Outline

• Introduction

• Direct RF feedback
  • Globally reduce a cavity impedance

• Long delay feedback
  • Reduce impedance at revolution frequency harmonics

• Global feedbacks
  • Detect and fight the effect of an instability
  • Time and frequency domain

• Summary
Introduction
Why feedback?

- Open loop system subject to
  - Imperfections
  - Perturbations

→ Feed output back to input → correction

→ New system with new dynamics
  - Control parameters of system
  - Make naturally unstable system stable again
Why RF feedback?

- Smaller longitudinal emittance
- Higher beam current
- Better longitudinal quality
- All bunches identical
- More luminosity
- More peak intensity

Image current of beam induces voltage surrounding structure

→ RF cavities particularly affected due to intentionally large impedance

→ Longitudinal instabilities

→ Degradation of longitudinal beam quality

How to improve?

→ RF feedbacks
Tree of RF feedbacks

- Control longitudinal parameters
  - Longitudinally unstable beam
  - Beam induced voltage

RF system identified as source

Feedback close to cavity
- Local, direct feedback

Far away (delay)
- Long delay feedback

Source unknown
- Global feedback
Onion model of RF feedbacks
Direct RF feedback
Objective of local impedance reduction

- Induced voltage in cavity may cause
  1. Dephasing of total cavity voltage
  2. Longitudinal instability

→ Reduce beam induced voltage
→ Reduce cavity impedance experienced by the beam

✓ Beam induced voltage reduced: \( \frac{R}{(R+R_{\text{shunt}})} \)
- Power for given voltage increased: \( \frac{(R+R_{\text{shunt}})}{R} \)
Direct feedback

- Use amplifier to counteract beam induced voltage
  → Decrease only apparent impedance experienced by beam
Direct feedback

- Use amplifier to counteract beam induced voltage
  → Decrease only apparent impedance experienced by beam

- Gap signal, $V$: **Beam and generator contributions**
- Drive signal, $V_{\text{drive}}$: **Pure generator**

→ Compare drive signal (no beam) with gap (beam and generator)
→ Amplify inverted difference
Direct feedback

- Use amplifier to counteract beam induced voltage
  → Decrease only apparent impedance experienced by beam

→ Feedback parameterized by
  - Open loop gain, $G$
  - Total loop delay, $\tau$ → frequency dependent phase shift
Issue with delay

- Dephasing due to physical delay

- Delay is natural enemy of every feedback system
  - Propagation delay in cables and electronics
  - Latency of conversion and signal processing

→ Phase rotation of complex signal: $e^{-i(\omega - \omega_0)\tau} = e^{-i\Delta\omega\tau}$
Direct feedback

• Use amplifier to counteract beam induced voltage
  → Decrease only apparent impedance experienced by beam

→ Total current in cavity ($V_{\text{drive}} = 0$):

\[ I_t(\omega) = I_b(\omega) + I_g(\omega) \]

\[ I_g(\omega) = -V_t(\omega)G e^{-i\Delta \omega \tau} \]
Impedance with direct feedback

- **Total cavity voltage:**
  \[ V_t(\omega) = \frac{I_b(\omega)Z(\omega)}{1 + Z(\omega)Ge^{-i\Delta\omega\tau}} \]

- **Impedance with feedback:**
  \[ Z_{fb}(\omega) = \frac{dV_t(\omega)}{dI_b(\omega)} = \frac{Z(\omega)}{1 + Z(\omega)Ge^{-i\Delta\omega\tau}} \]

\[ Z(\omega) \approx \frac{R}{1 + 2iQ\frac{\Delta\omega}{\omega_0}} \approx \frac{R}{2iQ\frac{\Delta\omega}{\omega_0}} \]
Stability with feedback

• Dephasing due to loop delay at $\Delta \omega_\tau$

$$\Delta \phi_\tau = \Delta \omega_\tau \cdot \tau$$

• Which dephasing results in unity absolute loop gain?

Open loop: $G|Z(\Delta \omega_\tau)| = G \frac{R}{2Q} \frac{\Delta \phi_\tau}{\omega_0 \tau} = 1$

$$\Delta \phi_\tau = G \frac{R \omega_0 \tau}{2Q}$$

• Phase margin defined as

$$\Delta \phi_m = \frac{\pi}{2} - \Delta \phi_\tau = \frac{\pi}{2} - G \frac{R \omega_0 \tau}{2Q}$$
Stability with feedback

→ Phase margin:

\[ \Delta \phi_m = \frac{\pi}{2} - \Delta \phi_T = \frac{\pi}{2} - G \frac{R \omega_0 \tau}{Q} \frac{1}{2} \]

\[ \Delta \phi_{\text{max}} = \frac{\pi}{4} \]

• Conventional stability limit defined for

→ Maximum stable gain:

\[ G_{\text{max}} = \frac{\pi}{2} \frac{1}{R/Q} \frac{1}{\omega_0 \tau} \]
Impedance with feedback

- **Normalized impedance:**

\[
Z_{\text{fb}}(\omega) = \frac{1}{G} \left( \frac{1}{GR} e^{-i\Delta\omega \tau} + \frac{4}{\pi} i \frac{G_{\text{max}}}{G} \Delta\omega \tau \right)
\]

Amplifier \( Z(\omega) \) \[\xrightarrow{\text{Cavity with direct feedback}}\] Beam \( Z_{\text{fb}}(\omega) \)

\[
Z_{\text{fb}}(\omega_0) = \frac{R}{1 + GR}
\]

**Phase margin:** 90.00°
Example: direct feedback lab experiment

- Coaxial cavity, $f_0 \approx 57$ MHz
- ‘Power’ amplifier: $\sim 10 \text{ mW}$

$\Rightarrow$ No risk of damage

$\Rightarrow$ Usually more: Tens to hundreds of kilowatts

[Graphs showing measured transfer function and phase with open loop and with feedback]
Example: 10 MHz RF system in CERN PS

Transfer function with and without feedback

More feedback gain

• Feedback gain of 24 dB
→ Equivalent impedance, \( Z_{fb}(\omega) \) reduced by more than order of magnitude
→ Impedance for amplifier remains unchanged, \( Z(\omega) \)
Example: CERN PS 10 MHz cavity feedback

- 10 + 1 ferrite loaded cavities, tunable from 2.8...10 MHz
- Two amplifiers excited in parallel by one amplifier

→ Realistic amplifier behaviour with higher order modes
Modelling a real cavity – time domain

- Time domain response of cavity and amplifier

Exciting bunch

Cavity response: closed loop

→ Comparing with measured response to beam excitation

→ No instantaneous damping due to inherent delay

→ Filling time significantly reduced with feedback
Limitations of direct feedback

• Contributions to maximum feedback gain

\[ G_{\text{max}} = \frac{\pi}{2} \frac{1}{R/Q} \frac{1}{\omega_0 \tau} \]

using \[ Q = \frac{\omega_0}{\Delta \omega_{-3dB}} \]

\[ = \frac{\pi}{2} \cdot \frac{1}{R} \cdot \frac{1}{\Delta \omega_{-3dB}} \cdot \frac{1}{\tau} \]

1. Increasing shunt impedance **not a good idea**
2. Decreasing delay has physical limits
   → How close can amplifier be to cavity?
   → Minimum delay of feedback chain?
3. Reduce **bandwidth**
   → Reduce bandwidth of feedback chain instead of cavity?
Feedbacks with delay
Why?

→ Loop delay cannot be made short: amplifier not close enough to cavity
→ Need impedance reduction beyond stability limit of direct feedback
→ Cavity to be damped has large bandwidth

How?

→ Cleverly use the properties of the beam spectrum
→ Profit from of slow synchrotron motion
Longitudinal beam spectrum
Longitudinal beam spectrum

- Circular accelerator
  \[ \rightarrow \text{Beam signal periodic with revolution frequency: } \omega_{\text{rev}} \]

\[ \rightarrow \text{Spectral components at: } \omega = n\omega_{\text{rev}} \]

\[ \omega_{\text{RF}} \]

Multi-bunch beam

Spectrum of single bunch
• Longitudinally unstable bunches may perform oscillations
  → Synchrotron frequency is basic periodicity: \( \omega_s \)

  \[ \omega = n \omega_{\text{rev}} \pm m \omega_s \]

  \( \omega_s \ll \omega_{\text{rev}} \)

  → Adds sidebands at \( \omega_{\text{rev}} \) harmonics:
  → Sidebands usually close to \( \omega_{\text{rev}} \) harmonic since
Beam spectrum

→ Beam can only induce voltage at frequencies

\[ \omega = n\omega_{\text{rev}} \pm m\omega_s \]

→ Relevant frequencies from RF point of view

\[ \omega = \omega_{\text{RF}} \pm n\omega_{\text{rev}} \pm m\omega_s \]

→ Feedback only needs to damp these frequency components

→ Can one profit from this property for RF feedbacks beyond conventional stability limit?
Periodic filters
Periodic notch and comb filters

- Transfer function periodic in frequency
  \[ H(\omega) = H(\omega \mod \omega_0) \]

- Niche application in communication technology
  → Who wants to listen to multiple radio stations at the same time?

- Very useful for circular accelerators thanks to properties of beam spectrum

→ How to build such filters?
Periodic notch and comb filters

- Add signal with itself, but delay by a fixed delay, $\tau$

\[
\begin{align*}
V_{\text{in}} & \quad \rightarrow \quad \text{Delay, } \tau \\
\rightarrow \quad + & \quad \rightarrow \quad V_{\text{out}}
\end{align*}
\]

- Addition (maxima) or subtraction (minima)

\[
y(t) = x(t) + x(t - \tau)
\]
Periodic notch and comb filters

• Add signal with itself, but delay by a fixed delay, $\tau$

$$x(t) = e^{i\omega t}$$

$$y(t) = x(t) + x(t - \tau)$$

$$= e^{i\omega t} + e^{i\omega(t-\tau)}$$

$$= e^{i\omega t} \left(1 + e^{-i\omega \tau}\right)$$

• Addition (maxima) or subtraction (minima)

$\rightarrow$ Filter to remove (notch) revolution frequency harmonics
Periodic notch and comb filters

- Delay output signal by $\tau$ and add to input signal

- Addition (maxima) or subtraction (minima)

$$y(t) = x(t) + y(t - \tau)$$
Periodic notch and comb filters

- Delay output signal by $\tau$ and add to input signal

$$y(t) = x(t) + y(t - \tau)$$

- Ansatz:

$$y(t) = ae^{i\omega t}$$

$$y(t) = e^{i\omega t} \cdot \frac{1}{1 - e^{i\omega \tau}}$$

- Addition (maxima) or subtraction (minima)

Amplitude and phase of filter transfer function

$\rightarrow$ Remove everything but revolution frequency harmonics
Digital implementation

- Replace analogue delay line by digital storage

\[ y[n] = x[n] + \alpha y[n - k] \rightarrow (1 + \alpha z^{-k})Y(z) = X(z) \]

\[
H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \alpha z^{-k}} = \frac{z^k}{z^k - \alpha}
\]

\[ z = e^{2\pi i \cdot \omega/\omega_{\text{clk}}} \]

\[ \alpha = 0.00 \]
Feedback with periodic filters
1-tuple delay feedback

1. **Comb filter** to extract revolution frequency harmonics
2. **Delay** to complete physical delay of cables and signal processing to 1 revolution period

![Diagram of 1-tuple delay feedback system]
1-turn delay feedback

1. **Comb filter** to extract revolution frequency harmonics
2. **Delay** to complete physical delay of cables and signal processing to 1 revolution period
1-turn delay feedback

1. **Comb filter** to extract revolution frequency harmonics
2. **Delay** to complete physical delay of cables and signal processing to 1 revolution period

\[ G \rightarrow GH(\omega) \]

\[ Z_{fb}(\omega) = \frac{Z(\omega)}{1 + Z(\omega)Ge^{-i\Delta\omega\tau}} \]

\[ \rightarrow Z_{1tfb}(\omega) = \frac{Z(\omega)GH(\omega)e^{-i\Delta\omega\tau}}{1 + Z(\omega)GH(\omega)e^{-i\Delta\omega\tau}} \]
Cavity transfer function with 1-turn delay FB

→ Transfer function with comb filter

\[ Z_{1\text{tfb}}(\omega) = \frac{Z(\omega)}{1 + Z(\omega)GH(\omega)e^{-i\Delta\omega\tau}} \]

Variation of feedback gain

Variation of feedback delay

→ Impedance between revolution frequency harmonics
→ Not excited by beam, but potential issue for stability
→ Total delay very critical
Example: long delay feedback lab experiment

- 1-turn delay feedback around 57 MHz resonator
  - Analogue comb filter with ~2.5 km optical fiber delay
  - Accelerator with $f_{rev} \approx 76$ kHz ($2\pi R \approx 4$ km circumference)

![Graph showing open/closed loop transfer function with 2/+2 ns delay error (+/-1.4 \cdot 10^{-4})]
Example: 1-turn delay in CERN PS

- Combination of direct and 1-turn delay feedbacks

- Fast wide-band feedback around amplifier (internal) → Gain limited by delay

- 1-turn delay feedback → High gain at $n \times f_{rev}$
Example: 1-turn delay in CERN PS

→ Reduce cavity impedance beyond stability limit of wide-band FB

Open/closed loop transfer functions

- Calculated
- Measured

Spectrum at cavity gap return

Feedback off

Feedback on

→ Important additional impedance reduction

→ Clever usage of beam periodicity in circular accelerator
Multi-harmonic feedback
Treat each harmonic independently

- Separate feedback loop by harmonic
  - Full flexibility of individual loop parameters
  - Empowered by processing power of modern digital hardware

![Diagram showing single harmonic processing: band-pass and dephasing]

Open loop transfer function for 4 adjacent harmonics
Example: Damping of wide-band cavity

- Multi-harmonic feedback reduces beam induced voltage
- First 12 revolution frequency harmonics damped

Spectrum of beam induced voltage

1 bunch

C1510.

6 bunches

Feedback off
Feedback on

→ Damping beyond stability limit of direct feedback
Global feedbacks

Global feedback systems: longitudinal bunch-by-bunch, mode-by-mode feedbacks
Global RF feedback

1. Detect derivation of beam
   → Transverse: position offset
   → Longitudinal: phase offset

2. Signal processing to filter relevant information

3. Amplify and apply correction
   → Drive dedicated kicker
   → Drive accelerating cavities as longitudinal kickers
Longitudinal oscillation of bunches

- Longitudinally unstable beam, but **driving source unknown**
- Each bunch oscillation, but not with the same phase

**Bunches oscillating (dipole, $2\pi \cdot 10/21$ phase advance)**

<table>
<thead>
<tr>
<th>Time domain</th>
<th>Frequency domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Measure phase of each bunch</td>
<td>• Measure spectral component corresponding to mode</td>
</tr>
<tr>
<td>→ <strong>Apply kick to bring phase</strong></td>
<td>→ <strong>Apply kick to remove that spectral component</strong></td>
</tr>
<tr>
<td>back to reference position</td>
<td></td>
</tr>
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<td>→ “Bunch-by-bunch”</td>
<td>→ “Mode-by-mode”</td>
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</table>
Time domain: Bunch-by-bunch feedback

- Intuitive: **Measure oscillation of each bunch and correct**

→ Multiple feedbacks in parallel
→ Flexible control (gain/phase) for each bunch
Example: Bunch-by-bunch RF feedback

- Multi-bunch feedback developed for electron storage rings:
  - Used at Advanced Light Source (ALS) at LBNL (ALS), PEP at SLAC, DAφNE at INFN-LNF, etc.

- Bunch phase detection
  - De-multiplexer
  - Signal processor
  - Multiplexer

- Longitudinal kicker
  - Power amp
  - Modes in frequency domain
  - FFT

- Bunches in time domain

- Master Oscillator
  - Phase-locked at 6° RF of Cavity

- Timing & Control
  - A/D Down-sampler
  - DSP
  - Farm of Digital Signal Processors

- Hold Buffer
  - D/A
  - QPSK Modulator
  - Kicker Oscillator 1.125 GHz
  - Phase-locked to Ring
Frequency domain: Mode-by-Mode

- Less intuitive: Suppress components in beam spectrum
- Fixed phase advance from bunch-to-bunch creates sideband at $n\omega_{\text{rev}}$

$$\omega = n\omega_{\text{rev}} \pm m\omega_S$$

$2\pi \cdot 10/21$ phase advance: $n = 10, m = 1$
Frequency domain: Mode-by-Mode

- Less intuitive: Suppress components in beam spectrum
- Fixed phase advance from bunch-to-bunch creates sideband at $n\omega_{\text{rev}}$
  \[ \omega = n\omega_{\text{rev}} \pm m\omega_S \]
- $2\pi\cdot10/21$ phase advance: $n = 10$, $m = 1$
- No sidebands at $\pm\omega_S$ → Dipole oscillations removed
- No sidebands at $\pm2\omega_S$ → Quadrupole oscillations removed
1. Filter synchrotron frequency side-bands
2. Inject correction to remove them

→ Stable beam

→ Multiple feedbacks in parallel
→ Optimum parameters (phase, gain) for each harmonic of $\omega_{\text{rev}}$
Example: CERN PS coupled-bunch feedback

- Mode-by-mode dipole feedback
- 10 parallel processing chains  → stabilize beam for LHC
Summary

1. Direct RF feedback
   → Globally reduce cavity impedance

2. Long delay feedback
   → Reduce impedance at revolution frequency harmonics

3. Global feedback
   → Just fix problems of (sometimes) not understood origin

• Chose feedback most appropriate to your problem
  → Prefer inner layers of feedback onion
  → Combination of different RF feedbacks

• Delay is principal enemy of almost every RF feedback
  → Keep it short, you cannot beat causality!
A big Thank You

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Thank you very much for your attention!
References

## Direct RF feedback on cavity

- You **know** the driving impedance → RF cavity
- You can be **close** to the cavity

<table>
<thead>
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<th>Advantages</th>
<th>Disadvantages</th>
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<tr>
<td>• Shunt impedance reduction of cavity resonance</td>
<td>• Local feedback</td>
</tr>
<tr>
<td>• Robust, performance does not depend on beam parameters</td>
<td>• Amplifier must be close to cavity</td>
</tr>
<tr>
<td>• Excellent transient response</td>
<td>• Feedback system per cavity</td>
</tr>
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# 1-turn delay/multi-harmonic feedback

- **You** *know* the driving impedance $\rightarrow$ RF cavity
- **You cannot** be *close* to the cavity

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You know the driving impedance to RF cavity. You cannot be close to the cavity.

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# Global feedback

- **You do not know** the source of the problem
- **You observe and analyse** the effect of an instability

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<td>• Globally reduced consequence of instability</td>
<td>• Treats consequence, not cause of a problem</td>
</tr>
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<td>• One feedback sufficient to control instability</td>
<td>• Narrow range of application</td>
</tr>
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<td></td>
<td>• Dedicated longitudinal kicker</td>
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</table>
RF system overview

Beam

Cavity

Power amplifier

Low-level RF system

Beam
Application of global corrections

- **Local feedbacks** → Act on individual RF stations
- **Global feedbacks** → Act on all RF stations simultaneously

→ RF distribution to compensate time of flight between stations
→ All RF stations applying *correction in unison*
Frequency and wavelength ranges

- **SPS 200 MHz**
  - CLIC 12 GHz
  - PS longitudinal damper
  - PS main RF system

- **100 kHz**
  - 3 km
- **1 MHz**
  - 300 m
- **10 MHz**
  - 30 m
- **100 MHz**
  - 3 m
- **1 GHz**
  - 30 cm
- **10 GHz**
  - 3 cm
- **100 GHz**
  - 3 mm

- **Long wave**
  - SPS main RF system

- **Medium/short wave**
  - PS longitudinal damper

- **VHF**
  - CLIC 12 GHz

- **Microwave links**
  - SPS 200 MHz