Beam-Beam Effects in colliders

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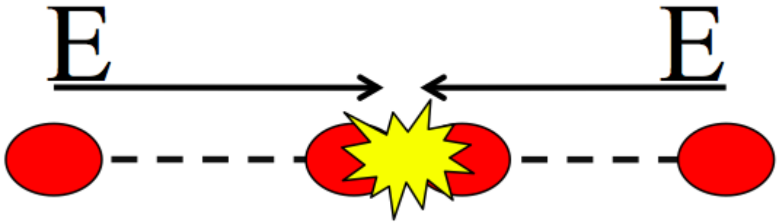
CERN BE-ABP-ICE

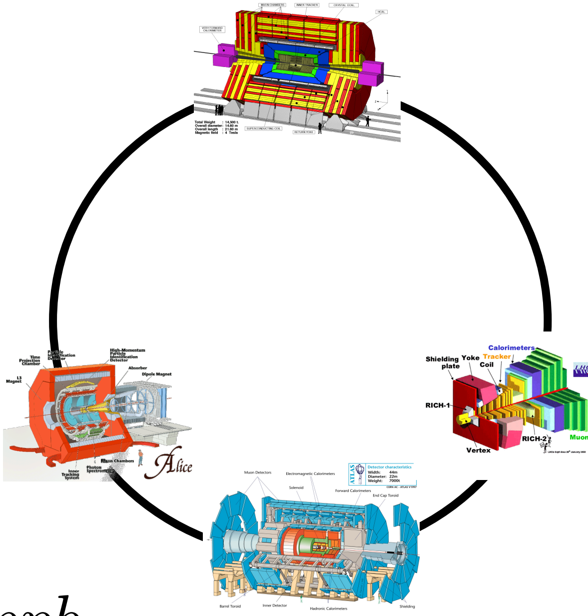
Hadron Colliders

$$E^* \approx 2 \times E$$



$$N_{event/s} = L \cdot \sigma_{event}$$

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$



Bunch intensity: $N_p = 1.15 - 1.65 \cdot 10^{11}\ ppb$

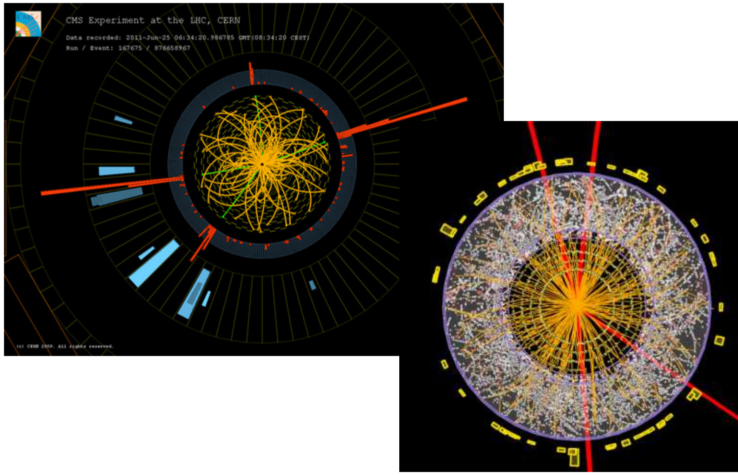
Transverse Beam size: $\sigma_{x,y} = 16 - 30\ \mu m$

Number of bunches $1370 - 2808$

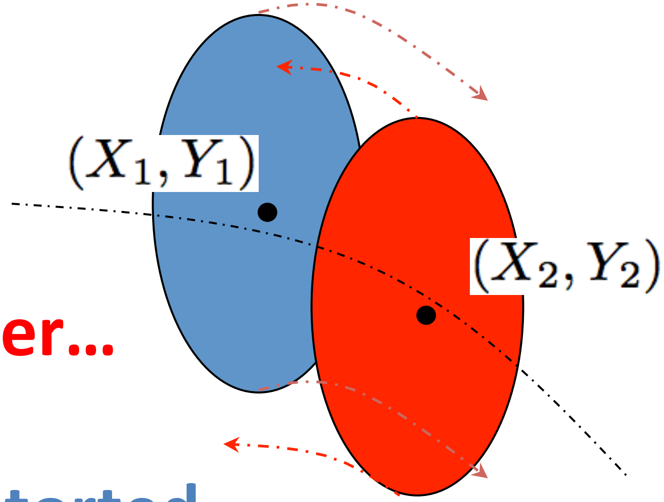
Revolution frequency $11\ kHz$

When do we have beam-beam effects?

- They occur when two beams get closer and collide



- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)



- **Unfortunately: usually both go together…**
- **0.001% (or less) of particles collide**
- **99.999% (or more) of particles are distorted**

Beam-beam effects: overview

- **Circular Colliders**: interaction occurs at every turn
  - Many effects and problems
  - Try to understand some of them

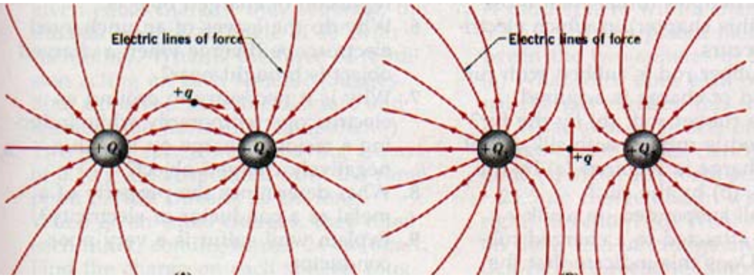
- Overview of selected effects (single particle and multi-particle effects)
- Qualitative and physical picture of effects
- Observations
- Mathematical derivations and more info in References [#] or at

Beam-beam webpage http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/

And CAS Proceedings

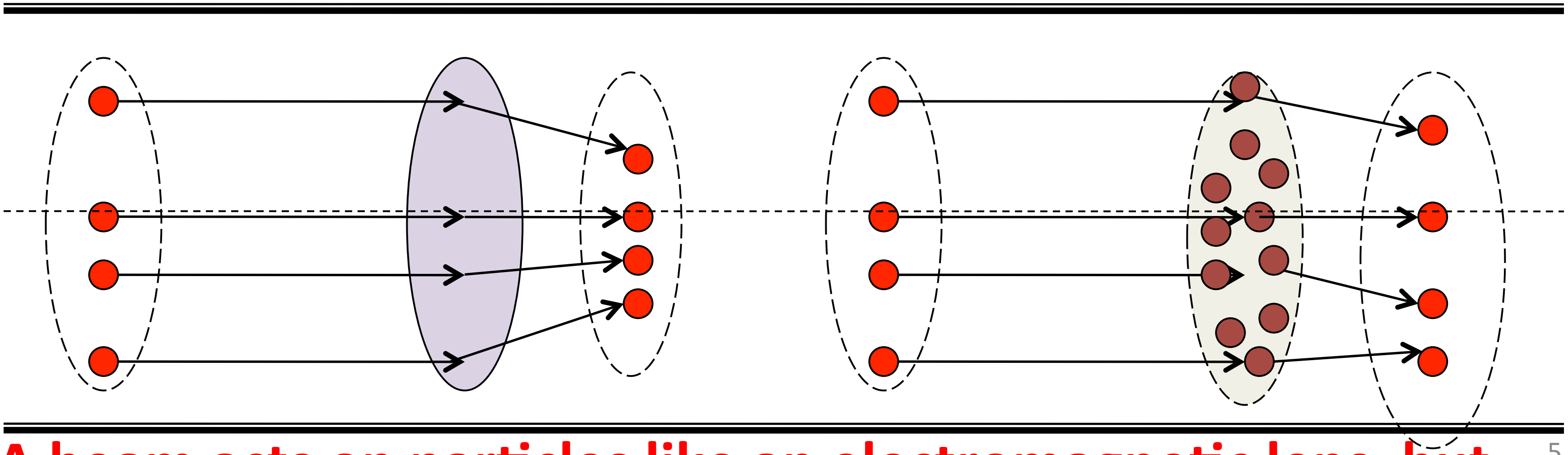
Beams EM potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges



Force on itself (space charge) and opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted


Focusing quadrupole

Opposite Beam

A beam acts on particles like an electromagnetic lens, but…

Beam-beam Mathematics

General approach in electromagnetic problems **Reference[5]** already applied to beam-beam interactions in **Reference[1,3, 4]**

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int \frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}$$

$$\overrightarrow{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force…

Round Gaussian distribution:

Gaussian distribution for charges:
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

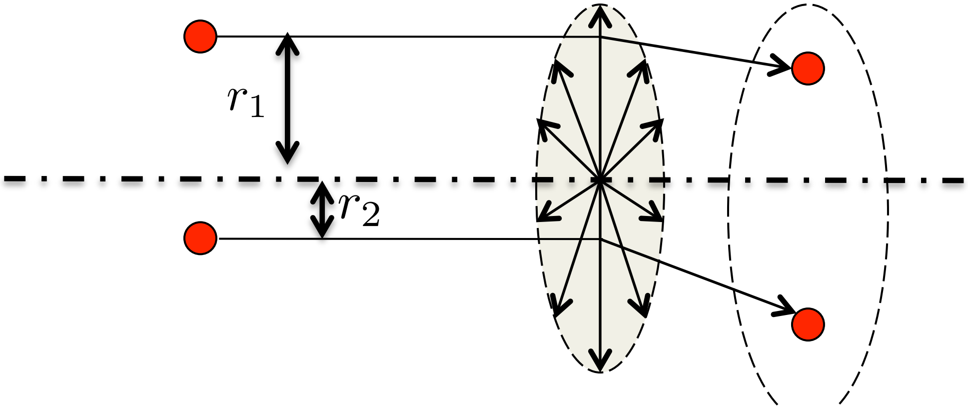
Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r,s,t)\ dt$$

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

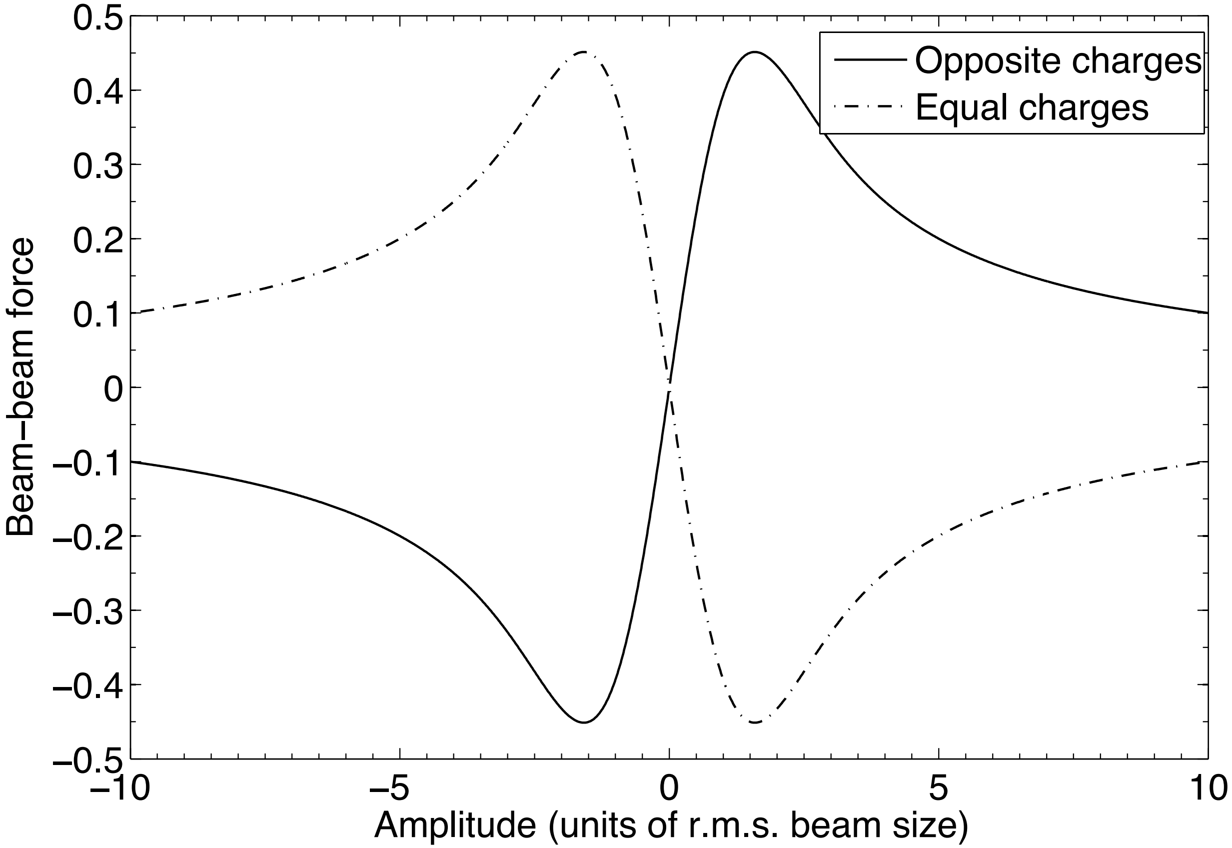
$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2}[1 - e^{-\frac{r^2}{2\sigma^2}}]$$

Only radial component in relativistic case



How does this force looks like?

Beam-beam Force



$$F_r(r) = \pm \frac{ne^2(1+\beta_{rel}^2)}{2\pi\epsilon_0}\frac{1}{r}[1-\exp(-\frac{r^2}{2\sigma^2})]$$

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Physics fill lasts for many hours 10h – 24h

Strongest non-linearity in a collider YOU CANNOT AVOID!

La Une | Mercredi 4 juillet 2012 | Dernière mise à jour 18:09

Mon journal numérique | Abonnements | Météo: Genève 26°

Tribune deGenève

CARNET NOIR
L'acteur de télévision Andy Griffith est mort à 86 ans

CHAMP-DOLLON
Pour s'être plaint sur Facebook, un gardien est puni

FORMULE 1
La pilote Maria De Villota grièvement blessée

PHYSIQUE

Une nouvelle particule a été découverte

Une nouvelle particule a été découverte par des chercheurs du CERN lancés sur la trace du boson de Higgs. Plus...

Mis à jour il y a 2 minutes

Bourse › SWISSQUOTE

SMI	6'191.43	-0.04%
Stoxx50	2'437.70	-0.21%
DJIA	12'943.82	+0.56%

Les plus lus ›

1 Le Conseil d'Etat a fait valser ses cadres. Une première
2 Les fontaines de Dubai pleurent Whitney Houston
3 Champ-Dollon: pour s'être plaint sur Facebook, un gardien est puni
4 La pilote Maria De Villota grièvement blessée
5 Sentier des Toblerones: Apple censure la ville de Gland

Genève au fil du temps

Aux origines du casino de Genève. Le Kursaal fut l'une des attractions de la ville pendant 80 ans.

Voir nos galeries photo

Cet été, bien informé rime avec mobilité

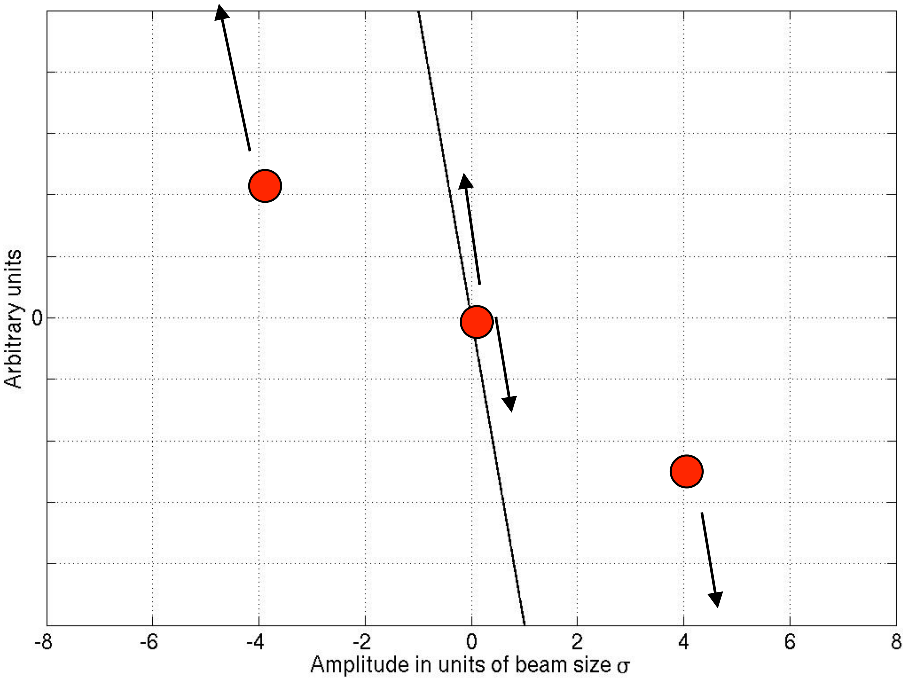
Two main questions:

What happens to a single particle?

What happens to the whole beam?

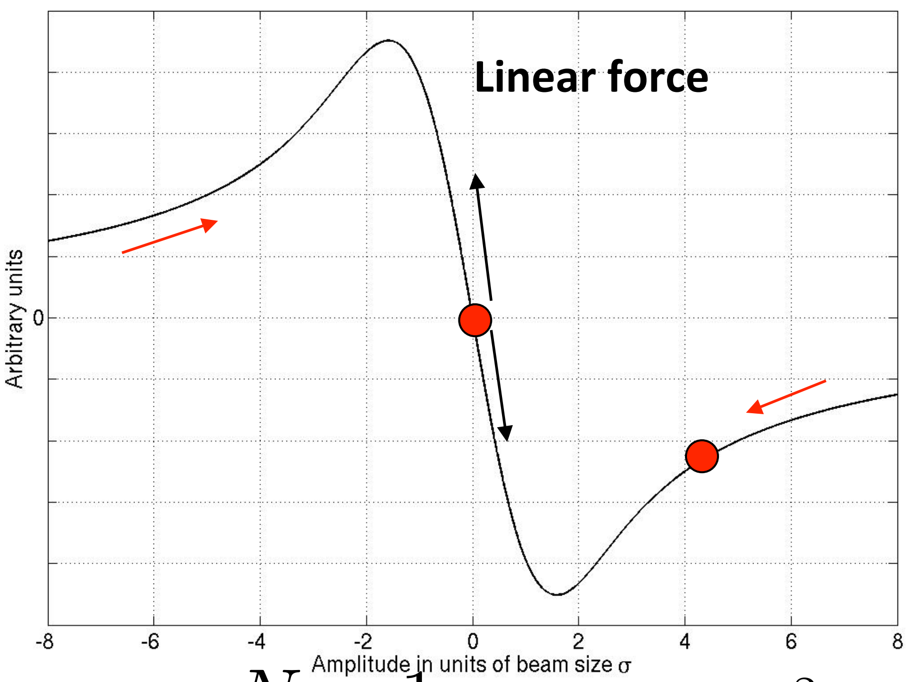
Beam-Beam Force: single particle…

Lattice defocusing quadrupole



$$F = -k \cdot r$$

Beam-beam force



$$\mathrm{F} \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}} \right]$$

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

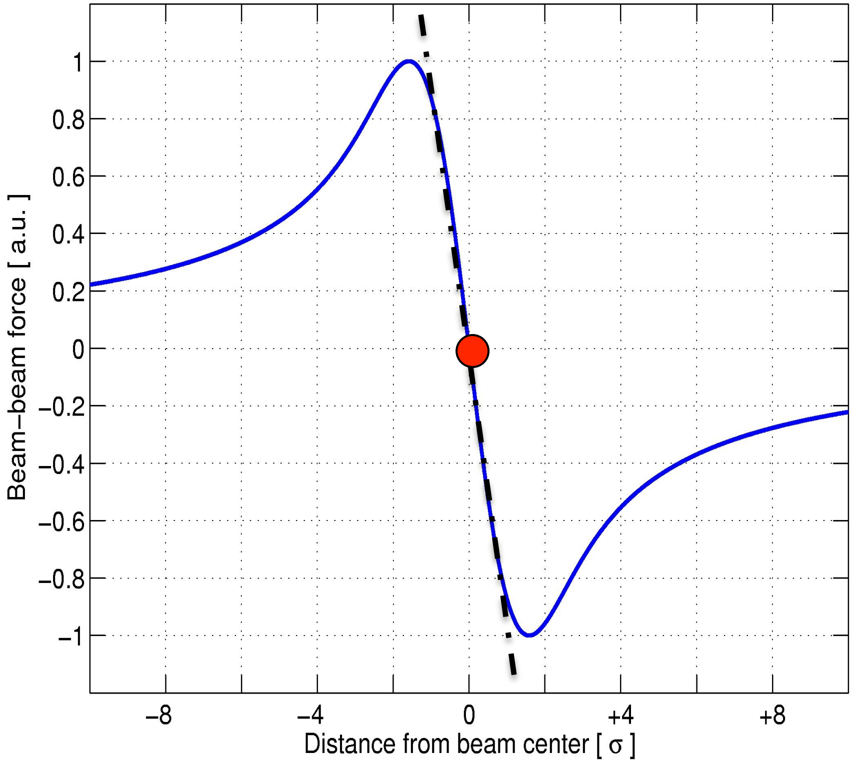
Can we quantify the beam-beam strenght?

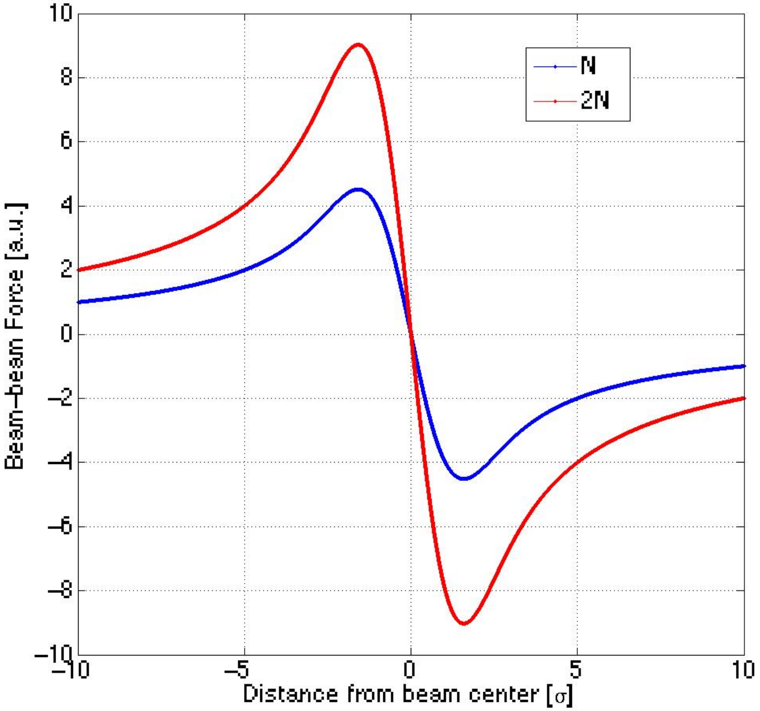
Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force

For small amplitudes: linear force

$$F \propto -\xi \cdot r$$

The slope of the force gives you the beam-beam parameter ξ


Beam-beam force



$$\Delta r' = -\frac{N_p r_0}{r}\cdot\frac{r}{r^2}\cdot\left[1-\mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma}\cdot\frac{1}{r}\cdot\left[1-\left(1-\frac{r^2}{2\sigma^2}+\ldots\right)\right]$$

Colliders:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r^{'})}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Examples:

Parameters	LEP (e^+e^-)	LHC(pp)
Intensity $N_{p,e}$/bunch	4 10^{11}	1.15 10^{11}
Energy GeV	100	7000
Beam size H	160-200 μm	16.6 μm
Beam size V	2-4 μm	16.6 μm
$\beta_{x,y}^*$ m	1.25-0.05	0.55-0.55
Crossing angle μrad	0	285
ξ_{bb}	**0.07**	**0.0037**

Linear Tune shift

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta *}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta *} & 1 \end{pmatrix}$$

Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β*:

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta *} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Linear tune

Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β*:

Tune is changed

$$cos(2\pi(Q+\Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} sin(2\pi Q)$$

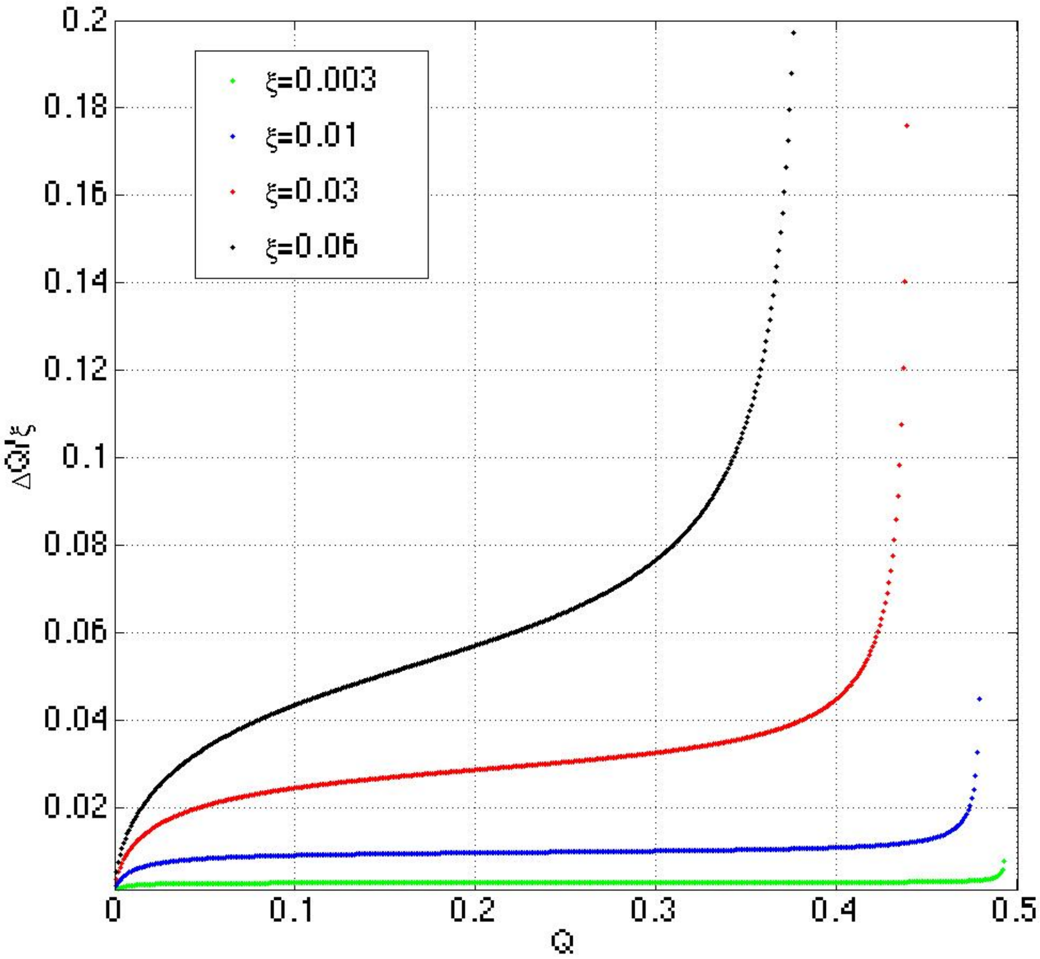
β-function is changed:

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q+\Delta Q))}$$

…how do they change?

Tune dependence of tune shift and dynamic beta

Tune shift as a function of tune

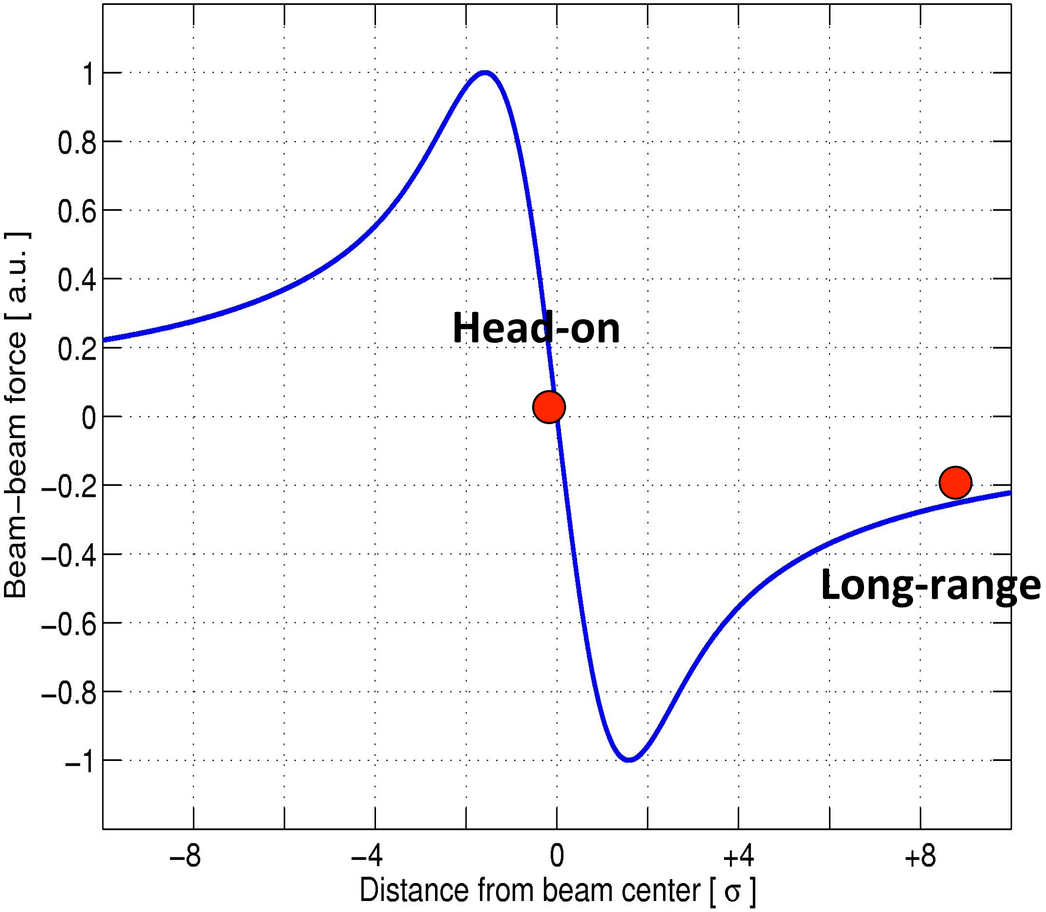


Larger ξ → **Strongest variation with Q**

Head-on and Long-range interactions

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$

Beam-beam force

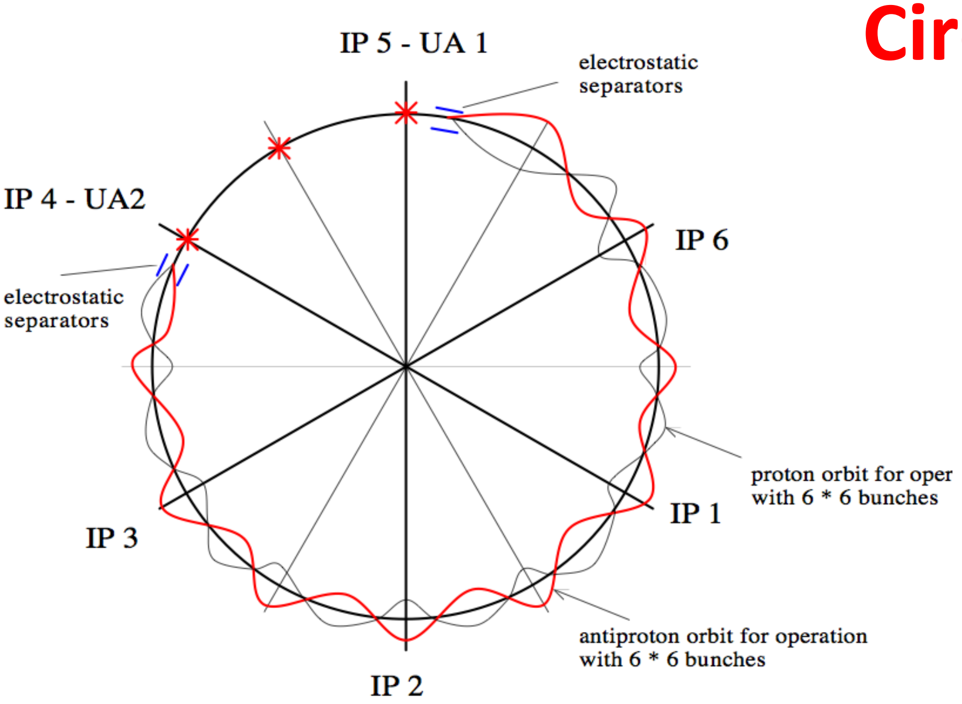


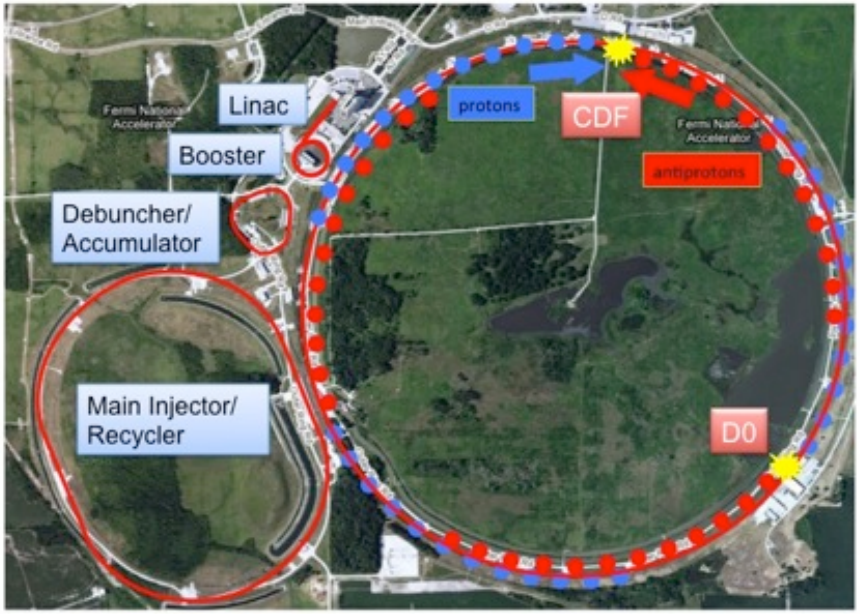
Other beam passing in the center force: HEAD-ON beam-beam interaction

Other beam passing at an offset of the force: LONG-RANGE beam-beam interaction

Circular colliders HO and LR

SPS collider: 6 bunches
3 HO and 9 LR



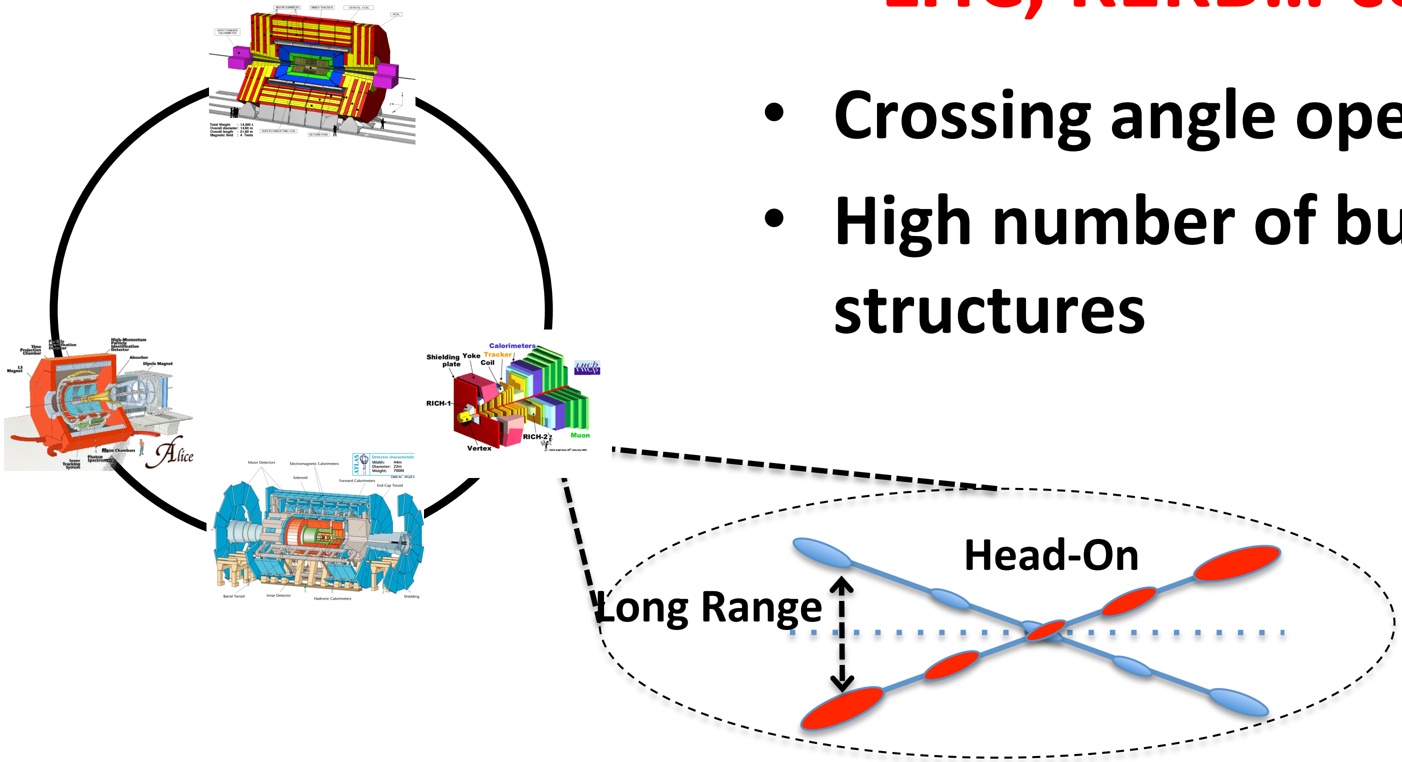


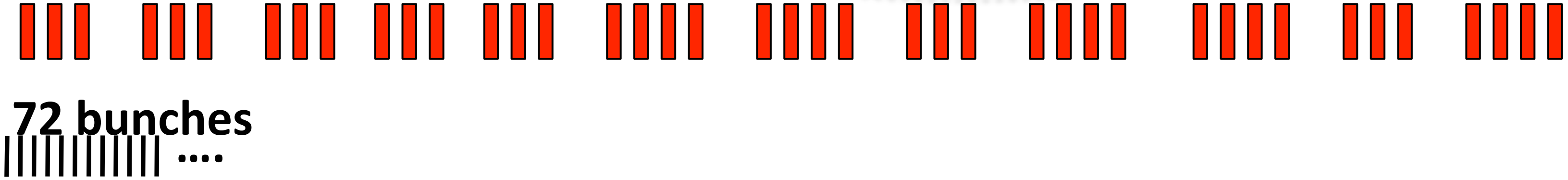
Tevatron: 36 bunches
2 BBIs Head-on and 72 Long-range

RHIC: 110 bunches
2 BBIs Head-on

LHC, KEKB… colliders

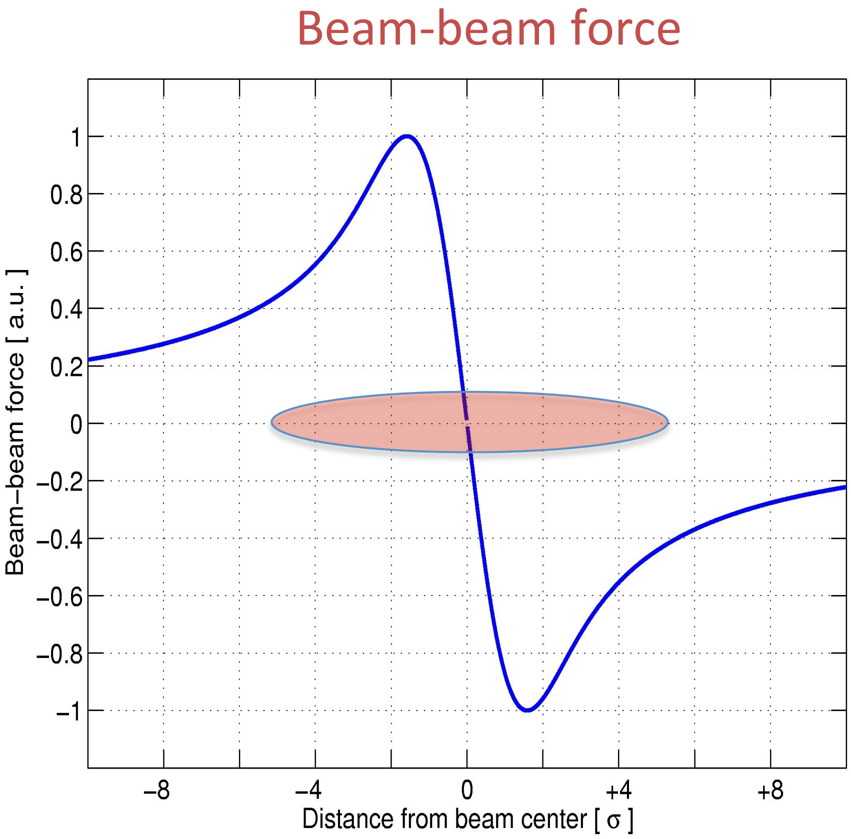
- **Crossing angle operation**
- **High number of bunches in train structures**





	SppS	Tevatron	RHIC	LHC
Number Bunches	6	36	109	2808
LR interactions	9	70	0	120/40
Head-on interactions	3	2	2	4

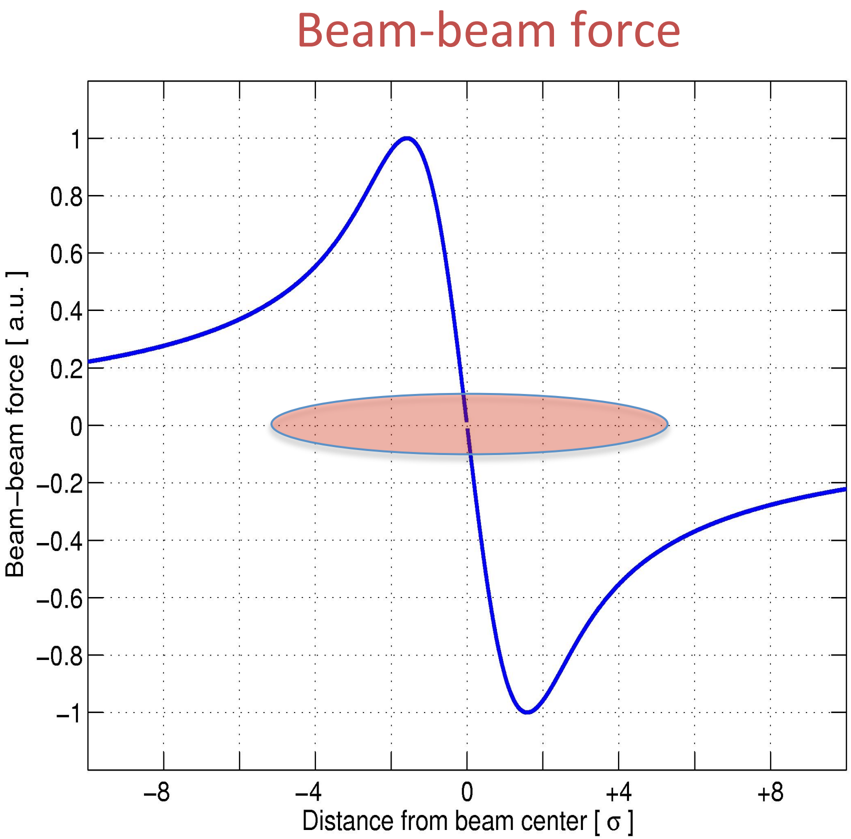
A beam is a collection of particles



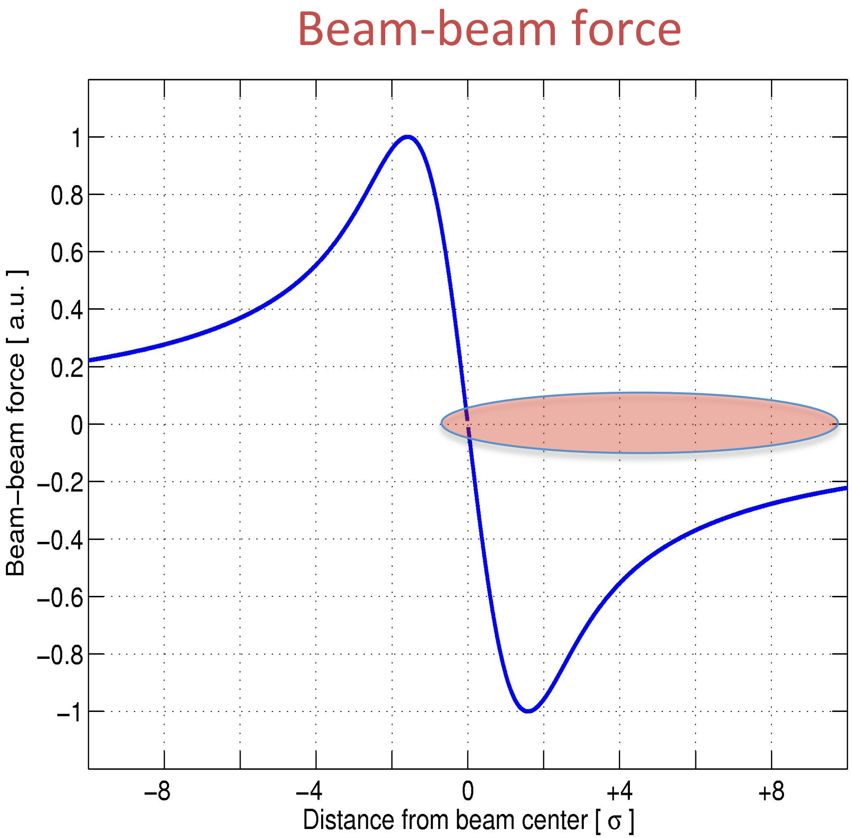
Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)

A beam will experience all the force range


Beam-beam force

Second beam passing in the center
HEAD-ON beam-beam interaction


Beam-beam force

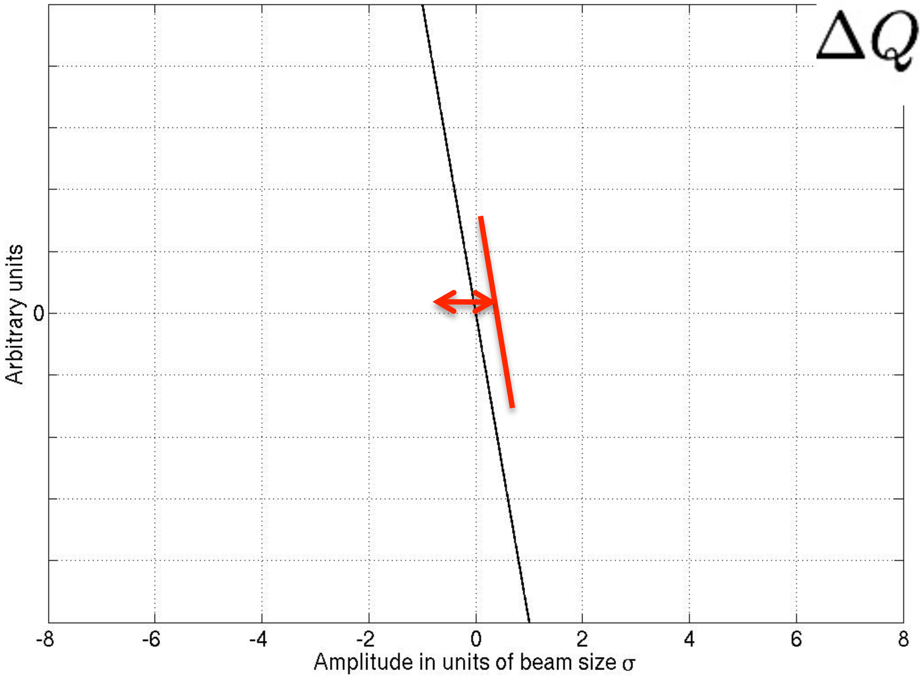
Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force

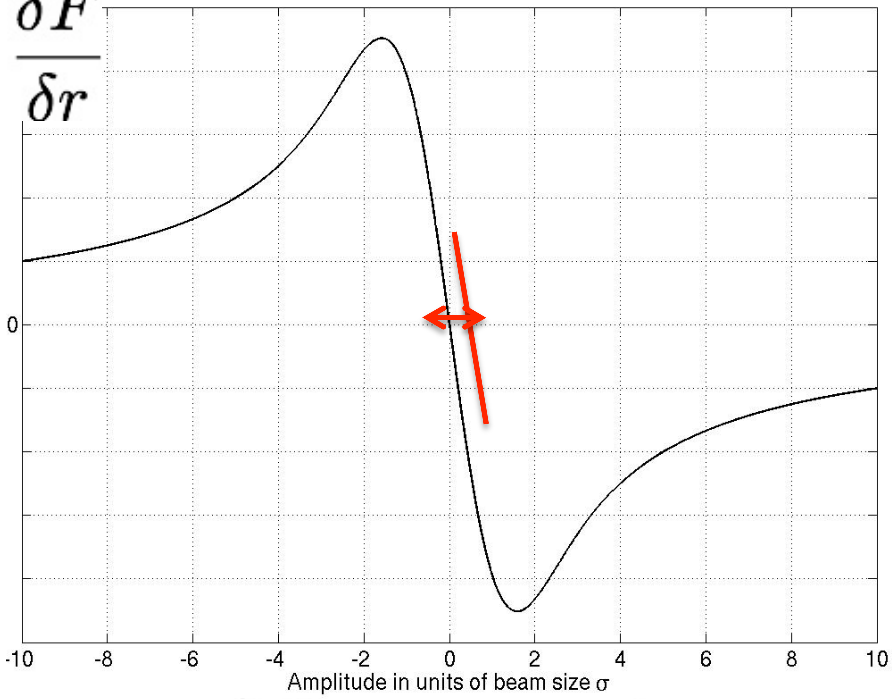
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude

$$\Delta Q \propto \frac{\delta F}{\delta r}$$



$$\Delta Q_{quad} = const$$



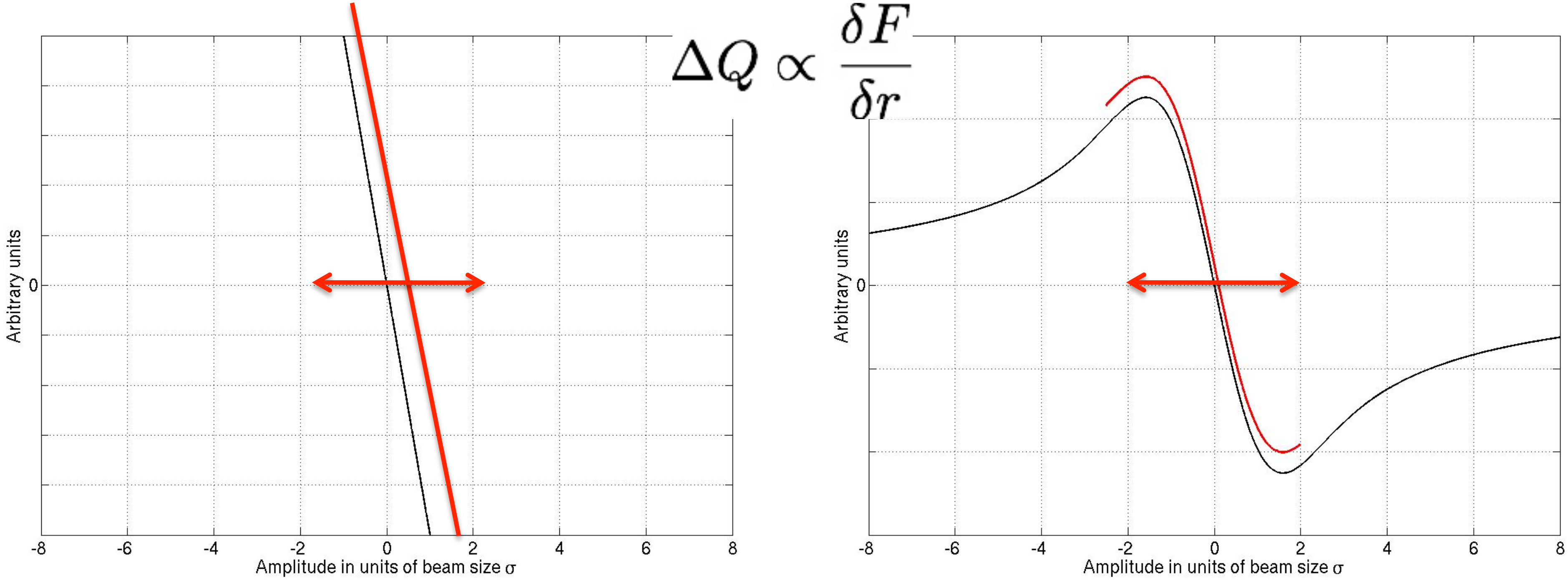
$$\Delta Q_{bb} \approx const$$

For small amplitude test particle linear tune shift

$$\lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread



$$\Delta Q \propto \frac{\delta F}{\delta r}$$

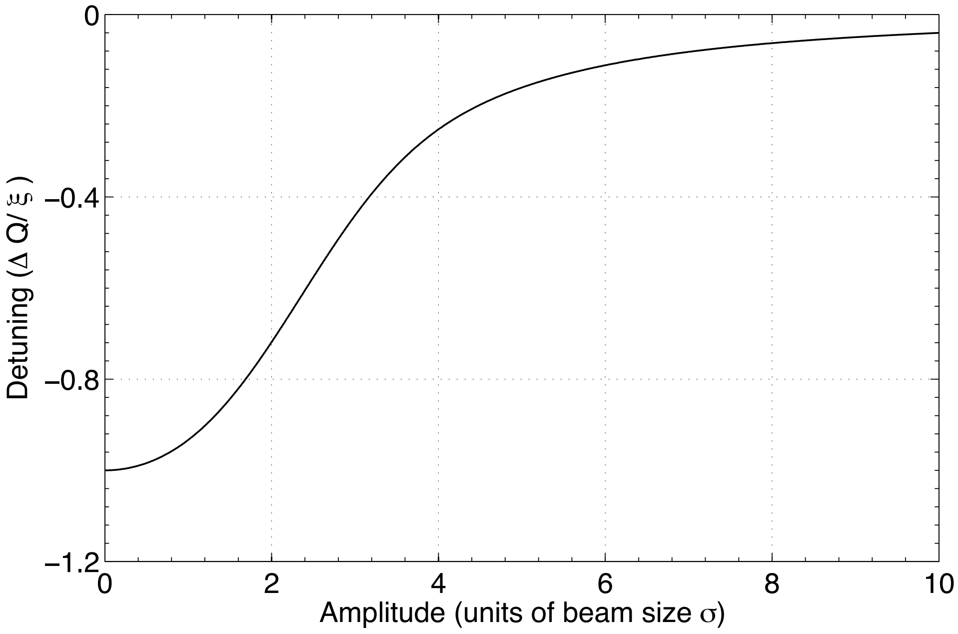
$$\Delta Q_{quad} = const \qquad \Delta Q_{bb} \neq const$$

$$\Delta Q(x) = \frac{N r_0 \beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0(\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude

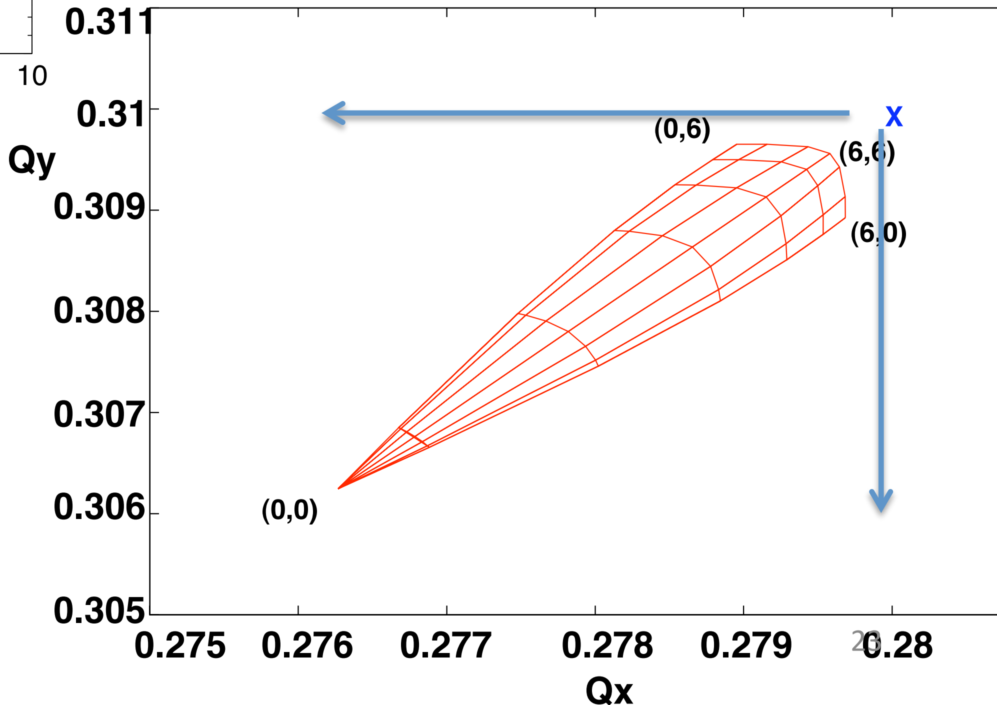


And in the other plane?

THE SAME DERIVATION

same tune spread

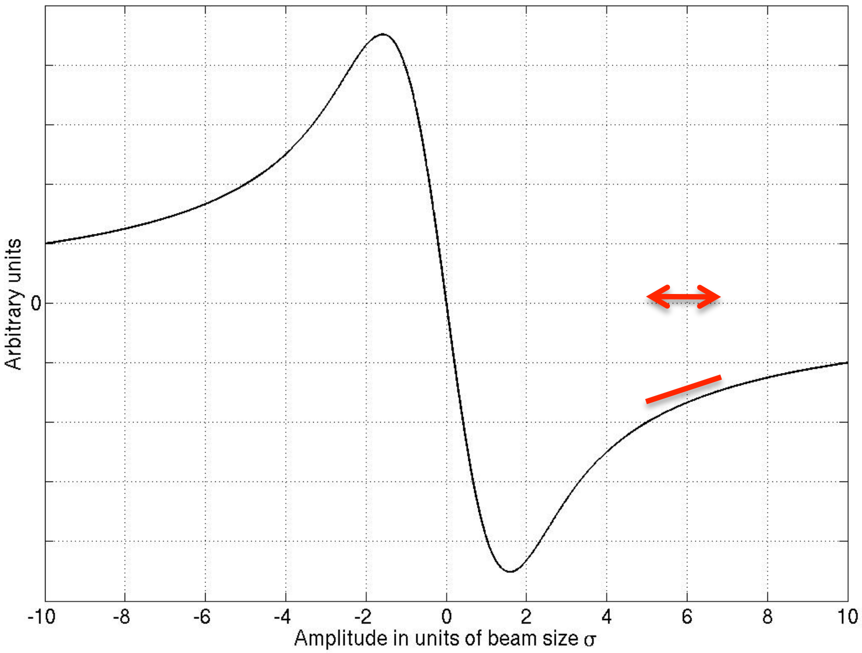
Tune footprint for head–on collision



FOOTPRINT

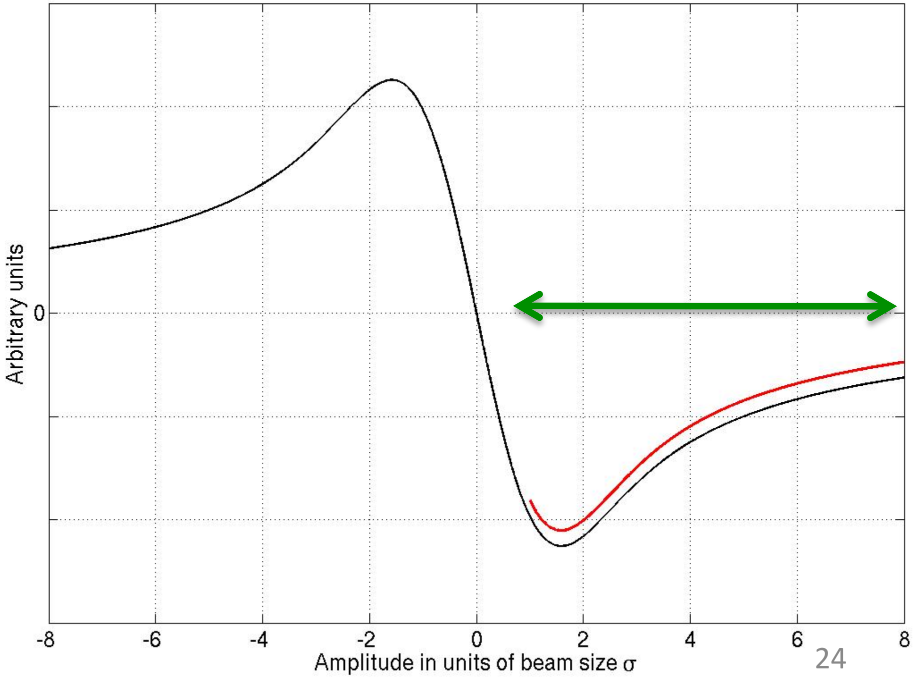
2-D mapping of the detuning with amplitude of particles

And for long-range interactions?



Second beam centered at d (i.e. 6σ)

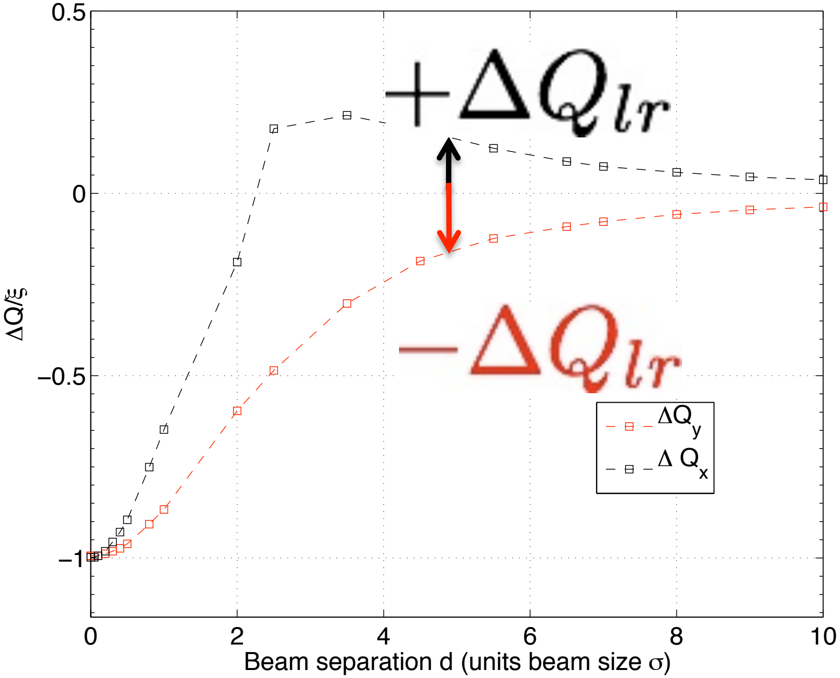
- **Small amplitude particles positive tune shifts**
- **Large amplitude can go to negative tune shifts**



Long range tune shift scaling for distances $d > 6\sigma$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

Long-range footprints



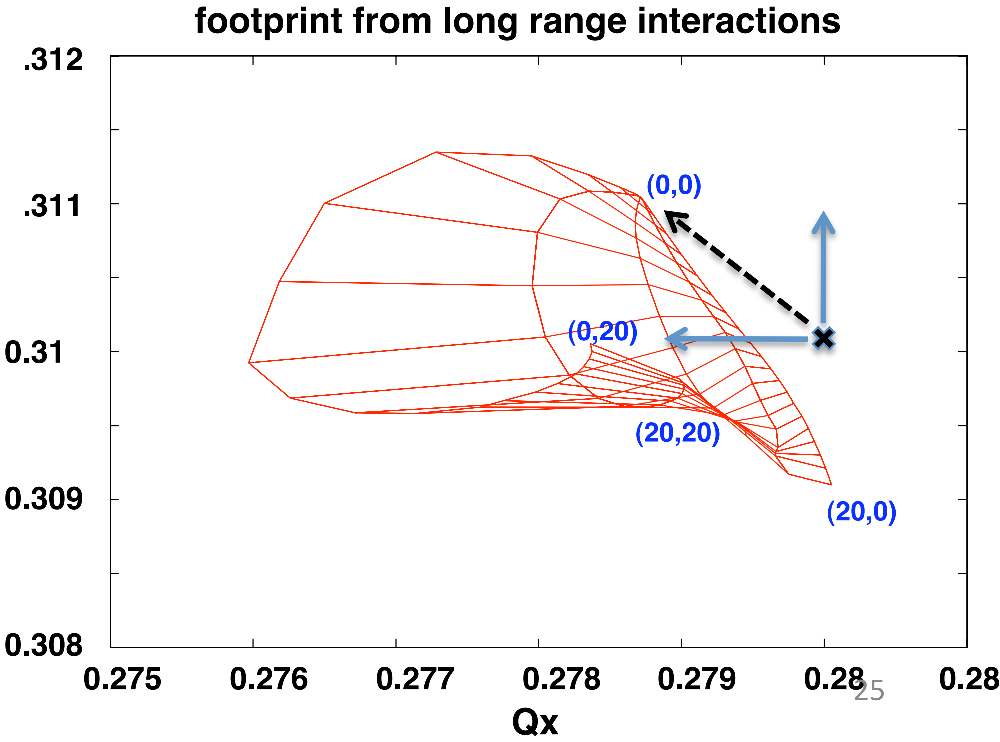
Separation in vertical plane!

And in horizontal plane?

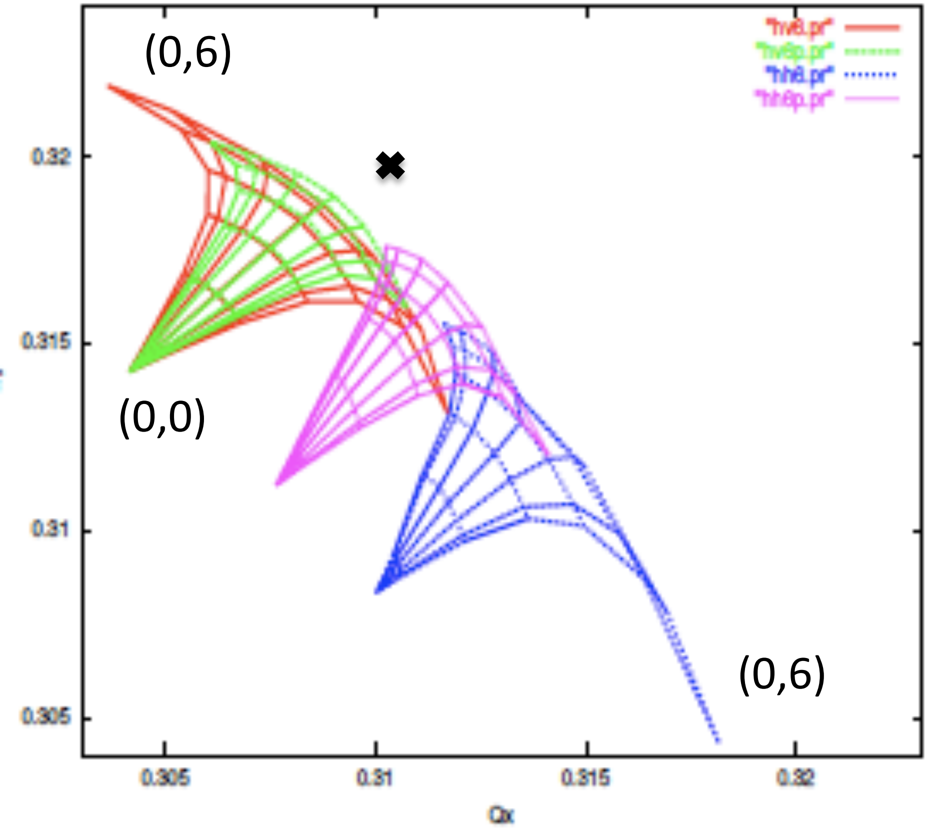
The test particle is centered with the opposite beam

tune spread more like for head-on at large amplitudes

The picture is more complicated now the LARGE amplitude particles see the second beam and have larger tune shift



Beam-beam tune shift and spread



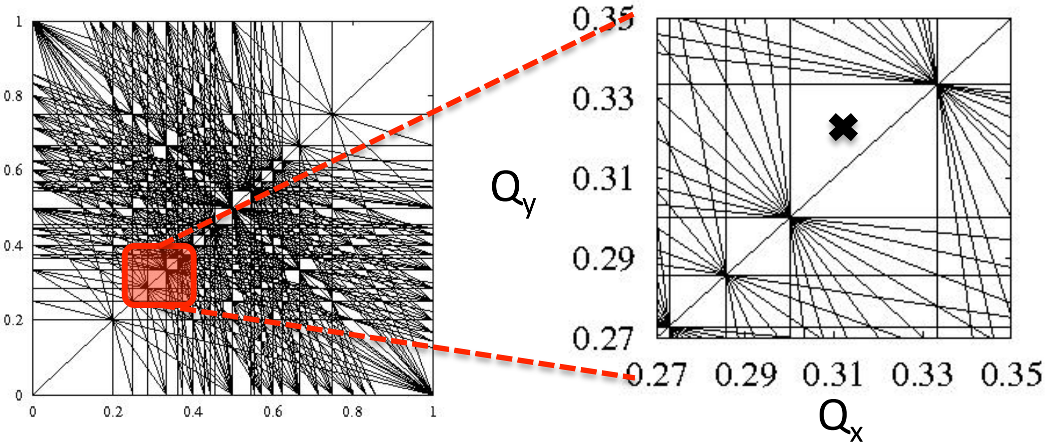
Footprints depend on:

- number of interactions
- Type (Head-on and long-range)
- Plane of interaction

When long-range effects become important footprint wings appear and alternating crossing important

Aim to reduce the area as much as possible!

Passive compensation of tune shift Ref[7]



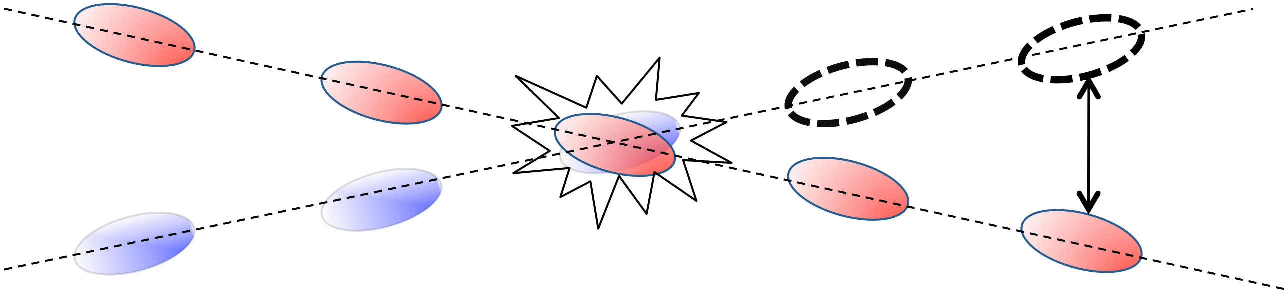
Complications

PACMAN and SUPER PACMAN bunches

Pacman:

miss long range BBI

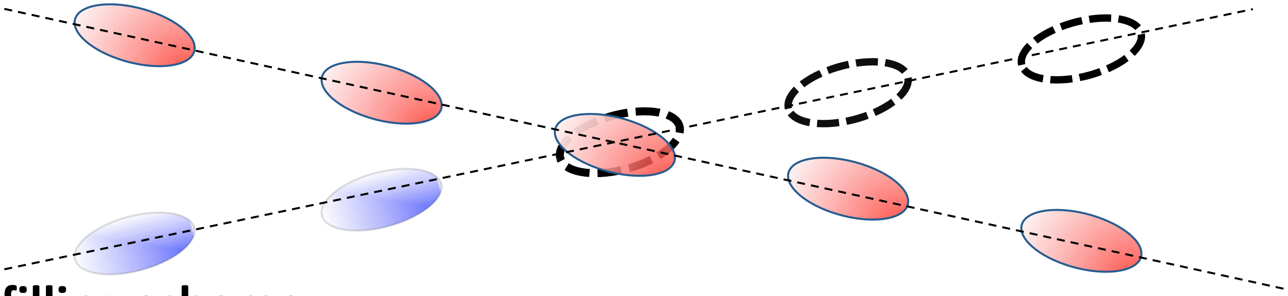
(120-40 LR interactions)



Super Pacman:

miss head-on BBI

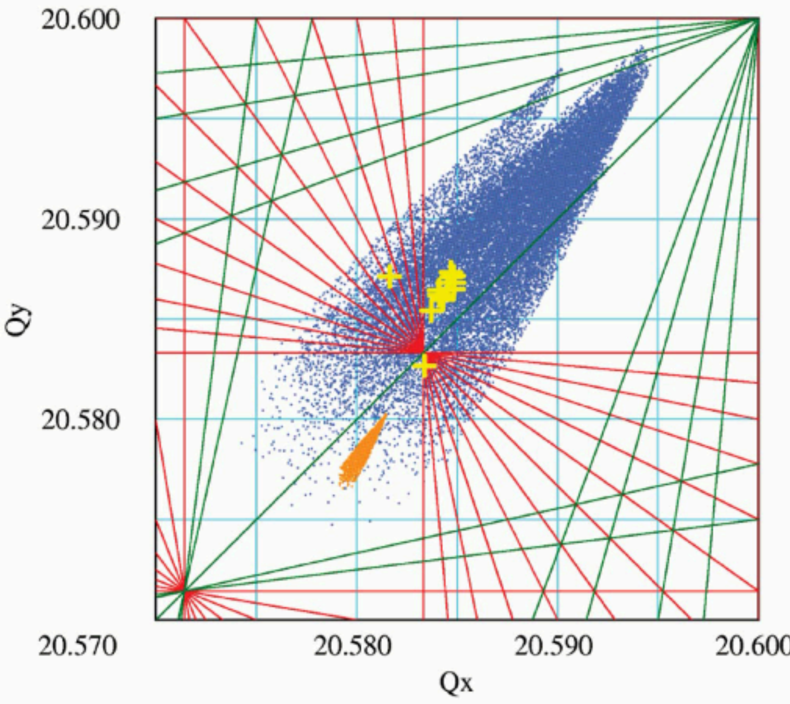
IP2 and IP8 depending on filling scheme



Different bunch families: Pacman and Super Pacman

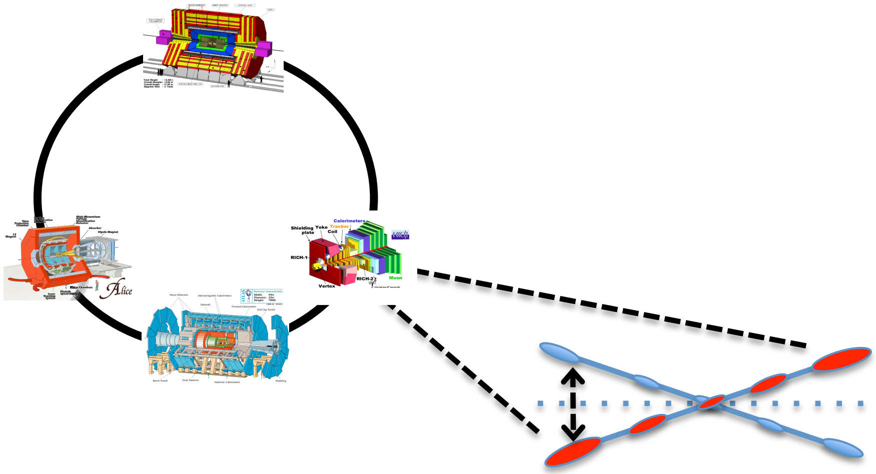
Pacman and Super-pacman

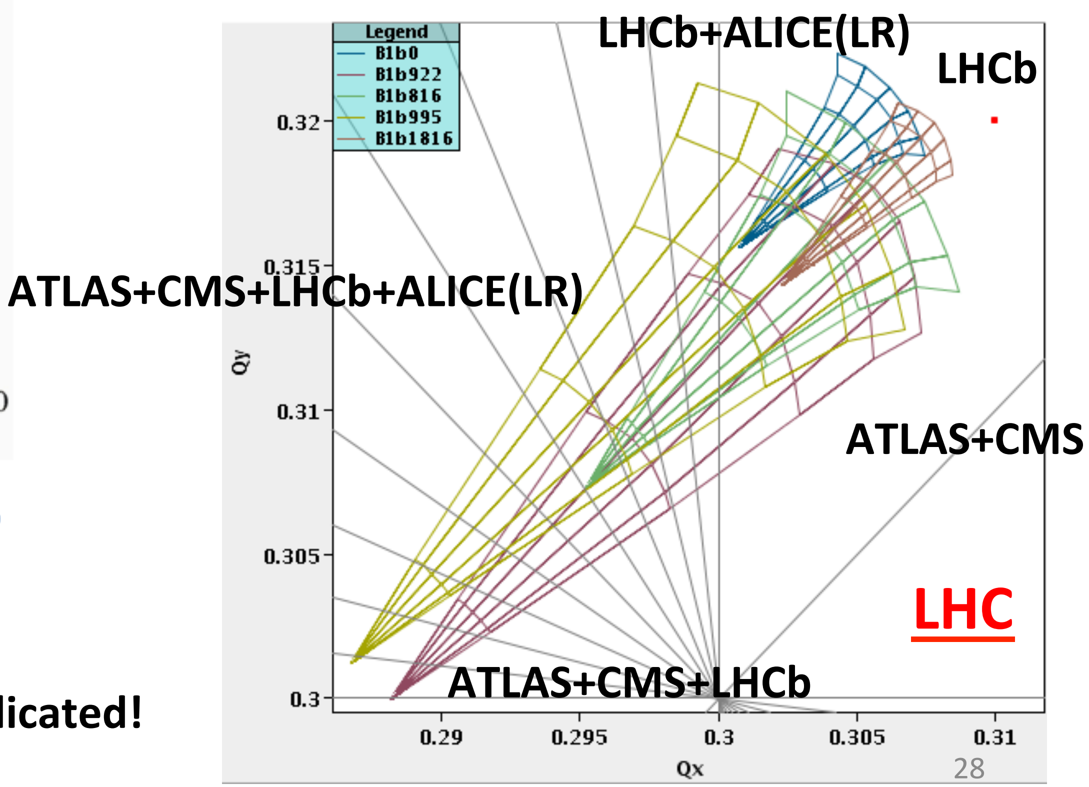
Tevatron



Antiproton bunches footprint (blue)
Proton bunches footprint (orange)

…operationally it is even more complicated!
…intensities, emittances…



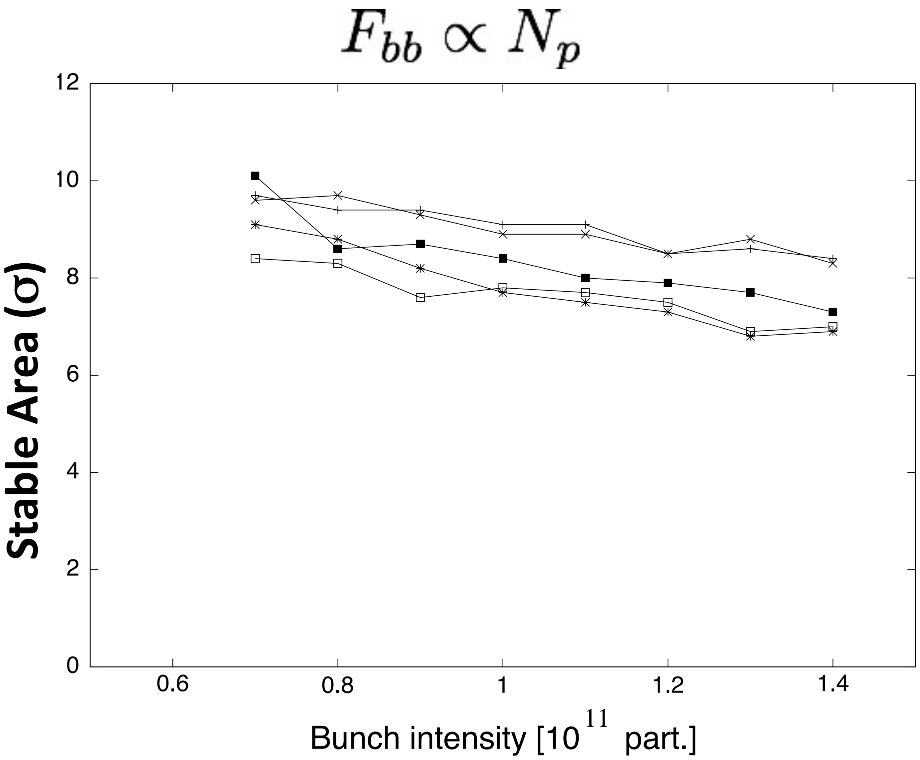


LHC

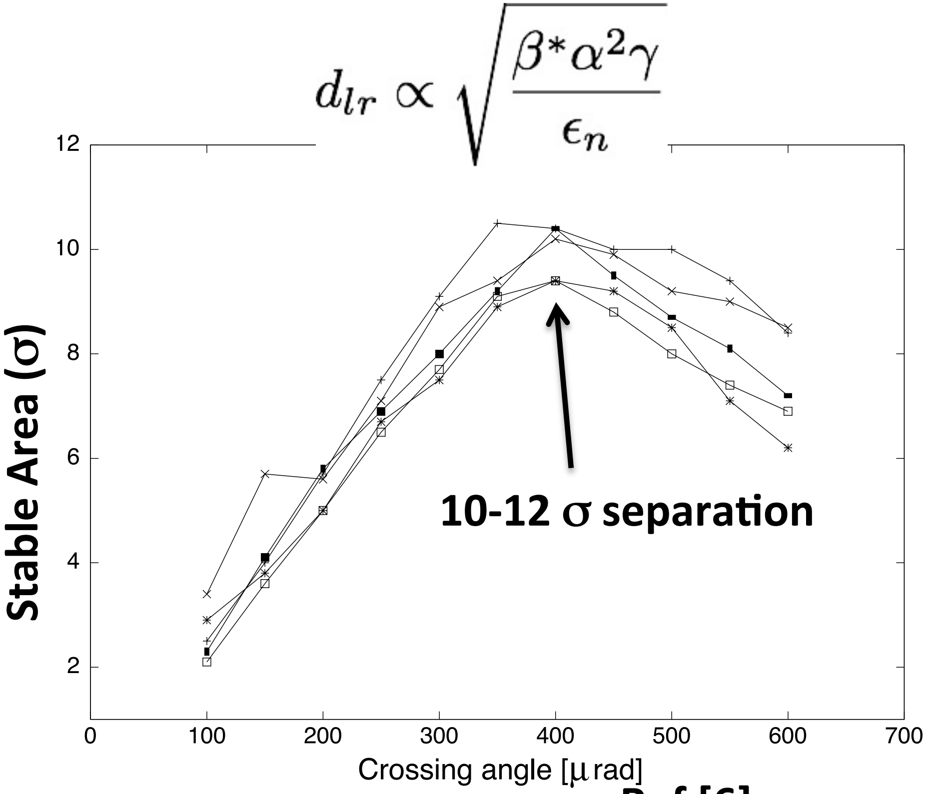
Particle Losses

Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle



$$F_{bb} \propto N_p$$



$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

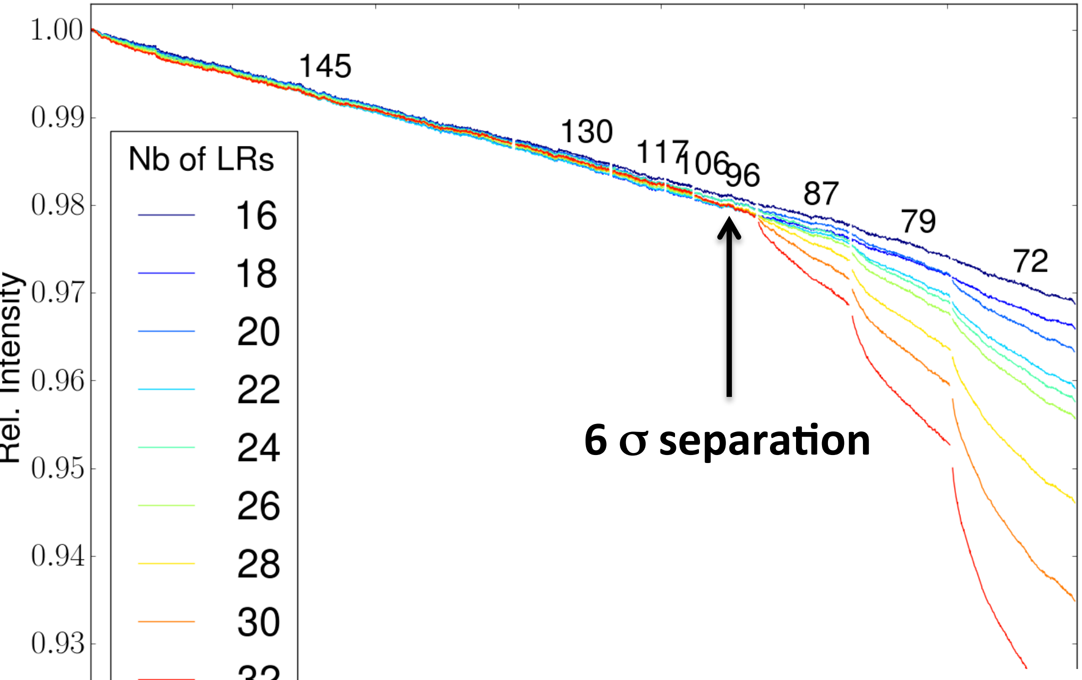
10-12 σ separation

Ref [6]

Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β*, intensity) is the result of careful study of different effects!

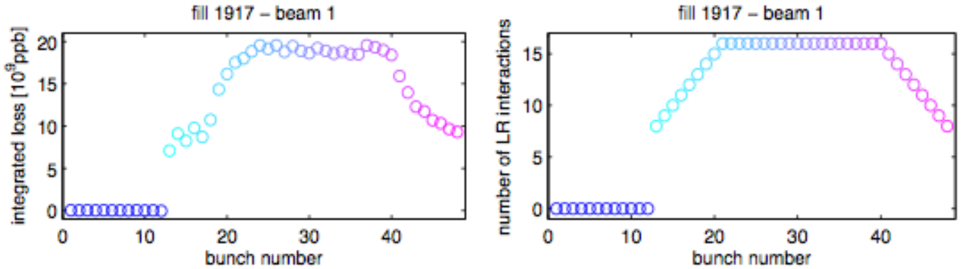
DO we see the effects of LR in the LHC?

Courtesy X. Buffat



Small crossing angle = small separation

If separation of long range too small particles become unstable and get lost



Courtesy G. Papotti

Particle losses follow number of Long range interactions

Nominal LHC will have twice the number of interactions

Observations in Leptons:

From our known formulas:

$$L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \qquad \xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

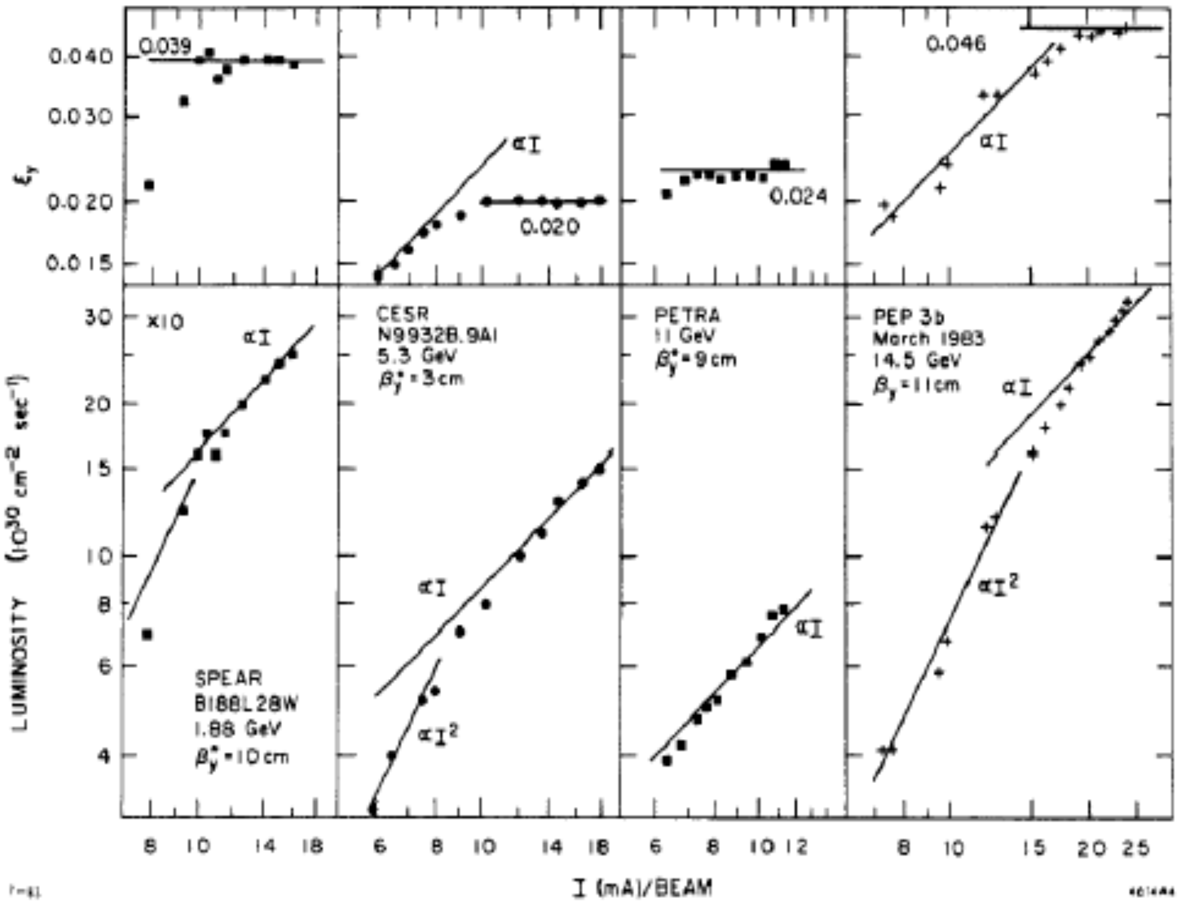
Increasing bunch population N_1 and N_2:

- luminosity should increase N_2 $$L \propto N_1 N_2$$
- beam-beam parameter linearly $$\xi \propto N_{1,2}$$

But…

Leptons beam-beam limit

First beam-beam limit (J. Seeman, 1983)



$$\xi \propto const$$

$$\xi \propto N_{1,2}$$

$$L \propto N_{1,2}$$

$$L \propto N_1 N_2$$

Luminosity and vertical tune shift parameter *vs.* beam current for SPEAR, CESR, PETRA & PEP.

What is happening?

Again….

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

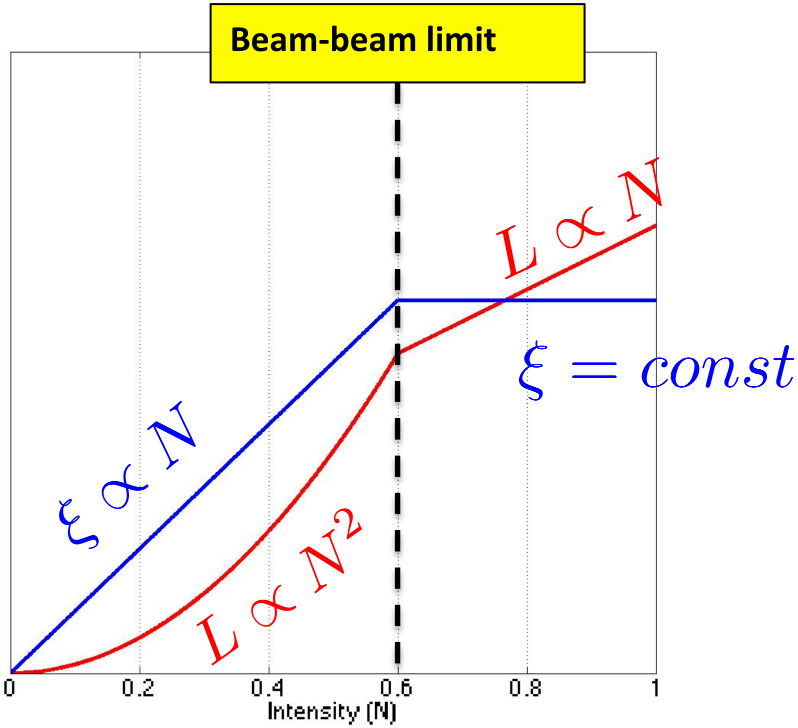
$$L = \frac{N^2 f n_b}{4\pi\sigma_x\sigma_y}$$

Lepton colliders $\sigma_x >> \sigma_y$

$$\xi_y \approx \frac{r_0\beta^*_y}{2\pi\gamma\sigma_x}\cdot\boxed{\frac{N}{\sigma_y}}$$

$$L = \frac{N f n_b}{4\pi\sigma_x}\cdot\boxed{\frac{N}{\sigma_y}}$$

As to be constant!



Above beam-beam limit:
σ_y increases when N increases to keep ξ constant

Equilibrium emittance

1. Synchrotron radiation: vertical plane damped, horizontal plane exited!
2. Horizontal beam size normally much larger than vertical (LEP 200 - 4 μm)
3. Vertical beam-beam effect depends on horizontal (larger) amplitude
4. Coupling from horizontal to vertical plane

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Equilibrium between horizontal excitation and vertical damping determines ξ_{limit}

Long-range BB and Orbit Effects

Long Range Beam-beam interactions lead to orbit effects

Long range kick

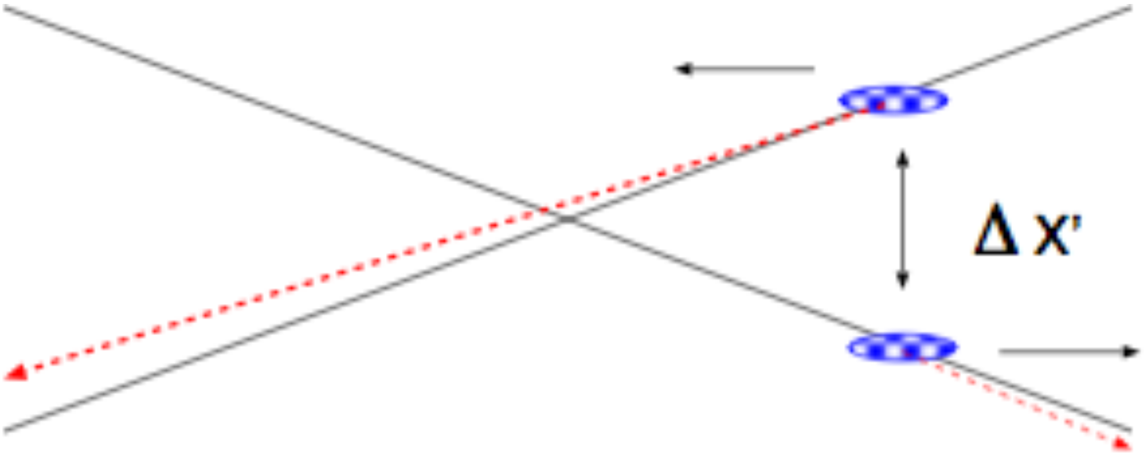
$$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}[1-\exp(-\frac{r^2}{2\sigma^2})$$

For well separated beams

$$d \gg \sigma$$

The force has an amplitude independent contribution: ORBIT KICK

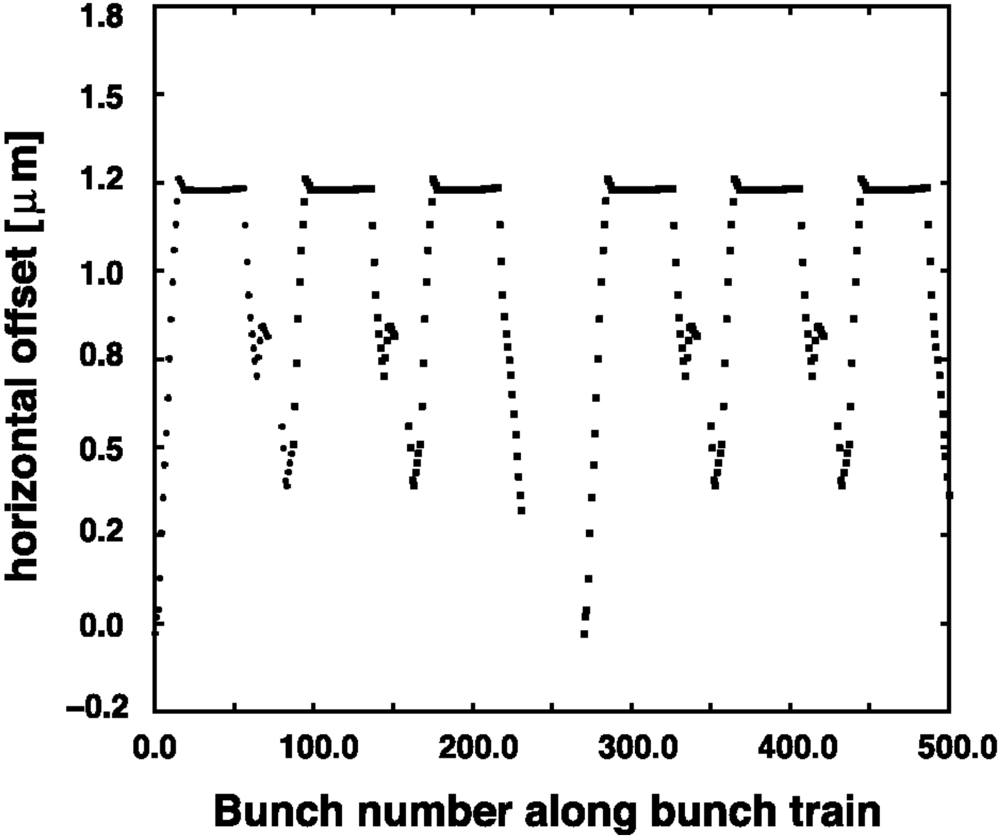
$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \ldots]$$



Orbit can be corrected but we should remember PACMAN effects

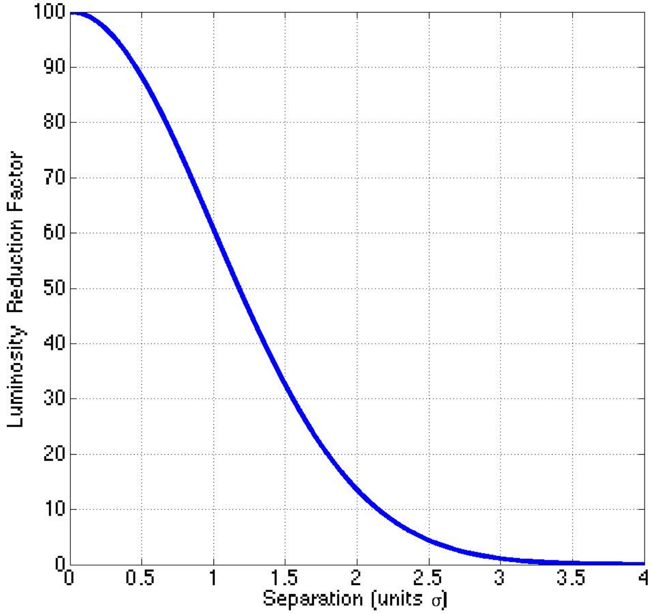
LHC orbit effects

Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect



d = 0 - 0.4 units of beam size

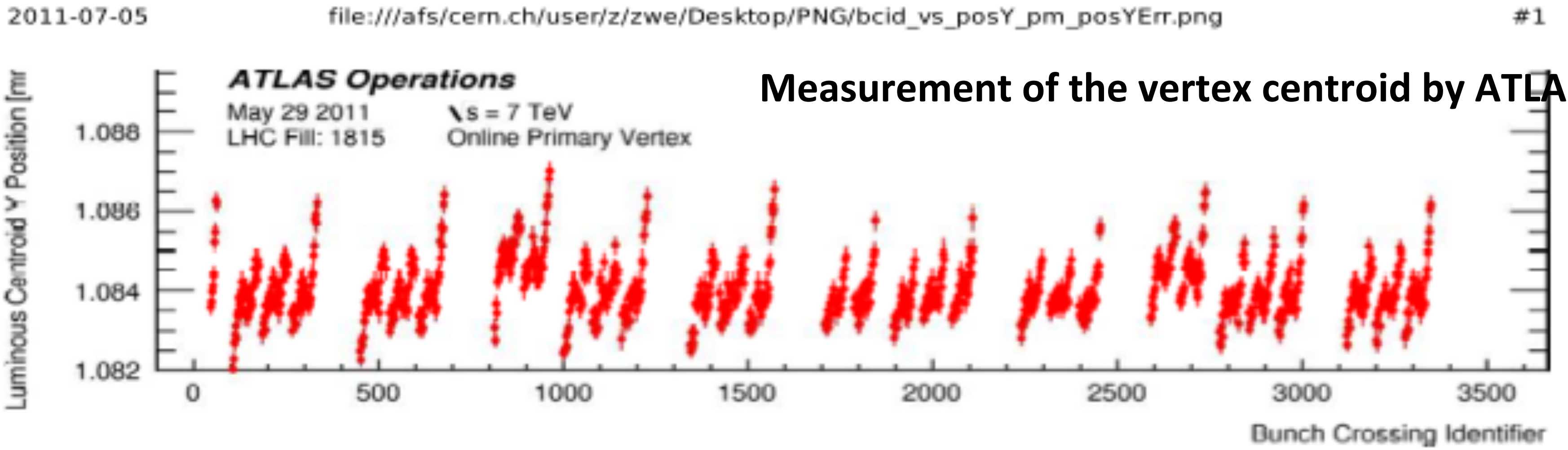
$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$



Ref [7]

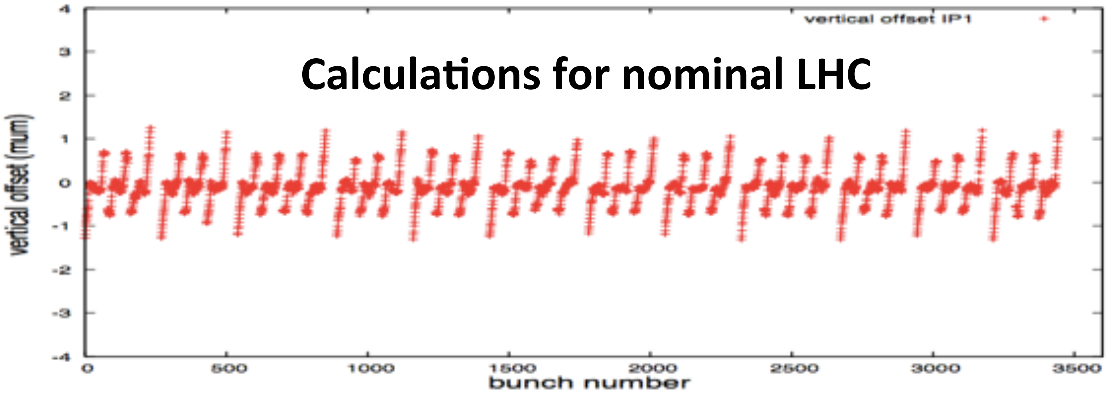
Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity



Measurement of the vertex centroid by ATLAS

Courtesy W. Kozanecki

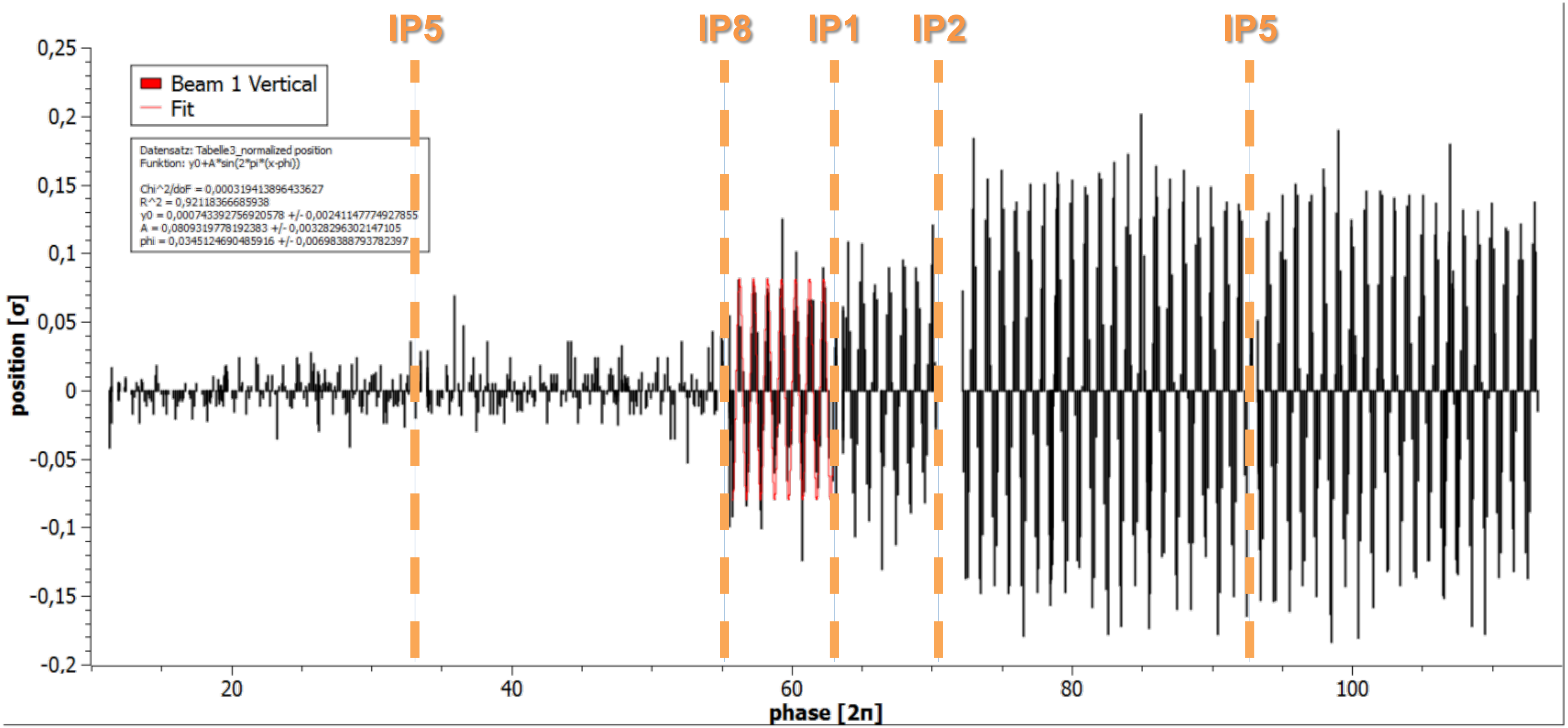


Calculations for nominal LHC

Effect is already visible with reduced number of interactions

Ref [7]

Long range orbit effect observations:



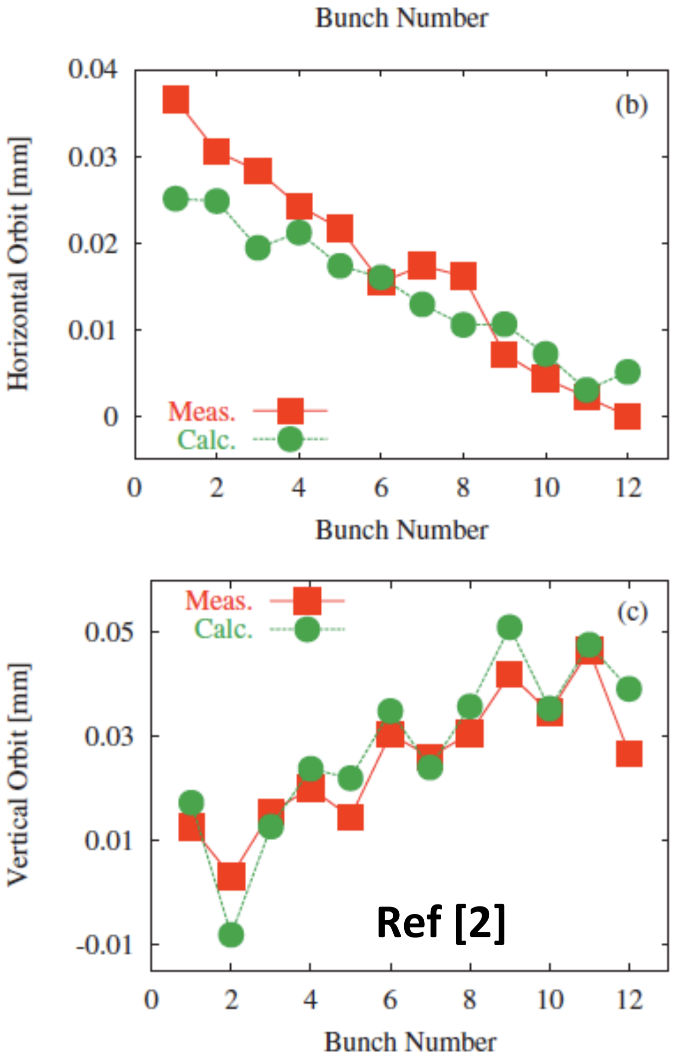
```
Datensatz: Tabelle3_normalized position
Funktion: y0+A*sin(2*pi*(x-phi))

Chi^2/doF = 0,000319413896433627
R^2 = 0,92118366685938
y0 = 0,000743392756920578 +/- 0,00241147774927855
A = 0,0809319778192383 +/- 0,00328296302147105
phi = 0,0345124690485916 +/- 0,00698388793782397
```

Courtesy T. Baer

Vertical oscillation starts when one beam is ejected and dumped

Tevatron orbit effects



Ref [2]

Beam-beam single bunch orbit can be well reproduced and measured also in LEP

Effects can become important (1 σ offset not impossible)

LUMINOSITY Deterioration

Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a whole test bunch during collision

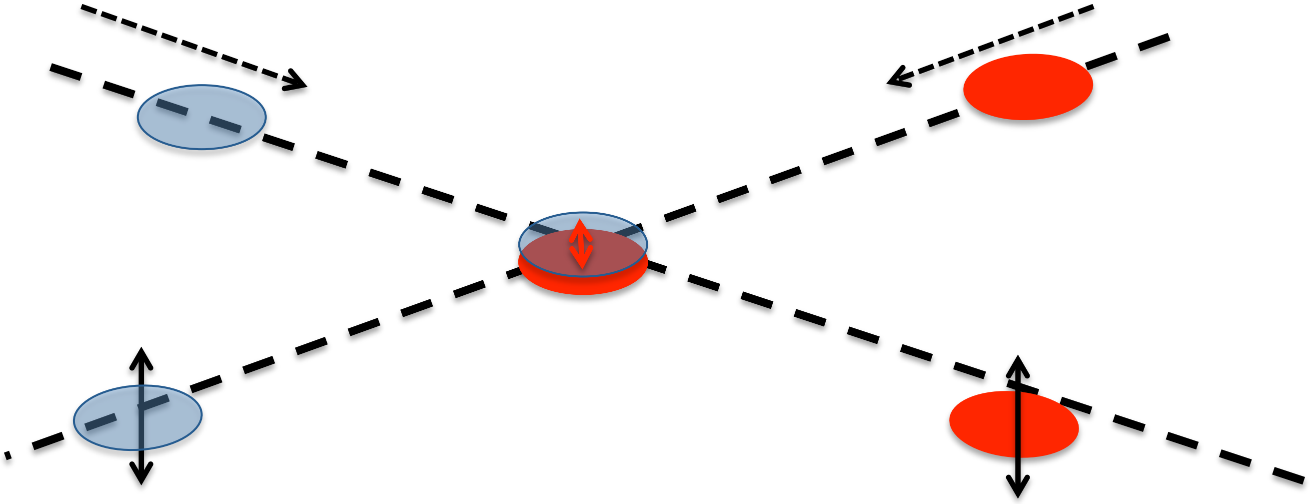
We study the collective behaviour of all particles of a bunch

Coherent motion requires an organized behaviour of all particles of the bunch

Coherent beam-beam force

- Beam distributions $\boldsymbol{\Psi_1}$ **and** $\boldsymbol{\Psi_2}$ mutually changed by interaction
- Interaction depends on distributions
  - Beam 1 Ψ_1 solution depends on beam 2 Ψ_2
  - Beam 2 Ψ_2 solution depends on beam 1 Ψ_1
- Need a self-consistent solution

Coherent beam-beam effects



- Whole bunch sees a kick as an entity (coherent kick)
- Coherent kick seen by full bunch different from single particle kick
- Requires integration of individual kick over particle distribution

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{4\sigma^2}}\right]$$

- Coherent kick of separated beams can excite coherent dipolar oscillations
- All bunches couple because each bunch “sees” many opposing bunches(LR): many coherent modes possible!

Coherent effects

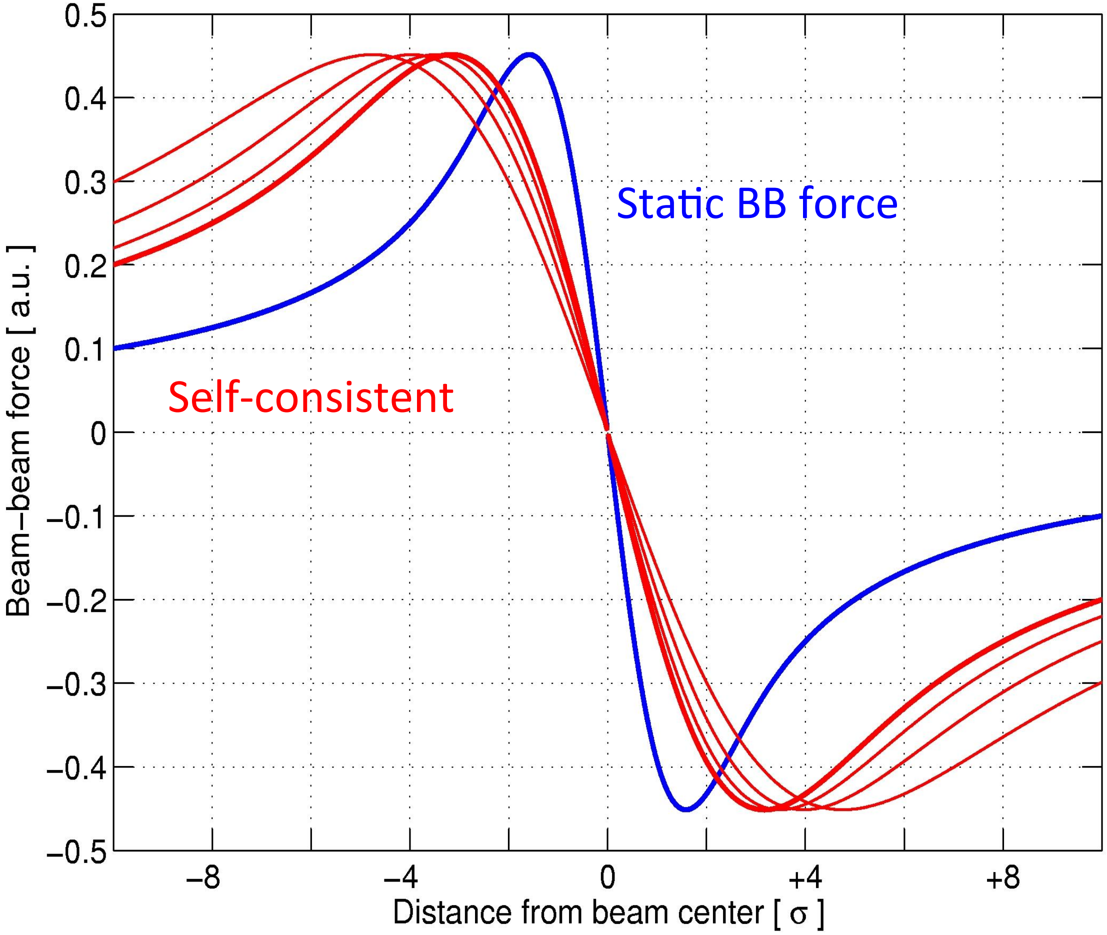
Self-consistent treatment needed

Perturbative methods

static source of distortion:
example magnet

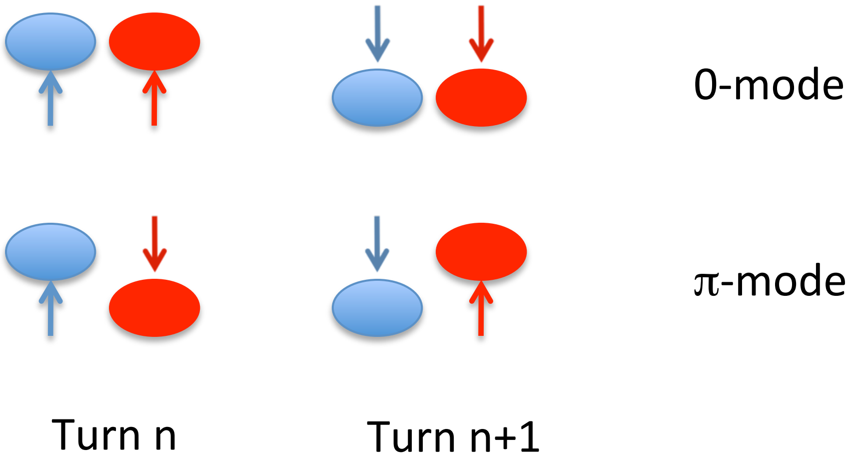
Self-consistent method

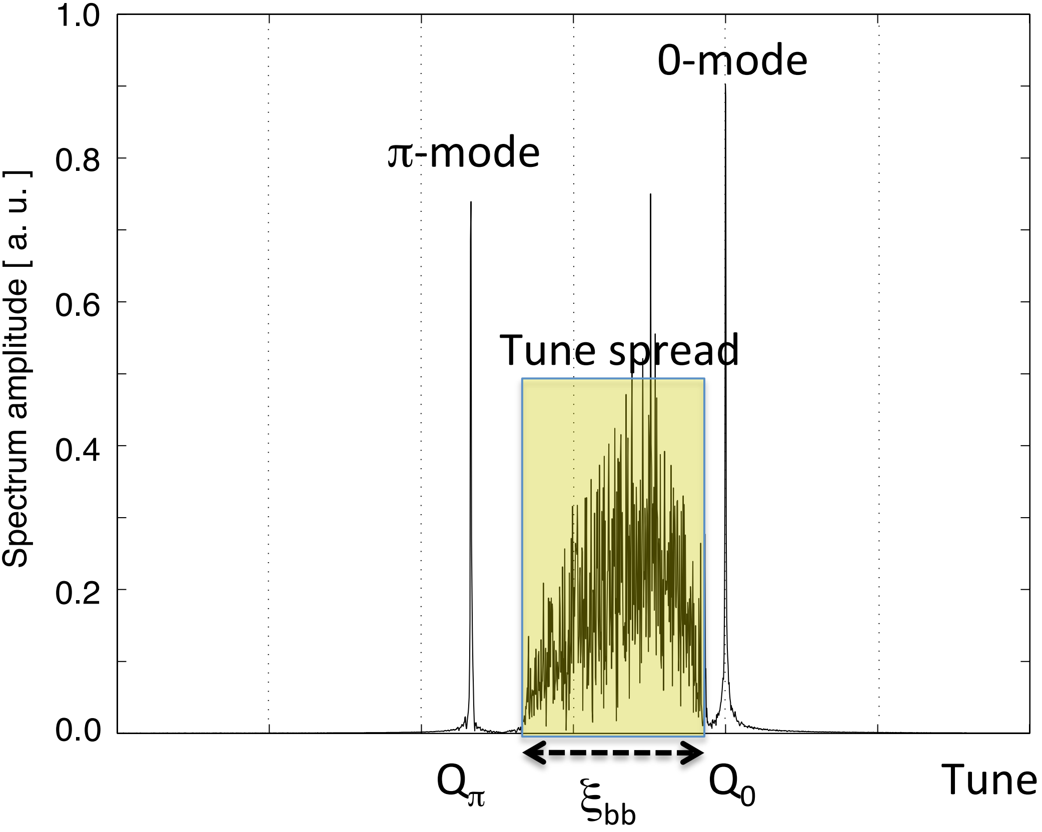
source of distortion changes
as a result of the distortion



For a complete understanding of BB effect a self-consistent treatment should be used

Simple case: one bunch per beam





MOVIE

0-mode at unperturbed tune Q_0

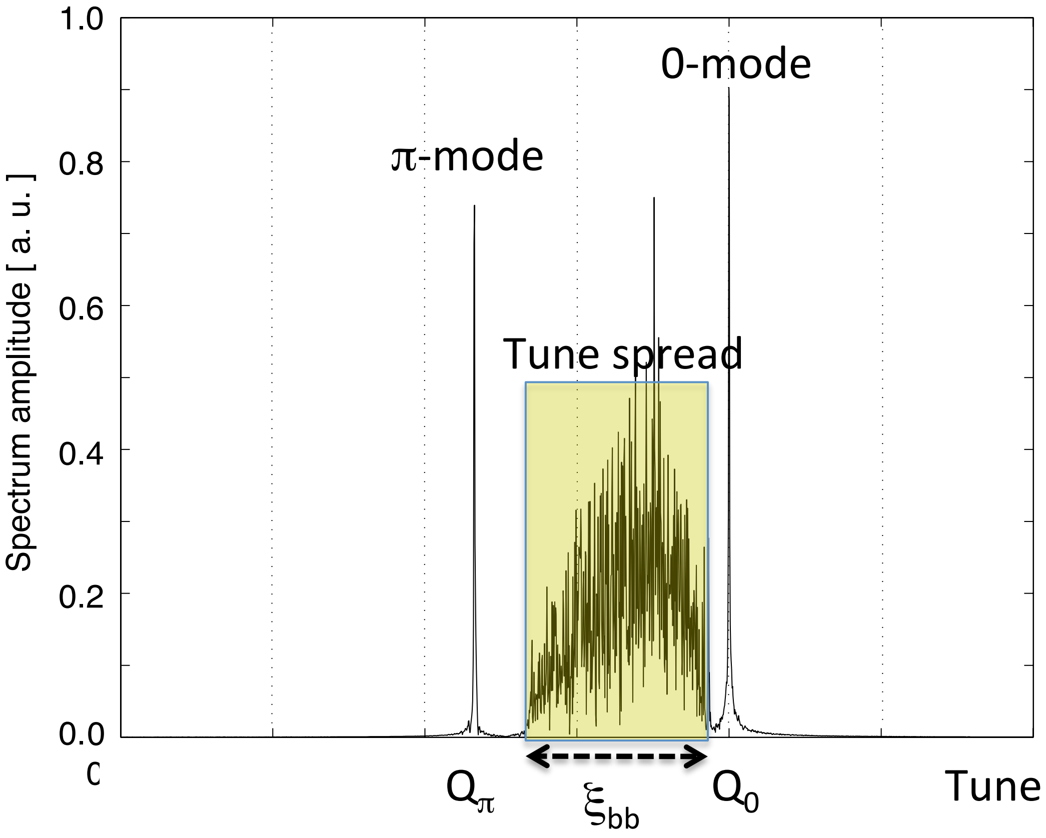
π-mode is shifted at $Q_\pi = 1.1\text{-}1.3\ \xi_{bb}$

Incoherent tune spread range $[0,-\xi]$

$$\Delta Q = Y \cdot \xi$$

- Coherent mode: two bunches are “locked” in a coherent oscillation
- 0-mode is stable (mode with NO tune shift)
- π-mode can become unstable (mode with largest tune shift)

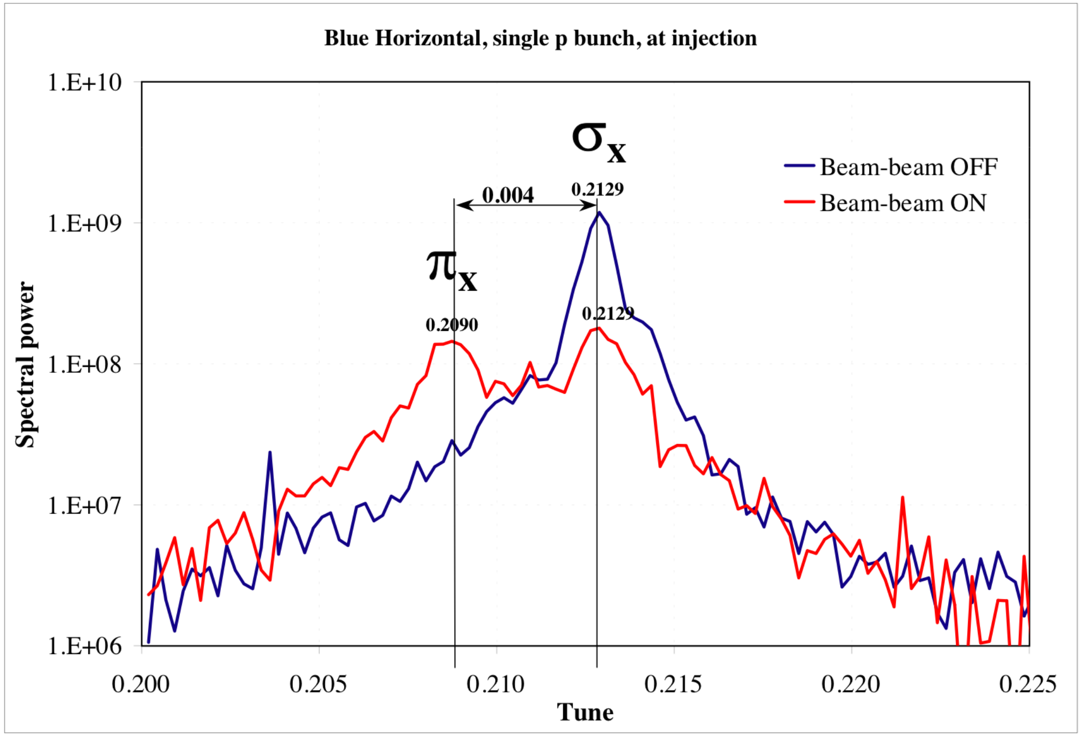
Simple case: one bunch per beam and Landau damping



Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop

- π-mode has frequency out of tune spread (Y) so it is not damped!

Coherent modes at RHIC

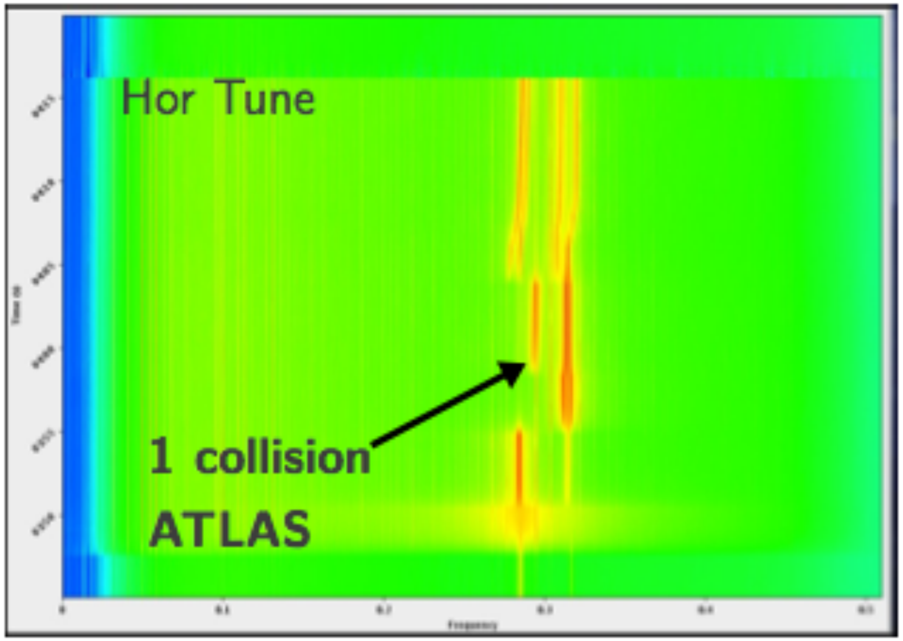


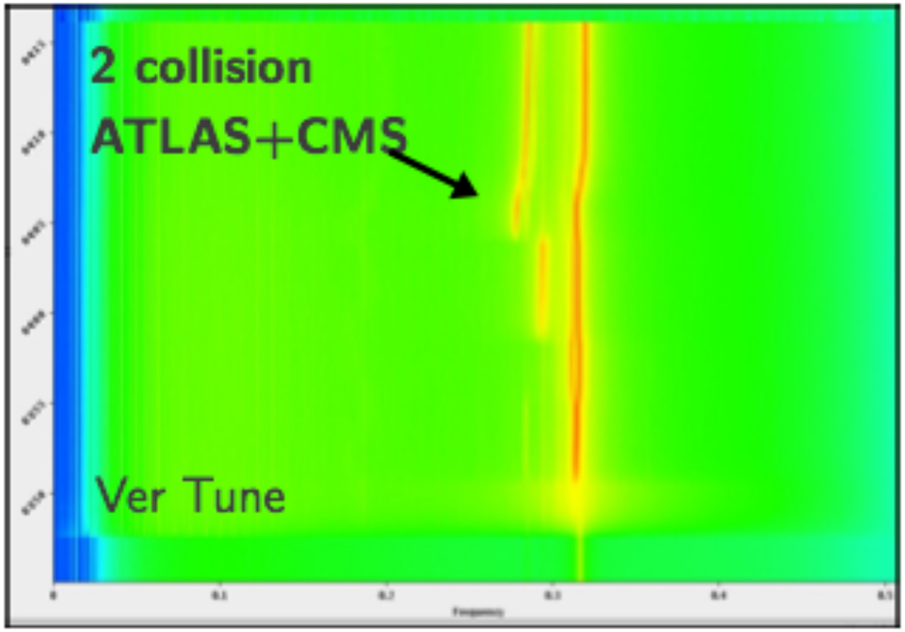
Courtesy W. Fischer (BNL)

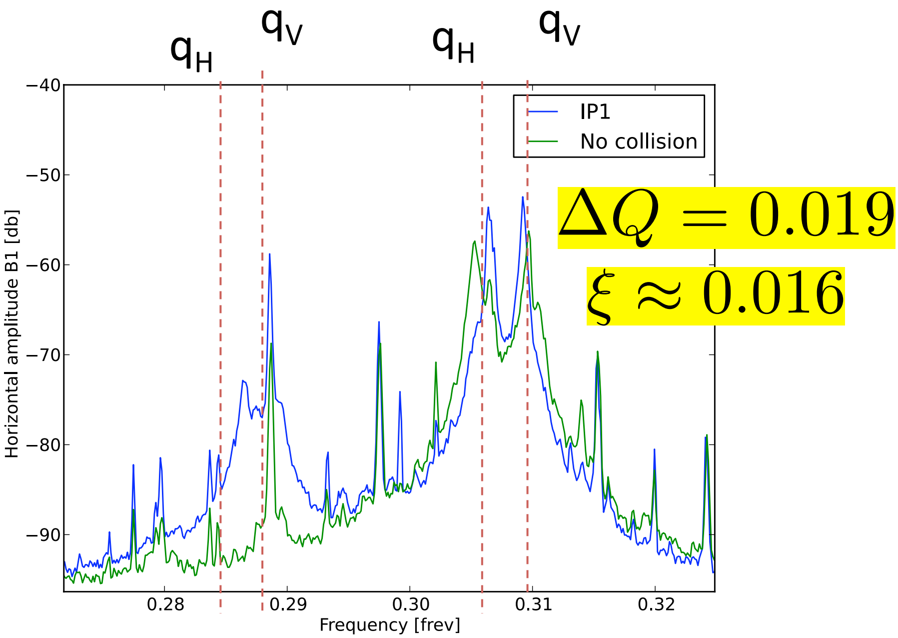
Tune spectra before collision and in collision two modes visible

Head-on beam-beam coherent mode: LHC

BBQ Signals

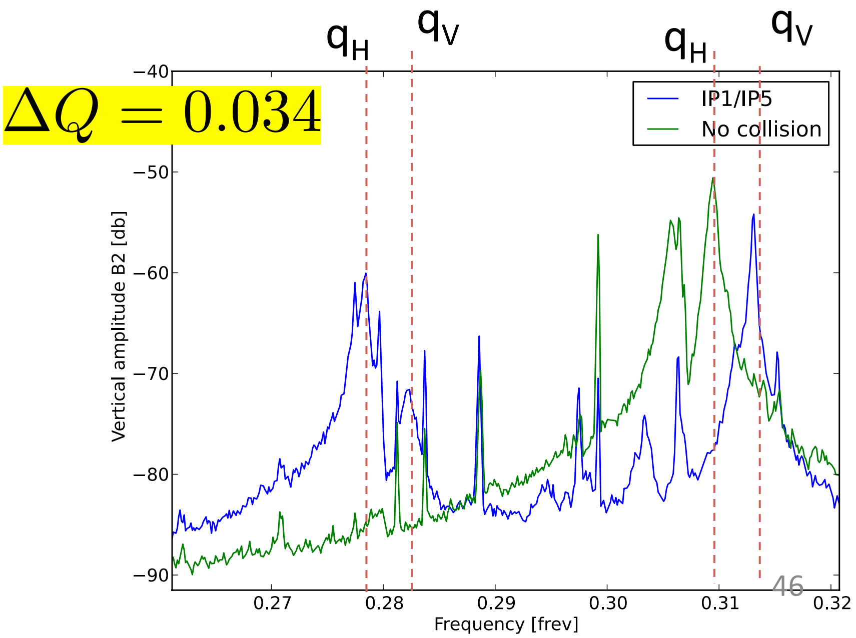






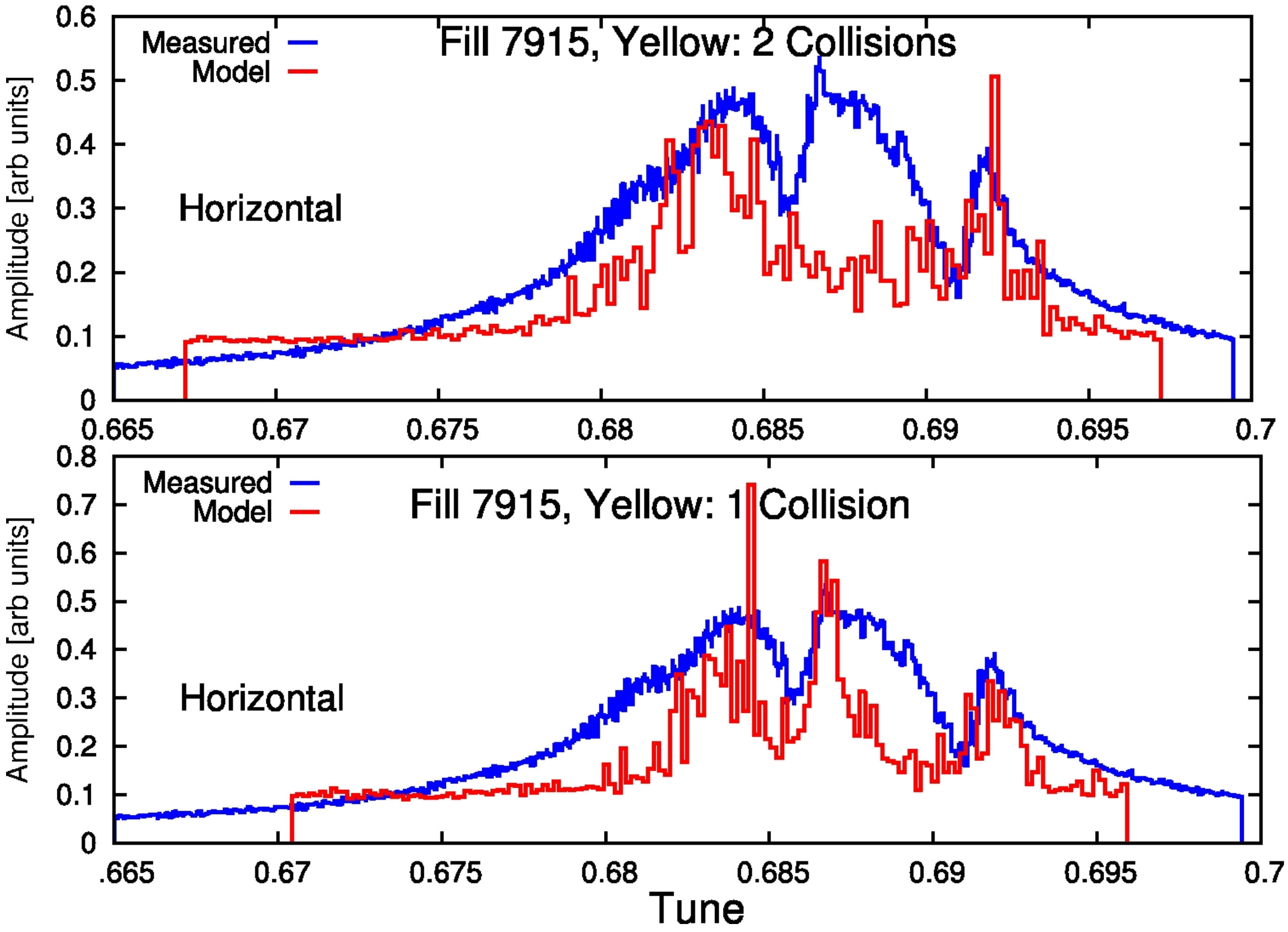
$$\Delta Q = 0.019$$

$$\xi \approx 0.016$$



$$\Delta Q = 0.034$$

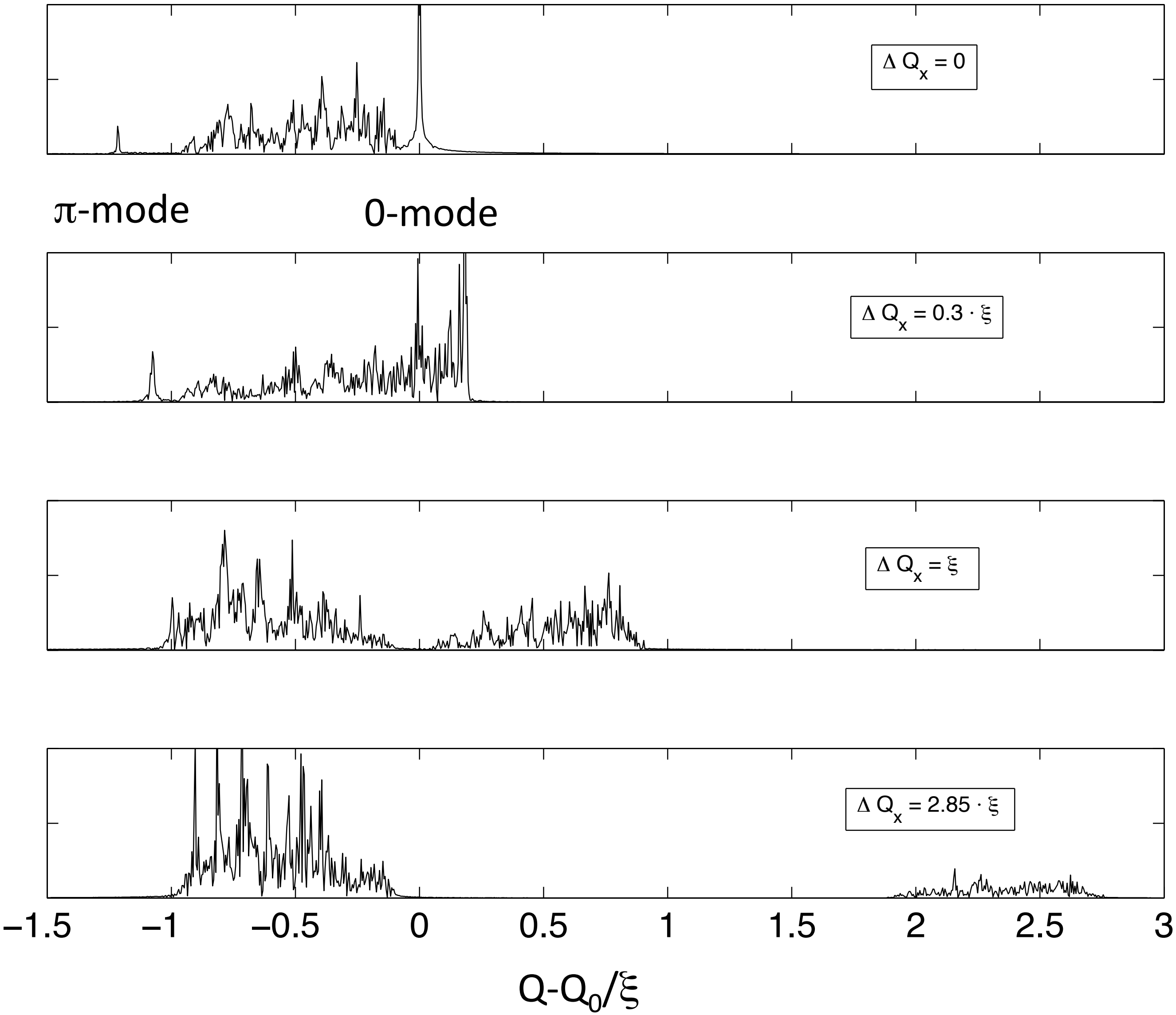
Beam-beam coherent modes and Landau Damping



Pacman effect on coherent modes

Single bunch diagnostic so important

Different Tunes

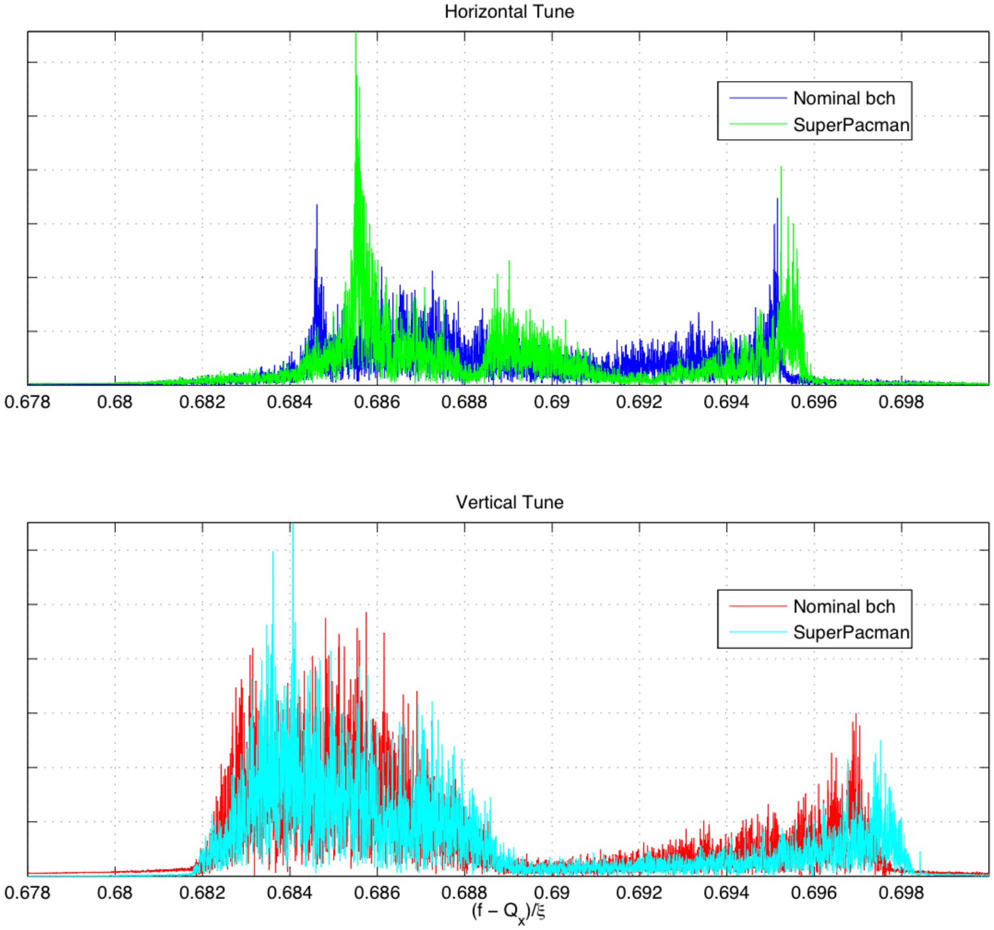


Tune split breaks symmetry and coherent modes disappear

Analytical calculations in Reference [8]

Different tunes or intensities

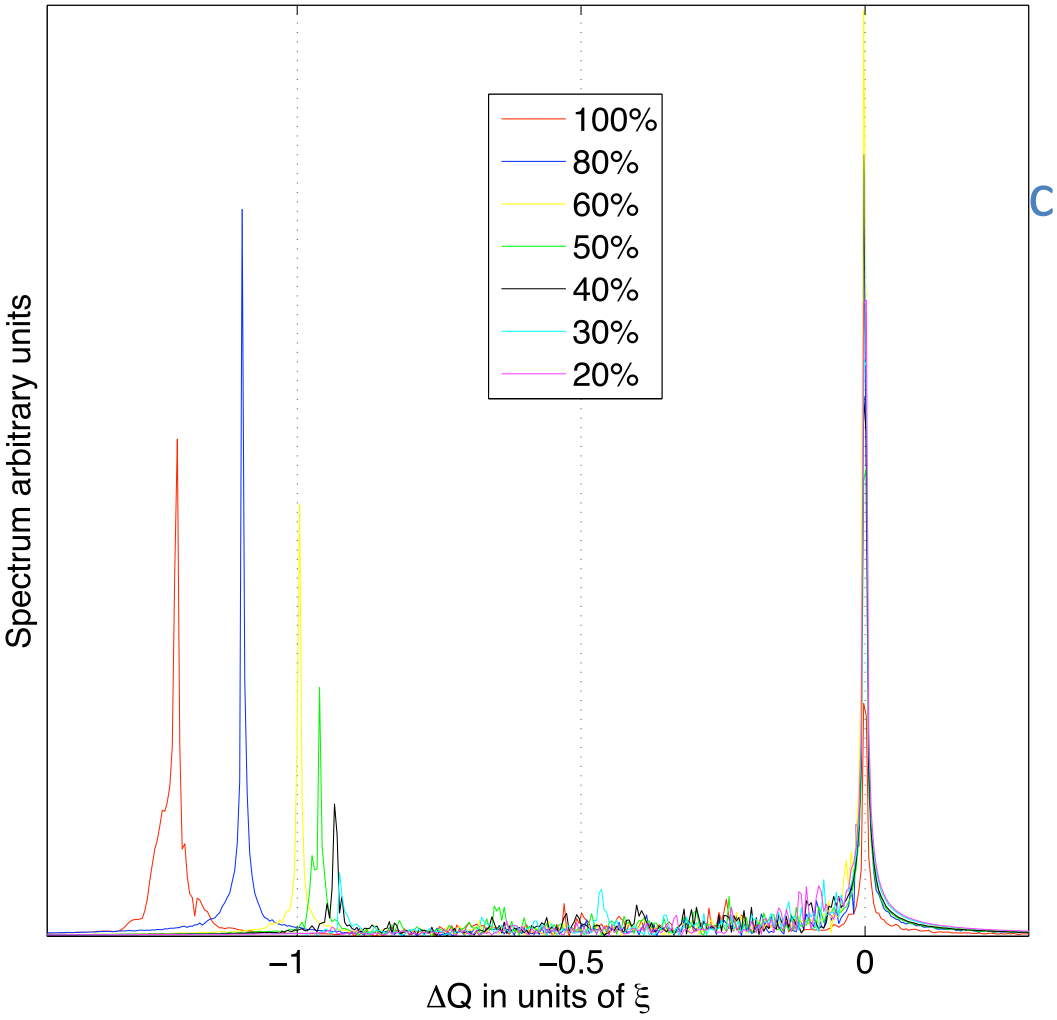
RHIC running with mirrored tune for years to break coherent oscillations



LHC has used a tune split to suppress coherent BB modes

2010 Physics Run

Different bunch intensities



For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55% **Reference[9]**

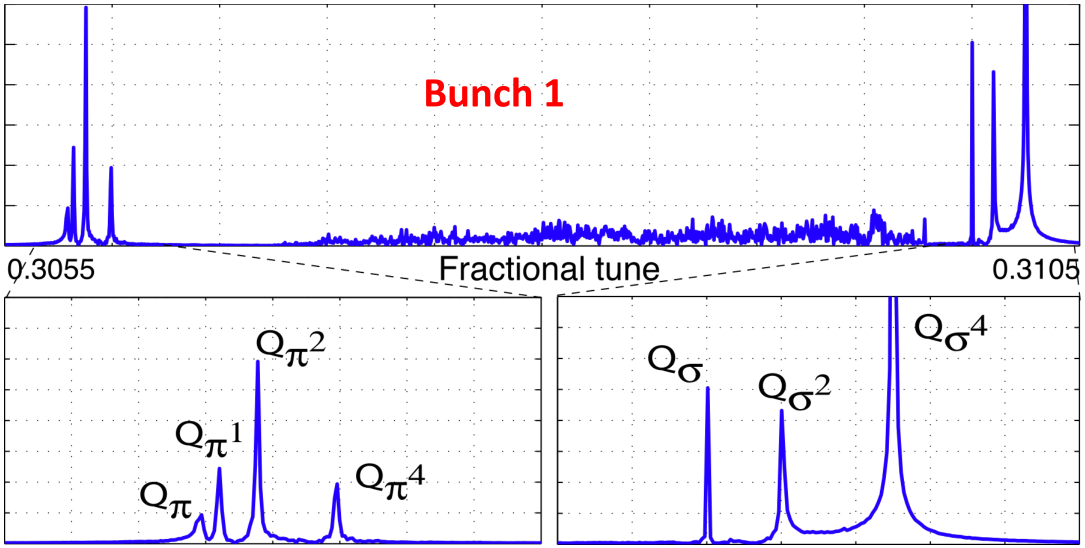
We assumed:

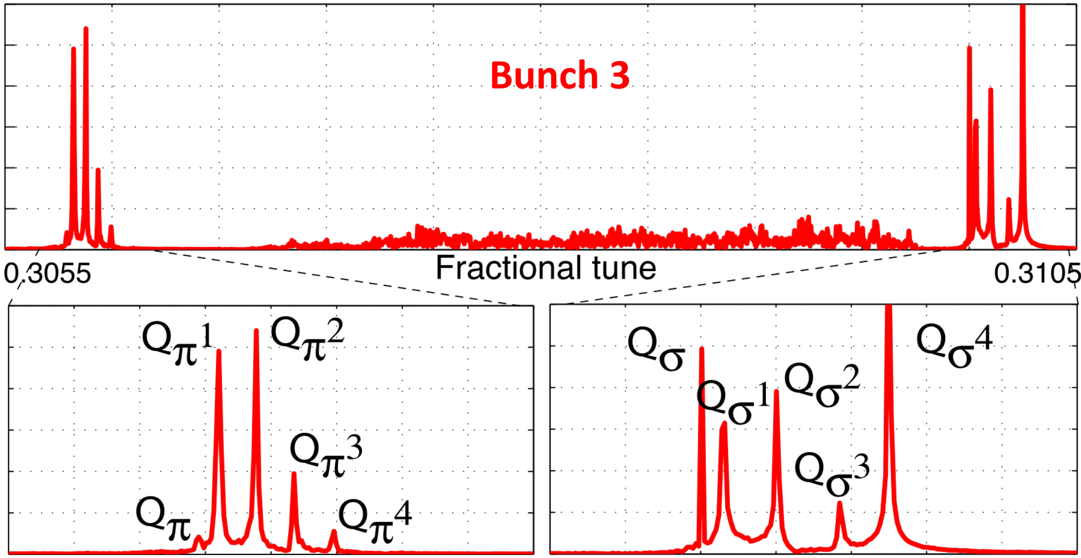
- equal emittances
- equal tunes
- NO PACMAN effects

(bunches will have different tunes)

For coherent modes the key is to break the simmetry in your coupled system…(tunes, intensities, collision patters…)

And Long range interactions?





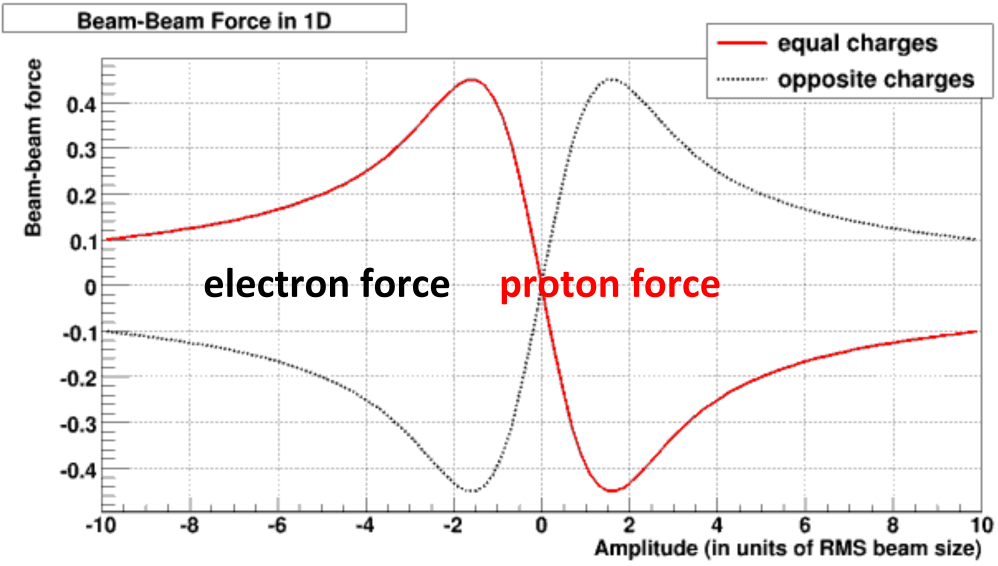
- **Each bunch will have different number of modes and tune spectra**
- **No Landau damping of long-range coherent modes**

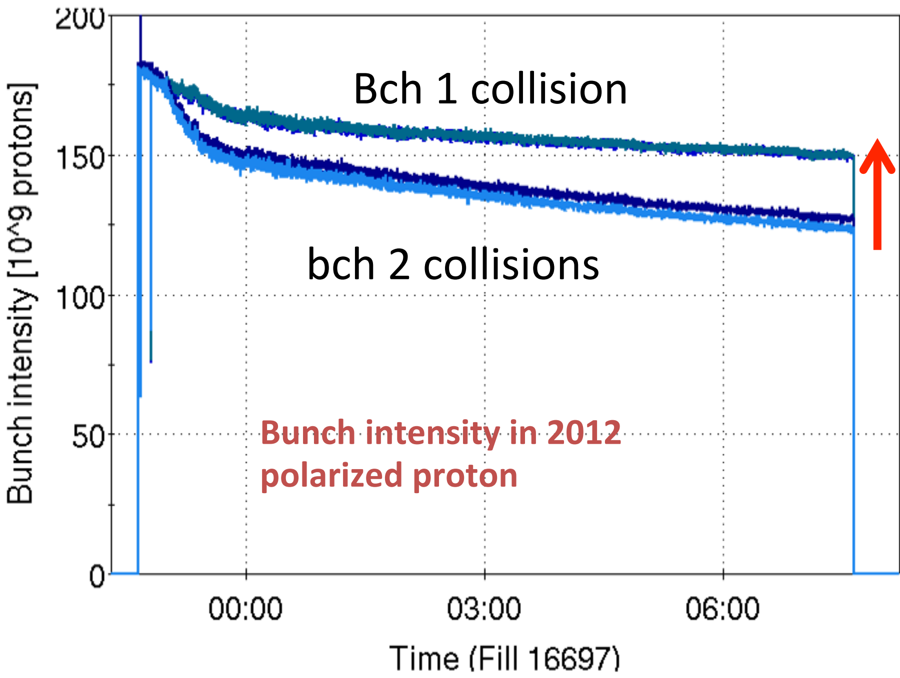
Single bunch diagnostic can make the difference

Beam-beam compensations:

Head-on

- Linear e-lens, suppress shift
- Non-linear e-lens, suppress tune spread





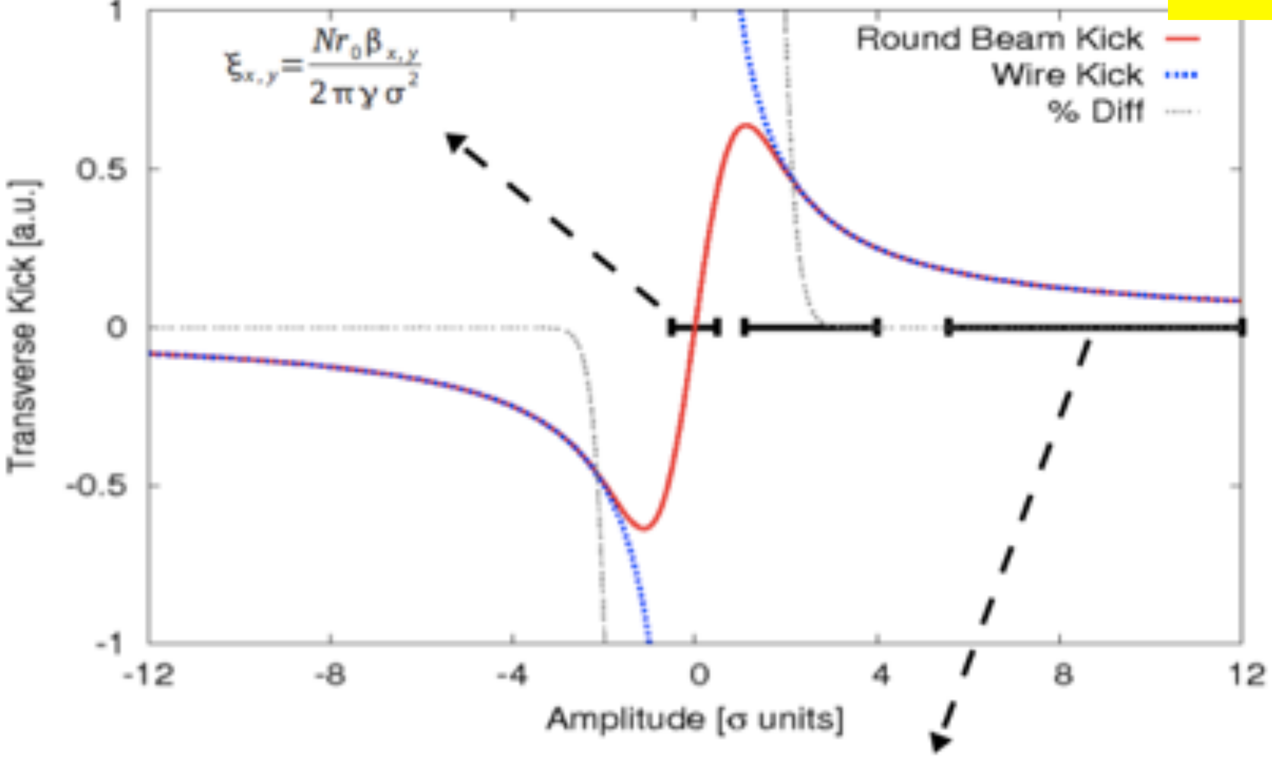
Bunch intensity in 2012 polarized proton

- Past exerience: at Tevatron linear and non-linear e-lenses, also hollow….
- Present: test for half compensation at RHIC with non-linear e-lens

Beam-beam compensations: long-range

Beam-beam wire compensation

R. Calaga



$\sigma << d$: $$\Delta x'(x,d)=-\frac{K}{d}.(1+\frac{x}{d}+\frac{x^2}{d^2}+...)$$

- Past exerience: at RHIC several tests till 2009…
- Present: simulation studies on-going for possible use in HL-LHC…

…not covered here…

- *Linear colliders special issues*
- *Asymmetric beams effects*
- *Coasting beams*
- *Beamstrahlung*
- *Synchrobetatron coupling*
- *Beam-beam experiments*
- *Beam-beam and impedance*
- *…*

References:

[1] http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf

[2] V. Shiltsev et al, "Beam beam effects in the Tevatron", Phys. Rev. ST Accel. Beams 8, 101001 (2005)

[3] Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)

[4] Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)

[5] J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.

[6] H. Grote, F. Schmidt, L. H. A. Leunissen,"LHC Dynamic Aperture at Collision", LHC-Project-Note 197, (1999).

[7] W. Herr,"Features and implications of different LHC crossing schemes", LHC-Project-Note 628, (2003).

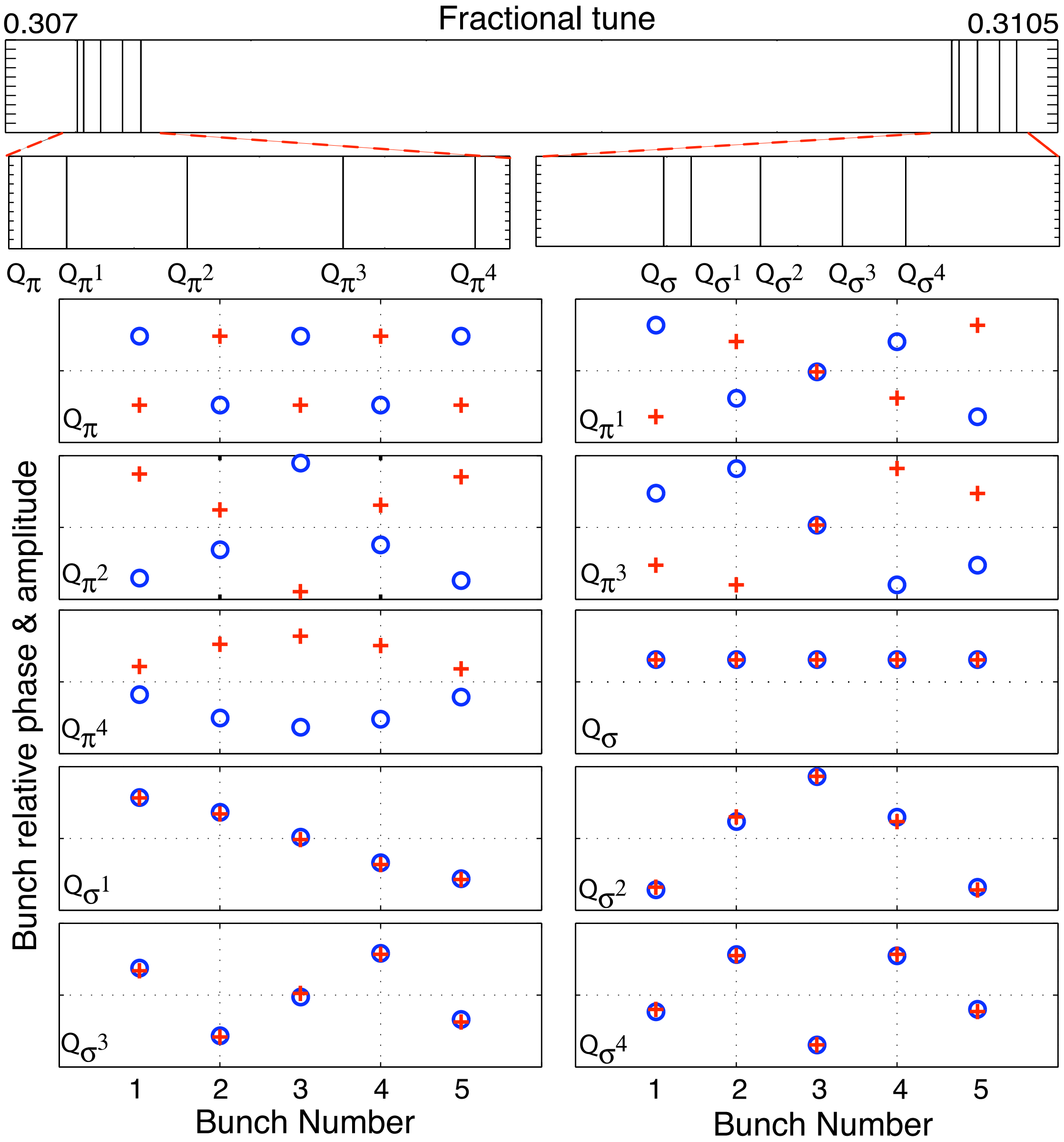
[8] A. Hofmann,"Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.

[9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams ", Part. Acc. 59, 43 (1996).

…much more on the LHC Beam-beam webpage:

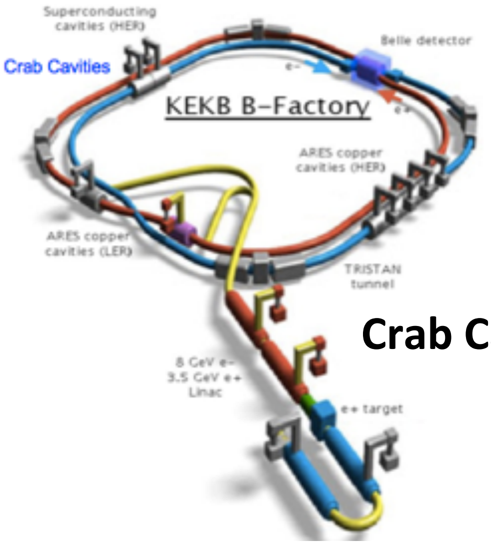
http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/

Bunch trains and more interactions

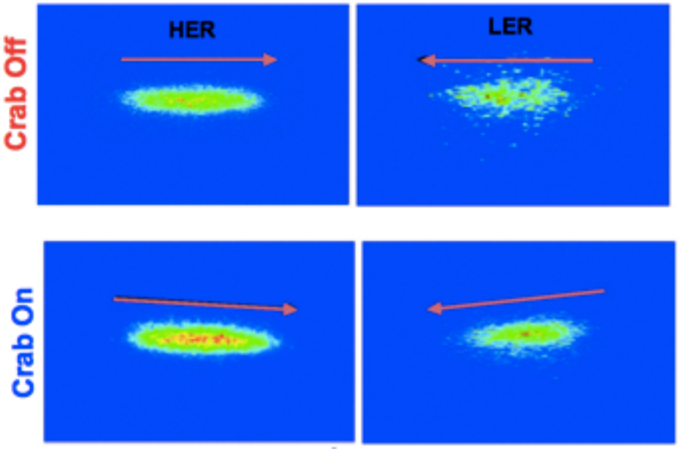


Eigenfunctions of coherent modes

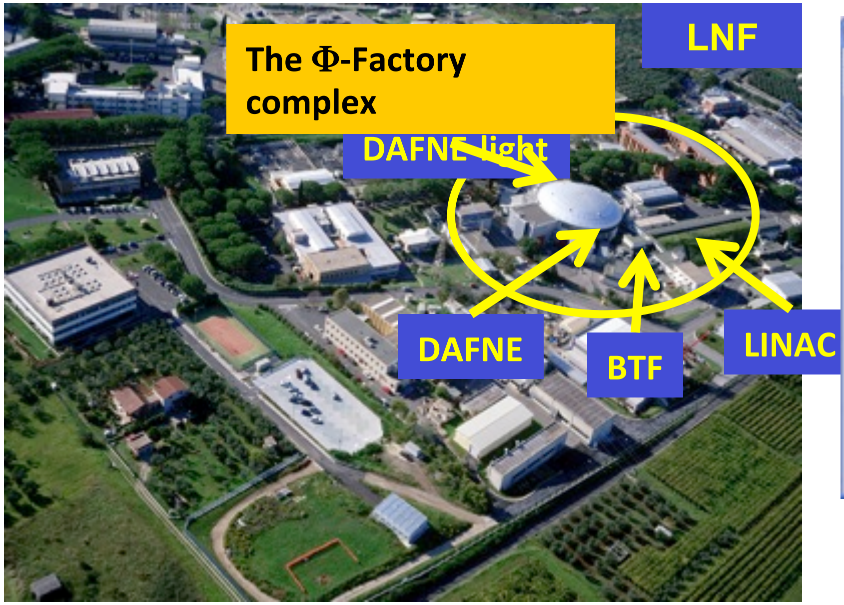
Lepton colliders and crab cavities



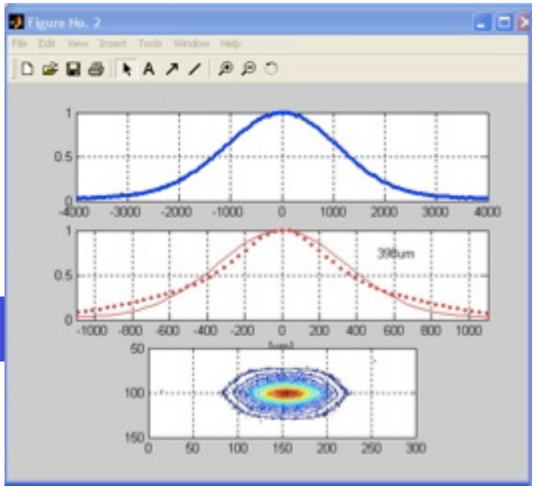
Crab Cavity Experience from KEKB



LPA and crabbed waist



CRAB OFF



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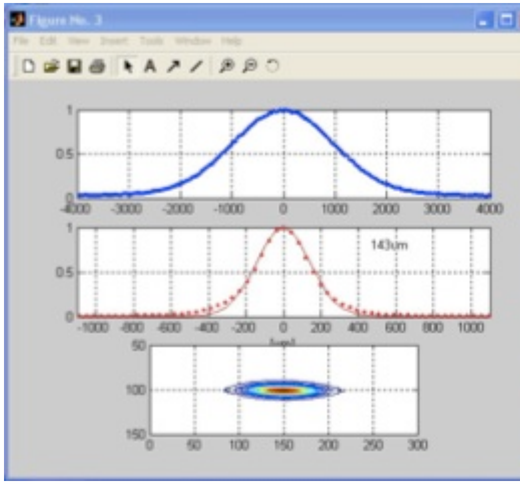
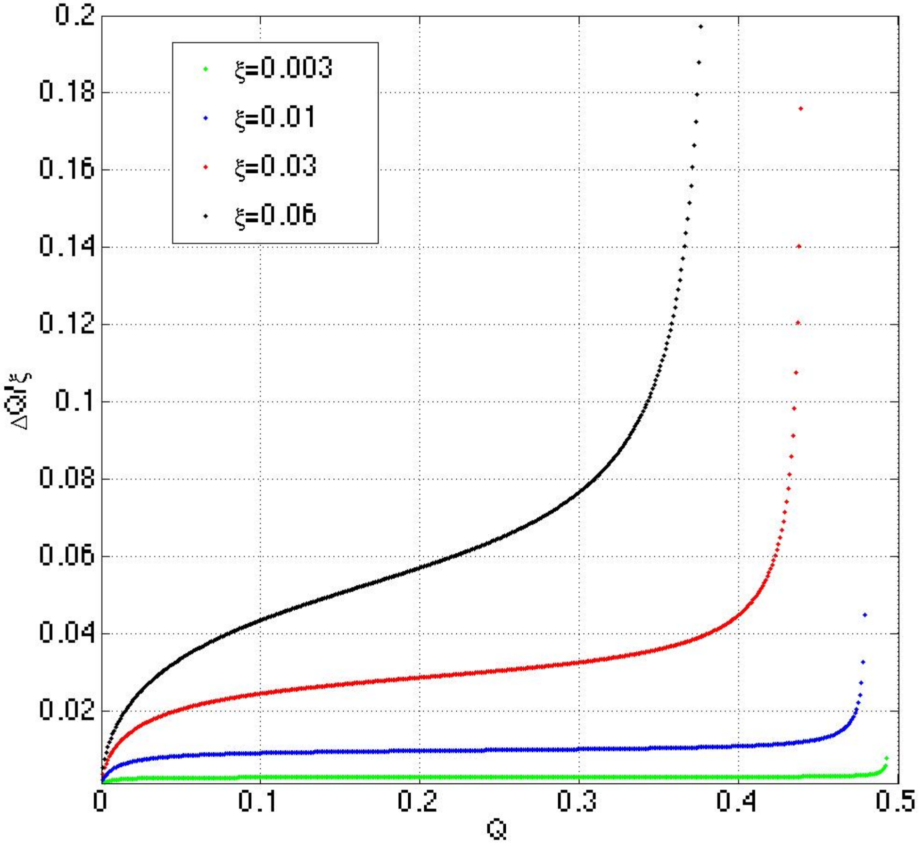
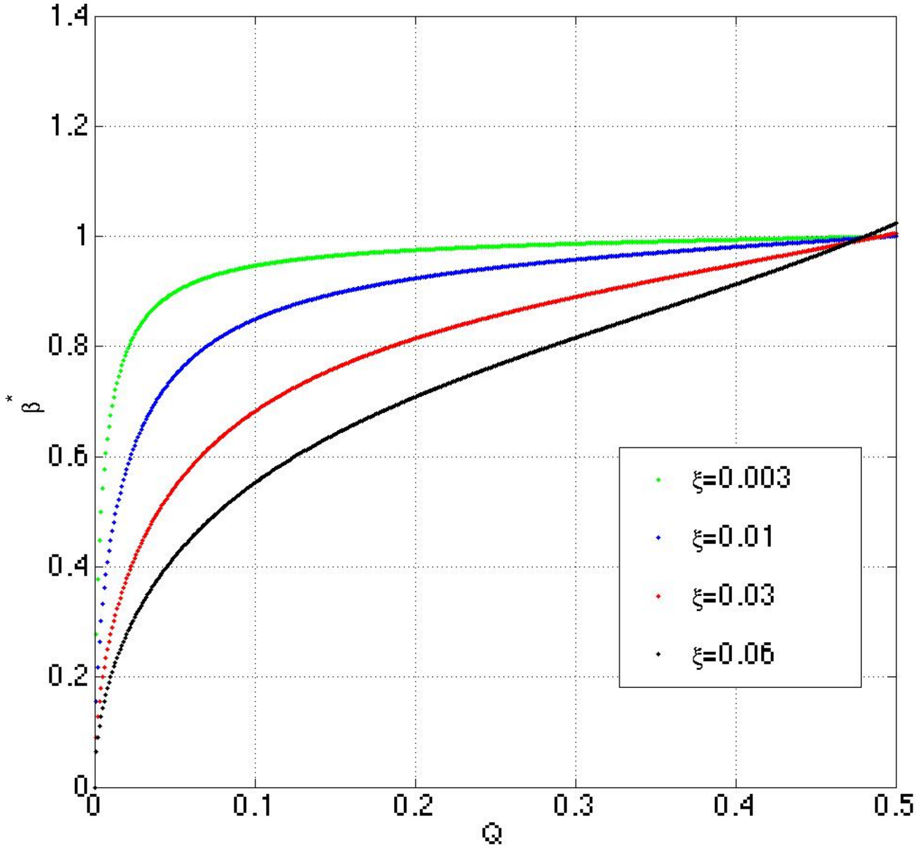


Tune dependence of tune shift and dynamic beta

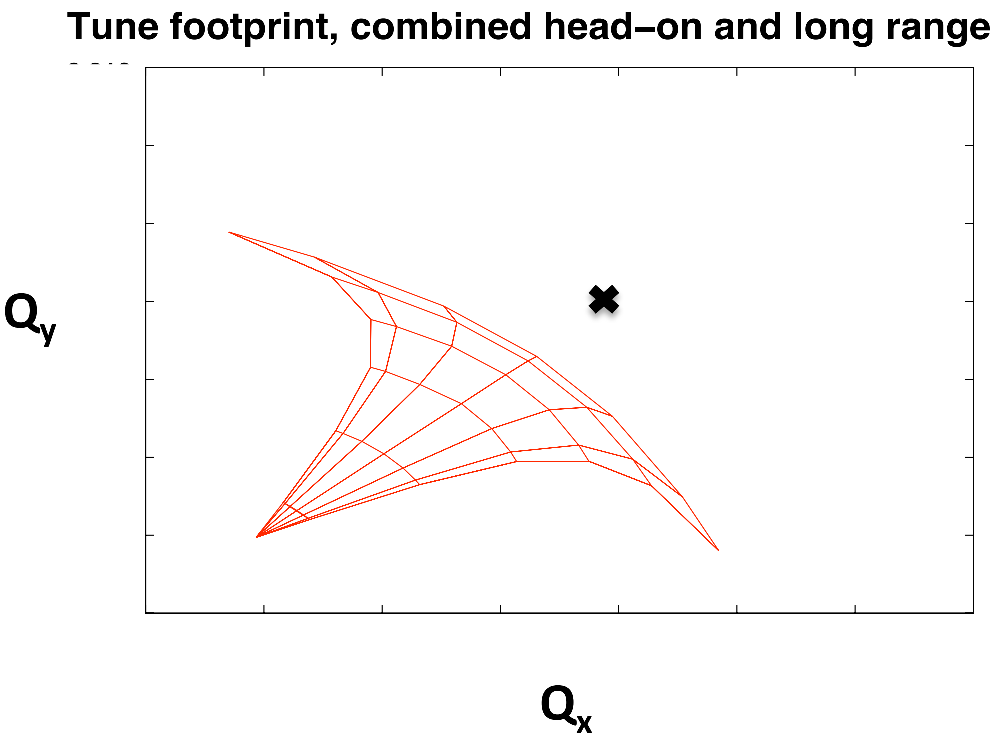
Tune shift



Dynamic beta



Beam-beam tune shift and spread



2 experiments
2 Head on collisions
30 long-range with horizontal separation
30 long-range with vertical separation
Experiments opposite in azimuthal