Collective Effects & its Diagnostics

part 1

Vladimir Kornilov
GSI Darmstadt, Germany
Introduction

Collective Effects:
- forces due to many-particle system
- “swarm” motion of a many-particle system

Single-particle (incoherent) oscillations

Collective (coherent) oscillations

observations
Oscillations in Beams

Incoherent oscillations

Coherent oscillations

Incoherent tune-shifts
$\Delta Q = Q - Q_0$

Coherent tune-shifts
$\Delta Q = Q - Q_0$

Large & important differences

Δy

Δz

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Oscillations in Beams

Incoherent oscillations

No coherent oscillations \( \langle y \rangle = 0 \)

coasting beam: \( L = C, \lambda(z) = \text{const} \), no synchrotron motion, \( \delta p = \text{const} \)

bunched beam: \( L_{\text{bunch}}, \lambda(z) \) profile, synchrotron oscillations \( Q_s : \delta p - z \)

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Oscillations, Forces

incoherent  coherent

\[ \Omega_{\text{inc}} \quad \text{oscillations} \quad \Omega_{\text{coh}} \]

interactions

\[ \overline{F}_{\text{inc}} = 0 \quad F_{\perp} = F_{\text{inc}} + F_{\text{coh}} \quad \overline{F}_{\perp} = F_{\text{coh}} \]
Collective oscillations are waves
Introduction

Water wave

Sound wave

Traveling oscillation in a medium. Very different from the medium particle motion.
Oscillations: waves

Waves can be unstable or damped

The wave frequency is complex:

\[ \omega = \omega_r + i\omega_i \]

The wave physical parameter:

\[ A(t) = A_0 \cos(\omega_r t) e^{\omega_i t} \]
We observe, and we are interested, only in special collective oscillations: EIGENMODES
Eigenmodes: intrinsic orthogonal oscillations of the dynamical system, with the fixed frequencies (eigenfrequencies).

\[ A\vec{x} = \lambda\vec{x} \]

We often talk about the shift:

\[ \Delta\Omega = \Omega - \Omega_{\text{eigenfrequency}} \]

Eigenmodes of a tuning fork. Pure tone at eigenfrequencies.
Eigenmodes

Example: Transverse eigenmodes in a coasting beam

\[ x(s, t) = x_0 e^{i n s / R - i \Omega t} \]

\( n \) is the mode index.
Wave length: \( C/n \)
Frequencies:
slow wave \( \Omega_s = (n - Q_{\beta}) \omega_0 \)
fast wave \( \Omega_f = (n + Q_{\beta}) \omega_0 \)

Angular rotation (\( \Omega_{ang} \)):
\[ \Omega_{ang} = \left( 1 - \frac{Q_{\beta}}{n} \right) \omega_0 \]
Eigenmodes in a coasting beam

Experimental observations of the coasting-beam waves

Space Structure:
\( n=4 \), as expected for \( Q=3.25 \), with correct \( Q_s \) and \( Q_{\text{ang}} \)

SIS18 synchrotron at GSI Darmstadt

V. Kornilov, O. Boine-Frankenheim,  
GSI-Acc-Note-2009-008, GSI Darmstadt (2009)
Eigenmodes in a coasting beam

Experimental observations of the coasting-beam waves

SIS18 synchrotron at GSI Darmstadt

Frequency Structure:
f = 159.9 kHz, as expected
(1 - 0.25)f₀ for Q = 3.25

V. Kornilov, O. Boine-Frankenheim,
GSI-Acc-Note-2009-008, GSI Darmstadt (2009)
Eigenmodes in bunched beams

Transverse collective oscillations in bunched beams: Head-Tail Modes

Eigenmode:

Eigenfrequencies:

BPM $\Delta$–signal along the bunch, overlapped over several turns: Wiggles and Nodes
Eigenmodes in bunched beams

Head-Tail Modes are measured in many machines, the first observation in CERN PSB (1974):

$k=0$

$k=1$

$k=2$
Particle, Beam Oscillations

Different types of coherent oscillations

Transverse, Longitudinal

Dipolar (m=1)  Quadrupolar (m=2)  Sextupolar (m=3)

Here we consider mostly the dipole transverse oscillations. For the others: the physics and the formalism are similar.
Impedances
Wakes & Impedances

- Leading charge
- Trailing charges
- Leading charge generates electromagnetic fields
- Leading charge is loosing energy
- Trailing charge is gaining/loosing energy

Electric field pattern for a resistive wall pipe
A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993

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Wakes & Impedances

Dipolar wakes: 
(F_x2 \sim \Delta x_1)
(driving)
the same for the whole trailing slice: coherent

Quadrupolar wakes: 
(F_x2 \sim \Delta x_2)
(detuning)
different for individual particles: incoherent

experiences the wake field

\[ \Delta x_1 \]

\[ \Delta x_2 \]

trailing leading

The facility impedances have coherent and incoherent effects

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In order to describe the effect of the wake fields, we define the “wake function”.

The energy loss/gain of the trailing charge.

The longitudinal Wake Function:

\[ \int F_{\|} \, ds = \Delta \mathcal{E}_2 = -q_1 q_2 W_{\|}(z) \]

\[ F_{\|}(s, z) = q_2 E_z(s, z) \]

The “lumped” (localized) impedance:
one interaction per turn.
Field integral is over the structure elements.
In order to describe the effect of the wake fields, we define the “wake function”.

The energy loss/gain of the trailing charge.

The transverse Wake Function:

\[ \int F_x ds = \mathcal{E}_0 \Delta x'_2 = -q_1 q_2 W_x(z) x_1 \]

\[ F_x(s, z) = q_2 E_x(s, z) + q_2 \left\{ v \times B \right\}_x(s, z) \]

The dipole impedance: the offset of the leading particle produces the wake, which does not depend on the trailing particle offset.
Wakes & Impedances

In the frequency domain:
**IMPEDANCE**
a complex function of the frequency

\[
Z_{\parallel}(\omega) = \int e^{-i\omega z/v} W_{\parallel}(z) \frac{dz}{v}
\]

\[
Z_{\perp}(\omega) = i \int e^{-i\omega z/v} W_{\perp}(z) \frac{dz}{v}
\]

Effect of an impedance on a collective eigenmode:

\[
\Omega = \Omega_{\text{Re}} + i\Omega_{\text{Im}} = \Omega_{\text{mode}} + \kappa Z_{\text{Im}}^{\text{eff}} + i\kappa Z_{\text{Re}}^{\text{eff}}
\]

beam oscillation  frequency shift  growth rate

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Impedances

An accelerator facility can have a complicated impedance spectrum

Example: real part of the transverse impedance in SIS100 of FAIR, Darmstadt
Impedances:

- an intensity effect (larger $q \rightarrow$ stronger fields)
- can be coherent and incoherent
- affect the coherent and the incoherent frequencies (both can be observed)
- a beam-external effect
Tune Shifts
Measurements

Different diagnostics for coherent and for incoherent oscillations

BPM Signal = 0 ; Schottky Monitor signal ≠ 0

BPM Signal ≠ 0

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Seems to be easy: by measuring $\Delta Q_{\text{coh}}$ the impedance is determined.

But then, how to understand this:

Single bunch tune measurements at the CERN SPS, J. Gareyte, EPAC2002

What has been measured? The horizontal impedance was surely non-zero.
**Tune Shifts**

Laslett coefficients for coasting beams:

\[
\Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\epsilon_1}{h^2}
\]

\[
\Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \epsilon_1}{h^2} + \frac{\xi_1}{\gamma^2 h^2} \right]
\]

- \(\xi_1\): symmetries, coherent
- \(\epsilon_1\): unsymmetries, incoherent
- \(1/\gamma^2\): \(E-B\) cancellation

Elliptical pipe, \(h=b_y\) is the half-height.
Perfectly conducting pipe.
Different terms for:
- Low frequencies (ac magnetic field)
- Magnet poles
- Partial neutralization

\[
\zeta = \frac{2r_p R^2}{\beta^2 \gamma Q_0}
\]

**Handbook of Acc. Physics and Eng. 2013, 2.4.5**
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62
Tune Shifts

Laslett coefficients for coasting beams:

\[
\Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\varepsilon_1}{\hbar^2}
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\]

For bunched beams \((B=I_{av}/I_{peak})\):

\[
\Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\varepsilon_1}{\hbar^2} \left( \beta^2 + \frac{1}{B \gamma^2} \right)
\]

\[
\Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{\hbar^2} + \frac{\xi_1}{B \gamma^2 \hbar^2} \right]
\]

The bunching factor, the cancellation, \(\beta^2\) appear in a non-straightforward way.

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62
Intensity Effects & Tune Shifts

From the first-order expansion of the forces for small perturbations, a symbolic relation:

\[ \Delta Q_{\text{coh}} = \Delta Q_{Z} + \Delta Q_{\text{inc}} \]

- **Coherent tune shift:** everything affecting \( \langle x \rangle \)
- **Tune shifts due to impedance:** \( \langle x \rangle \) acting on individual particles. Include incoherent effects.
- **Facility- and lattice-related incoherent tune shifts.** No space charge.

This is why there are incoherent effects in the coherent tune shift:

\[ \Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \epsilon_1}{h^2} + \frac{\xi_1}{B \gamma^2 h^2} \right] \]

**References:**
- Handbook of Acc. Physics and Eng. 2013, 2.4.5
- K.Y. Ng, Phys. of Intensity Dep. Beam Instab., 2006
- P. Bryant, CAS1986, CERN 87-10, p.62

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\[ \Delta Q_{\text{coh}} = \Delta Q_{Z-} + \Delta Q_{\text{inc}} \]

\[ \Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \epsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2} \right] \]

Incoherent effect of the unsymmetries reduces \( \Delta Q_Z \) horizontally and enhances \( \Delta Q_Z \) vertically

Single bunch tune measurements at the CERN SPS, J.Gareyte, EPAC2002

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Space Charge
Space Charge

Electric field due to the charged beam particles
Transverse “betatron” oscillations of a single particle, amplitudes $a_x$, $a_y$, frequencies: tunes $Q_x$, $Q_y$.

Due to the electric field of the “space charge”, tunes are decreased by $\Delta Q_x$, $\Delta Q_y$: tune shifts.

- is a collective effect: many particles produce the field
- affects the incoherent oscillations: the field moves with the beam center
- is a beam-internal interaction
Space charge is a beam-internal interaction. The effects of space charge are different from the external ones.

This is space charge

Baron Münchhausen
Space Charge

\[ \Delta Q_{sc} = -g_a \frac{\lambda_0 r_p R}{4 \gamma^3 \beta^2 \varepsilon_x} \]

Space charge tune shift:
1. is always negative
2. proportional to the intensity (\( \lambda \))
3. depends on the transverse distribution (\( g_a \))
4. \( 1/\gamma^2 \): \( E-B \) cancellation
5. \( 1/\sigma^2 \): transverse beam size (emittance \( \varepsilon \))
6. different for every particle (tune spread)

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Example for tune spreads: SIS100 (FAIR, Darmstadt)

Resonances in the transverse oscillations:
$kQ_x + mQ_y = n$

2\textsuperscript{nd} order, quadrupole
3\textsuperscript{rd} order, sextupole
4\textsuperscript{th} order, octupole

Black dot: set tunes
$Q_x = 18.84$, $Q_y = 18.73$

Green area: tune spread due to the chromaticity $\xi$

$$\Delta Q = Q\xi \frac{\Delta p}{p}$$

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Space Charge

Example: Tune spread due to space-charge in SIS100 (FAIR, Darmstadt)

$f_{\text{longitudinal}} \approx 1.5 \text{ kHz} \\
(T \approx 600 \text{ us})$

$f_{\text{transverse}} \approx 5 \text{ MHz} \\
(T \approx 0.2 \text{ us})$

Resonance crossing due to space charge: very different from the usual (set tunes) and still an unsolved issue.
Space Charge

Image Charges: often confused as another “space charge”

Vacuum pipe

beam

affects the beam as a whole, shifts $Q_{coh}$, does not shift $Q_{inc}$

Very different from space charge:
• coherent
• beam-external

Actually, is an impedance
Compare the incoherent (space-charge) tune shift and the coherent (due to impedance) tune shift

\[ \Delta Q_{sc} = -\frac{\lambda_0 r_p}{\gamma Q_0} \frac{1}{\gamma \beta} \frac{Q_0 R}{4 \varepsilon_{xn}} \quad \Delta Q_{coh} = \frac{\lambda_0 r_p}{\gamma Q_0} \frac{i Z_1}{Z_0/R} \]

- both depend linearly on the intensity
- decrease at the ramp as \(1/\gamma\)
- space-charge: additional \(1/\gamma \beta\)

\(\varepsilon_{xn}\): normalized rms emittance

- \(r_p = q^2/(4\pi \varepsilon_0 mc^2)\)
- \(Z_0 = 1/(\varepsilon_0 c)\)

Special impedance: image charges

\[ Z_{IC} = -i \frac{Z_0 R \xi_{geom}}{\beta^2 \gamma^2 h^2} \]

- decreases faster than space-charge: \(1/\gamma^2 \beta^2\)
- related to space-charge: induced fields in the pipe
- should not be confused with space-charge
Beam Transfer Function (BTF)
Beam Transfer Function

an excitation:

\[ x'' + \omega_{\beta_i}^2 x = \hat{G} e^{-i\Omega t} \]

beam forced response:

\[ \langle x \rangle = A e^{-i\Omega t + \Delta \phi} \]

\[ R(\Omega) = 2\omega_{\beta_0} \frac{\langle x(t) \rangle}{\hat{G} e^{-i\Omega t}} \]

kicker

beam

network analyser

pickup

exciting signal

\( \Omega, \varphi, A^{in} \)

response

\( \Omega, \varphi, A^{out} \)

BTF: amplitude(\( \Omega \)), phase(\( \Omega \))
Beam Transfer Function

BTF is:

- Useful diagnostics; the tune, $\delta p$, chromaticity, beam distribution
- A fundamental function in the beam dynamics
- Necessary to describe the beam signals and Landau damping

$$R_0(\Omega) = \int \frac{f(\omega)d\omega}{\omega - \Omega}$$

$\Delta Q = (\Omega - (m\pm Q_f)f_0)/f_0$

$\delta Q_\xi = |m\eta\pm(Q_f\eta - Q_0\xi)|\delta p/p$

J. Borer, et al, PAC1979
D. Boussard, CAS 1993, CERN 95-06, p.749
A. Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
Handbook of Acc. Physics and Eng. 2013, 7.4.17
Beam Transfer Function

BTF: a standard measurement with a network analyser

- Collective response to the excitation
- Observe the incoherent spectrum
- Still, the beam is stable: Landau Damping!

Beam Transfer Function

BTF provides a direct measure of Landau Damping

\[ \Delta Q_{\text{coh}} R(\Omega) = 1 \]

Measured BTF in SIS18

Resulting Stability Diagram

Beam Transfer Function

BTF provides a direct measure of the impedance & space charge

\[
\frac{1}{R(\Omega)} = \frac{1}{R_0(\Omega)} - Z_\perp(\Omega)
\]

resulting BTF:

BTF with space charge is more tricky, but 
\(1/R(\Omega)\) behaves like a shift due to \(\text{Im}(Z_\perp)\)

BTF\(^{-1}\) with \(Z_\perp\)

\[
\frac{1}{R(\Omega)}
\]

“intrinsic” BTF\(^{-1}\)

\[
\frac{1}{R_0(\Omega)}
\]

J. Borer, et al, PAC1979
D. Boussard, CAS 1993, CERN 95-06, p.749
A. Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
Handbook of Acc. Physics and Eng. 2013, 7.4.17

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Beam Transfer Function

Measurements of the transverse BTF in the Fermilab Main Ring

Stability diagram \((1/BTF)\) is shifted by a real impedance

Beam Transfer Function

Measurements of the transverse BTF in the SIS18, GSI

Stability diagram (1/BTF) is shifted by space charge

Part 2 tomorrow:

- Shottky signals with collective effects
- Head-Tail modes
- Decoherence
- Instabilities