

Lattice Design in Particle Accelerators

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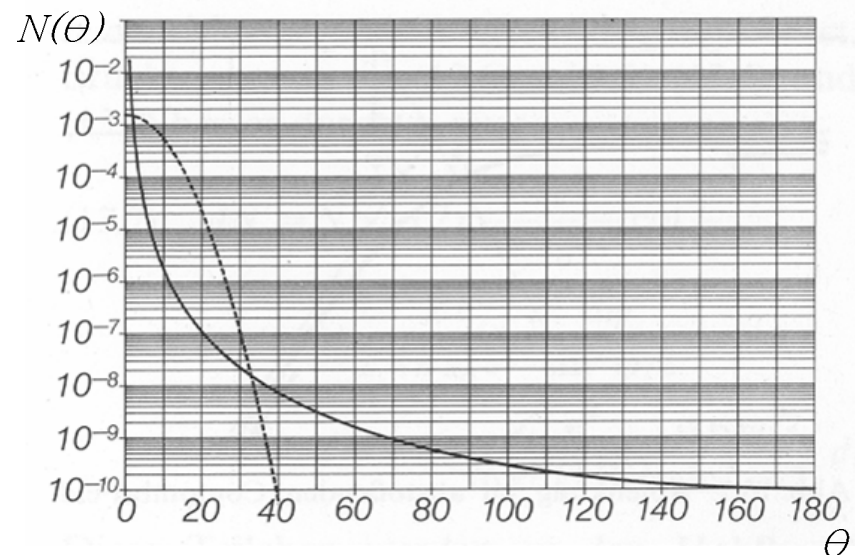
Historical note:

... Particle acceleration where **lattice design is not needed**

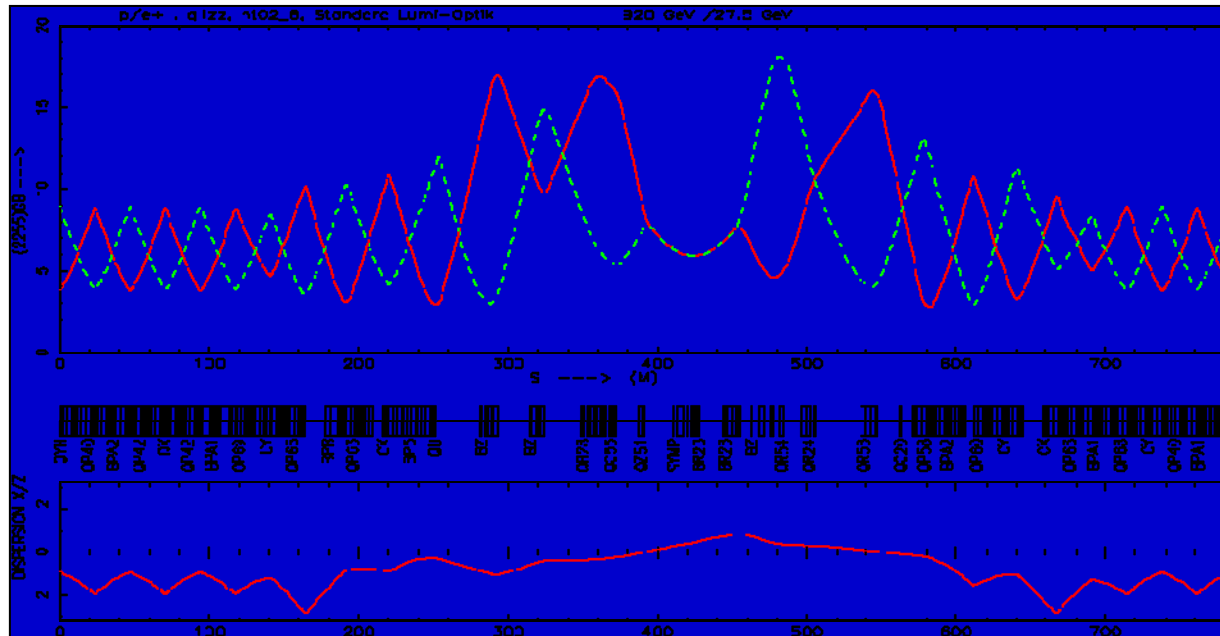
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\epsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$

Rutherford Scattering, 1906

Using radioactive particle sources:
 α -particles of some MeV energy



Lattice design: design and optimisation of the principle elements of an accelerator ... *the lattice cells*



1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Lattice Design: „... how to build a storage ring“

High energy accelerators → **circular machines**

somewhere in the lattice we need a number of **dipole magnets**,
that are bending the design orbit to a **closed ring**

Geometry of the ring:

centrifugal force = Lorentz force

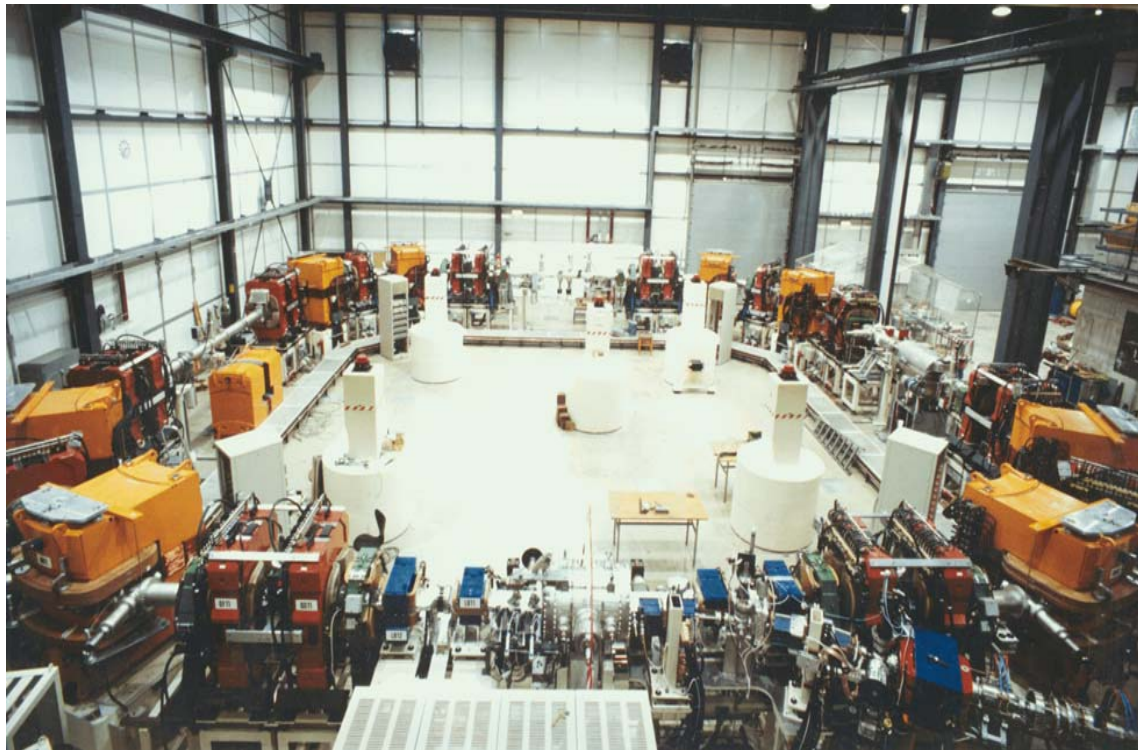
$$e * v * B = \frac{m v^2}{\rho}$$

$$\rightarrow e * B = \frac{m v}{\rho} = p / \rho$$

$$\rightarrow B * \rho = p / e$$

p = momentum of the particle,
ρ = curvature radius

Bρ = beam rigidity



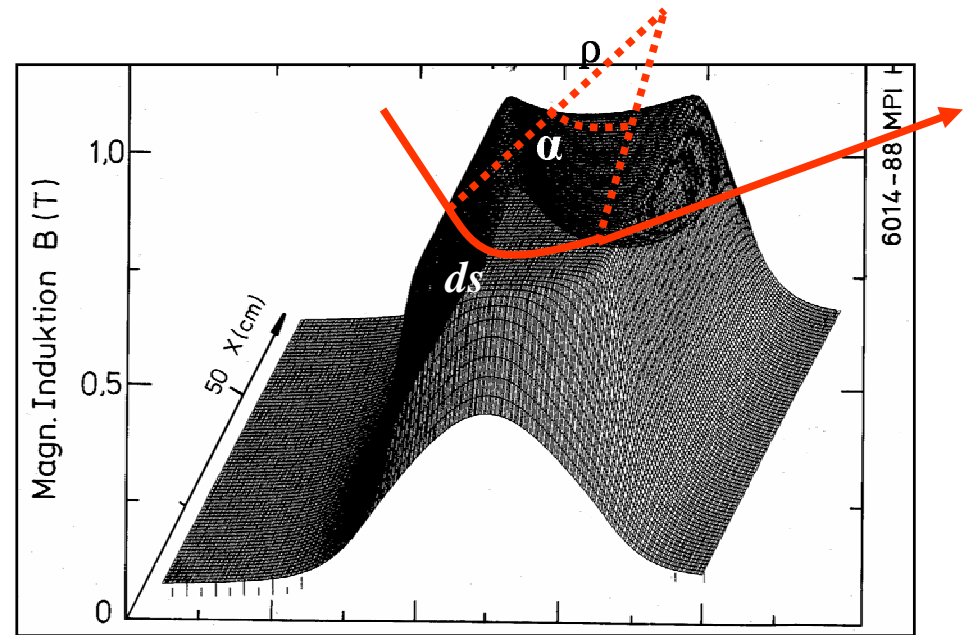
Example: heavy ion storage ring TSR
8 dipole magnets of equal bending strength

Circular Orbit:

„... defining the geometry“

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho}$$

$$\alpha = \frac{B^* dl}{B^* \rho}$$



field map of a storage ring dipole magnet

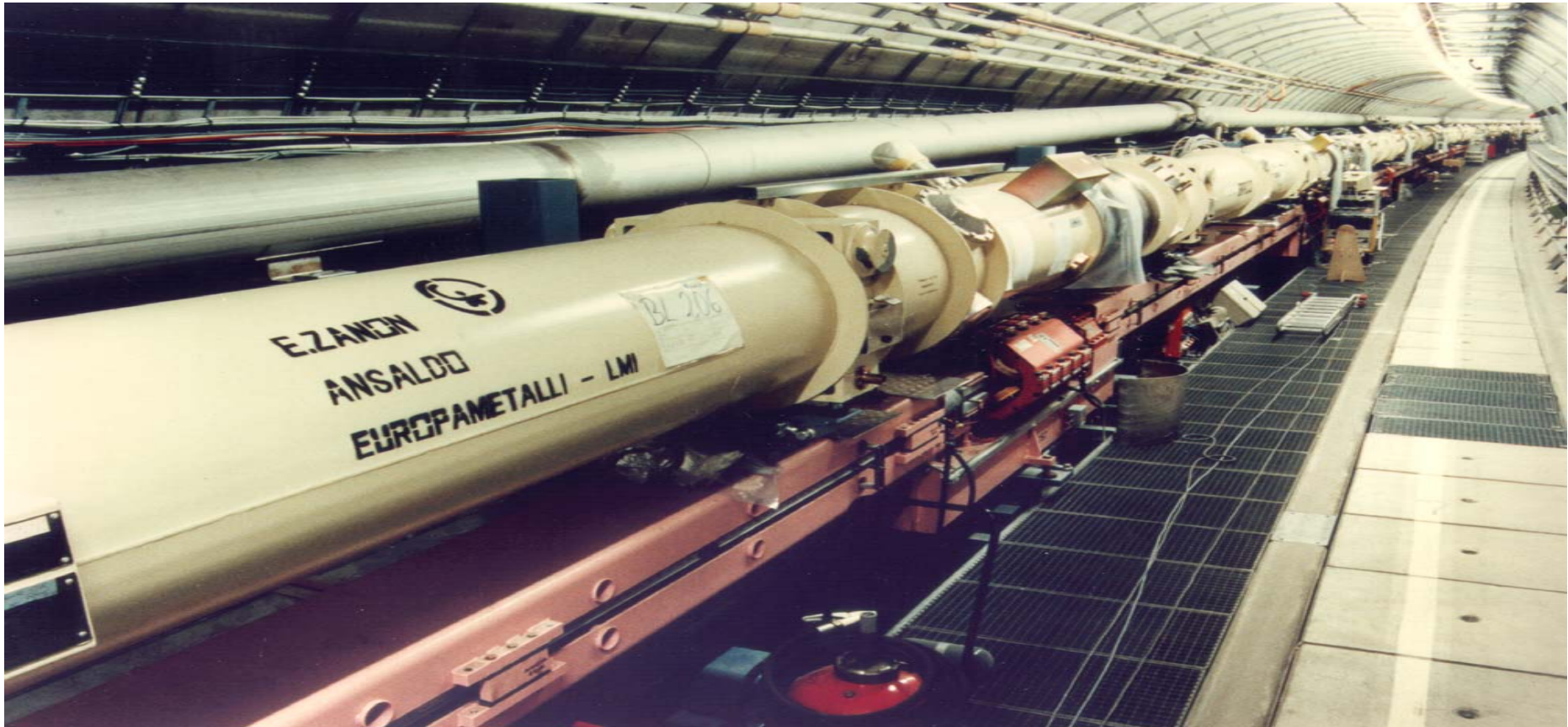
The angle swept out in one revolution must be 2π , so

... for a full circle

$$\alpha = \frac{\int B dl}{B^* \rho} = 2\pi \quad \rightarrow \quad \int B dl = 2\pi^* \frac{p}{q}$$

Nota bene: $\frac{\Delta B}{B} \approx 10^{-4}$ is usually required !!

Example HERA:



920 GeV Proton storage ring
dipole magnets $N = 416$

$l = 8.8\text{m}$

$q = +1 e$

$$\int Bdl \approx N * l * B = 2\pi p / q$$

$$B \approx \frac{2\pi * 920 * 10^9 eV}{416 * 3 * 10^8 \frac{m}{s} * 8.8m * e} \approx \underline{\underline{5.15 \text{ Tesla}}}$$

Focusing forces and particle trajectories:

Equation of motion

$$x'' + K * x = 0$$

$$K = -k + 1/\rho^2$$

hor. plane

$$K = k$$

vert. plane

$$\text{dipole magnet: } \frac{1}{\rho} = \frac{B}{p/e}$$

$$\text{quadrupole lens: } k = \frac{g}{p/e}$$
$$\text{focal length: } f = \frac{1}{k * l}$$

Example: HERA Ring:

Circumference: $C_0 = 6335 \text{ m}$

Bending radius: $\rho = 580 \text{ m}$

Quadrupol Gradient: $G = 110 \text{ T/m}$

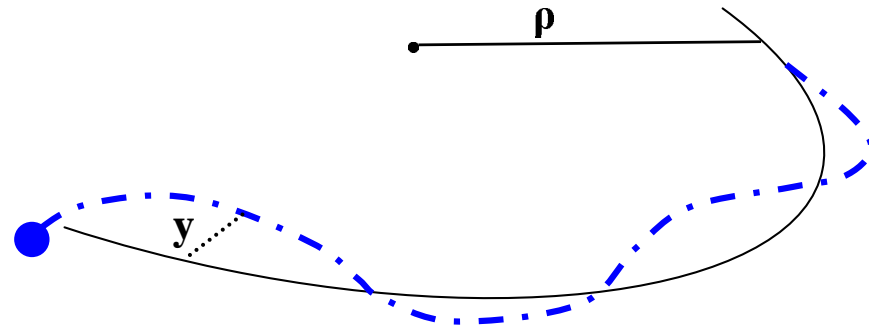
$$\rightarrow k = 33.64 * 10^{-3} / \text{m}^2$$

$$\rightarrow 1/\rho^2 = 2.97 * 10^{-6} / \text{m}^2$$

- ! For estimates in large accelerators *the weak focusing term $1/\rho^2$ can in general be neglected*

„... single particle trajectories“

$$y'' + K * y = 0$$



Solution for a focusing magnet

$$y(s) = y_0 * \cos(\sqrt{K} * s) + \frac{y'_0}{\sqrt{K}} * \sin(\sqrt{K} * s)$$

$$y'(s) = -y_0 * \sqrt{K} * \sin(\sqrt{K} * s) + y'_0 * \cos(\sqrt{K} * s)$$

**Or written more convenient
in matrix form:**

$$\begin{pmatrix} y \\ y' \end{pmatrix}_s = M * \begin{pmatrix} y \\ y' \end{pmatrix}_0$$

Matrices of lattice elements

Hor. **focusing** Quadrupole Magnet

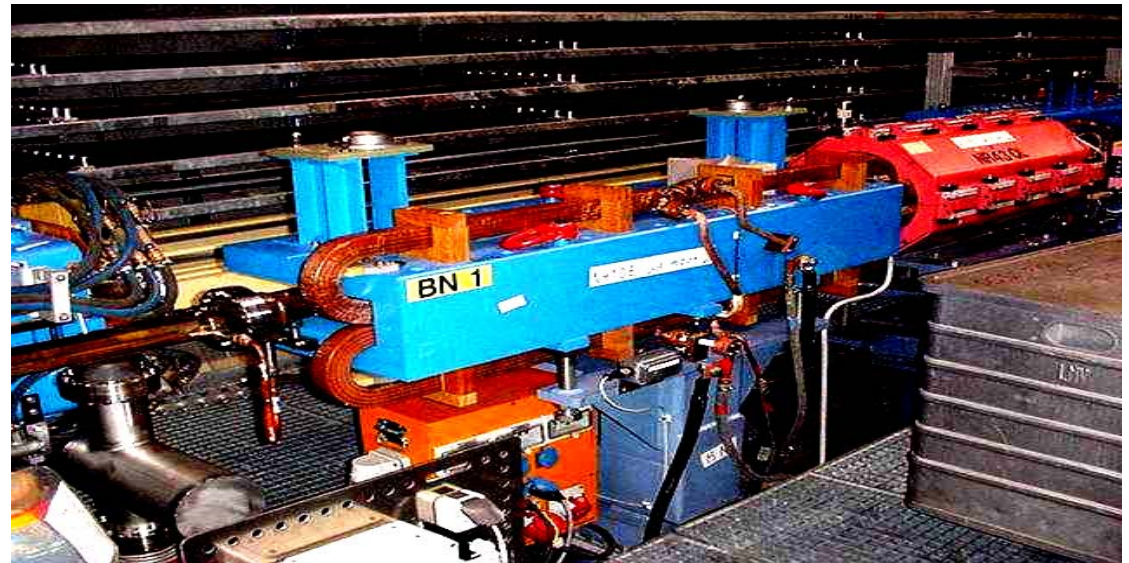
$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l) \\ -\sqrt{K} \sin(\sqrt{K} * l) & \cos(\sqrt{K} * l) \end{pmatrix}$$

Hor. **defocusing** Quadrupole Magnet

$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sinh(\sqrt{K} * l) \\ \sqrt{K} \sinh(\sqrt{K} * l) & \cosh(\sqrt{K} * l) \end{pmatrix}$$

Drift space

$$M_{Drift} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$



$$M_{lattice} = M_{QF1} * M_{D1} * M_{QD} * M_{D1} * M_{QF2} \dots$$

Periodic Lattices

In the case of periodic lattices the transfer matrix can be expressed as a function of a set of periodic parameters α, β, γ

$$M(s) = \begin{pmatrix} \cos \mu + \alpha_s \sin \mu & \beta_s \sin \mu \\ -\gamma_s \sin \mu & \cos(\mu) - \alpha_s \sin \mu \end{pmatrix}$$

$$\mu = \int_s^{s+L} \frac{dt}{\beta(t)}$$

$\mu =$ phase advance
per period:

For stability of the motion in periodic lattice structures it is required that

$$|\text{trace}(M)| < 2$$

In terms of these new periodic parameters the **solution of the equation of motion** is

$$y(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\Phi(s) - \delta)$$

$$y'(s) = \frac{-\sqrt{\varepsilon}}{\sqrt{\beta}} * \{ \sin(\Phi(s) - \delta) + \alpha \cos(\Phi(s) - \delta) \}$$

The new parameters α , β , γ can be transformed through the lattice via the matrix elements defined above.

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_s = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} * \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

Question: „ What does that mean ????? “

... and here starts the **lattice design !!!**

Most simple example: drift space

$$M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

particle coordinates

$$\begin{pmatrix} x \\ x' \end{pmatrix}_l = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

→

$$\begin{aligned} x(l) &= x_0 + l * x_0' \\ x'(l) &= x_0' \end{aligned}$$

transformation of twiss parameters:

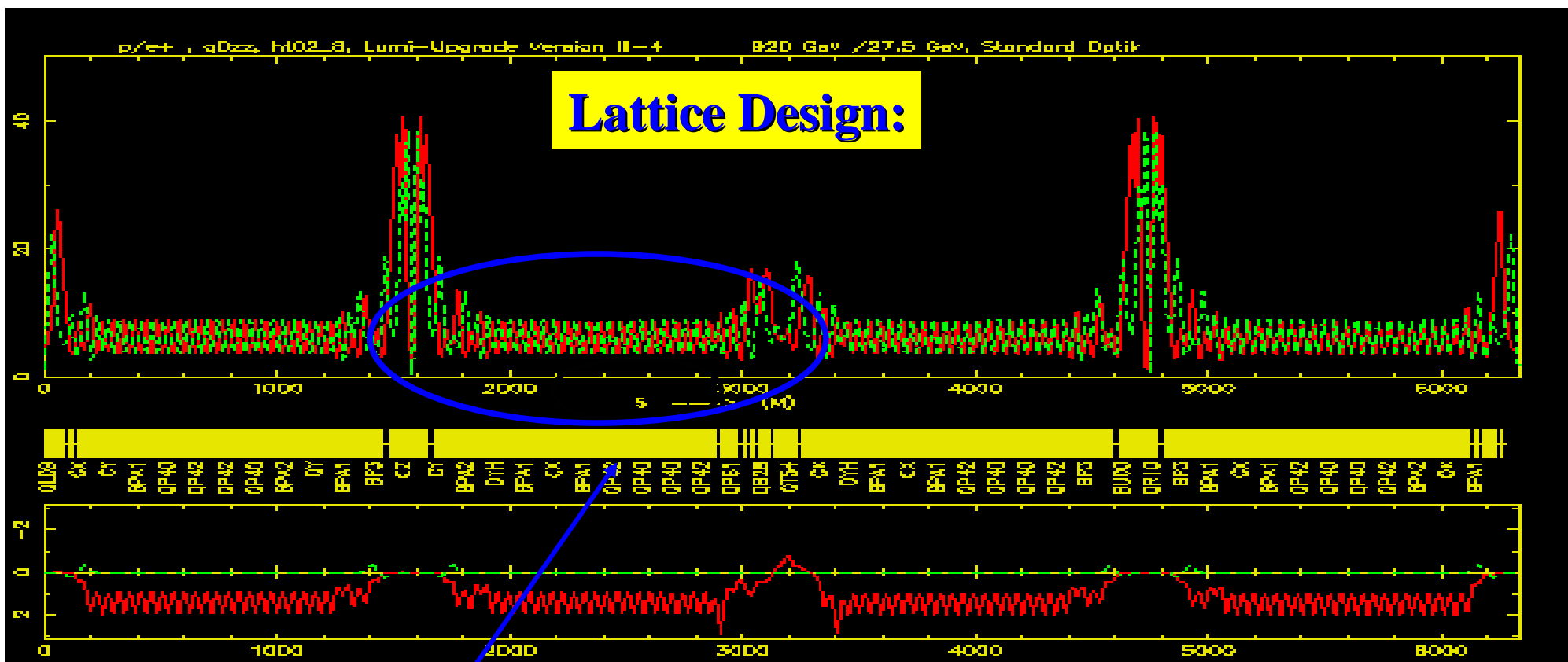
$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_l = \begin{pmatrix} 1 & -2l & l^2 \\ 0 & 1 & -l \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

$$\beta(s) = \beta_0 - 2l * \alpha_0 + l^2 * \gamma_0$$

Stability ...?

$$\text{trace}(M) = 1 + 1 = 2$$

→ A periodic solution doesn't exist in a lattice built exclusively out of drift spaces.



Arc: regular (periodic) magnet structure:

bending magnets \rightarrow define the energy of the ring
 main focusing & tune control, chromaticity correction,
 multipoles for higher order corrections

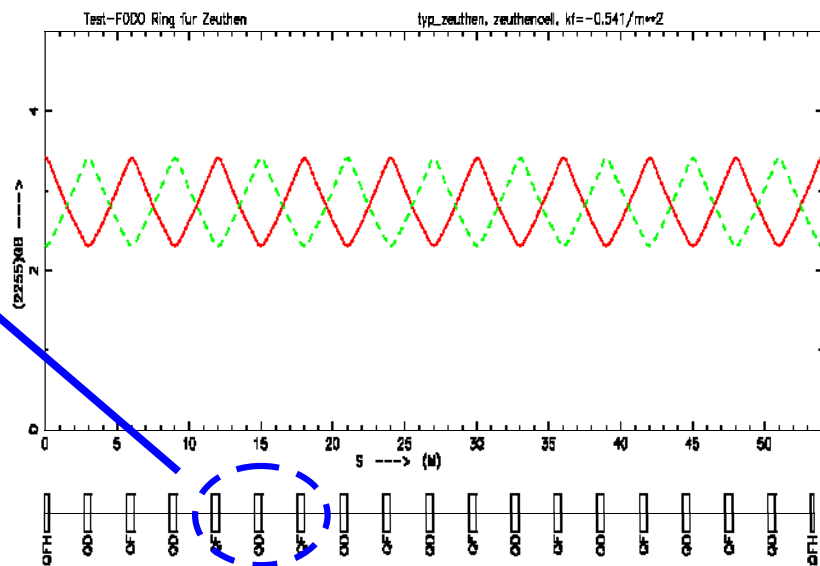
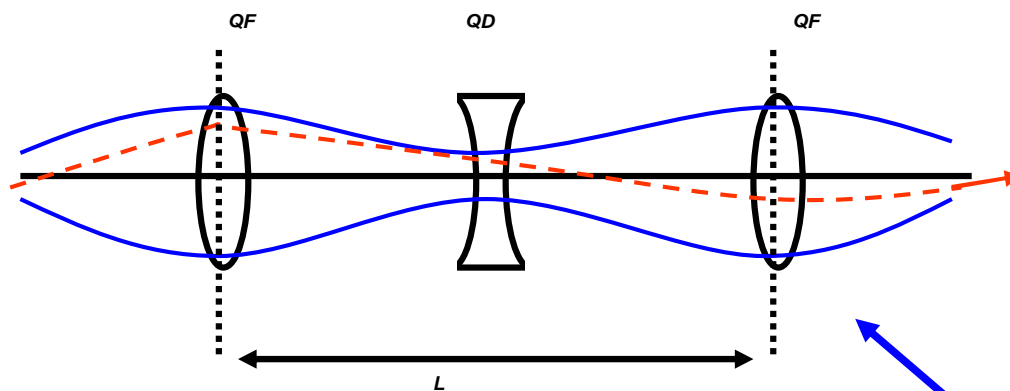
Straight sections: drift spaces for injection, dispersion suppressors,
 low beta insertions, RF cavities, etc....

... and the high energy experiments if they cannot be avoided

The FoDo-Lattice

A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with **nothing** in between.

(**Nothing** = elements that can be neglected on first sight: drift, bending magnets, RF structures ... **and especially experiments...**)

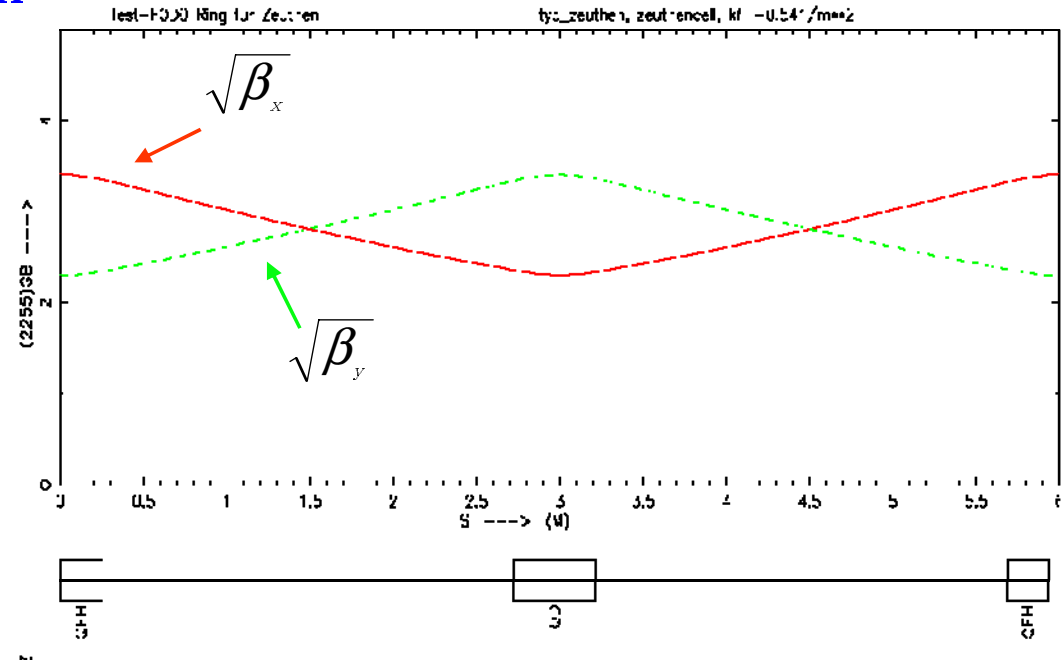


Starting point for the calculation: in the middle of a focusing quadrupole

Phase advance per cell $\mu = 45^\circ$,

→ calculate the twiss parameters for a periodic solution

Periodic solution of a FoDo Cell



Output of the optics program:

Nr	Type	Length <i>m</i>	Strength <i>1/m²</i>	β_x <i>m</i>	α_x	φ_x <i>1/2π</i>	β_z <i>m</i>	α_z	φ_z <i>1/2π</i>
0	IP	0,000	0,000	11,611	0,000	0,000	5,295	0,000	0,000
1	QFH	0,250	-0,541	11,228	1,514	0,004	5,488	-0,781	0,007
2	QD	3,251	0,541	5,488	-0,781	0,070	11,228	1,514	0,066
3	QFH	6,002	-0,541	11,611	0,000	0,125	5,295	0,000	0,125
4	IP	6,002	0,000	11,611	0,000	0,125	5,295	0,000	0,125

$QX = 0,125$ $QZ = 0,125$

$0,125 * 2\pi = 45^\circ$

Can we understand, what the optics code is doing?

matrices

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l_q) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l_q) \\ -\sqrt{K} \sin(\sqrt{K} * l_q) & \cos(\sqrt{K} * l_q) \end{pmatrix}, \quad M_{Drift} = \begin{pmatrix} 1 & l \\ 0 & 1_d \end{pmatrix}$$

strength and length of the FoDo elements

$$K = +/- 0.54102 \text{ m}^{-2}$$

$$l_q = 0.5 \text{ m}$$

$$l_d = 2.5 \text{ m}$$

The matrix for the **complete cell** is obtained by multiplication of the element matrices

$$M_{FoDo} = M_{qfh} * M_{ld} * M_{qd} * M_{ld} * M_{qfh}$$

Putting the numbers in and **multiplying out** ...

$$M_{FoDo} = \begin{pmatrix} 0.707 & 8.206 \\ -0.061 & 0.707 \end{pmatrix}$$

The transfer matrix for 1 period gives us all the information that we need !

1.) is the motion stable?

$$\text{trace}(M_{FoDo}) = 1.415 \rightarrow$$

< 2

2.) Phase advance per cell

$$M(s) = \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\gamma \sin \mu & \cos(\mu) - \alpha \sin \mu \end{pmatrix} \rightarrow$$

$$\cos(\mu) = \frac{1}{2} * \text{trace}(M) = 0.707$$

$$\mu = \arccos\left(\frac{1}{2} * \text{trace}(M)\right) = 45^\circ$$

3.) hor β -function

$$\beta = \frac{M(1,2)}{\sin(\mu)} = 11.611 \text{ m}$$

4.) hor α -function

$$\alpha = \frac{M(1,1) - \cos(\mu)}{\sin(\mu)} = 0$$

Can we do it a little bit easier ?

We can: ... the „thin lens approximation“

Matrix of a focusing quadrupole magnet:

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l) \\ -\sqrt{K} \sin(\sqrt{K} * l) & \cos(\sqrt{K} * l) \end{pmatrix}$$

If the focal length f is much larger than the length of the quadrupole magnet,

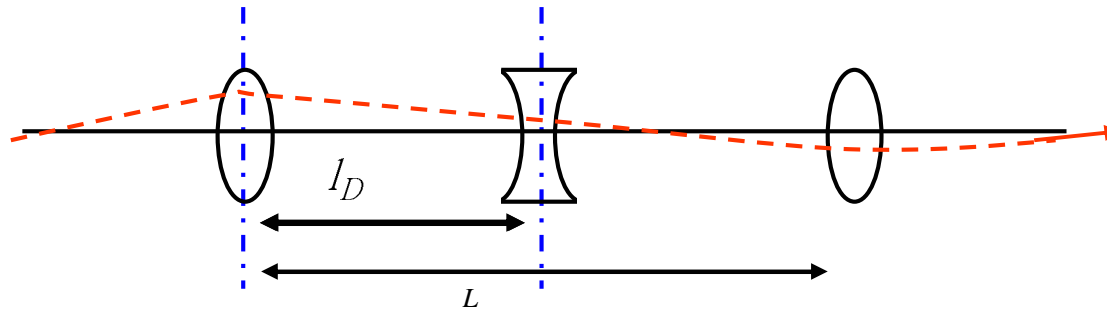
$$f = \frac{1}{kl_q} \gg l_q$$

the transfer matrix can be approximated using

$$kl_q = \text{const}, \quad l_q \rightarrow 0$$

$$M = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

FoDo in thin lens approximation



$$l_D = L/2,$$

$$\tilde{f} = 2f$$

Calculate the matrix for a half cell, starting in the middle of a foc. quadrupole:

$$M_{halfCell} = M_{QD/2} * M_{ID} * M_{QF/2}$$

$$M_{halfCell} = \begin{pmatrix} 1 & 0 \\ 1/\tilde{f} & 1 \end{pmatrix} * \begin{pmatrix} 1 & l_D \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ -1/\tilde{f} & 1 \end{pmatrix}$$

$$M_{halfCell} = \begin{pmatrix} 1 - l_D/\tilde{f} & l_D \\ -l_D/\tilde{f}^2 & 1 + l_D/\tilde{f} \end{pmatrix}$$

for the second half cell set $f \rightarrow -f$

FoDo in thin lens approximation

Matrix for the complete FoDo cell:

$$M = \begin{pmatrix} 1 + \frac{l_D}{\tilde{f}} & l_D \\ -\frac{l_D}{\tilde{f}^2} & 1 - \frac{l_D}{\tilde{f}} \end{pmatrix} * \begin{pmatrix} 1 - \frac{l_D}{\tilde{f}} & l_D \\ -\frac{l_D}{\tilde{f}^2} & 1 + \frac{l_D}{\tilde{f}} \end{pmatrix}$$

$$M = \begin{pmatrix} 1 - \frac{2l_D^2}{\tilde{f}^2} & 2l_D \left(1 + \frac{l_D}{\tilde{f}}\right) \\ 2\left(\frac{l_D^2}{\tilde{f}^3} - \frac{l_D}{\tilde{f}^2}\right) & 1 - 2\frac{l_D^2}{\tilde{f}^2} \end{pmatrix}$$

Now we know, that the **phase advance is related to the transfer matrix** by

$$\cos \mu = \frac{1}{2} \text{trace} (M) = \frac{1}{2} * \left(2 - \frac{4l_D^2}{\tilde{f}^2}\right) = 1 - \frac{2l_D^2}{\tilde{f}^2}$$

After some beer and with a little bit of trigonometric gymnastics

$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

we can calculate the phase advance as a function of the FoDo parameter ...

$$\cos(\mu) = 1 - 2 \sin^2(\mu/2) = 1 - \frac{2l_D^2}{\tilde{f}^2}$$

$$\sin(\mu/2) = l_D / \tilde{f} = \frac{L_{Cell}}{2\tilde{f}}$$

$$\sin(\mu/2) = \frac{L_{Cell}}{4f}$$

Example:

45-degree Cell

$$L_{Cell} = l_{QF} + l_D + l_{QD} + l_D = 0.5m + 2.5m + 0.5m + 2.5m = 6m$$

$$1/f = k * l_Q = 0.5m * 0.541 m^{-2} = 0.27 m^{-1}$$

$$\sin(\mu/2) \approx \frac{L_{Cell}}{4f} = 0.405$$

$$\rightarrow \mu \approx 47.8^\circ$$

$$\rightarrow \beta \approx 11.4m$$

Remember:

Exact calculation yields:

$$\mu = 45^\circ$$

$$\beta = 11.6m$$

Stability in a FoDo structure



SPS Lattice

$$M_{FoDo} = \begin{pmatrix} 1 - \frac{2l_D^2}{\tilde{f}^2} & 2l_D \left(1 + \frac{l_D}{\tilde{f}}\right) \\ 2\left(\frac{l_D^2}{\tilde{f}^3} - \frac{l_D}{\tilde{f}^2}\right) & 1 - 2\frac{l_D^2}{\tilde{f}^2} \end{pmatrix}$$

Stability requires: $\text{trace}(M) < 2$

$$\text{trace}(M) = \left| 2 - \frac{4l_D^2}{\tilde{f}^2} \right| < 2$$

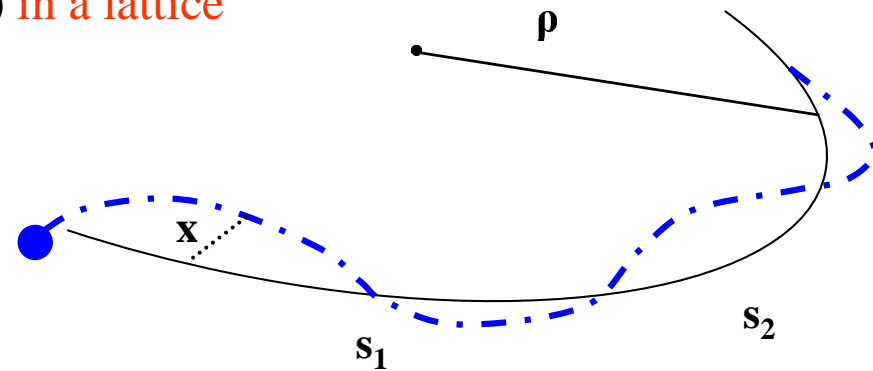
$$\rightarrow f > \frac{L_{cell}}{4}$$

For stability the focal length has to be larger than a quarter of the cell length !!

Transformation matrix in terms of the Twiss parameters

Transformation of the coordinate vector (x, x') in a lattice

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M_{s_1, s_2} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$



General solution of the equation of motion

$$x(s) = \sqrt{\varepsilon * \beta(s)} * \cos(\phi(s) + \varphi)$$

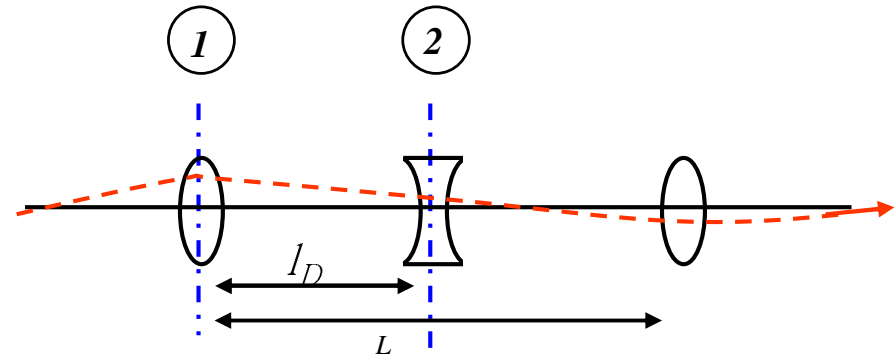
$$x'(s) = -\sqrt{\varepsilon / \beta(s)} * \{ \alpha(s) \cos(\phi(s) + \varphi) + \sin(\phi(s) + \varphi) \}$$

Transformation of the coordinate vector (x, x') expressed as a function of the twiss parameters

$$M_{1 \rightarrow 2} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}} (\cos \phi_{12} + \alpha_1 \sin \phi_{12}) & \sqrt{\beta_2 \beta_1} \sin \phi_{12} \\ \frac{(\alpha_1 - \alpha_2) \cos \phi - (1 + \alpha_1 \alpha_2) \sin \phi_{12}}{\sqrt{\beta_2 \beta_1}} & \sqrt{\frac{\beta_2}{\beta_1}} (\cos \phi_{12} - \alpha_2 \sin \phi_{12}) \end{pmatrix}$$

Transfer matrix for half a FoDo cell:

$$M_{\text{halfCell}} = \begin{pmatrix} 1 - l_D/\tilde{f} & l_D \\ -l_D/\tilde{f}^2 & 1 + l_D/\tilde{f} \end{pmatrix}$$



Compare to the twiss parameter form of M

$$M = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_2}} (\cos \phi_{12} + \alpha_2 \sin \phi_{12}) & \sqrt{\beta_1 \beta_2} \sin \phi_{12} \\ \frac{(\alpha_1 - \alpha_2) \cos \phi_{12} - (1 + \alpha_1 \alpha_2) \sin \phi_{12}}{\sqrt{\beta_1 \beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}} (\cos \phi_{12} - \alpha_2 \sin \phi_{12}) \end{pmatrix}$$

In the *middle of a foc (defoc) quadrupole of the FoDo we always have $\alpha = 0$* , and the *half cell will lead us from β_{\max} to β_{\min}*

$$M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\hat{\beta}}{\beta}} \cos \frac{\mu}{2} & \sqrt{\hat{\beta} \beta} \sin \frac{\mu}{2} \\ \frac{-1}{\sqrt{\hat{\beta} \beta}} \sin \frac{\mu}{2} & \sqrt{\frac{\hat{\beta}}{\beta}} \cos \frac{\mu}{2} \end{pmatrix}$$

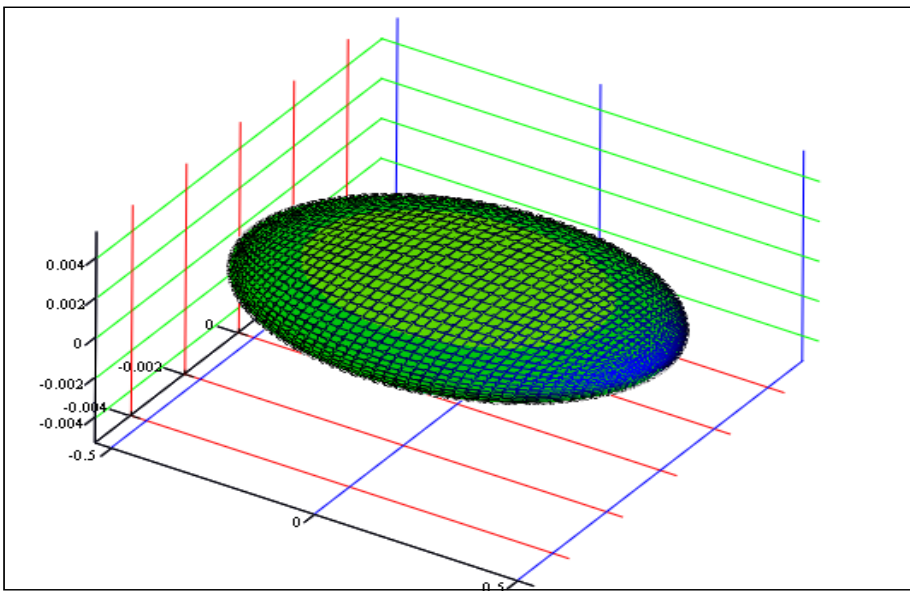
Solving for β_{max} and β_{min} and remembering that $\sin \frac{\mu}{2} = \frac{l_D}{\tilde{f}} = \frac{L}{4f}$

$$\left. \begin{aligned} \frac{S'}{C} &= \frac{\hat{\beta}}{\check{\beta}} = \frac{1+l_D/\tilde{f}}{1-l_D/\tilde{f}} = \frac{1+\sin \frac{\mu}{2}}{1-\sin \frac{\mu}{2}} \\ \frac{S}{C'} &= \hat{\beta} \check{\beta} = \tilde{f}^2 = \frac{l_D^2}{\sin^2 \frac{\mu}{2}} \end{aligned} \right\}$$



$$\hat{\beta} = \frac{(1 + \sin \frac{\mu}{2})L}{\sin \mu} \quad !$$

$$\check{\beta} = \frac{(1 - \sin \frac{\mu}{2})L}{\sin \mu} \quad !$$



(Z, X, Y)

The maximum and minimum values of the β -function are solely determined by the phase advance and the length of the cell.

Longer cells lead to larger β

typical shape of a proton bunch in the HERA FoDo Cell

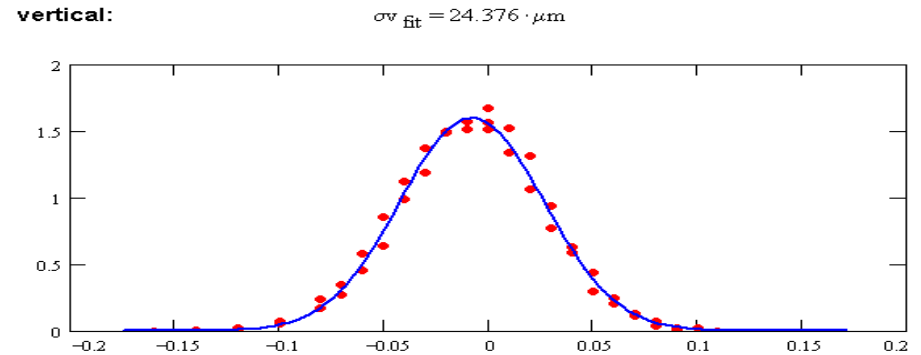
Beam dimension:

Optimisation of the FoDo Phase advance:

In both planes a **gaussian particle distribution** is assumed, given by the beam emittance ε and the β -function

$$\sigma = \sqrt{\varepsilon\beta}$$

HERA beam size

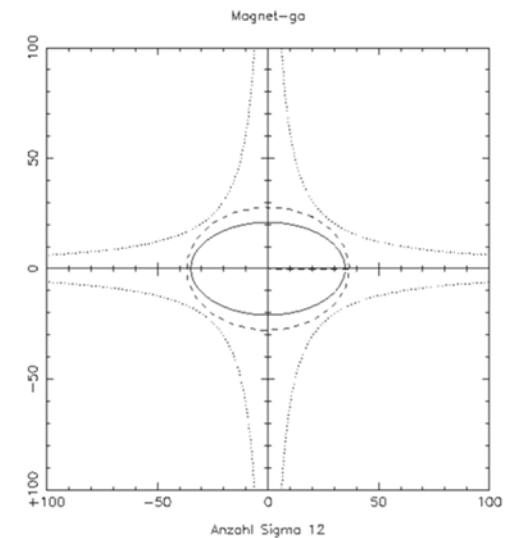


In general **proton beams are „round“** in the sense that

$$\varepsilon_x \approx \varepsilon_y$$

So for highest aperture we have to **minimise the β -function** in both planes:

$$r^2 = \varepsilon_x \beta_x + \varepsilon_y \beta_y$$



typical beam envelope, vacuum chamber and pole shape in a foc. Quadrupole lens in HERA

Optimising the FoDo phase advance

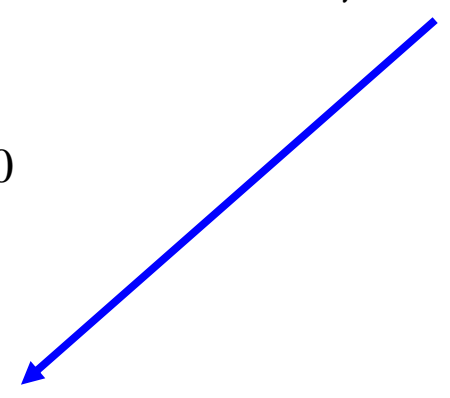
$$r^2 = \varepsilon_x \beta_x + \varepsilon_y \beta_y$$

search for the phase advance μ that results in a minimum of the sum of the beta's

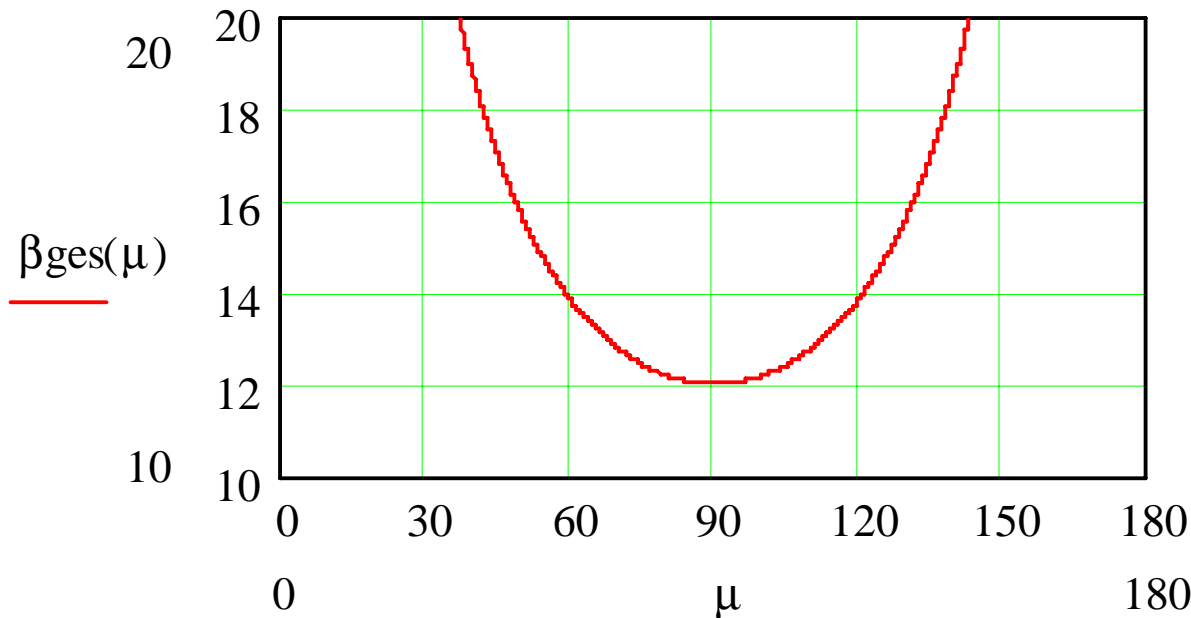
$$\hat{\beta} + \check{\beta} = \frac{(1 + \sin \frac{\mu}{2}) * L}{\sin \mu} + \frac{(1 - \sin \frac{\mu}{2}) * L}{\sin \mu}$$

$$\hat{\beta} + \check{\beta} = \frac{2L}{\sin \mu}$$

$$\frac{d}{d\mu} \left(\frac{2L}{\sin \mu} \right) = 0$$



$$\frac{L}{\sin^2 \mu} * \cos \mu = 0 \rightarrow \mu = 90^\circ$$

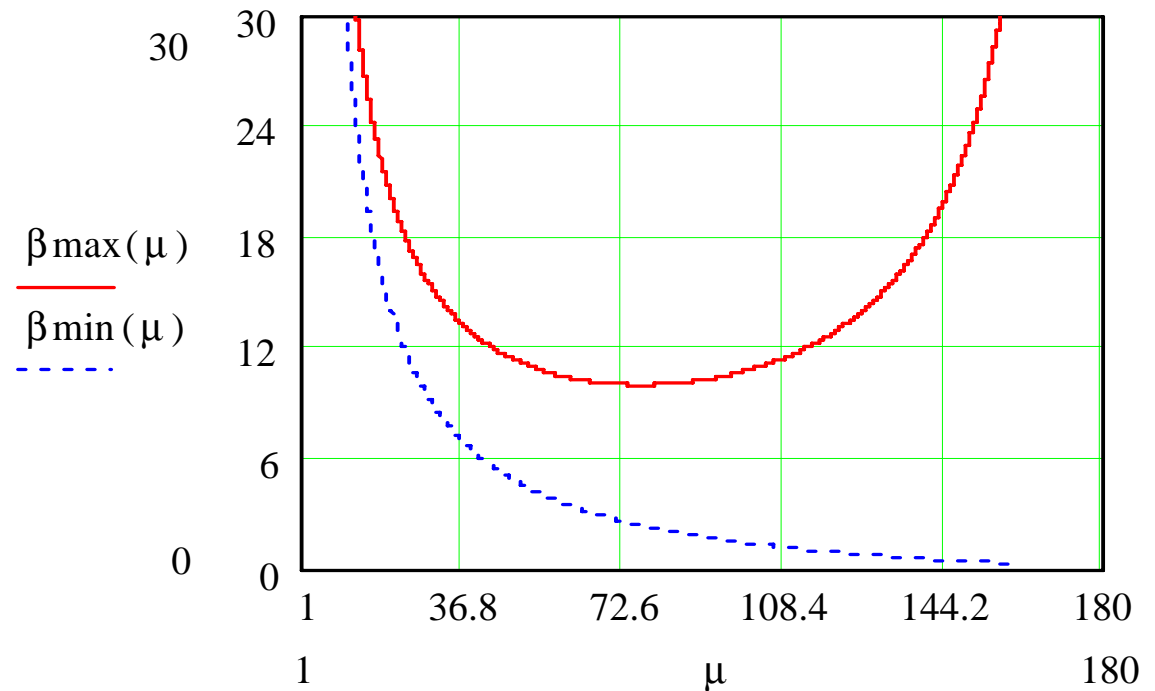


Electrons are different

electron beams are usually flat, $\varepsilon_y \approx 2 - 10 \% \varepsilon_x$
→ optimise only β_{hor}

$$\frac{d}{d\mu}(\hat{\beta}) = \frac{d}{d\mu} \frac{L(1 + \sin \frac{\mu}{2})}{\sin \mu} = 0 \rightarrow \mu \approx 76^\circ$$

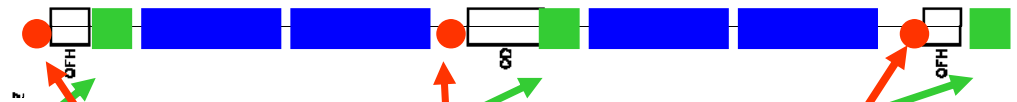
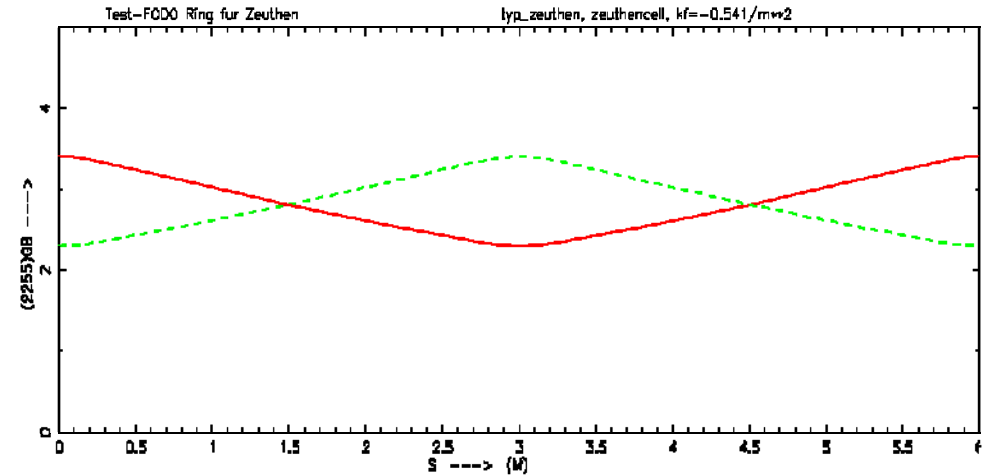
red curve: β_{max}
blue curve: β_{min}
as a function of the phase advance μ



Orbit distortions in a periodic lattice

field error of a dipole/distorted quadrupole

$$\rightarrow \delta(\text{mrad}) = \frac{ds}{\rho} = \frac{\int B ds}{p/e}$$



the particle will follow a new closed trajectory, the distorted orbit:

$$x(s) = \frac{\sqrt{\beta(s)}}{2 \sin \pi Q} * \oint \frac{\sqrt{\beta(\tilde{s})}}{\rho(\tilde{s})} \cos(|\phi(s) - \phi(\tilde{s})| - \pi Q) d\tilde{s}$$

* the orbit amplitude will be large if the β function at the location of the kick is large
 $\beta(\tilde{s})$ indicates the sensitivity of the beam \rightarrow here orbit correctors should be placed in the lattice

* the orbit amplitude will be large at places where in the lattice $\beta(s)$ is large \rightarrow here beam position monitors should be installed

Orbit Correction and Beam Instrumentation in a storage ring



Elsa ring, Bonn

*

Resumé:

1.) Dipole strength:

$$\int B ds = N * B_0 * l_{eff} = 2\pi \frac{p}{q}$$

l_{eff} effective magnet length, N number of magnets

2.) Stability condition:

$$Trace(M) < 2$$

for periodic structures within the lattice / at least for the transfer matrix of the complete circular machine

3.) Transfer matrix for periodic cell

$$M(s) = \begin{pmatrix} \cos \mu + \alpha(s) \sin \mu & \beta(s) \sin \mu \\ -\gamma(s) \sin \mu & \cos(\mu) - \alpha(s) \sin \mu \end{pmatrix}$$

α, β, γ depend on the position s in the ring, μ (phase advance) is independent of s

4.) Thin lens approximation:

$$M_{QF} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_Q} & 1 \end{pmatrix} \quad f_Q = \frac{1}{k_Q l_Q}$$

focal length of the quadrupole magnet $f_Q = 1/(k_Q l_Q) \gg l_Q$

5.) Tune (rough estimate):

$$\mu = \int_s^{s+L} \frac{dt}{\beta(t)}$$

*Tune = phase advance
in units of 2π*

$$Q := N * \frac{\mu}{2\pi} = \frac{1}{2\pi} * \oint \frac{ds}{\beta(s)} \approx \frac{1}{2\pi} * \frac{2\pi \bar{R}}{\bar{\beta}} = \bar{R} / \bar{\beta}$$

\bar{R} , $\bar{\beta}$ average radius
and β -function

$$Q \approx \frac{\bar{R}}{\bar{\beta}}$$

6.) Phase advance per FoDo cell

(thin lens approx)

$$\sin \frac{\mu}{2} = \frac{L_{Cell}}{4f_Q}$$

L_{Cell} length of the complete FoDo cell, f_Q focal length of the quadrupole, μ phase advance per cell

7.) Stability in a FoDo cell

(thin lens approx)

$$f_Q > \frac{L_{Cell}}{4}$$

8.) Beta functions in a FoDo cell

(thin lens approx)

$$\hat{\beta} = \frac{(1 + \sin \frac{\mu}{2}) L_{Cell}}{\sin \mu} \quad \check{\beta} = \frac{(1 - \sin \frac{\mu}{2}) L_{Cell}}{\sin \mu}$$

L_{Cell} length of the complete FoDo cell, μ phase advance per cell

Conclusion:

- * *„the arc“ of a storage ring is usually built out of a periodic sequence of single magnet elements eg. FoDo sections*
- * *a first guess of the main parameters of the beam in the arc is obtained by the settings of the quadrupole lenses in this section*
- * *we can get an estimate of the beam parameters using a selection of „rules of thumb“*

Usually the real beam properties will not differ too much from these estimates and we will have a nice storage ring and a beautiful beam and everybody is happy around.

And then someone comes and spoils it all by saying something stupid like installing a tiny little piece of detector in our machine ...

INSERTIONS

