Instabilities Part I: Introduction – multiparticle systems and dynamics

Giovanni Rumolo and Kevin Li
In this introductory part, we will provide a qualitative description of collective effects and their impact on particle beams.

We will introduce multiparticle systems and investigate multiparticle effects. This will be the first step towards a more involved understanding of collective effects and their effect (next lectures).

• Part 1: Introduction – dynamics of multiparticle systems

  • Introduction to beam instabilities
  • Instabilities examples
  • Basic concepts
    • Beam matching
    • Multiparticle effects – filamentation and decoherence
What are collective effects?

- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.
What are collective effects?

• We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

• First step → **Coordinates system** we will use throughout this set of lectures
  • The origin O is moving along with the “synchronous particle”, i.e. a reference particle that has the design momentum and follows the design orbit
  • Transverse coordinates x (Horizontal) and y (Vertical) relative to reference particle (x,y << R), typically x is in the plane of the bending
  • Longitudinal coordinate z relative to reference particle
  • Position along accelerator is described by independent variable s = vt
What are collective effects?

- A charged particle beam is generally described as a **multiparticle system** via the **coordinates** and the **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional phase space which can be described by a **particle distribution function**.

- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):

\[
\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s)
\]
What are collective effects?

- A charged particle beam is generally described as a **multiparticle system** via the **coordinates** and the **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional phase space which can be described by a **particle distribution function**.

- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):
  - Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
  - Collective effects add to this **distribution dependent force fields** (space charge, wake fields)

\[
\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s) \propto f (F_{\text{extern}} + F_{\text{coll}} (\psi))
\]
What are collective effects?

- A charged particle beam is generally described as a multiparticle system via the coordinates and the canonically conjugate momenta of all of its particles — this makes up a distribution in the 6-dimensional phase space which can be described by a particle distribution function.
- Hence, we will study the evolution of the phase space occupied by this particle distribution (and described by its particle distribution function):
  - Optics defined by the machine lattice provides the external force fields (magnets, electrostatic fields, RF fields), e.g., for guidance and focusing.
  - Collective effects add to this distribution dependent force fields (space charge, wake fields).

For a multiparticle system this self-consistent equation becomes arbitrarily complex and practically impossible to solve.

Obtaining the multiparticle dynamics very often requires computer simulation codes.

\[
\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s) \propto f (F_{\text{extern}} + F_{\text{coll}} (\psi))
\]
A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!

\[
N = \int \psi(x, x', y, y', z, \delta) \, dx \, dx' \, dy \, dy' \, dz \, d\delta
\]

\[
\langle x \rangle = \frac{1}{N} \int x \cdot \psi(x, x', y, y', z, \delta) \, dx \, dx' \, dy \, dy' \, dz \, d\delta
\]

\[
\sigma_x^2 = \frac{1}{N} \int (x - \langle x \rangle)^2 \cdot \psi(x, x', y, y', z, \delta) \, dx \, dx' \, dy \, dy' \, dz \, d\delta
\]

and similar definitions for \(\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z\)
The instability loop

Equations of motion of the beam particles

Interaction with the external environment

\[ (\vec{E}, \vec{B}) \]

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields
The instability loop

When the loop closes, either the beam will find a new stable equilibrium configuration ...

Equations of motion of the beam particles

Interaction with the external environment

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

\( s = vt \)

\( (\vec{E}, \vec{B}) \)
The instability loop

Multi-bunch beam

$s = vt$

Equations of motion of the beam particles

Interaction with the external environment

$\left( \vec{E}, \vec{B} \right)$

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

... or it might develop an instability along the bunch train ...
The instability loop

Equations of motion of the beam particles

\[ (\vec{E}, \vec{B}) \]

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

Interaction with the external environment

Multi-bunch beam

\[ s = vt \]

... or also an instability affecting different bunches independently of each other
The instability loop

Act on the right side of the loop → Minimise the creation of EM fields potentially detrimental to the beam

This will translate eventually into reducing/controlling/optimising the **impedance** of the single components of an accelerator ring
The instability loop

Multi-bunch beam

\[ s = vt \]

Interaction with the external environment

Equations of motion of the beam particles

Act on the left side of the loop

- Introduce stabilising terms in the equations of motion

\[ \left( \vec{E}, \vec{B} \right) \]

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

This will translate eventually adding cooling/damping terms or nonlinear driving terms that can provide stabilisation of the system through Landau damping.
The instability loop

... or we actively create another loop in which we first detect a selected moment of the beam distribution and we then kick the particles accordingly to cancel deviations from the desired steady state

\[ (E, B) \]

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

Equations of motion of the beam particles

Interaction with the external environment

Detect moments

Kick beam particles
Formal description of collective beam motion

• Formally, instead of investigating the full set of equations for a multiparticle system, we typically instead describe the latter by a particle distribution function:

\[ \psi = \psi(x, x', y, y', z, \delta, s) \]

where

\[ dN(s) = \psi(x, x', y, y', z, \delta, s) \, dx \, dx' \, dy \, dy' \, dz \, d\delta \]

• The accelerator environment together with the multiparticle system forms a Hamiltonian system for which the Hamilton equations of motion hold:

\[ \frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \quad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x} \]
Formal description of collective beam motion

We can now derive the **Vlasov equation** which forms the **foundation of the theoretical treatment** of beam dynamics with collective effects:

- Consider an infinitesimal volume element \(d\Omega\) containing a finite number of particles \(dN\) in phase space which evolve in time:
  - \(dN\) is conserved as no particles can enter or leave the area (Picard-Lindelöf)
  - \(d\Omega\) is conserved by means of the Hamilton equations of motion

- It follows that:
  \[
  \frac{d}{ds} \psi = \frac{\partial \psi}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial \psi}{\partial x'} \frac{\partial x'}{\partial s} + \frac{\partial}{\partial s} \psi = \frac{\partial \psi}{\partial \mathbf{q}} \frac{\partial \mathbf{H}}{\partial \mathbf{p}} - \frac{\partial \psi}{\partial \mathbf{p}} \frac{\partial \mathbf{H}}{\partial \mathbf{q}} + \frac{\partial}{\partial s} \psi = 0
  \]

  \([\psi, H]\): Poisson bracket

- The accelerator environment together with the multiparticle system forms a **Hamiltonian system** for which the **Hamilton equations of motion** hold:

\[
\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'} , \quad \frac{\partial x'}{\partial s} = - \frac{\partial H}{\partial x}
\]
The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

\[
\frac{\partial}{\partial s} \psi = [H, \psi]
\]

With the Hamiltonian composed of an **external** and a **collective part**, and the particle distribution function decomposed into an **unperturbed part** and a **small perturbation** one can write

\[
\frac{\partial}{\partial s} \psi = [H_0 + H_1, \psi_0 + \psi_1]
\]

This becomes to **first order**

\[
\frac{\partial}{\partial s} \psi_1 = [H_0, \psi_1] + [H_1(\psi_0 + \psi_1), \psi_0]
\]

**Linearization in \( \psi_1 \):**

\[
\ldots \propto \hat{\Lambda} \psi_1
\]

We are looking for the EV of the evolution \( \rightarrow \) **becomes an EV problem!**

\[
\psi_1 = \sum_k a_k \psi_k \exp\left(\frac{i\Omega_k}{\beta c} s\right)
\]
The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

\[ \frac{\partial}{\partial s} \psi = [H, \psi] \]

With the Hamiltonian composed of an external and a collective part, and the particle distribution function decomposed into an unperturbed part and a small perturbation one can write

\[ \frac{\partial}{\partial s} \psi = [H_0 + H_1, \psi_0 + \psi_1] \]

We call these distinct eigenvalues \( \psi_k \) the **coherent k-mode**.

The mode and thus for example also an instability is fully characterized by a single number:

**the complex tune shift** \( \Omega_k \)

We are looking for the EV of the evolution \( \Rightarrow \text{becomes an EV problem!} \)
Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

\[ \frac{\partial}{\partial s} \psi = [H, \psi] \]

- **Remark:**
  - Hamiltonian composed of an external and a collective part, and the particle distribution function decomposed into an unperturbed part and a small perturbation one can write
  
  - The stationary distribution \( \psi_0 \) is the distribution where
    
    \[ \frac{\partial}{\partial s} \psi_0 = [H_0, \psi_0] = 0 \]

- This becomes to first order

  - In particular, a distribution is always stationary if
    
    \[ \psi_0 = \psi_0(H_0), \text{ as } [H_0, \psi_0(H_0)] = 0 \]

**Solving for or finding the stationary solution for a given \( H_0 \) (which in fact represents the machine 'potential') will be later referred to as **matching**.**
• We have seen the difference between external forces and self-induced forces and how these lead to collective effects.

• We have seen schematically how these collective effects can induce coherent beam instabilities and some knobs to avoid them.

• We have briefly sketched the theoretical framework within which the beam dynamics of collective effects is usually treated – we have encountered the Vlasov equation, bunch / beam eigenmodes and the complex tune shift.

• Part 1: Introduction – dynamics of multiparticle systems

  • Introduction to beam instabilities
  • Instabilities examples
  • Basic concepts
    • Beam matching
    • Multiparticle effects – filamentation and decoherence
Why worry about beam instabilities?

• Why study beam instabilities?
  • The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)
  • Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    • Allows identifying the source and possible measures to mitigate/suppress the effect
    • Allows dimensioning an active feedback system to prevent the instability
The CERN accelerator complex

- ALICE (2008, 27 km)
- LHCb
- ATLAS (1976, 7 km)
- ELENA (2016, 31 m)
- AD (1999, 182 m)
- SPS (1972, 157 m)
- ISOLDE (1989)
- PS (1959, 628 m)
- LEIR (2005, 78 m)
- HiRadMat (2011)
- CERN Accelerator School

Legend:
- p (proton)
- ion
- neutrons
- $\bar{p}$ (antiproton)
- electron
- proton/antiproton conversion
Coupled bunch instability in the SPS

- Injection of 4 batches of 72 bunches trains into the SPS
- Later trains feature strong losses (intensity) and large blow-up (emittance) – this leads to a strong loss of beam brightness
Coupled bunch instability in the SPS

- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits **strong coherent coupled bunch oscillations**

**Bunch # 60**

**72 bunches**

**Vertical bunch pos. [mm]**

**Horizontal bunch pos. [mm]**

**Turn**

**Bunch no.**

**Vertical bunch pos. [mm]**

26/06/2019

Beam Instabilities I - Giovanni Rumolo and Kevin Li - Slangerup
Coupled bunch instability in the SPS

- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits coherent coupled bunch oscillations
• Loss of more than 30% of the bunch intensity due to a slow transverse mode coupling instability (TMCI).
Loss of more than 30% of the bunch intensity due to a slow transverse mode coupling instability (TMCI) can be mitigated by a wideband feedback system.
• We now understand that collective effects can have a **huge detrimental impact** on the machine performance and why, therefore, the study and the understanding of instabilities is important.

• We have encountered some **real world examples** of instabilities observed throughout the CERN accelerator chain.

• Before moving on to a more detail view of collective effects, we will have a quick look at some **distinct characteristics of multi-particle beam dynamics**.

• **Part 1: Introduction** – dynamics of multiparticle systems

  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence
Beam matching

- As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian $H$) the stationary solution is given by:

$$\frac{\partial}{\partial s} \psi = [H, \psi] = 0$$

and can be constructed via matching:

- In real life, an injected beam ought to be matched to the machine for best performance.
- Given a particle distribution function and a machine optics locally described by a Hamiltonian we ensure matching by targeting for:

$$\psi \rightarrow \psi = \psi(H) \rightarrow H$$
Matching examples

We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

- Betatron motion

$$H = \frac{1}{2} x'^2 + \left(\frac{Q_x}{R}\right)^2 x^2$$

$$H_0 = \sigma^2_{x'} = \left(\frac{Q_x}{R}\right)^2 \sigma^2_x \implies \frac{\sigma_x}{\sigma_{x'}} = \frac{R}{Q_x} = \beta_x$$

- Synchrotron motion - linear

$$H(z, \delta) = -\frac{1}{2} \eta \beta c \delta^2 + \frac{eV\hbar}{4\pi R^2 p_0} z^2$$

$$H_0 = \eta \beta c \sigma^2_\delta = \frac{eV\hbar}{2\pi R^2 p_0} \sigma^2_z \implies \frac{\sigma_z}{\sigma_\delta} = R\eta \sqrt{\frac{2\pi \beta^2 E_0}{eV \eta h}} = \frac{R\eta}{Q_s} \sigma_\delta = \beta_z$$
Matching examples

We take the example of Gaussian distribution functions

\[ \psi(H) = \exp\left( \frac{H}{H_0} \right) \]

In reality the synchrotron motion is described by the Hamiltonian:

\[ H(z, \delta) = -\frac{1}{2} \eta \beta c \delta^2 + \frac{eV}{2\pi \hbar p_0} \left( \cos \left( \frac{h\delta}{R} \right) - \cos \left( \frac{h\delta^c}{R} \right) + \frac{\Delta E}{eV} \left( \frac{h\delta}{R} - \frac{h\delta^c}{R} \right) \right) \]

This leads to **nonlinear equations** and the matching procedure becomes more involved.
Matching illustration – matched beams

• Betatron motion – **linear**
  \[ \frac{\sigma_x}{\sigma_{x'}} = \beta_x \]

Matched beams **maintain their beam moments** and their shape in phase space
Matching illustration – mismatched beams

- Betatron motion – linear
  \[ \frac{\sigma_x}{\sigma_x'} = \beta_x \]

Mismatched beams show oscillations in their beam moments and may change their shape due to filamentation.
Matching illustration – linear vs. nonlinear

- Betatron motion – **linear**
  \[ \frac{\sigma_x}{\sigma_{x'}} = \beta_x \]

Nonlinearities lead to **detuning with amplitude**. This is visible as the **characteristic spiraling** of larger amplitude particles.
• We have learned about the **meaning of matching** a beam to the machine optics.

• We have seen how to **formally match a beam** to a given description of a machine.

• We have seen **examples of matched and mismatched beams** and have seen the difference between **linear and non-linear motion**.

• **Part 1: Introduction – dynamics of multiparticle systems**

  • Introduction to beam instabilities
  • Instabilities examples
  • Basic concepts
    • Beam matching
    • Multiparticle effects – filamentation and decoherence
Sources and impact of transverse nonlinearities

• We have learned or we may know from operational experience that there are a set of **crucial machine parameters to influence beam stability** – among them **chromaticity and amplitude detuning**

• Chromaticity
  o Controlled with sextupoles – provides **chromatic shift** of bunch spectrum wrt. impedance
  o Changes interaction of beam with impedance
  o Damping or excitation of **headtail modes**

• Amplitude detuning
  o Controlled with octupoles – provides (incoherent) **tune spread**
  o Leads to absorption of coherent power into the incoherent spectrum → **Landau damping**
Example: filamentation as result of detuning
Example: filamentation as result of detuning

- Taking an **FFT of the centroid motion** (black curve) reveals the tune – interestingly there is a spread
- In the simulation we have access to the trajectory of each individual particle – we can equally perform an **FFT of every particle** and plot the horizontal vs. vertical tune to obtain the **tune footprint**
Example: chromaticity – de- & recoherence

- Chromatic detuning:
  \[ \Delta Q_x = Q'_x \delta \]
  \[ \delta = \delta \sin(\varphi) \]

- Consider a particle in 6d phase space performing both betatron and synchrotron oscillations.

- The accumulated betatron detuning after one half, resp. one full synchrotron period reads:
  \[ \Delta Q_{x, \text{acc}} \bigg|_{T_s/2} = \hat{\delta} \int_0^\pi \sin(\varphi) \, d\varphi = 2\hat{\delta} \]
  \[ \Delta Q_{x, \text{acc}} \bigg|_{T_s} = \hat{\delta} \int_0^{2\pi} \sin(\varphi) \, d\varphi = 0 \]

- After one full synchrotron period all tune shifts have vanished (i.e., also the tune spread has vanished – the beam has recohered).
Example: chromaticity – de- & recoherence

• Chromatic detuning:

spread has vanished – the beam has recohered
• Sources for transverse nonlinearities are, e.g., chromaticity and detuning with amplitude from octupoles.

• Transverse nonlinearities can lead to decoherence and emittance blow-up.

• The effects seen so far are characteristics for multiparticle systems but are not collective effects.

• Part 1: Introduction – dynamics of multiparticle systems
  
  • Introduction to beam instabilities
  • Instabilities examples
  • Basic concepts
    • Beam matching
    • Multiparticle effects – filamentation and decoherence
• We have learned about some of the peculiarities of collective effects. We have also introduced multi-particle systems and have seen how these can be described and treated theoretically.

• We have seen some real-world example of collective effects manifesting themselves as coherent beam instabilities.

• We have looked at some specific features of multi-particle beam dynamics such as matching, decoherence and emittance blow-up due to filamentation. These are not to be confused with collective effects.

• Part 1: Introduction – dynamics of multiparticle systems

  • Introduction to beam instabilities
  • Instabilities examples
  • Basic concepts
    • Beam matching
    • Multiparticle effects – filamentation and decoherence
End part 1
Backup
We have learned about the **particle description** of a beam.

We have seen **macroparticles** and **macroparticle models**.

We have seen how **macroparticle models** are **mapped and represented in a computational environment**.

**Part 1: Introduction** – multiparticle systems, macroparticle models and wake functions

- **Introduction to beam instabilities**
- **Basic concepts**
  - Particles and macroparticles – macroparticle distributions
  - Beam matching
  - Multiparticle effects – filamentation and decoherence
  - Wakefields as sources of collective effects
The particle description

• As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a **particle distribution function**.

• In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.

• A **macroparticle** is a numerical **representation** of a **cluster of neighbouring physical particles**.

• Thus, instead of solving the system for the $N$ ($\sim 10^{11}$) physical particles one can significantly **reduce the number of degrees of freedom** to $N_{MP}$ ($\sim 10^6$). At the same time one must be aware that this **increases of the granularity** of the system which gives rise to numerical noise.
Macroparticle representation of the beam

- Macroparticle models permit a **seamless mapping** of realistic systems into a **computational environment** – they are fairly easy to implement.

Beam:

\[
\begin{pmatrix}
  x_i \\
  x'_i \\
  y_i \\
  y'_i \\
  z_i \\
  \delta_i
\end{pmatrix}, \quad i = 1, \ldots, N
\]

**Macroparticle number**

\[
\begin{pmatrix}
  q_i \\
  m_i
\end{pmatrix}
\]

Canonically conjugate coordinates and momenta

**In [6]:**

\[
df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
\]

**Out[6]:**

<table>
<thead>
<tr>
<th></th>
<th>dp</th>
<th>x</th>
<th>xp</th>
<th>y</th>
<th>yp</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001590</td>
<td>0.000566</td>
<td>-2.285393e-05</td>
<td>-0.001980</td>
<td>4.283152e-06</td>
<td>0.353427</td>
</tr>
<tr>
<td>1</td>
<td>0.001978</td>
<td>0.000370</td>
<td>1.954404e-05</td>
<td>-0.000359</td>
<td>5.543904e-05</td>
<td>0.159670</td>
</tr>
<tr>
<td>2</td>
<td>0.003492</td>
<td>-0.000829</td>
<td>-2.773707e-05</td>
<td>0.000291</td>
<td>6.627340e-05</td>
<td>-0.251489</td>
</tr>
<tr>
<td>3</td>
<td>0.002195</td>
<td>-0.001688</td>
<td>-2.317633e-05</td>
<td>0.001878</td>
<td>-1.870926e-05</td>
<td>-0.038597</td>
</tr>
<tr>
<td>4</td>
<td>0.000572</td>
<td>0.000990</td>
<td>5.493907e-05</td>
<td>0.000152</td>
<td>-1.951051e-05</td>
<td>0.492968</td>
</tr>
<tr>
<td>5</td>
<td>-0.000418</td>
<td>0.001088</td>
<td>4.778027e-05</td>
<td>0.003320</td>
<td>-7.716856e-06</td>
<td>0.415582</td>
</tr>
<tr>
<td>6</td>
<td>-0.000114</td>
<td>-0.000194</td>
<td>1.065400e-05</td>
<td>0.001798</td>
<td>-4.984276e-07</td>
<td>-0.349064</td>
</tr>
<tr>
<td>7</td>
<td>0.001100</td>
<td>-0.001257</td>
<td>6.873217e-05</td>
<td>-0.002374</td>
<td>5.657645e-06</td>
<td>-0.023157</td>
</tr>
<tr>
<td>8</td>
<td>0.002706</td>
<td>0.005351</td>
<td>1.867898e-07</td>
<td>-0.000765</td>
<td>3.012523e-05</td>
<td>-0.291095</td>
</tr>
<tr>
<td>9</td>
<td>0.003508</td>
<td>0.000499</td>
<td>1.865768e-05</td>
<td>-0.001032</td>
<td>5.363820e-05</td>
<td>0.211726</td>
</tr>
<tr>
<td>10</td>
<td>-0.001711</td>
<td>-0.003168</td>
<td>4.372560e-05</td>
<td>-0.001933</td>
<td>-2.151020e-05</td>
<td>-0.145358</td>
</tr>
<tr>
<td>11</td>
<td>-0.002150</td>
<td>-0.000565</td>
<td>1.853825e-05</td>
<td>-0.003959</td>
<td>-6.192450e-06</td>
<td>0.072499</td>
</tr>
<tr>
<td>12</td>
<td>0.002059</td>
<td>0.003453</td>
<td>3.808703e-05</td>
<td>0.000118</td>
<td>3.179588e-05</td>
<td>-0.001816</td>
</tr>
<tr>
<td>13</td>
<td>0.002709</td>
<td>0.000241</td>
<td>3.457535e-05</td>
<td>0.000474</td>
<td>5.057865e-05</td>
<td>-0.005464</td>
</tr>
<tr>
<td>14</td>
<td>-0.001593</td>
<td>0.000711</td>
<td>1.667091e-05</td>
<td>-0.002523</td>
<td>-3.804168e-05</td>
<td>-0.089881</td>
</tr>
<tr>
<td>15</td>
<td>-0.000830</td>
<td>-0.000393</td>
<td>7.473946e-05</td>
<td>0.000839</td>
<td>5.057865e-05</td>
<td>-0.005464</td>
</tr>
<tr>
<td>16</td>
<td>-0.001743</td>
<td>-0.003034</td>
<td>-3.457535e-05</td>
<td>0.000474</td>
<td>5.057865e-05</td>
<td>-0.005464</td>
</tr>
</tbody>
</table>
Macroparticle representation of the beam

- Initial conditions of the beam/particles

<table>
<thead>
<tr>
<th>Profile</th>
<th>Size</th>
<th>Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>Emittance</td>
<td>Optics</td>
</tr>
<tr>
<td>Parabolic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- We use random number generators to obtain random distributions of coordinates and momenta

- Example transverse Gaussian beam in the SPS with normalized emittance of 2 um (0.35 eVs longitudinal)

\[
\varepsilon_\perp = \beta \gamma \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} = \beta \gamma \sigma_x \sigma_{x'}
\]

\[
\varepsilon_\parallel = 4\pi \sigma_z \sigma_\delta \frac{p_0}{e}
\]
Wake fields illustrative examples

- Resonator wake: $f_r = 200$ MHz, $Q = 20$ – Gaussian bunch charge profile
- The plots show how the bunch moments and the wake function **convolve into an integrated deflecting kick** at the different positions along the bunch.
Scrubbing run in 2015 – early stage

- Injection of multiple bunch batches from the SPS into the LHC.
- Violent **instabilities during initial stages of scrubbing** – clear e-cloud signature
- Very hard to control in the beginning – **slow and staged ramp-up of intensity** (24 → 36 → 48 → 60 → 72 → 144 bpi)
- At later stages dumps under control but still **emittance blow-up and serious beam quality degradation**.
- Beam and e-cloud induced **heating of kickers and collimators**.
Headtail instabilities in the LHC

• The **impedance in the LHC** can give rise to coupled and single bunch instabilities which, when left untreated, can lead to **beam degradation and beam loss**.

• As an example, **headtail instabilities** are predicted from **macroparticle simulations** using the LHC impedance model.

• These simulations help to understand and to predict unstable modes which are observed in the real machine.
Headtail instabilities in the LHC

\[ m = 0 \quad \text{and} \quad m = -1 \]

Macroparticle simulations (PyHEADTAIL)

- These simulations help to understand and to **predict instabilities** which are observed in the real machine.
• The **wake function** is the **electromagnetic response** of an object to a charge pulse. It is an intrinsic property of any such object.

• The wake function **couples two charge distributions** as a function of the distance between them.

• The response depends on the boundary conditions and can occur e.g. due to **finite conductivity** (resistive wall) or more or less sudden **changes in the geometry** (e.g. resonator) of a structure.
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

\[
\Delta E_2 = \int F(x_1, x_2, z, s) \, ds = -q_1 q_2 \mathbf{w}(x_1, x_2, z)
\]

\(w\) is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)
Wake potential for a distribution of particles

Definition as the **integrated force** associated to a change in energy:

- For an extended particle distribution this becomes

\[
\Delta E_2(z) \propto \int \lambda_1(x_1, z_1) w(x_1, x_2, z - z_1) \, dx_1 \, dz_1
\]

Forces become dependent on the **particle distribution function**.
Wake fields – impact on the equations of motion

\[ \Delta E_2(z) \propto \int \lambda_1(x_1, z_1) w(x_1, x_2, z - z_1) \, dx_1 \, dz_1 \]

- We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

\[ H = \frac{1}{2} x' + \frac{1}{2} \left( \frac{Q_x}{R} \right)^2 x^2 + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) \, dx_1 \, dz_1 \, dx \]

- The equations of motion become:

\[ x'' + \left( \frac{Q_x}{R} \right)^2 x + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) \, dx_1 \, dz_1 = 0 \]

The presence of wake fields adds an additional excitation which depends on
1. The moments of the beam distribution
2. The shape and the order of the wake function
How are wakes and impedances computed?

• Analytical or semi-analytical approach, when geometry is simple (or simplified)
  • Solve Maxwell’s equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
  • Find closed expressions or execute the last steps numerically to derive wakes and impedances

• Numerical approach
  • Different codes have been developed over the years to solve numerically Maxwell’s equations in arbitrarily complicated structures
  • Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the ICFA mini-Workshop on “Electromagnetic wake fields and impedances in particle accelerators”, Erice, Sicily, 23-28 April, 2014

• Bench measurements based on transmission/reflection measurements with stretched wires
  • Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations