Collective Effects & its Diagnostics

part 1

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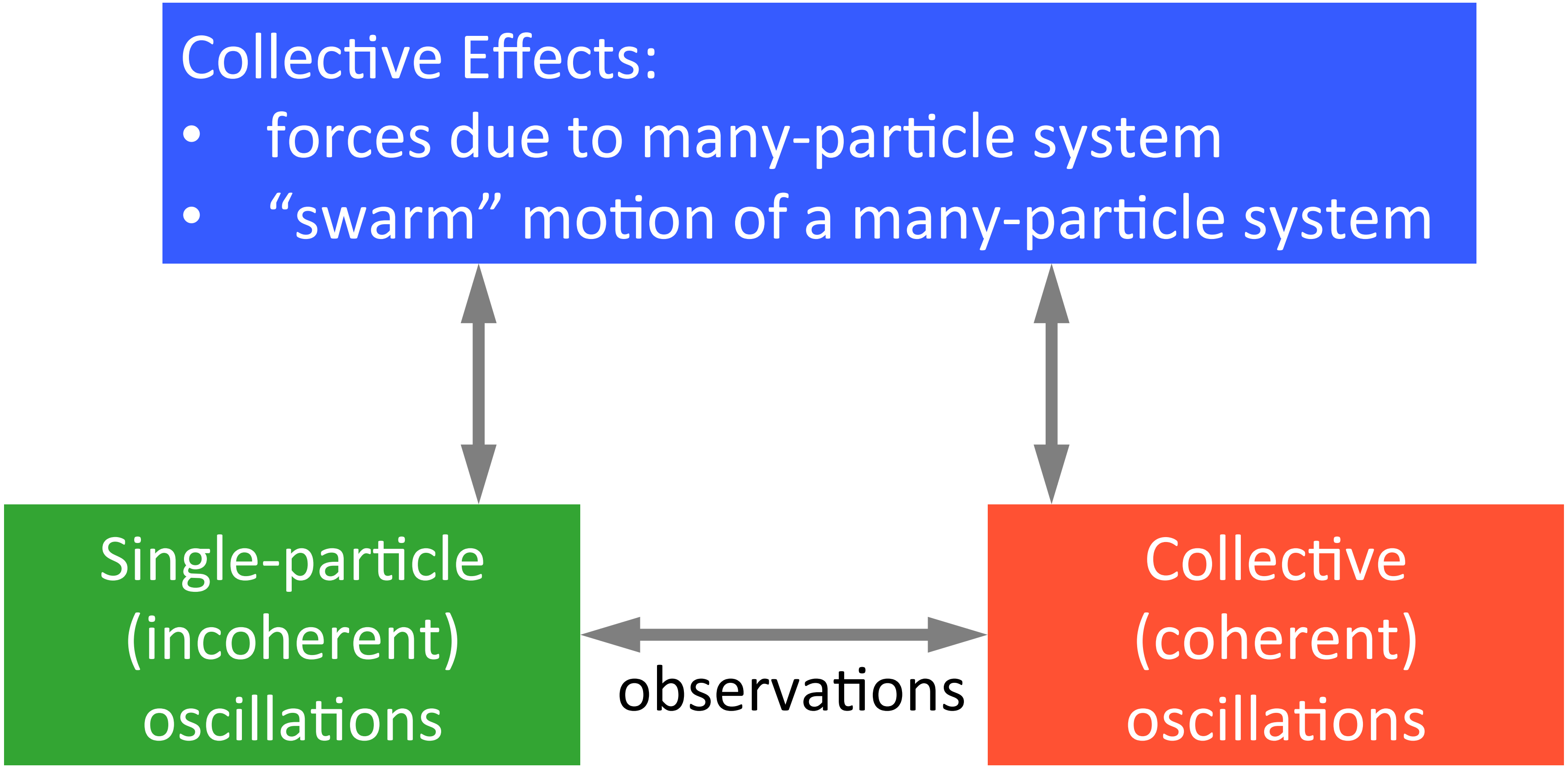
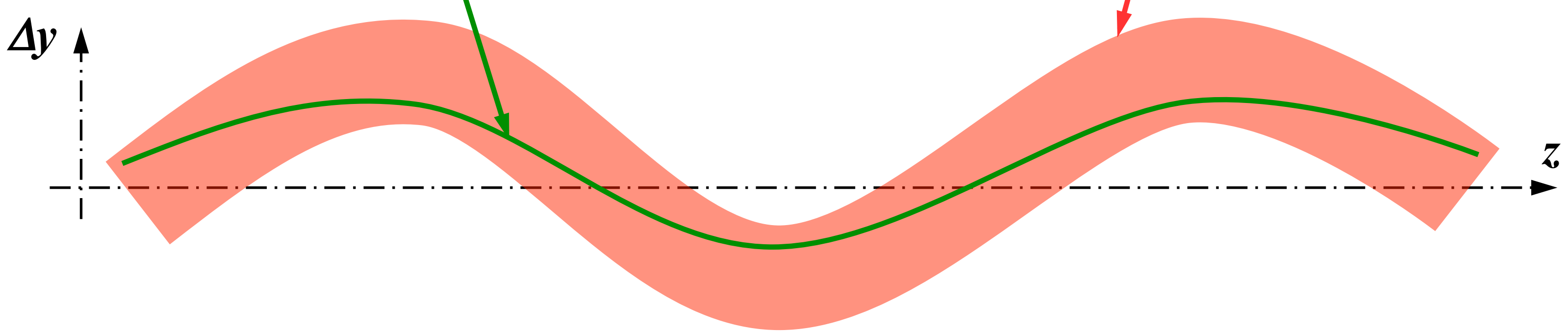
Introduction



Oscillations in Beams

Incoherent oscillations

Coherent oscillations



Incoherent tune-shifts

$\Delta Q = Q - Q_0$

Large & important differences

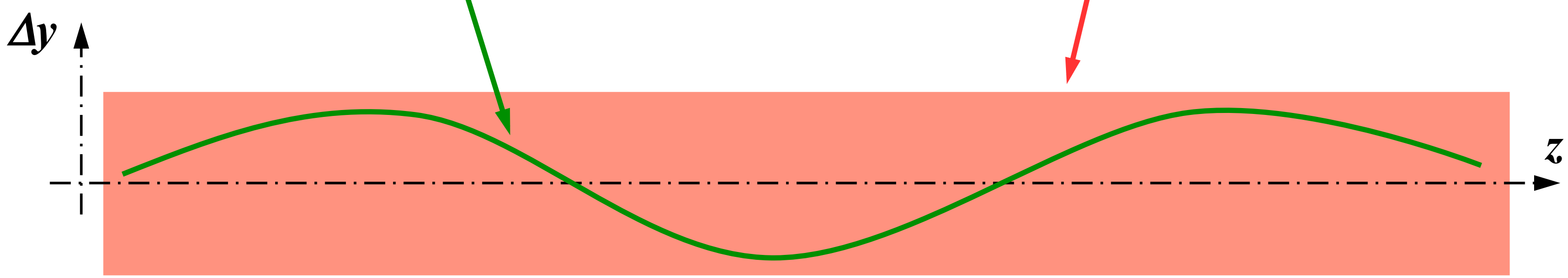
Coherent tune-shifts

$\Delta Q = Q - Q_0$

Oscillations in Beams

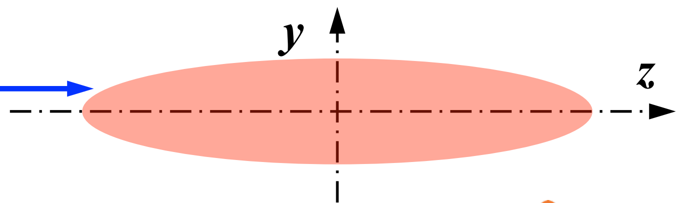
Incoherent oscillations

No coherent oscillations
$\langle y \rangle = 0$



coasting beam: L=C, λ(z)=const,
no synchrotron motion, δp=const

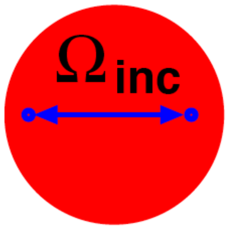
bunched beam: L_{bunch}, λ(z) profile,
synchrotron oscillations Q_s : δp−z

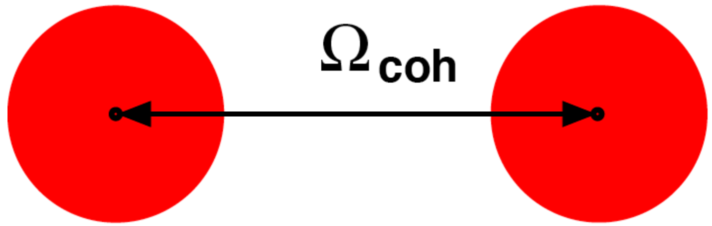


Oscillations, Forces

incoherent **coherent**

oscillations





interactions

$$\overline{F^{\perp}_{\mathrm{inc}}} = 0$$

$$F^{\perp} = F^{\perp}_{\mathrm{inc}} + F^{\perp}_{\mathrm{coh}}$$

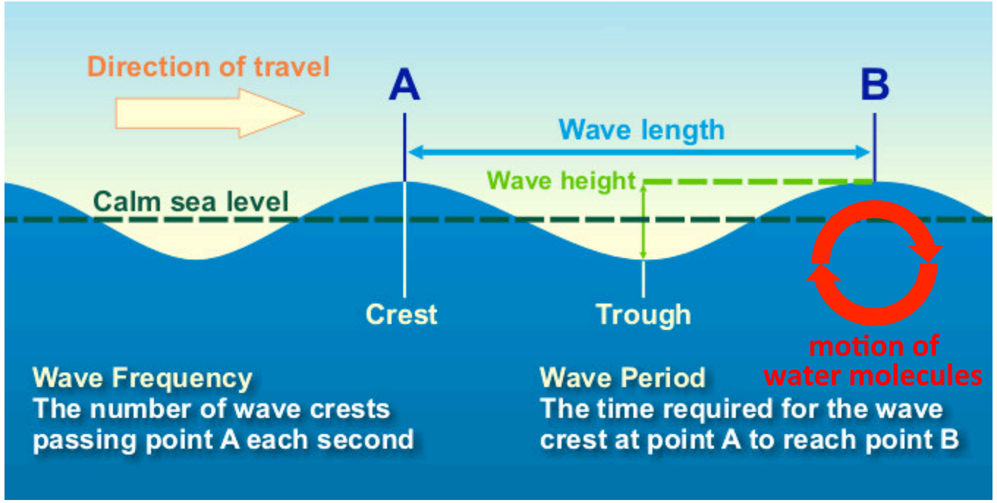
$$\overline{F^{\perp}} = F^{\perp}_{\mathrm{coh}}$$

Introduction

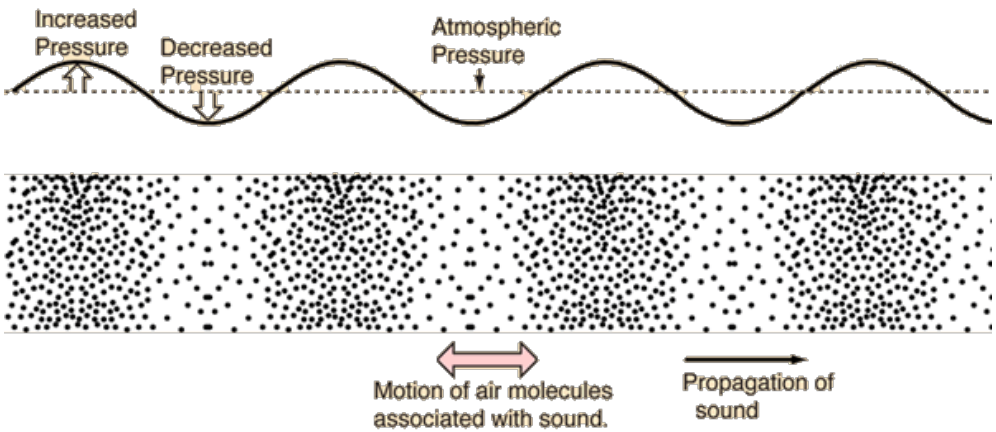
Collective oscillations are waves

Introduction

Water wave



Sound wave



Traveling oscillation in a medium.
Very different from the medium particle motion.

Oscillations: waves

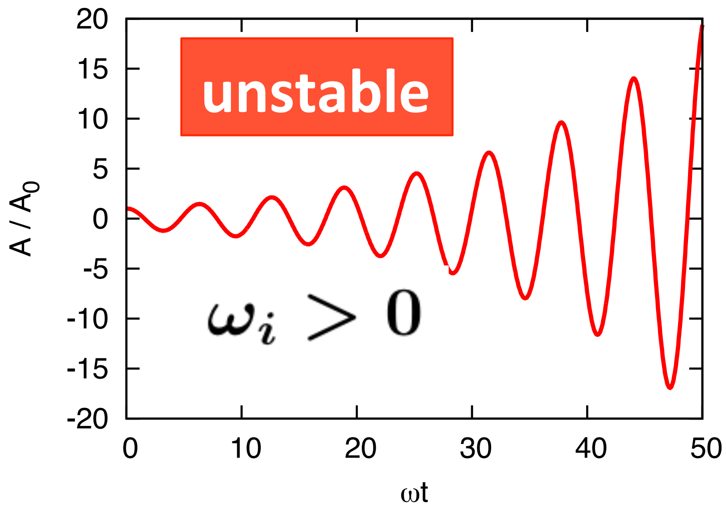
Waves can be
unstable or damped

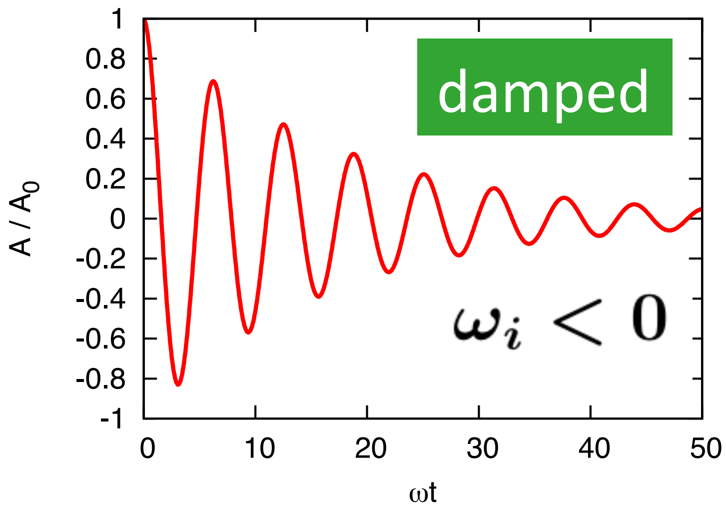
The wave frequency is complex:

$$\omega = \omega_r + i\omega_i$$

The wave physical parameter:

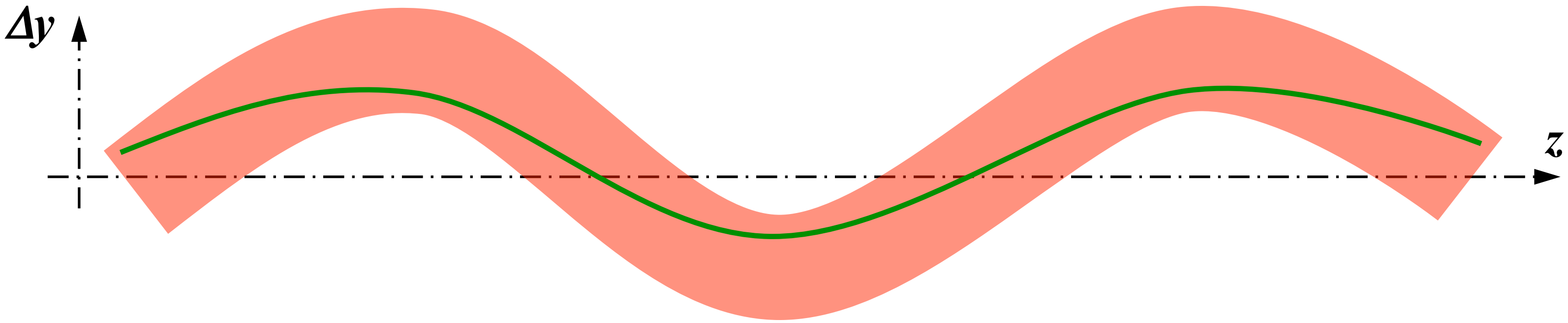
$$A(t) = A_0 \cos(\omega_r t)\ e^{\omega_i t}$$





Collective Oscillations

We observe, and we are interested,
only in special collective oscillations:
EIGENMODES



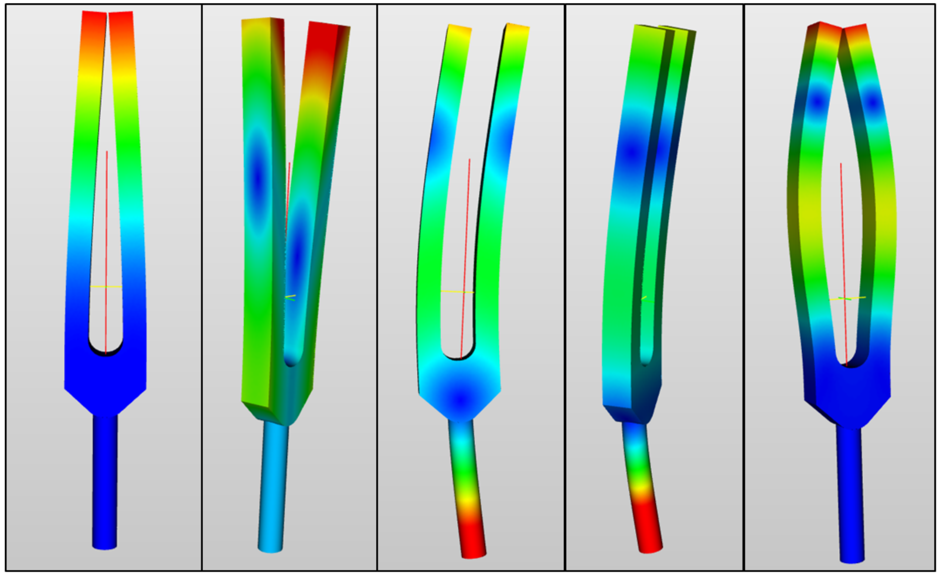
Eigenmodes

Eigenmodes: intrinsic orthogonal oscillations of the dynamical system, with the fixed frequencies (eigenfrequencies)

$$A\vec{x} = \lambda\vec{x}$$

eigenvalue → λ

eigenmode → $\vec{x}$



We often talk about the shift:

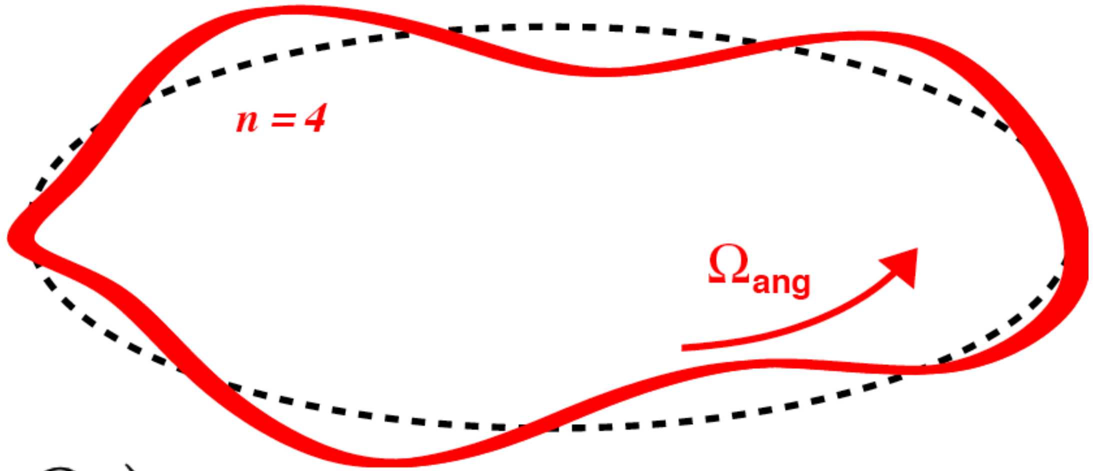
$$\Delta\Omega = \Omega - \Omega_{\mathrm{eigenfrequency}}$$

Eigenmodes of a tuning fork.
Pure tone at eigenfrequencies.

Eigenmodes

Example: Transverse eigenmodes in a coasting beam

$$x(s,t) = x_0 e^{ins/R - i\Omega t}$$



n is the mode index.

Wave length: *C/n*

Frequencies:

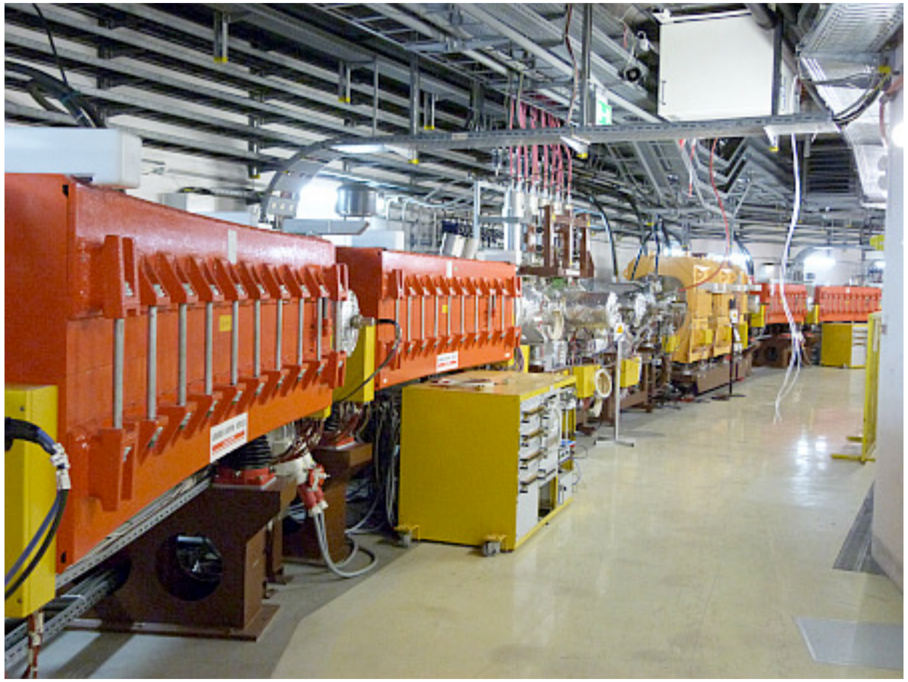
$$\begin{aligned} \text{slow wave} \quad \Omega_\text{s} &= (n - Q_\beta)\omega_0 \\ \text{fast wave} \quad \Omega_\text{f} &= (n + Q_\beta)\omega_0 \end{aligned}$$

Angular rotation (Ω_s):

$$\Omega_\text{ang} = \left(1 - \frac{Q_\beta}{n}\right)\omega_0$$

Eigenmodes in a coasting beam

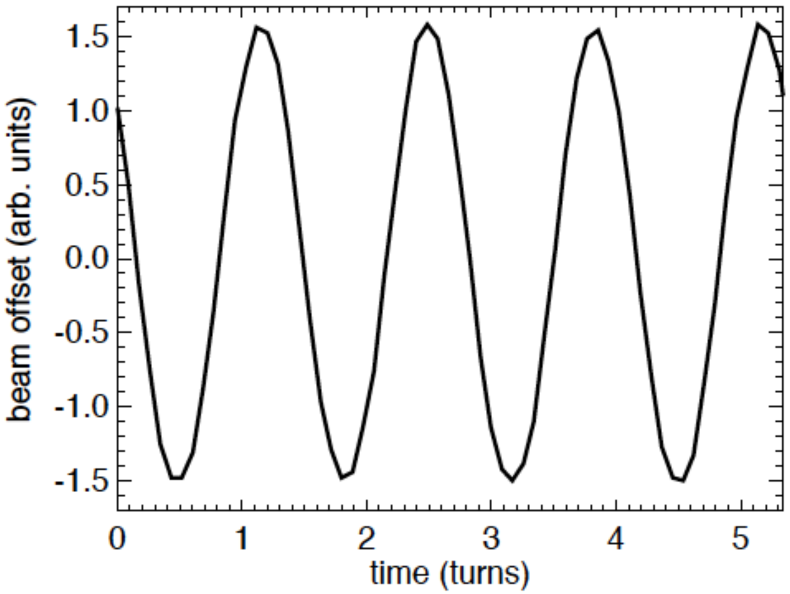
Experimental observations of the coasting-beam waves



SIS18 synchrotron at GSI Darmstadt

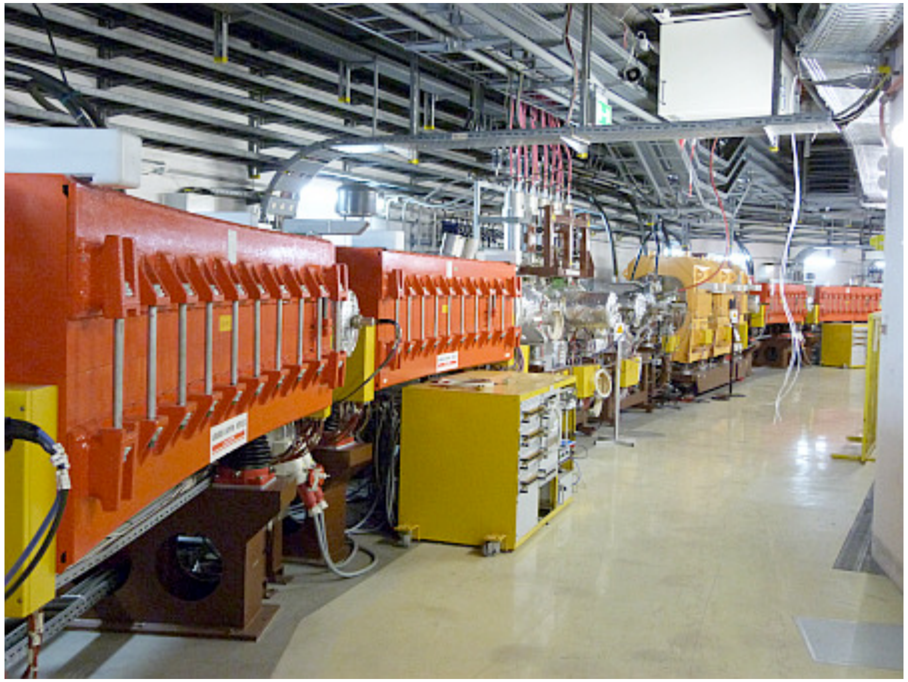
V. Kornilov, O. Boine-Frankenheim,
GSI-Acc-Note-2009-008, GSI Darmstadt (2009)

Space Structure:
n=4, as expected for Q=3.25,
with correct Ω_s and Ω_{ang}



Eigenmodes in a coasting beam

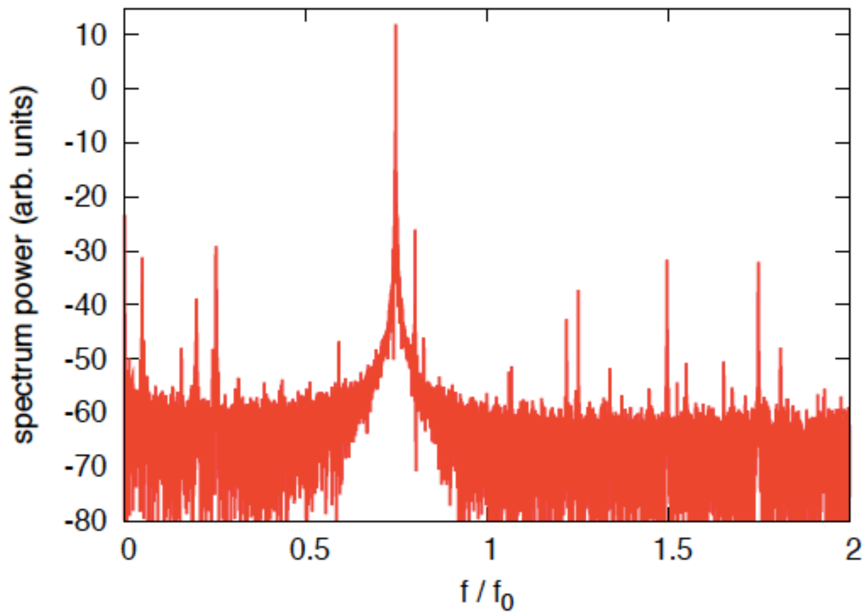
Experimental observations of the coasting-beam waves



SIS18 synchrotron at GSI Darmstadt

V. Kornilov, O. Boine-Frankenheim,
GSI-Acc-Note-2009-008, GSI Darmstadt (2009)

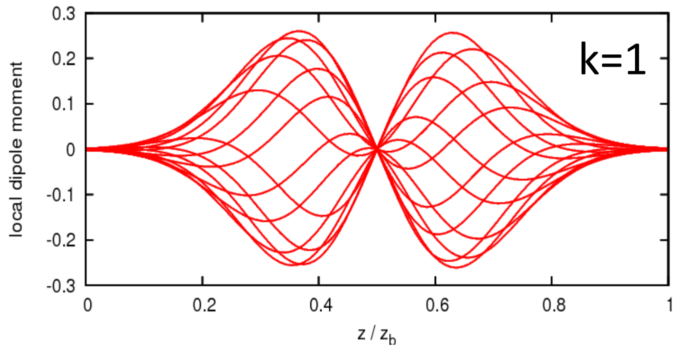
Frequency Structure:
f=159.9kHz, as expected
$(1\text{-}0.25)f_0$ for Q=3.25

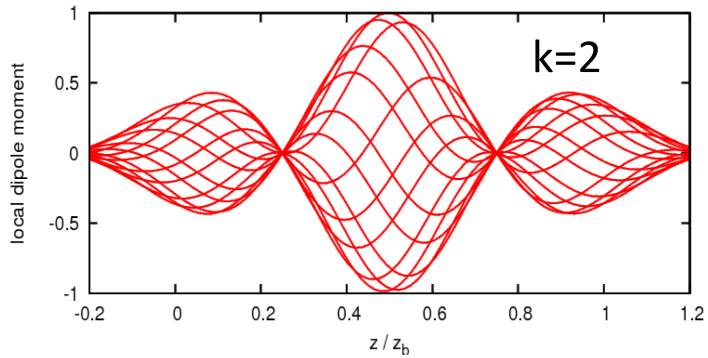


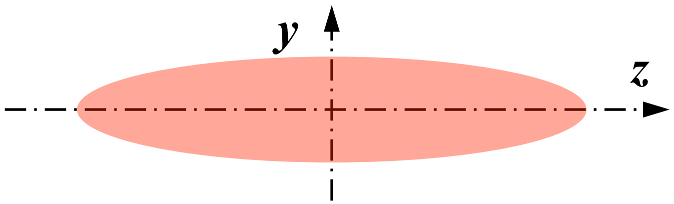
Eigenmodes in bunched beams

Transverse collective oscillations in bunched beams:
Head-Tail Modes

Eigenmode:

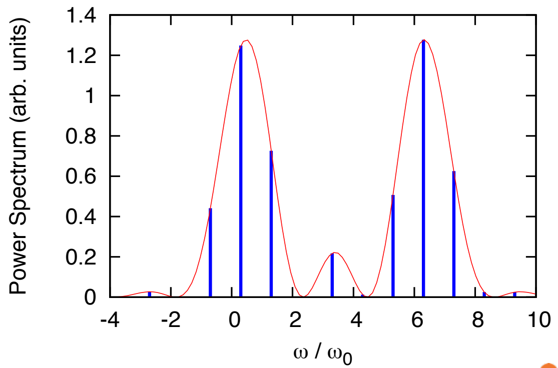






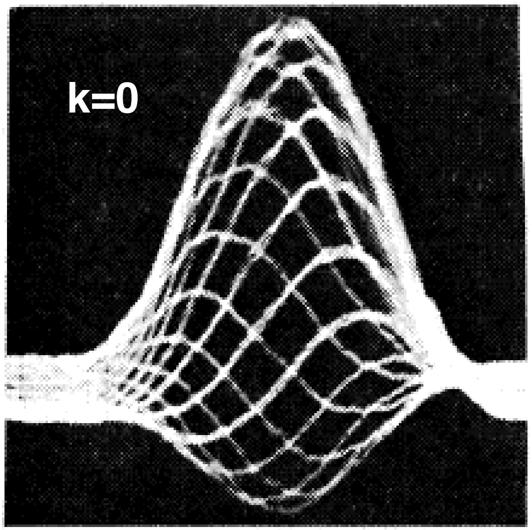
BPM Δ−signal along the bunch, overlapped over several turns: Wiggles and Nodes

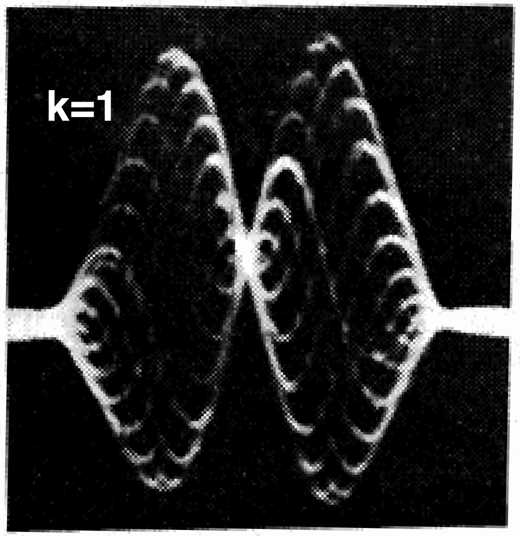
Eigenfrequencies:

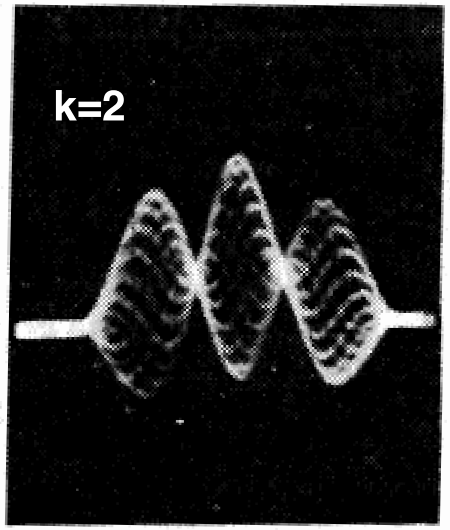


Eigenmodes in bunched beams

Head-Tail Modes are measured in many machines,
the first observation in CERN PSB (1974):





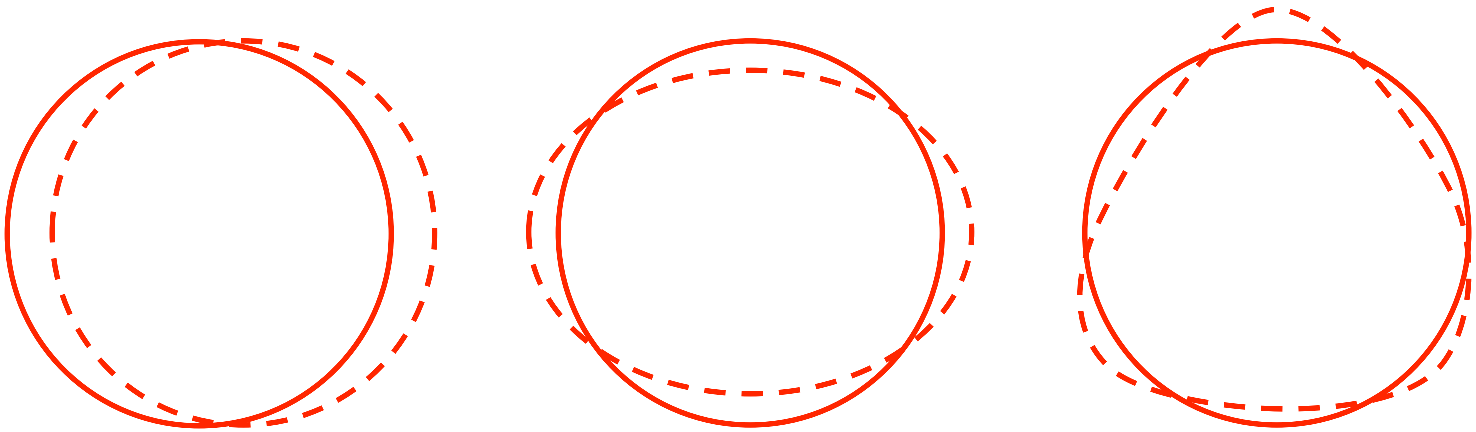


Particle, Beam Oscillations

Different types of coherent oscillations

Transverse, Longitudinal

Dipolar (m=1)	Quadrupolar (m=2)	Sextupolar (m=3)

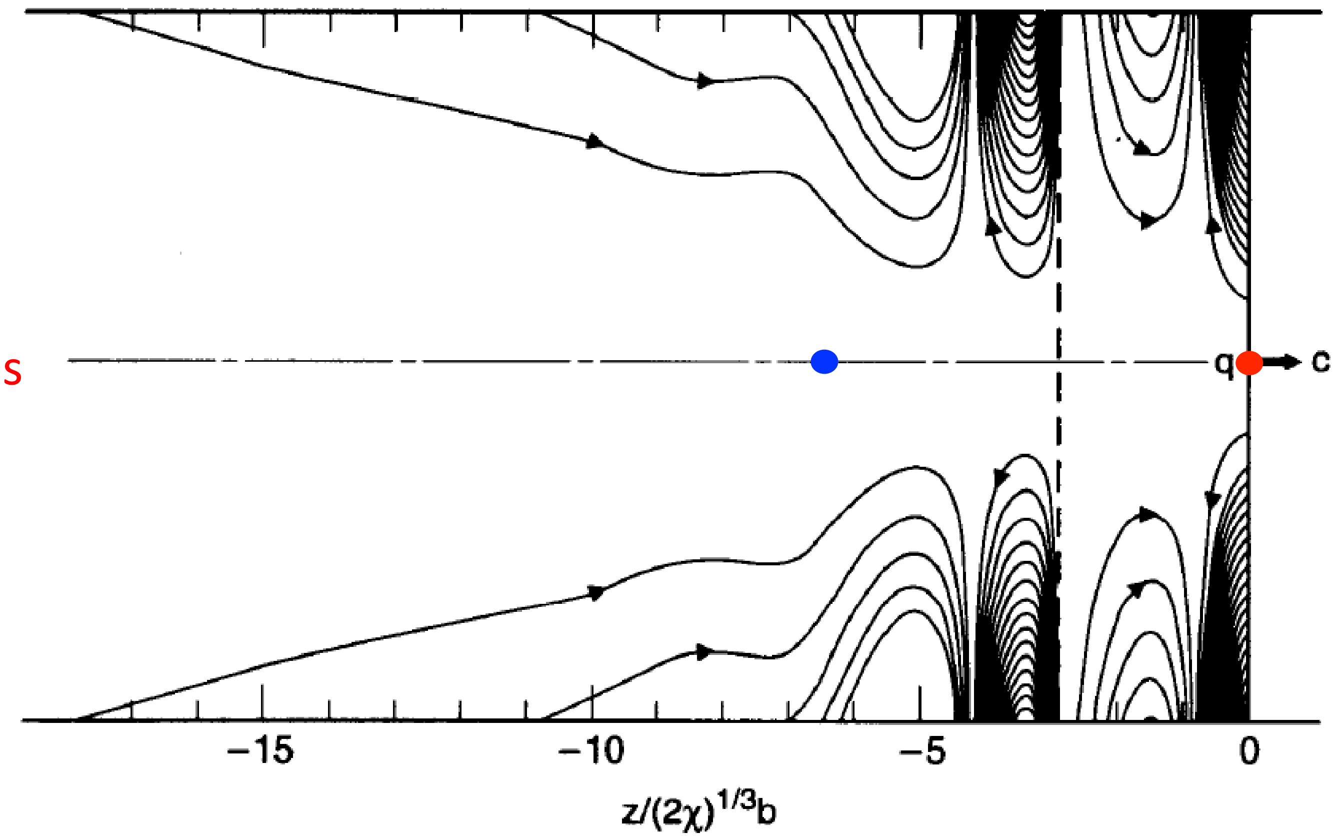


Here we consider mostly the dipole transverse oscillations.
For the others: the physics and the formalism are similar.

Impedances

Wakes & Impedances

- Leading charge
- Trailing charges
- Leading charge generates electromagnetic fields
- Leading charge is loosing energy
- Trailing charge is gaining/loosing energy



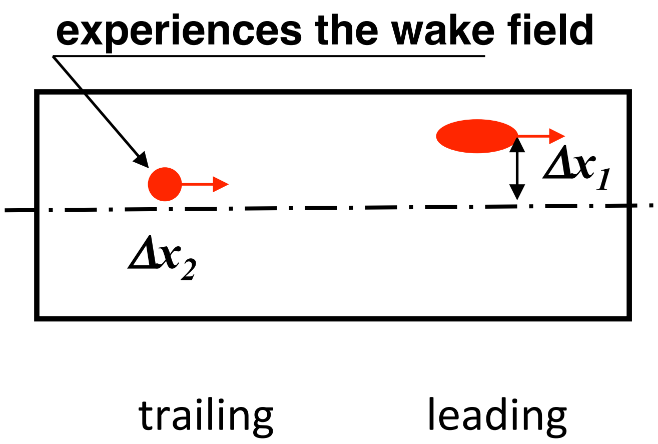
Electric field pattern for a resistive wall pipe

A.Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993

Wakes & Impedances

Dipolar wakes: $F_{x2} \sim \Delta x_1$
(driving) the same for the whole trailing slice: coherent

Quadrupolar wakes: $F_{x2} \sim \Delta x_2$
(detuning) different for individual particles: incoherent



The facility impedances have coherent and incoherent effects

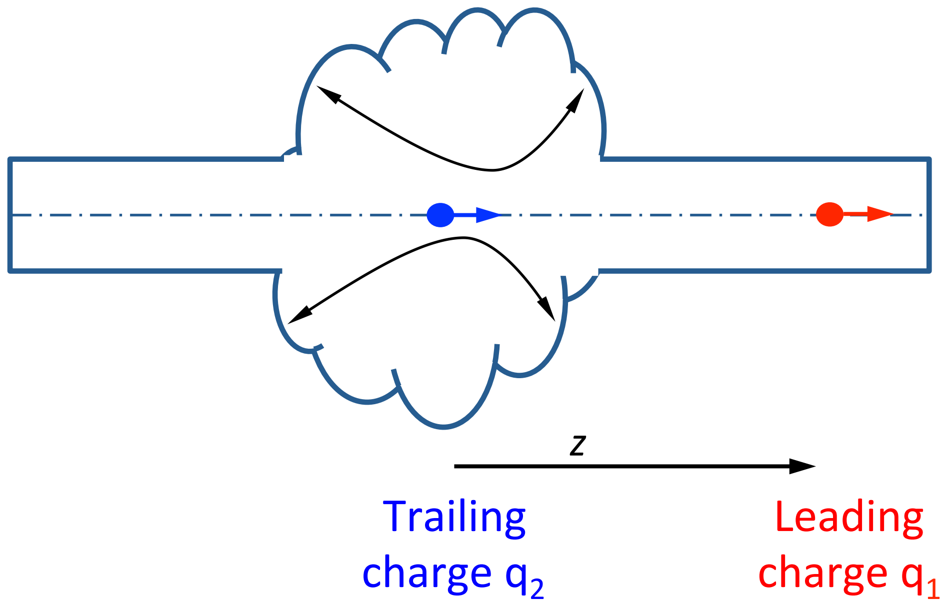
Wakes & Impedances

In order to describe the effect of the wake fields, we define the "wake function".

The energy loss/gain of the trailing charge.

The longitudinal Wake Function:

$$\int F_{\|} ds = \Delta \mathcal{E}_2 = -q_1 q_2 W_{\|}(z)$$
$$F_{\|}(s,z) = q_2 E_z(s,z)$$



The "lumped" (localized) impedance:
one interaction per turn.
Field integral is over the structure elements.

Wakes & Impedances

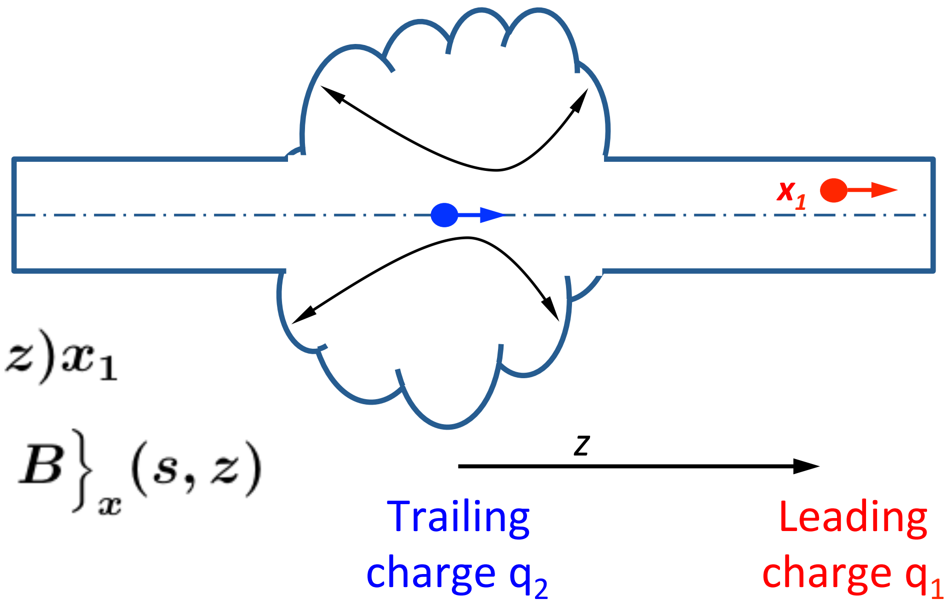
In order to describe the effect of the wake fields, we define the "wake function".

The energy loss/gain of the trailing charge.

The transverse Wake Function:

$$\int F_x ds = \mathcal{E}_0 \Delta x_2' = -q_1 q_2 W_x(z) x_1$$

$$F_x(s,z) = q_2 E_x(s,z) + q_2 \{v \times B\}_x (s,z)$$



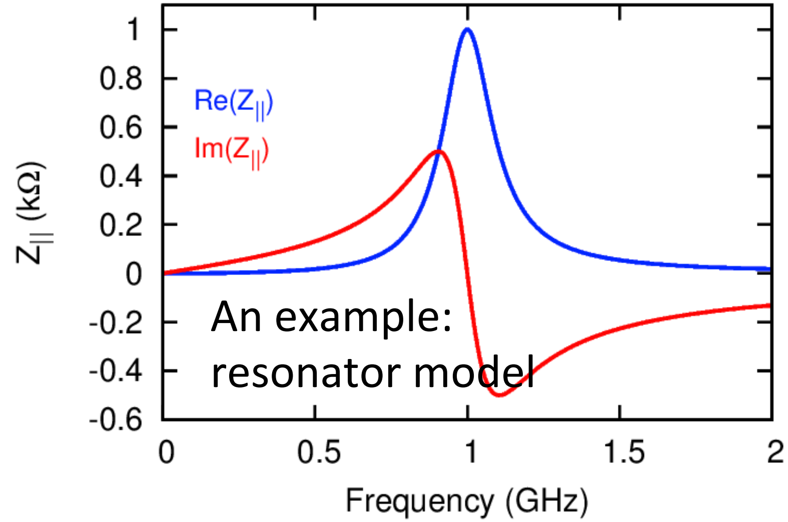
The dipole impedance: the offset of the leading particle produces the wake, which does not depend on the trailing particle offset.

Wakes & Impedances

In the frequency domain:
IMPEDANCE
a complex function of the frequency

$$Z_{\|}(\omega) = \int e^{-i\omega z/v} W_{\|}(z) \frac{dz}{v}$$

$$Z_{\perp}(\omega) = i \int e^{-i\omega z/v} W_{\perp}(z) \frac{dz}{v}$$



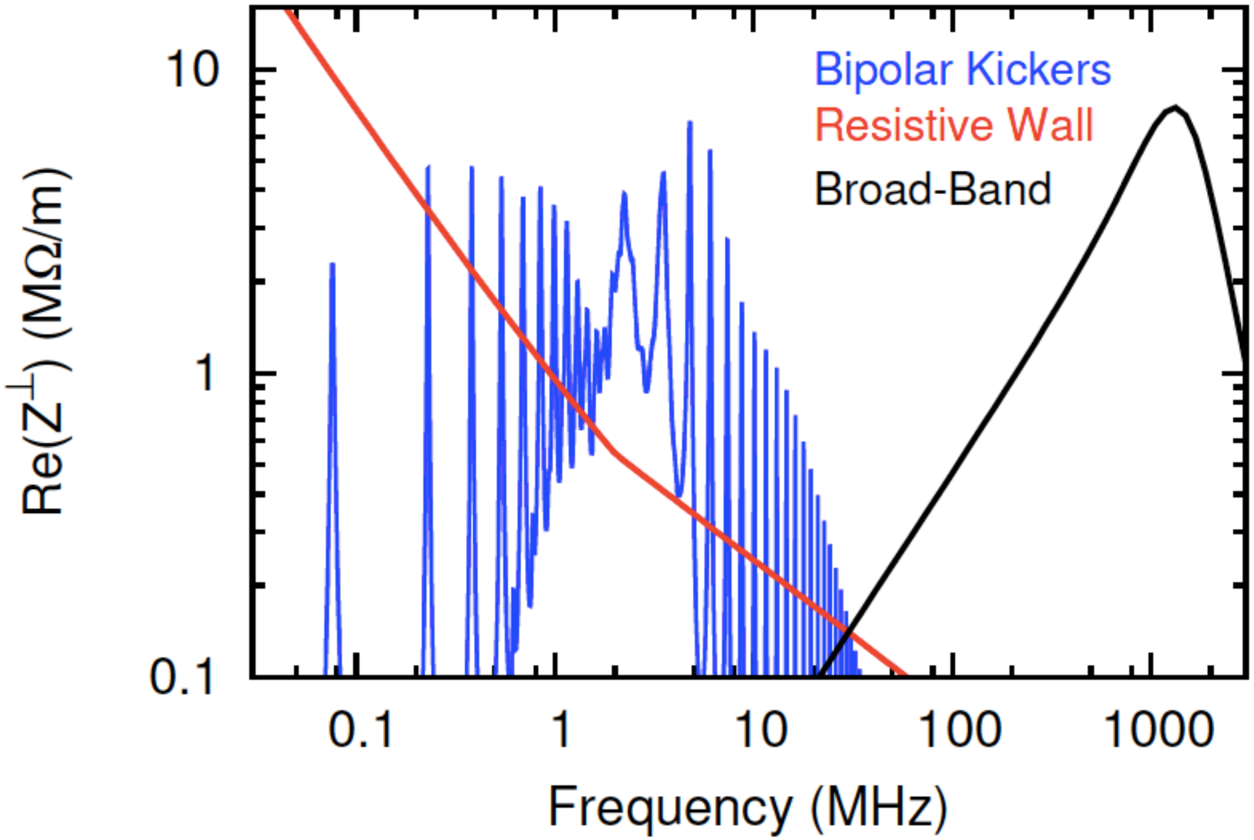
Effect of an impedance on a collective eigenmode:

$$\Omega = \Omega_{\mathrm{Re}} + i\Omega_{\mathrm{Im}} = \Omega_{\mathrm{mode}} + \kappa Z_{\mathrm{Im}}^{\mathrm{eff}} + i\kappa Z_{\mathrm{Re}}^{\mathrm{eff}}$$

beam oscillation | frequency shift | growth rate

Impedances

An accelerator facility can have a complicated impedance spectrum

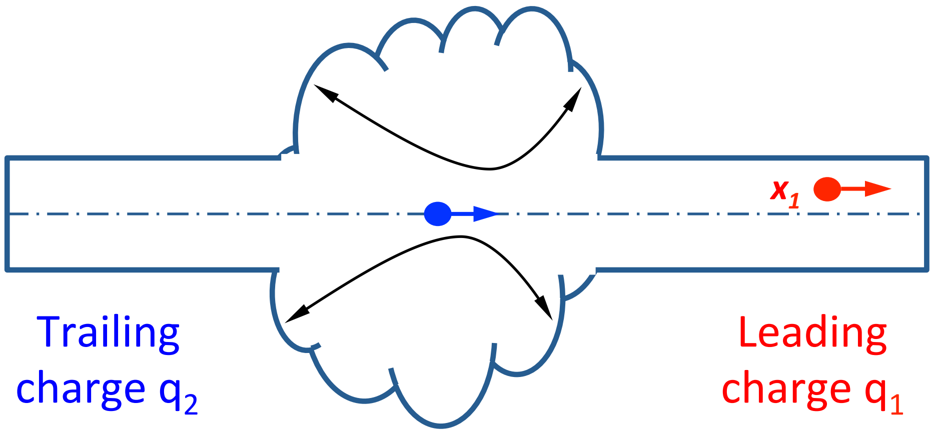


Example: real part of the transverse impedance in SIS100 of FAIR, Darmstadt

Impedances

Impedances:

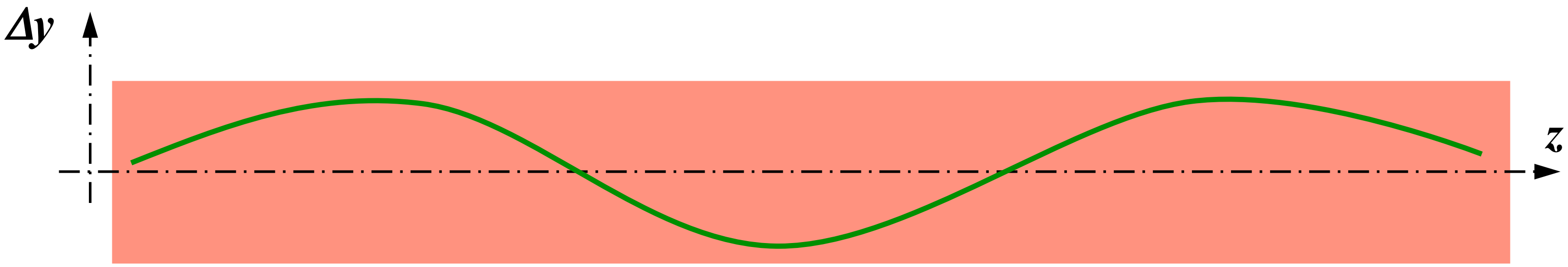
- an intensity effect (larger q → stronger fields)
- can be coherent and incoherent
- affect the coherent and the incoherent frequencies (both can be observed)
- a beam-external effect



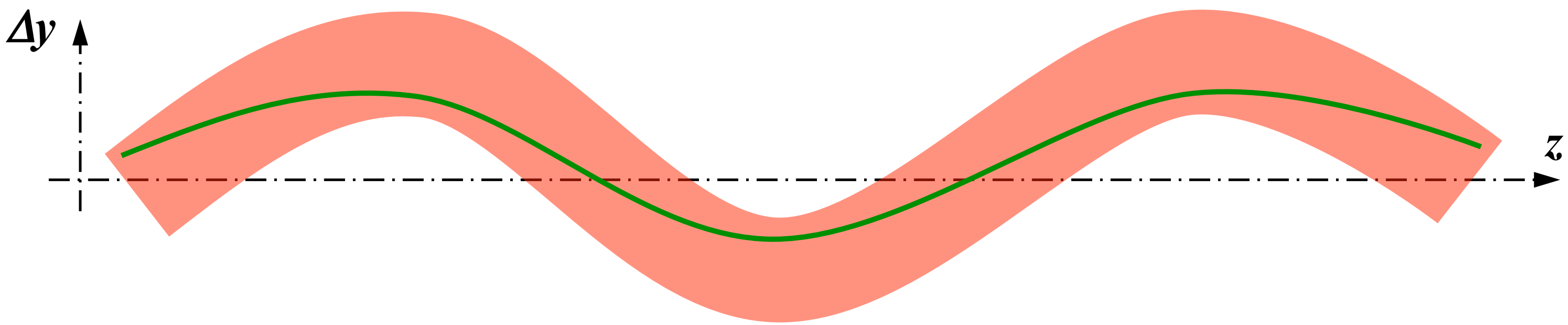
Tune Shifts

Measurements

Different diagnostics for coherent and for incoherent oscillations



BPM Signal = 0 ; Schottky Monitor signal ≠ 0

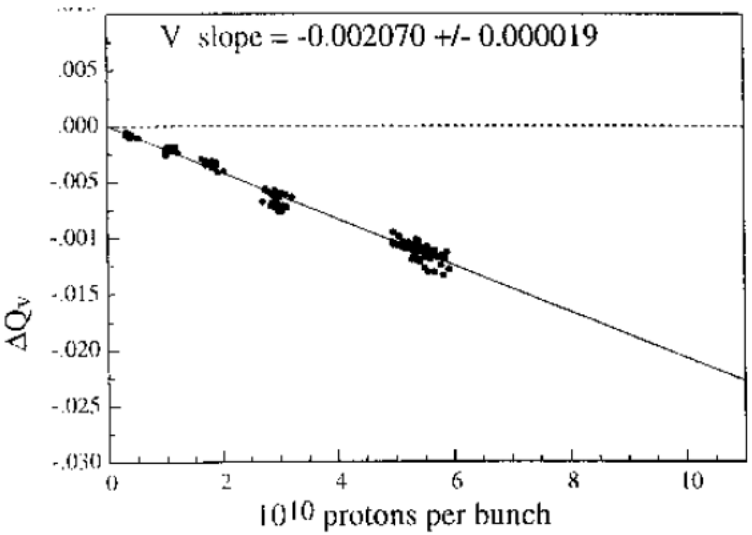


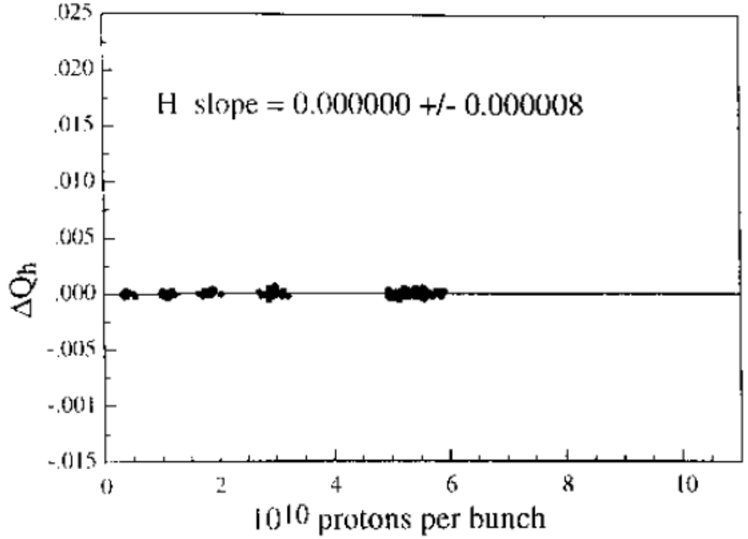
BPM Signal ≠ 0

Tune Shifts

Seems to be easy: by measuring ΔQ_{coh} the impedance is determined

But then, how to understand this:





Single bunch tune measurements at the CERN SPS, J. Gareyte, EPAC2002

What has been measured?
The horizontal impedance was surely non-zero.

Tune Shifts

Laslett coefficients for coasting beams:

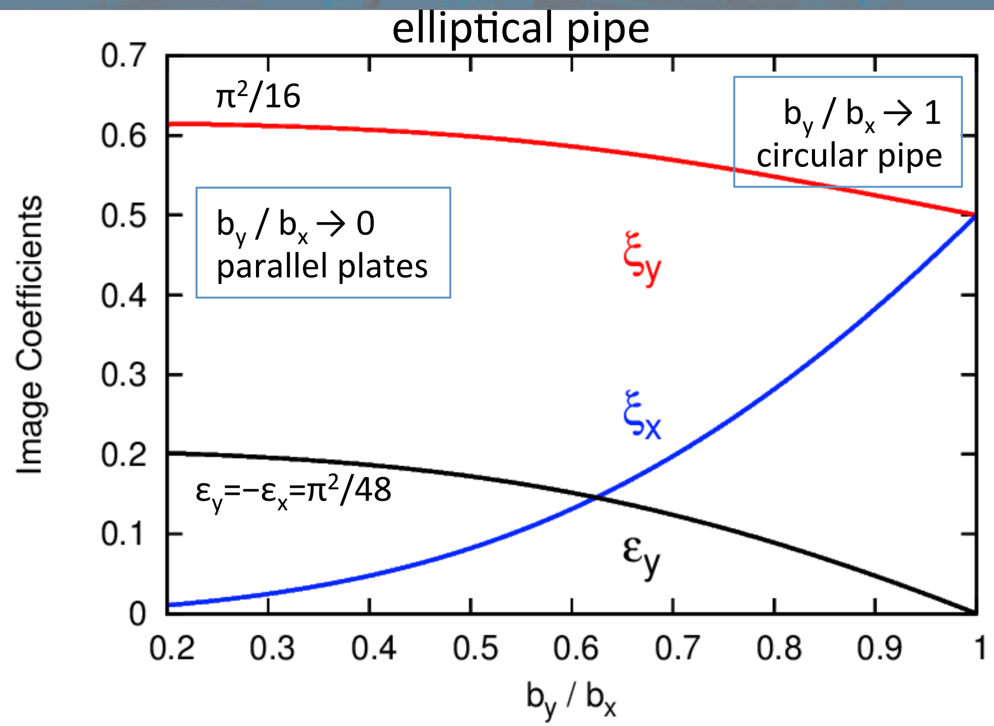
$$\Delta Q_{\text{inc}} = -\zeta\lambda_0 \frac{\varepsilon_1}{h^2}$$

$$\Delta Q_{\text{coh}} = -\zeta\lambda_0 \left[\frac{\beta^2\varepsilon_1}{h^2} + \frac{\xi_1}{\gamma^2 h^2}\right]$$

ξ_1 : symmetries, coherent

ε_1 : unsymmetries, incoherent

$1/\gamma^2$: ***E–B*** cancellation



Elliptical pipe, $h=b_y$ is the half-height.
Perfectly conducting pipe.
Different terms for:

Low frequencies (ac magnetic field)
Magnet poles
Partial neutralization

$$\zeta = \frac{2r_p R^2}{\beta^2\gamma Q_0}$$

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62

Tune Shifts

Laslett coefficients for coasting beams:

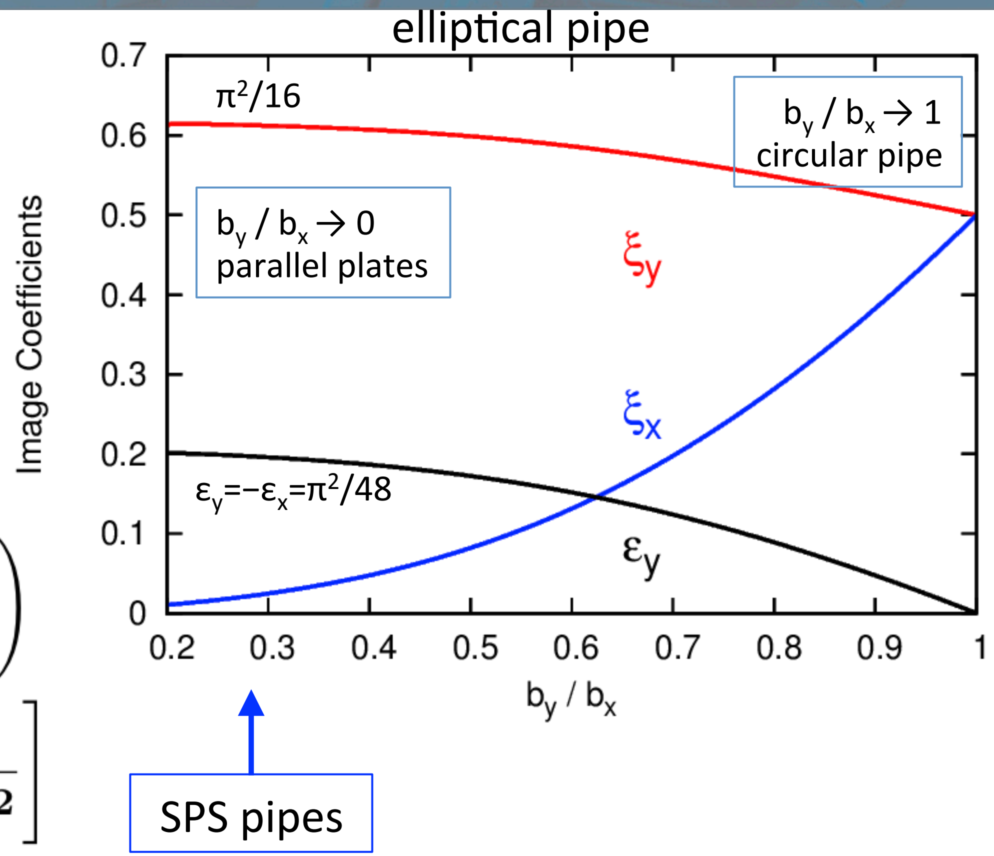
$$\Delta Q_{\text{inc}} = -\zeta\lambda_0 \frac{\varepsilon_1}{h^2}$$

$$\Delta Q_{\text{coh}} = -\zeta\lambda_0 \left[\frac{\beta^2\varepsilon_1}{h^2} + \frac{\xi_1}{\gamma^2 h^2}\right]$$

For bunched beams ($B=I_{av}/I_{peak}$):

$$\Delta Q_{\text{inc}} = -\zeta\lambda_0 \frac{\varepsilon_1}{h^2}\left(\beta^2 + \frac{1}{B\gamma^2}\right)$$

$$\Delta Q_{\text{coh}} = -\zeta\lambda_0 \left[\frac{\beta^2\varepsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2}\right]$$



The bunching factor, the cancellation, β^2 appear in a non-straightforward way

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62

Intensity Effects & Tune Shifts

From the first-order expansion of the forces for small perturbations, a symbolic relation:

$$\Delta Q_{\mathrm{coh}} = \Delta Q_{Z^{\perp}} + \Delta Q_{\mathrm{inc}}$$

- ΔQ_{coh}: Coherent tune shift: everything affecting $\langle \mathbf{x} \rangle$
- $\Delta Q_{Z^{\perp}}$: Tune shifts due to impedance: $\langle \mathbf{x} \rangle$ acting on individual particles. Include incoherent effects.
- ΔQ_{inc}: Facility- and lattice-related incoherent tune shifts. No space charge.

this is why there are incoherent effects in the coherent tune shift

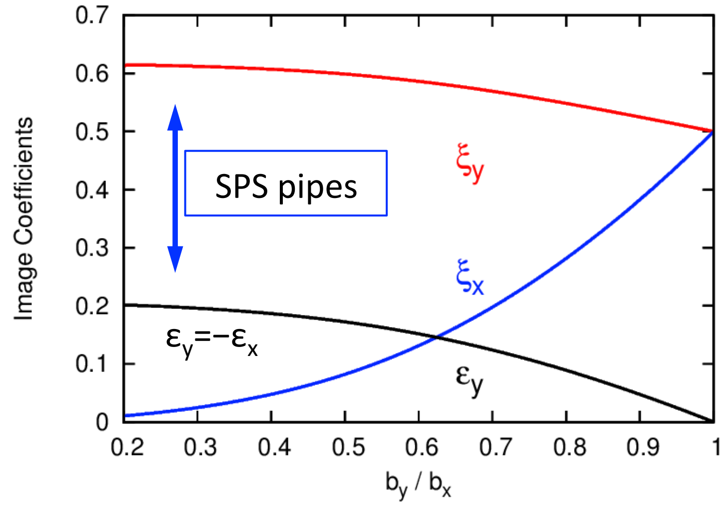
$$\Delta Q_{\mathrm{coh}} \quad = \quad -\zeta\lambda_0 \left[\frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2} \right]$$

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62

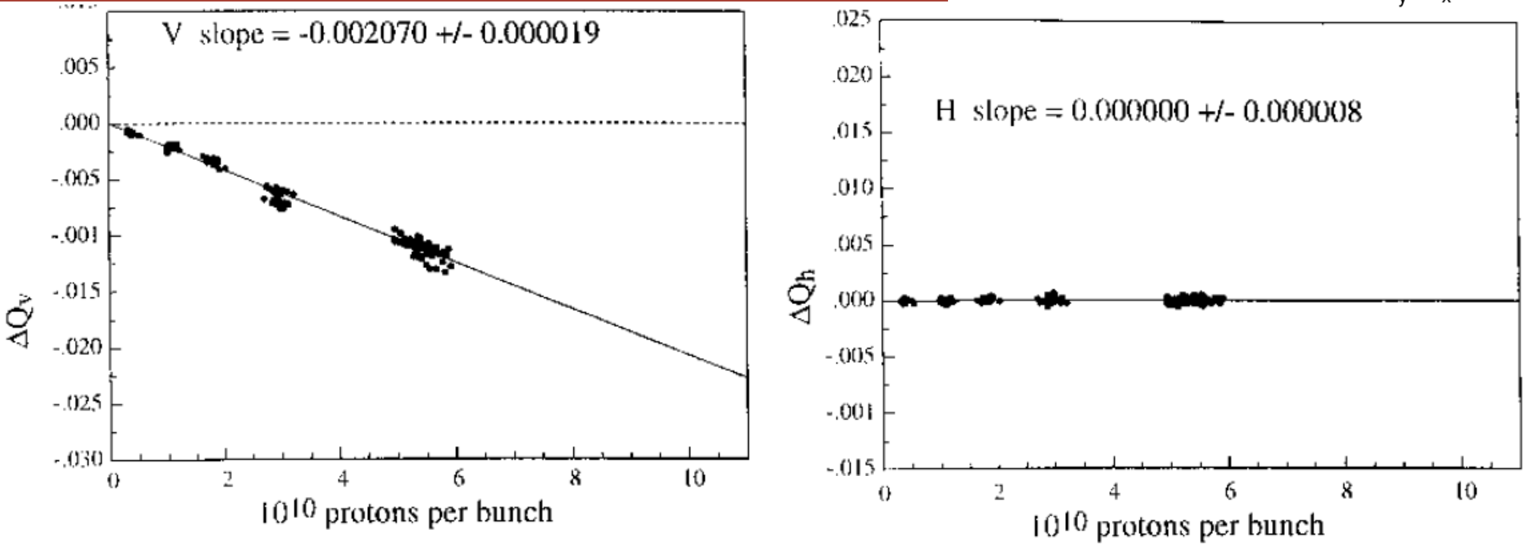
Intensity Effects & Tune Shifts

$$\Delta Q_{\text{coh}} = \Delta Q_{Z^{\perp}} + \Delta Q_{\text{inc}}$$

$$\Delta Q_{\text{coh}} = -\zeta\lambda_0\left[\frac{\beta^2\varepsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2}\right]$$



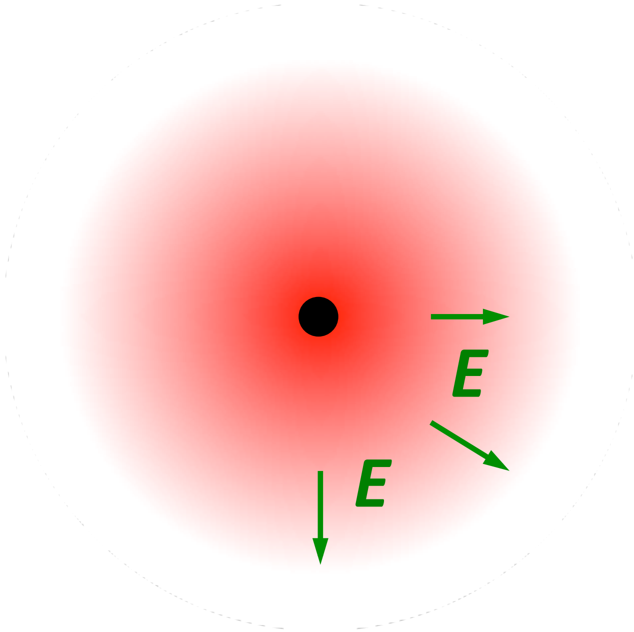
Incoherent effect of the unsymmetries reduces ΔQ_Z horizontally and enhances ΔQ_Z vertically



Single bunch tune measurements at the CERN SPS, J.Gareyte, EPAC2002

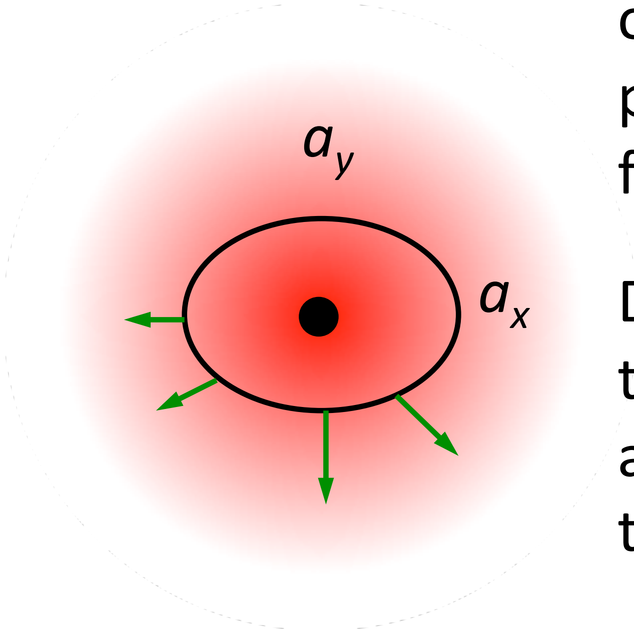
Space Charge

Space Charge



Electric field due to the charged beam particles

Space Charge



Transverse "betatron" oscillations of a single particle, amplitudes a_x, a_y, frequencies: tunes Q_x, Q_y

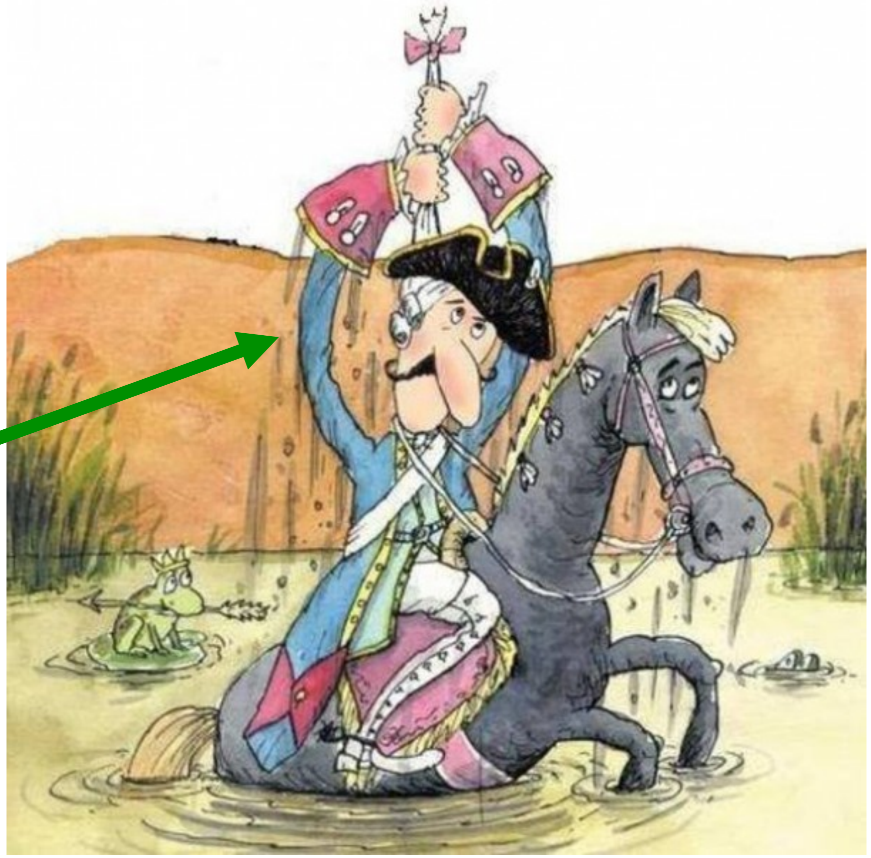
Due to the electric field of the "space charge", tunes are decreased by ΔQ_x, ΔQ_y : tune shifts

- is a collective effect: many particles produce the field
- affects the incoherent oscillations: the field moves with the beam center
- is a beam-internal interaction

Space Charge

Space charge is a beam-internal interaction.
The effects of space charge are different from the external ones.

This is space charge



Baron Münchhausen

Space Charge

$$\Delta Q_{\mathrm{sc}} = -g_a \frac{\lambda_0 r_p R}{4\gamma^3\beta^2\varepsilon_x}$$

Space charge tune shift:

1. is always negative
2. proportional to the intensity (λ)
3. depends on the transverse distribution (g_a)
4. $1/\gamma^2$: ***E–B*** cancellation
5. $1/a^2$: transverse beam size (emittance ε)
6. different for every particle (tune spread)

Space Charge

Example for tune spreads: SIS100 (FAIR, Darmstadt)

Resonances in the transverse oscillations:
$kQ_x+mQ_y = n$

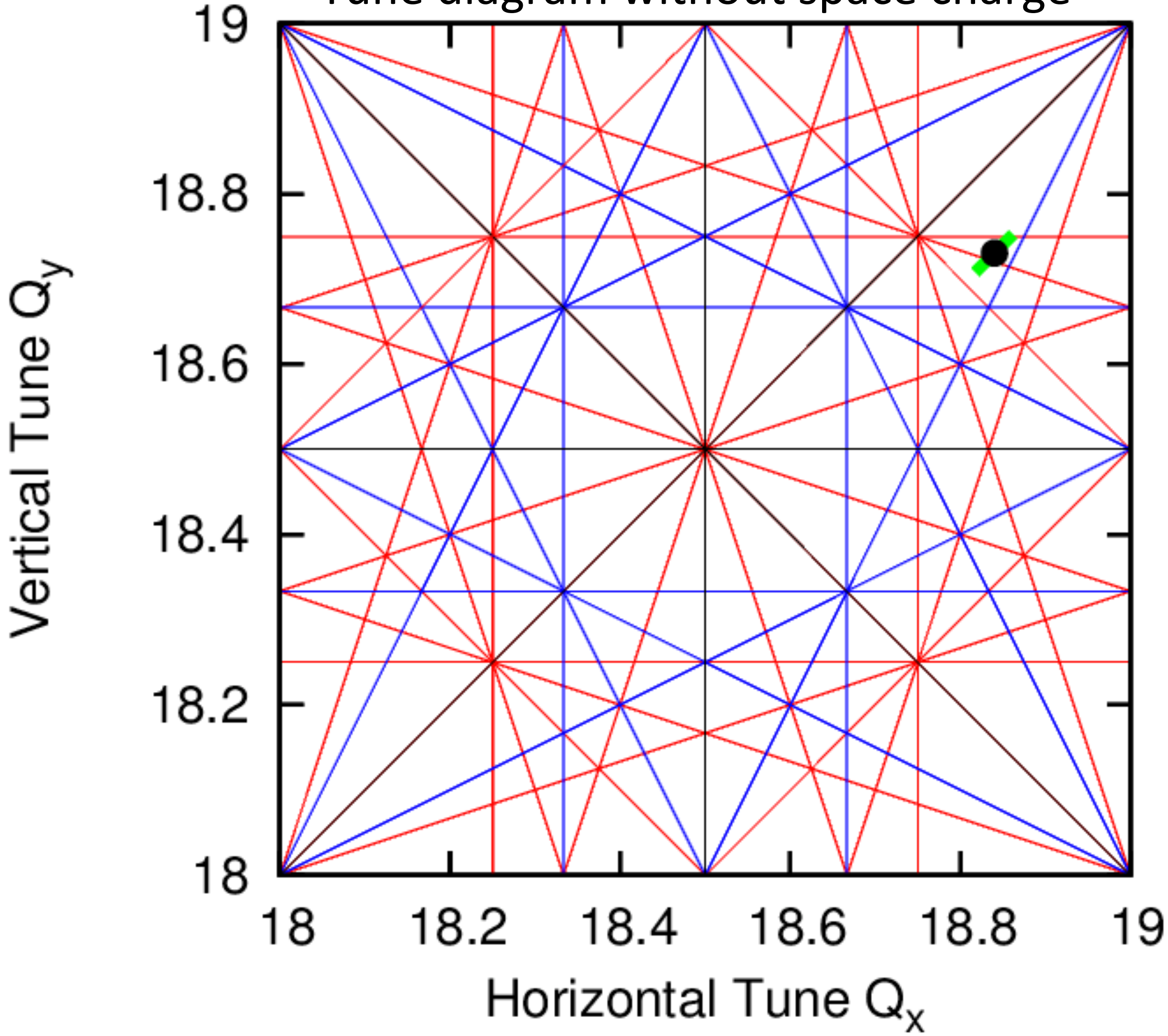
2^{nd} order, quadrupole
3^{rd} order, sextupole
4^{th} order, octupole

Black dot: set tunes
Q_x=18.84, Q_y=18.73

Green area: tune spread due to the chromaticity ξ

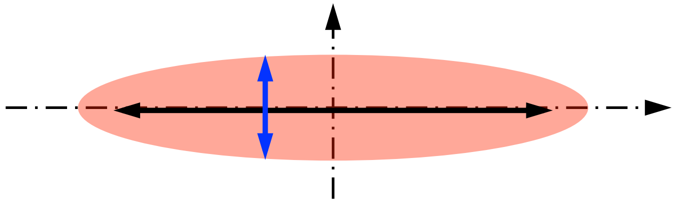
$$\Delta Q = Q\xi \frac{\Delta p}{p}$$

Tune diagram without space charge



Space Charge

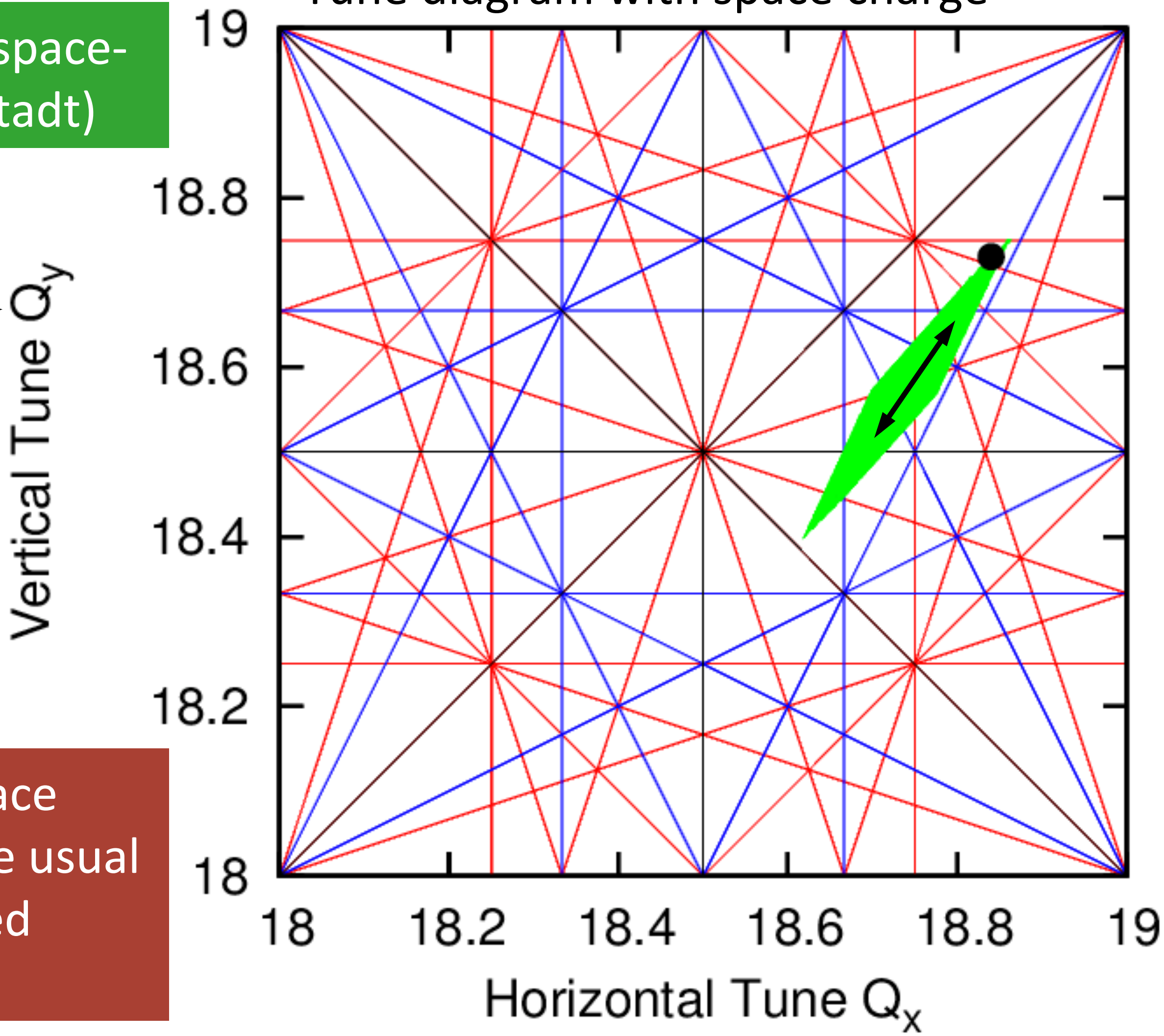
Example: Tune spread due to space-charge in SIS100 (FAIR, Darmstadt)



$f_{longitudinal} \approx 1.5$ kHz
(T ≈ 600 us)
$f_{transverse} \approx 5$ MHz
(T ≈ 0.2 us)

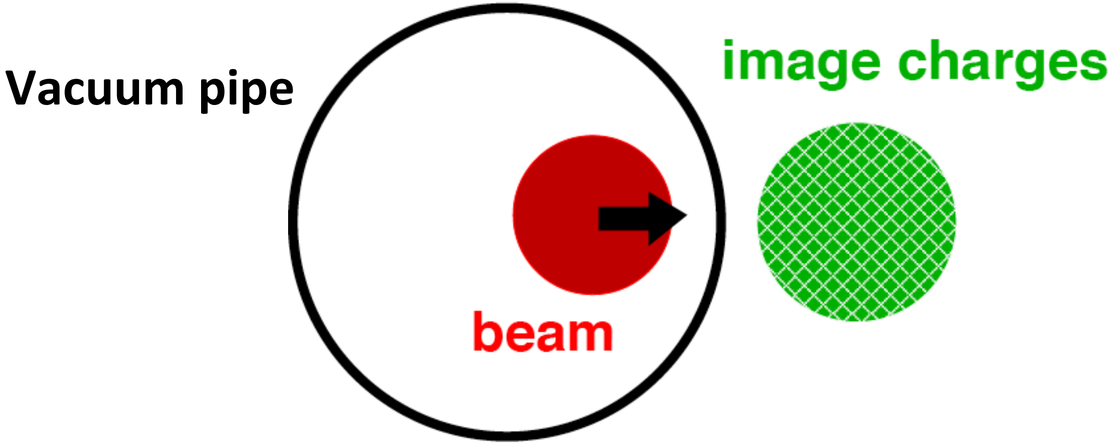
Resonance crossing due to space charge: very different from the usual (set tunes) and still an unsolved issue.

Tune diagram with space charge



Space Charge

Image Charges:
often confused as another “space charge”



affects the beam as a whole, shifts Q_{coh} ,
does not shift Q_{inc}

Very different from space charge:
- coherent
- beam-external

Actually, is an impedance

Tune Shifts

Compare the incoherent (space-charge) tune shift and the coherent (due to impedance) tune shift

$$\Delta Q_{\mathrm{sc}} = -\frac{\lambda_0 r_p}{\gamma Q_0}\frac{1}{\gamma\beta}\frac{Q_0 R}{4\varepsilon_{x\mathrm{n}}} \qquad \Delta Q_{\mathrm{coh}} = \frac{\lambda_0 r_p}{\gamma Q_0}\frac{i Z^{\perp}}{Z_0/R}$$

- both depend linearly on the intensity
- decrease at the ramp as $1/\gamma$
- space-charge: additional $1/\gamma\beta$

ε_{xn} : normalized rms emittance

$r_p = q^2/(4\pi\varepsilon_0 mc^2)$

$Z_0 = 1/(\varepsilon_0 c)$

Special impedance: image charges

$$Z_{\mathrm{IC}}^{\perp} = -i\frac{Z_0 R \xi_{\mathrm{geom}}}{\beta^2\gamma^2 h^2}$$

- decreases faster than space-charge: $1/\gamma^2\beta^2$
- related to space-charge: induced fields in the pipe
- should not be confused with space-charge

Beam Transfer Function (BTF)

Beam Transfer Function

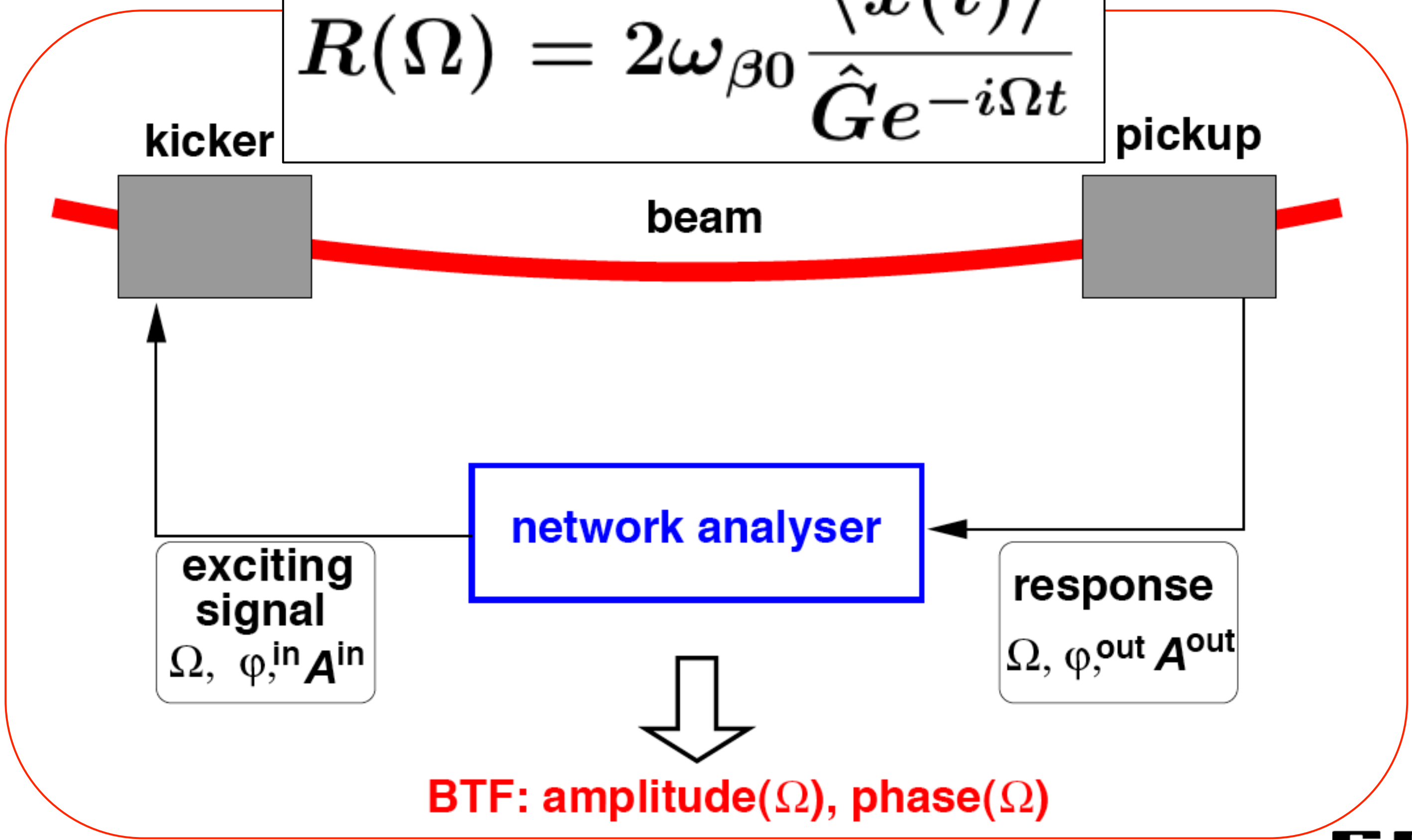
an excitation:

$$x'' + \omega_{\beta i}^2 x = \hat{G} e^{-i\Omega t}$$

beam forced response:

$$\langle x \rangle = A\, e^{-i\Omega t + \Delta\phi}$$

$$R(\Omega) = 2\omega_{\beta 0} \frac{\langle x(t) \rangle}{\hat{G} e^{-i\Omega t}}$$

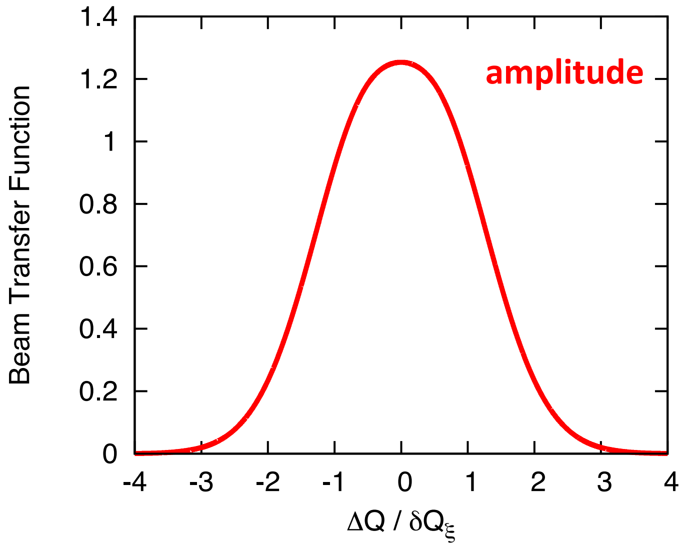


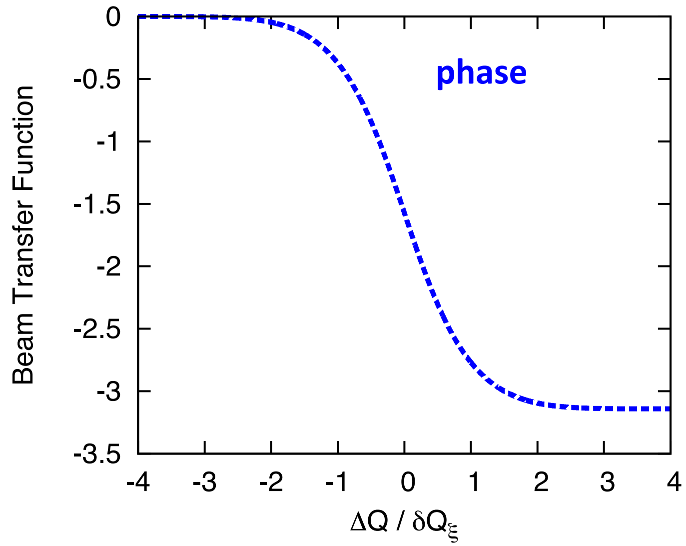
Beam Transfer Function

BTF is:

- Useful diagnostics; the tune, δp, chromaticity, beam distribution
- A fundamental function in the beam dynamics
- Necessary to describe the beam signals and Landau damping

$$R_0(\Omega) = \int \frac{f(\omega)d\omega}{\omega - \Omega}$$





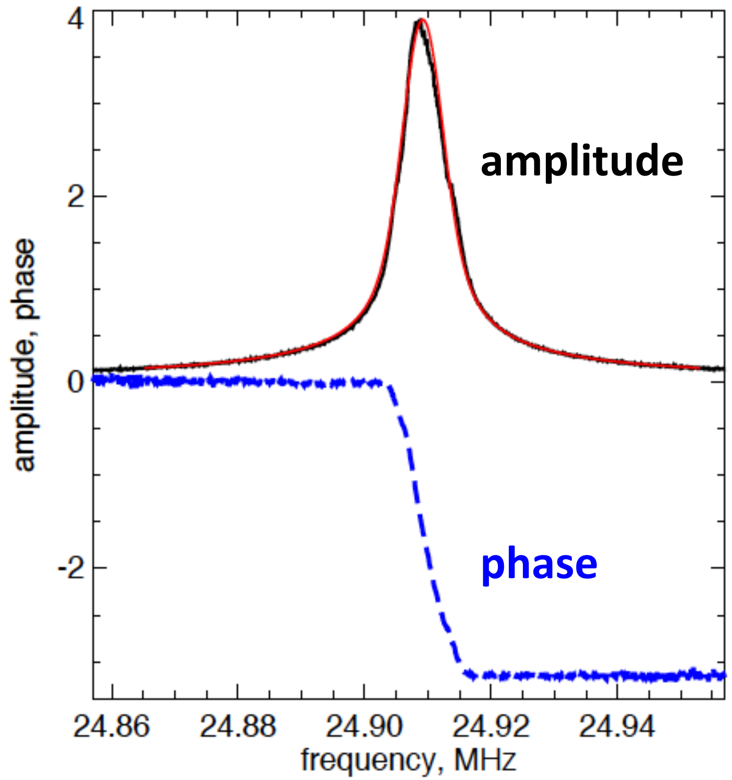
$\Delta Q=(\Omega-(m\pm Q_f)f_0)/f_0$
$\delta Q_\xi=|m\eta\pm(Q_{f\eta}\eta-Q_0\xi)|\delta p/p$

J.Borer, et al, PAC1979
D.Boussard, CAS 1993, CERN 95-06, p.749
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
Handbook of Acc. Physics and Eng. 2013, 7.4.17

Beam Transfer Function

BTF: a standard measurement with a network analyser

- Collective response to the excitation
- Observe the incoherent spectrum
- Still, the beam is stable: Landau Damping!



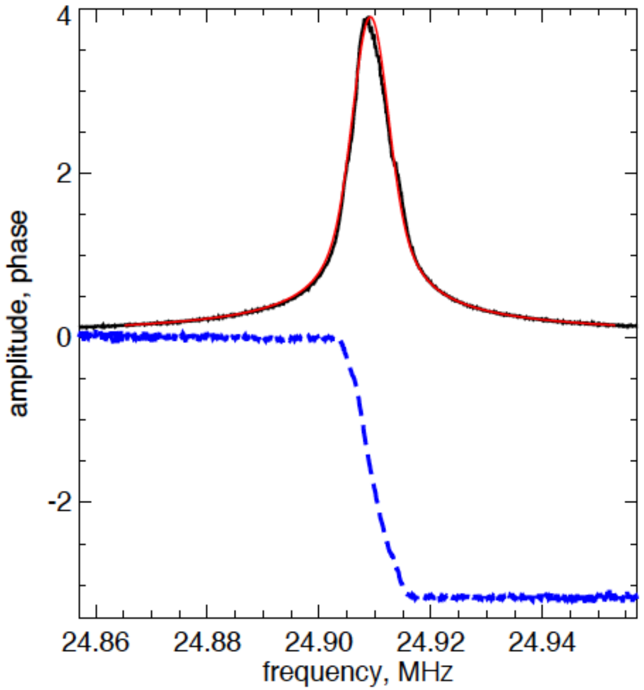
V.Kornilov, et al, GSI-Acc-Note-2006-12-001, GSI Darmstadt (2006)

Beam Transfer Function

BTF provides a direct measure of Landau Damping

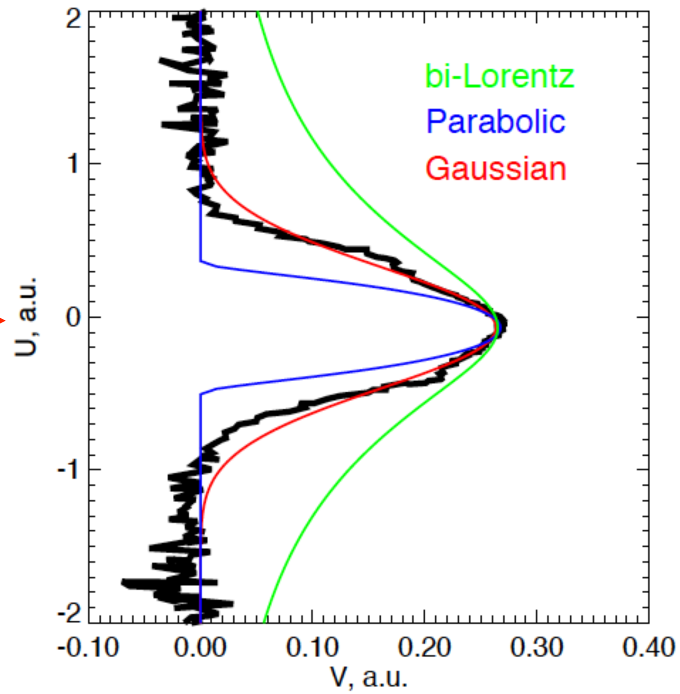
$$\Delta Q_{\mathrm{coh}} R(\Omega) = 1$$

Measured BTF in SIS18



$$\frac{1}{R(\Omega)}$$

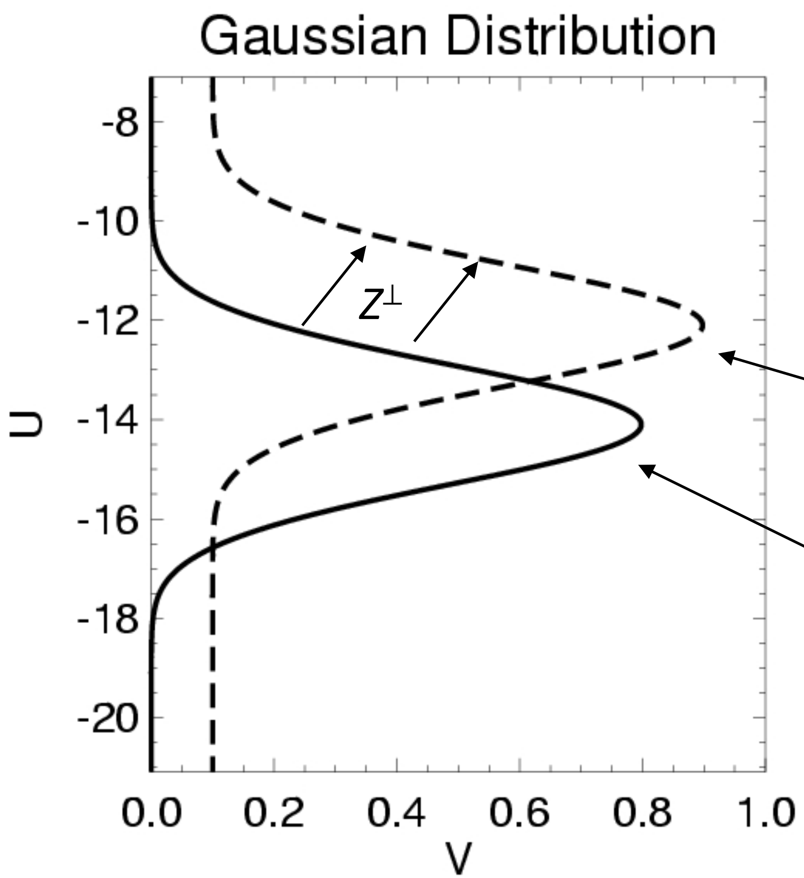
Resulting Stability Diagram



V.Kornilov, et al, GSI-Acc-Note-2006-12-001, GSI Darmstadt (2006)

Beam Transfer Function

BTF provides a direct measure of the impedance & space charge



resulting BTF:

$$\frac{1}{R(\Omega)} = \frac{1}{R_0(\Omega)} - Z^{\perp}(\Omega)$$

BTF with space charge is more tricky, but 1/R(Ω) behaves like a shift due to Im($Z^{\perp}$)

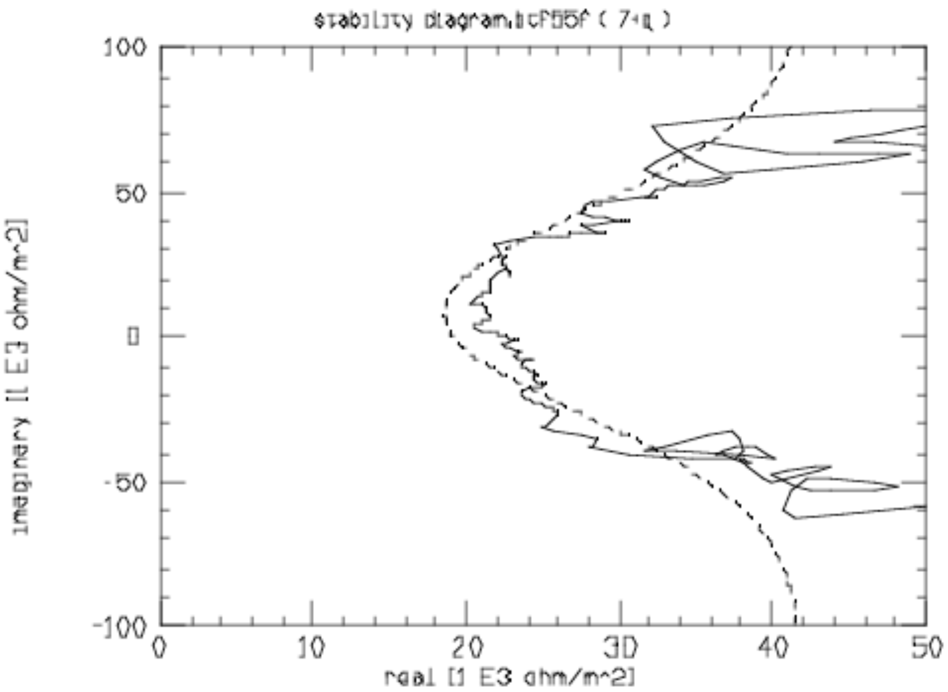
BTF^{-1} with $Z^{\perp}$ $\frac{1}{R(\Omega)}$

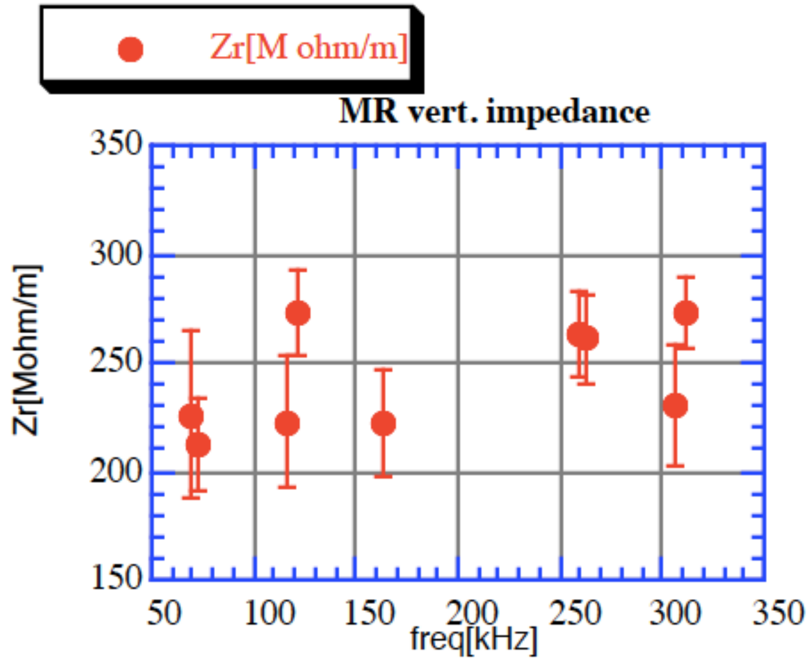
"intrinsic" BTF^{-1} $\frac{1}{R_0(\Omega)}$

J.Borer, et al, PAC1979
D.Boussard, CAS 1993, CERN 95-06, p.749
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
Handbook of Acc. Physics and Eng. 2013, 7.4.17

Beam Transfer Function

Measurements of the transverse BTF in the Fermilab Main Ring



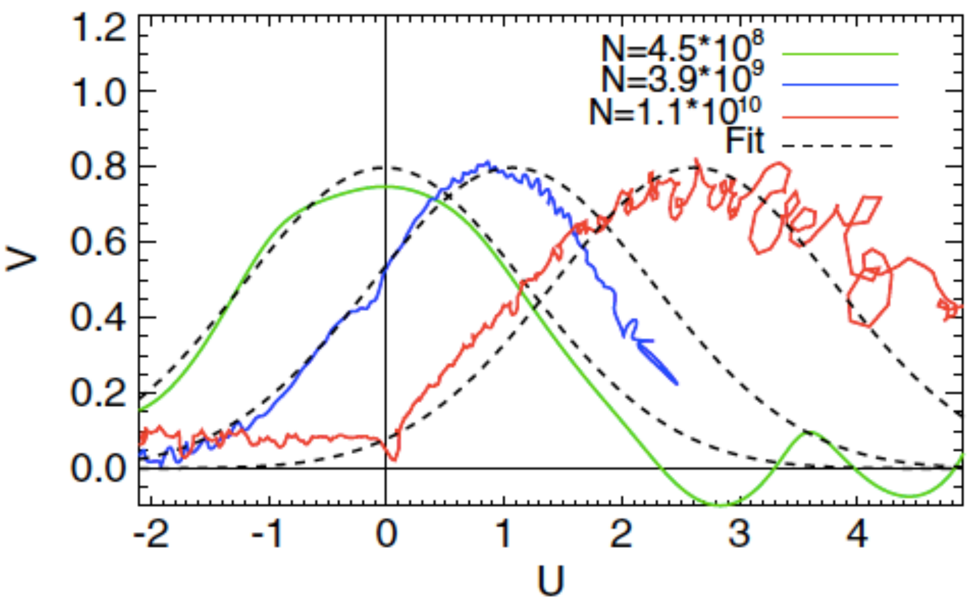


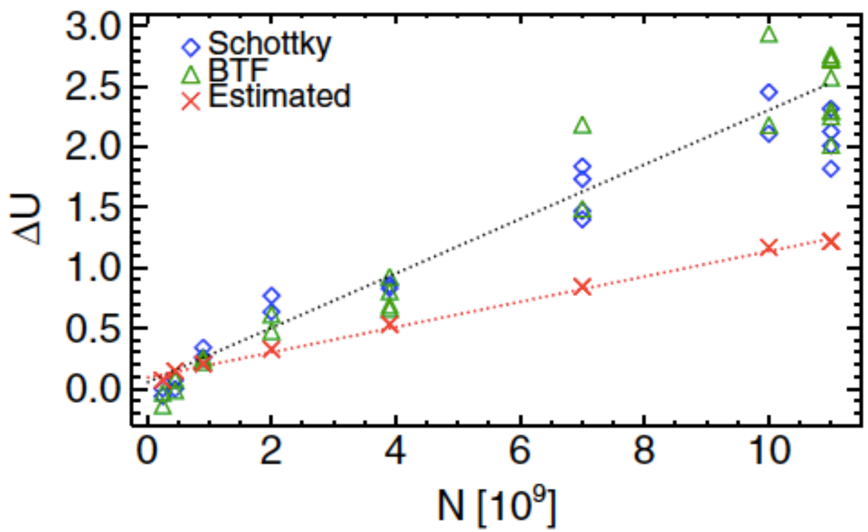
Stability diagram (1/BTF) is shifted by a real impedance

P.Chou, G.Jackson, Proc. PAC1995, p. 3088

Beam Transfer Function

Measurements of the transverse BTF
in the SIS18, GSI





Stability diagram (1/BTF) is shifted by space charge

S.Paret, et al, PRSTAB 13, 022802 (2010)

Part 2 tomorrow:

- Shottky signals with collective effects
- Head-Tail modes
- Decoherence
- Instabilities