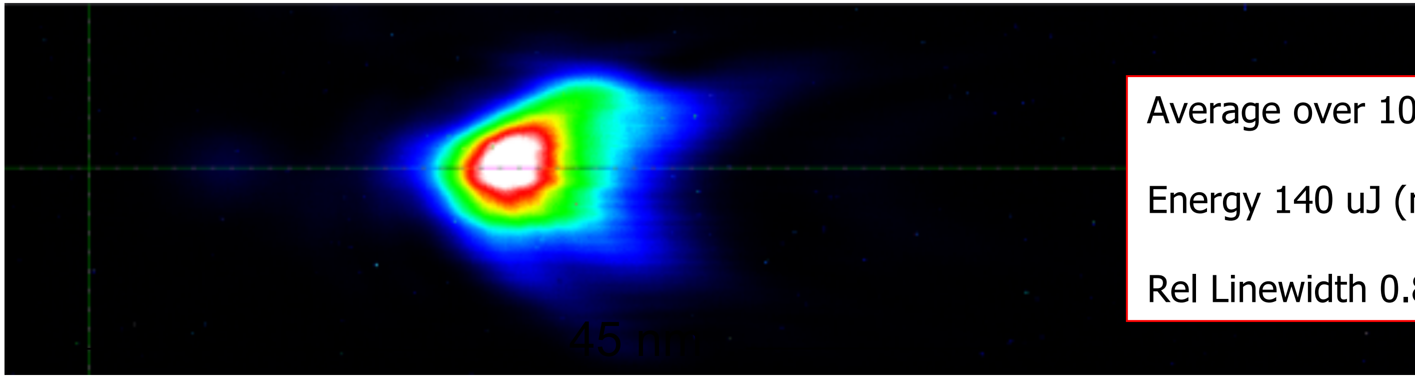
Chirp $\bar{\gamma} = \bar{\gamma}(s) = \gamma_0 + \alpha(s - s_0)$

$$\lambda_l \approx \frac{\lambda_u}{2\gamma(s)_r^{\,2}}\left(1 + \frac{K(z)^2}{2}\right)$$

Taper $K = K(z) = K_0 + \alpha_k(z - z_0)$

Resonance is maintained by tuning the undulator taper





Average over 100 spectra:

Energy 140 uJ (max 380 uJ)

Rel Linewidth 0.8% rms

Single cooperation length observed in many spectra (as the one shown above)

Average energy per pulse 18 times higher !!!

… in a narrower bandwidth (~1/2)