

Overview of Linacs

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What?
When ?
How?

What is a linac

- **LIN**ear **AC**celerator : single pass device that increases the energy of a charged particle by means of a (radio frequency) electric field and it is equipped with magnetic elements (quadrupoles, solenoids, bending magnets) to keep the charged particle on a given trajectory.
- Motion equation of a charged particle in an electromagnetic field

$$\frac{d\vec{p}}{dt} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

$\vec{p} = \text{momentum} = \gamma m_0 \vec{v}$
$q, m_0 = \text{charge, mass}$
$\vec{E}, \vec{B} = \text{electric, magnetic field}$
$t = \text{time}$
$\vec{x} = \text{position vector}$
$\vec{v} = \frac{d\vec{x}}{dt} = \text{velocity}$

What is a linac-cont'ed

Relativistic or not

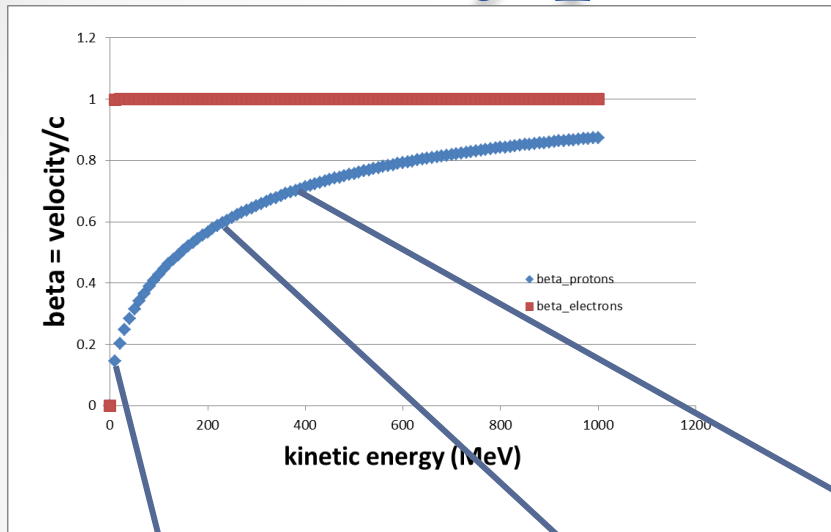
$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

type of particle :
charge couples with the
field, mass slows the
acceleration

type of RF
structure

type of focusing

Type of particles



- electron, mass 0.511 MeV, quickly relativistic, easier to accelerate
- proton, mass 938.28 MeV, $q/m=1$
- carbon ion, mass 11.3 GeV, $q/m=1/3$ and then $1/2$.

At 7 MeV
beta =0.12,
gamma=1.01

At 250 MeV
beta =0.6 ,
gamma=1.26

At 450 MeV
beta =0.74 ,
gamma=1.48

Electron linacs

To order these specialized brazed products contact us today.

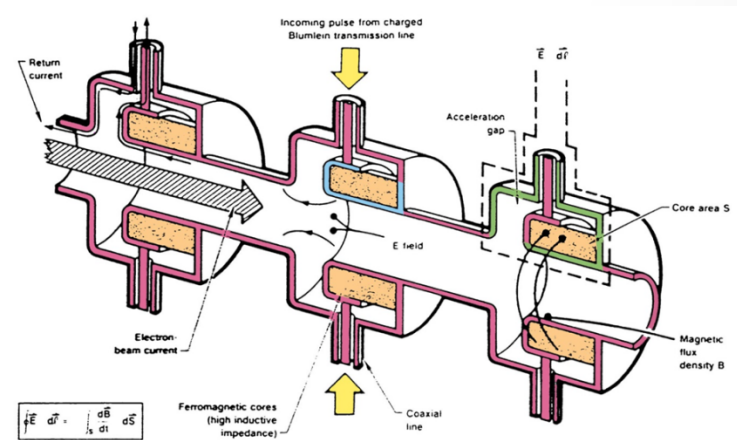


- Energy range of linacs: 4-25 MeV
- Electrons are accelerated by microwaves (10^3 - 10^4 MHz)
- Philips SL-75/5: S-band 2856 MHz, MW cavities dimensions - length 3 cm, radius 5 cm, electrons 5 MeV, tungsten target



For sake of completeness

- Static- Efield Linacs :
 - device which provides a constant potential difference (and consequently electric field)..
 - acceleration is limited to few MeV. Limitation comes from electric field breakdown
 - still used in the very first stage of acceleration when ions are extracted from a source.
- Time varying- Efield Linacs :
 - Induction linacs : based on the magnetic induction principle.



The magnetic induction accelerator principle

Electrostatic field



750 keV Radio Frequency
accelerator (2m long, 0.5 m across)

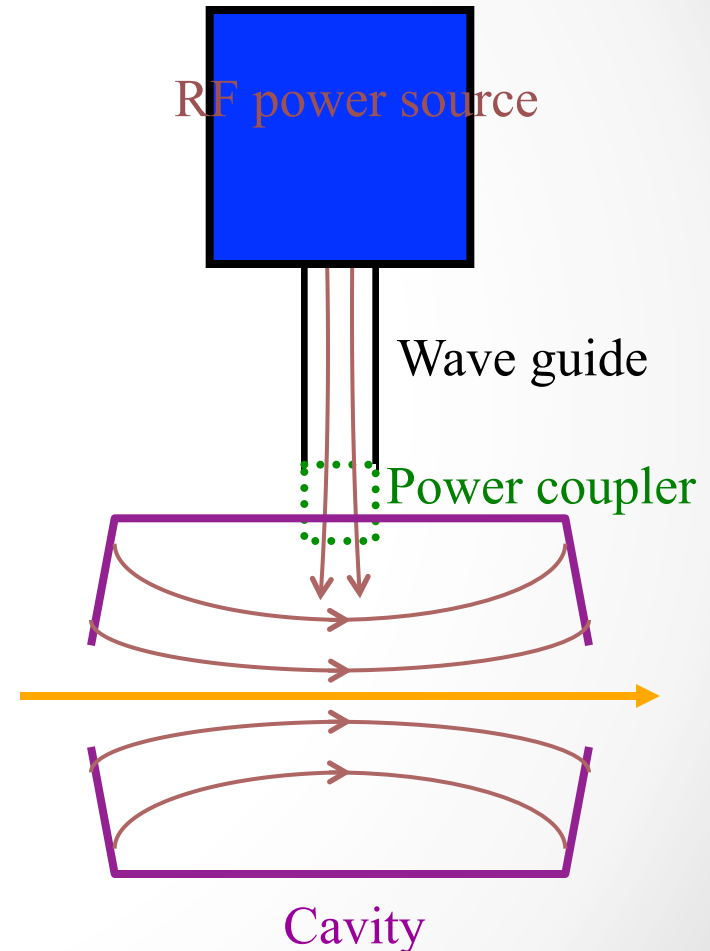
750 kV Cockcroft-Walton

When ? A short history

- Acceleration by **time varying** electromagnetic field overcomes the limitation of static acceleration
- First experiment towards an RF linac : Wideroe linac 1928 on a proposal by Ising dated 1925. A bunch of potassium ions were accelerated to 50 keV in a system of drift tubes in an evacuated glass cylinder. The available generator provided 25 keV at 1 MHz.
- First realization of a linac : 1931 by Sloan and Lawrence at Berkeley laboratory
- From experiment to a practical accelerator : Wideroe to Alvarez
 - to proceed to higher energies it was necessary to increase by order of magnitude the frequency and to enclose the drift tubes in a RF cavity (resonator)
 - this concept was proposed and realized by Luis Alvarez at University of California in 1955 : A 200 MHz 12 m long Drift Tube Linac accelerated protons from 4 to 32 MeV.
 - the realization of the first linac was made possible by the availability of high-frequency power generators developed for radar application during World War

How? principle of an RF linac

- 1) **RF power source**: generator of electromagnetic wave of a specified frequency. It feeds a
- 2) **Cavity**: space enclosed in a metallic boundary which resonates with the frequency of the wave and tailors the field pattern to the
- 3) **Beam**: flux of particles that we push through the cavity when the field is maximized as to increase its
- 4) **Energy**.



Brief description of Radio Frequency Linear Accelerators for protons and light ions for medical applications with the purpose of understanding

- 1) Why medical linacs are designed the way they are
- 2) what are the important issues should you ever attempt to design one.

What is a linac-cont'ed

From 1 to 1.26

$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

$q/m=1/1$ to $q/m=1/3$

type of RF structure

type of focusing

Types of RF structures

Type of structure	Used
Radio Frequency Quadrupole	HIT,CNAO, MEDAUSTRON, ADAMS,TULIP2.0
Interdigital-H structure	HIT CNAO MEDAUSTRON
Drift Tube Linac	IMPLART (ENEA FRASCATI) / TULIP2.0
Cell Coupled Linac also called Side Coupled Linac	ADAMS, TULIP

wave equation -recap

- Maxwell equation for E and B field:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = 0$$

- In **free space** the electromagnetic fields are of the *transverse electromagnetic*, TEM, type: the electric and magnetic field vectors are \perp to each other and to the direction of propagation.
- In a **bounded medium** (cavity) the solution of the equation must satisfy the boundary conditions :

$$\begin{aligned} \vec{E}_{//} &= \vec{0} \\ \vec{B}_{\perp} &= \vec{0} \end{aligned}$$

TE or TM modes

- TE (=transverse electric) : the electric field is perpendicular to the direction of propagation. in a cylindrical cavity

TE_{nml}

n : azimuthal,

m : radial

l longitudinal component

- TM (=transverse magnetic) : the magnetic field is perpendicular to the direction of propagation

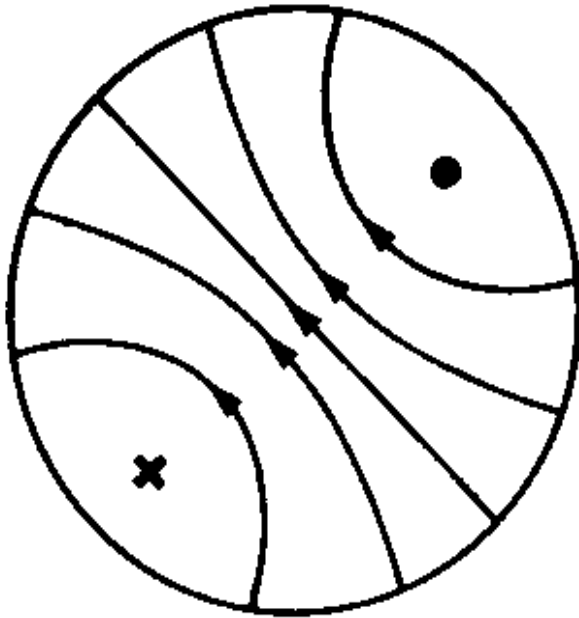
TM_{nml}

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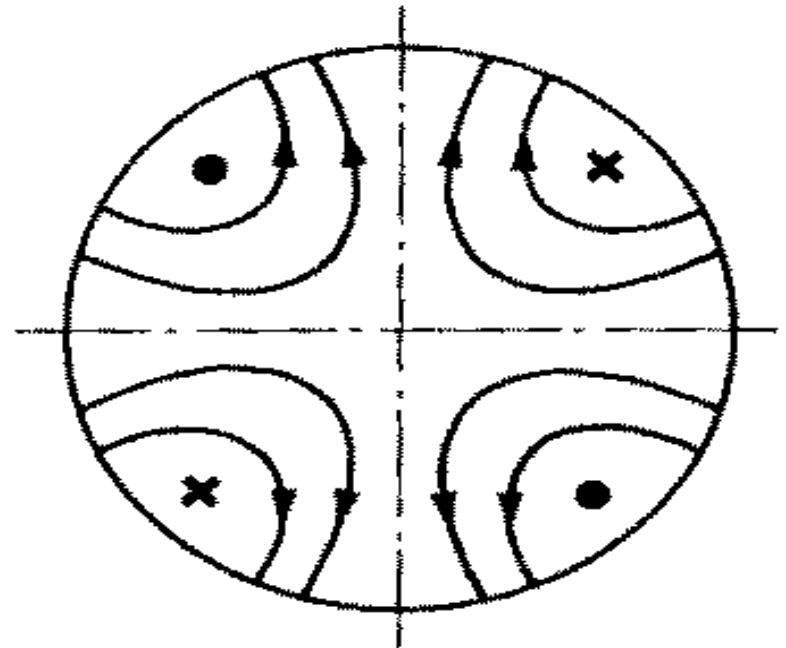
l longitudinal component

TE modes



Empty cavity; mode TE_{11}

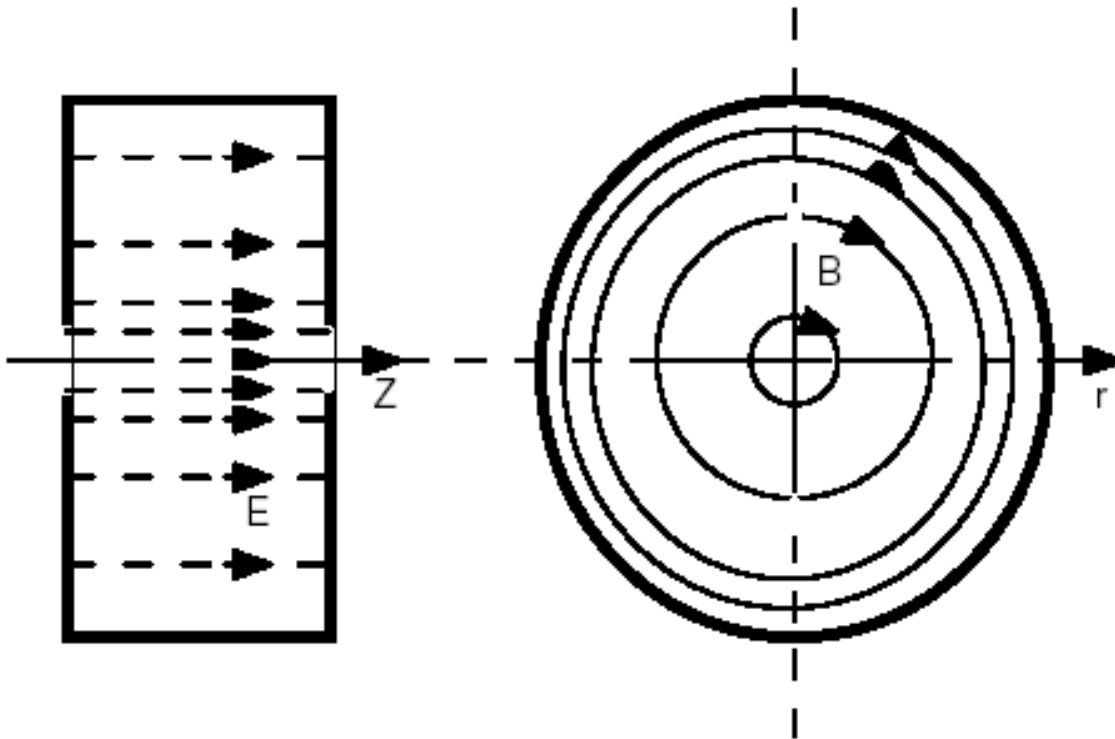
dipole mode used in the IH structures



Empty cavity; mode TE_{21}

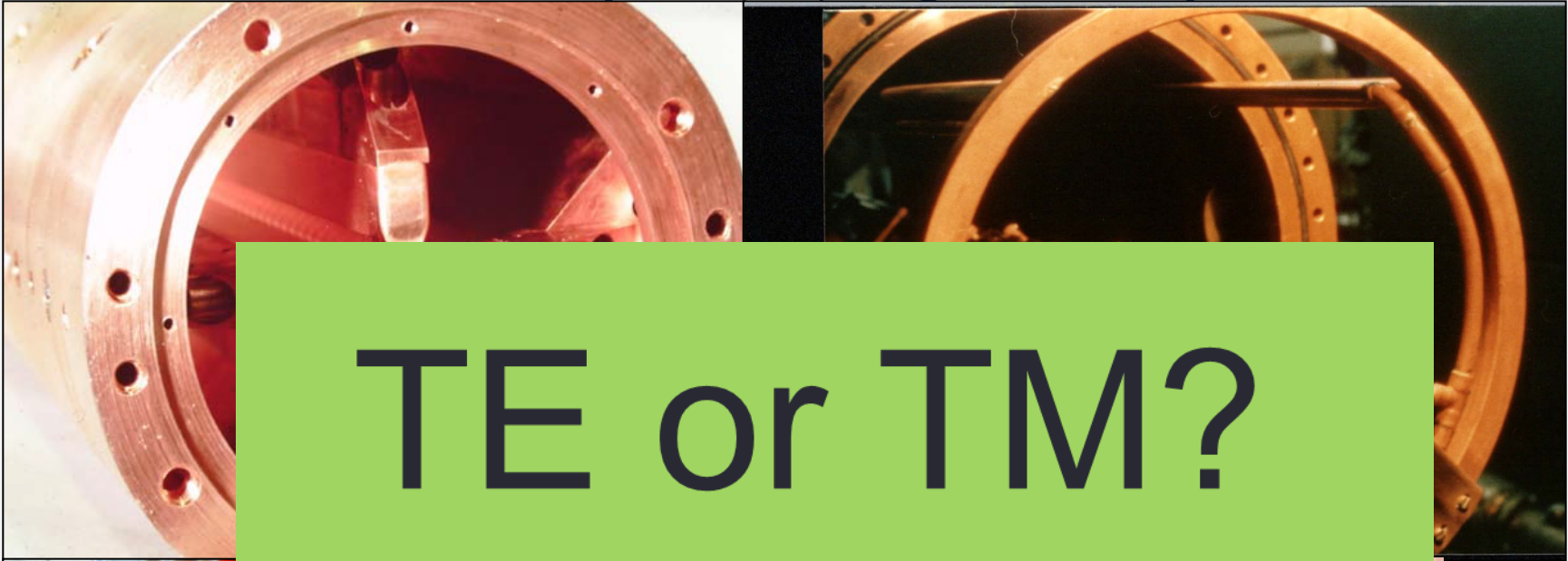
quadrupole mode used in Radio Frequency Quadrupole

TM modes

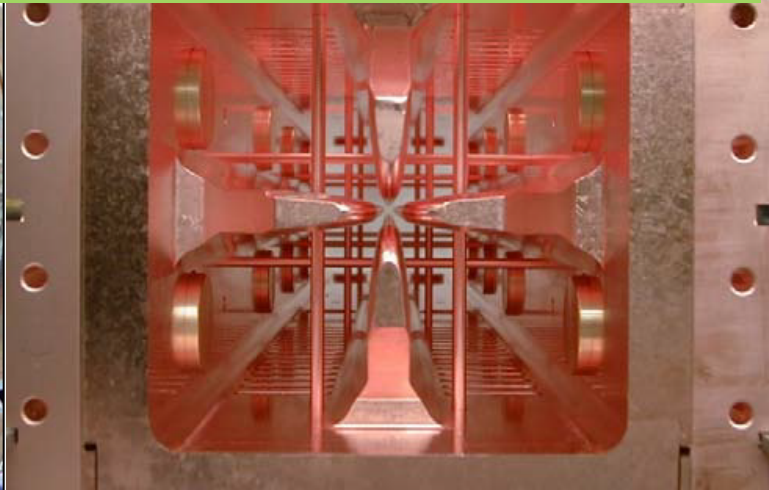


TM₀₁₀ mode , most commonly used accelerating mode

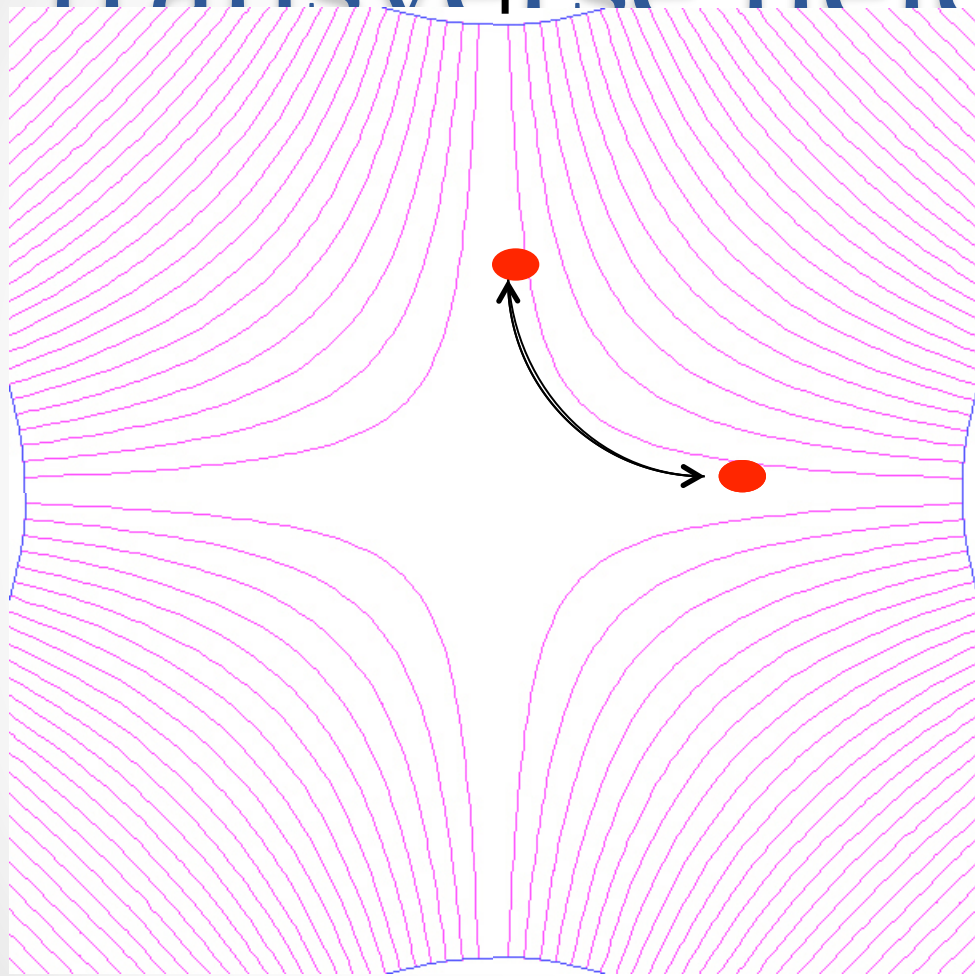
Radio Frequency Quadrupoles



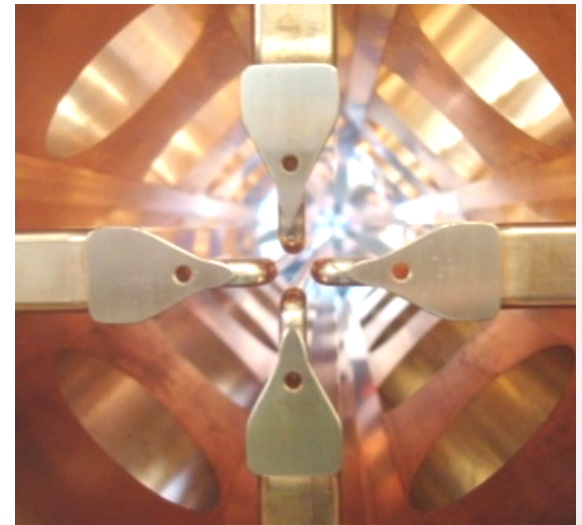
TE or TM?



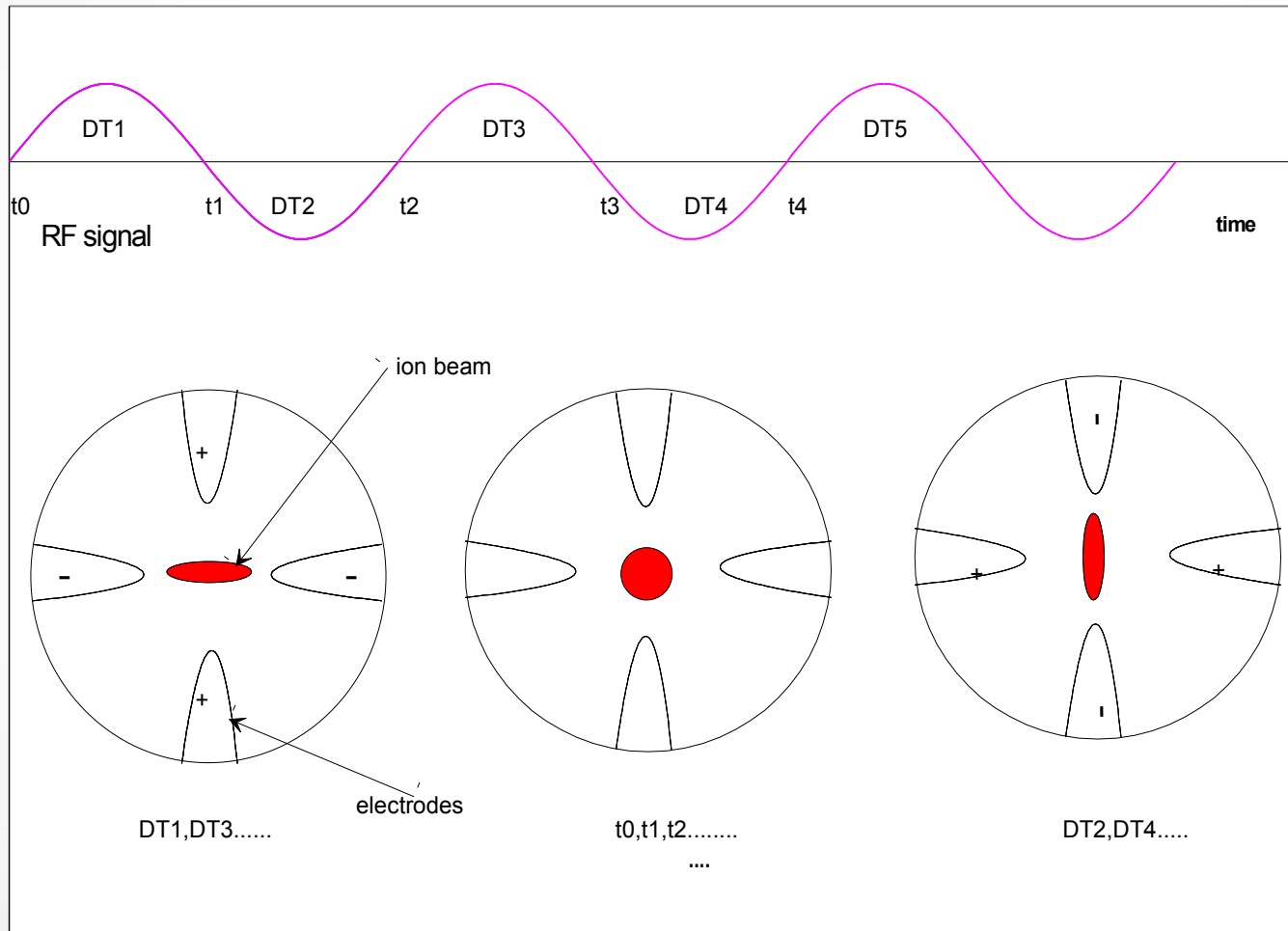
transverse field in an RFQ



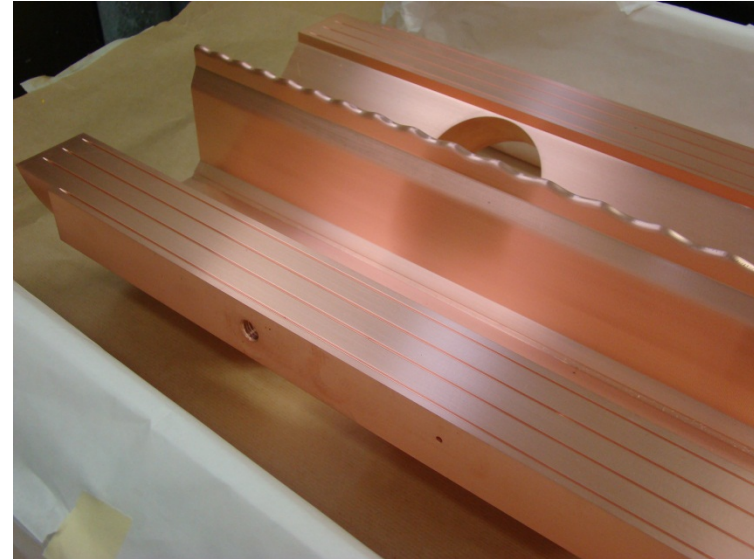
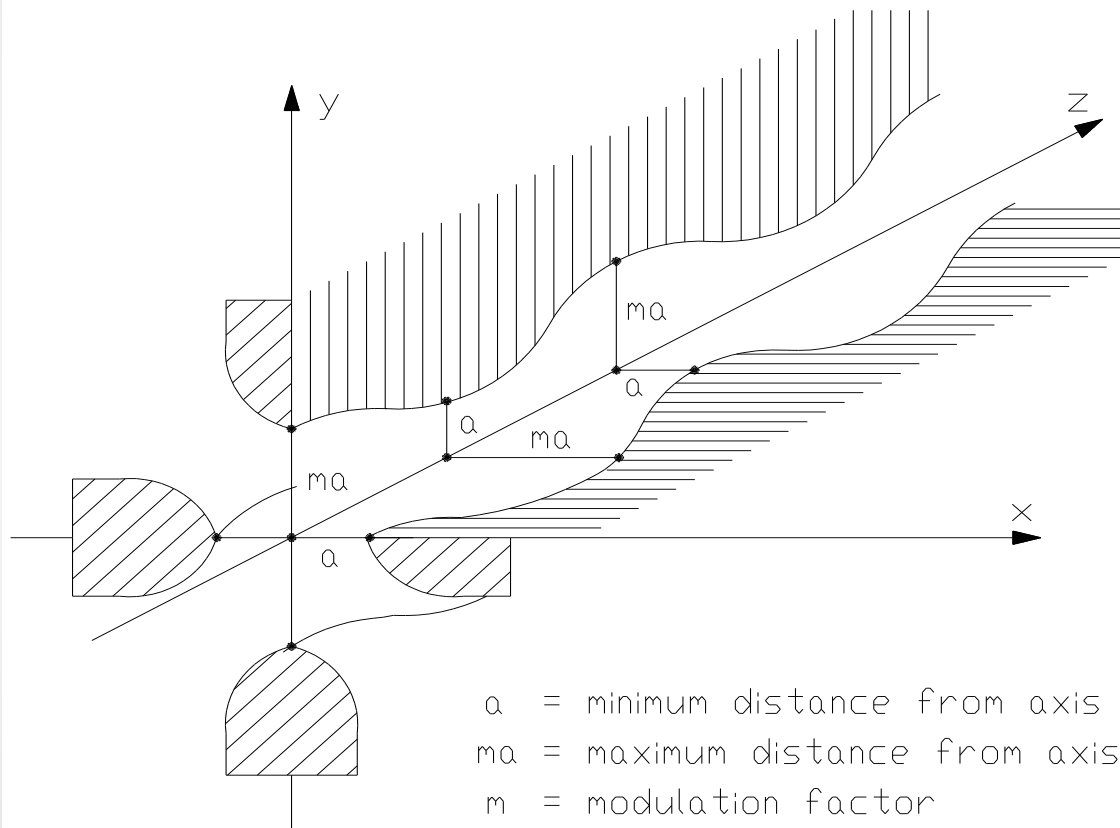
alternating gradient
focussing structure with
period length $\beta\lambda$
(in half RF period the
particles have travelled a
length $\beta\lambda/2$)



transverse field in an RFQ



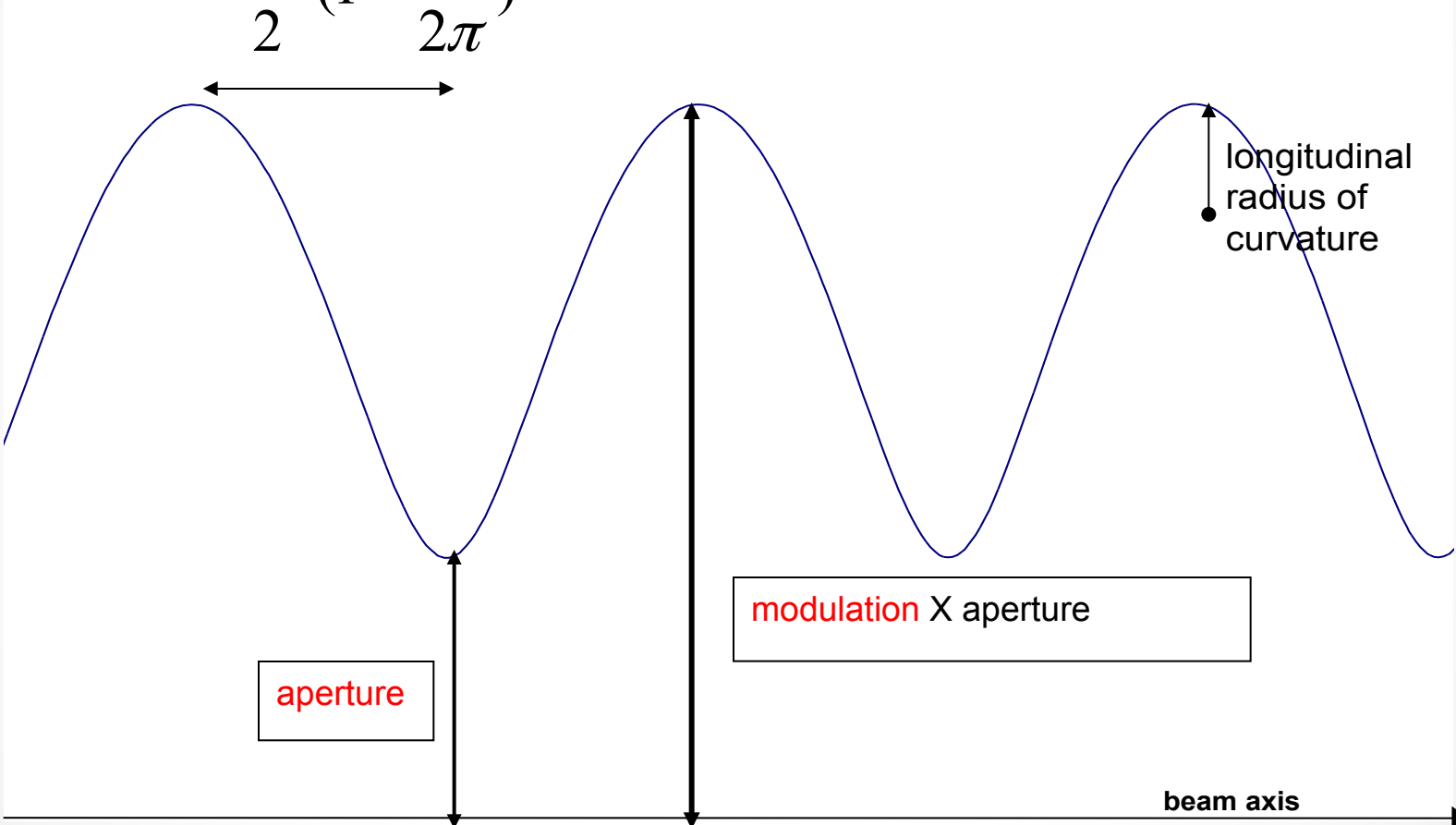
acceleration in RFQ



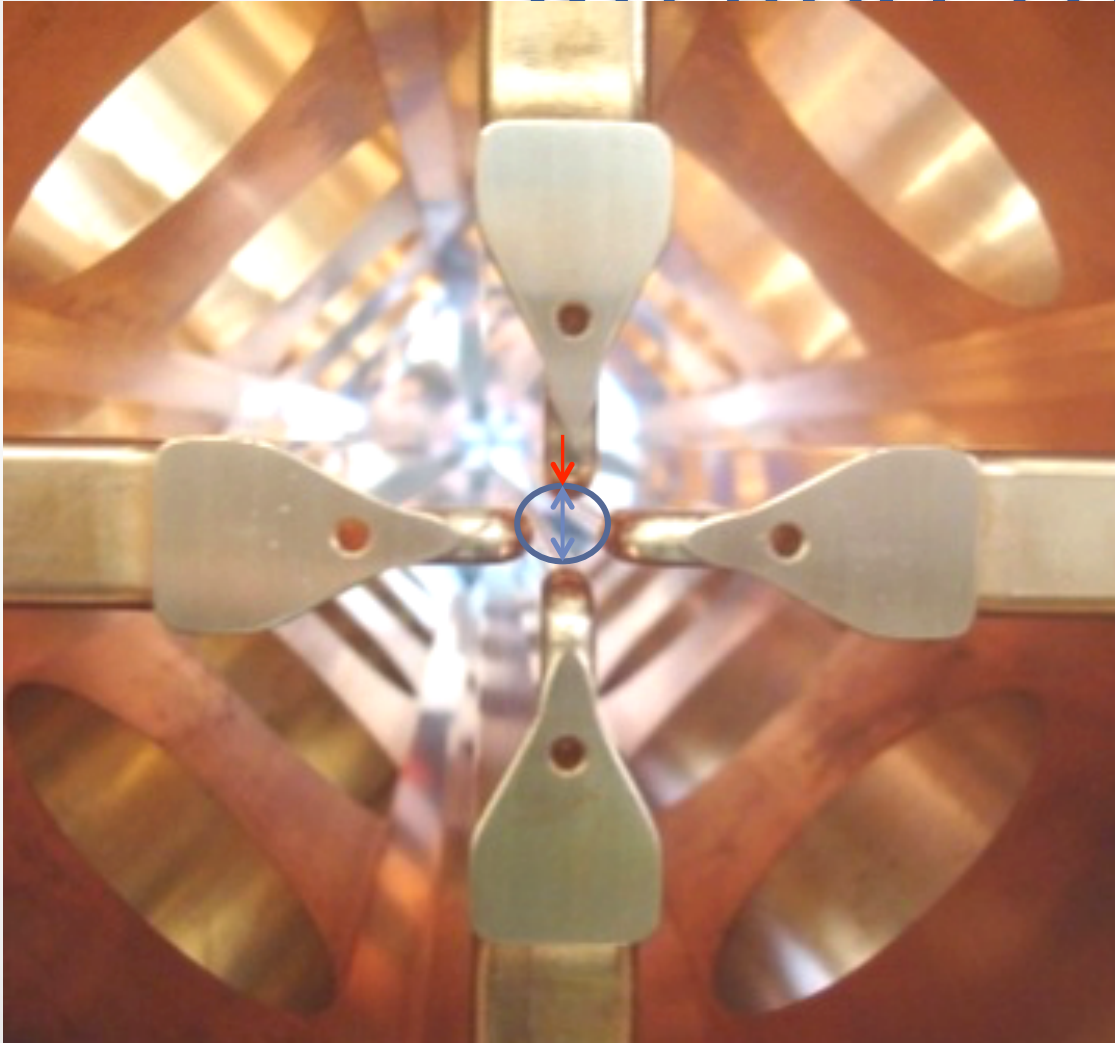
longitudinal modulation on the electrodes creates a longitudinal component in the TE mode

acceleration in an RFQ

$$\frac{\beta\lambda}{2} \left(1 - \frac{\Delta\phi}{2\pi}\right)$$



R0 and rho



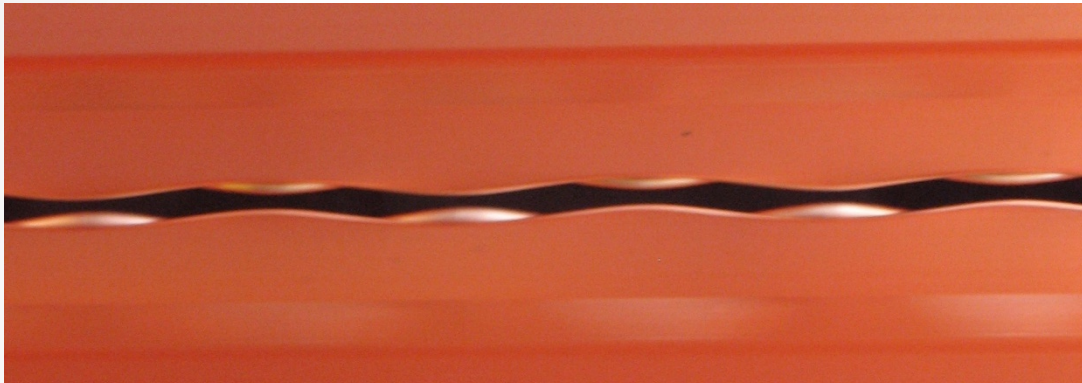
$2R_0$ = average distance between opposite vanes



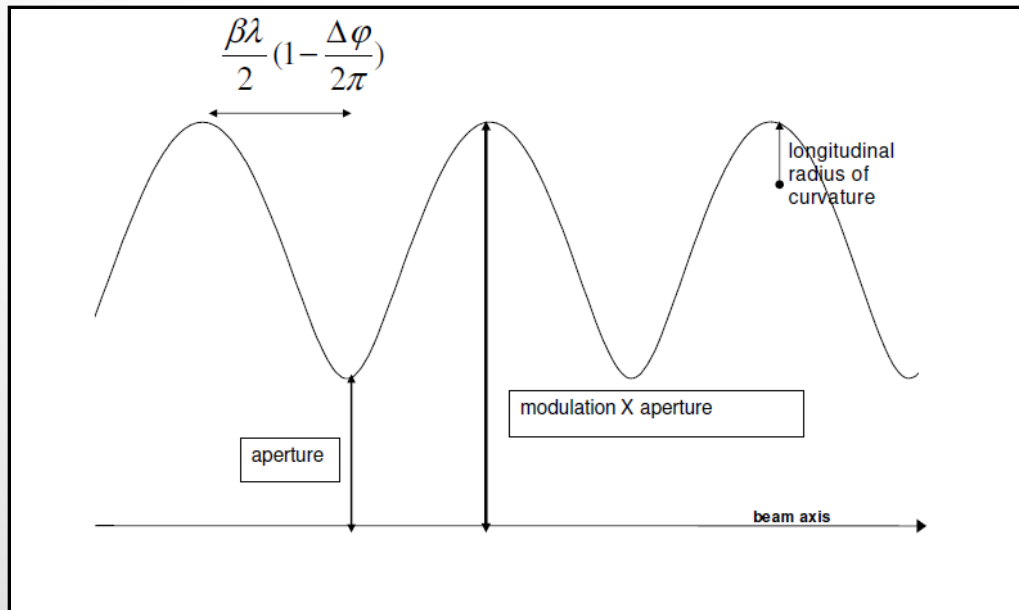
ρ : Transverse radius of curvature of the vane-tip.

Beam goes into the paper in between the 4 vanetips

Modulation and Rhol



Looking in from the RF port : these are adjacent



important parameters of the RFO

$$B = \left(\frac{q}{m_0} \right) \left(\frac{V}{a} \right) \left(\frac{1}{f^2} \right) \frac{1}{a} \left(\frac{I_o(ka) + I_o(mka)}{m^2 I_o(ka) + I_o(mka)} \right)$$

type of particle

limited by sparking

Transverse field distortion due to modulation (=1 for un-modulated electrodes)

$$E_0 T = \frac{m^2 - 1}{m^2 I_o(ka) + I_o(mka)} \cdot V \frac{2 \pi}{\beta \cdot \lambda 4}$$

Accelerating efficiency : fraction of the field deviated in the longitudinal direction (=0 for un-modulated electrodes)

cell length

transit time factor

.....and their relation

$$\left(\frac{I_0(ka) + I_0(mka)}{m^2 I_0(ka) + I_0(mka)} \right) + \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)} \cdot I_0(ka) = 1$$

focusing
efficiency

accelerating
efficiency

a =bore radius, β, γ =relativistic parameters, c =speed of light, f = rf frequency, I_0, I_1 =zero, first order Bessel function, k =wave number, λ =wavelength, m =electrode modulation, m_0 =rest mass, q =charge, r = average transverse beam dimension, r_0 =average bore, V =vane voltage

RFQ

- The resonating mode of the cavity is a focusing mode
- Alternating the voltage on the electrodes produces an alternating focusing channel
- A longitudinal modulation of the electrodes produces a field in the direction of propagation of the beam which bunches and accelerates the beam
- Both the focusing as well as the bunching and acceleration are performed by the RF field
- Not very efficient accelerator
- The RFQ is the only linear accelerator that can accept a low energy CONTINUOUS beam of particles
- 1970 Kapchinskij and Teplyakov propose the idea of the radiofrequency quadrupole (I. M. Kapchinskii and V. A. Teplyakov, Prib.Tekh. Eksp. No. 2, 19 (1970))

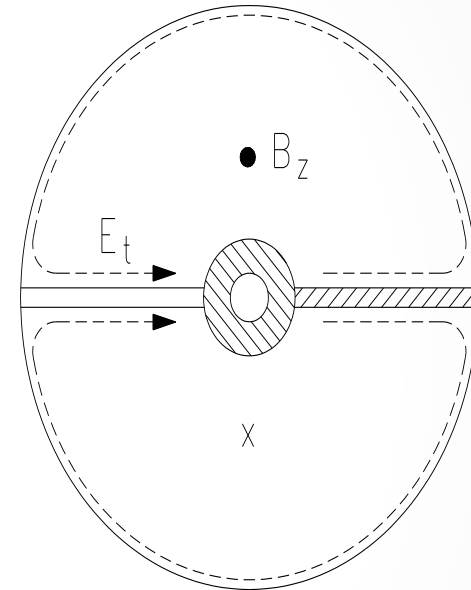
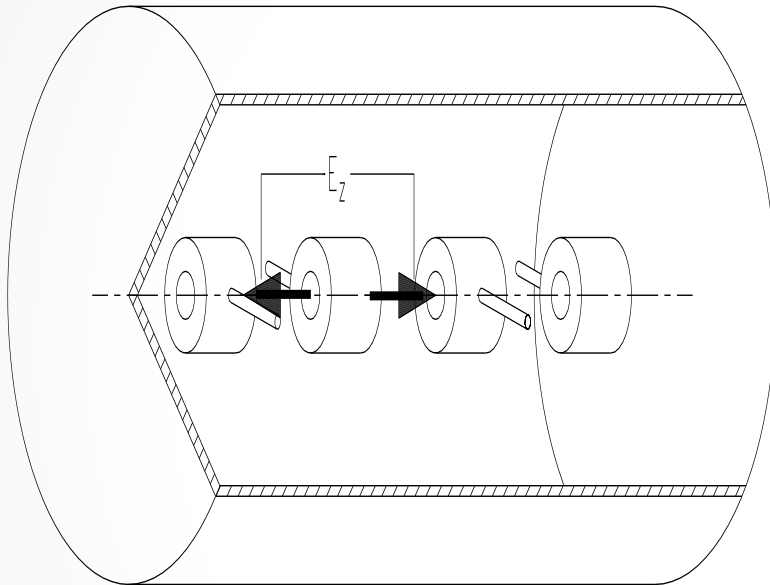
Interdigital H structure



CNAO IH

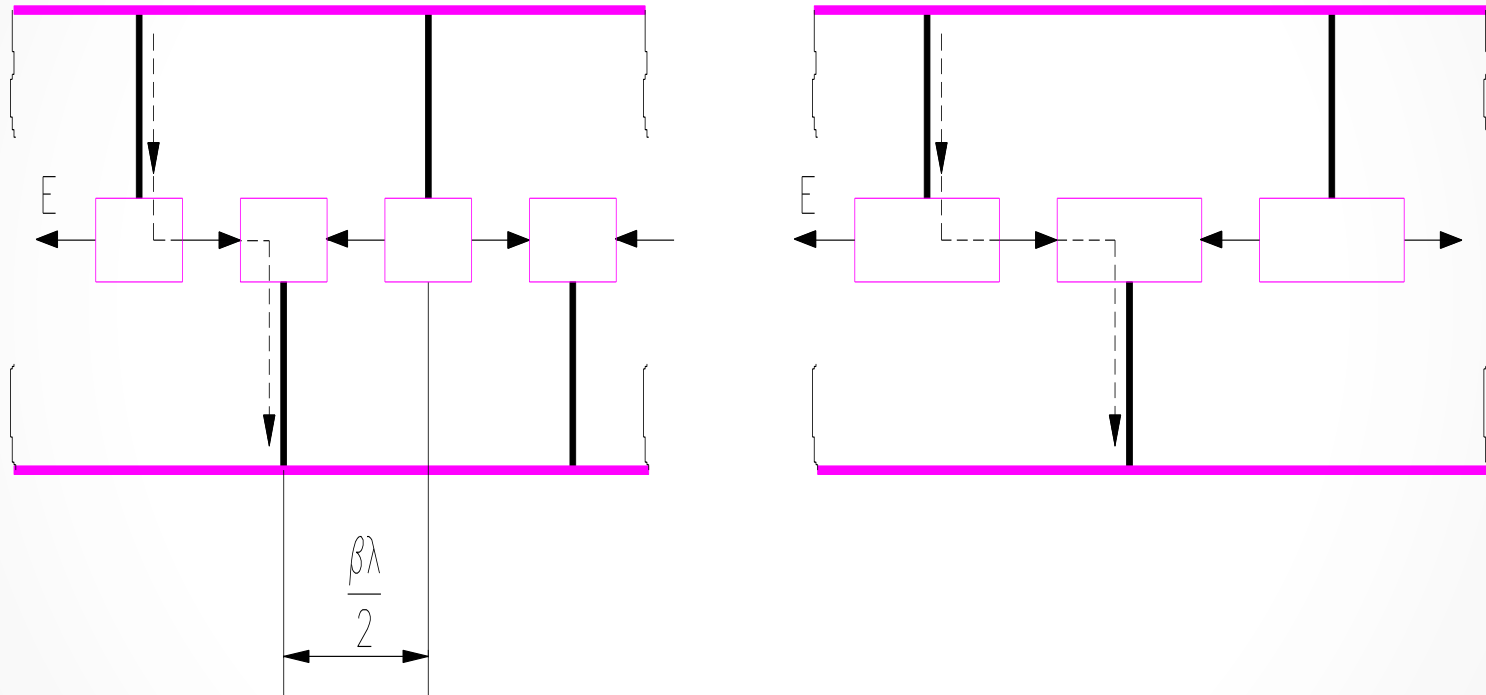


Interdigital H structure



the mode is the TE₁₁₀

Interdigital H structure



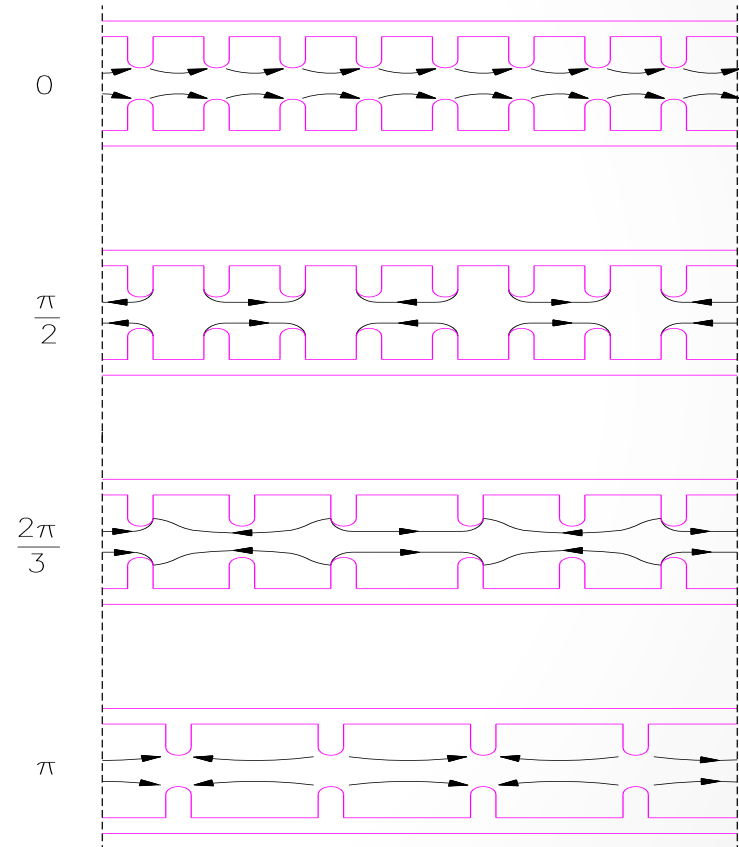
- stem on alternating side of the drift tube force a longitudinal field between the drift tubes
- focalisation is provided by quadrupole triplets places OUTSIDE the drift tubes or OUTSIDE the tank

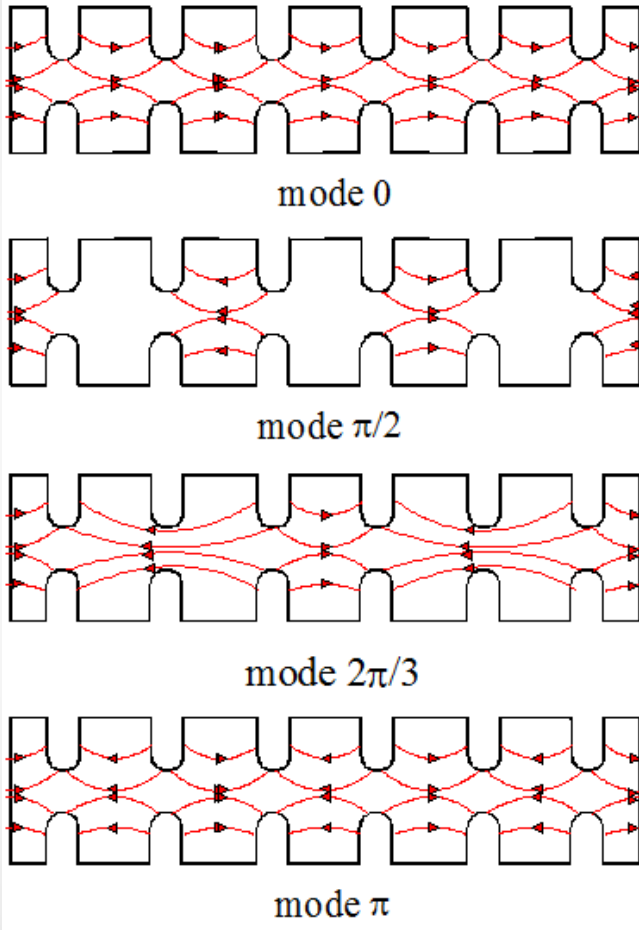
IH

- The resonating mode of the cavity is a dipole mode mode
- The cavity is equipped with thin drift tubes.
- Alternating the stems on each side of the drift tubes produces a field in the direction of propagation of the beam which accelerates the beam
- Focusing is provided by quadrupole triplets located inside the tank in a dedicated section
- Very efficient in the low beta region ($\beta \cong 0.02$ to 0.08) and low frequency (up to 200MHz)
- not for high intensity beam due to long focusing period
- ideal for low beta ion acceleration

cavity modes

- **0-mode** Zero-degree phase shift from cell to cell, so fields adjacent cells are in phase. Best example is DTL.
- **π -mode** 180-degree phase shift from cell to cell, so fields in adjacent cells are out of phase. Best example is multicell superconducting cavities.
- **$\pi/2$ mode** 90-degree phase shift from cell to cell. In practice these are biperiodic structures with two kinds of cells, accelerating cavities and coupling cavities. The CCL operates in a $\pi/2$ structure mode. This is the preferred mode for very long multicell cavities, because of very good field stability.





Mode 0 also called mode 2π .

For synchronicity and acceleration, particles must be in phase with the E field on axis (will be discussed more in details in part.3).

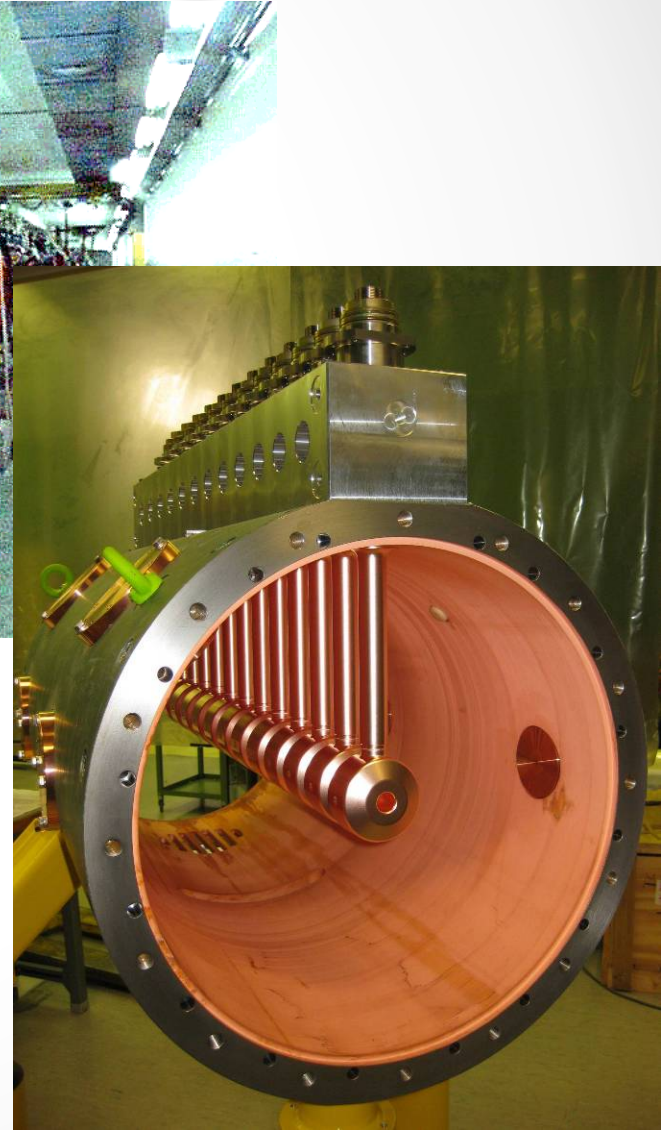
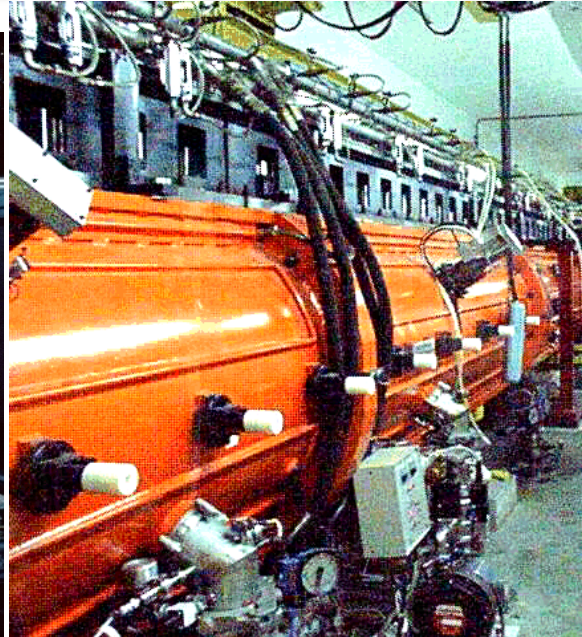
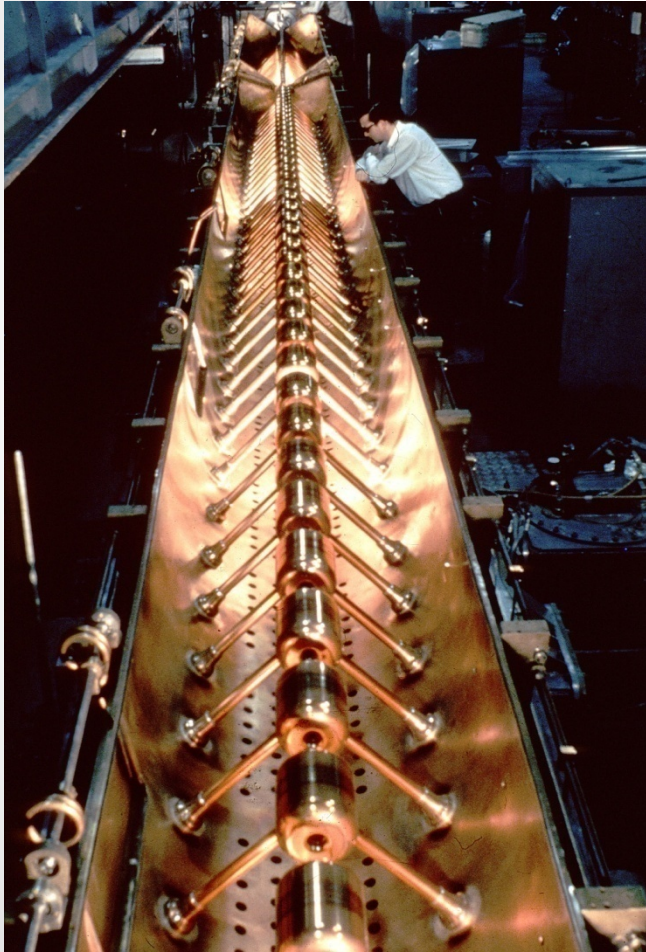
During 1 RF period, the particles travel over a distance of $\beta\lambda$.

The cell L length should be:

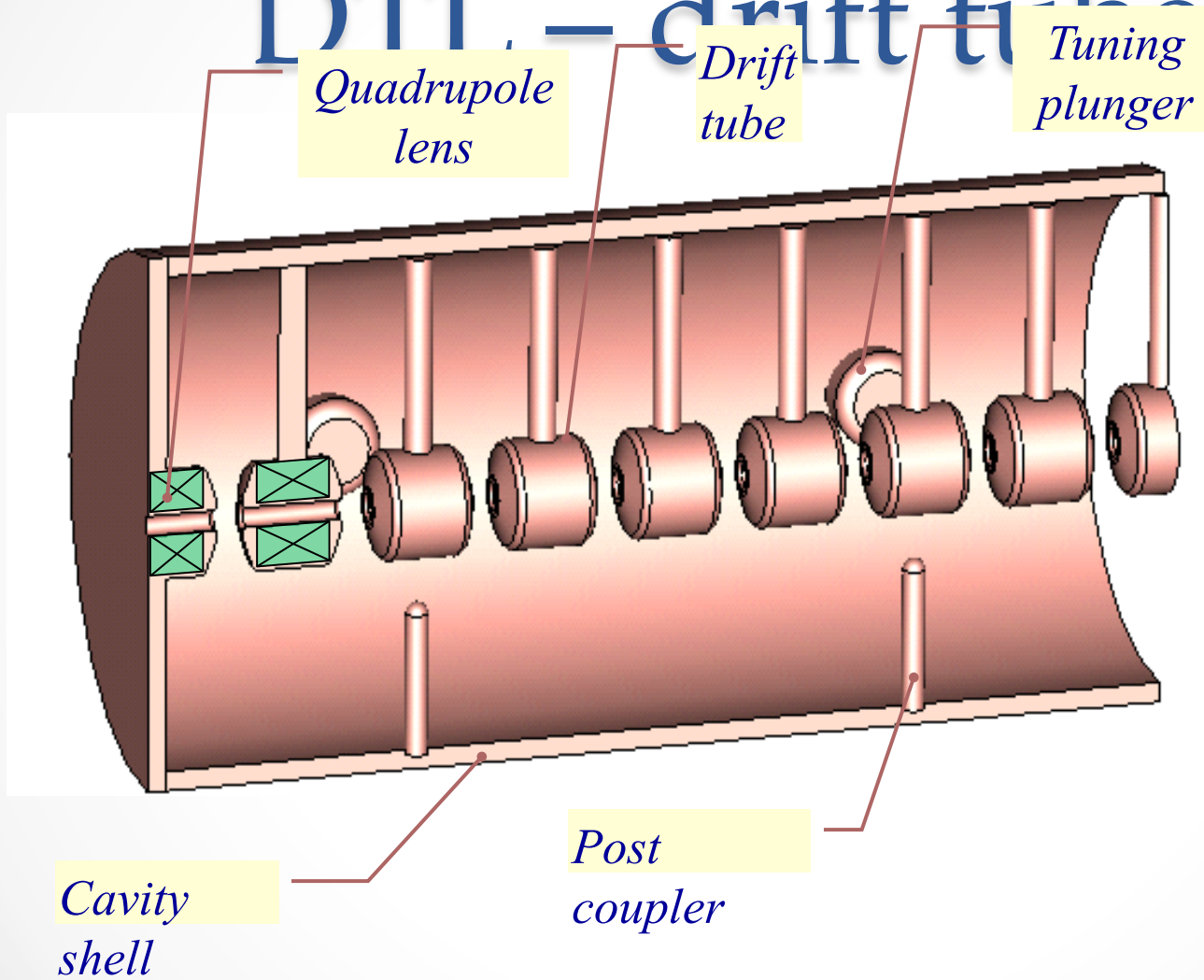
Mode	L
2π	$\beta\lambda$
$\pi/2$	$\beta\lambda/4$
$2\pi/3$	$\beta\lambda/3$
π	$\beta\lambda/2$

Named from the phase difference between adjacent cells.

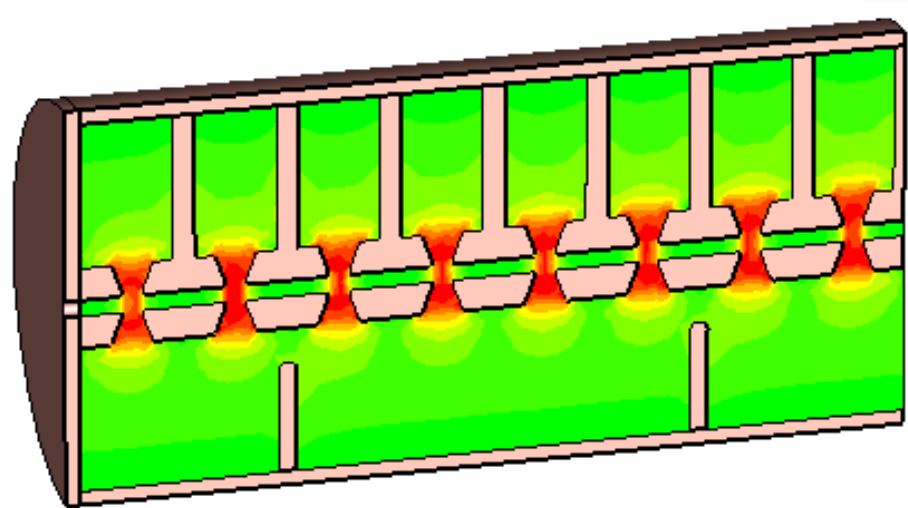
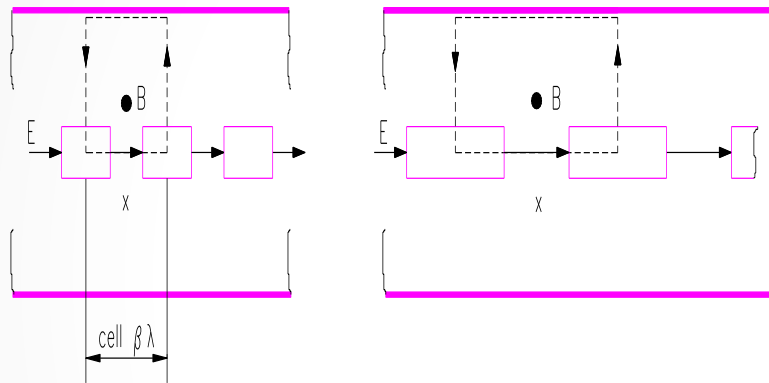
Drift Tube Linac



DTI – drift tubes



DTL : electric field



Mode is TM₀₁₀

DTL

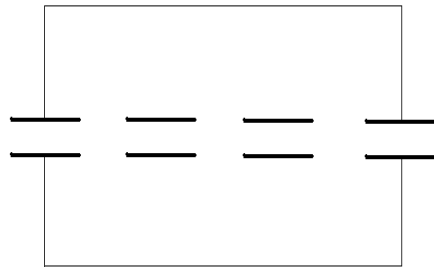
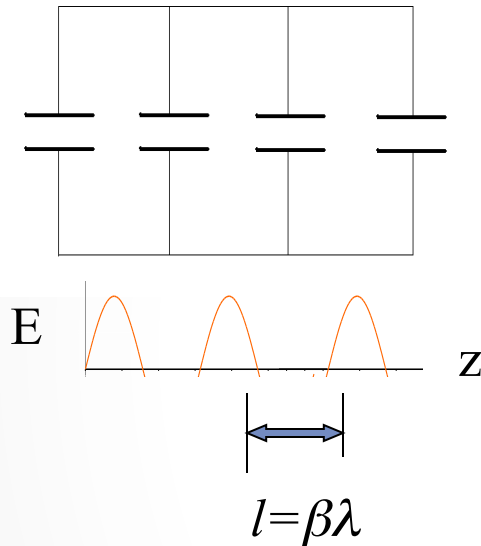
The DTL operates in **0 mode** for protons and heavy ions in the range $\beta=0.04-0.5$ (750 keV - 150 MeV)

Synchronism condition (0 mode):

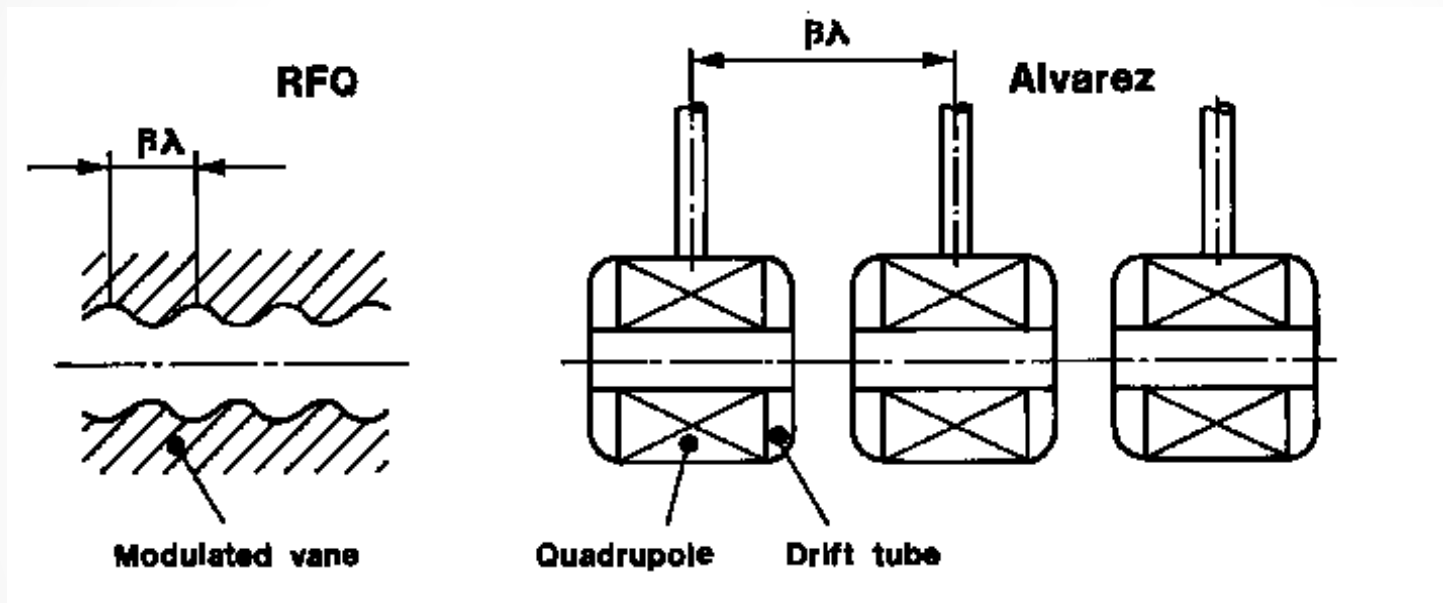
$$l = \frac{\beta c}{f} = \beta \lambda$$

The beam is inside the “drift tubes” when the electric field is decelerating

The fields of the 0-mode are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher shunt impedance

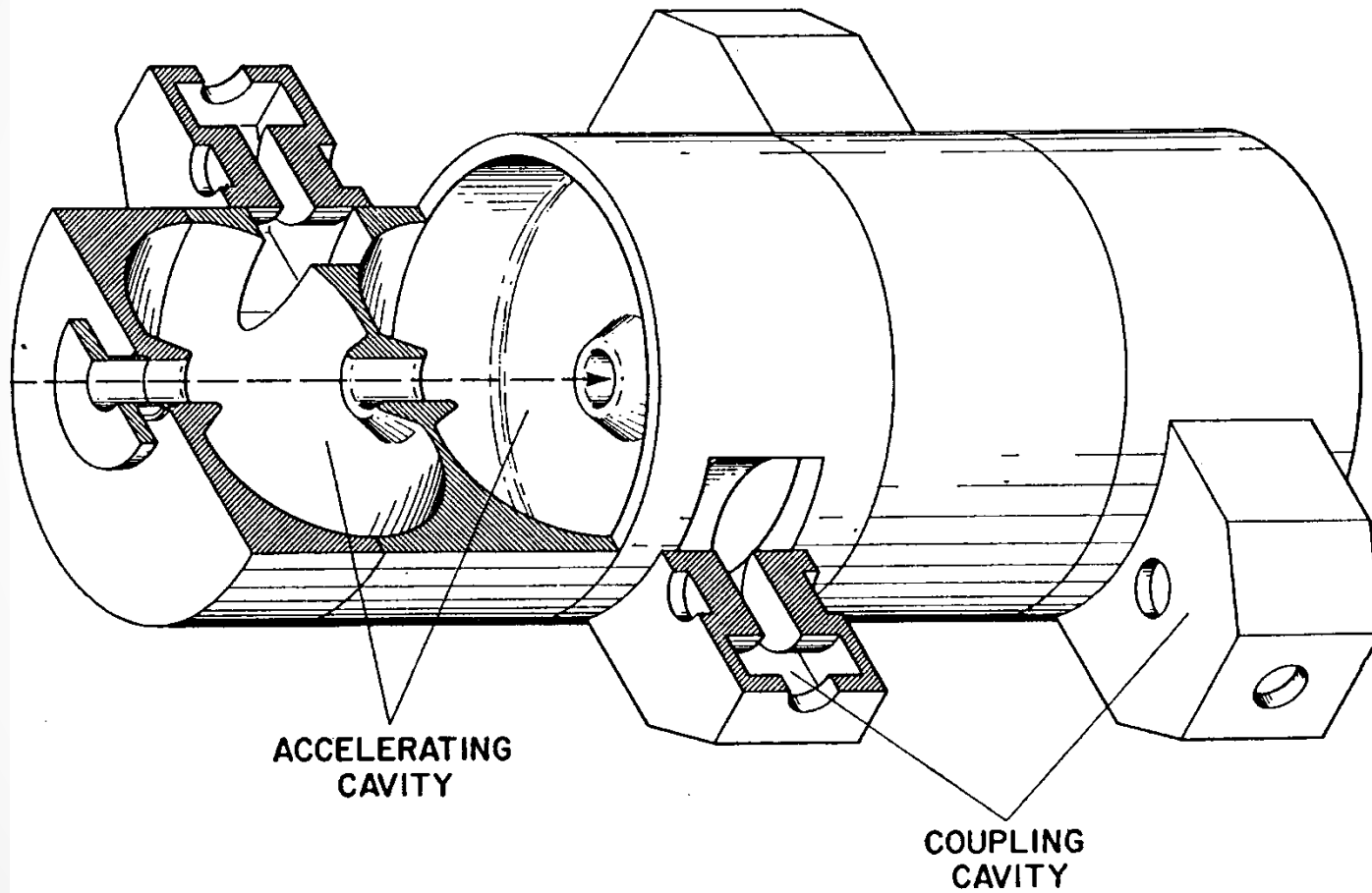


RFQ vs. DTL

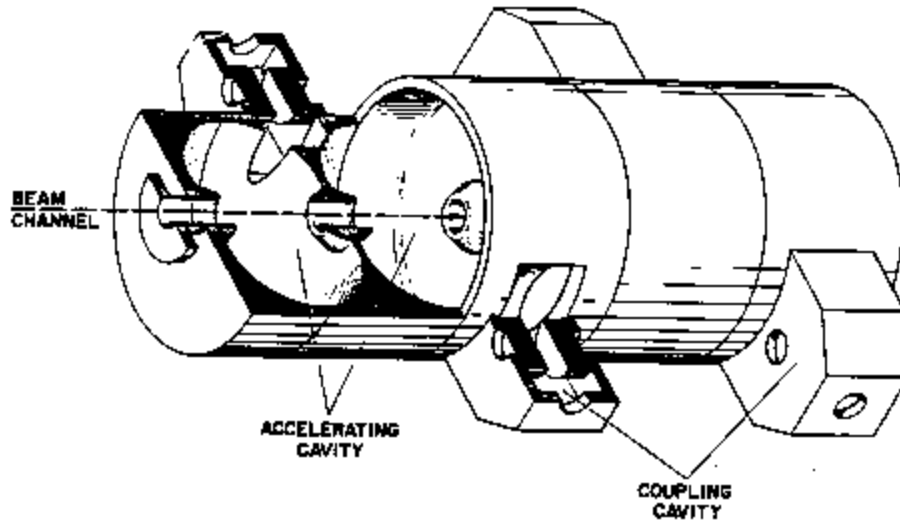


DTL can't accept low velocity particles, there is a minimum injection energy in a DTL due to mechanical constraints

Side Coupled Linac



The Side Coupled Linac



multi-cell Standing Wave
structure in **$\pi/2$ mode**
frequency 800 - 3000 MHz
for protons ($\beta=0.3 - 1$)

Rationale: high beta \Rightarrow cells are longer \Rightarrow advantage for high frequencies

- at high f , high power (> 1 MW) klystrons available \Rightarrow long chains (many cells)
- long chains \Rightarrow high sensitivity to perturbations \Rightarrow operation in $\pi/2$ mode

Side Coupled Structure:

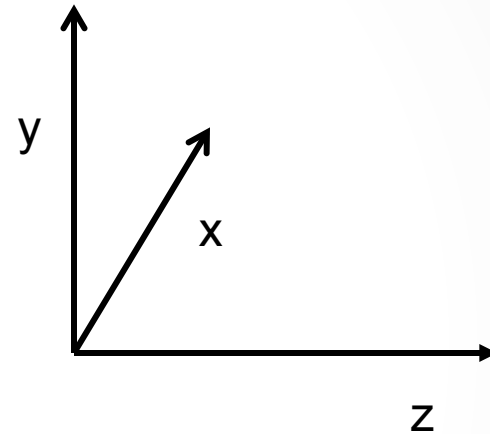
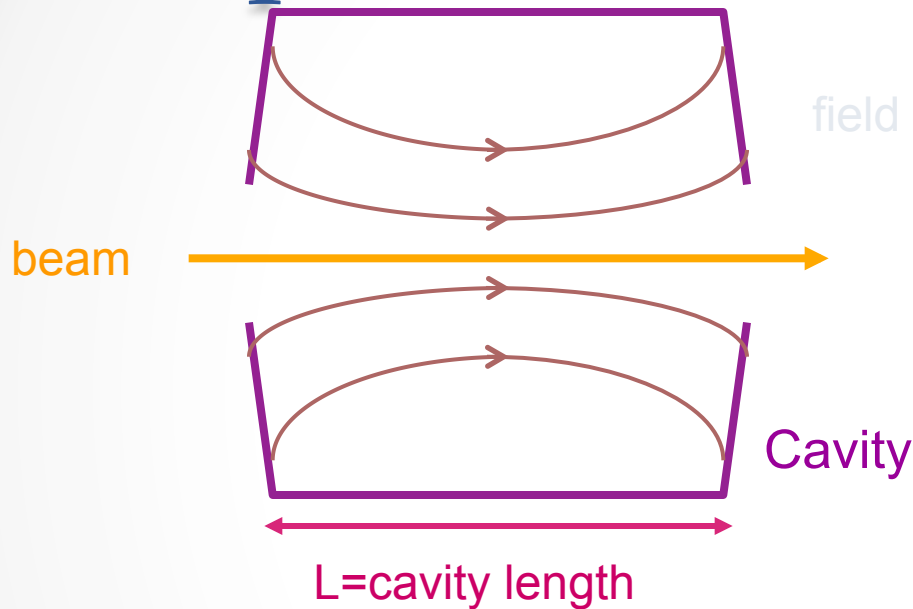
- from the wave point of view, $\pi/2$ mode
- from the beam point of view, π mode

SDTL –mix DTL and SCL



Choices choices

cavity geometry and related parameters definition



1-Maximum field/average field

2-Shunt impedance

3-Quality factor

4-Filling time

5-Transit time factor

6-Effective shunt impedance

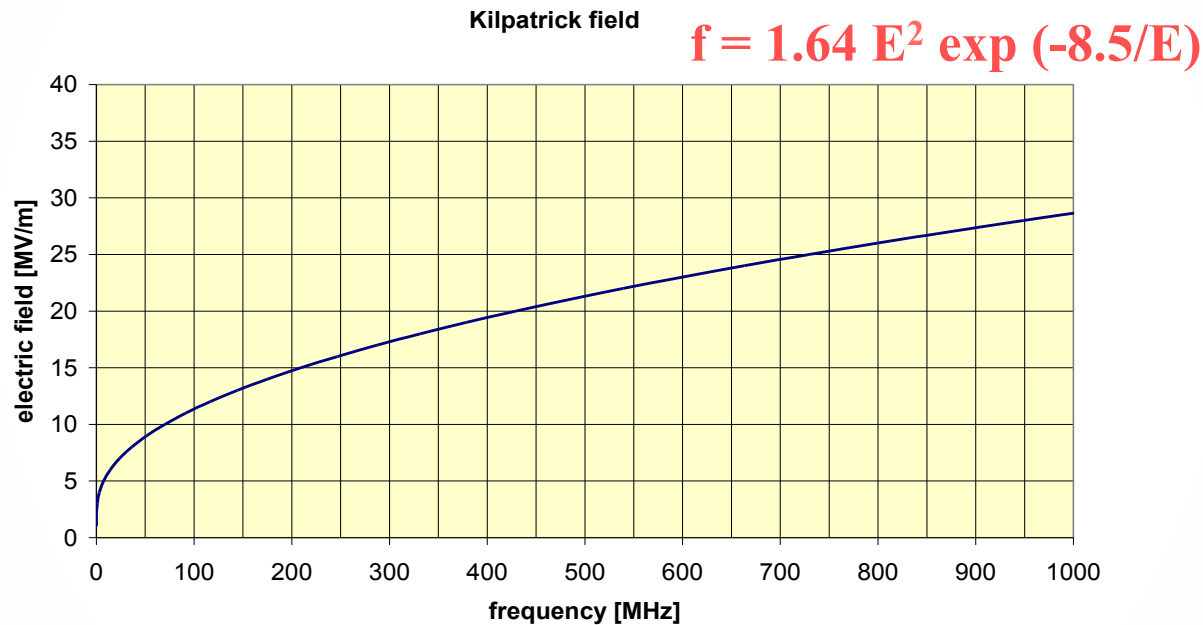
Average electric field

- **Average electric field** (E_0 measured in V/m) is the space average of the electric field along the direction of propagation of the beam in a given moment in time when $F(t)$ is maximum.

$$E_0 = \frac{1}{L} \int_0^L E_z(x=0, y=0, z) dz$$
$$E(x, y, z, t) = E(x, y, z) \cdot e^{-j\omega t}$$

- physically it gives a measure how much field is available for acceleration
- it depends on the cavity shape, on the resonating mode and on the frequency

Kilpatrick sparking criterion



GUIDELINE

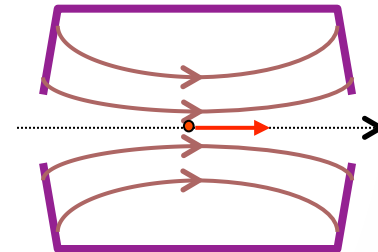
nowadays : peak
surface field up to
2*kilpatrick field

Quality factor for normal
conducting cavity is $E_{\text{peak}}/E_0 T$

Transit time factor

- transit time factor (T, dimensionless) is defined as the maximum energy gain per charge of a particles traversing a cavity over the average voltage of the cavity.
- Write the field as

$$E_z(x, y, z, t) = E_z(x, y, z)e^{-i(\omega t)}$$



- The energy gain of a particle entering the cavity on axis at phase ϕ is

$$\Delta W = \int_0^L qE_z(o, o, z)e^{-i(\omega t + \phi)} dz$$

Transit time factor

- assume constant velocity through the cavity (APPROXIMATION!!) we can relate position and time via

$$z = v \cdot t = \beta ct$$

- we can write the energy gain as

$$\Delta W = qE_0 L T \cos(\phi)$$

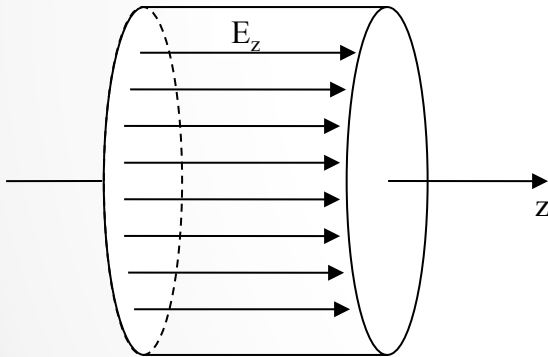
- and define transit time factor as

$$T = \frac{\left| \int_0^L E_z(z) e^{-j\left(\frac{\omega z}{\beta c}\right)} dz \right|}{\int_0^L E_z(z) dz}$$

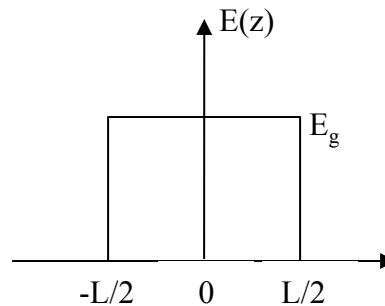
T depends on the particle velocity and on the gap length. IT DOESN'T depend on the absolute value field

Transit time factor

- NB : Transit time factor depends on x, y (the distance from the axis in cylindrical symmetry). By default it is meant the transit time factor on axis
- Exercise!!! If $E_z = E_0$ then



TM₀₁₀ mode in a pillbox cavity



Square-wave electric field distribution

$$T = \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\left(\frac{\pi L}{\beta\lambda}\right)}$$

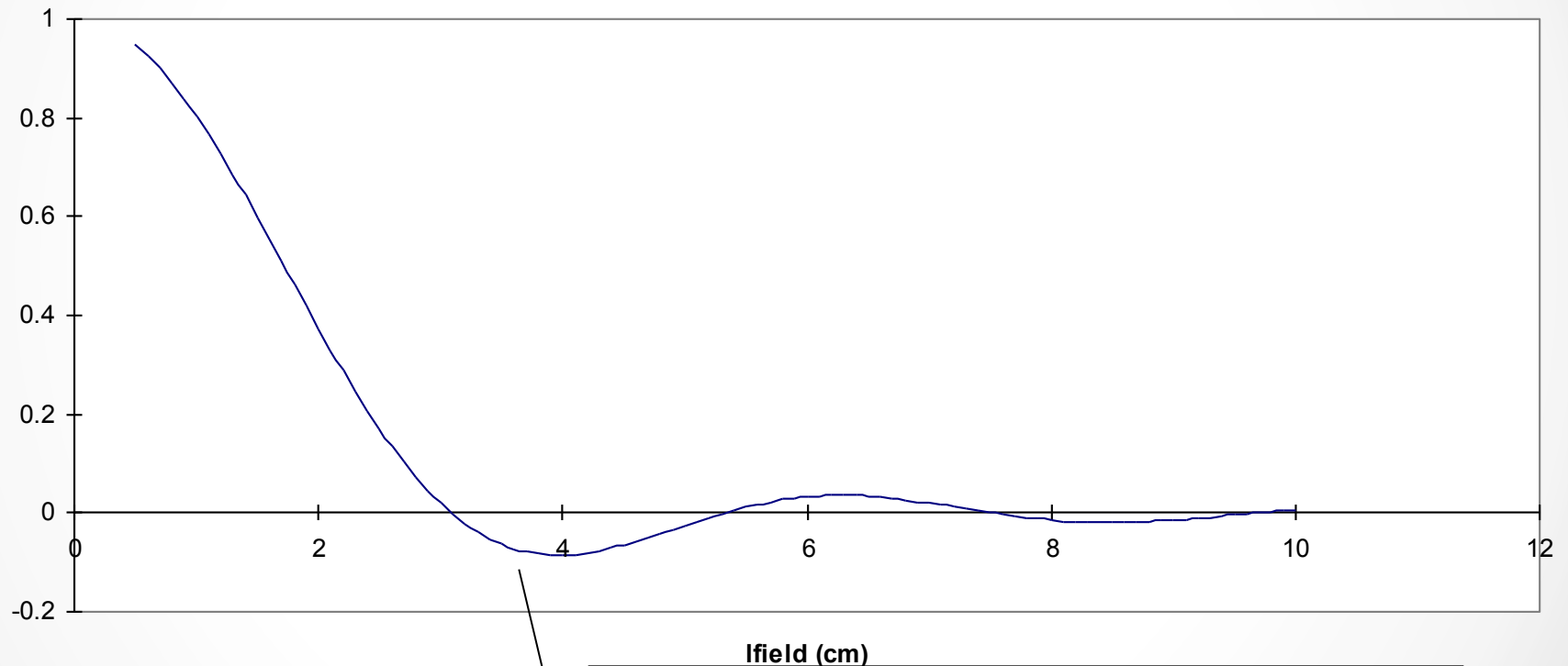
L =gap length

β =relativistic parametre

λ =RF wavelenght

Transit time factor

tff for 100 keV protons, 200 MHz., parabolic distribution



if we don't get the length right we can end up decelerating!!!

effective shunt impedance

- **Effective shunt impedance** (Z measured in Ω / m) is defined as the ratio of the average effective electric field squared ($E_0 T$) to the power (P) per unit length (L) dissipated on the wall surface.
- it is independent of the field level and cavity length, **it depends on the cavity mode and geometry and on the velocity of the particle to be accelerated**

$$Z_{TT} = \frac{(E_0 T)^2}{P} \cdot L$$

measure if the structure is optimized and adapted to the velocity of the particle to be accelerated

Measure of how much energy a charged particle can gain for 1 w of power when travelling over 1 m of structure.

overview



take with
CAUTION!

	Ideal range of beta	frequency	Effective gradient	
RFQ	Low!!! - 0.05	40-400 MHz	1 MV/m (350MHz)	Ions / protons
IH	0.02 to 0.08	40-200 MHz	4.5 MV/m (200MHz)	Ions and also protons
DTL	0.04-0.5	100-400 MHz	3.5 MV/m (350MHz)	Ions / protons
SCL	Ideal Beta=1 But as low as beta 0.3	800 - 3000 MHz	20 MV/m (3000MHz)	protons

What is a linac-cont'ed

Relativistic or not

$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

type of particle :
charge couples with the
field, mass slows the
acceleration

type of RF
structure

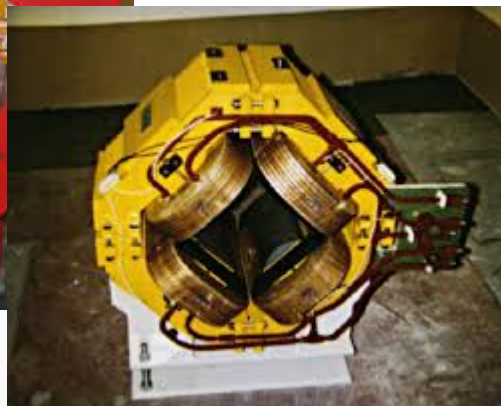
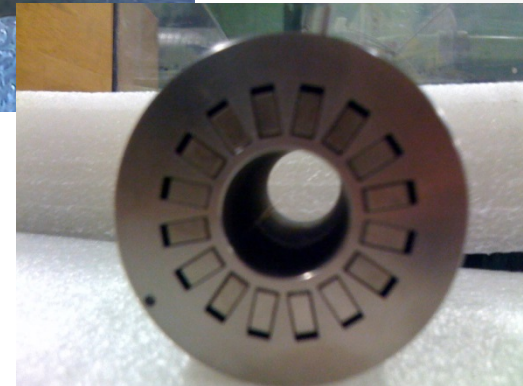
type of focusing

Magnetic quadrupoles

ElectroMQ



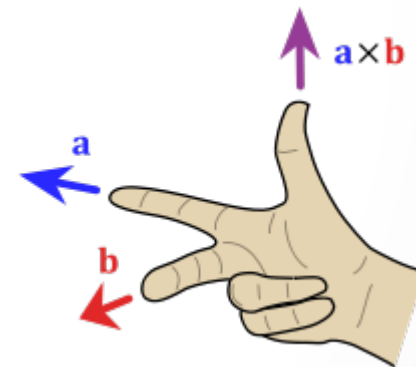
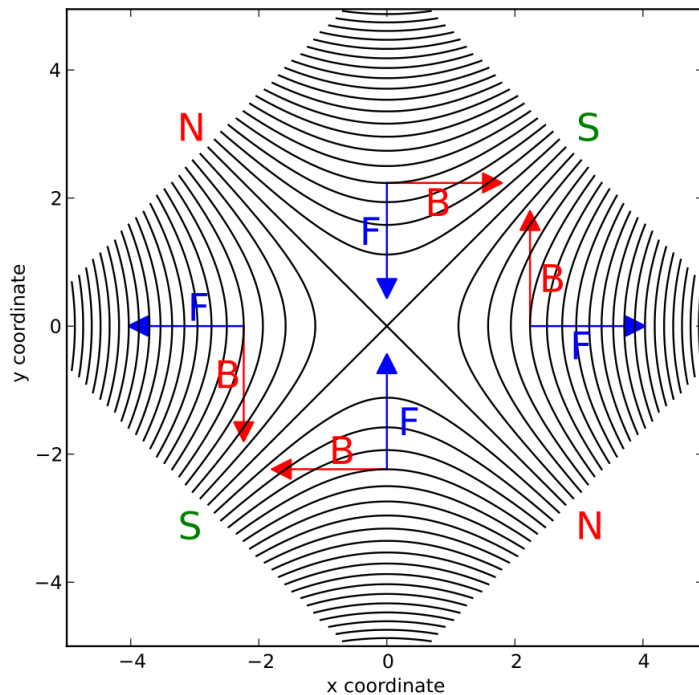
Permanent MQ



Focusing force

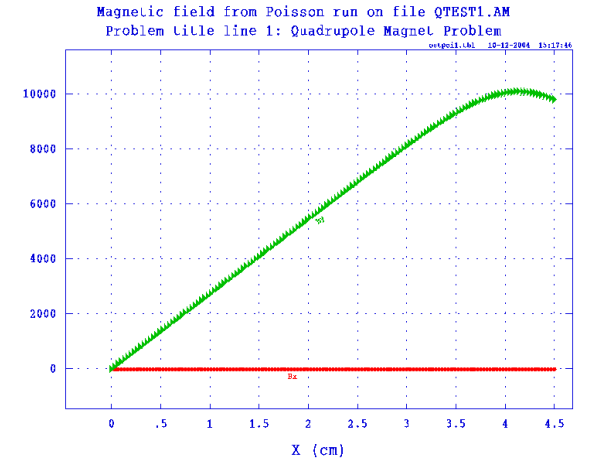
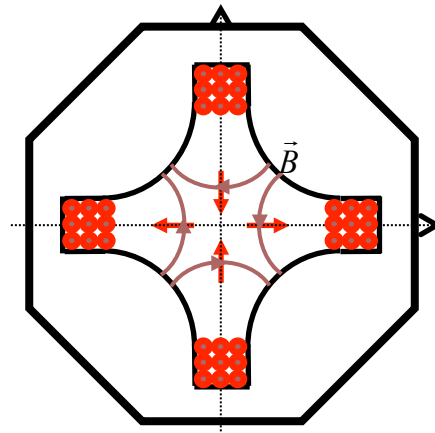
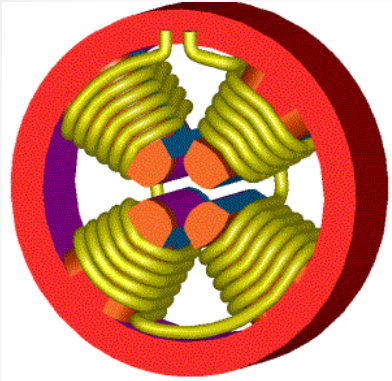
B =magnetic field/ F =force

Positively charged particles going into the screen

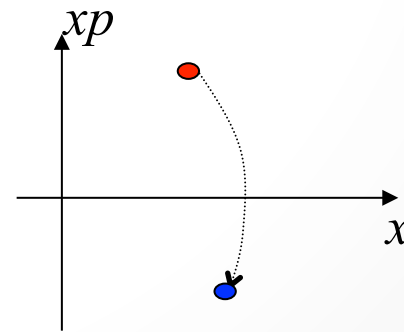
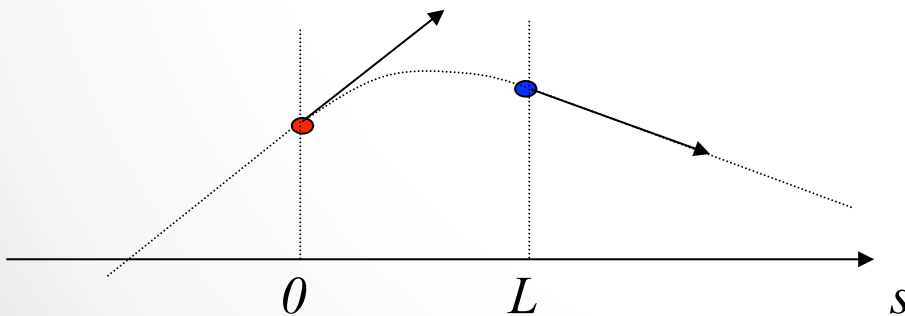


POLARITY
CHANGED WRT
PREVIOUS SLIDE

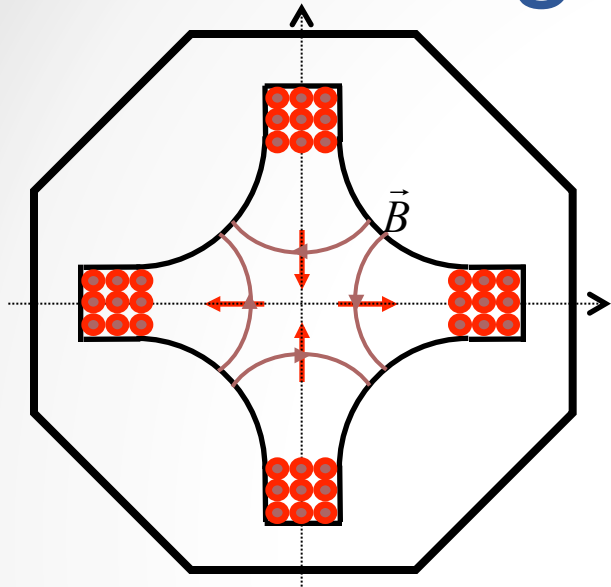
Quadrupole



Length L ; gradient $G=B/a$; a = aperture



Magnetic quadrupole



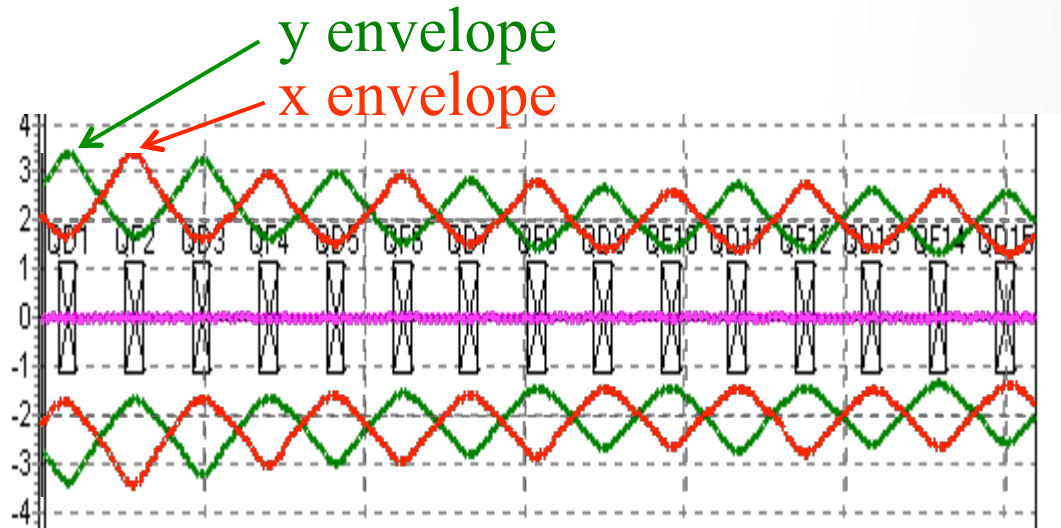
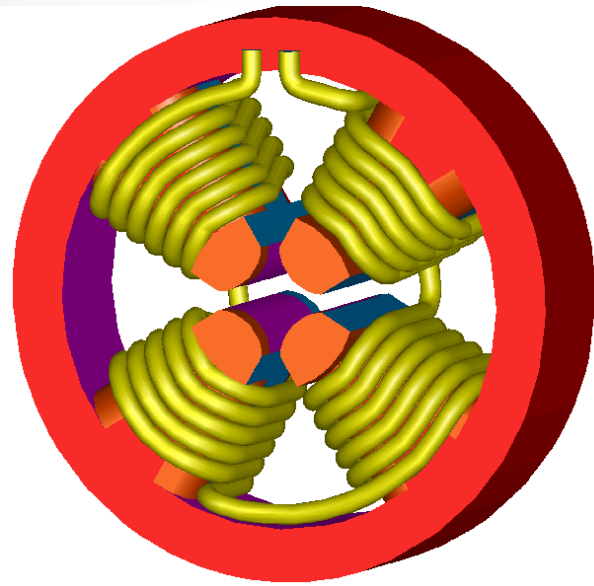
Magnetic field

$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases}$$

Magnetic force

$$\begin{cases} F_x = -q \cdot v \cdot G \cdot x \\ F_y = q \cdot v \cdot G \cdot y \end{cases}$$

Focusing in one plan, defocusing in the other



sequence of focusing and defocusing quadrupoles