INTRODUCTION TO FEEDBACK SYSTEMS

1. Tasting the benefits of feedback: a Static Case
2. Feedback control of Dynamic Systems
3. Digital Feedback Systems
4. Local Orbit Feedback
5. Transverse Multi-Bunch Feedback

D. Bulfone
- Control is a very common concept:

  > Manual Control

  > Automatic Control: involves ‘machine’ only

- **Control**: Control is the process of causing a process variable to conform to some desired value, called a reference value.

- **Feedback Control**: Feedback is the scheme of measuring the **controlled variable** and using that information to influence the value of the **controlled variable itself** (closed-loop control).

- The overall goal of feedback control is to use the principle of feedback to cause the controlled variable of a **dynamic process** to follow a desired reference regardless of any external disturbances or changes in the process dynamics.
1. TASTING THE BENEFITS OF FEEDBACK: A STATIC CASE
Example of a ‘static’ case

Beam Position x

steerer magnet
power supply

beam position monitor

power supply current change $\Delta I = 1A \rightarrow$ kick angle $\theta = 0.5$ mrad

temperature variation $\Delta T = 1C \rightarrow$ power supply current drift $\Delta I = -0.2$ A

(assume relations above linear and invariant with time)
Component & Functional Block Diagrams

- Component Block Diagram

```
<table>
<thead>
<tr>
<th>Desired beam position</th>
<th>Controller</th>
<th>Actuator</th>
<th>Process</th>
<th>Actual beam position</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
</tbody>
</table>
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- Functional Block Diagram

\[ r = \text{reference (desired) beam position [mm]} \]
\[ u = \text{power supply current [A]} \]
\[ y = \text{actual beam position [mm]} \]
\[ w = \text{temperature change [°C]} \]

\[ r \rightarrow \Sigma \rightarrow u \rightarrow + \rightarrow 0.2 \rightarrow 5 \rightarrow y \]

\[ (1°C \rightarrow 0.2 A) \]
\[ (1A \rightarrow 0.5 \text{ mrad} \times 10 \text{ m} \rightarrow 5 \text{ mm}) \]
Open and Closed-loop Control

- Open-loop Control

\[ y_{ol} = 5(u - 0.2w) = 5u - w \]

\[ w = 0; \text{ set } r = 5 \text{ mm } \rightarrow u = \frac{1}{5} r \rightarrow y_{ol} = r \]

\[ w = 1 \rightarrow y_{ol} = 4 \text{ mm } \rightarrow |(y_{ol} - r)/r| = 20\% \]

- Closed-loop Control

\[ \left\{ \begin{array}{l}
    y_{cl} = 5(u - 0.2w) = 5u - w \\
    u = 100(r - y_{cl}) \\
    y_{cl} = 0.998r - 0.002w \\
    w = 0 \rightarrow y_{cl} = 4.99 \text{ mm } \rightarrow |(y_{ol} - r)/r| = 0.2\% \\
    w = 1 \rightarrow y_{cl} = 4.988 \text{ mm } \rightarrow |(y_{ol} - r)/r| = 0.24\%
\end{array} \right. \]
Open and Closed-loop Control

**- Open-loop Control**

\[ y_{ol} = r - w \]

| w  | \(|(y-r)/r| \ [%] \) |
|----|-----------------|
| 0  | 0               |
| 1  | 20              |
| 2  | 40              |
| 3  | 60              |

**- Closed-loop Control**

\[ y_{cl} = 0.998r - 0.002w \]

- (Great) reduction of sensitivity to temperature (and plant) changes
- (Small) steady-state error when there is no perturbation (w=0)
2. FEEDBACK CONTROL OF DYNAMIC SYSTEMS [1-3]
Feedback Control of Dynamic Systems

- How to characterize and how to implement feedback control on dynamic systems?

**Step 1. Development of the System Dynamic (Mathematical) Model:**
- set of differential equations that describe the dynamic behaviour of the system

**Step 2. Analysis of the System Dynamics:**

2A. **TRANSFER FUNCTION** methods:
- system pulse response ( \( \rightarrow \) time domain )
- system frequency response ( \( \rightarrow \) frequency domain )

2B. **STATE SPACE** method:
- time domain
- system described by a set of first-order differential equations rather then by one or more \( n^{th} \)-order differential equations
- basis for solving broader classes of control system problems, including most advanced and recent developments (non-linear, adaptive, optimal control, etc….)
Feedback Control of Dynamic Systems

**Step 3. Design of the Appropriate Feedback Control:**

- While an open-loop Controller has no effect on the dynamics of the system, it does change it in a closed-loop feedback configuration.

- Design the controller that gives the closed-loop system the specified characteristics:
  1. Reject effect of disturbances on the system output
  2. Reduce steady-state errors
  3. Reduce the sensitivity to plant parameter and their changes
  4. Improve transient response: e.g. increase speed of response – increase system bandwidth
  5. Increase relative stability

- Effect of increasing feedback gain: pros (😊) and cons (🙁)

  → Trade-off exists between beneficial and detrimental effects

**Step 4. Simulate**
**Step 1. Development of the System Dynamic Model**

- Linear time-constant systems → use Laplace transform to analyze diff. equations:

\[
F(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt \quad \text{for } f(t) \text{ defined for } t > 0; \quad s = \sigma + j\omega
\]

- for zero initial conditions:

\[
RI_{\text{out}}(s) + LI_{\text{out}}(s) = RI_{\text{in}}(s)
\]

\[
I_{\text{out}}(s) = \frac{R}{R + sL}I_{\text{in}}(s)
\]
Step 1. Development of the System Dynamic Model

- From Component Diagrams to Block Diagrams:

\[ \frac{Y}{R} = \frac{K \cdot R \cdot B(s)}{R + sL + [K \cdot R \cdot B(s) \cdot M(s)]} \]

\( R, E, U, W, Y, B: \) Laplace-transformed signals
\( K, B(s), M(s): \) transfer functions

- Closed-Loop Feedback Does Change System Dynamics
**Step 2. Analysis of the System Dynamics**

- Instead of solving for $Y(s)$ and anti-transforming, different techniques (for ex. root-locus method, Bode-plot representation, Nyquist diagrams, Nichols charts) exist to predict system dynamical behavior from the analysis of its transfer function.

- **Pulse Response (time domain):**
  - Being the Laplace transform of $\delta(t)$ equal to 1, the transfer function is the Laplace transform of the system impulse response.
  - The pulse response is characterized by the location of the transfer function poles (and zeros).
**Step 2. Analysis of the System Dynamics**

**Step 3. Design of the Feedback Controller**

- **Goal**: design an appropriate controller (compensator) that changes location of poles and zeros to obtain desired system performance.

- **Controller**: passive (resistors, capacitors, etc.) and/or active components (amplifiers, computers).

\[
\frac{1}{s^2 + as + b}
\]
Step 2. Analysis of the System Dynamics

- **Frequency Response (frequency domain):**
  - Frequency response is a representation of the system’s steady-state response to sinusoidal inputs at varying frequencies.
  - Response of a system with transfer function $G(s)$ to an input $u(t) = U_0 \sin(\omega t)$ is:

  $$y(t) = U_0 |G(j\omega)| \sin(\omega t + \theta) \quad \text{with} \quad |G(j\omega)| = |G(s)| \bigg|_{s=j\omega} \quad \theta = \tan^{-1} \frac{\text{Im}[G(j\omega)]}{\text{Re}[G(j\omega)]}$$

- **Bode Diagrams:** frequency on logarithmic scale, phase in degrees and magnitude in decibels $= 20 \log_{10}(|G(j\omega)|)$

- Frequency response and pulse response curves contain the same ‘dynamic’ information (system rise time, peak overshoot, etc…)

\[
\frac{50}{s^3 + 9s^2 + 30s + 40}
\]
**Step 2. Analysis of the System Dynamics**

- Closed-loop behavior can be estimated from the experimental measurement of the open-loop frequency response.

\[- u(t) = U_0 \sin(\omega t) \]

\[ y(t) = U_0 \frac{K_G K_H G(j\omega) H(j\omega)}{1 + K_G K_H G(j\omega) H(j\omega)} \sin(\omega t + \theta) \]

- Instability occurs when \( |1 + D(j\omega)| = 0 \)

- For most real-life systems, magnitudes decrease and phase lags increase with increasing frequencies

→ stability conditions:

\[ |D(j\omega)| < 1 \text{ at } \angle D(j\omega) = -180^\circ \text{ or } \angle D(j\omega) > -180^\circ \text{ at } |D(j\omega)| = 1 \]
Step 3. Design of the Feedback Controller

- **The Proportional, Integral, Derivative (PID) Controller:**

- The PID is the most widely used controller type that allows reaching a good compromise of the overall closed-loop performance.

- **Proportional term** increases disturbance rejection, lowers sensitivity to parameter changes and steady-state errors (never to zero), but reduces stability

- **Integral term** eliminates the steady-state error, but also reduces stability

- **Derivative term** improves stability, reducing the overshoot and improving the transient response.

\[
u(t) = K_p e(t) + K_i \int_0^t e(\tau) \, d\tau + K_d \frac{de}{dt}
\]
3. DIGITAL FEEDBACK SYSTEMS [4-7]
- **Digital Control**: Employment of a digital computer in the control of a process.

- **Digital Feedback Systems**: Digital Feedback Systems

  - Employment of a digital computer in the control of a process.

- $T_s = \text{sampling period}, \quad \omega_s = \text{sampling frequency}$

- $r(kT), e(kT), y(kT), u(kT), k = 0, 1, 2 \ldots$ are sequences of numbers, called discrete variables - also $r(k), e(k), \text{etc…}$

- Aliasing effect of sampling process:

  $$|A(\omega)|$$

  ![Diagram](attachment://diagram.png)
Digital Feedback Systems

**- The z-transform:** The z-transform of the discrete variable \( f(k) \) \( k=0, 1, 2 \ldots \) is

\[
F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}
\]

- The z-transform plays the same role for discrete-time systems that the Laplace transform does for continuous-time systems:

  → Can define the transfer function of a digital system as the ratio of the z-transforms of the output and input discrete signals
  → Can use block diagrams

\[
\begin{align*}
R & \quad \Sigma \quad E
\end{align*}
\]

\[
\begin{align*}
Y(z) &= \frac{G(z) H(z)}{1 + G(z) H(z)} \\
B \quad \frac{Y(z)}{R(z)}
\end{align*}
\]

- Relationship between s-plane and z-plane

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Digital Feedback Systems

- Digital Filters:
- The Control Algorithm executed by the digital computer can be written in the most general form as:

\[ u(k) = a_0 e(k) + a_1 e(k-1) + a_2 e(k-2) + \ldots + a_N e(k-N) - b_1 u(k-1) - b_2 u(k-2) \ldots - b_M u(k-M) \]

\[
\frac{U(z)}{E(z)} = H(z) = \frac{\sum_{k=0}^{N} a_k z^{-k}}{1 + \sum_{k=1}^{M} b_k z^{-k}}
\]

- What is the dynamic behavior of the digital filter? What the frequency response?
- Given the digital filter z-transform \( H(z) \), its frequency response is \( H(e^{j\omega T}) \):
  - It is periodic with period \( \omega_s \)
  - With \( a_k \) and \( b_k \) real,
    \[
    |H(e^{j\omega T})| = |H(e^{-j\omega T})|
    \]
    \[
    \angle H(e^{j\omega T}) = -\angle H(e^{-j\omega T})
    \]
  - Plotted only between 0 and the Nyquist frequency \( \omega_s/2 \)
Digital Feedback Systems

- **Advantages:**
  - Immune to variations induced by the environment (e.g. temperature) or aging
  - Reproducibility
  - Accuracy specified by controller A/D resolution, word length, fixed/floating point arithmetic, does not depend on components tolerances.
  - Flexibility: modifications done by software
  - Digital filters typically have higher performance in terms of attenuation and frequency selectivity
  - Implementation of sophisticated control algorithms
  - Combination of dynamic control algorithms with logic/decision making capabilities of computers
  - Effective integration with control system: logging of system data, diagnostics in parallel to feedback operation

- **Disadvantages:**
  - Loop delays induced by the sampling mechanism and computation can affect system stability
  - Always electric systems (by definition): cannot be used, for ex., in explosion risky environments

- Where digital circuits are available and have sufficient speed to perform the signal processing, they are usually preferable.
4. LOCAL ORBIT FEEDBACK [8-9]
Local Orbit Feedback

- Used, for example, in synchrotron radiation sources to stabilize the electron beam position and angle at the center of the Insertion Device (Undulator or Wiggler), without affecting the rest of the orbit.

- Local orbit bump

- 2 BPMs and 4 corrector magnets (typically)
Local Orbit Feedback

- 4 inputs (correctors) and 2 outputs (BPMs): local feedback is a MIMO (Multi-Input-Multi-Output) system, usually analyzed with the state space formalism.

- Given the 4 available degrees of freedom (the corrector settings), we can define a (4x2) ‘Bump Matrix’ B that satisfies 4 constraints: closure of orbit bump + desired position at BPM#1 + desired position at BPM#2

- The Bump Matrix transforms the (4 inputs x 2 outputs) system into a (2 inputs x 2 outputs) system with decoupled channels, with independent controllers.
Local Orbit Feedback

- **Design Specifications:**
  - Stabilize beam with sub-micron accuracy from DC to ~100 Hz with respect to external disturbances, including noise components at 50 Hz mains frequency and harmonics.
  - Cancel steady-state errors
  - Reduce sensitivity to plant parameter and their changes
  - Stability
- Machine dynamic behavior dominated by corrector magnet + power supply: -3dB cut-off frequency of 70 Hz.

- Negligible phase delay induced by eddy currents in stainless steel vacuum chamber: 1deg@60Hz, 2deg@100Hz.

- **Digital Controller:**
  - Delay from BPM to corrector D/A (in $T_s$)
  - PID Controller
Control Algorithms

- **The PID Controller:**

- **Harmonic Suppressor(s):**
  - Remove specific periodic noise components induced by the mains (50 Hz + harmonics).
  - Digital resonator selects the noise frequency, delay shifts phase to produce 180° at the output.

![PID Controller Diagram]

Closed-loop ratio of output position over noise with PID controller ($K_P=3$, $K_I=0.01$, $K_D=10$): ~3dB point at 150 Hz
operation & performance

- beam position spectra:

- horizontal and vertical beam position spectra measured by a BPM:
  - LOF OFF (blue)
  - LOF ON with PID only (green)
  - LOF ON with PID + HS at 50, 100, 150, 200, 250, 300 Hz (red)
**Operation & Performance**

- **Time Domain Measurements**:

Horizontal (left) and vertical (right) beam position measured by a low-gap BPM with LOF OFF / ON (8 kHz acquisition rate, low-pass filtered at 250 Hz)

- *rms* of position and angle of source point at ID centre over 8-hour period with LOF ON, calculated from the two low-gap BPM readings:

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>10 - 250 Hz</td>
<td>0.85</td>
<td>0.47</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>0 - 10 Hz</td>
<td>0.15</td>
<td>0.06</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

[8 kHz BPM acquisition rate] [20 Hz BPM acquisition rate]
Local Orbit Feedback Operation

- Real-time observation of Local Orbit Feedback effect:

- PID ($K_p=3$, $K_i=0.01$, $K_d=5$) + 5 Harmonic Suppressors at 23, 50, 100, 150 and 200 Hz

FEEDBACK ON
5. TRANSVERSE MULTI-BUNCH FEEDBACK [10-12]
Coupled-Bunch Instabilities

- Several advanced applications of circular accelerators rely on intense beams with hundreds of bunches at high current. For example:

  Particle Factories (B, Φ, etc…) ➞ “Luminosity”
  Synchrotron Radiation Sources ➞ “Brightness”

- Electro-magnetic fields generated by the beam interact with the surroundings and act back on the beam itself, producing **Coupled Bunch Instabilities (CBI)** on both the longitudinal and transverse plane.

**Example:** interaction with an RF cavity can excite its Higher Order Modes

Bunch

“Zoomed” Beam Pipe
**Coupled-Bunch Instabilities**

- **A Basic CBI Model: the Coupled Mechanical Pendulums**

  - Motion of the n-th bunch in a beam of M bunches:

    \[
    \ddot{y}_n(t) + d \dot{y}_n(t) + \omega^2 y_n(t) = F(t) \quad \ddot{y}_n(t) + (d - A) \dot{y}_n(t) + \omega^2 y_n(t) = 0
    \]

  - Natural damping mechanisms
  - CBI driving force
  - Betatron/Synchrotron frequency for Transverse/Longitudinal plane: \textit{tune} \times \textit{revolution frequency} \( \omega_0 \)

- **Vertical Plane Example**
Coupled-Bunch Instabilities

- **A Basic CBI Model: the Coupled Mechanical Pendulums**
  - Motion of the n-th bunch in a beam of M bunches:
    \[
    \ddot{y}_n(t) + d \dot{y}_n(t) + \omega^2 y_n(t) = F(t)
    \]
    \[
    \ddot{y}_n(t) + (d - A) \dot{y}_n(t) + \omega^2 y_n(t) = 0
    \]

- **Natural damping mechanisms**
- **Betatron/Synchrotron frequency for Transverse/Longitudinal plane:**
  - tune \( \times \) revolution frequency \( \omega_0 \)

- **Vertical Plane Example**
  - All modes appear once in a \( M\omega_0/2 \) freq. span

- **Vertical Betatron Sidebands**

**Introduction to Feedback Systems, D. Bulfone**

CERN Accelerator School
ICTP - Trieste
02 - 14 October 2005
Coupled-Bunch Instabilities

- **A Basic CBI Model: the Coupled Mechanical Pendulums**
  - Motion of the n-th bunch in a beam of M bunches:
    \[
    \ddot{y}_n(t) + d \dot{y}_n(t) + \omega^2 y_n(t) = F(t)
    \]
    \[
    \ddot{y}_n(t) + (d-A) \dot{y}_n(t) + \omega^2 y_n(t) = 0
    \]

- **Vertical Plane Example**
  - In case A>d:

  ![Diagram of a beam with bunches and mechanical pendulums]
Feedback Operating Principle

- The Bunch-by-Bunch Approach:

- “Bunch-by-bunch” feedback individually detects and corrects motion of each bunch

\[ \dot{y}_n(t) + (d + G - A) y_n(t) + \omega^2 y_n(t) = 0 \]

- The closed-loop system adds a damping term to the bunch motion

- The controller determines a kick signal for each bunch that is shifted by \( \pi/2 \) phase with respect to the oscillation signal of the same bunch when it passes through the kicker

- The feedback acts on all the bunches. It damps all instability modes
Feedback Operating Principle

- **Analyzing the Bunch System Dynamics**:

\[
\ddot{y}_n(t) + (d-A)\dot{y}_n(t) + \omega^2 y_n(t) = u(t) \quad \text{Laplace transform} \rightarrow \quad s^2Y(s) + s(d-A)Y(s) + \omega^2 = U(s)
\]

- “Bunch” transfer function \( B(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + s(d-A) + \omega^2} \)

- Real part of complex conjugate poles is \((A - d)/2\)

\[ \propto \text{feedback gain} \]

![Graphs showing LHP, 0, and RHP regions with points for different values of d and G]
Feedback System Architecture

- **ELETTRA / Swiss Light Source Design Specifications:**
  - Bunch-by-bunch feedback, acting on 432/480 (ELETTRA/SLS) 2-ns spaced bunches
  - Bandwidth 250 MHz

  - Reject the DC “stable beam” component from the BPM signal

- **Additional Controller Requirements:**
  - Availability of diagnostic tools
  - Flexibility
  - Same processing electronics for the transverse and longitudinal systems

→**Digital Controller, based on software programmable DSPs**

![Feedback System Architecture Diagram](image-url)
- **Digital Filters:**
  - Can provide any phase → feedback operation independent from BPM-kicker relative betatron phase

- 3-coefficient Finite Impulse Response (FIR) filter (nominal vertical fractional tune 0.177)

- 5-coefficient FIR filter with compensation of tune variations (energy ramping, closing/opening IDs)
5.1 MEASUREMENTS AND BEAM MANIPULATION TOOLS
High-resolution Wide-band Spectra

- 250MHz-wide spectra for complete multi-bunch mode analysis. 1 kHz resolution with repetition rate in control room of 0.5 Hz.
- Max. resolution 5.2 Hz.

- Base-band 250MHz-wide multi-bunch spectra (with rejected DC closed orbit component), TMBF off/on

- 12MHz-wide zoom on lower frequency part of left side spectra
- Growth/damp transients are created by switching the feedback off/on or antidamping/damping through the proper setting of the digital filter coefficients.

- Transient frequency domain analysis
- Rise times and damping rates of coupled-bunch modes can be measured by fitting the acquired data.

- Spontaneous growth of oscillation amplitudes in the bunch train
- Filter coefficients set to zero and restored back after a specified 3.6 ms interval.
Growth/Damp Transients

‘Movie’ sequence:
1. TMBF off
2. TMBF on after about 5.2 ms
‘Camera’ view slice is 50 turns (about 43 μs)
Betatron Tune Measurement

- When beam is affected by CBIs, spectrum of turn-by-turn position data of a given bunch provides fractional betatron tune

![Graph of Turn-by-turn vertical position of a single bunch: 200000 turns](image1)

![Graph of Spectrum of the position signal of a single bunch](image2)
- **Objective:** measure betatron tune without affecting Users experimental activities (TMBF on)

- **Technique:**
  - Excite one (or few) selected bunch(es): antidamping/damping transients with arbitrary downloadable waveforms (for ex. white, pink noise…)
  - Acquire data and perform FFT
Anti-Damping/Damping Transient on 24 Selected Bunches

‘Movie’ sequence:
1. Anti-damping TMBF
2. Damping TMBF after about 3.5 ms

‘Camera’ view slice is 50 turns (about 43 μs)
**Betatron Tune Measurement by Excitation of Single Bunches**

- **Objective:** measure betatron tune without affecting Users experimental activities (TMBF on)
- **Technique:**
  - Excite one (or few) selected bunch(es): antidamping/damping transients with arbitrary downloadable waveforms (for ex. white, pink noise…)
  - Acquire data and perform FFT

- **Betatron Tune Tracking**
  - **Objective:** Adaptive technique to keep TMBF operation at its optimum working point, irrespectively of betatron tune changes.
  - **Technique:**
    a. Measure tune (see above)
    b. Calculate feedback digital filter coefficients according to the updated tune value
    c. Download coefficients into the running DSPs.
REFERENCES


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