Introduction to Optics: basics, components, diffraction

The CERN Accelerator School

Beam Instrumentation
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Introduction to Optics: overview

Outline

- ***Motivation:*** why study optics?
- ***Geometric Optics***
  - Basics of refractive systems
  - Designing components, ray tracing
- ***Interference***
  - Principles of Interferometry
  - Michelson, Mach Zehnder & FSI
- ***Diffraction:***
  - Fourier optics, convolution theory.
  - Applications in diagnostics

Disclaimer: focus on optics relevant for Beam Diagnostics

Why study optics?

Light matters…

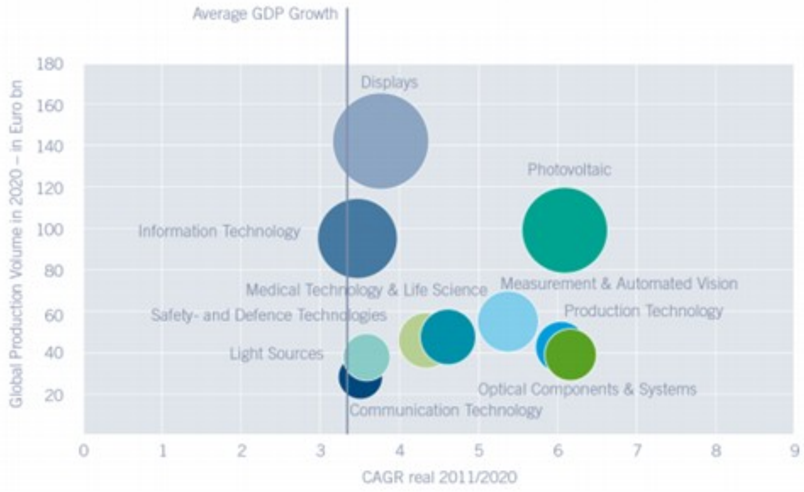
Optics: **the study of the behaviour and properties of light, including the transmission and deflection of radiation**

underpins

photonics: **the science and technology of generating, controlling, and detecting photons**

Photonics *market is € 300 billion: double that by 2020.*

Expected Growth of Global Photonics Segments 2011–2020 compared to GDP Growth



Bubble size indicates worldwide production volume in 2020

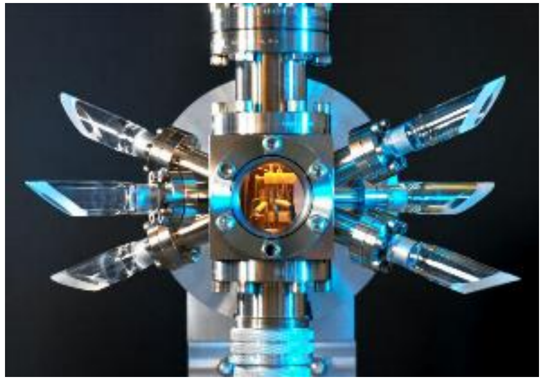
Source: BMBF, SPECTARIS, VDMA, ZVEI (pub.), 'Branchenreport Photonik 2013', Optech Consulting, Study 'Photonik 2013'/Own calculations

- Our modern world relies on light-based technologies:
  - Smart phones, laptops, displays and data storage
  - Fast internet, fibre-optic and satellite telecommunications
  - Medical applications, advanced imaging, metrology
  - Media production and broadcasting, 3D cinema
  - Energy from solar power, lighting technology…

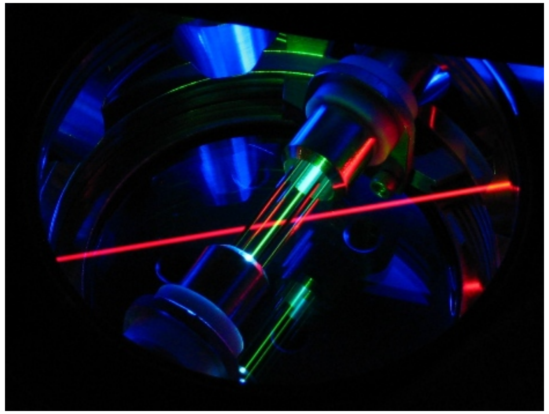
Why study optics?

Centrality to modern physics…

Strontium ion traps for optical frequency standards

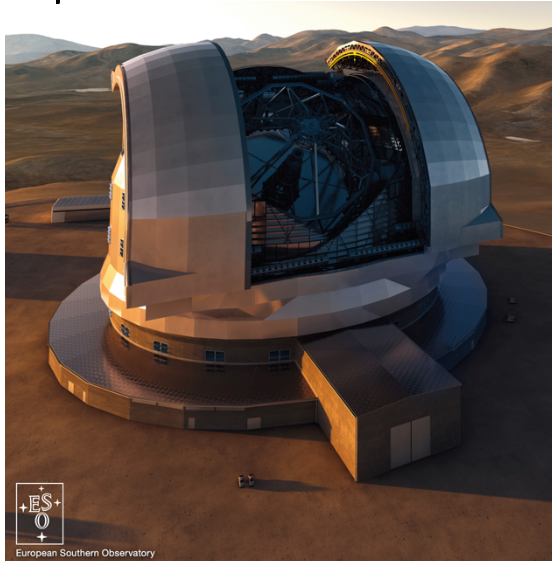


Laser cooling in atomic traps:

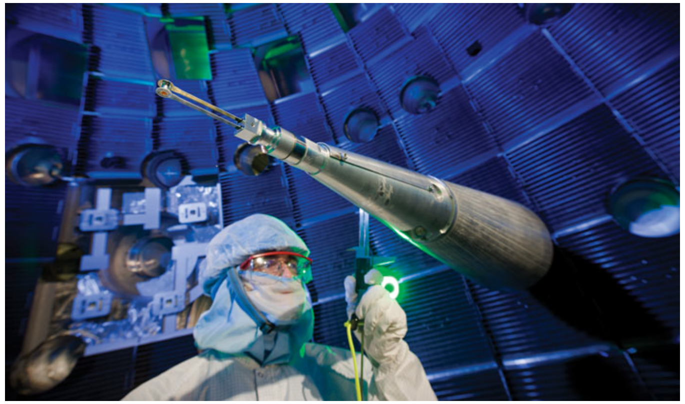


- **Optics is an essential for most research in physics:**
  - Astronomy and cosmology
  - Microscopy and crystallography
  - Spectroscopy and atomic theory
  - Quantum theory
  - Quantum optics, quantum computing
  - Relativity theory
  - Ultra-cold atoms
  - Laser nuclear ignition
  - Particle accelerators present and future
  - Holographic imaging

European Southern Observatory



National Ignition Facility, US



Why study optics?

Beauty of optical phenomena

Why study optics?

… giant lenses are awesome?!

Spotted on a visit to KEK

CAS optics course aims

These 3 lectures aim to equip you with enough knowledge of optics, lasers and practical setups to understand and start to develop your own versatile and precise beam diagnostics.

- ***Lecture 1 [Wed 12h]: Introduction to Optics: basics, components, diffraction***
  - *Fundamental concepts, how light behaves in different circumstances.*
  - *How to calculate, and create good optics design.*
- ***Lecture 2 [Thurs 11h]: Lasers, technologies and setups***
  - *How lasers work, different types, understanding their parameters and cost.*
  - *Including optical fibres for data transmission and readout.*
- ***Lecture 2 [Fri 12h]: Applications of lasers in beam instrumentation***
  - *Examples of some optical and laser based beam diagnostics and what type of precision is achievable.*

…and there was light

- Starting from James Clerk ***Maxwell's equations*** (1865) for electric **E** and magnetic **B** fields, in the absence of charge (ρ=0) and currents (J=0):

Gauss's law for electricty: $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

Faraday's law of induction: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Ampère's law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}.$

- *Take the curl and use vector identity* $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

to show:

$$\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

…and there was light

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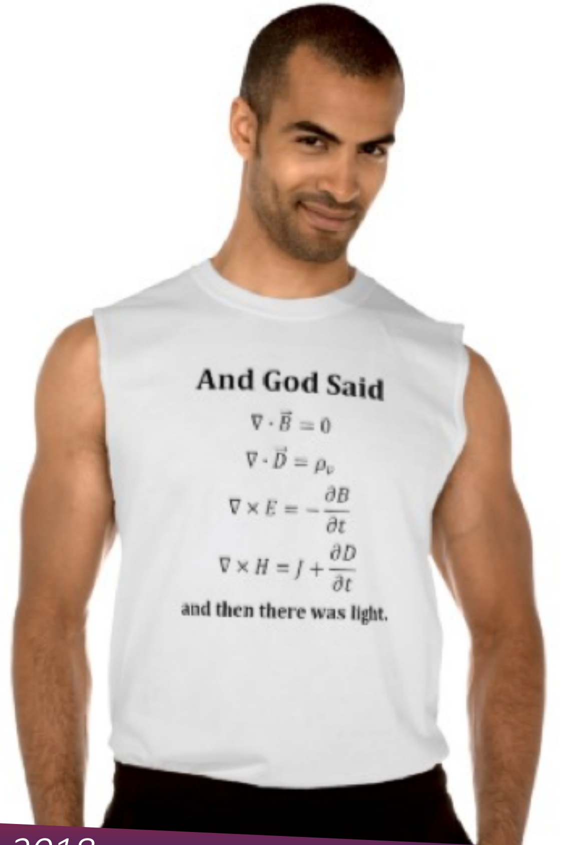
to show:

$$\nabla^2\mathbf{E} = \epsilon_0\mu_0\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad \nabla^2\mathbf{B} = \epsilon_0\mu_0\frac{\partial^2\mathbf{B}}{\partial t^2} \qquad \nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

- *These are wave equations with velocity:* $v = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3\times10^8\ \mathrm{m\ s^{-1}}$

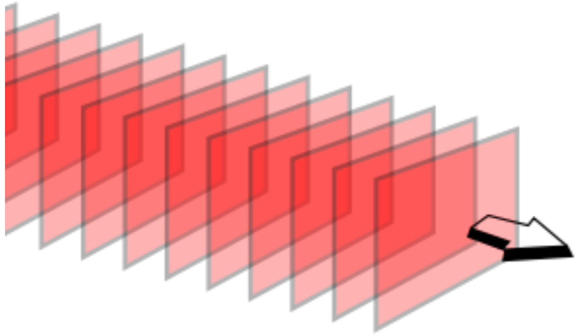
Light is an electromagnetic wave



Basis of Geometric Optics

- One solution of the 3-dimensional wave equation is plane waves

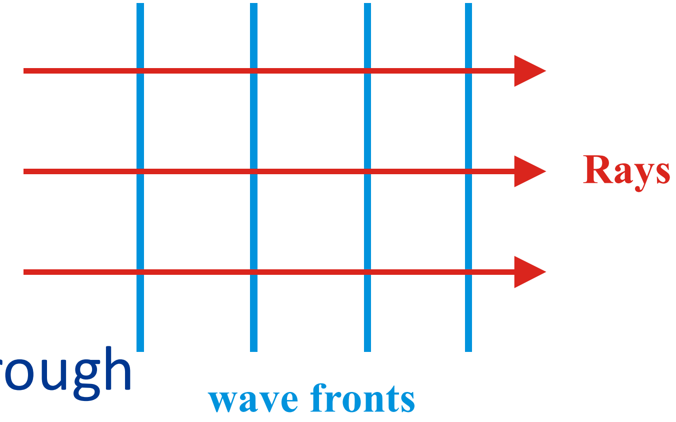
$$U(x, y, z, t) = U_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$



- In optics, typically consider simplified solution to a 1D wave equation:

k is wave number, $2\pi/\lambda$
Z is the direction of travel
λ the wavelength
c, speed of light
$\omega = 2\pi c/\lambda$, the angular frequency
[Note, no phase offset in this solution]

$$U(z, t) = U_0 \cos[\tfrac{2\pi}{\lambda}(z - ct)] = U_0 \cos[(kz - \omega t)]$$



- In an isotropic media, light travels in straight lines, known as rays.
- Geometric optics is a technique for determining the light path through multiple interfaces between media of different refractive indices.

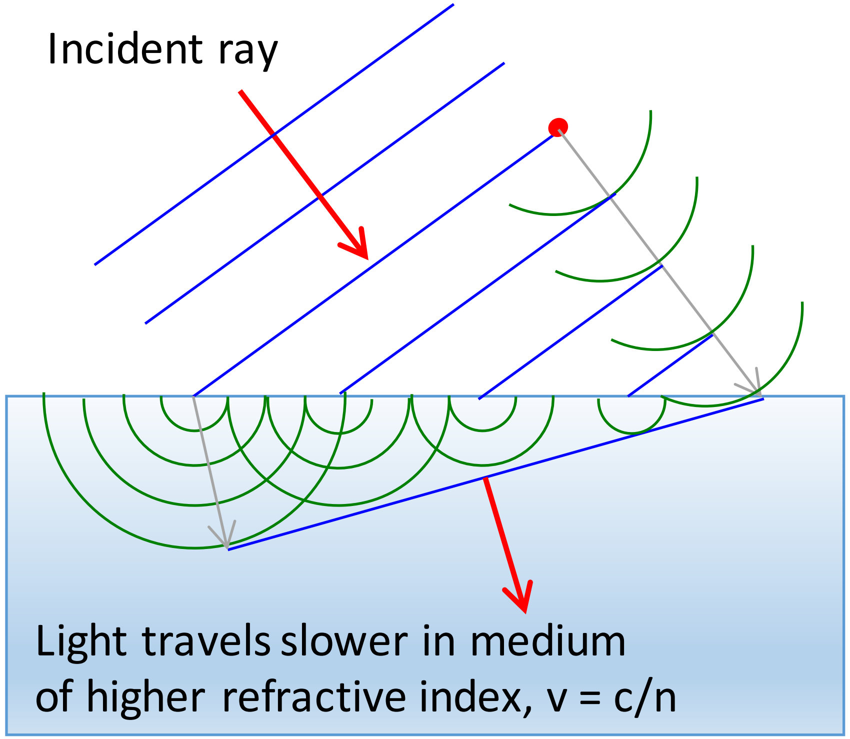
Ray theory and refraction

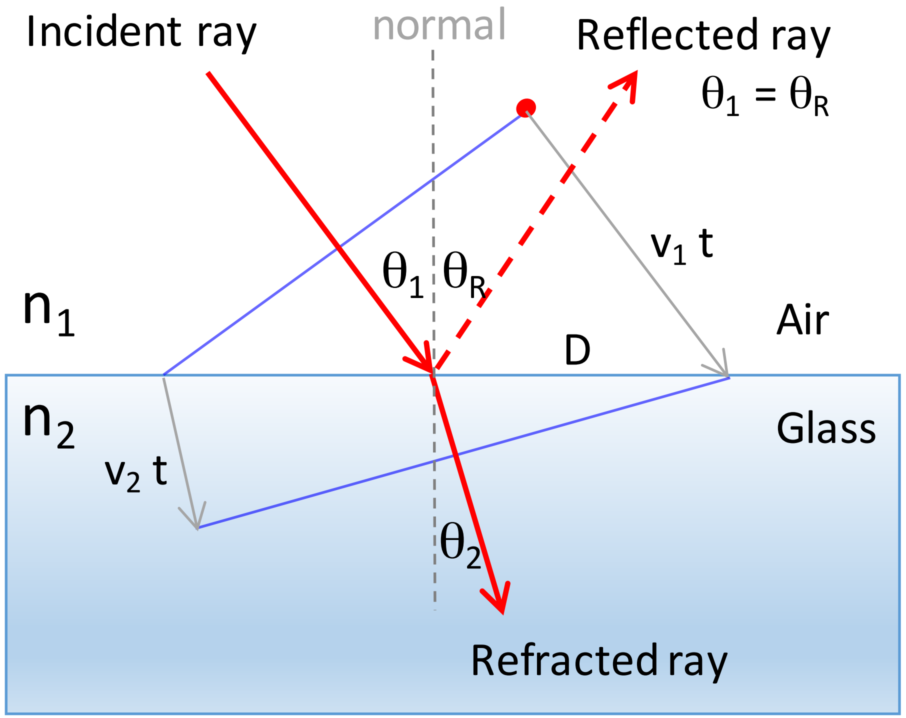
Two basic assumptions:

1. light travels in straight lines, known as **rays**, in each uniform medium.
2. light **reflects** and/or **refracts** at an interface between different media

Valid for isotropic media and apertures much larger than the wavelength of light.

- Huygens' construction can be used to derive Snell's law of refraction at an interface:





$n_1 < n_2$ in this example

Light has different speeds in each medium $v = c/n$.

Distances travelled are $v_1 t$ and $v_2 t$ in same time t.

$$\sin \theta_1 = v_1 t / D$$

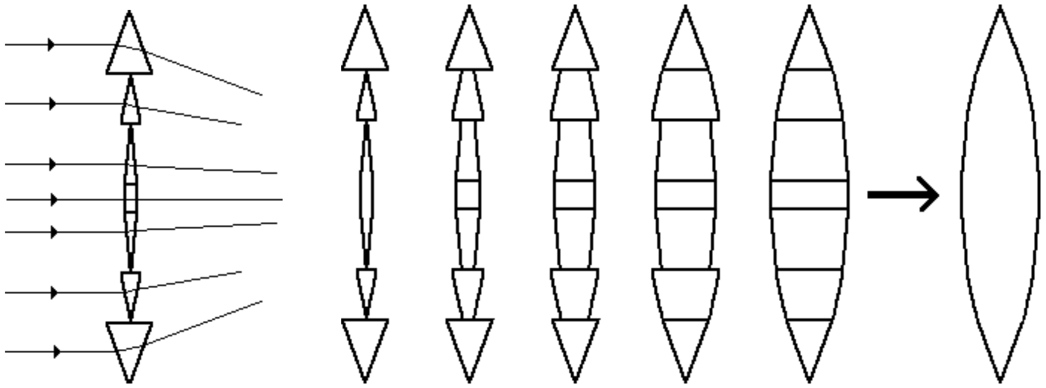
$$\sin \theta_2 = v_2 t / D$$

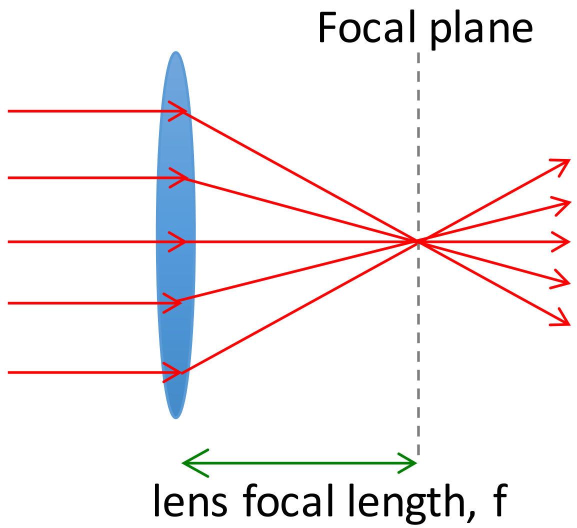
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's Law of refraction

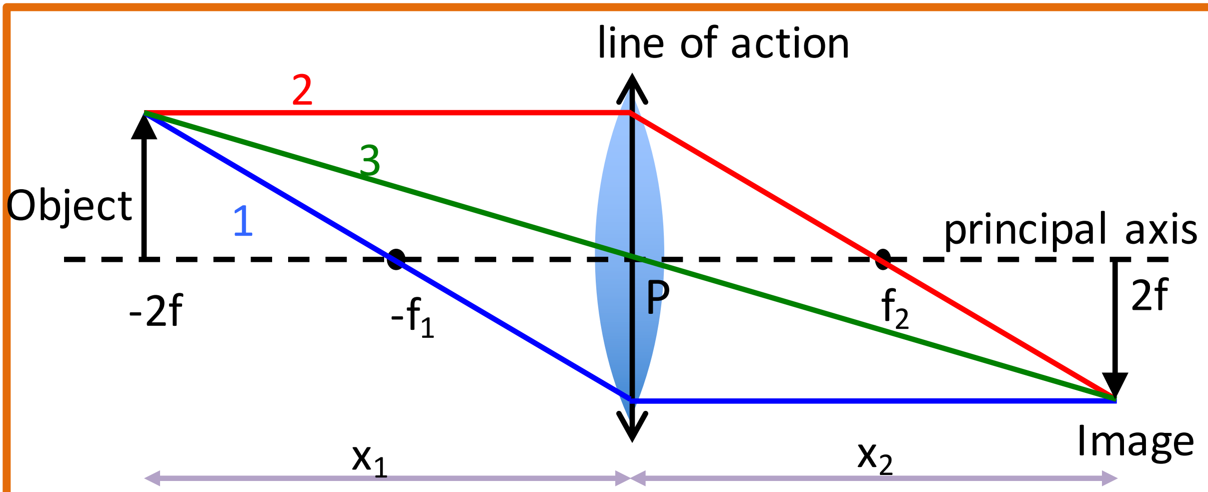
Basic components: lenses

- *A converging lens is basically a stack of prisms, such that paraxial rays converge in the focal plane*





- *The location and magnification of an image can be found by ray tracing:*



Lens equation (note x_1 is negative)

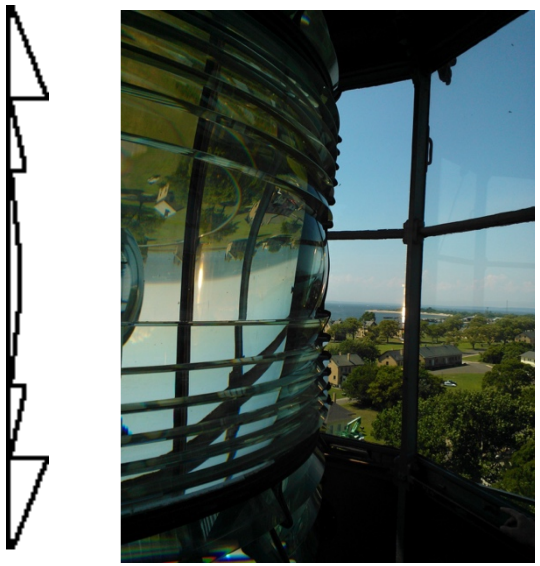
$$\frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{f_2}$$

Lateral magnification

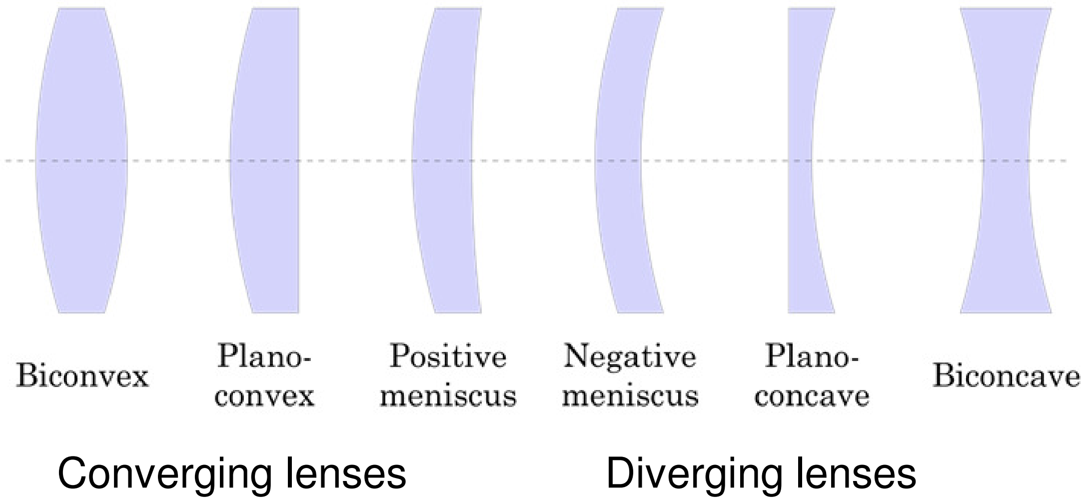
$$m = \frac{h_2}{h_1} = \frac{x_2}{x_1}$$

1. *A ray passing through f_1 before refraction, is parallel to the principal axis after refraction.*
2. *A ray parallel to the principal axis before refraction, travels through F_2 after refraction.*
3. *A ray passing through P is undeviated.*

Fresnel Lens

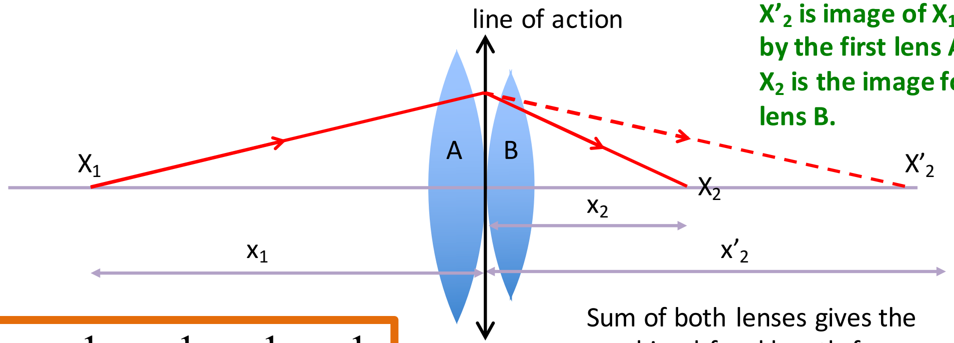


Basic components: Lens types and systems



- *Constructing an optical instrument typically requires multiple lenses.*
- *One can apply the lens equation multiple times, or use the effective focal length of the combination.*
- ***However, there is a better way…***

Two thin lenses in contact:



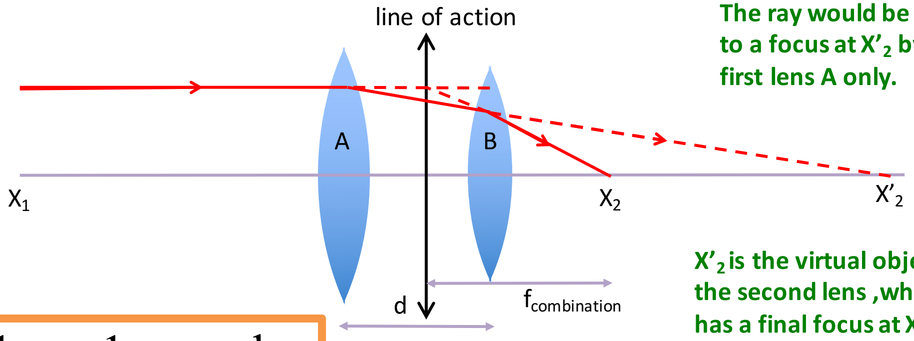
X'_2 is image of X_1 formed by the first lens A only. X_2 is the image formed by lens B.

$$\frac{1}{f_{2A}} + \frac{1}{f_{2B}} = \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{f_C}$$

Sum of both lenses gives the combined focal length f_c:

The combined power is the sum of the individual powers for thin lenses in contact

Two thin lenses separated by distance d :



The ray would be brought to a focus at X'_2 by the first lens A only.

X'_2 is the virtual object for the second lens ,which has a final focus at X_2

$$\frac{1}{f} = \frac{1}{f_{2A}} + \frac{1}{f_{2B}} - \frac{d}{f_{2A} f_{2B}}$$

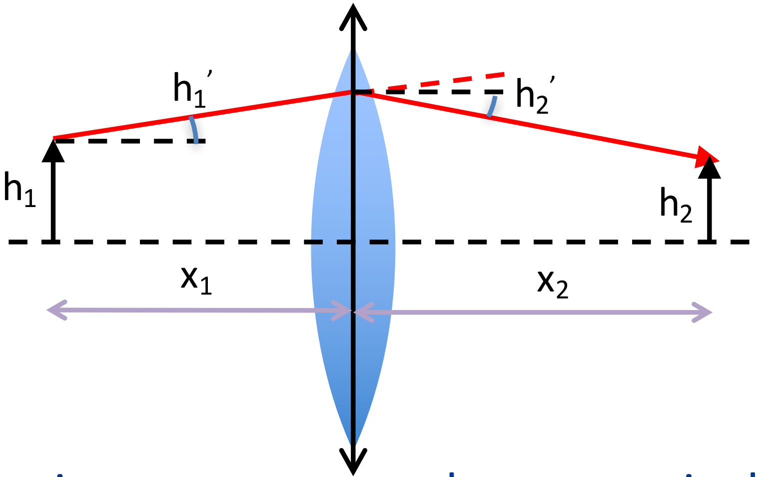
The power of the combination is less than the sum of the powers.

Matrix method of ray tracing

- *A ray is described by the height h_1 from the optical axis and angle h_1'*
- *Optical components described by their transfer matrix:*

Free space drift $$M_D(x_1) = \begin{pmatrix} 1 & x_1 \\ 0 & 1 \end{pmatrix}$$

Action at thin lens $$M_L(F) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -F & 1 \end{pmatrix}$$



Example of drift-lens-drift:

$$\begin{pmatrix} h_2 \\ h_2' \end{pmatrix} = M_D(x_2) M_L(F) M_D(-x_1) = \begin{pmatrix} h_1 \\ h_1' \end{pmatrix}$$

$$M_D(x_2) M_L(F) M_D(-x_1) = \begin{pmatrix} 1 & x_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -F & 1 \end{pmatrix} \begin{pmatrix} 1 & -x_1 \\ 0 & 1 \end{pmatrix}$$

$$M_{TR} = \begin{pmatrix} 1 & x_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -x_1 \\ -F & Fx_1 + 1 \end{pmatrix}$$

$$M_{TR} = \begin{pmatrix} 1 - Fx_2 & \boxed{-x_1 + Fx_1x_2 + x_2} \\ -F & Fx_1 + 1 \end{pmatrix}$$

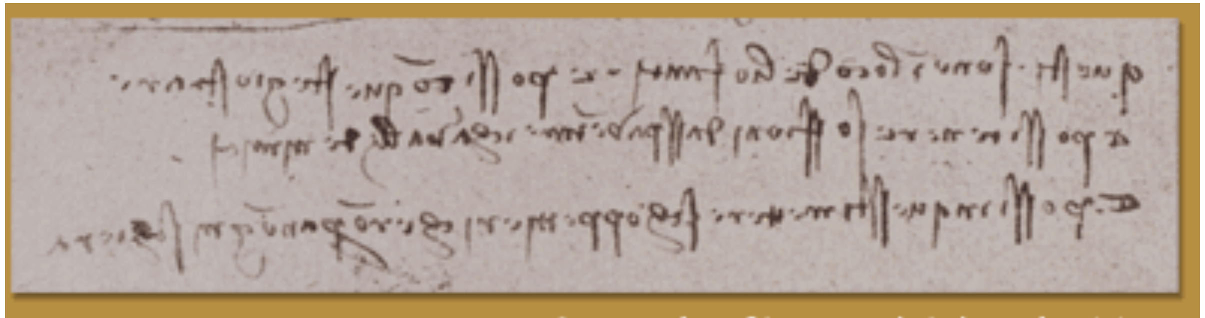
- Similar in concept to accelerator optics lattice, note lenses typically focus in both planes simultaneously (unlike quadrupoles)

Angle independent image formation: $$\boxed{-x_1 + Fx_1x_2 + x_2 = 0} \Rightarrow \frac{1}{x_2} - \frac{1}{x_1} = F = \frac{1}{f}$$

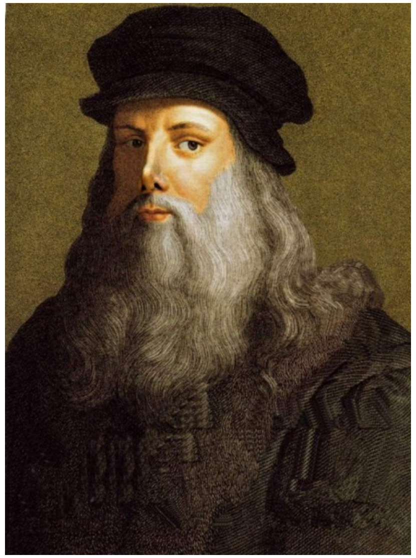
The lens equation!

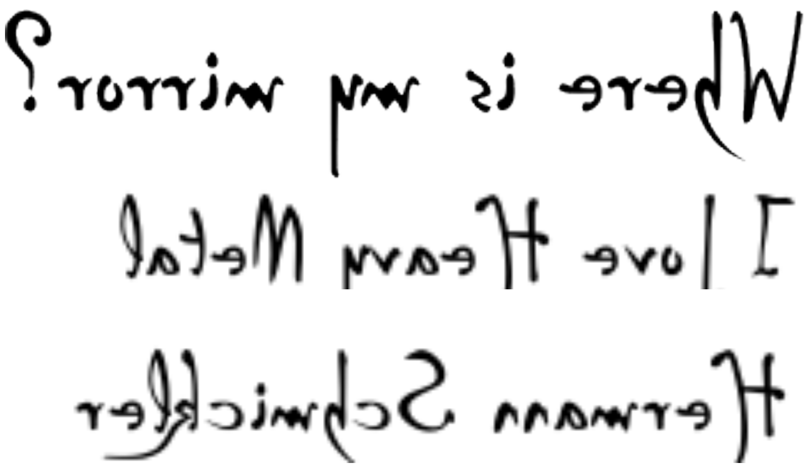
Reflection transformations

Leonardo da Vinci famously used mirror writing to obfuscate his notes (he was also left-handed)



A sample of Leonardo's handwriting

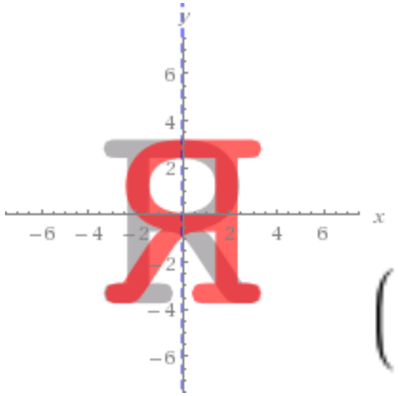




Reflect across x=0

$$(x, y) \longrightarrow (-x, y)$$

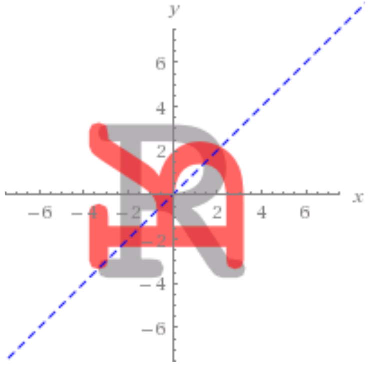
$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Reflect across x=y

$$(x, y) \longrightarrow (y, x)$$

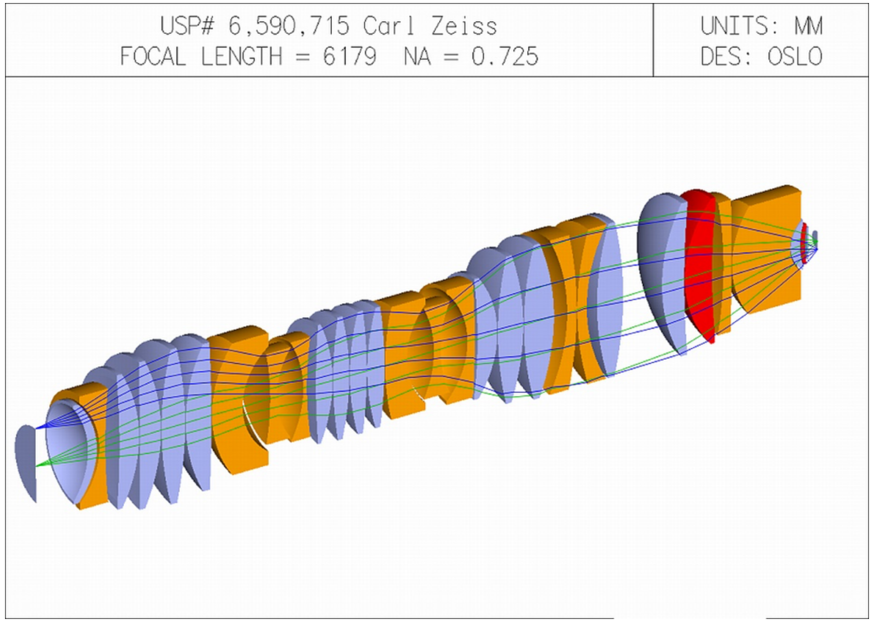
$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Optical design with ray tracing software

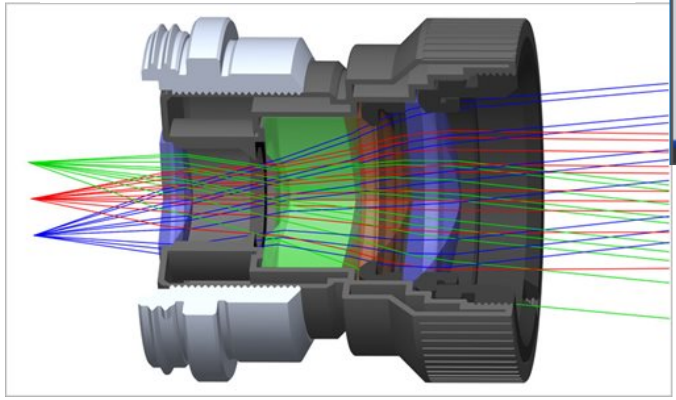
- *Ray tracing divides the real light field into discrete monochromatic rays that are propagated through the system. Can input real light distribution.*
- *Several professional software suites available, e.g.*

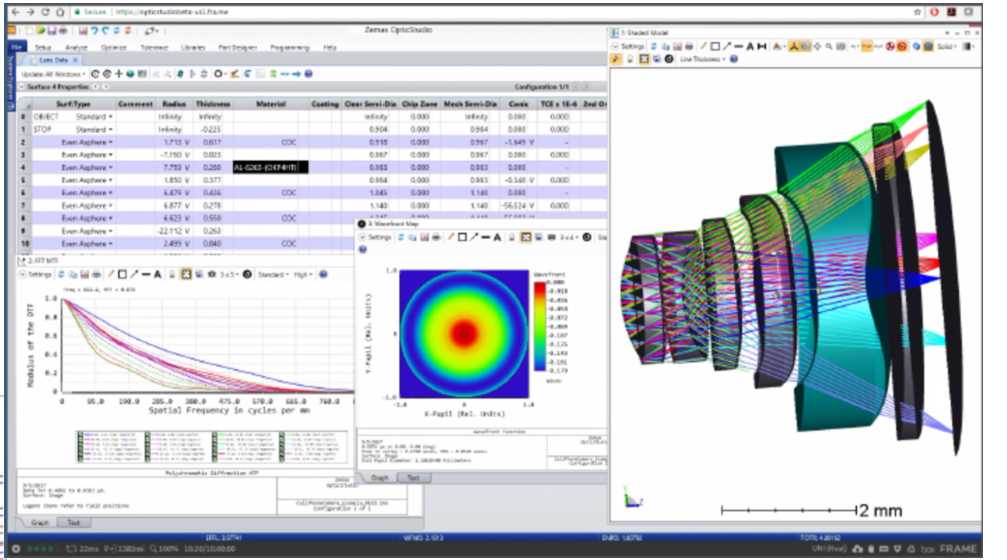
OSLO: Optics Software for Layout and Optimization
https://www.lambdares.com/oslo/

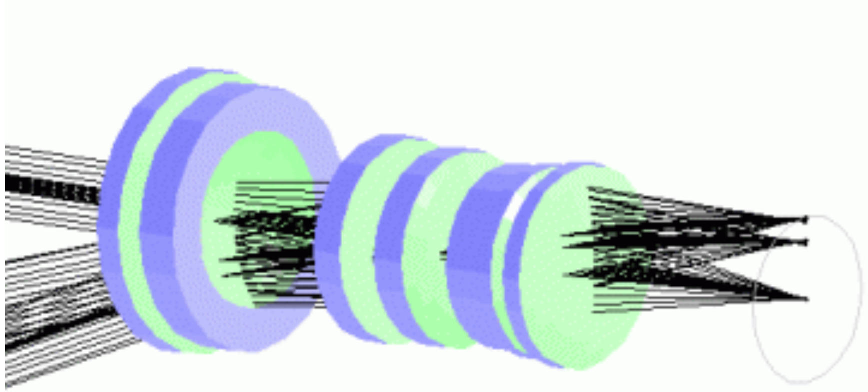


ZEMAX
https://www.zemax.com/

LensMechanix®







WinLens3D - lens design & optimization software
http://www.opticalsoftware.net/index.php/how_to/lens_design_software/winlens3d/

Physical optics: Interference basics

- *The wave properties of light gives rise to interference between multiple paths, where each path has a phase advance.*

$$\delta = \frac{2\pi}{\lambda} d = \frac{2\pi n}{\lambda_0} d$$

- *Consider two sinusoidal disturbances at a point at time t, having travelled different distances, x_1 and x_2:*

$$E_1 = a_1 e^{i(\omega t - kx_1)} = a_1 e^{i(\omega t - \delta_1)}$$
$$E_2 = a_2 e^{i(\omega t - kx_2)} = a_2 e^{i(\omega t - \delta_2)}$$

$$E_1 = a_1 \mathrm{e}^{i\phi_1} \text{ and } E_2 = a_2 \mathrm{e}^{i\phi_2}$$

instantaneous phase, $\phi = \omega t - kx$

- *By the principle of superposition the resulting disturbance is the sum of the complex spatial amplitudes $E = E_1 + E_2$. We measure the intensity, the square of the sum of E-fields:*

$$I = |E_1 + E_2|^2$$

$$I = |E|^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\delta_2 - \delta_1)$$

Note for identical amplitudes $a_1 = a_2$

$$I = 4a^2 \cos^2\left(\frac{\delta_2 - \delta_1}{2}\right)$$

Constructive o.p.d. = $m\lambda$

Destructive o.p.d. = $(m+1/2)\lambda$

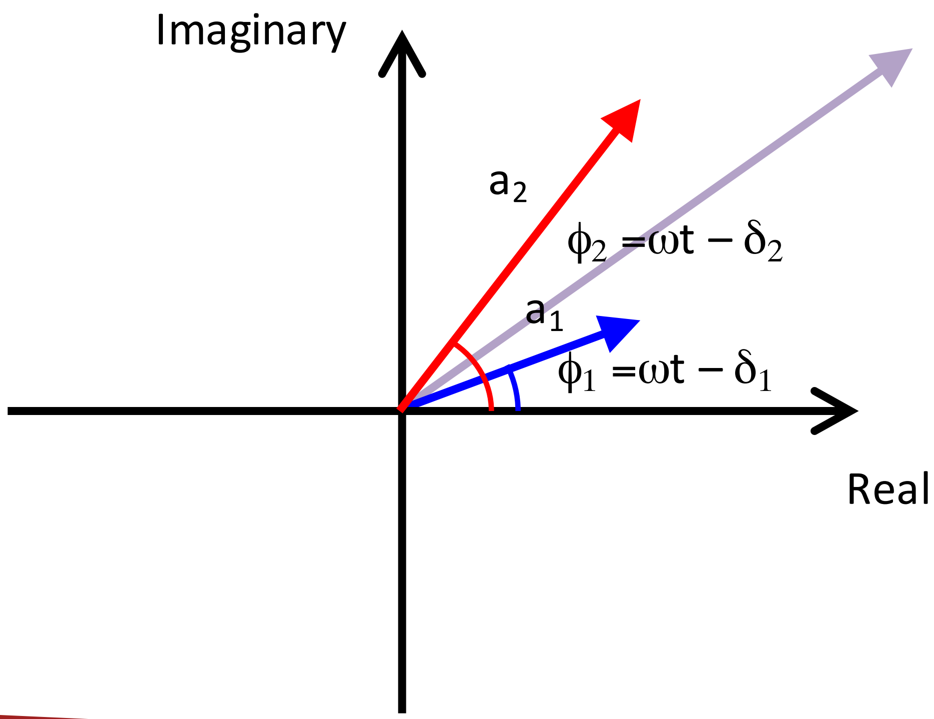
Physical optics: Phasors

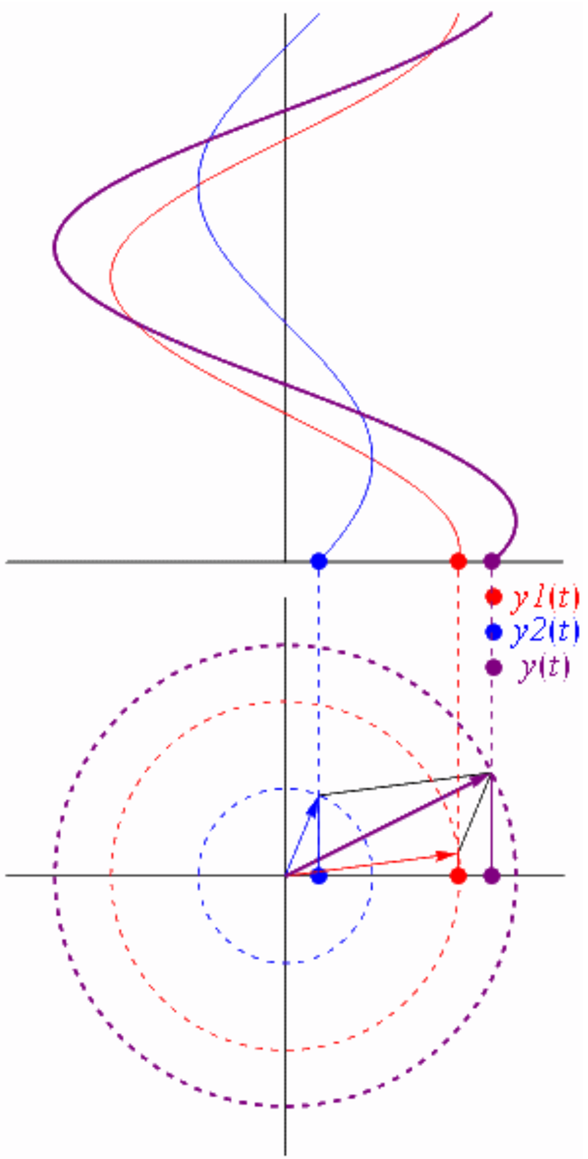
- *Reminder of phasors, visualisation of the superposition principle,*

$$E_1 = a_1 e^{i(\omega t - kx_1)} = a_1 e^{i(\omega t - \delta_1)} = a_1[\cos(\phi_1) + i\sin(\phi_1)]$$

$$E_2 = a_2 e^{i(\omega t - kx_2)} = a_2 e^{i(\omega t - \delta_2)} = a_2[\cos(\phi_2) + i\sin(\phi_2)]$$

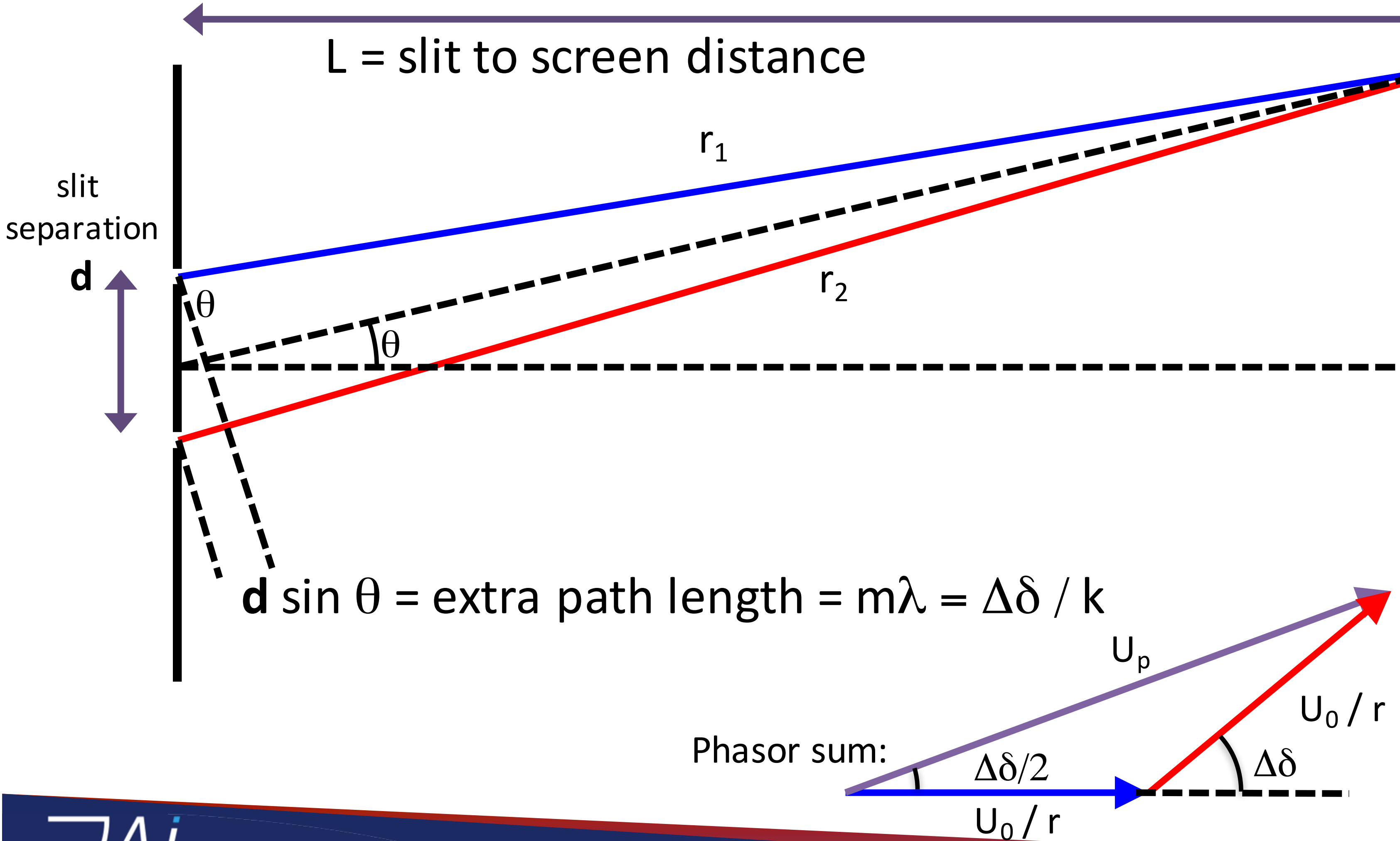
  - $E = E_1 + E_2$

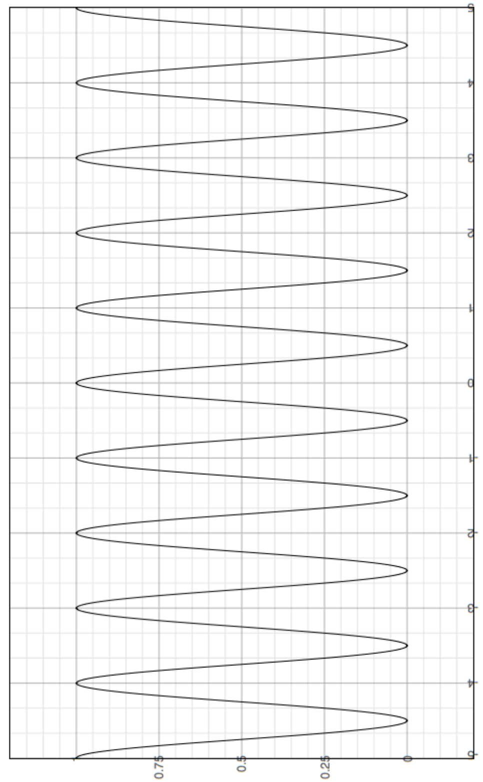




Physical optics: double slit interference

- *For infinitesimal slit size, see interference fringes in far field:*

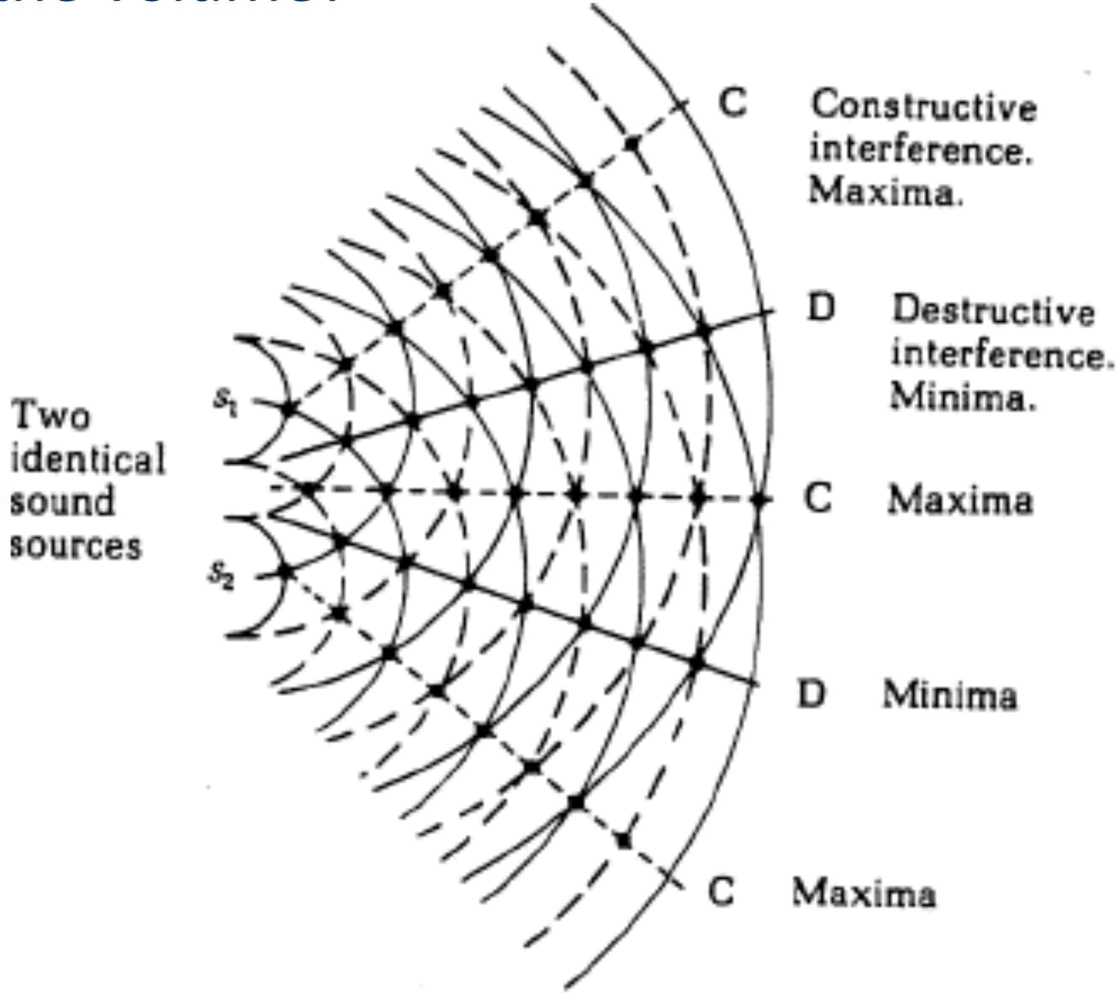




$$I_P = 4\left(\frac{U_0}{r}\right)^2 \cos^2\left(\frac{1}{2}kd\sin\theta\right)$$

Physical optics: 2 source interference

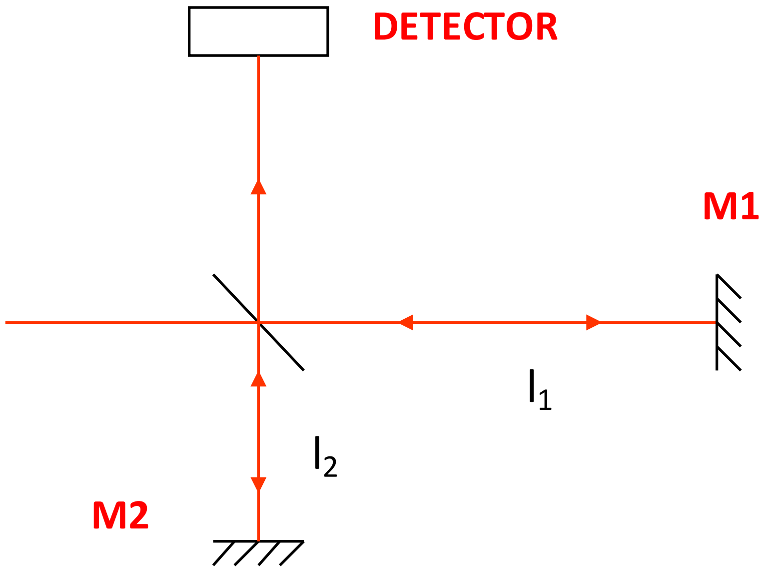
- *Where should Hermann sit to maximize the volume?*

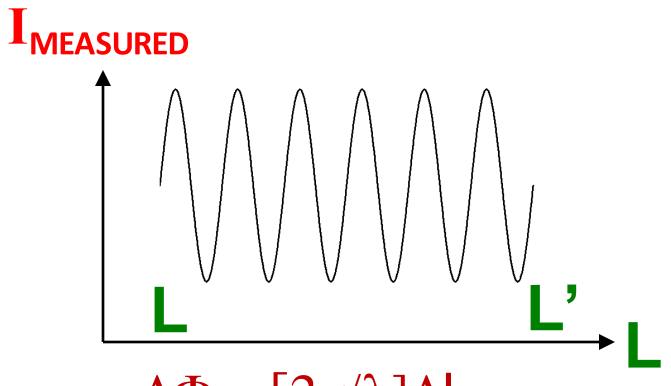


?

Michelson Interferometer

- *Interferometers are used widely for accurate distance measurements:*
  - *If the length of each interferometer arm is fixed we observe some phase Φ at the detector, due to the optical path difference, $L = l_1 - l_2$*
  - *If one mirror is moved some distance x, we observe a phase change at the detector:*





$$\Delta\Phi = [2\pi/\lambda]\Delta L$$

$$\phi_1 = \frac{2\pi}{\lambda} l_1$$

$$\phi_2 = \frac{2\pi}{\lambda} l_2$$

$$\Phi = \frac{2\pi}{\lambda}(l_1 - l_2) = \frac{2\pi}{\lambda} L$$

Essentially we count fringes as the path difference is changed.

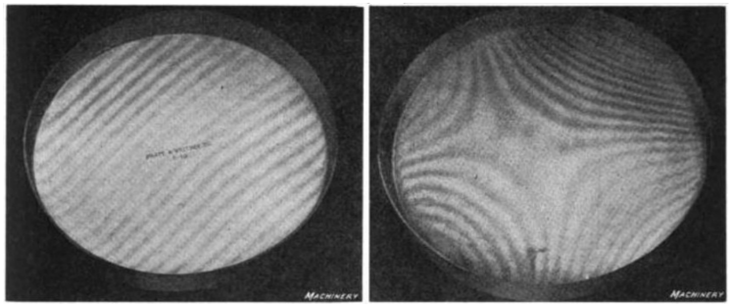
Interference fringe counting:
change in phase proportional to change in optical path length

Φ is the detected phase
L is the optical path difference (n=1)

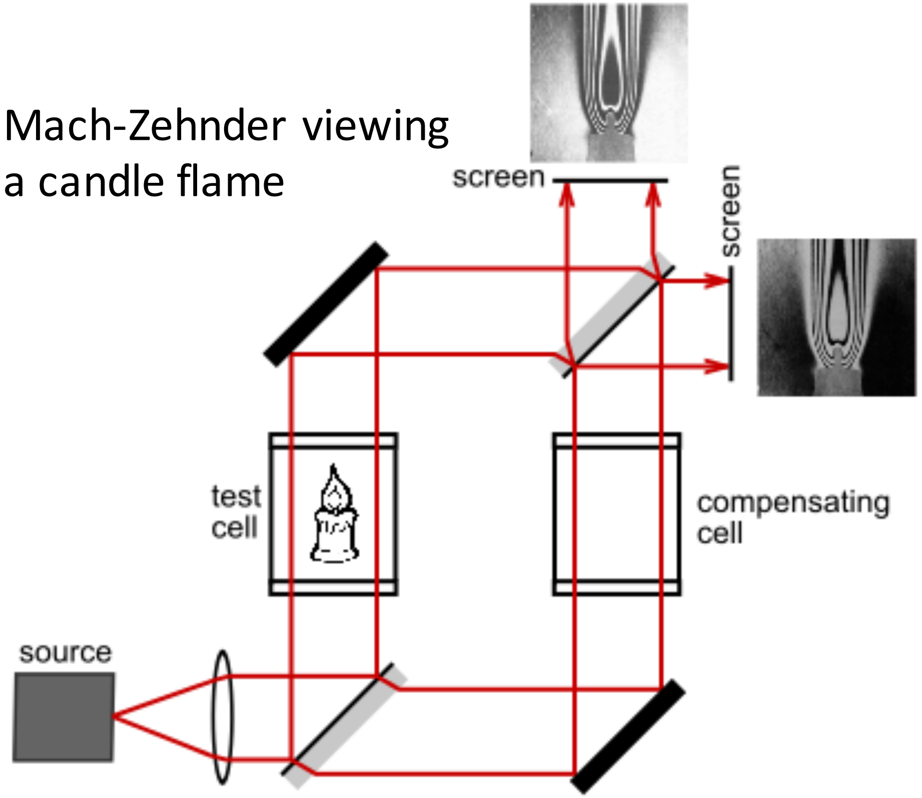
Interferometer Types

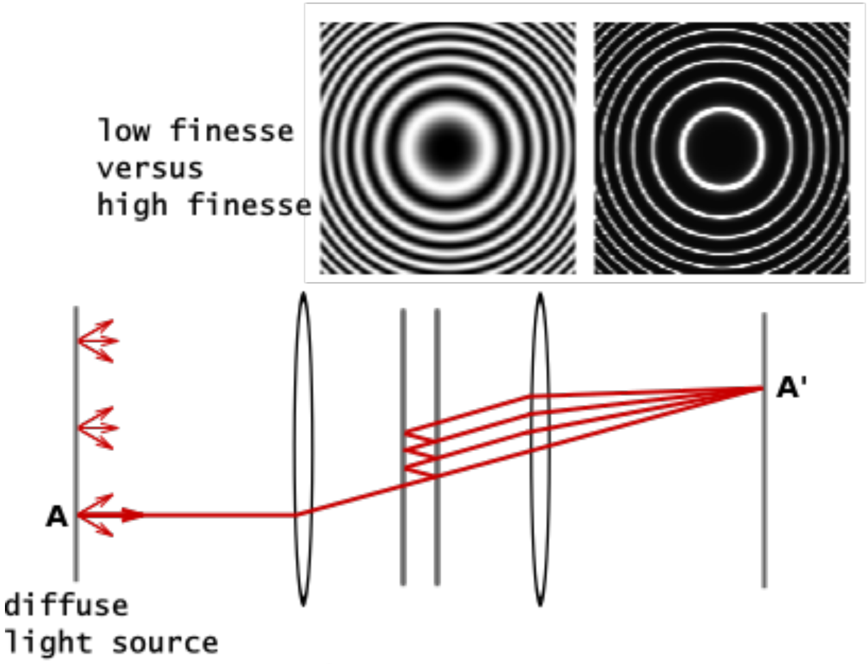
- *The interferometer is an amazingly versatile instrument*
- *Various configurations to create interference by division of amplitude, e.g.:*

Check surface quality of a lens or optical flat can be tested with interference fringes.

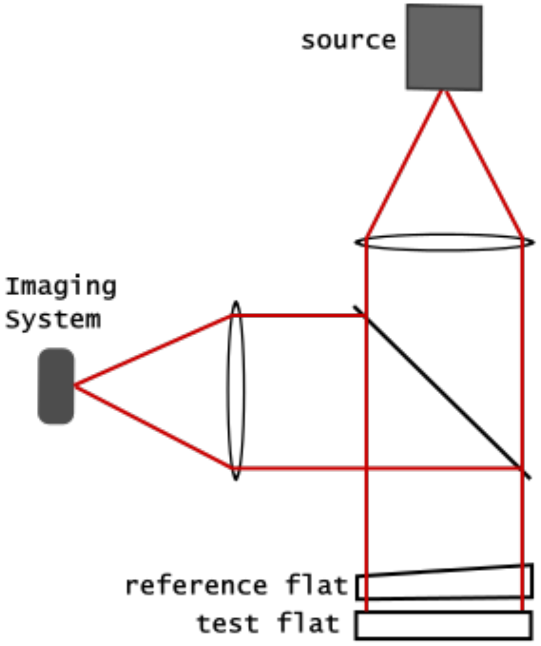


Mach-Zehnder viewing a candle flame





Fabry Perot

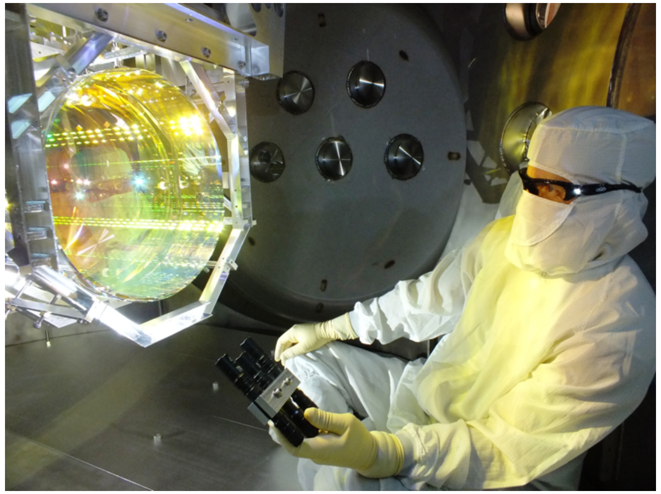


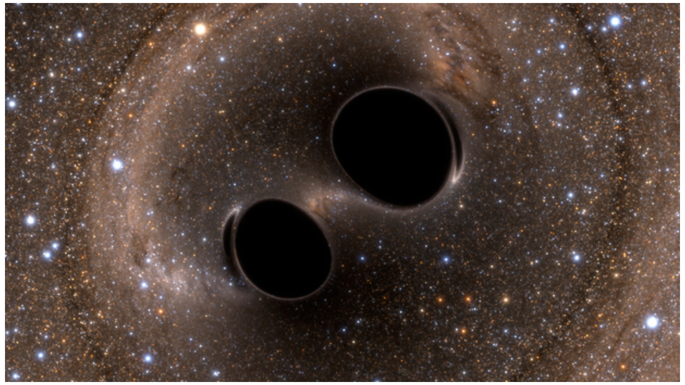
Fizeau

Direct detection of Gravitational Waves

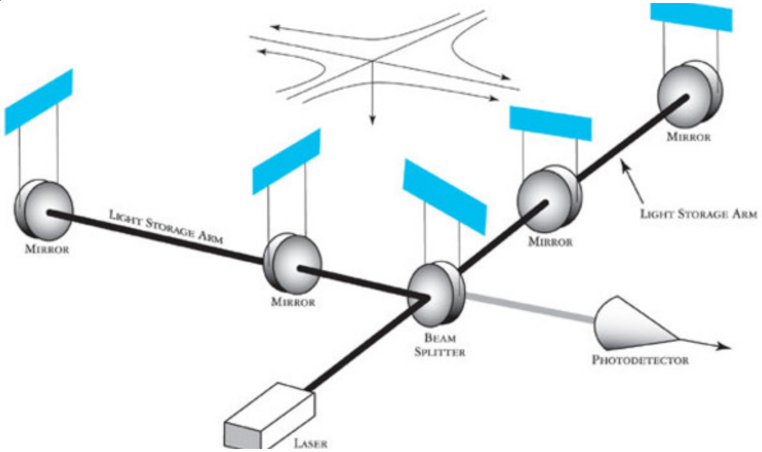
- *Exquisite sensitivity: gravitational wave typically lengthens and contracts each arm of the interferometer by length of 10^{-21} * arm length*

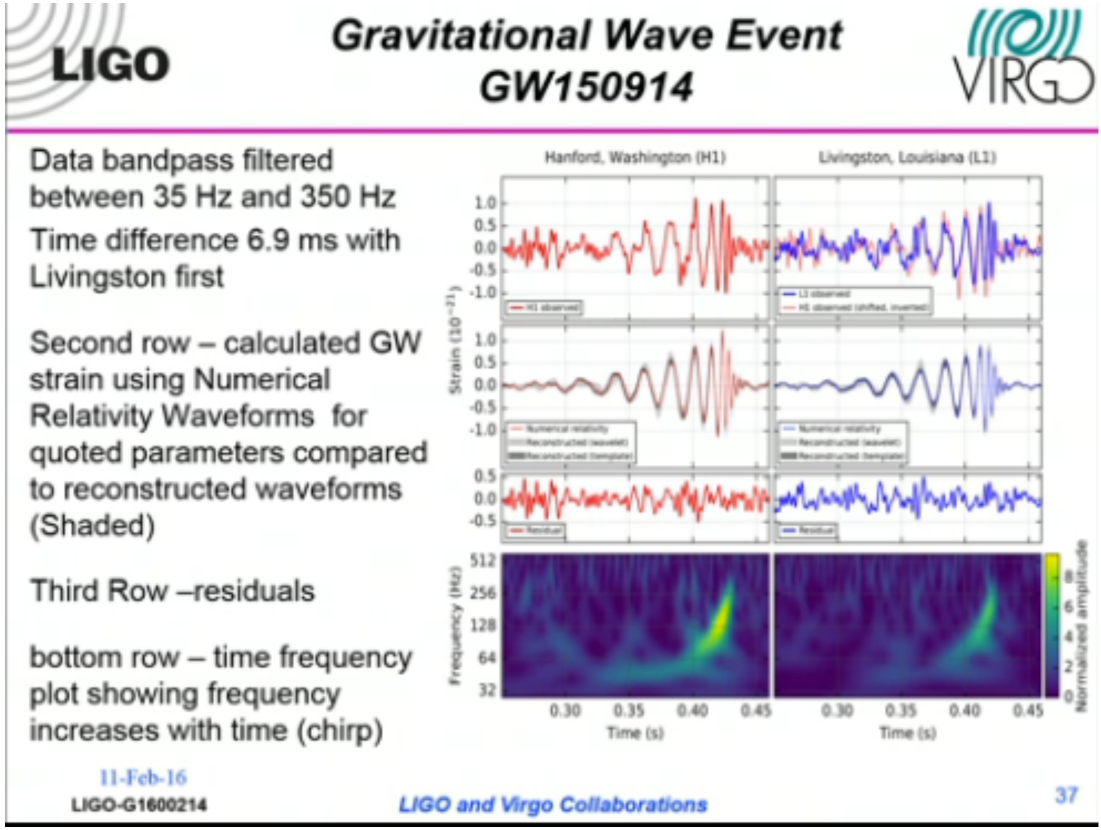
First signal from a binary black-hole





Quadrupole oscillation of space-time





Barry Barish (LIGO) CERN seminar 11/2/16

www.ligo.caltech.edu/video/ligo20160211v10

Finnish Diffraction

- *What happens when waves meet an aperture or obstacle in Finland?*

The calm before the storm in Tuusula…

Finnish Diffraction

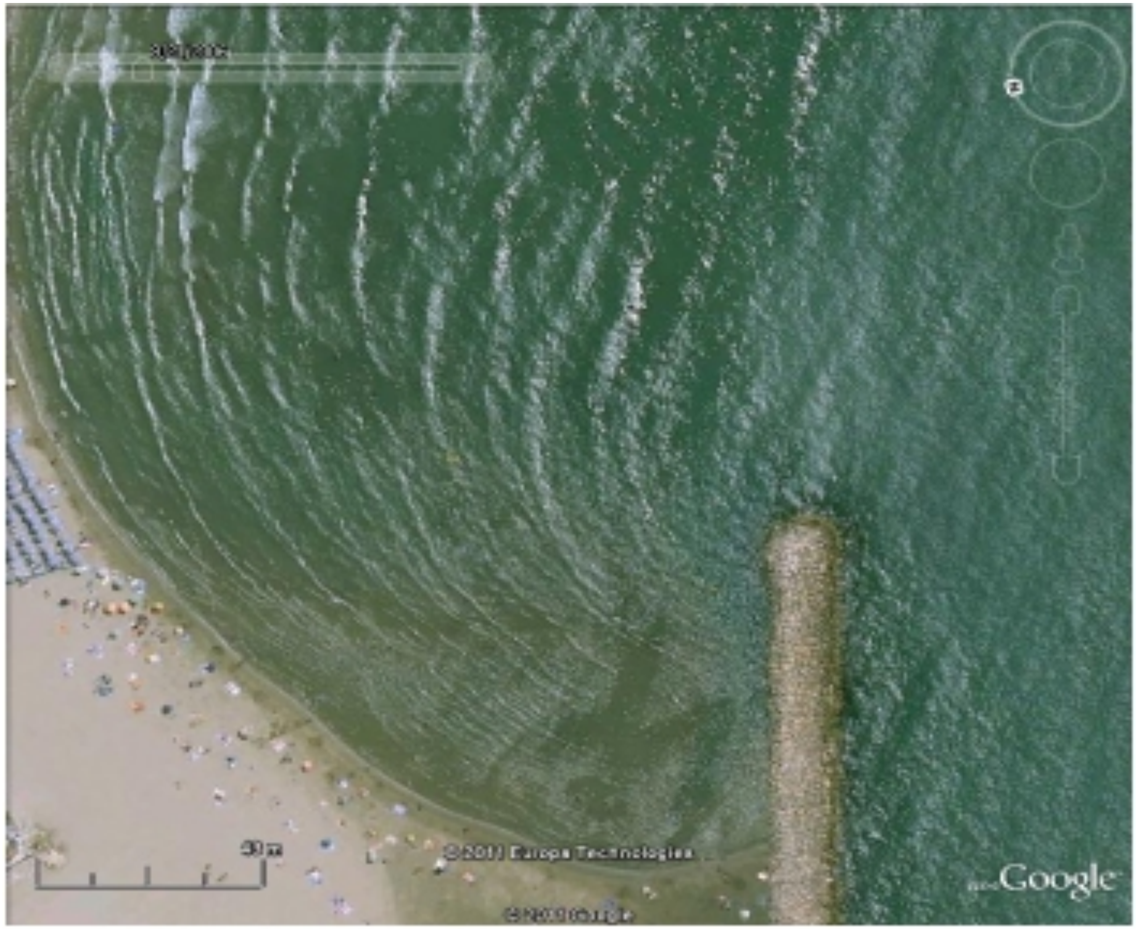
- *What happens when waves meet an aperture or obstacle in Finland?*

Finnish Diffraction

- *What happens when waves meet an aperture or obstacle in Finland?*

Wave Diffraction

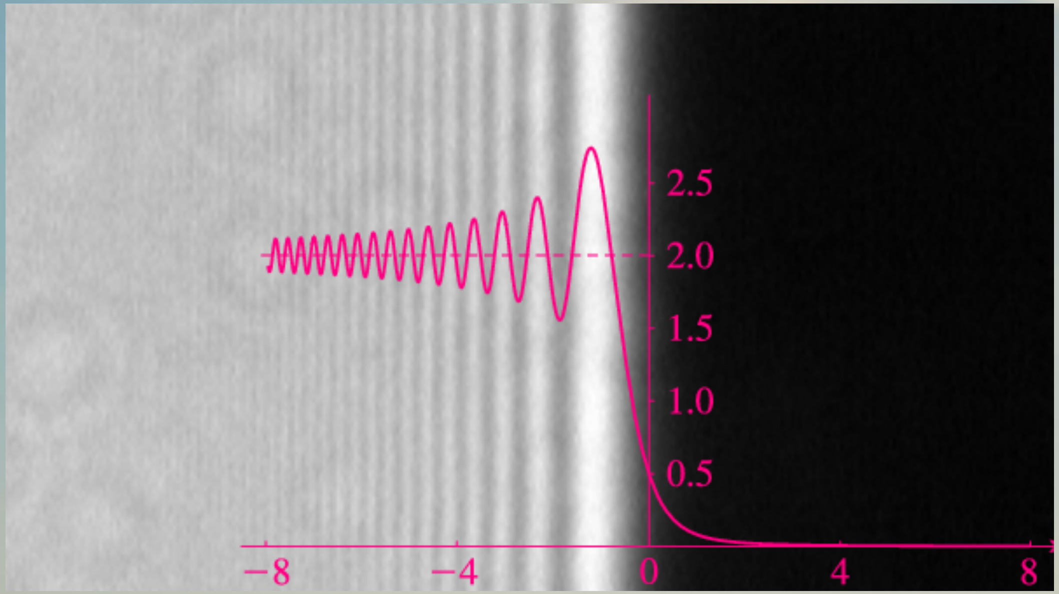
- *Diffraction occurs wherever there is an obstacle or aperture*



From Teaching waves with Google Earth
doi:10.1088/0031-9120/47/1/73

Light on the edge

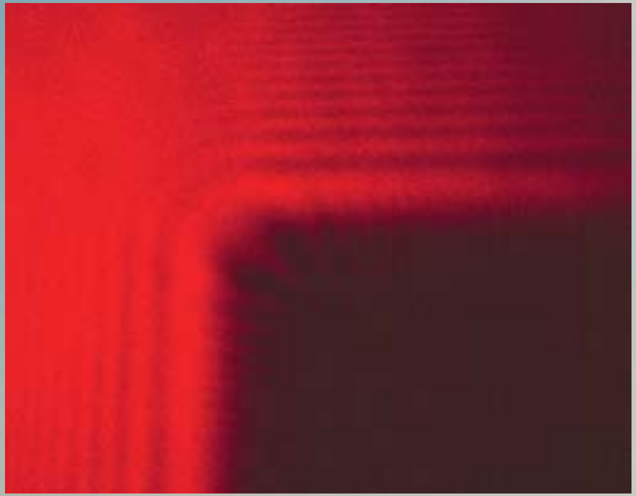
- *Diffraction fringes at a razor's straight edge:*



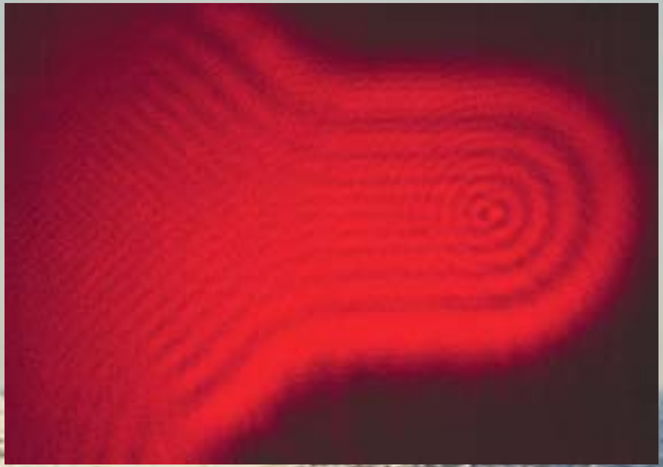
Light on the edge

- *Diffraction fringes at a razor's straight edge:*

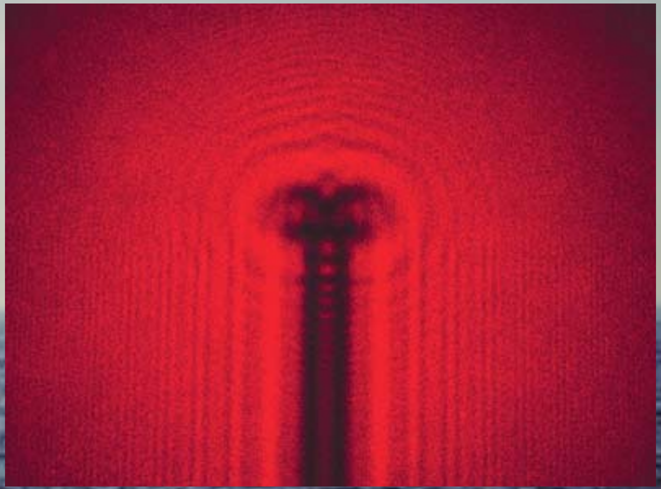
We see similar fringes at the corner…



…and at the curved cut out in the centre of the razor:



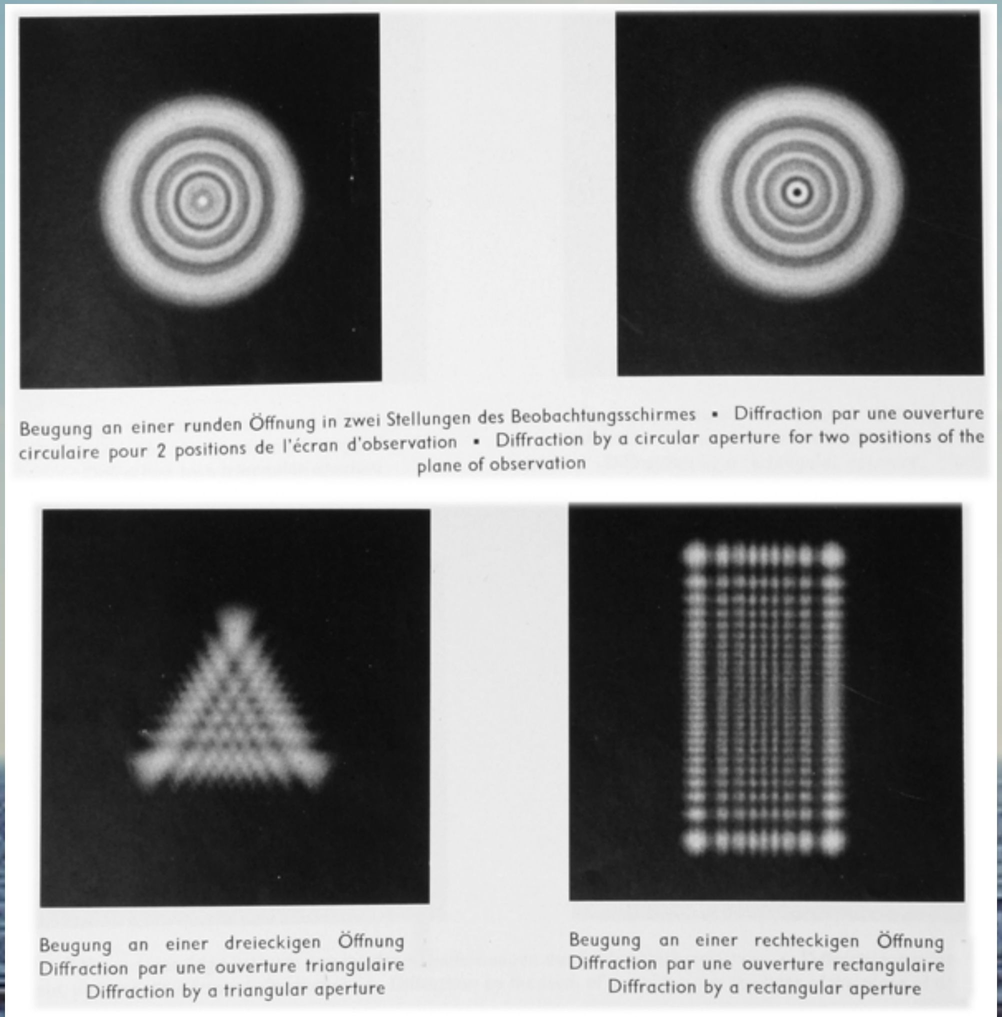
Diffraction at a pinhead



- *Diffraction effects may be helpful or problematic when constructing optical instruments*

Light on the edge

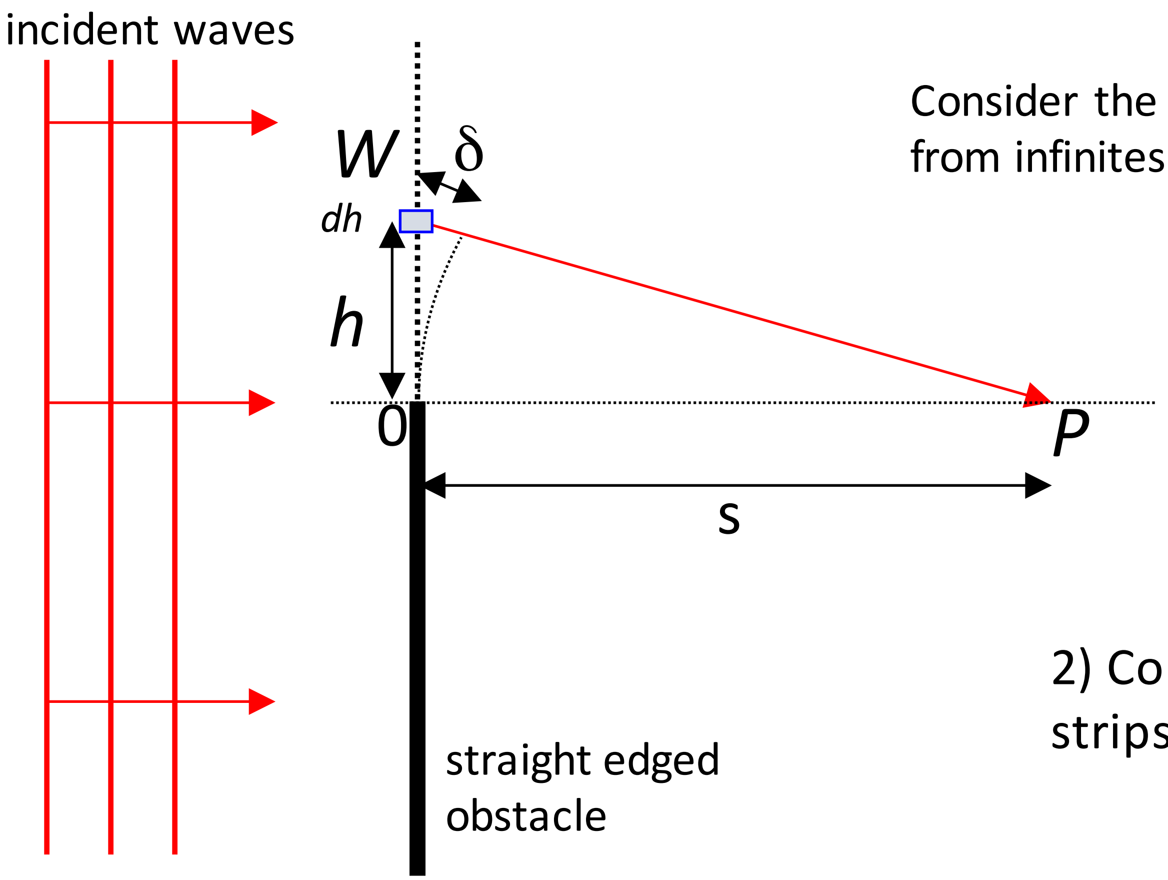
- *Diffraction fringes: circular, triangular and rectangular apertures:*



"Atlas of optical phenomena"; Michel Cagnet, Maurice Françon, Shamlal Mallick; Springer-Verlag, 1971.

Fresnel diffraction at a straight edge

- *Consider plane waves incident on a straight-edged obstacle:*
- *We aim to evaluate the intensity at P, by summing all contributions that pass the obstacle*



Consider the intensity at P due to contributions from infinitesimal strip, dh, at height, h.:

1) Compared to the phase of a wave from O, the extra phase of the wave from W is

$$\phi(h) = \frac{2\pi}{\lambda}\left[\left(s^2 + h^2\right)^{1/2} - s\right] \approx \frac{\pi h^2}{\lambda s}$$

Approximation valid for $h^2 << s^2$

2) Construct a phasor, dx+idy = dh[exp(iϕ(h)], due to infinitesimal strips of height dh at h:

$$dx = dh\cos\frac{\pi h^2}{\lambda s} \qquad and \qquad dy = dh\sin\frac{\pi h^2}{\lambda s}$$

A thing of beauty: the Cornu spiral

- The phasor will trace out a spiral (with tangent at phase angle $\phi \sim h^2$)

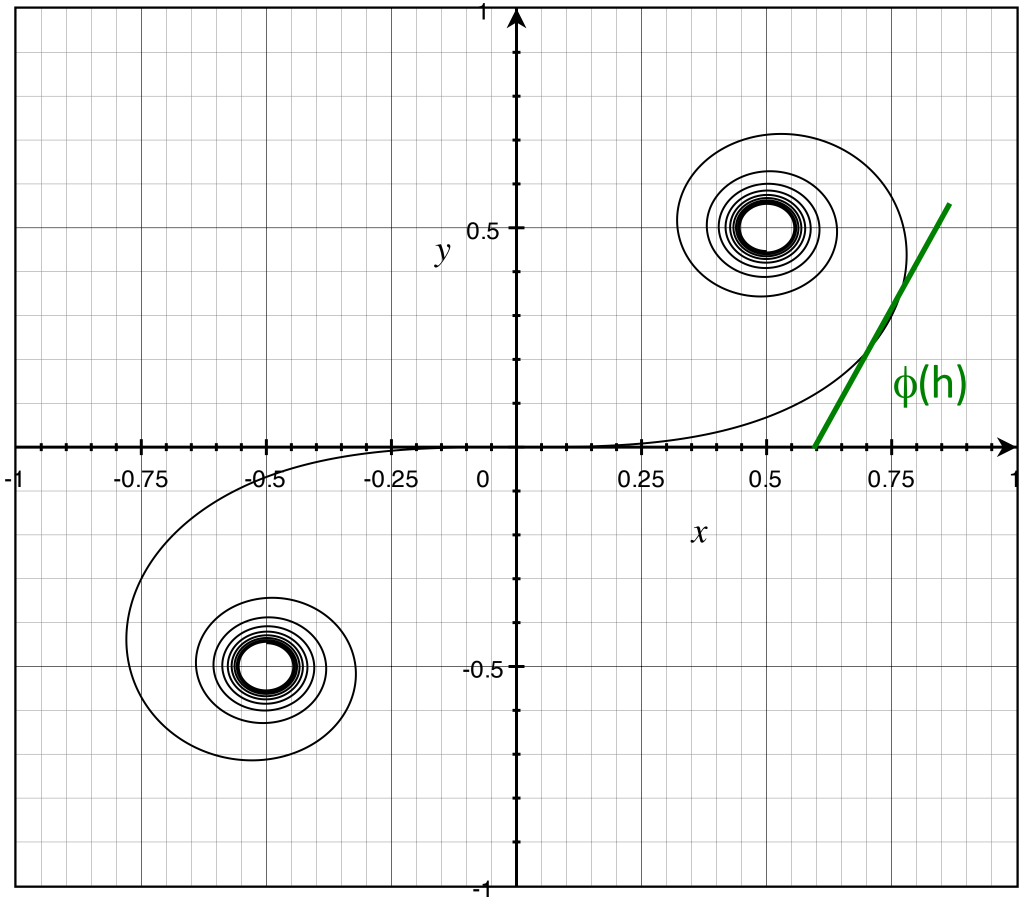
Usually define a dimensionless variable which represents the distance along the spiral,

$$v = h\left(\frac{2}{\lambda s}\right)^{1/2}$$

- The spiral coordinates are given by the Fresnel integrals

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$



Marie Alfred Cornu (1841-1902)

Note if phase were linear in h we would have a circle

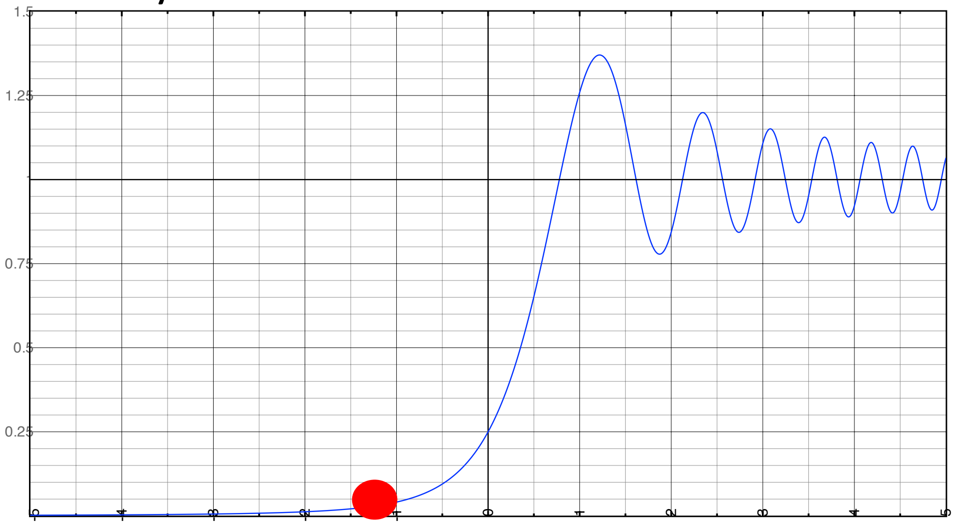
Fresnel diffraction at a straight edge

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

$I = (x^2+y^2)/2$

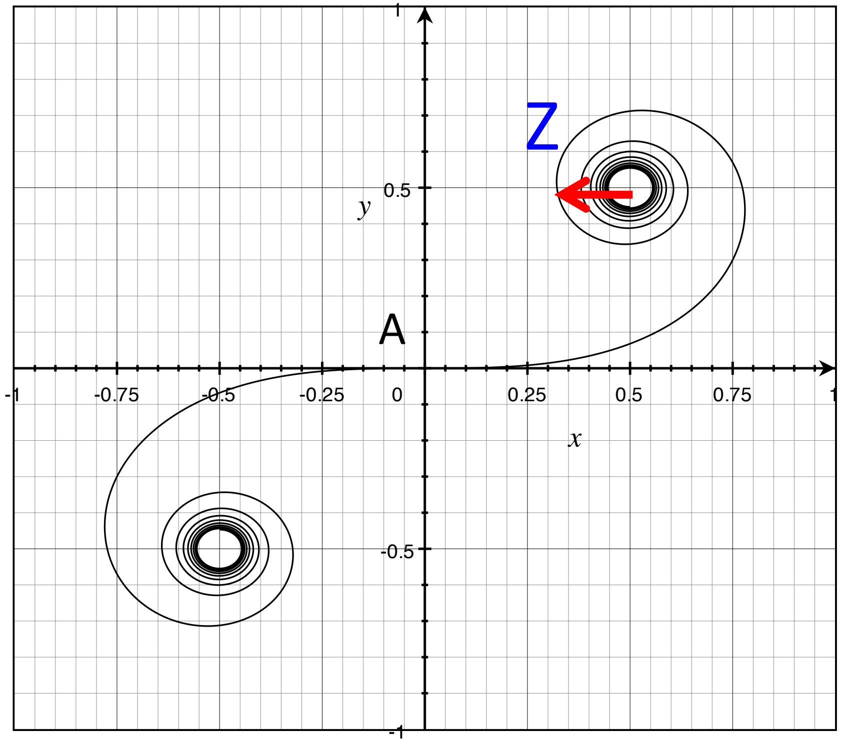
Intensity



Straight edge

In geometric shadow

Not in geometric shadow



Start in geometric shadow…
length of arrow grows as we move within the shadow

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$

ROYAL HOLLOWAY UNIVERSITY OF LONDON

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

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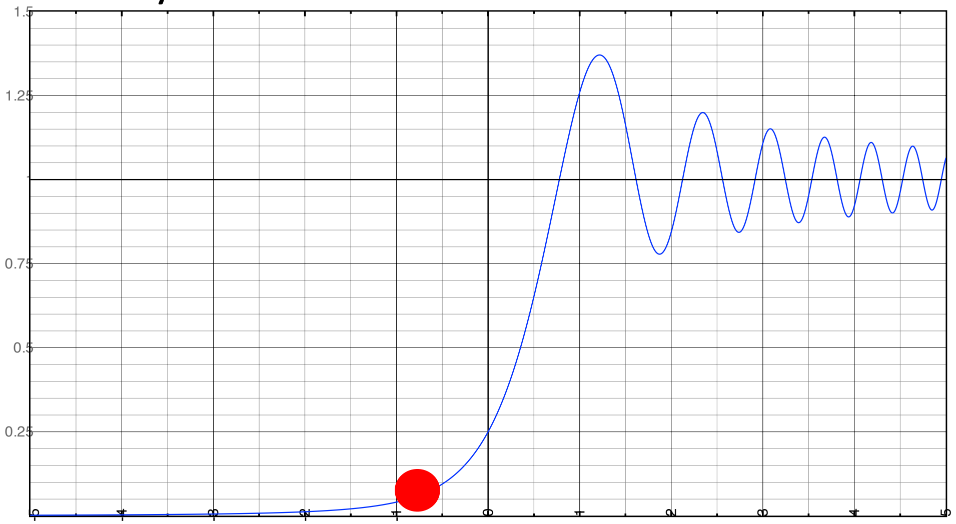
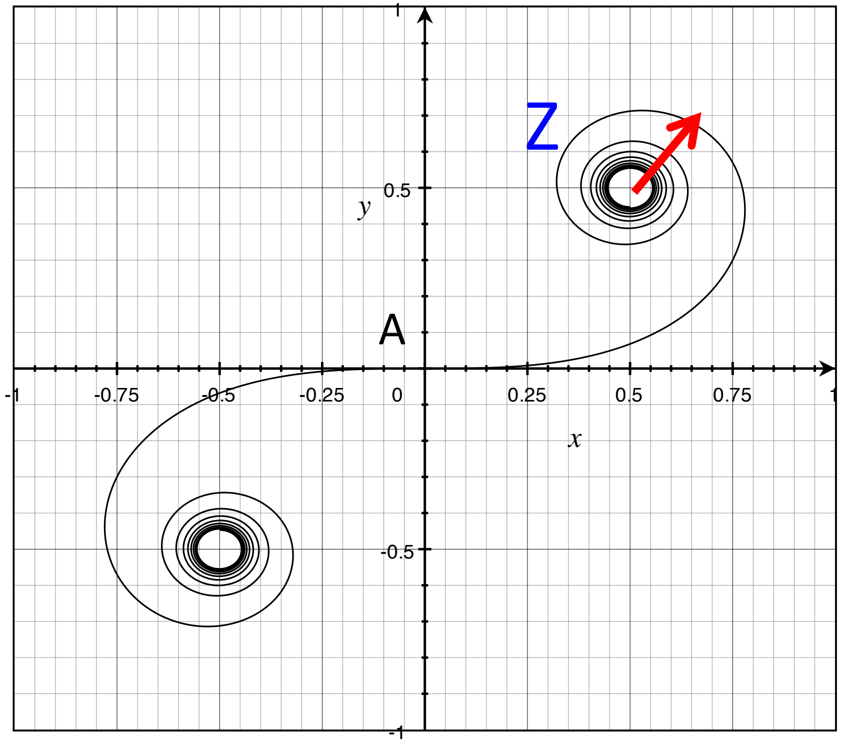
Intensity



Straight edge

In geometric shadow Not in geometric shadow



Start in geometric shadow…
length of arrow grows as we move within the shadow

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$

Fresnel diffraction at a straight edge

ROYAL HOLLOWAY UNIVERSITY OF LONDON

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

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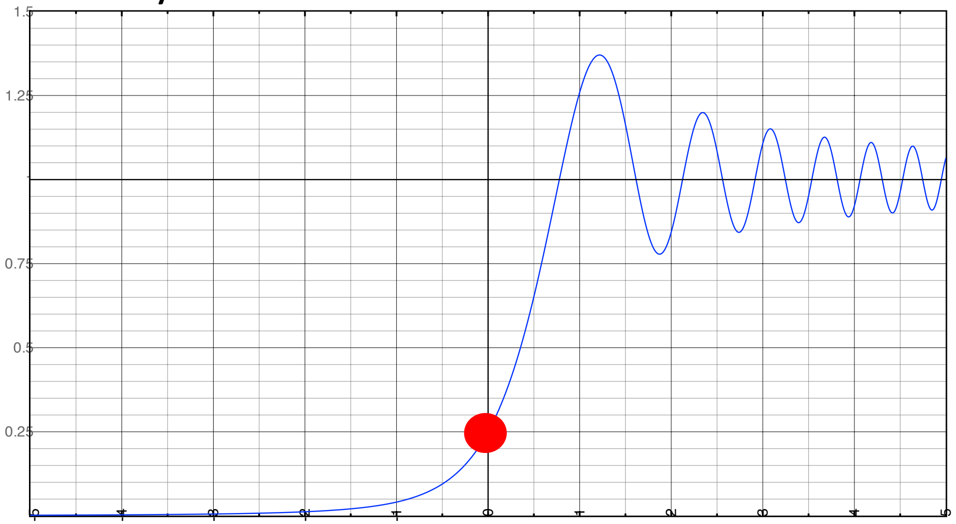
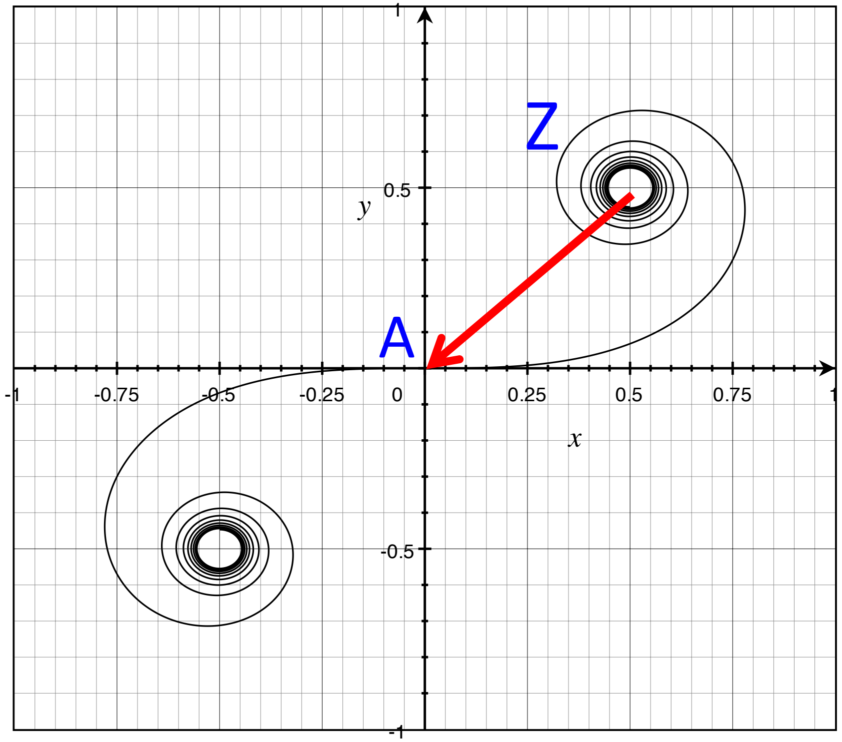


Straight edge

In geometric shadow Not in geometric shadow



Predicts that the intensity at P, in line the straight edge is ½(AZ)²=0.25, or one quarter of that when no obstacle is present.

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

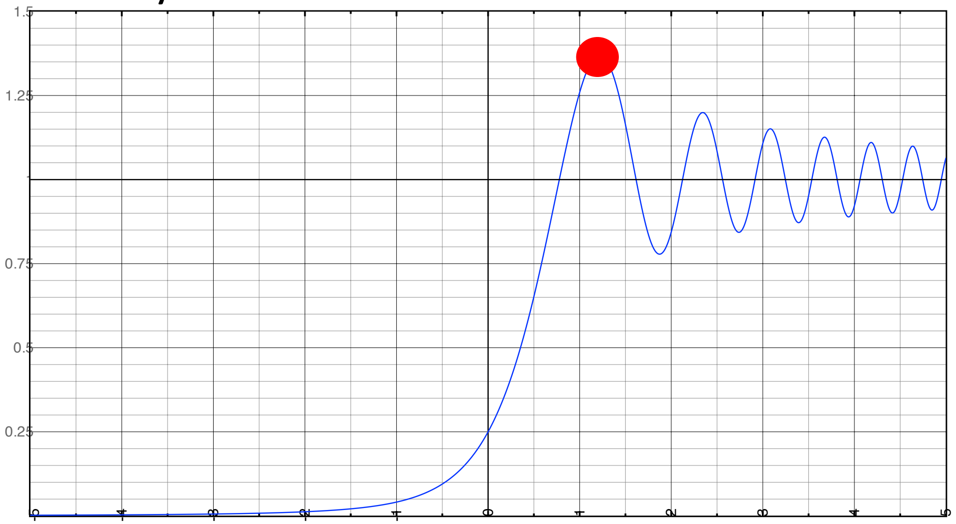
$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$

Fresnel diffraction at a straight edge

A thing of beauty: the Cornu spiral

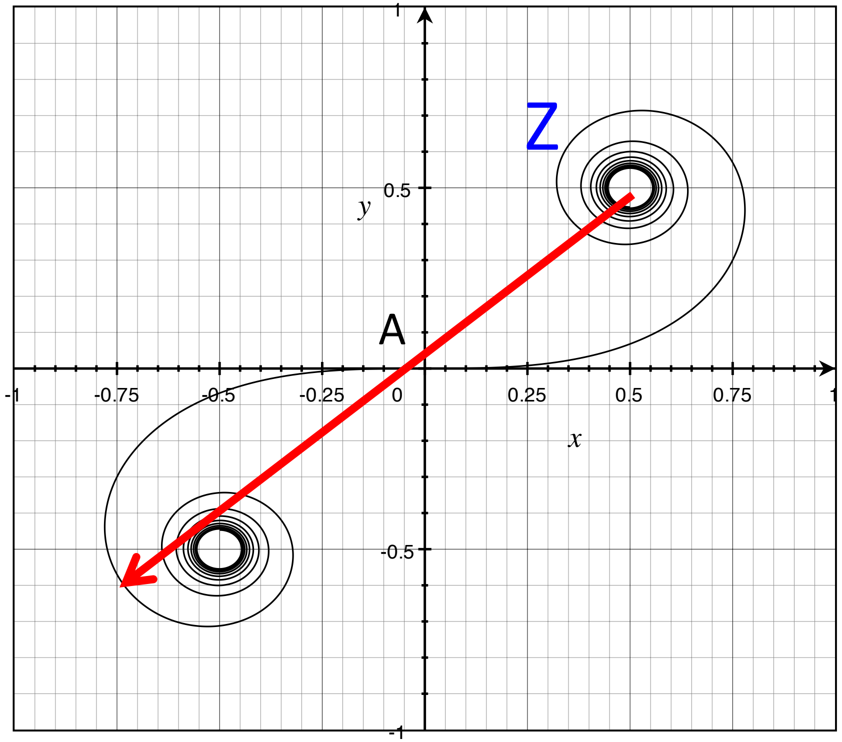
The arrow length traces out the straight edge pattern, with resultant normalized intensity

$I = (x^2+y^2)/2$



Straight edge

In geometric shadow Not in geometric shadow



$$x = \int_0^v \cos\frac{\pi v'^2}{2} dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2} dv'$$

Reach first maximum:
Note it is larger than if the obstacle were not present!

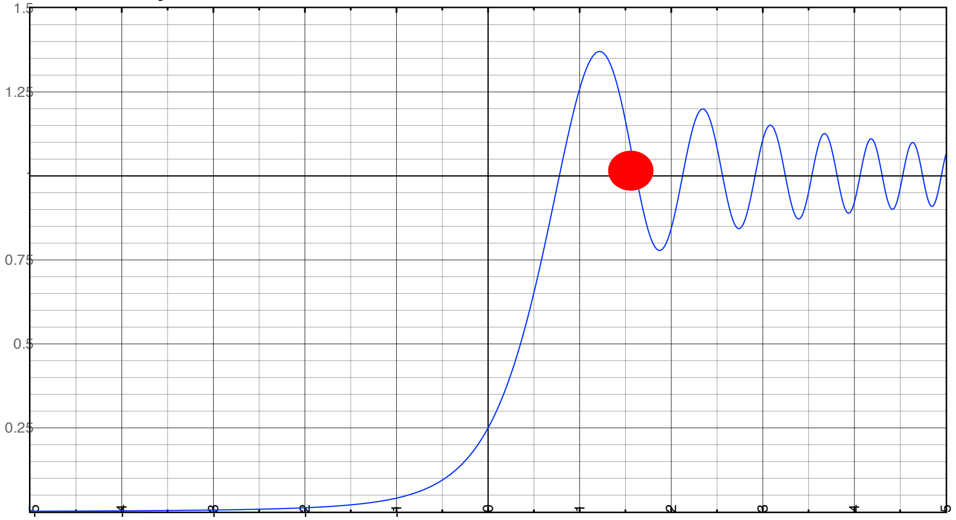
Fresnel diffraction at a straight edge

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

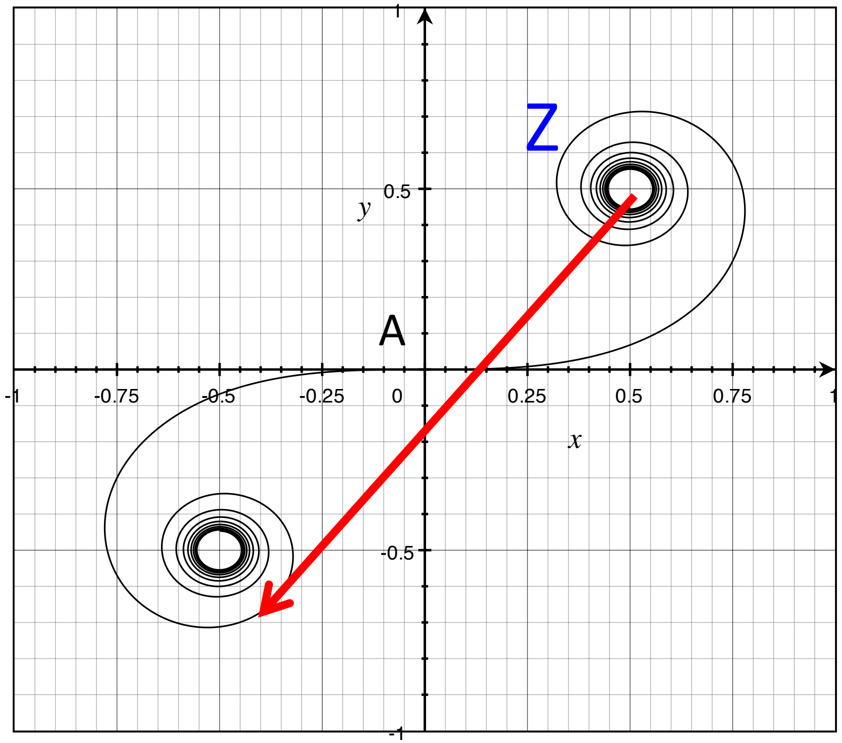
$I = (x^2+y^2)/2$

Intensity



Straight edge

In geometric shadow   Not in geometric shadow



Just past the first maximum:

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$

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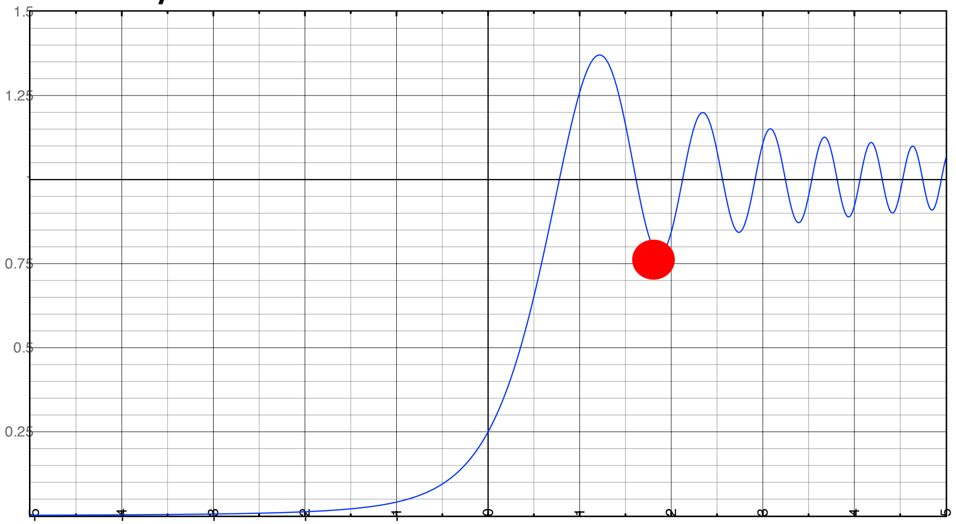
A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

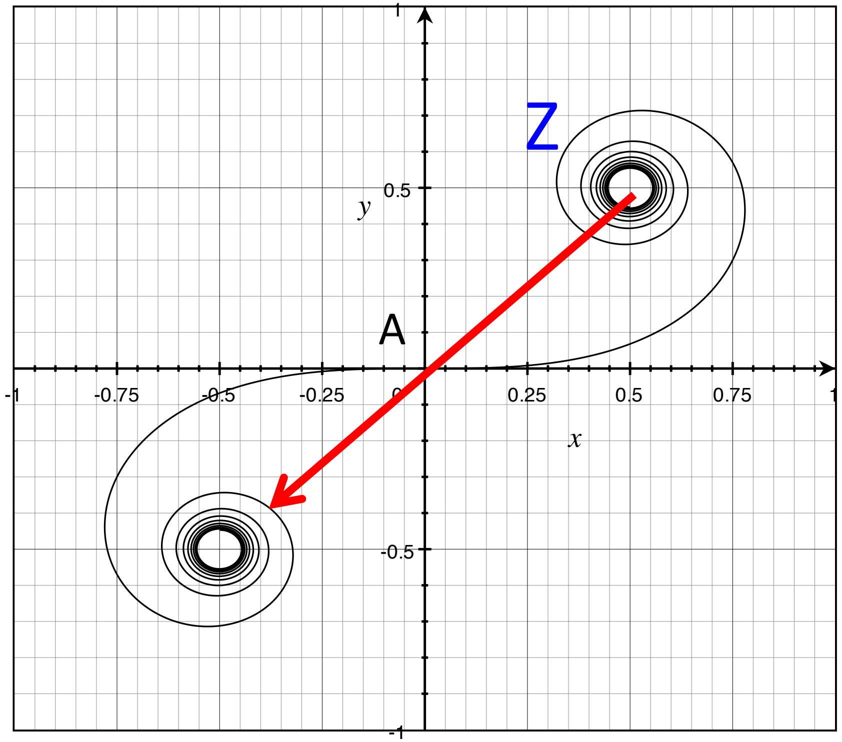
$I = (x^2+y^2)/2$

Intensity



Straight edge

In geometric shadow Not in geometric shadow



Reach first minimum

$$x = \int_0^v \cos\frac{\pi v'^2}{2}\,dv'$$

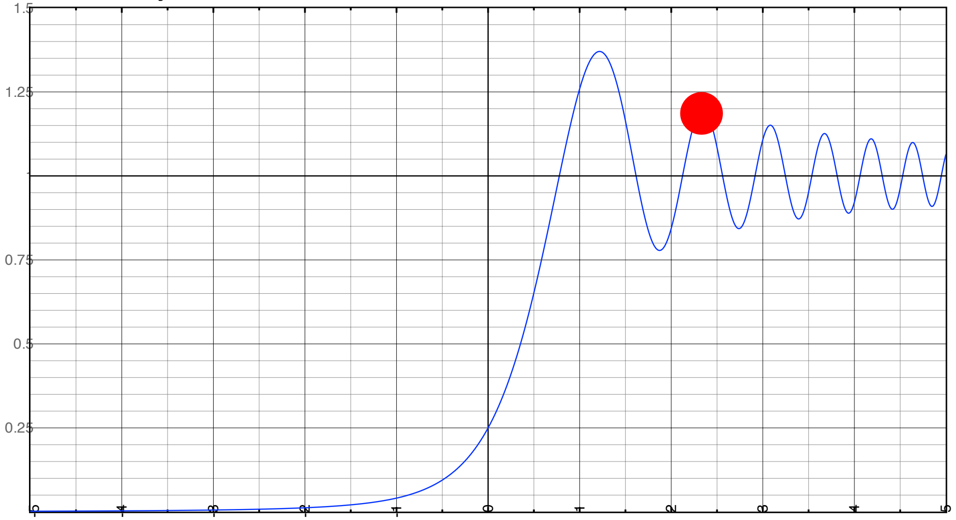
$$y = \int_0^v \sin\frac{\pi v'^2}{2}\,dv'$$

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

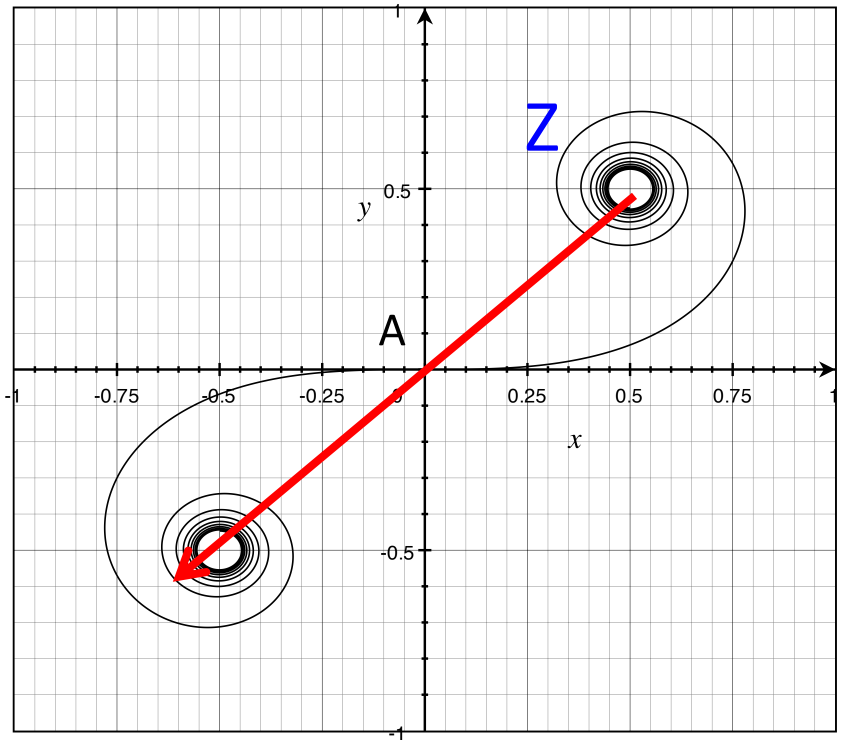
$I = (x^2+y^2)/2$

Intensity



Straight edge

In geometric shadow   Not in geometric shadow



Reach second maximum

$$x = \int_0^v \cos \frac{\pi v'^2}{2} dv'$$

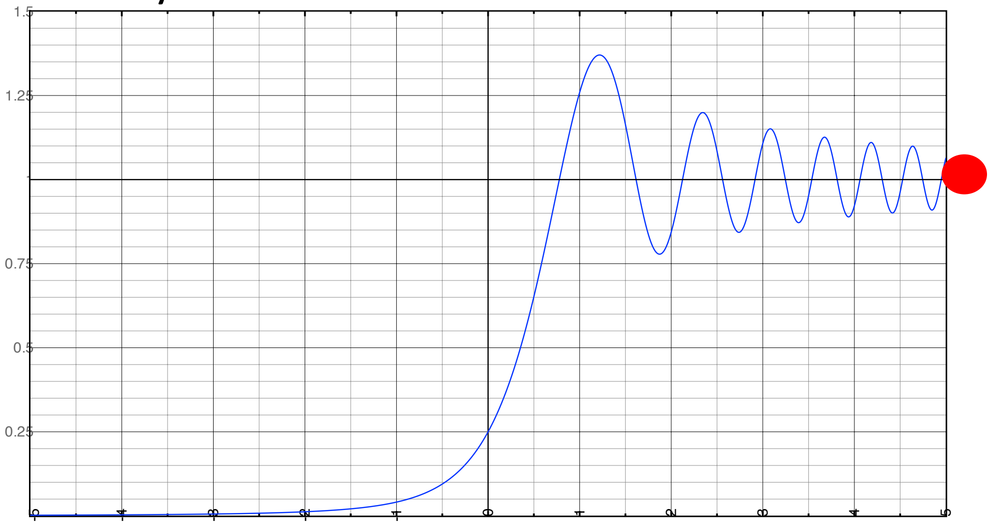
$$y = \int_0^v \sin \frac{\pi v'^2}{2} dv'$$

A thing of beauty: the Cornu spiral

The arrow length traces out the straight edge pattern, with resultant normalized intensity

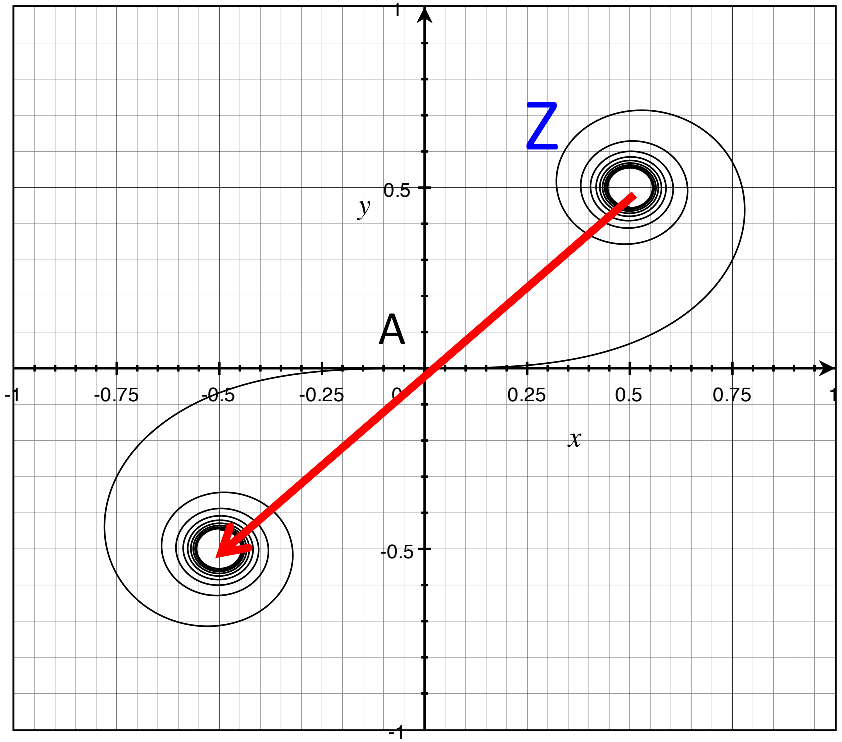
$I = (x^2+y^2)/2$

Intensity



Straight edge

In geometric shadow   Not in geometric shadow

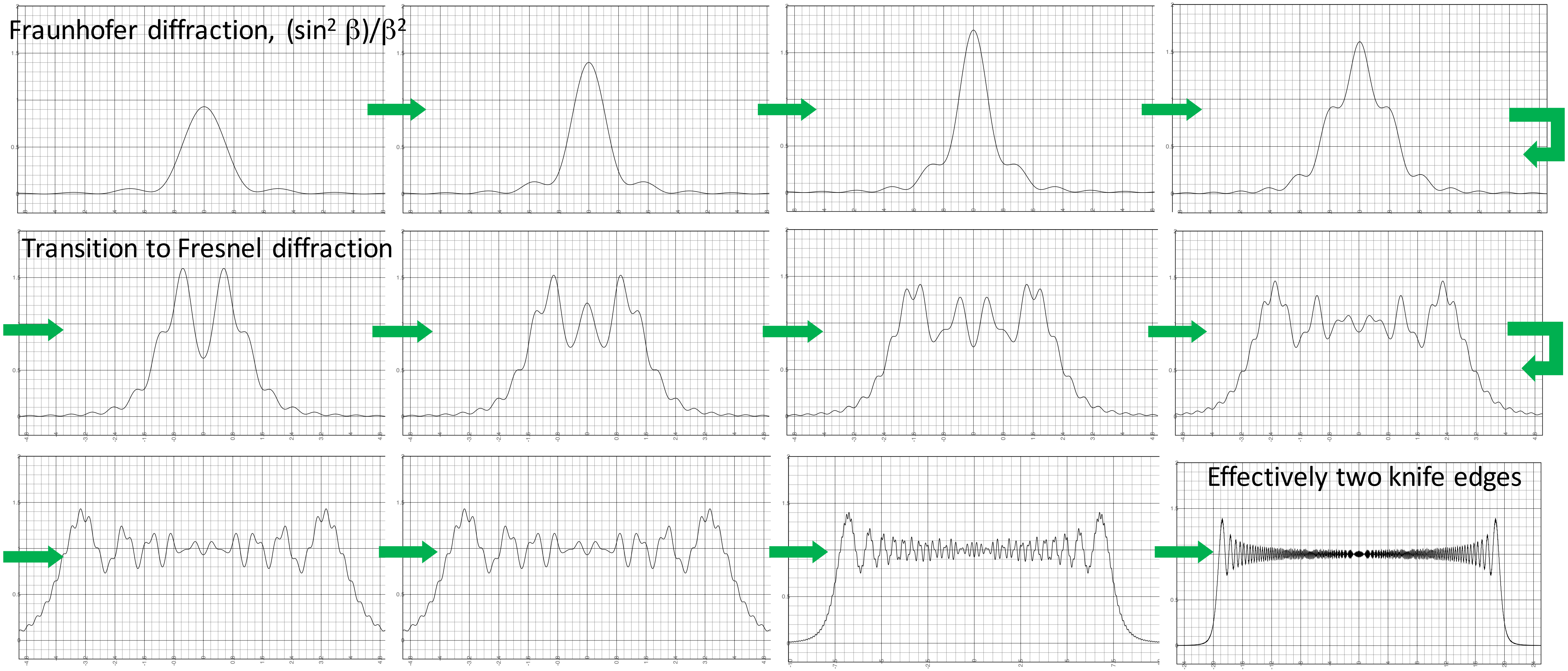


Tends to the centre of the spiral, where the intensity = 1.

$$x = \int_0^v \cos\frac{\pi v'^2}{2} dv'$$

$$y = \int_0^v \sin\frac{\pi v'^2}{2} dv'$$

Far field intensity for a widening slit:

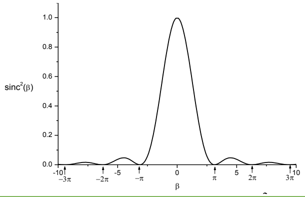
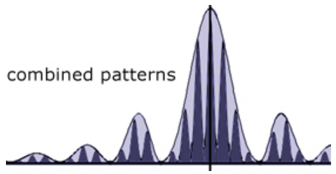
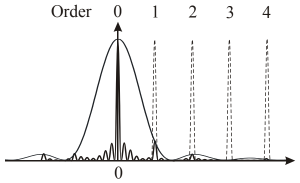


General Fraunhofer Diffraction in 1D

- *To calculate the far field diffraction pattern take the **Fourier Transform** of the transmission function of the diffracting aperture:*

$$I(\theta_x) = \left|E_{res}(\theta_x)\right|^2 = \left|\int_S A(x_s)\exp\left[-ikx_s\sin\theta_x\right]dx_s\right|^2$$

Common examples

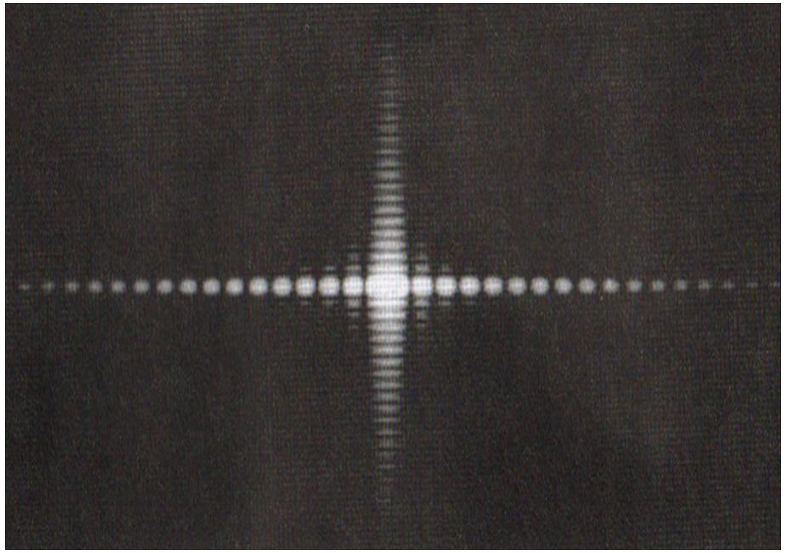
Intensity integral, FT of aperture function	Solution	Definitions
Single slit: $I(\theta_x) = \left|E_{res}(\theta_x)\right|^2 = \left|\int_{-a/2}^{a/2} A_0\exp\left[-ikx_s\sin\theta_x\right]dx_s\right|^2$	$I = A_0^2\dfrac{\sin^2\alpha}{\alpha^2}$ 	$\alpha = \dfrac{\pi}{\lambda}a\sin\theta_x$
Double slit: $I(\theta_x) = \left|\int_{-b/2-a}^{-b/2} A(x_s)\exp\left[-ikx_s\sin\theta_x\right]dx_s + \int_{b/2}^{b/2+a} A(x_s)\exp\left[-ikx_s\sin\theta_x\right]dx_s\right|^2$	$I = A_0^2\dfrac{\sin^2\alpha}{\alpha^2}\cos^2\dfrac{\delta}{2}$ 	$\alpha = \dfrac{\pi}{\lambda}a\sin\theta_x$ and $\delta = \dfrac{2\pi}{\lambda}(a+b)\sin\theta_x$
N-slit grating: $I(\theta_x) = \left|\int_{-a/2}^{a/2} Ae^{-ikx_s\sin\theta_x}dx_s + \int_{d-a/2}^{d+a/2} Ae^{-ikx_s\sin\theta_x}dx_s + \int_{2d-a/2}^{2d+a/2} Ae^{-ikx_s\sin\theta_x}dx_s + \ldots + \int_{(N-1)d-a/2}^{(N-1)d+a/2} Ae^{-ikx_s\sin\theta_x}dx_s\right|^2$	$I = A_0^2\dfrac{\sin^2\alpha}{\alpha^2}\dfrac{\sin^2 N\beta}{\sin^2\beta}$ 	$\alpha = \dfrac{\pi}{\lambda}a\sin\theta$ *and* $\beta = \dfrac{\delta}{2} = \dfrac{\pi}{\lambda}d\sin\theta$

Diffraction in 2D

- *Rectangular slit:*

$$I(\theta_x) = \left| \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} A(x_s) \exp[-ikx_s \sin\theta_x] \exp[-iky_s \sin\theta_y] dx_s\, dy_s \right|^2$$

$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2\beta}{\beta^2}, \quad \text{where } \alpha = \frac{\pi}{\lambda} a \sin\theta_x \text{ and } \beta = \frac{\pi}{\lambda} b \sin\theta_y$$

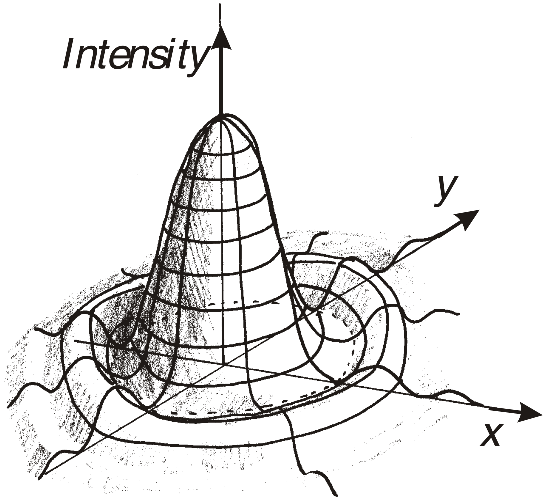


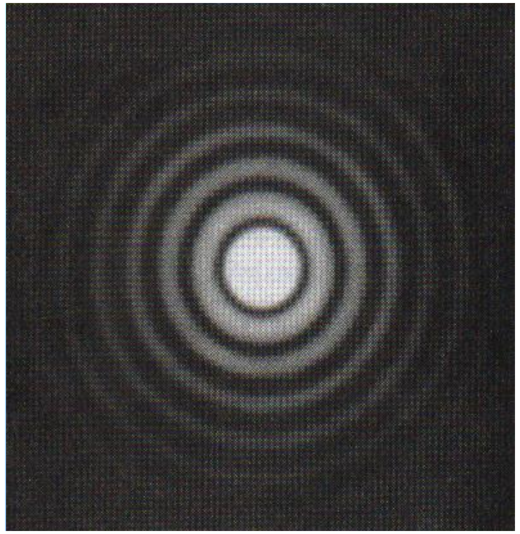
- *Circular aperture:*

$$E_{res}(\rho_d, \theta_d) = \int_0^{\bar{\rho}} \int_0^{2\pi} A(x_s) \rho_s \exp\left[-\frac{ik}{L} \rho_s \cdot \rho_d \cdot \cos(\theta_s - \theta_d) \right] d\rho_s\, d\theta_s$$

$$E_{res} = A_0^2 \frac{J_1^2(\alpha)}{\alpha^2}, \text{ where } \alpha = \frac{ka\rho_d}{L}$$

$$\theta = \frac{1.22 \cdot \lambda}{\rho_d}$$





Convolution visualized

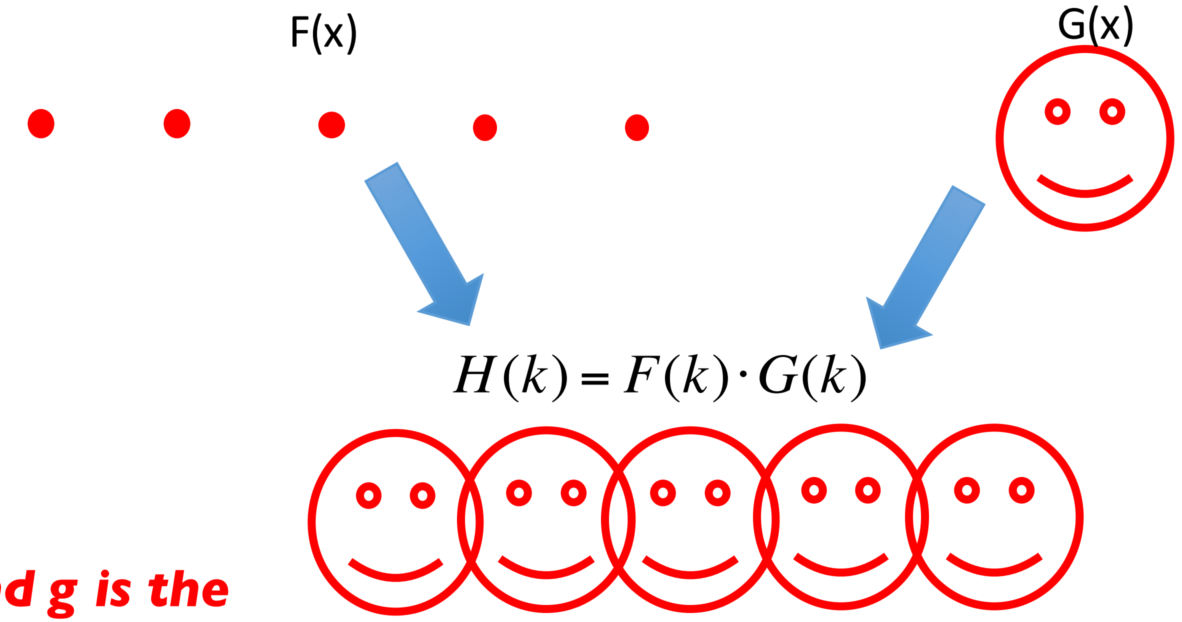
- *The convolution function:*

$$h(x) = f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(x')g(x'-x)\,dx'$$

- *The convolution theorem:*
  - F(k) is the Fourier Transform of f(x)
  - G(k) is the Fourier Transform of g(x)
  - H(k) is the Fourier Transform of h(x)
  - Then:

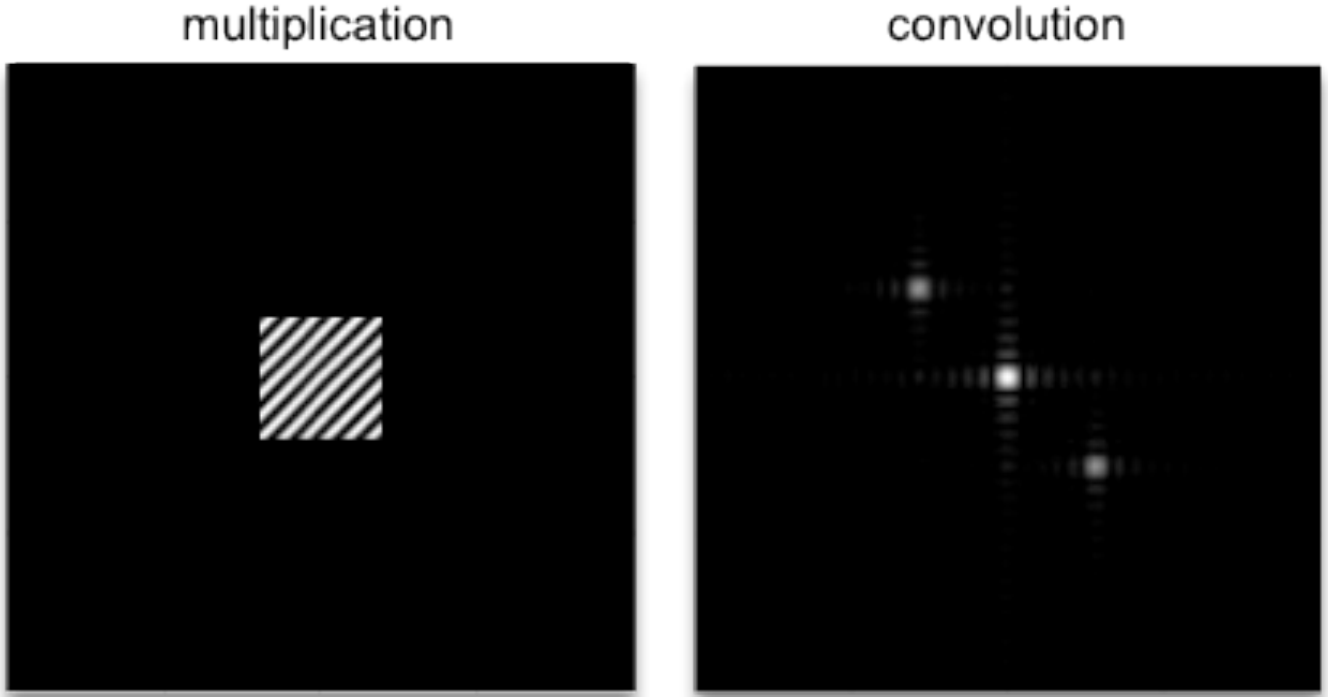
$$H(k) = F(k) \cdot G(k)$$

  - ***The Fourier transform of a convolution of f and g is the product of the Fourier transforms of f and g***



Convolution theorem

Spatial domain 1 | Fourier transform 1 | Spatial domain 2 | Fourier transform 2



Spatial domain

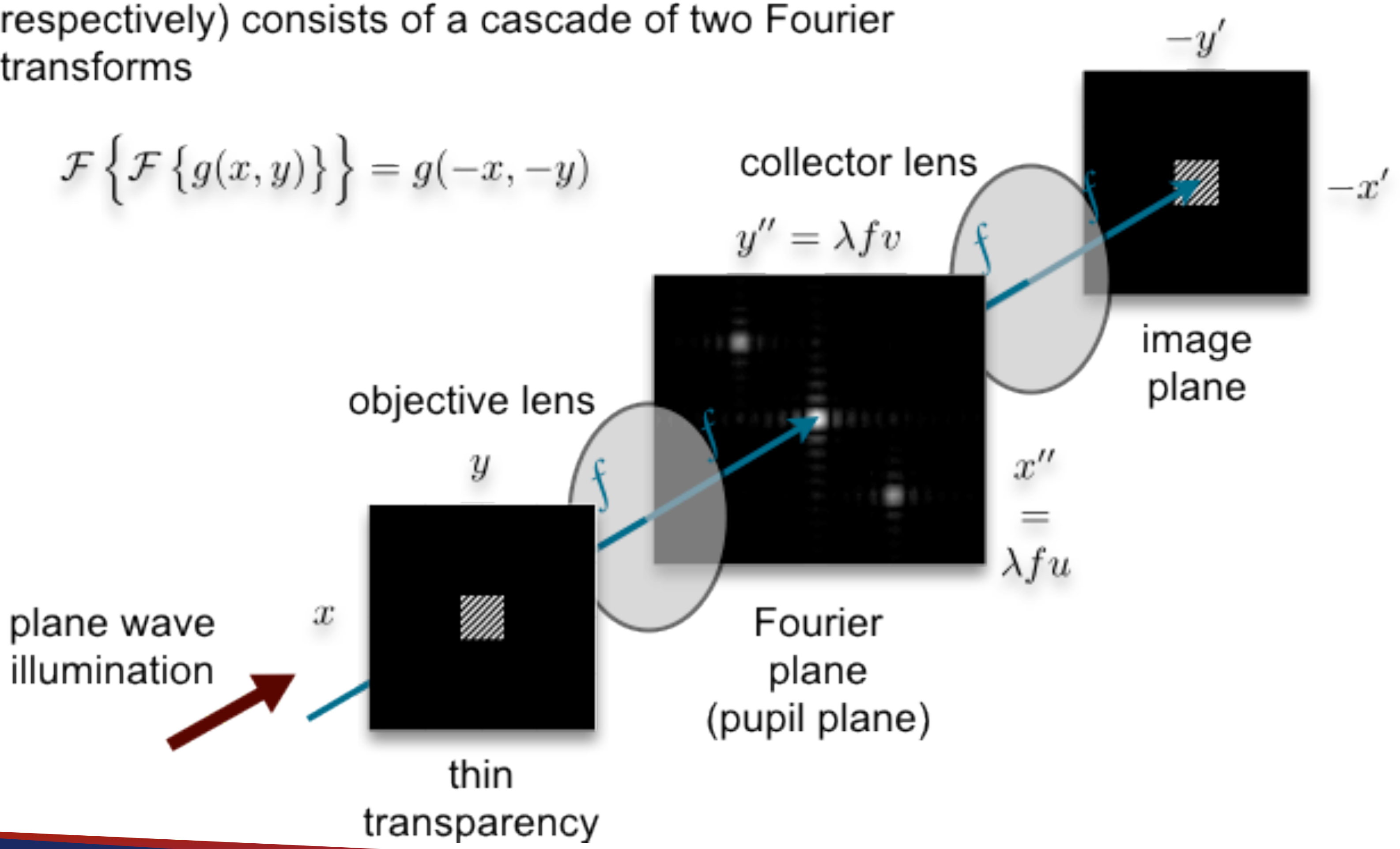
Fourier transform (frequency domain)

MIT OpenCourseWare
http://ocw.mit.edu

Spatial filtering

The 4F system (telescope with finite conjugates one focal distance to the left of the objective and one focal distance to the right of the collector, respectively) consists of a cascade of two Fourier transforms

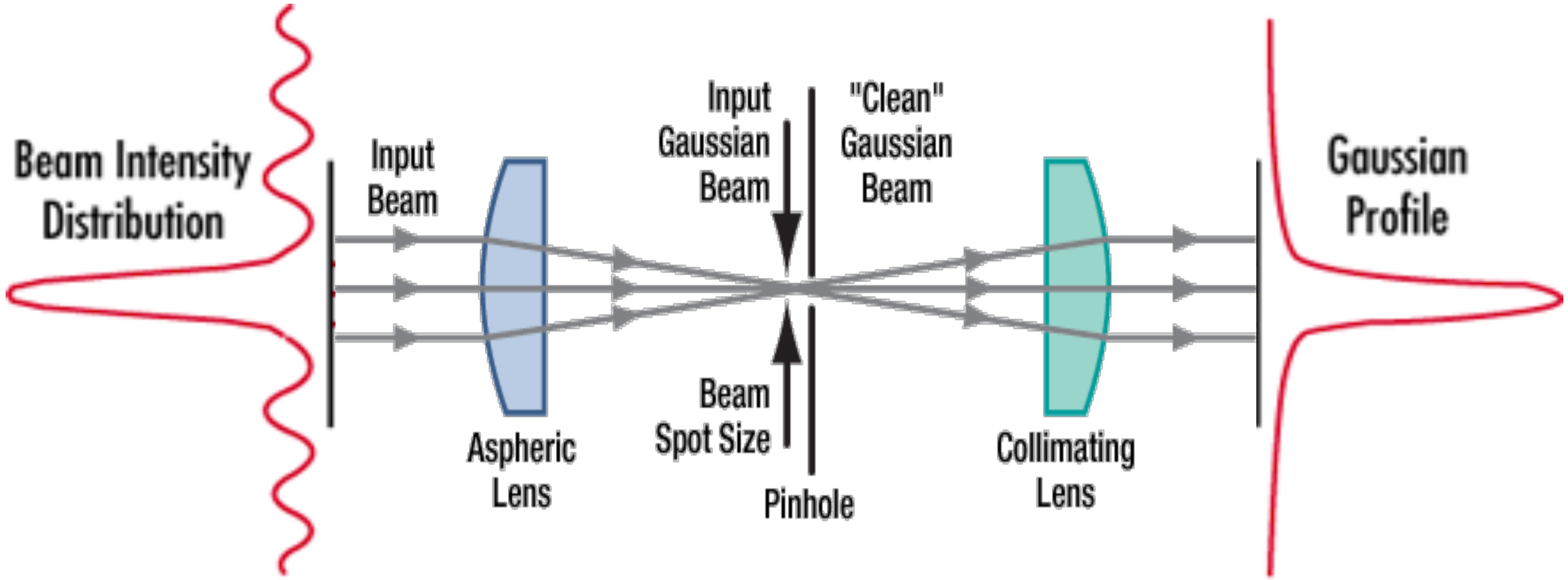
$$\mathcal{F}\left\{\mathcal{F}\left\{g(x,y)\right\}\right\} = g(-x,-y)$$



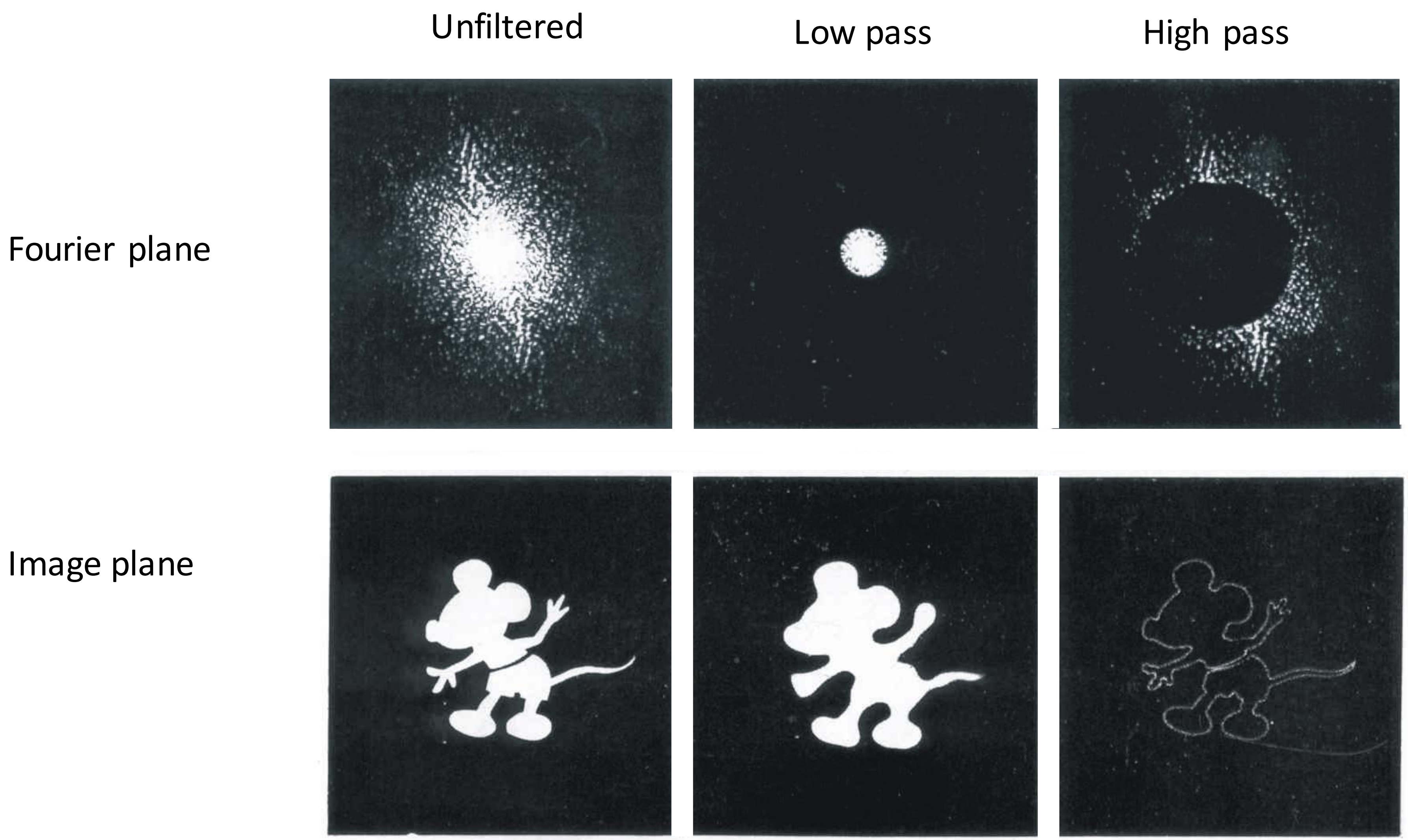
MIT OpenCourseWare
http://ocw.mit.edu

Pin hole as a low pass filter

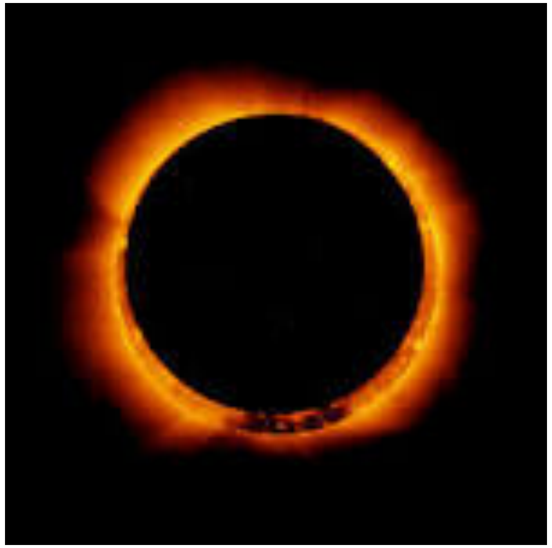
- *A pinhole aperture placed at the focus of the lens acts in the Fourier plane:*
  - This eliminates structure with higher spatial frequencies, which produce light furthest from the central position.
  - A microscope objective and pinhole is typically used to remove aberrations and improve the quality of a Gaussian laser beam.



Spatial filtering in image processing



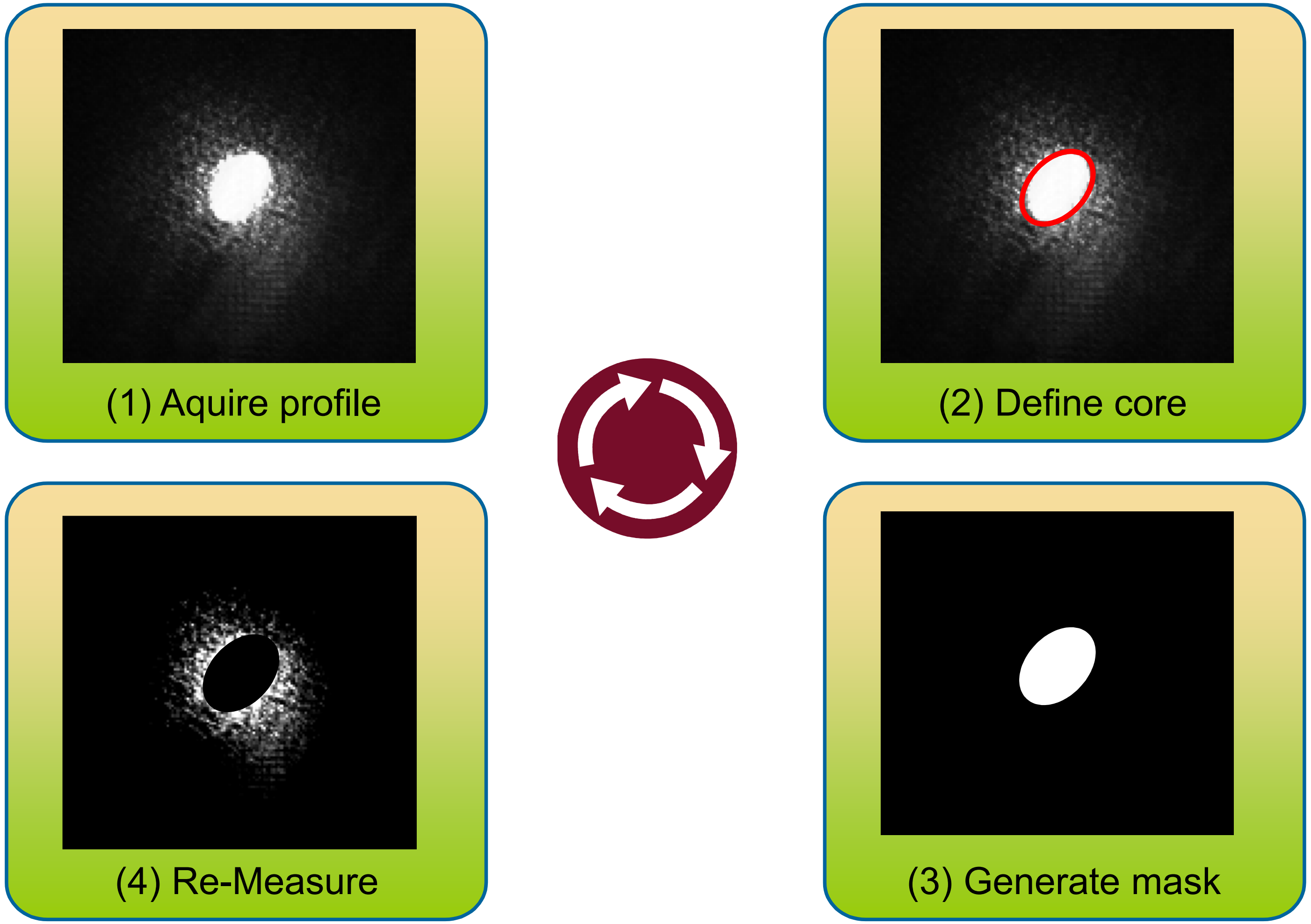
Halo monitoring: core masking



The Sun's chromosphere is 4 orders of magnitude less dense than the photosphere (which itself is three to four orders less dense than air at sea level).

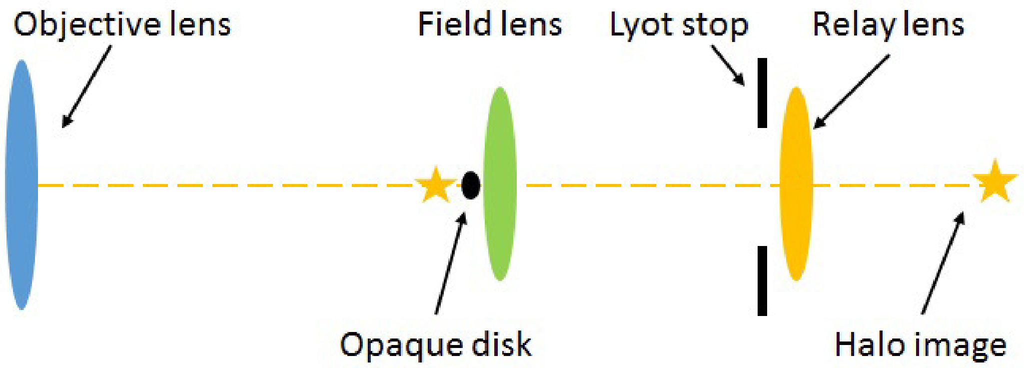
The chromosphere becomes directly visible during an eclipse.

J. Egberts, et al.,
JINST **5** P04010 (2010)
H. Zhang, R. Fiorito, et al.,
Phys. Rev. STAB 15 (2012)



Application: Coronagraph for LHC beam halo

A. Goldbatt et al. MOPG74m IBIC2016



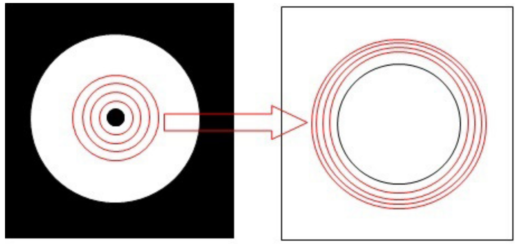


Figure 2: Sketch of diffraction pattern at the objective lens

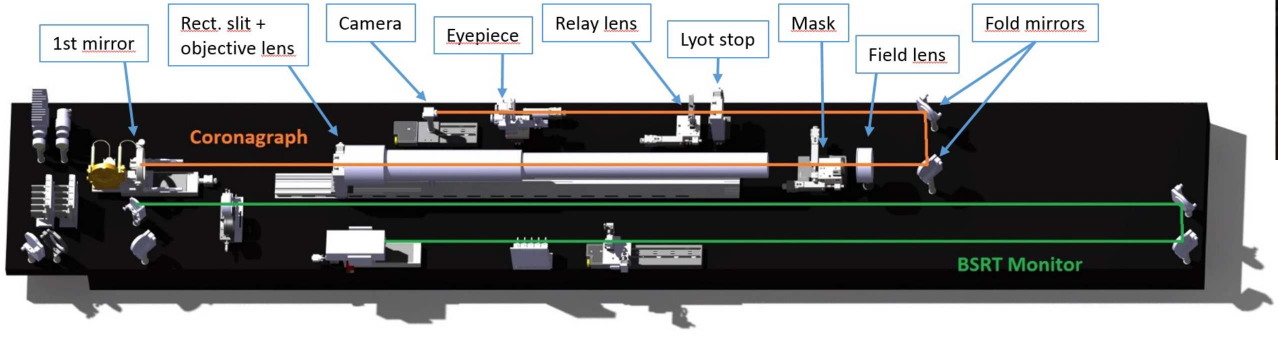
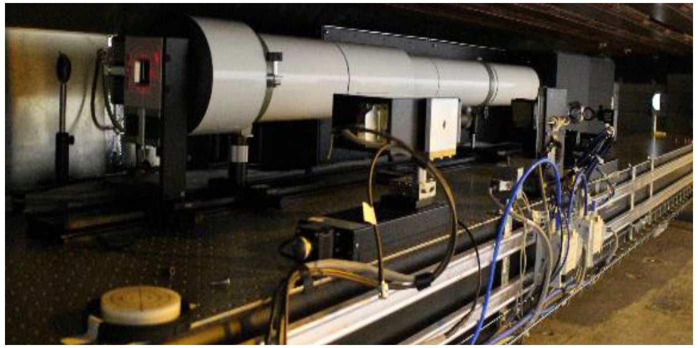


Figure 4: 3D drawing of the LHC halo monitor.

- *Observe synchrotron light form LHC:*
  - Opaque disk blocks the beam core.
  - However, the limited diameter of the object lens creates unwanted diffraction, which overlays the halo.
  - By adding the field lens to image the objective lens, the unwanted diffraction moves radially out.
  - A Lyot stop is then used to block the diffraction, allowing only the LHC halo to be imaged.



G. Trad, **T. Mitsuhashi**, E. Bravin, A. Godblatt, F. Roncarolo
First Observation of the LHC Beam Halo Using a Synchrotron Radiation Coronagraph
http://inspirehep.net/record/1626217/files/tuoab2.pdf

Summary of ‘Introduction to Optics’

- *The simple **refractive nature** of electromagnetic waves enables complex optical instruments to be designed from multiple elements:*
  - *Light propagation is typically calculated by dedicated ray tracing software, based on matrix methods.*
- ***Interference** is a powerful tool for precise displacement measurements with sensitivities at a fraction of the wavelength of light*
  - *we we explore some relevant examples in the following lectures.*
- ***Diffraction** effects must be considered when designing instruments, with numerical calculations based on the Fourier Transform of the transmission function of the aperture.*
  - *Spurious effects can typically be spatially filtered in the Fourier plane, or by applying a mask on the Fourier Transform in software to reconstruct only the image of interest.*

- *Next time: lasers, fibre optics and applications.*