Beam Beam Effects

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Motivation of Colliders

• The advantage of a collider is to reach higher energy. The center of mass energy of a two head-on collision particles is

\[ E_{cm} = \sqrt{m_1^2 + m_2^2 + 2E_1E_2(1 + \beta_1\beta_2)} \]

• where E1, E2 are the energy of the two colliding beams, respectively. And, β1 and β2 are the Lorentz β of each beam. For two relativistic beam of the same particle with the same energy of E, the effective energy is simply \( \sqrt{s} = 2E \)

• Typically, collider is used to
  • To discover new particles: Higgs@LHC, Top quark@Tevatron
  • To explore the inner structure of matter quark-gluon plasma@RHIC, proton spin structure@RHIC and HERA
Figure of merit of a typical collider

- Peak luminosity: # of collisions per unit area and per unit time
- For the case of head-on collisions

\[ L \sim f \frac{N_1 N_2}{A} \]

- Ways to increase the peak luminosity
  - Increase # of particles in each beam, i.e. bunch intensity
  - Increase # of bunches
  - Make each bunch more bright, i.e. shrink the size of the bunch at the collision point
Figure of merit of a typical collider

Integrated luminosity is defined as total number of collision events within the duration of a store

\[ L_{int} = \int_{0}^{T_{store}} L(t) dt \]

The unit of integrated luminosity is the inverse of cross-section unit, and typically expressed in inverse barn (\(10^{-24} cm^{-2}\)). For instance, RHIC delivered about 540 pb\(^{-1}\) of about 4 month polarized proton operation in 2013.

In additional to the direct burn-off rate of collisions, the integrated luminosity is directly affected by

- how effective is the detector: vertex distribution, detector ramp-up time, etc
- beam emittance growth during store due to various diffusion mechanisms such as intra-beam scattering, beam-beam effect, orbital resonance, etc.
- overall percentage of time-in-store
Effect of Collisions on Beam Dynamics

• Each beam consists of many than millions of charged particles
  • Space charge: effect on particle’s motion by its electromagnetic field
  • Always defocusing!
• Beam-beam: impact on particle’s motion by the electromagnetic field from the opposite beam
Beam-beam interaction

• For a Gaussian beam with charge distribution

\[ \rho(x, y, z) = \frac{Nq}{4\pi\varepsilon_0} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} + \frac{z^2}{2\sigma_z^2}\right)} \]

• The potential of its electric field in the beam’s rest frame

\[ \phi(x, y, z) = \frac{Nq}{\sqrt{\pi}} \int_0^\infty e^{-\left(\frac{x^2}{2\sigma_x^2+t} + \frac{y^2}{2\sigma_y^2+t} + \frac{z^2}{2\sigma_z^2+t}\right)} \frac{dt}{\sqrt{(2\sigma_x^2 + t)(2\sigma_y^2 + t)(2\sigma_z^2 + t)}} \]
Beam-beam interaction

- Follow the classical way, one can then get the electrostatic field in the particle’s rest frame

\[ \vec{E}(x, y, z) = -\nabla \phi(x, y, z) \]

- For the 2-D Gaussian beam, one can then

\[
E_x - iE_y = -\frac{inq}{2\varepsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \left[ \text{erf} \left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}} \text{erf} \left( \frac{x\sigma_y - iy\sigma_x}{\sigma_x \sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]
\]

where \( n \) is the line density
Beam-beam for round beams

• For infinitely long round beam, the field is given by

\[ 2\pi r E_r = \frac{1}{\varepsilon_0} \int_0^r 2\pi r' \rho(r') dr', \text{ and } 2\pi B_\phi = \mu_0 \int_0^r 2\pi r' \beta c \rho(r') dr' \]

• For a round beam with 2-D Gaussian distribution, i.e.

\[ \rho(x, y) = \frac{nq}{2\pi \sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}}, \] where \( n \), is the line density in the lab frame, the field then becomes

\[ 2\pi r E_r = \frac{1}{\varepsilon_0} \int_0^r \frac{2\pi n q r'}{2\pi \sigma^2} e^{-r'^2/2\sigma^2} dr' = \frac{nq}{\varepsilon_0} \left[ 1 - e^{-r^2/2\sigma^2} \right] \]
Beam-beam for round beams

• And, the beam-beam force is

\[ \vec{F} = q(\vec{E} + \vec{V} \times \vec{B}) = q(E_r + \beta c B_\phi) \times \hat{r} \]

• Plug in the electromagnetic field, we then get the beam-beam force

\[ F_r = \frac{nq^2(1+\beta^2)}{4\pi \epsilon_0 \sigma^2 r} \left[ 1 - e^{-r^2/2\sigma^2} \right] \]
Beam-beam for round beams

- For particles nearby the beam center, i.e. $r \ll \sigma$

\[ F_r = \frac{nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} r \]

- a linear focusing/defocusing effect with a gradient of

\[ k_x = \frac{nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} \text{ and } k_y = \frac{nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} \]

- a quadruple-like kick! $\Rightarrow$ linear tune shift!
- same sign in both planes!!!
Beam-beam for round beams

- For particles far from the center, i.e. $r \gg \sigma$

$$F_r = \frac{nq^2(1+\beta^2)}{2\pi\varepsilon_0 r}$$

$$\frac{dF_r}{dr} = -\frac{nq^2(1+\beta^2)}{2\pi\varepsilon_0 r^2}$$

⇒ For a particle far from the center, the induced tune spread

- amplitude dependent
- opposite sign w.r.t. to the particle near the center
Beam-beam kick

- In general, \( F_r = \frac{nq^2(1+\beta^2)}{2\pi\epsilon_0 r} \left[ 1 - e^{-r^2/2\sigma^2} \right] \)
- Highly non-linear!
- Beam-beam kick for a test particle traverses through a longitudinal gaussian

\[
\gamma m\beta c \Delta r' = \int_{-\infty}^{\infty} F_r(r, s, t) dt
\]

\[
F_r(r, s, t) = \frac{Nq^2(1 + \beta^2)}{2\pi\epsilon_0 r\sigma_s} \left[ 1 - e^{-r^2/2\sigma^2} \right] \left[ e^{-\left(s + vt\right)^2/2\sigma^2_s} \right]
\]

- for relativistic case, the beam-beam kick

\[
\Delta r' = \frac{2Nr_0}{\gamma r} \left[ 1 - e^{-r^2/2\sigma^2} \right]
\]
Linear Beam-Beam Tune Shift

- Linear beam-beam kick: $\Delta r' \approx \frac{2Nr_0}{\gamma r} \frac{r}{2\sigma^2} = \frac{Nr_0}{\gamma \sigma^2} r = kr$

- In the presence of one collision point, i.e. one beam-beam lens, the one turn transfer matrix

$$OTM = \begin{pmatrix} \cos 2\pi Q & \beta_0^* \sin 2\pi Q \\ -\frac{1}{\beta_0^*} \sin 2\pi Q & \cos 2\pi Q \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{Nr_0}{\gamma \sigma^2} & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos 2\pi (Q + \Delta Q) & \beta_0^* \sin 2\pi (Q + \Delta Q) \\ -\frac{1}{\beta_0^*} \sin 2\pi (Q + \Delta Q) & \cos 2\pi (Q + \Delta Q) \end{pmatrix}$$

This then yields

$$\cos 2\pi (Q + \Delta Q) = \cos 2\pi Q - \beta_0^* \frac{Nr_0}{2\gamma \sigma^2} \sin 2\pi Q$$
Linear Beam-Beam Tune Shift

- For $Q \neq m$ or $\frac{m}{2}$ and $\Delta Q \ll 1$, beam-beam tune shift

$$\Delta Q = \frac{1}{4\pi} \frac{\beta^* N r_0}{\gamma \sigma^2} = \xi = \frac{1}{4\pi} \frac{N r_0}{\gamma \epsilon}$$

Here, $m$ is integer

Beam-beam parameter: only function of intensity and normalized beam emittance!
**Effects from beam-beam**

- Linear tune shift for a beam passing through the opposite beam
  \[ \xi = \Delta Q = \frac{1}{4\pi} \frac{\beta^* N r_0}{\gamma \sigma^2}, \text{a.k.a beam-beam parameter} \]

- Luminosity
  \[ L = k f_{rev} \frac{N^2}{4\pi \sigma^2} = k f_{rev} \frac{N}{r_0 \beta^*} \xi \gamma, \text{and } k \text{ is number of bunches in collision.} \]

- For beam stability, \( \xi < 0.5 \),
  \[ L = k f_{rev} \frac{N}{r_0 \beta^*} \xi \gamma < k f_{rev} \frac{N}{2r_0 \beta^*} \gamma \]
  - For RHIC pp@250GeV, \( \beta^* = 1 \), 112 bunches, \( 2 \times 10^{11} \) protons per bunch, IP=1, \( L < 1.5 \times 10^{34} cm^{-2}s^{-1} \)
  - For IP>1, \( \xi \Rightarrow \xi/\text{number of IPs.} \)

- In reality, \( \xi \) is limited also by the resonance mechanisms in the lattice as well as other contributions
  - RHIC-pp@255GeV: \( \sim 0.007 \) per IP, LEP@100GeV: 0.05-0.08 per IP
Effects from beam-beam

- Non-linear beam-beam kick

\[ \Delta r' = -\frac{2N r_0}{\gamma r} \left[ 1 - e^{-r^2/2\sigma^2} \right] \]

- Large spread of tune spread

\[ \Delta Q(a) = \xi \frac{2}{\alpha} \left[ 1 - e^{-\alpha/2} I_0(\alpha/2) \right], \]

where \( a \) is the betatron amplitude, and

\[ \alpha = \frac{\varepsilon \beta^*}{2\sigma^2}, \]

and \( \varepsilon \) is the particle action

- Non-linear resonance

⇒ **Beam instability**
  - Poor beam lifetime

⇒ **Emittance growth**
  - Poor luminosity lifetime
Beam Transfer Function with Beam-Beam
Beam Transfer Function of a typical RHIC store
A Typical BTF of RHIC Beam in Collision

Snake resonance at $5Q_y = 0.48 + 3$

Snake resonance at $5Q_y = 0.5 + 3$
Average Store Polarization vs. vertical tune

- The closer the vertical tune towards 0.7, the lower the beam polarization.
- The data also shows that the direct beam-beam contribution to polarization loss during store is weak.

![Graph showing average store polarization vs. vertical tune]
Beam beam crossing scheme

- When two beams crossing at RHIC
Measure closed orbit

- Distribute beam position monitors around ring.
Coherent Beam-beam dipole Oscillation

• Before collision, the two beams pass through interaction region with a separation

• For the case of vertical separation, the center of each beam experiences a dipole kick

\[ \Delta y' = \frac{N r_0 y + d}{\gamma \sigma^2 r^2} \left[ 1 - e^{-r^2/2\sigma^2} \right] \]

and

\[ \Delta x' = \frac{N r_0 x}{\gamma \sigma^2 r^2} \left[ 1 - e^{-r^2/2\sigma^2} \right] \]

• This in turn induces additional dipole oscillation of the bunch
Coherent Beam-beam dipole Oscillation

• This in turn induces additional dipole oscillation of the bunch
• This also means the orbital motion of the two colliding bunches can influence each other, which in turn can lead to coherent beam-beam oscillations

\( \sigma \) mode

\( \pi \) mode

Coherent Beam-beam dipole Oscillation

- Large crossing angle
  - Needed to have beams well separated in situations, such as LHC
  - Typically, $6 \sim 12 \sigma$
  - In this situation, the bunches of counter-rotating beam experience a long-range beam-beam kick

![Diagram showing beam-beam interaction](image)

Courtesy of W. Herr
Long Range Beam-Beam

• What’s the big deal?
  • Different from head-on beam-beam effect! The tune shift under this circumstance bears the opposite sign from the tune shift from the head-on collision.

![Graph of r/σ vs. F]

Courtesy of W. Herr

Tune footprint, head-on and long range

Vertical separation

2 head-on

Horizontal separation

Qx

Qy

0.272 0.274 0.276 0.278 0.28 0.282 0.284 0.286
Coherent Beam-beam modes observed at RHIC

W. Fischer et al, Observation of Coherent Beam-Beam Modes in RHIC, C-A/AP/#75 June 2002
Mitigations on various beam-beam effect

• Carefully choose the working point, i.e. betatron tunes for collision
  – In an area which offers the maximum space between harmful resonances

• Correct the harmful resonances
  – High order resonance correction is quite difficult and often not perfect

• Compensate the beam-beam effect
  – Electron lens for proton-proton collider
Luminosity Improvement: ELens

- Electron lens: W. Fischer, Y. Luo and et al
  - Low energy electron beam to provide a focusing lens to compensate the beam-beam induced tune spread
  - Allows higher bunch intensity

Simulation by Y. Luo and et al
Luminosity Improvement: ELens

- RHIC Electron lens: W. Fischer, Y. Luo and et al
  - One 2.5m long 6 T super-conducting solenoid with field lines straight within ~ 0.2σ
  - Electron gun of current intensity 14A/cm^2
  - Received $4 million ARRA fund
  - Expect a factor of 2 gain in luminosity
  - Expect to commission in RUN 2012
Beam-beam compensation using ELens

Successfully demonstrated at RHIC!

Electron lenses for head-on beam-beam compensation in RHIC

X. Gu, W. Fischer, Z. Altinbas, M. Anerella, E. Bajon, M. Bannon, D. Bruno, M. Costanzo,
Crab Crossing

- large crossing angle can reduce beam-beam effect. However, it also reduces luminosity
- use RF cavity on either side of the collision point to align the bunch shape of the two beams to recover luminosity reduction due to geometric factor. Such a cavity is called crab cavity
Crab cavity

- First introduced by Dr. R. Palmer (BNL) in 1988 and first demonstrated at KEK B-factory in 2007

- An RF device operates at TM110 mode that provides phase dependent transverse kicks to tilt the bunch. The size of the tilt is proportionally to the strength of the maximum field of the cavity and distance between cavity to IP
KEKB crab cavity

SRF “Squashed cell cavity” at 2.8K with crabbing mode at 500 MHz (2.8 MV defl voltage)

- ~ 3 years operation under high current
- $l_{\text{peak}} = 21.1 \times 10^{33} \text{ cm}^2/\text{s}$ (with crabs)

KEKB operation terminated in June 2010 for the upgrade towards SuperKEKB

Oide, ICFA08
Crab waist

- for a collider with two flat beams in collision, one can use large horizontal crossing angle to reduce parasitic collision as well as beam-beam tune shift

- one can use a sextupole on either side of the collision point to re-distribute the beta squeeze waist in the overlap area of the two beams. The beta function at the waist then becomes

\[
\beta(s) = \beta^* + \frac{(s - x/\theta)^2}{\beta^*}
\]
Summary

- The beam-beam interaction is one of the most dominant effect limiting the performance of collider
  - The higher the luminosity, the larger beam-beam parameter
  - The beam-beam parameter only depends on the (normalized) brightness of the beam! It is independent of beta*, while luminosity is directly proportional to the beta*!

- Only the full understanding of its rich beam dynamics led to the tremendous progress in continuous pushing towards higher and higher luminosity
Reference


