Beam dynamics and layout

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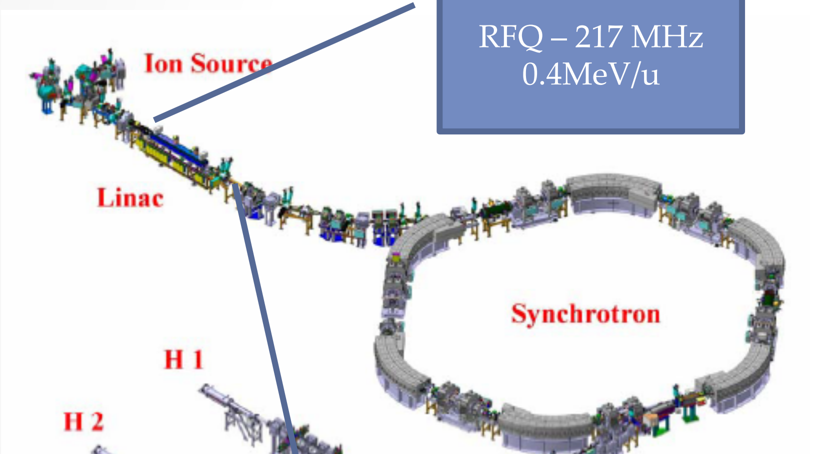
Two designs

- 5m long linac + Synchro with radius about 10 m

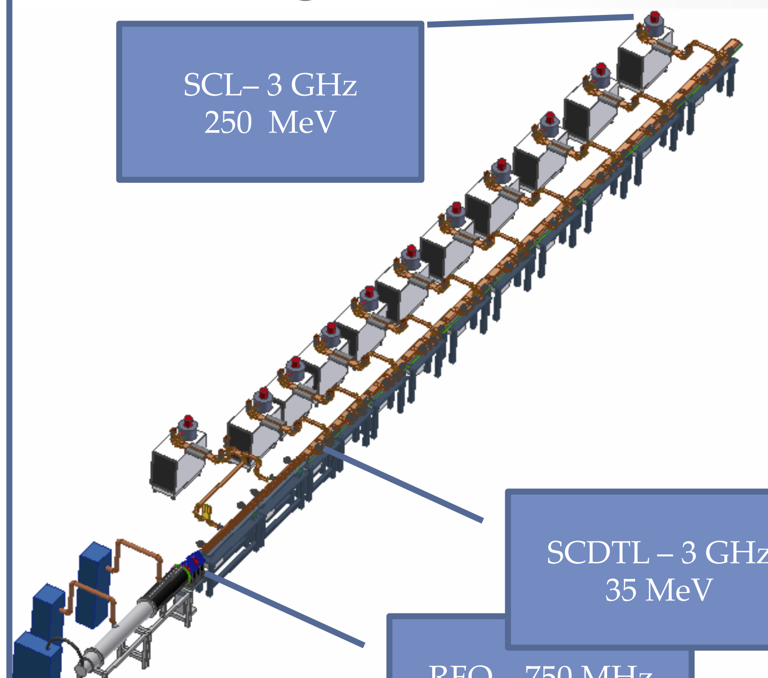


RFQ – 217 MHz
0.4MeV/u

IH 7 – 217 MHz
7MeV/u

- 30 m long linac

SCL– 3 GHz
250 MeV



SCDTL – 3 GHz
35 MeV

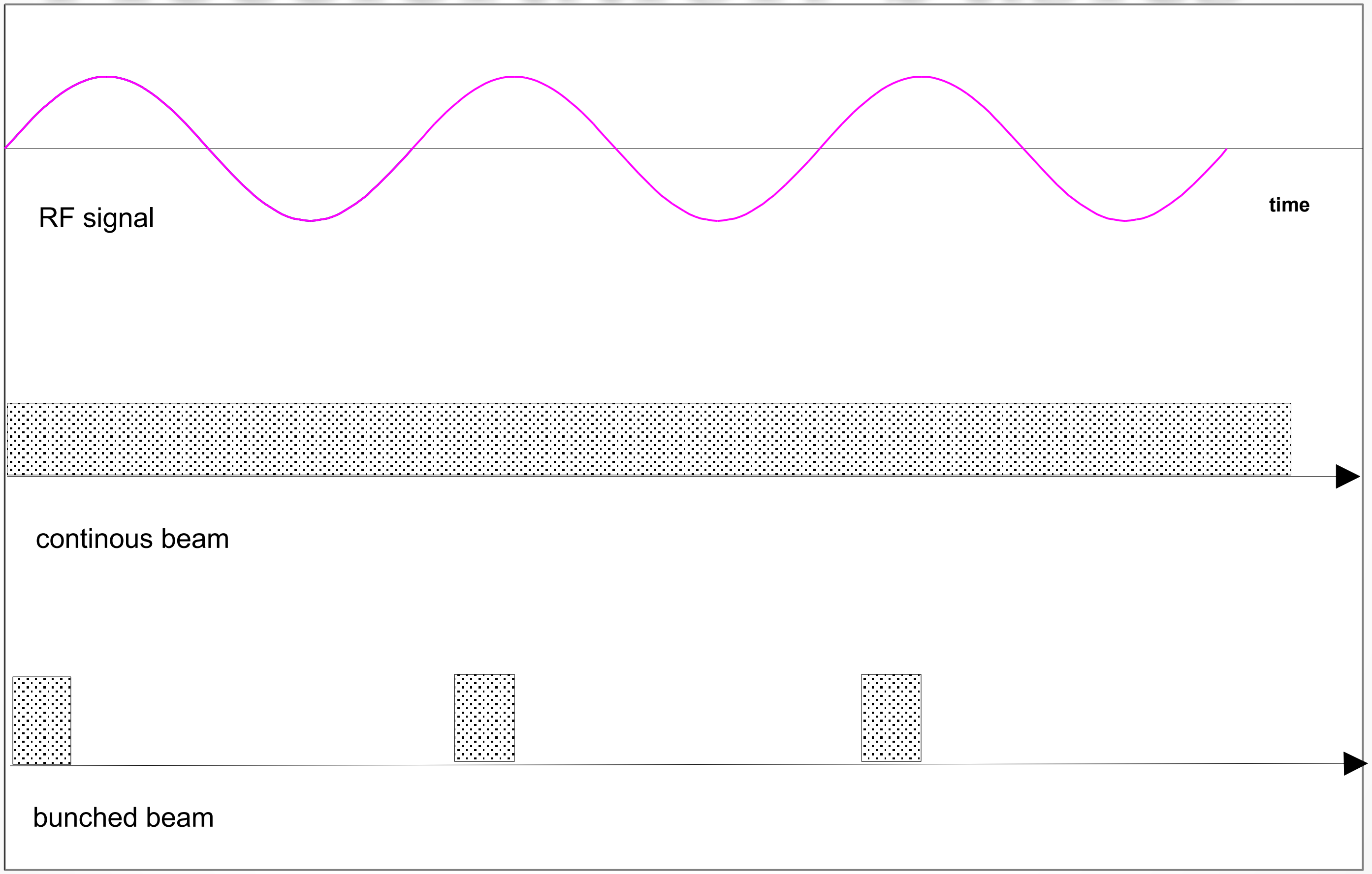
RFQ – 750 MHz
5 MeV

A medical accelerator should …

- Increase the energy of a bunch of protons (carbon ions) from 50keV to 250 MeV (450MeV/u) : NEED RF CAVITIES!
- Keep the beam volume small : NEED SOLENOIDS and QUADRUPOLES
- Conserve the beam emittance : we need to proceed "adiabatically" , i.e. without abrupt transitions and respecting a few rules.
- Besides all the above, be

Compact –Cheap- Reliable Easy to operate and Modular

Acceleration-basics

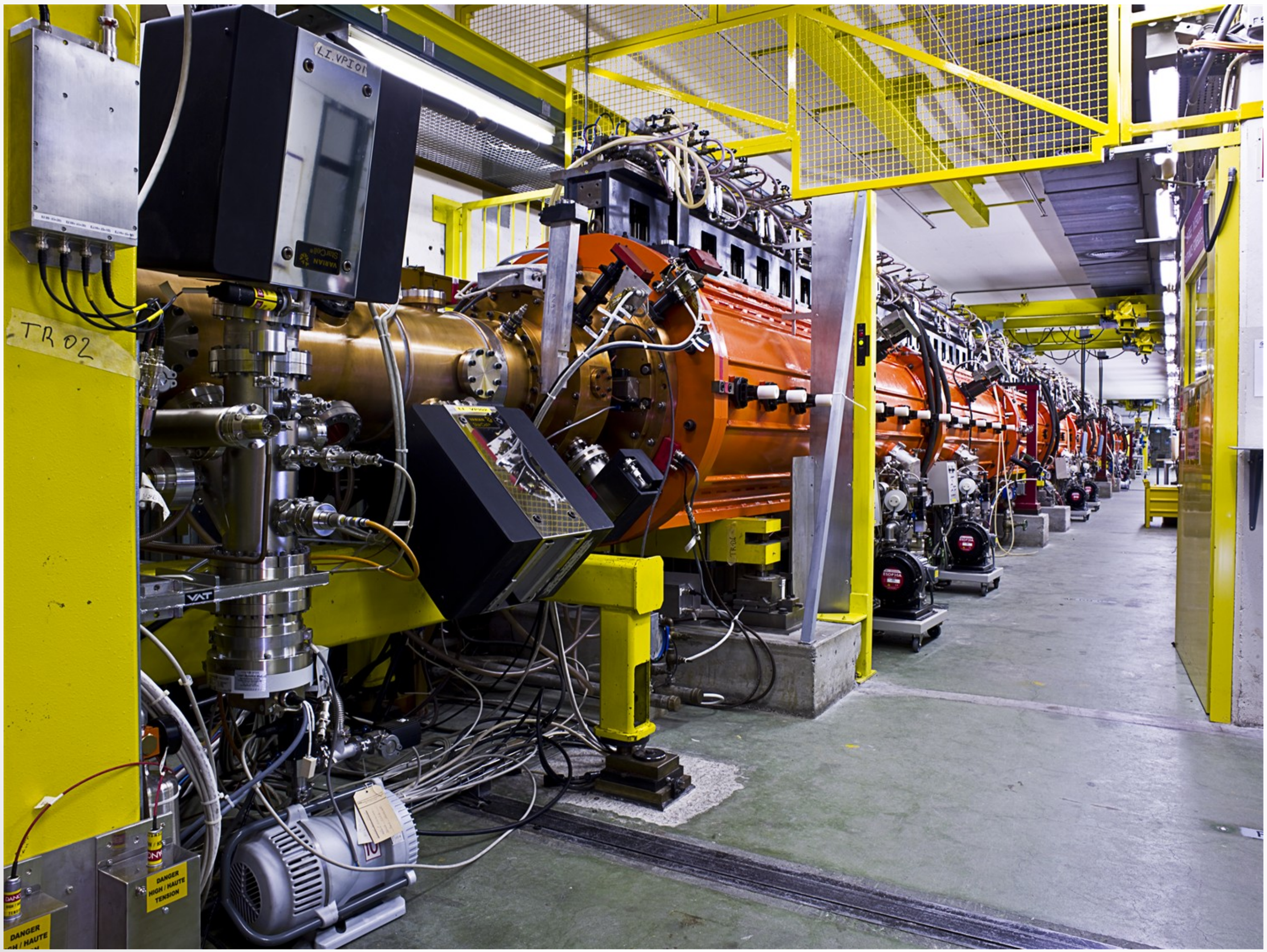


It is not possible to transfer energy to an un-bunched beam

We need a pre-injector

- System that prepares a continuous beam from the source for RF acceleration:
    - To accelerate the beam to the input energy of the injector (optional).
    - Shape the longitudinal emittance to match it to the injector acceptance.
    - All the above with minimal losses and minimal emittance growth.
- Typically few MeV /few meters : not very efficient
- It comes after the particle source (electrostatic acceleration)
- Pre-injector is where the LONGITUDINAL emittance is formed (we go from 2d to 3d)

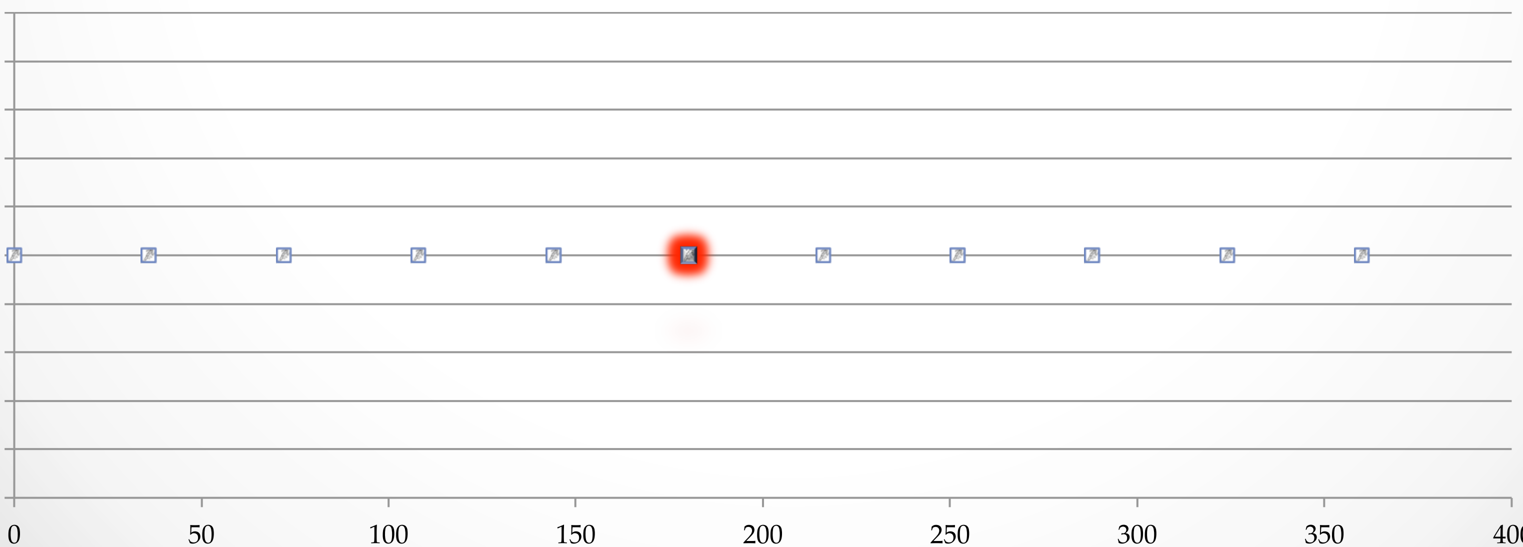
Pre-injectors



How to group a flow of people

- Think of a line of people running single file at the same speed:

- Two ways :
    - Send them on different path (400 m track)
    - Tell to half of them to slow down and half of them to speed up



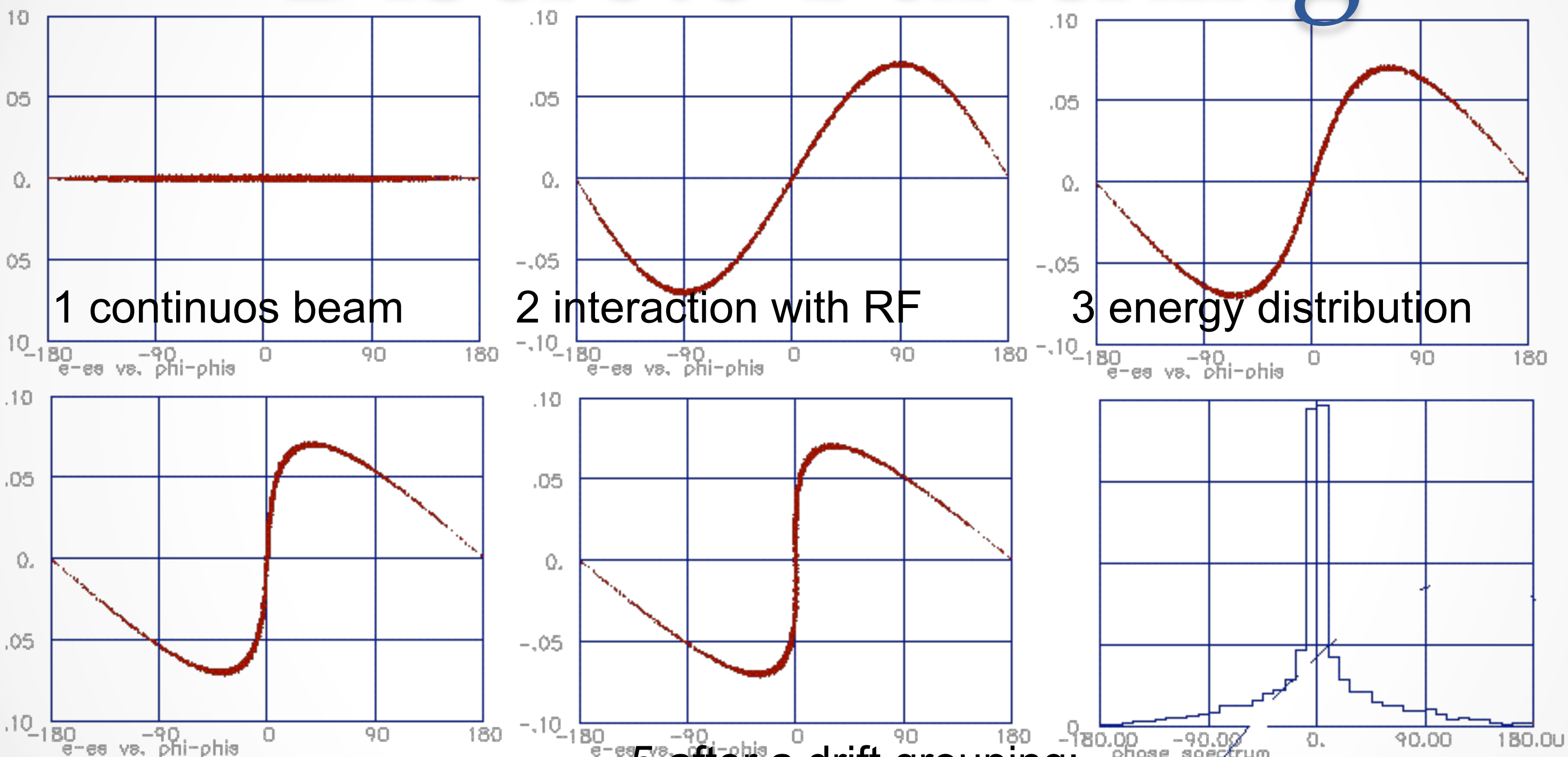
How to bunch a particle beam

- For a beam of particles
  - Send them through a dispersive system

  - Use a RF cavity to generate a velocity spread inside the beam

    - generate a velocity spread inside the beam by letting it through an RF cavity
    - let the beam distribute itself around the particle with the average velocity

Discrete Bunching



1 continuos beam

2 interaction with RF

3 energy distribution

4 earlier particles are slowed down,
later particles are accelerated

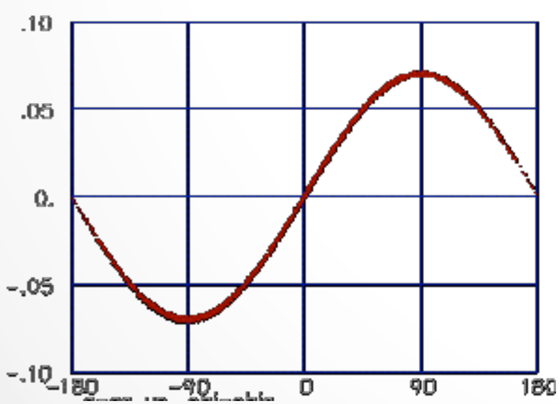
5.after a drift grouping: around the “central” particle

6 distribution of the particles

Discrete bunching - efficiency

Single harmonic

- Single cavity efficiency is 50% at best : ½ of the beam sees the wrong slope….

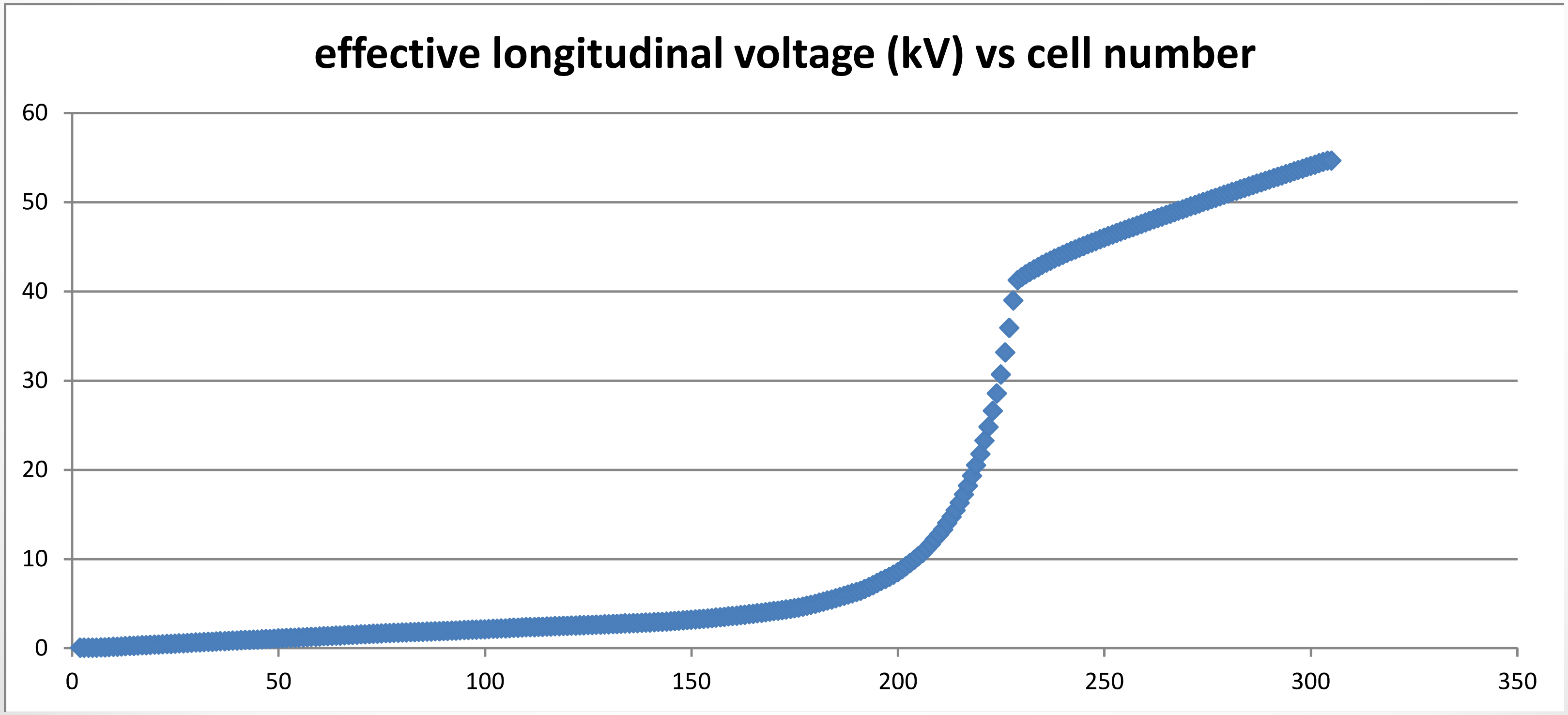


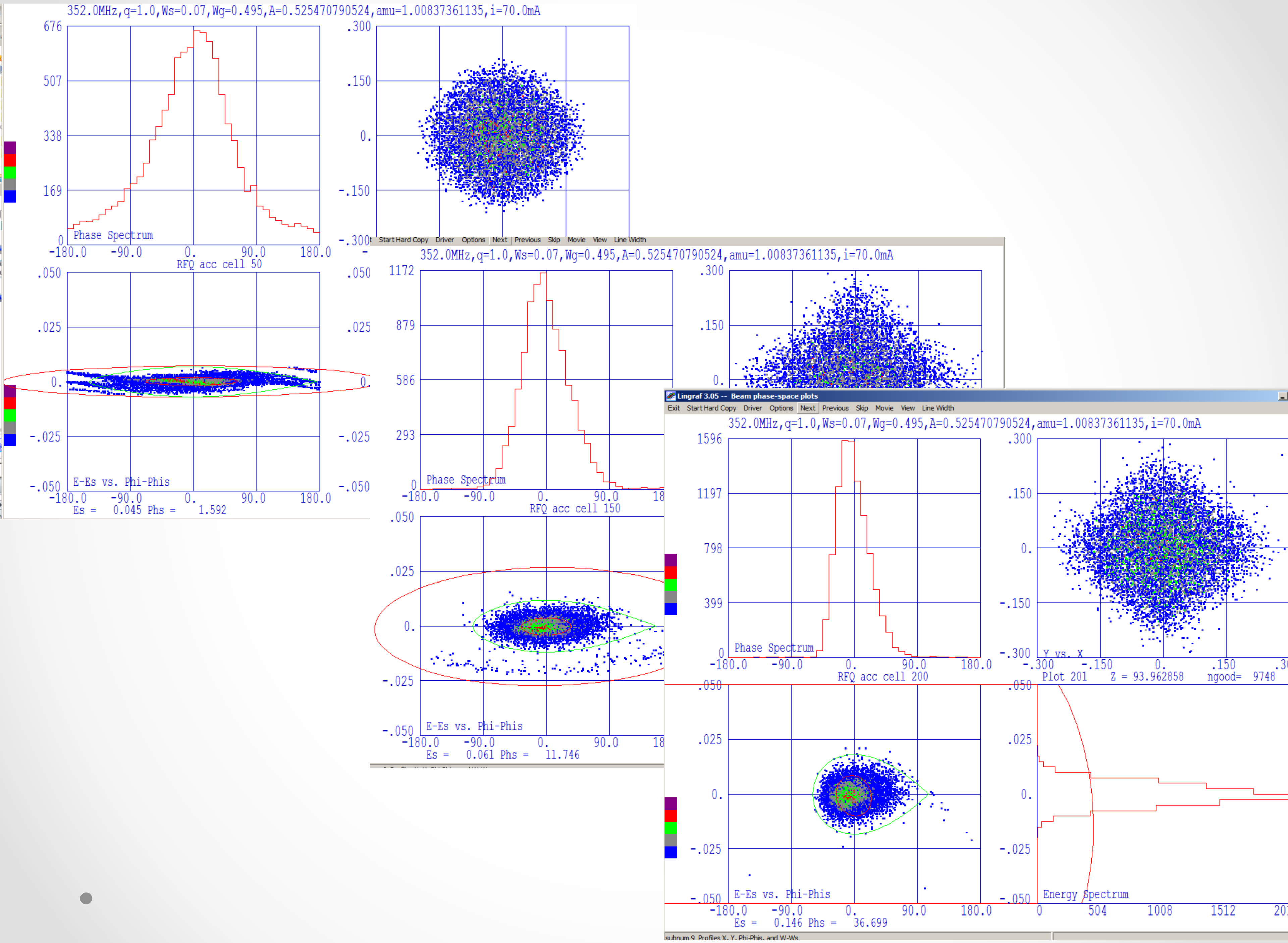
Double harmonic

- Maybe we can add a higher harmonic and improve things (double harmonic buncher)

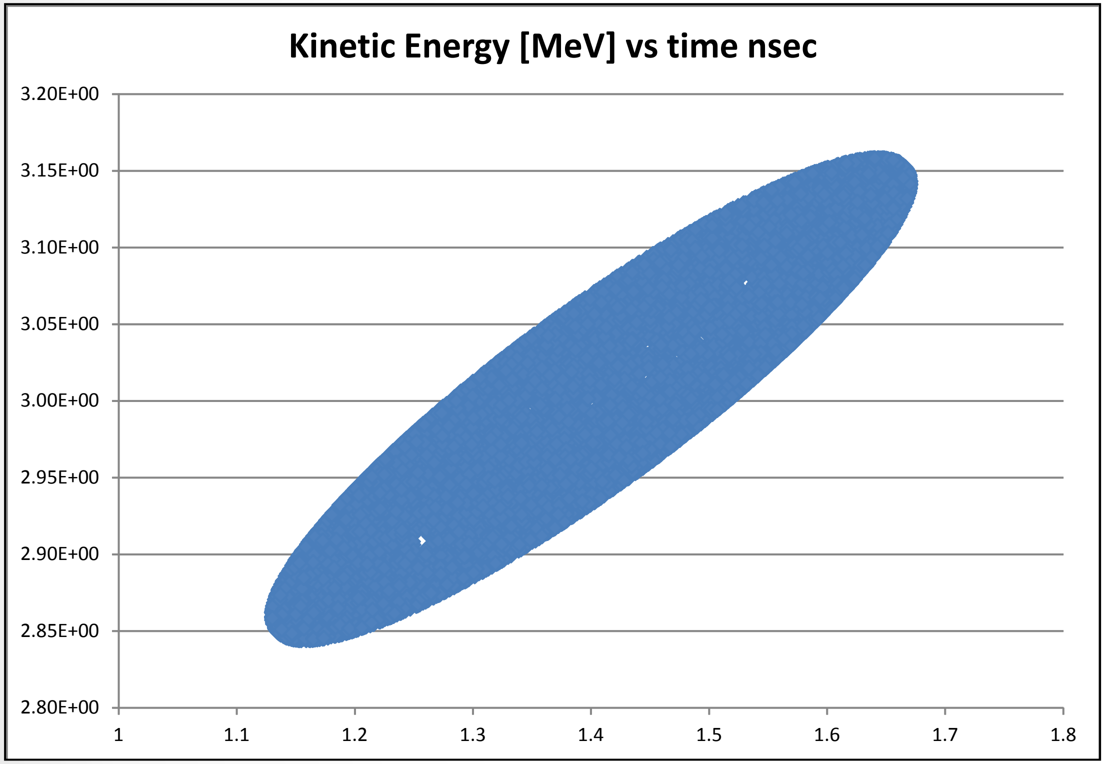
Can we improve the efficiency

- Adiabatic bunching : continuous bunching at very low voltage which gives time to the beam to wrap around the synchronous phase. generate the velocity spread continuously with small longitudinal field : bunching over several oscillation in the phase space (up to 100!) allows a better capture around the stable phase : 95% capture vs 50 %
- in an RFQ by slowly increasing the depth of the modulation along the structure it is possible to smoothly bunch the beam and prepare it for acceleration.



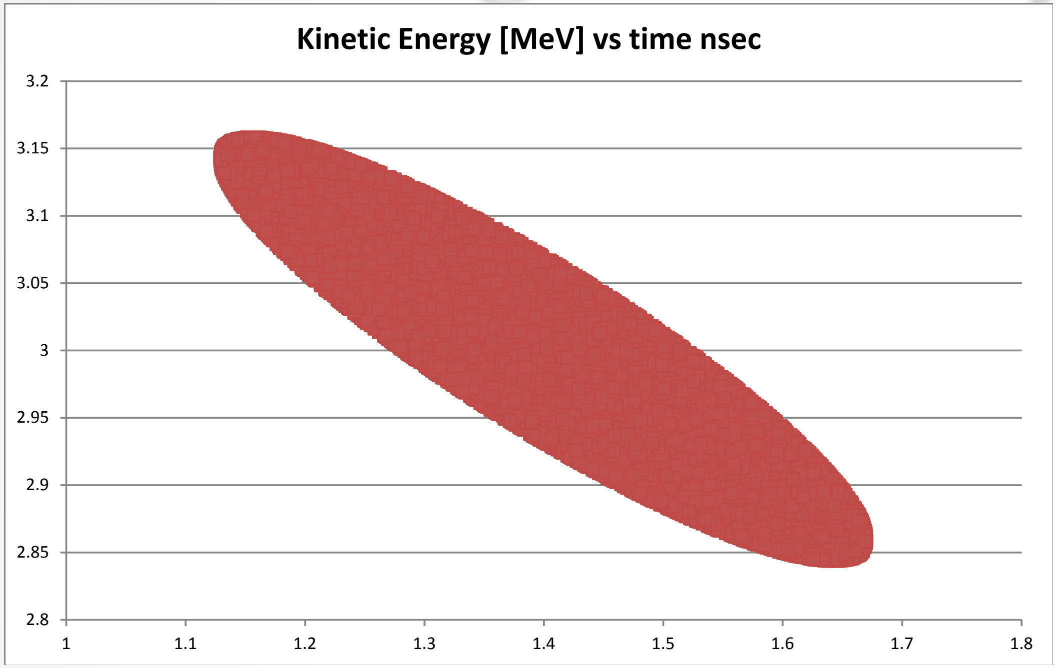

352.0MHz,q=1.0,Ws=0.07,Wg=0.495,A=0.525470790524,amu=1.00837361135,i=70.0mA
Phase Spectrum
RFQ acc cell 50
E-Es vs. Phi-Phis
Es = 0.045 Phs = 1.592
Start Hard Copy Driver Options Next Previous Skip Movie View Line Width
352.0MHz,q=1.0,Ws=0.07,Wg=0.495,A=0.525470790524,amu=1.00837361135,i=70.0mA
Phase Spectrum
RFQ acc cell 150
E-Es vs. Phi-Phis
Es = 0.061 Phs = 11.746
Lingraf 3.05 -- Beam phase-space plots
Exit Start Hard Copy Driver Options Next Previous Skip Movie View Line Width
352.0MHz,q=1.0,Ws=0.07,Wg=0.495,A=0.525470790524,amu=1.00837361135,i=70.0mA
Phase Spectrum
RFQ acc cell 200
Y vs. X
Plot 201 Z = 93.962858 ngood= 9748
E-Es vs. Phi-Phis
Es = 0.146 Phs = 36.699
Energy Spectrum
subnum 9 Profiles X, Y, Phi-Phis, and W-Ws

Ck-longitudinally focused



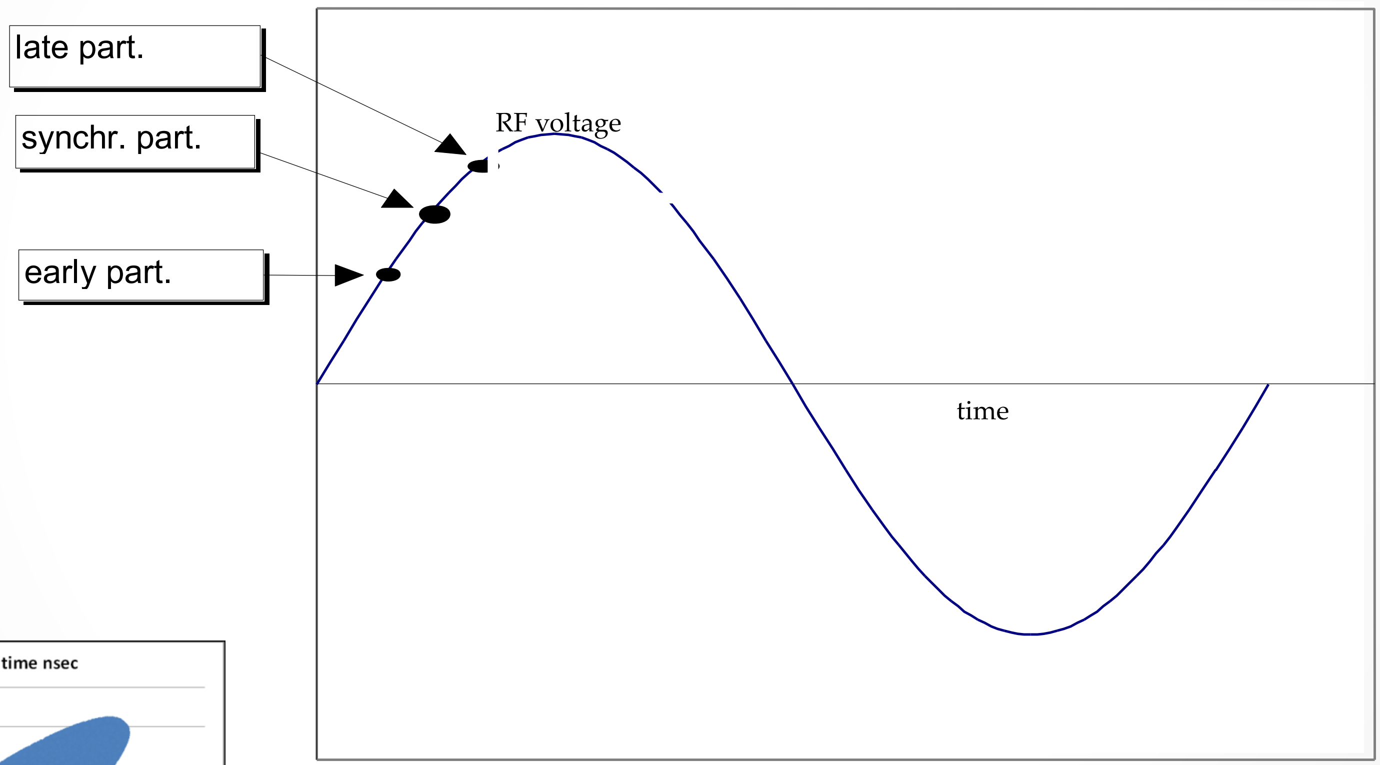
- Assume we are on the frequency of 352MHz, T = 2.8 nsec
- Q1 : is this beam bunched?
- Q2 : is this beam going to stay bunched ?

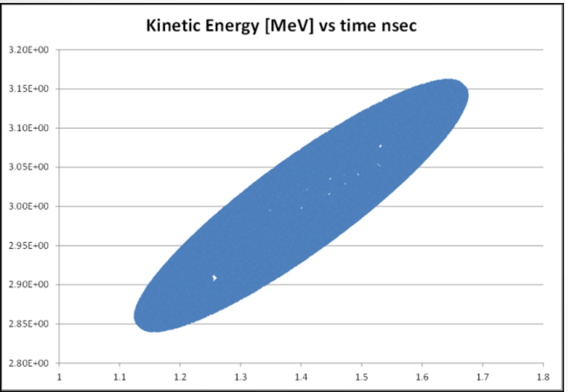
Ck-longitudinally defocused



- Assume we are on the frequency of 352MHz, T = 2.8 nsec
- Q3 :how will this beam look after say 20 cm?

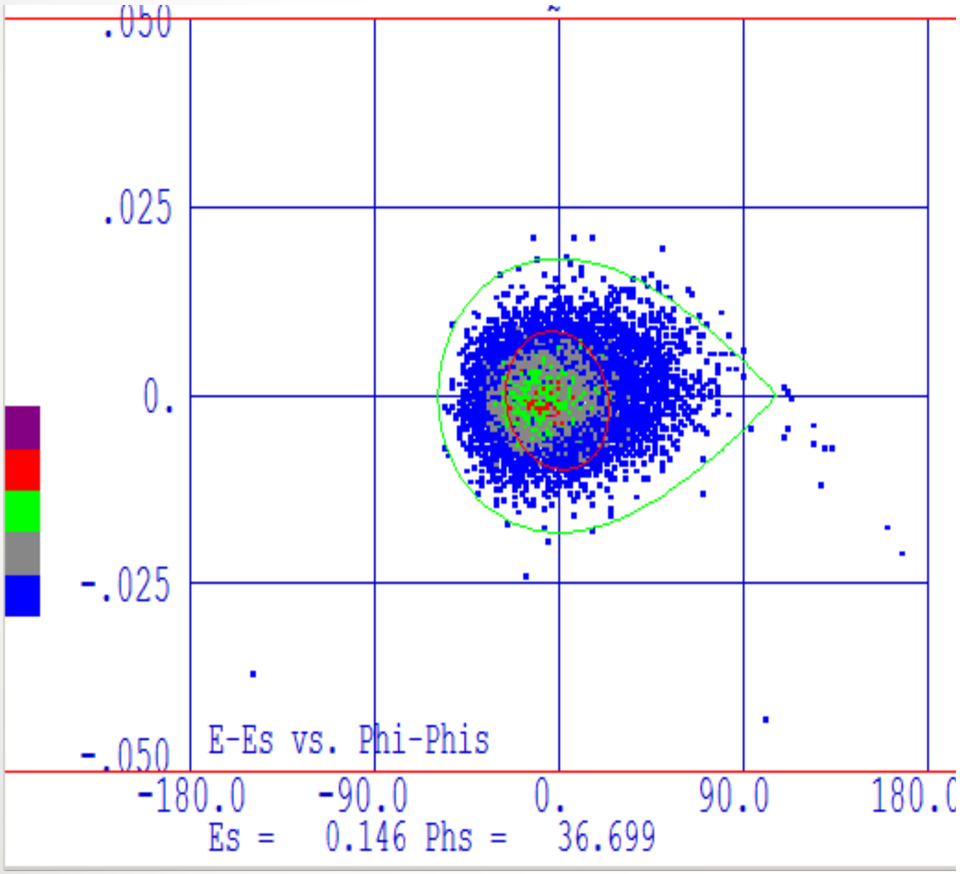
Keep bunching during acceleration





for phase stability we need to accelerate when dEz/dz >0 i.e. on the rising part of the RF wave

Synchronous particle



- it's the (possibly fictitious) particle that we use to calculate and determine the phase along the accelerator. It is the particle whose velocity is used to determine the synchronicity with the electric field. Design for that particle and provide longitudinal focusing so that the other stick with it!

Synchronous particle and geometrical beta β_g.

- design a linac for one "test" particle. This is called the "synchronous" particle.

- the length of each accelerating element determines the time at which the synchronous particles enters/exits a cavity.
- For a given cavity length there is an optimum velocity (or beta) such that a particle traveling at this velocity goes through the cavity in half an RF period.
- The difference in time of arrival between the synchronous particles and the particle traveling with speed corresponding to the geometrical beta determines the phase difference between two adjacent cavities
- in a synchronous machine the geometrical beta is always equal to the synchronous particle beta and EACH cell is different

Perfect synchronicity

- The field has to be always positive in the cavity
- The length of the accelerating gap is either
  $L=\beta\lambda/2$ or $L=\beta\lambda$
- Each cavity is adapted to the speed of the particle (geometrical beta, measure of the cavity length in terms of optimal speed of the particle going through)
- Best possible longitudinal beam dynamics
- Full control of the longitudinal phase space

Case : $\beta_s = \beta_g$

- The absolute phase φ_i and the velocity β_{i-1} of this particle being known at the entrance of cavity i, its RF phase ϕ_i is calculated to get the wanted synchronous phase ϕ_{si}, $\boxed{\phi_i = \varphi_i - \phi_{si}}$
- the new velocity β_i of the particle can be calculated from, $\boxed{\Delta W_i = qV_0T \cdot \cos\phi_{si}}$

  ① if the phase difference between cavities i and $i+1$ is given, the distance D_i between them is adjusted to get the wanted synchronous phase ϕ_{si+1} in cavity $i+1$.

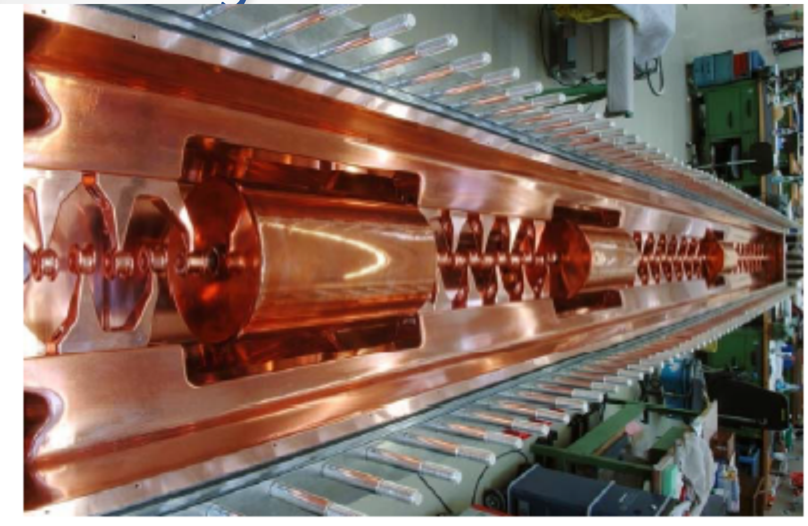
  ② if the distance D_i between cavities i and $i+1$ is set, the RF phase ϕ_i of cavity $i+1$ is calculated to get the wanted synchronous phase ϕ_{si+1} in it.

RF phase	ϕ_{i-1}		ϕ_i		ϕ_{i+1}
Particle velocity		β_{si-1}		β_{si}	
Distances		D_{i-1}		D_i	
Synchronous phase	ϕ_{si-1}		ϕ_{si}		ϕ_{si+1}
Cavity number	$i-1$		i		$i+1$

Synchronism condition :

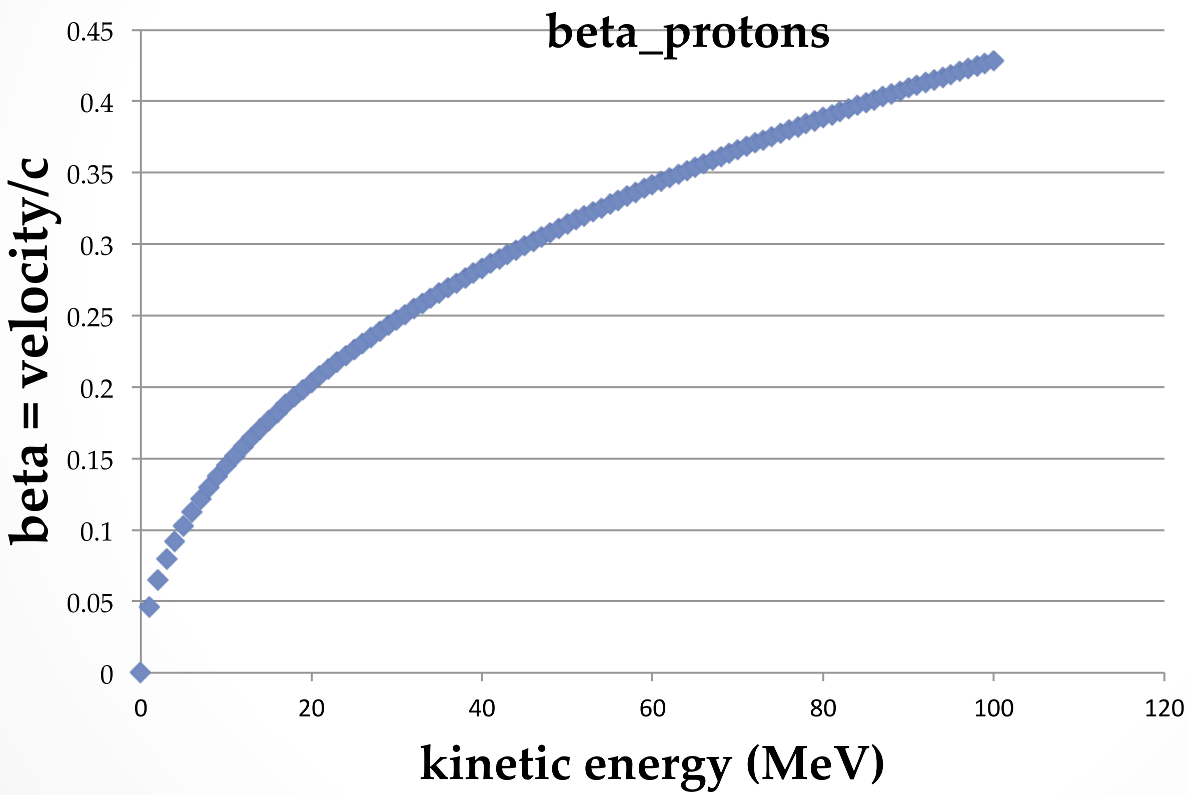
$$\boxed{\phi_{si+1} - \phi_{si} = \omega \cdot \frac{D_i}{\beta_{si} c} + \phi_{i+1} - \phi_i + 2\pi n}$$

Synchronous structures



Compact –Cheap- Reliable
Easy to operate and Modular

Beta vs W

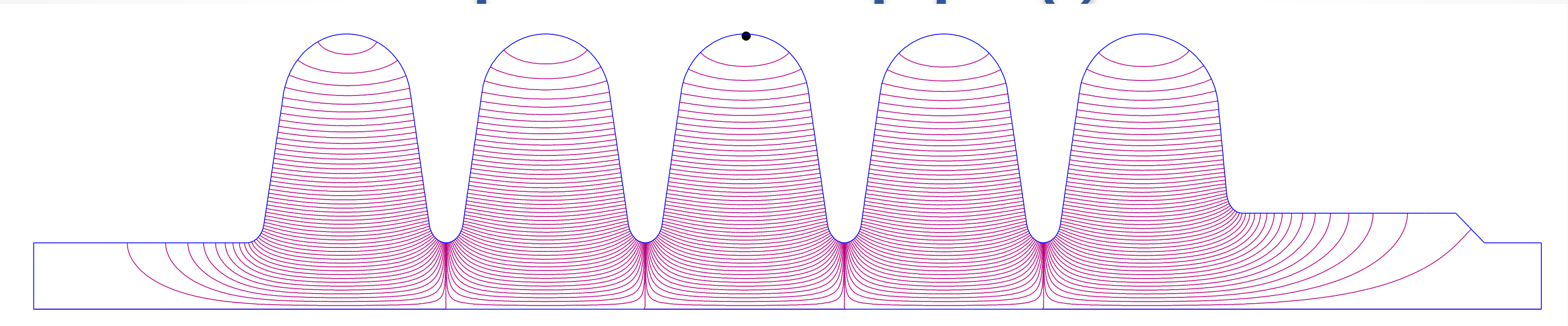


beta_protons
beta = velocity/c
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
120
kinetic energy (MeV)

Case : $\beta_s \sim \beta_g$

- for simplifying construction and therefore keeping down the cost, cavities are not individually tailored to the evolution of the beam velocity but they are constructed in blocks of identical cavities (tanks). several tanks are fed by the same RF source.
- This simplification implies a "phase slippage" i.e. a motion of the centre of the beam . The phase slippage is proportional to the number of cavities in a tank and it should be carefully controlled for successful acceleration.

phase slippage



Lcavity = $\beta_g\lambda/2$

particle enters the cavity with $\beta_s < \beta_g$. It is accelerated

the particle has not left the cavity when the field has changed sign : it is also a bit decelerated

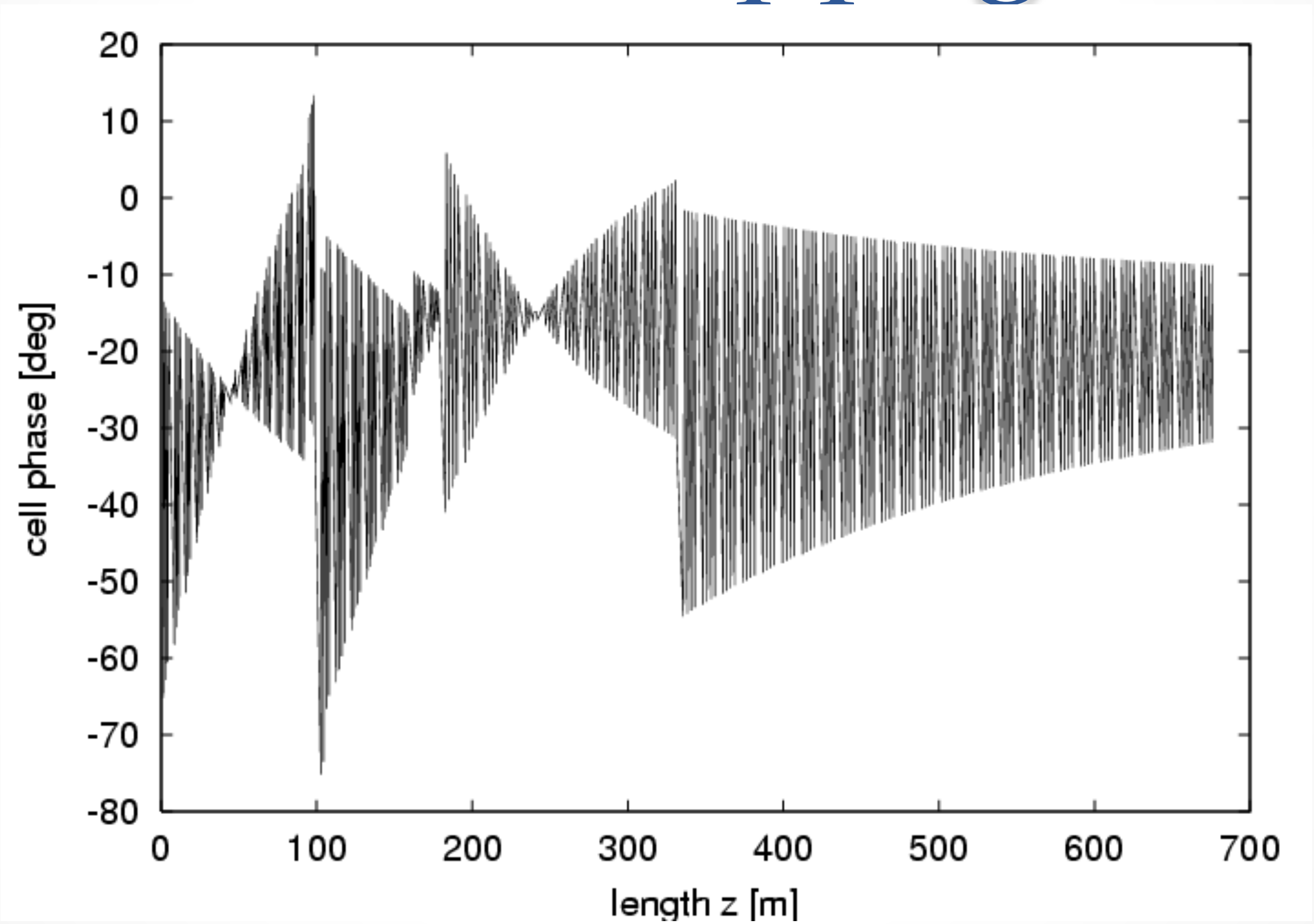
the particle arrives at the second cavity with a “delay”

........and so on and so on

we have to optimize the initial phase for minimum phase slippage

for a given velocity there is a maximum number of cavity we can accept in a tank

Phase slippage



Acceleration

- to describe the motion of a particle in the longitudinal phase space we want to establish a relation between the energy and the phase of the particle during acceleration
- energy gain of the synchronous particle $$\Delta W_s = qE_0 LT \cos(\phi_s)$$
- energy gain of a particle with phase Φ $$\Delta W = qE_0 LT \cos(\phi)$$
- assuming small phase difference Δ Φ =Φ-Φ$_s$

$$\frac{d}{ds}\Delta W = qE_0 T \cdot [\cos(\varphi_s + \Delta\varphi) - \cos\varphi_s]$$

- and for the phase

$$\frac{d}{ds}\Delta\varphi = \omega\left(\frac{dt}{ds} - \frac{dt_s}{ds}\right) = \frac{\omega}{c}\left(\frac{1}{\beta} - \frac{1}{\beta_s}\right) \cong -\frac{\omega}{\beta_s c}\frac{\Delta\beta}{\beta_s} = -\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\Delta W$$

Acceleration-Separatrix

- Equation for the canonically conjugated variables phase and energy with Hamiltonian (total energy of oscillation):

$$\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\left\{\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2 + qE_0T\left[\sin(\varphi_s+\Delta\varphi) - \Delta\varphi\cos\varphi_s - \sin\varphi_s\right]\right\} = H$$

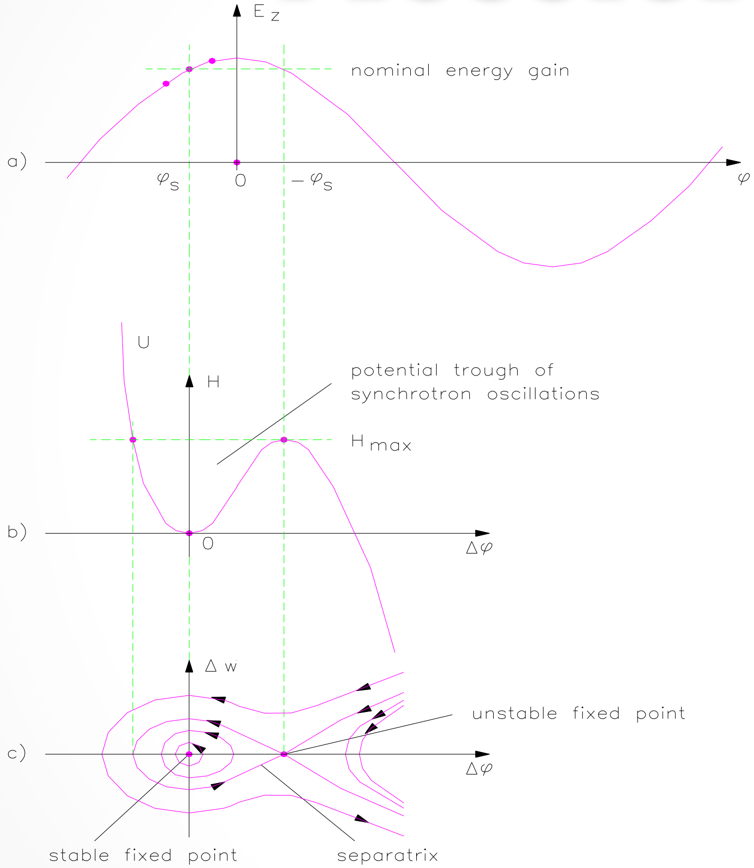
- For each H we have different trajectories in the longitudinal phase space .Equation of the separatrix (the line that separates stable from unstable motion)

$$\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2 + qE_0T\left[\sin(\varphi_s+\Delta\varphi) + \sin\varphi_s - (2\varphi_s+\Delta\varphi)\cos\varphi_s\right] = 0$$

- Maximum energy excursion of a particle moving along the separatrix

$$\Delta\hat{W}_{\max} = \pm 2\left[\frac{qmc^3\beta_s^3\gamma_s^3E_0T(\varphi_s\cos\varphi_s - \sin\varphi_s)}{\omega}\right]^{\frac{1}{2}}$$

Acceleration

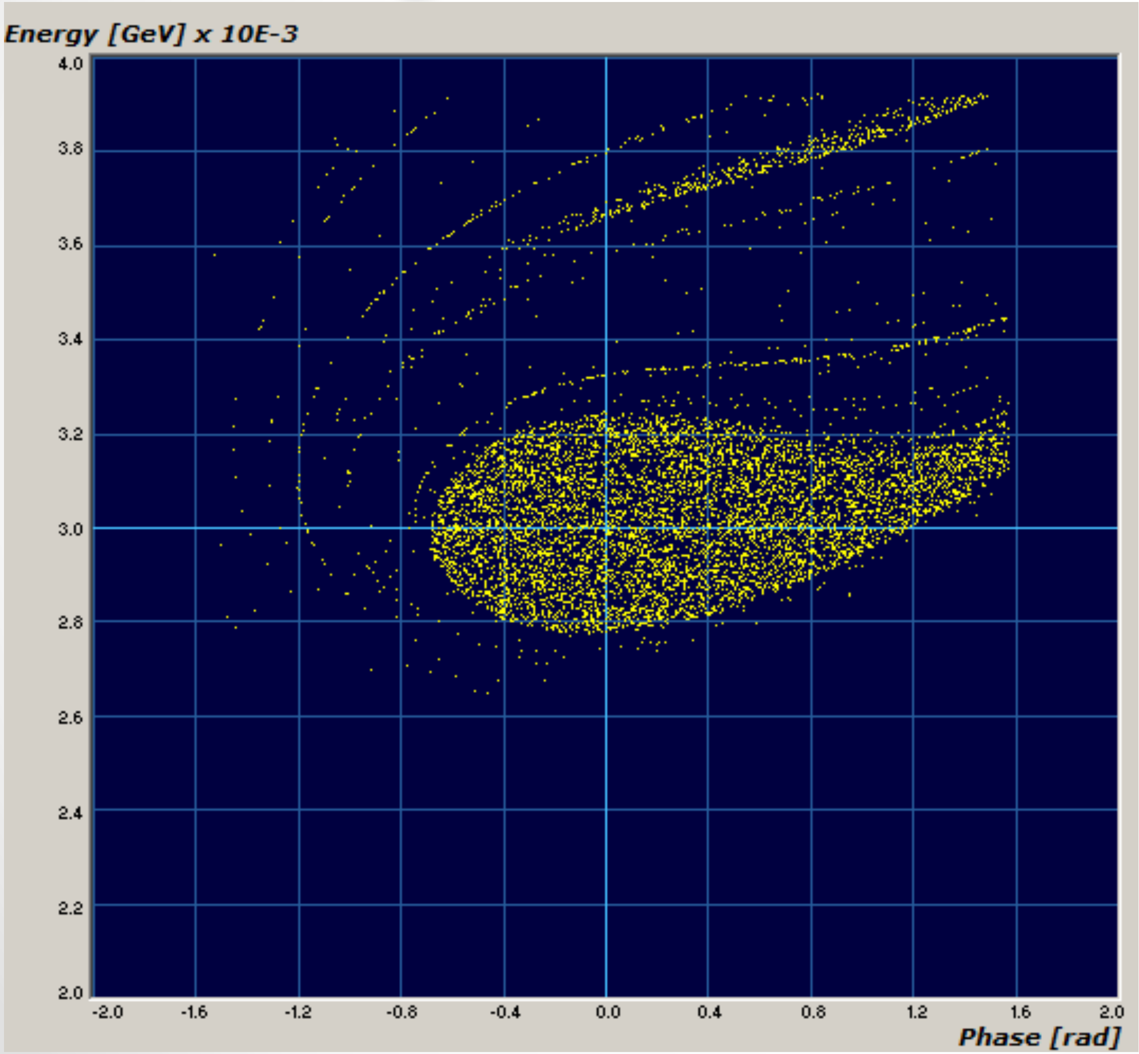


RF electric field as function of phase.

Potential of synchrotron oscillations

Trajectories in the longitudinal phase space each corresponding to a given value of the total energy (stationary bucket)

Longitudinal acceptance



Plot of the longitudinal acceptance of the CERN LINAC4 DTL (352 MHz, 3-50 MeV).
Obtained by plotting the survivors of very big beam in long phase space.

IH beam dynamics- KONUS

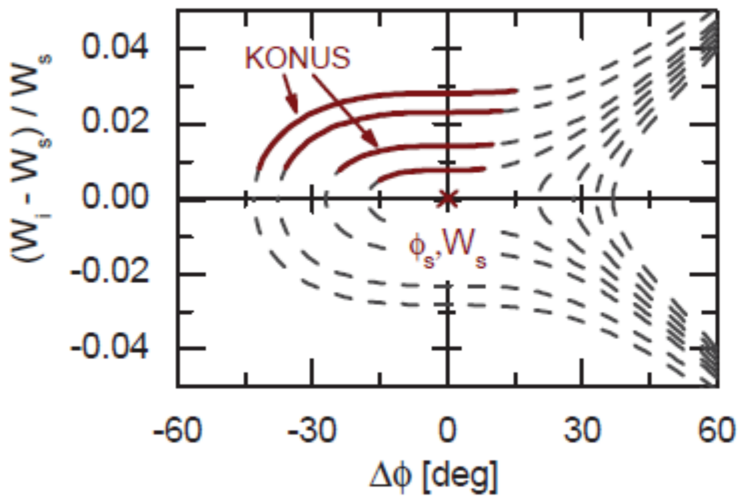


Figure 2: Single particle orbits in $\Delta W/W_s - \Delta\phi$ phase space at $\phi_s = 0°$ with color marking of the area used by KONUS.

Higher accelerating efficiency

Less RF defocusing (see later) – allow for longer accelerating sections w/o transverse focusing

Need re-bunching sections

Exceptions, exceptions…….

Longitudinal phase advance

- if we accelerate on the rising part of the positive RF wave we have a LONGITUDINAL FORCE keeping the beam bunched. The force (harmonic oscillator type) is characterized by the LONGITUDINAL PHASE ADVANCE

$$k_{0l}^2 = \frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda} \left[\frac{1}{m^2}\right]$$

- long equation

$$\frac{d^2 \Delta\varphi}{ds^2} + k_{0l}^2 \left(\Delta\varphi - \frac{\Delta\varphi^2}{2\tan(-\varphi_s)} \right) = 0$$

Longitudinal phase advance

- Per meter

$$k_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda}} \left[\frac{1}{m}\right]$$

Length of focusing period
L=(Number of RF gaps) βλ

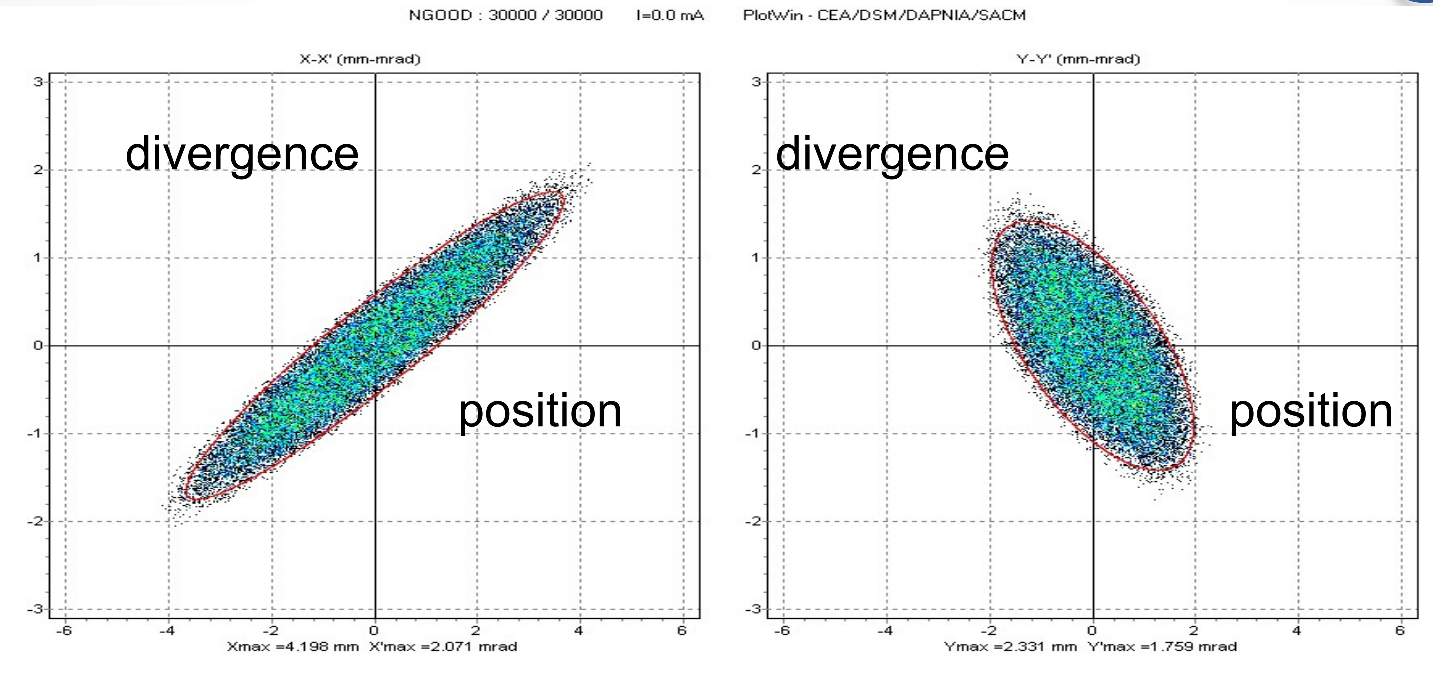
- Per focusing period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T N^2 \lambda \sin(-\varphi_s)}{mc^2 \beta_s \gamma^3}}$$

- Per RF period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{m \beta_s \gamma^3 \lambda}} \left[\frac{1}{s}\right]$$

Transverse focusing



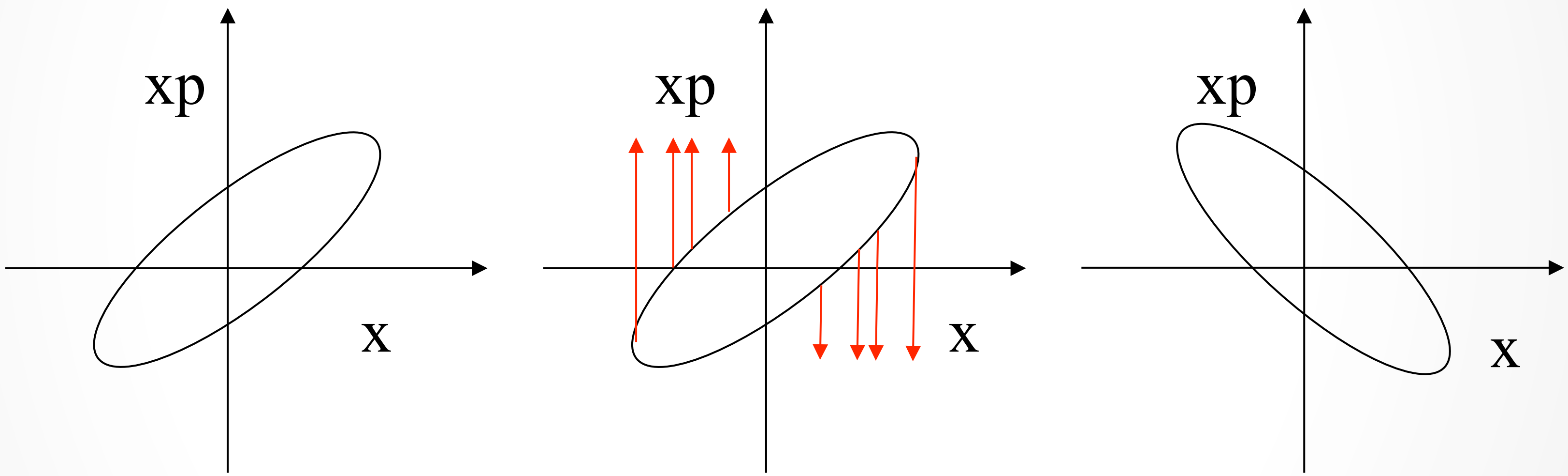
Bet = 6.3660 mm/Pi.mrad
Alp = -2.8807

DEFOCUSED

Bet = 1.7915 mm/Pi.mrad
Alp = 0.8318

FOCUSED

Focusing force



defocused beam

apply force towards the axis proportional to the distance from the axis

focused beam

$$F(x)=-K\,x$$

Focusing

- MAGNETIC FOCUSING

(dependent on particle velocity)

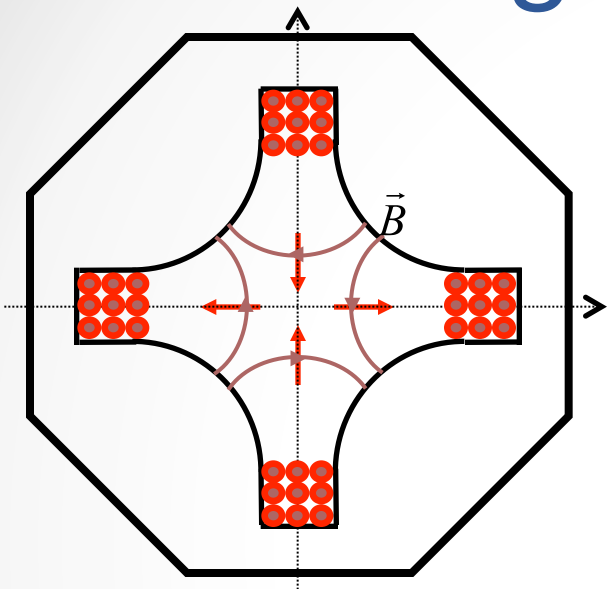
$$\vec{F} = q\vec{v} \times \vec{B}$$

- ELECTRIC FOCUSING (RFQ)

(independent of particle velocity)

$$\vec{F} = q \cdot \vec{E}$$

Magnetic quadrupole



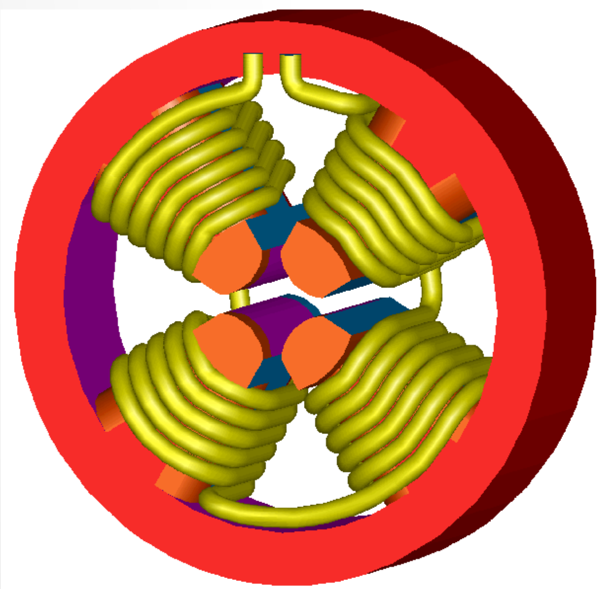
Magnetic field

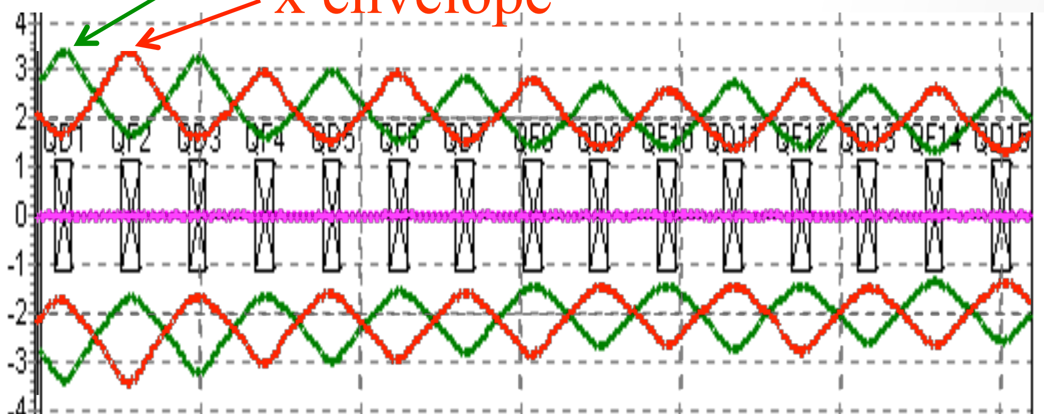
$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases}$$

Magnetic force

$$\begin{cases} F_x = -q \cdot v \cdot G \cdot x \\ F_y = q \cdot v \cdot G \cdot y \end{cases}$$

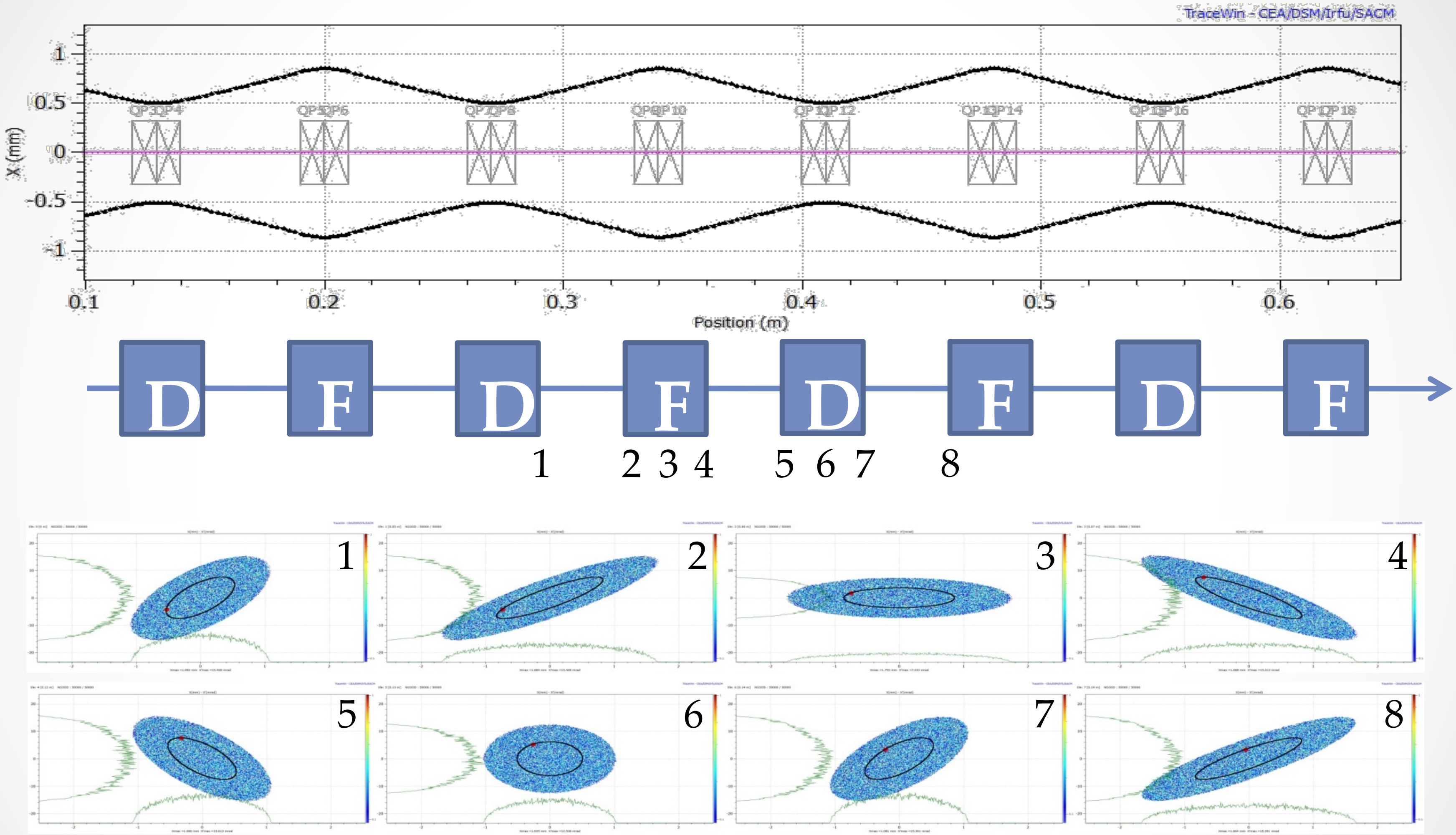
Focusing in one plan, defocusing in the other





sequence of focusing and defocusing quadrupoles

F0D0 focusing



The beam is matched, after every period, the twiss parameters are identical.

FODO

This is the only transparency where β is the twiss parameter and not the velocity/c

- periodic focusing channel : the beam 4D phase space is identical after each period
- Equation of motion in a periodic channel (Hill's equation) has periodic solution :

$$x(z) = \sqrt{\varepsilon_0 \beta(z)} \cdot \cos(\sigma(z))$$

emittance

beta function , has the periodicity of the focusing period

transverse phase advance

$$\beta(z+l) = \beta(z)$$

$$\sigma(z) = \int_0^z \frac{dz}{\beta(z)}$$

quadrupole focusing

$$\sigma_{0t} = \sqrt{\frac{\theta_0^4}{8\pi^2} + \Delta_{rf}}$$

zero current phase advance per period in a LINAC

G magnetic quadrupole gradient, [T/m]
N= number of magnets in a period

$$\theta_0^2 = \frac{qG\lambda^2 N^2 \beta\chi}{m_0 c\gamma}$$

for + - (*N=2*)

$$\chi = \frac{4}{\pi}\sin(\frac{\pi}{2}\Gamma)$$

for ++ - - (*N=4*)

$$\chi = \frac{8}{\sqrt{2}\pi}\sin(\frac{\pi}{4}\Gamma)$$

Γ is the quadrupole filling factor (quadrupole length relative to period length).

RF defocusing

Maxwell equations

$$\nabla \cdot E = 0 \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

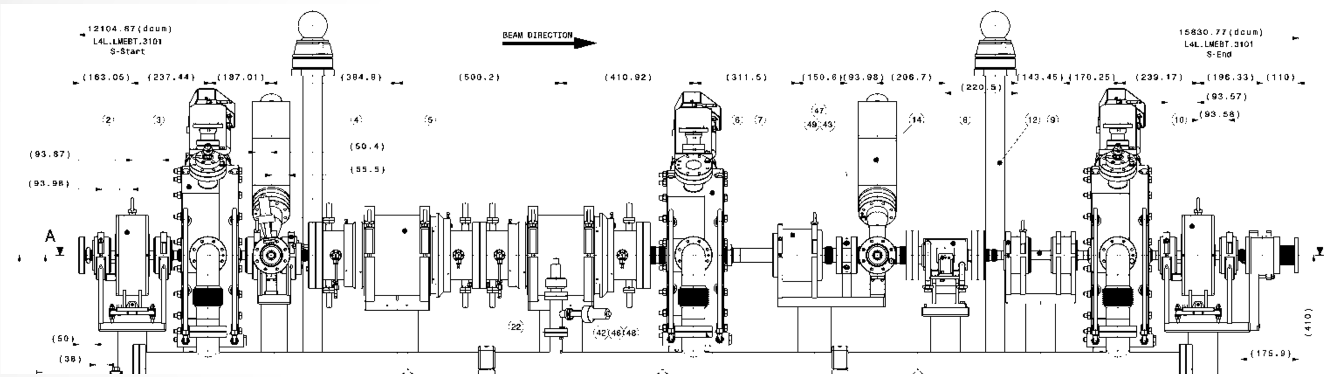
when longitudinal focusing (phase stability) , there is defocusing (depending on the phase) in the transverse planes

$$\Delta_{rf} = \frac{1}{2}\sigma_{0l}^{2} = \frac{\pi q \lambda N^2 E_0 T \sin\phi_s}{m_0 c^2 \beta\gamma^3}$$

Number of RF gap in a transverse focusing period

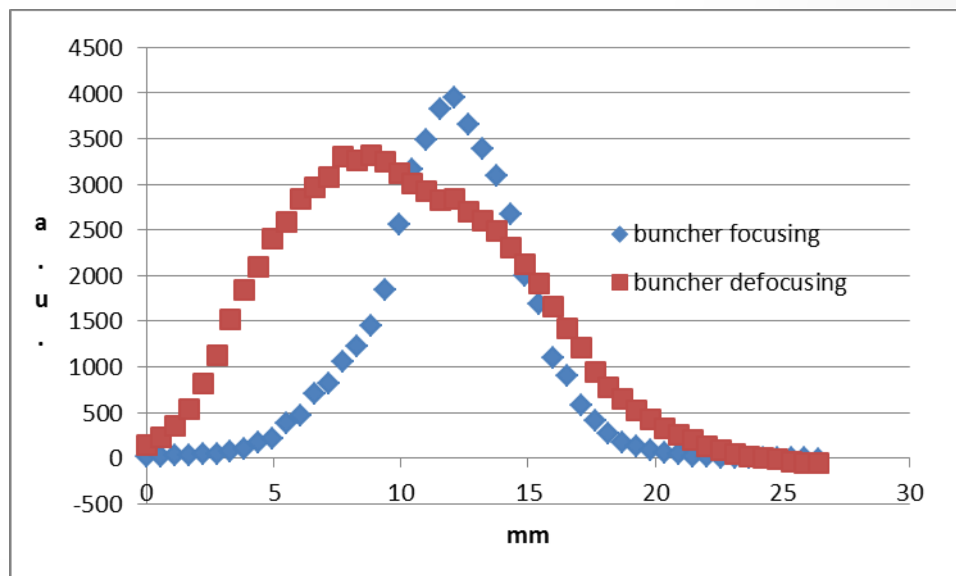
RF defocusing is MEASURABLE

(not only in text books)



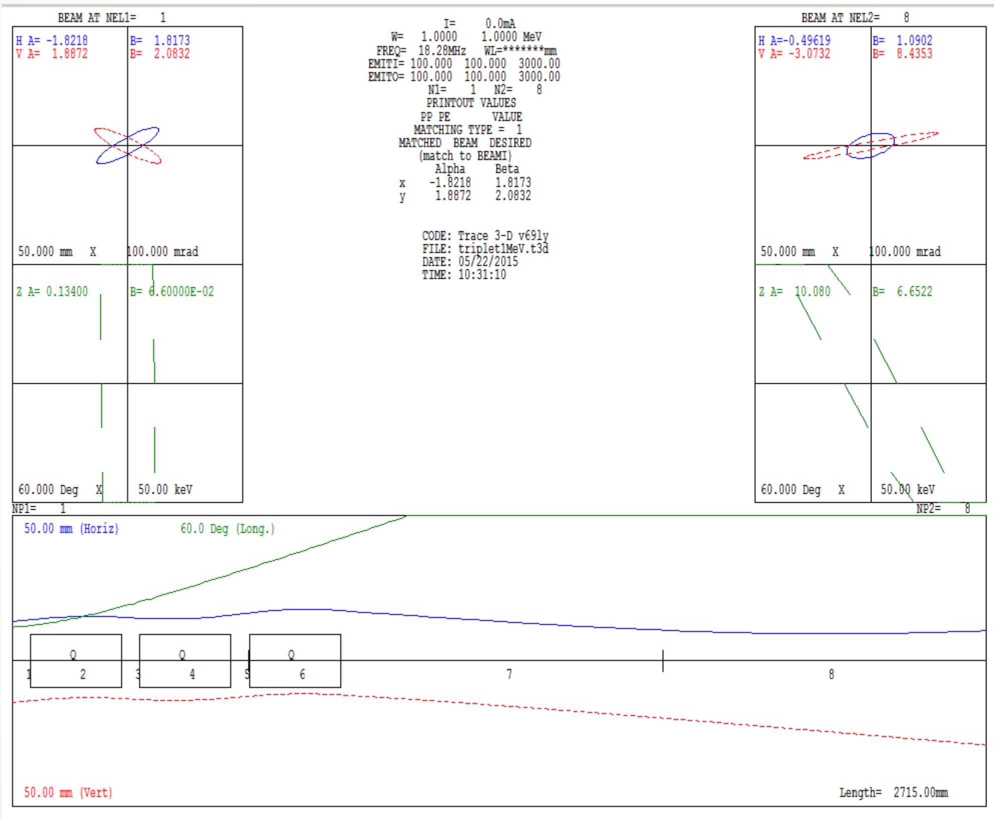
Change the buncher phase and measure the transverse beam profile

Effect of the phase-dependent focusing is visible and it can be used to set the RF phase in absence of longitudinal measurements.



triplets

- Arrangement of 3 quadrupoles

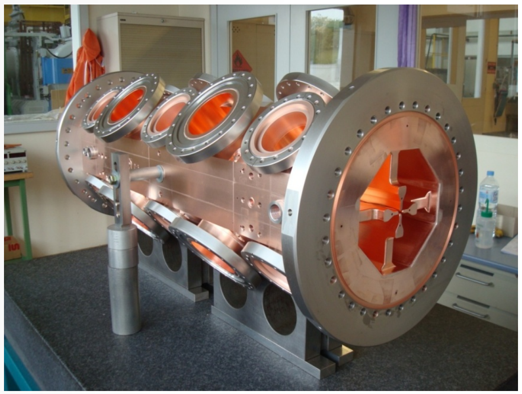


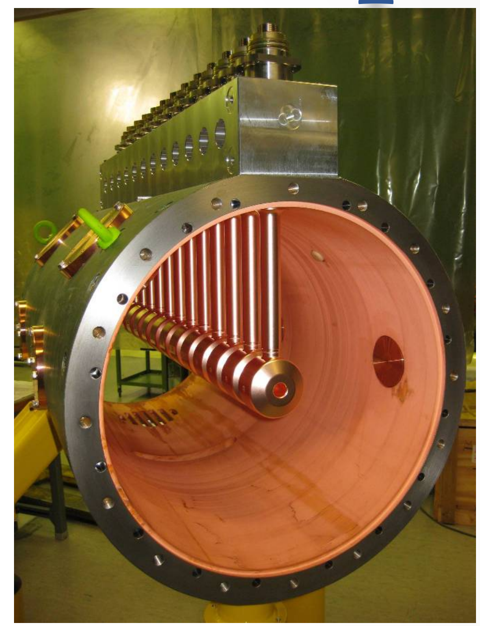
Not as strong focusing as FODO, but allows goind a long distance with a quasi parallel beam

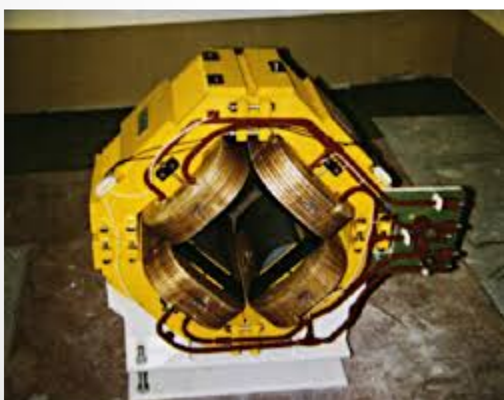
Ideal for low current beams

Ideal for IH structures in KONUS mode (no RF defocusing)

Building blocks-recap







Putting transverse and longitudinal together

- How many focusing elements per rf gap? Balancing exercise :
  - Real estate gradient
  - Bore aperture (hence RF efficiency)
  - Transverse acceptance (source performance)

- How many rf gap per cavity ? Balancing exercise :
  - Phase slippage
  - Power distribution
  - Transverse acceptance (source performance)

Choices / Questions

Frequency?	**Frequency and size** **Frequency and acceptance** **Frequency and maximum accelerating field** **Frequency and duty cycle**
RFQ output energy?	Into which structure do we inject How long is the RFQ (compared to wavelenght)
Continue with TE mode or switch to TM mode?	Start thinking about transverse focusing Think about the final energy Think about the energy at the transition (NB treshold for copper activation is around few MeV)
At what energy we start standardising the RF structures	Quasi-synchronous condition
PMQ or EMQ?	Cheap and easy or maximum flexibility