Multi-bunch Feedback Systems

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Outline

- Coupled-bunch instabilities
- Basics of feedback systems
- Feedback system components
- Theory of feedback control
- Digital signal processing
- Diagnostics features
- Feedback system setup and optimization
- Effects of multi-bunch feedbacks
Coupled-bunch instabilities at a glance

- Bunched beam in a storage ring
- Transverse (betatron) and longitudinal (synchrotron) oscillations normally damped by natural damping
- Interaction of the electromagnetic field with metallic surroundings (wake fields)
- Wake fields act back on the beam and produces growth of oscillations
- If the growth rate is stronger than the natural damping the oscillation gets unstable

Example: interaction with an RF cavity can excite its Higher Order Modes (HOM)
Objective of storage ring based particle accelerators

- High brightness in synchrotron light sources
- High luminosity in high energy physics experiments

- High currents
  - Many bunches

The interaction of these beams with the surrounding metallic structures gives rise to collective effects called “coupled-bunch instabilities”

- Large amplitude instabilities can cause beam loss
  - Limitation of the stored current to low values
  - If the growth of instability saturates, the beam may stay in the ring
  - Large instabilities degrade the beam quality: brightness or luminosity
Sources of instabilities

**Cavity High Order Modes (HOM)**
High Q spurious resonances excited by the beam act back on the beam itself
Each bunch affects the following bunches through the wake fields excited in the cavity
The cavity HOM can couple with a beam oscillation mode at the same frequency and give rise to an instability

**Resistive wall impedance**
Interaction of the beam with the vacuum chamber (skin effect)
Particularly strong in low-gap chambers and in-vacuum insertion devices (undulators and wigglers)

**Interaction of the beam with other objects**
Discontinuities in the vacuum chamber, small cavity-like structures, ...
Ex. BPMs, vacuum pumps, bellows, ...

**Ion instabilities**
Gas molecules ionized by electron beam
Positive ions remain trapped in the negative electric potential
Produce electron-ion coherent oscillations
Passive cures

Cavity High Order Modes (HOM)
- Thorough design of the RF cavity
- Mode dampers with antennas and resistive loads
- Tuning of HOMs frequencies through plungers or changing the cavity temperature

Resistive wall impedance
- Usage of low resistivity materials for the vacuum pipe
- Optimization of vacuum chamber geometry

Interaction of the beam with other objects
- Proper design of the vacuum chamber and of the various installed objects

Ion instabilities
- Ion cleaning with a gap in the bunch train

Landau damping by increasing the tune spread
- Higher harmonic RF cavity (bunch lengthening)
- Modulation of the RF
- Octupole magnets (transverse)

Active Feedbacks
Equation of motion of one particle: harmonic oscillator analogy

“x” is the oscillation coordinate (longitudinal or transverse displacement)

\[
\dot{x}(t) + 2D \ddot{x}(t) + \omega^2 x(t) = 0
\]

If \( \omega \gg D \), an approximated solution of the differential equation is a damped sinusoidal oscillation:

\[
x(t) = e^{-\frac{t}{\tau_D}} \sin(\omega t + \varphi)
\]

where \( \tau_D = 1/D \) is the “damping time constant” (D is called “damping rate”)

Externally excited uncorrelated oscillations (ex. quantum excitation) are damped by natural damping (synchrotron radiation damping or Landau damping). The oscillation of the individual particles is uncorrelated and shows up as an emittance growth.
Coupling with other bunches through the interaction with surrounding metallic structures add a “driving force” term $F(t)$. The oscillation of individual particles becomes correlated and the centroid of the bunch oscillates giving rise to coherent bunch oscillations (coupled bunch).

Equation of motion of one bunch: 

$$\ddot{x}(t) + 2D\dot{x}(t) + \omega^2 x(t) = F(t)$$

If the driving force is sinusoidal with frequency $\omega$ and the amplitude is proportional to the bunch oscillation amplitude, the equation becomes:

$$\ddot{x}(t) + 2(D-G)\dot{x}(t) + \omega^2 x(t) = 0$$

where $\tau_G = 1/G$ is the “growth time constant” ($G$ is called “growth rate”).

If $D > G$ the oscillation amplitude decays exponentially

If $D < G$ the oscillation amplitude grows exponentially

as:

$$x(t) = e^{-\frac{t}{\tau}} \sin(\omega t + \phi)$$

where

$$\frac{1}{\tau} = \frac{1}{\tau_D} - \frac{1}{\tau_G}$$

Since $G$ is proportional to the beam current $I_b$, if $I_b$ is lower than a given threshold $I_{th}$ the coupled bunch oscillation does not show up, if higher a coupled bunch oscillation arises.
Feedback systems act on the beam by adding a damping term $D_{fb}$ to the equation of motion:

$$\ddot{x}(t)+2(D-G+D_{fb}) \dot{x}(t)+\omega^2 x(t)=0$$

Such that $D-G+D_{fb} > 0$

The feedback is made of a sensor called detector and an actuator called kicker.

In order to add damping, the feedback must provide a kick proportional to the derivative of the bunch oscillation.

Since the oscillation is sinusoidal, the kick signal for each bunch is generated by shifting by $\pi/2$ the oscillation signal of the same bunch when it passes through the kicker.
Multi-bunch modes

Typically, transverse oscillation frequencies (horizontal and vertical) are higher than the revolution frequency, while the longitudinal frequency is lower than the revolution frequency.

Although each bunch oscillates at the tune frequency, there can be different modes of oscillation, called multi-bunch modes depending on how each bunch oscillates with respect to the others.
Multi-bunch modes

$M$ bunches equally spaced around the ring

Each multi-bunch mode is characterized by a bunch-to-bunch phase difference of:

$$\Delta \Phi = m \frac{2\pi}{M}$$

$m$ = multi-bunch mode number (0, 1, .., $M-1$)

Each multi-bunch mode is associated to a characteristic set of frequencies:

$$\omega = p M \omega_0 \pm (m+\nu) \omega_0$$

Where:

$p$ is an integer number $-\infty < p < \infty$

$\omega_0$ is the revolution frequency: $M\omega_0$ is the RF frequency (bunch repetition freq.)

$\nu$ is the tune

Two sidebands at $\pm (m+\nu) \omega_0$ for each multiple of the RF frequency
Multi-bunch modes: example 1

Vertical plane. One single stable bunch

Every time the bunch passes through a pickup (▼) placed at coordinate 0 a pulse is generated. If we think it as a Dirac impulse, the spectrum is a repetition of frequency lines at multiple of the revolution frequency: \( p\omega_0 \) for \(-\infty < p < \infty\)
Multi-bunch modes: example 2

One single bunch oscillating at the tune frequency $\nu \omega_0$: for simplicity we consider a vertical tune $\nu < 1$, ex. $\nu = 0.25$. $M = 1 \rightarrow$ only mode $\# 0$ exists

The Dirac impulse is modulated in amplitude with frequency $\nu \omega_0$
Two sidebands at $\pm \nu \omega_0$ appear at each of the spectrum frequency lines
Multi-bunch modes: example 3

Ten identical equally-spaced stable bunches filling all the ring buckets

The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency: \[ \omega_{rf} = 10 \omega_0 \quad \text{(RF frequency)} \]

Multi-bunch modes: example 4

10 identical equally-spaced bunches oscillating at the tune frequency $\omega_N$ ($N = 0.25$)

$M = 10 \rightarrow$ there are 10 possible modes of oscillation

Ex.: mode #0 ($m = 0$) $\Delta \Phi = 0$ i.e. all bunches oscillate with the same phase
Multi-bunch modes: example 4

The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency, modulated in amplitude with frequency $\nu \omega_0$. Sidebands at $\pm \nu \omega_0$ appear at each of the spectrum frequency lines:

$$\omega = p \omega_{rf} \pm \nu \omega_0$$

$-\infty < p < \infty$ ($\nu = 0.25$)

Since the spectrum is periodic and each mode appears twice (upper and lower side band) in a $M \omega_0$ frequency span, we can limit the spectrum analysis to a $M \omega_0/2$ frequency range.
Multi-bunch modes: example 5

Ex.: mode #1 \( (m = 1) \) \( \Delta \Phi = 2\pi/10 \) \( (\nu = 0.25) \)

\[ \omega = p\omega_{rf} \pm (\nu+1)\omega_0 \quad -\infty < p < \infty \]
Multi-bunch modes: example 6

Ex.: mode #2 \((m = 2)\) \(\Delta \Phi = 4\pi/10\) \((\nu = 0.25)\)

\[
\omega = p\omega_{rf} \pm (\nu+2)\omega_0 \quad -\infty < p < \infty
\]

Multi-bunch modes: example 7

Ex.: mode #3 (m = 3) \( \Delta \Phi = 6\pi/10 \) (\( \nu = 0.25 \))

\[
\omega = p\omega_{rf} \pm (\nu+3)\omega_0, \quad -\infty < p < \infty
\]
Multi-bunch modes: example 8

Ex.: mode #4 (m = 4) \( \Delta \Phi = \frac{8\pi}{10} \) (\( \nu = 0.25 \))

\[
\omega = p\omega_{rf} \pm (\nu+4)\omega_0 \quad -\infty < p < \infty
\]

Multi-bunch modes: example 9

Ex.: mode #5 (m = 5) \( \Delta \Phi = \pi \) \( (\nu = 0.25) \)

\[ \omega = p\omega_{rf} \pm (\nu+5)\omega_0 \quad -\infty < p < \infty \]

Marco Lenza, “Multi-bunch Feedback Systems”  
Multi-bunch modes: example 10

Ex.: mode #6 \((m = 6)\) \(\Delta \Phi = 12\pi / 10\) \((v = 0.25)\)

\[
\omega = p\omega_{rf} \pm (v+6)\omega_0 \quad -\infty < p < \infty
\]
Multi-bunch modes: example11

Ex.: mode #7 (m = 7)  \( \Delta \Phi = 14\pi/10 \) (\( \nu = 0.25 \))

\[ \omega = p\omega_{rf} \pm (\nu+7)\omega_0 \quad -\infty < p < \infty \]
Multi-bunch modes: example 12

Ex.: mode #8 \((m = 8)\) \(\Delta \Phi = 16\pi/10\) \((\nu = 0.25)\)

\[\omega = p\omega_{rf} \pm (\nu + 8)\omega_0\]
\(-\infty < p < \infty\)
Multi-bunch modes: example 13

Ex.: mode #9 ($m = 9$)  $\Delta \Phi = 18\pi/10$  ($\nu = 0.25$)

$$\omega = p\omega_{rf} \pm (\nu+9)\omega_0$$  $-\infty < p < \infty$
Multi-bunch modes: uneven filling and longitudinal modes

Any $\omega_{\text{rf}}/2$ portion of the beam spectrum contains the information of all potential modes and can be used to detect the presence of an instability and measure its amplitude.

If the bunches have not the same charge, i.e. the buckets are not equally filled (uneven filling), the spectrum also has frequency components at the revolution harmonics (multiples of $\omega_0$). The amplitude of each revolution harmonic depends on the filling pattern of one machine turn.

In case of longitudinal modes, we have a phase modulation of the stable beam spectrum. Components at $\pm n\omega_0$, $\pm 2n\omega_0$, $\pm 3n\omega_0$, ... can appear aside the revolution harmonics. Their amplitude depends on the depth of the phase modulation (Bessel series).
Multi-bunch modes: coupled-bunch instability

The resonance of a metallic structure surrounding the beam (ex. RF cavity) can couple with a multi-bunch mode and give rise to a coupled-bunch instability that increases the amplitude of the spectral line corresponding to that mode.

Effects of coupled-bunch modes:
- Increase of the beam dimensions
- Increase of the effective emittance
- Risk of beam loss
- Increase of lifetime due to decreased Touschek scattering (more diluted bunches)
Real example: Elettra synchrotron light source

\[ f_{\text{rf}} = 499.654 \text{ Mhz}, \ 432 \text{ bunches}, \ \text{bunch spacing} \approx 2 \text{ns}, \ f_0 = 1.15 \text{ MHz} \]

\[ \nu_{\text{hor}} = 12.30 \text{(fractional tune frequency}=345\text{kHz}), \ \nu_{\text{vert}} = 8.17 \text{(fractional tune frequency}=200\text{kHz}) \]

\[ \nu_{\text{long}} = 0.0076 \ (8.8 \text{ kHz}) \]

\[ \omega = pM \omega_0 \pm (m+n)\omega_0 \]

Spectral line at 512.185MHz

Lower sideband 200 KHz apart from the revolution harmonics

→ vertical mode #413

Spectral line at 604.9MHz

Upper sideband 8.8KHz apart from the 91st revolution harmonic

→ longitudinal mode #91
Feedback systems

Multi-bunch feedback systems detect the instability using one or more Beam Position Monitors (BPM) and act back on the beam to damp the oscillation through an electromagnetic actuator called kicker.

BPM and detector measure the beam oscillations.

The feedback processing unit generates the correction signal.

RF power amplifier and kicker act on the beam.
Mode-by-mode feedback

A mode-by-mode (frequency domain) feedback acts separately on each unstable mode.

An analog wideband electronics generates the position error signal from two BPM buttons.

A narrow band-pass filter selects a given mode.

The filtered signal is phase shifted by an adjustable delay line to produce a negative feedback.

One channel per unstable mode: all the channels work in parallel.

For machines with many bunches and several potentially unstable modes, the mode-by-mode feedback is not the appropriate choice.
Bunch-by-bunch feedback

A bunch-by-bunch (time domain) feedback acts individually on each bunch.

The correction signal for a given bunch is computed based only on the motion of that bunch.

There are as many processing channels as the number or bunches.

Every bunch is measured and corrected at every machine turn but, due to the delay of the feedback chain, the correction kick corresponding to a given measurement is applied to the bunch one or more turns later.

Damping the oscillation of each bunch is equivalent to damping all the multi-bunch modes.
Bunch-by-bunch analog implementation: one-BPM feedback

Transverse feedback

The correction signal applied to a given bunch must be proportional to the derivative of the bunch oscillation at the kicker, thus a sampled sinusoid shifted $\pi/2$ with respect to the oscillation of the bunch when it passes through the kicker.

The signal from a BPM with the appropriate betatron phase advance with respect to the kicker can be used to generate the correction signal.

The detector down converts the high frequency (typically a multiple of the bunch frequency $f_{rf}$) BPM signal into base-band (range 0 - $f_{rf}/2$).

The delay line assures that the signal of a given bunch passing through the feedback chain arrives at the kicker when, after one machine turn, the same bunch passes through it.
Analog implementation: two-BPM feedback

Transverse feedback case

The two BPMs can be in any ring position with respect to the kicker providing that they are separated by $\pi/2$ in betatron phase.

Their signals are combined with variable attenuators in order to provide any required phase of the resulting signal.
Analog implementation: revolution harmonics suppression

**Transverse feedback case**

The revolution harmonics (frequency components at multiples of $\omega_0$) are useless signal components that have to be eliminated in order not to saturate the RF amplifier.

A similar feedback architecture has been used to built the ALS transverse multi-bunch feedback system, also used at PLS and BessyII light sources.
Digital bunch-by-bunch feedback

Transverse and longitudinal case

The combiner generates the $X$, $Y$ or $\Sigma$ signal from the BPM button signals
The detector demodulates the position signal
The “stable beam components” are suppressed by the DC rejection unit
The resulting signal is digitized, processed and re-converted to analog by the processing unit
The modulator translates the correction signal to the frequency of the kicker (longitudinal)
The delay line adjusts the timing of the signal to match the bunch arrival time
The RF power amplifier supplies the power to the kicker
Digital vs. analog feedbacks

ADVANTAGES OF DIGITAL FEEDBACKS 😃

- Reproducibility: not subject to temperature or environment changes, aging, ...
- Flexibility: modification made by software/firmware (ex. control algorithms, transverse/longitudinal FB, ...)  
- Higher performance of feedback controllers and implementation of sophisticated control algorithms
- Efficient control: combination of basic control algorithms and additional control features (ex. saturation control, down sampling, etc.)
- Effective integration with control system: feedback setup and optimization, data acquisition, easy and automated operations, ...
- Availability of diagnostic features for both feedback commissioning and optimization, and for machine physics studies

DISADVANTAGE OF DIGITAL FEEDBACKS 😞

- High group delay due to ADC, digital processing and DAC
BPM and Combiner

The four signals from a standard four-button BPM can be opportunely combined to obtain the wide-band $X$, $Y$ and $\Sigma$ signals used respectively by the transverse horizontal, vertical and by the longitudinal feedback systems.

Usually BPM and combiner work around a multiple of $f_{rf}$, where the overall BPM-cables transfer function has maximum amplitude.

Moreover, a higher $f_{rf}$ harmonic is preferred for the longitudinal feedback because of the higher sensitivity of the phase detection system.

The SUM ($\Sigma$) signal only contains bunch arrival time information since the sum of the buttons has almost constant amplitude.

\[ X = (A + D) - (B + C) \]
\[ Y = (A + B) - (C + D) \]
\[ \Sigma = A + B + C + D \]
Detector: transverse feedback

Any $f_{rf}/2$ portion of the beam spectrum contains the information of all potential multi-bunch modes and can be used to detect an instability and measure its amplitude.

The detector (or RF front-end) translates the wide-band signal into base-band (0-$f_{rf}/2$ range): the operation is an amplitude demodulation.

**Heterodyne technique**: the “local oscillator” signal is derived from the RF by multiplying its frequency by an integer number corresponding to the chosen harmonic of $f_{rf}$.
Detector: longitudinal feedback

The detector generates the base-band longitudinal position (phase error) signal (0-$f_{rf}/2$ range) by processing the wide-band signal: the operation is a phase demodulation.

The phase demodulation can be obtained with the same heterodyne technique but using a “local oscillator” signal shifted by $\pi/2$ with respect to the bunches.

**Amplitude demodulation:**

$$A(t) \sin(3\omega_{rf} t) \cdot \sin(3\omega_{rf} t) \propto A(t) (\cos(0) - \cos(6\omega_{rf} t)) \approx A(t)$$

**Phase demodulation:**

$$\sin(3\omega_{rf} t + \varphi(t)) \cdot \cos(3\omega_{rf} t) \propto \sin(6\omega_{rf} t + \varphi(t)) + \sin(\varphi(t)) \approx \varphi(t)$$
Detector: time domain considerations

The base-band signal can be seen as a sequence of “pulses” each with amplitude proportional to the position error $X$ ($Y$ or $\Phi$) and to the charge of the individual bunches.

By sampling this signal with an A/D converter synchronous to the bunch frequency, one can measure $X$ ($Y$ or $\Phi$)

$$\text{Sampling Period} = \frac{1}{f_{rf}}$$

The band-pass and low-pass filters design is a compromise between maximum flatness of the top of the pulses and cross-talk between bunches due to long rise/fall times.

The multi-bunch-mode number $M/2$ is the one with higher frequency ($\approx f_{rf}/2$): the pulses have the same amplitude but alternating signs.

Ideal case: max flatness with min cross-talk

$\textbf{NO}$

$\textbf{YES}$
Rejection of stable beam signal

The base-band signal of each bunch can have a constant offset (DC component) due to:

- **transverse case**: closed orbit or unbalanced BPM electrodes or cables
- **longitudinal case**: beam loading, i.e. different stable beam phases for each bunch

In the frequency domain, the stable beam signal shows up as non-zero revolution harmonics.

These components have to be suppressed because they are useless for the feedback, reduce the dynamic range of the A/D converter and can saturate the D/A converter and the amplifier.

**Variable attenuators** on the BPM electrodes to make the signals identical in amplitude.

**Correlation filter**: $1 - z^{-1} \left( T=T_{\text{rev}}=1/f_0 \right)$ analog FIR filter made of delay lines and combiners to suppress all the revolution harmonics (DC included).

**DC rejection module**: the signal is sampled at $f_{rf}$, the mean value is calculated for each bunch, integrated, converted to analog and subtracted to the bunch signal.
The A/D converter samples and digitizes the signal at the bunch repetition frequency: each sample corresponds to the position (X, Y or Φ) of a given bunch. Precise synchronization of the sampling clock with the bunch signal must be provided.

The digital samples are then de-multiplexed into M channels (M is the number of bunches): in each channel the turn-by-turn samples of a given bunch are processed by a dedicated digital filter to calculate the correction samples.

The processing consists in DC component suppression and phase shift at the betatron/synchrotron frequency.

After processing, the correction sample streams are recombined and eventually converted to analog by the D/A converter.
Digital processor: the implementation

ADC: existing multi-bunch feedback systems usually employ 8-bit ADCs at up to 500 Msample/s; some implementations use a number of ADCs with higher resolution (14 bits) and lower rate working in parallel and sampling the same signal with shifted sampling clocks.

ADCs with more resolution have some advantages:
- lower quantization noise (crucial for low-emittance machines)
- higher dynamic range (in some cases rejection of stable beam signal is not necessary)

DAC: usually employed DACs convert samples at up to 500 Msample/s and 14-bit resolution.

Digital Processing: the feedback processing can be executed by DSPs or FPGAs.

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<td>A number of DSPs are necessary</td>
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Implementations and examples

- **ALS** (PLS, BessyII): completely analog transverse FB

- **ALS, PEP-II, DaΦne** (SPEAR, PLS, BessyII): digital longitudinal FB, a number of VME crates with DSP boards

- **PEP-II**: transverse digital FB, delay line with ADC-FPGA-DAC

- **KEKB** digital transverse and longitudinal FB: two-tap FIR filter performing DC rejection, filtering and delay

- **SPring-8** (TLS, KEK Photon Factory, Soleil): digital transverse FB, smart analog de-multiplexer + a number of ADC-FPGA channels and one DAC
Implementations and examples

- Elettra, SLS: digital transverse and longitudinal FB with VME ADC/DAC and commercial DSP boards
- CESR: digital longitudinal FB with one DSP, transverse digital FB with S/Hs and a number of ADC-FIFO channels and one DAC
- HERA-p: longitudinal digital FB with ADC-FPGA-DAC
- ESRF and Diamond: digital transverse and longitudinal FB using Libera (Instrumentation Technologies) with four ADCs one FPGA and one DAC
- DaΦne, KEK Photon Factory: digital transverse feedback using iGp (Dimtel) with ADC-FPGA-DAC
Kicker and Amplifier

The kicker is the feedback actuator. It creates a transverse/longitudinal electromagnetic field that kicks the bunches as they pass through the kicker at every turn. The overall effect is the damping of the betatron/synchrotron oscillations.

The amplifier must provide the necessary RF power to the kicker by amplifying the signal from the DAC (or from the modulator in the case of longitudinal feedbacks).

A bandwidth of at least $f_{rf}/2$ is necessary: from DC (all kicks of the same sign) to $f_{rf}/2$ (kicks of alternating signs).

The bandwidth of amplifier-kicker must be sufficient to correct each bunch with the appropriate kick without affecting the neighbour bunches. The amplifier-kicker design has to maximize the kick strength while minimizing the cross-talk between corrections given to different bunches.

Important issue: the group delay of the amplifier must be as constant as possible, e.g. the phase response must be linear, otherwise the feedback efficiency is reduced for some modes or can even excite some high frequency modes.
Kicker and Amplifier: transverse FB

For the transverse kicker a stripline geometry is usually employed. Amplifier and kicker work in the \(~\text{DC} - \frac{f_{rf}}{2}\) frequency range.

The ELETTRA/SLS transverse kicker (design by Micha Dehler-PSI)
Kicker and Amplifier: longitudinal FB

A "cavity like" kicker is usually preferred. Higher shunt impedance and smaller size.

The operating frequency range is typically $f_{rf}/2$ wide and placed on one side of a multiple of $f_{rf}:

ex. from $3f_{rf}$ to $3f_{rf}+f_{rf}/2$

A "pass-band" instead of "base-band" device.

The base-band signal from the DAC must be modulated, e.g. translated in frequency.

A SSB (Single Side Band) amplitude modulation or similar techniques (ex. QPSK) can be adopted.

The ELETTRA/SLS longitudinal kicker (design by Micha Dehler-PSI)
Theory of feedback control: transverse feedback

The motion of a particle can be described as a pseudo-harmonic oscillator with amplitude proportional to the square root of the $\beta$-function:

\[ x(s) = a \sqrt{\beta(s)} \cos \varphi(s), \quad \text{where} \quad \varphi(s) = \int_0^s \frac{ds}{\beta(s)} \]

The derivative of the position, i.e. the angle of the trajectory is:

\[ x' = -\frac{a}{\sqrt{\beta}} \sin \varphi + \frac{a\beta'}{2\sqrt{\beta}} \cos \varphi, \quad \text{with} \quad \varphi' = \frac{1}{\beta} \]

By introducing $\alpha = -\frac{\beta'}{2}$ we can write:

\[ x' = \frac{a}{\sqrt{\beta}} \sqrt{1+\alpha^2} \sin(\varphi + \arctan \alpha) \]

At the coordinate $s_k$, the electromagnetic field of the kicker deflects the particle bunch which varies its angle by $k$; as a consequence the bunch starts another oscillation:

\[ x_i = a_i \sqrt{\beta} \cos \varphi_i \]

which must satisfy the following constraints:

\[ \begin{cases} x(s_k) = x_i(s_k) \\ x'(s_k) = x_i'(s_k) + k \end{cases} \]

By introducing $A = a \sqrt{\beta}$, $A_i = a_i \sqrt{\beta}$ the two-equation two-unknown-variables system becomes:

\[ \begin{cases} A \cos \varphi = A_i \cos \varphi_i \\ A \sqrt{1+\alpha^2} \sin(\varphi + \arctan \alpha) = A_i \sqrt{1+\alpha^2} \sin(\varphi_i + \arctan \alpha) + k \end{cases} \]

The solution of the system gives amplitude and phase of the new oscillation:

\[ \begin{cases} A_i = \sqrt{(A \sin \varphi - k \beta)^2 + A^2 \cos^2 \varphi} \\ \varphi_i = \arccos \left( \frac{A}{A_i} \cos \varphi \right) \end{cases} \]
Theory of feedback control: transverse feedback

From \( A_i = \sqrt{(A \sin \varphi - k \beta)^2 + A^2 \cos^2 \varphi} \) when the kick is small \( k << \frac{A}{\beta} \) then 
\[ \frac{\Delta A}{A} = \frac{A - A_i}{A} \approx \frac{\beta}{A} k \sin \varphi \]

In the linear feedback case, e.g. when the turn-by-turn kick signal is a sinusoid proportional to the bunch oscillation amplitude, in order to maximize the damping rate the kick signal must be in-phase with \( \sin \varphi \):

\[ k = g \frac{A}{\beta} \sin \varphi \quad \text{with} \quad 0 < g < 1 \]

The gain \( g \) is determined by the max allowed kicker value which is generated when the oscillation amplitude at the kicker \( A \) is max:

\[ g = \frac{k_{\text{max}}}{A_{\text{max}}} \beta \quad \text{therefore} \quad k = \frac{k_{\text{max}}}{A_{\text{max}}} A \sin \varphi \]

For small kicks
\[ \frac{\Delta A}{A} \approx \frac{k_{\text{max}}}{A_{\text{max}}} \beta \sin^2 \varphi \]
the relative amplitude decrease is monotonic and its average is:

\[ \langle \frac{\Delta A}{A} \rangle \approx \frac{\beta k_{\text{max}}}{2 A_{\text{max}}} \]

Therefore, the average relative decrease is constant, which means that, in average, the amplitude decrease is exponential with time constant \( \tau \) (damping time) given by:

\[ \frac{1}{\tau} = \langle \frac{\Delta A}{A} \rangle \frac{1}{T_0} = \frac{\beta k_{\text{max}}}{2 A_{\text{max}} T_0} \quad \text{where} \quad T_0 \text{ is the revolution period.} \]

By referring to the oscillation at the BPM location, \( A_{B_{\text{max}}} \) is the max oscillation amplitude at the BPM:

\[ \frac{1}{\tau} = \frac{k_{\text{max}}}{2 T_0 A_{B_{\text{max}}}} \sqrt{\beta_k \beta_B} \]
The change in the transverse momentum $p$ of the bunch passing through the kicker can be expressed by:

$$\Delta p = \frac{e}{c} V_{\perp}$$

where

$$V_{\perp} = \int_{0}^{L} (\bar{E} + c \times \bar{B})_{\perp} \, dz$$

is the kick voltage and

$$p = \frac{E_{B}}{c}$$

$e$ = electron charge, $c$ = light speed, $\bar{E}, \bar{B}$ = fields in the kicker, $L$ = length of the kicker, $E_{B}$ = beam energy

$V_{\perp}$ can be derived from the definition of kicker shunt impedance:

$$R_{k} = \frac{V_{\perp}^{2}}{2 P_{k}}$$

The max deflection angle in the kicker is given by:

$$k_{\text{max}} = \frac{\Delta p}{p} = e \frac{V_{\perp}}{E_{B}} = \left( \frac{e}{E_{B}} \right) \sqrt{2 P_{k} R_{K}}$$

From the previous equations we can obtain the power required to damp the bunch oscillation with damping rate $\tau$:

$$P_{k} = \frac{2}{R_{K} \beta_{K}^{2}} \left( \frac{E_{B}}{e} \right)^{2} \left( \frac{T_{0}}{\tau} \right)^{2} \left( \frac{A_{B \text{max}}}{\sqrt{\beta_{B}}} \right)^{2}$$
Kicker power: transverse feedback

\[ P_K = \frac{2}{R_K \beta_K} \left( \frac{E_B}{e} \right)^2 \left( \frac{T_0}{\tau} \right)^2 \left( \frac{A_{B_{\text{max}}}}{\sqrt{\beta_B}} \right)^2 \]

Ex.: (Elettra):
\[ R_k = 15 \text{ k}\Omega \text{ (mean value)} \]
\[ E_B/e = 2\text{GeV} \]
\[ T_0 = 864 \text{ ns} \]
\[ \tau = 120 \mu\text{s} \]
\[ \beta_{B,H,V} = 5.2, 8.9 \text{ m} \]
\[ \beta_{K,H,V} = 6.5, 7.5 \text{ m} \]
**Kicker power: longitudinal feedback**

\[
P_k = \frac{2}{R_k} \left( \frac{\omega_S E_B \phi_{\text{max}}}{\omega_0 \alpha f_{\text{RF}} \tau} \right)^2
\]

- \( \tau \) = feedback damping time
- \( \omega_0 \) = revolution frequency
- \( \omega_S \) = synchrotron frequency
- \( \alpha \) = momentum compaction factor
- \( f_{\text{RF}} \) = RF frequency
- \( R_k \) = kicker shunt impedance
- \( E_B \) = beam energy
- \( \phi_{\text{max}} \) = maximum oscillation amplitude

The power is proportional to the square of the maximum phase oscillation.

If we switch on the feedback when the oscillation is small, the required power is lower.

The feedback gain must be kept high (a small oscillation corresponds to the entire dynamic range of DAC/amplifier).
Control system integration

- It is desirable that each component of the feedback system that needs to be configured and adjusted has a control system interface.
- Any operation must be possible from remote to facilitate the system commissioning and the optimization of its performance.
- An effective data acquisition channel has to provide fast data transfer of large amounts of data for analysis of feedback performance and beam dynamics studies.
- It is preferable to have a direct connection to a mathematical tool (e.g., Matlab) to develop measurement procedures using a script language and acquire data for post processing and data visualization.
M (number of bunches) filters each dedicated to one bunch

To damp the bunch oscillations the kick must be the derivative of the bunch position at the kicker: for a given oscillation frequency, a $\pi/2$ phase shifted signal must be generated

In the computation of the required filter phase response, the relative position of BPM and kicker must be taken into account (transverse feedback) as well as any additional delay due to the feedback latency (multiple of one machine revolution period)

The digital processing must also reject any residual constant offset (DC component) from the bunch signal to avoid DAC saturation

Digital filters can be implemented with FIR (Finite Impulse Response) or IIR (Infinite Impulse Response) architectures

A filter on the full-rate data stream can compensate for amplifier/kicker unwanted behaviour
Digital filter design: 3-tap FIR filter

The minimum requirements are:
1. DC rejection (coefficients sum = 0)
2. Amplitude response at the tune frequency
3. Phase response at the tune frequency

A 3-tap FIR filter fulfills these requirements: the filter coefficients can be calculated analytically.

Example:
- Fractional tune = 0.2
- Amplitude response at tune = 0.8
- Phase response at tune = 225°

\[ H(z) = -6.5 + 5.4 z^{-1} + 1.1 z^{-2} \]

**Z transform of the FIR filter response**

In order to have zero amplitude at "0" frequency, we must put a "zero" in \( z=1 \). Then another zero in \( z=c \) is added to fulfill the phase requirements.

"c" can be calculated analytically:

\[ H(z) = k(1 - z^{-1})(1 - cz^{-1}) \]

\[ H(z) = k(1 - (1 + c)z^{-1} + cz^{-2}) \]

\[ z = e^{j\omega} \]

\[ H(\omega) = k(1 - (1 + c)e^{-j\omega} + ce^{-2j\omega}) \]

\[ e^{-j\omega_0} = \sin \omega_0 - j \cos \omega_0 \]

\[ \alpha = \text{ang}(H(\omega_0)) \]

\[ \tan(\alpha) = \frac{c(\sin(\omega_0) - \sin(2\omega_0)) + \sin(\omega_0)}{c(\cos(2\omega_0) - \cos(\omega_0)) + 1 - \cos(\omega_0)} \]

\[ c = \frac{\tan(\alpha)(1 - \cos(\omega_0)) - \sin(\omega_0)}{(\sin(\omega_0) - \sin(2\omega_0)) - \tan(\alpha)(\cos(2\omega_0) - \cos(\omega_0))} \]

\( K \) is determined by the required amplitude of the transfer function at \( \omega = \omega_0 \)
Digital filter design: 5-tap FIR filter

With more degrees of freedom additional features can be obtained.

Ex.: transverse feedback. The tune frequency of the accelerator can significantly change during machine operations. The filter response (amplitude and angle) must guarantee the same feedback efficiency in a frequency range by performing an automatic compensation.

In this example, given the feedback processing delay, the kick is applied to the bunch after 4 machine turns. When the tune frequency increases, the phase of the filter must also increase, e.g. the phase response must have a positive slope around the working point.

In this case the filter design can be made using the Matlab function `invfreqz()`.

This function calculates the filter coefficients that best fit the required frequency response using the least squares method.

The desired response is provided to `invfreqz()` by defining amplitude and phase at three different frequencies: 0, \( f_1 \) and \( f_2 \).
Digital filter design: selective FIR filter

A FIR digital filter often used in longitudinal feedback systems is a “correlator” filter, which impulse response (the filter coefficients) is a sampled sinusoid with frequency equal to the bunch oscillation frequency (synchrotron tune).

This filter has a maximum of amplitude at the tune frequency and linear phase.

The more filter coefficients we use the more selectivity we obtain, thus rejection of the unwanted spectrum components.

Samples of the filter impulse response (= filter coefficients)

Amplitude and phase response of the filter
Digital filter design: IIR filters

Using IIR digital filters opens the door to a number of design techniques allowing for additional interesting features of the controller that can be implemented.

The disadvantage is a more difficult filter design and an increased complexity in the filter implementation.

Examples of additional features:

- Increase of the amplitude response selectivity for better noise rejection.
- Possibility to simultaneously stabilize different frequencies (e.g., dipole/quadrupole, horizontal/vertical, ...).
- Increase of the working frequency range with reduced performance degradation.
- Reduction of the amplitude response at frequencies that must not be fed back.

... . . . .

Various techniques are used: i.e., frequency domain design and model based design.

More sophisticated techniques can improve performance and robustness of the feedback under parametric changes of the accelerator and feedback components (i.e., optimal control, robust control, etc.).
Longitudinal feedback: down sampling

The synchrotron frequency is usually much lower than the betatron frequency: one complete synchrotron oscillation is accomplished in many machine turns (ex. 100)

In order to be able to properly filter the bunch signal, down sampling is necessary

One out of $D$ samples are taken: $D$ is the dawn sampling factor

The processing is performed on the down sampled digital signal and the filter design is done in the down sampled frequency domain (enlarged by the down sampling factor $D$)

The reduced data rate also allows for more time available to perform filter calculations and more complex filters can therefore be implemented

The correction signal at full rate is reconstructed by a hold buffer that keeps the same correction value for $D$ turns
Diagnostic features

The feedback system can implement a number of diagnostic capabilities useful for both commissioning and optimization of the feedback system and for machine physics studies:

1. **Input data recording for data analysis**: acquisition of large number of samples in parallel with the feedback processing.

2. **Changing filter parameters on the fly with required timing and individually for each bunch**: switching ON/OFF the feedback, generation of grow/damp transients, optimization of feedback performance.

3. **Generation of output arbitrary waveforms**: multi-bunch and individually for each bunch.

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![Diagram of Diagnostic controller](https://via.placeholder.com/150)

**Diagram Description**

- **Control system interface**
- **Timing**
- **Diagnostic controller**
- **ADC**
- **DEMUX**
- **Filter #n**
- **Filter #n+1**
- **Filter #n+2**
- **MUX**
- **DAC**

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*Marco Lonza, “Multi-bunch Feedback Systems”*  
Diagnostic features: data recording

Ex. 1: acquisition of ADC data (500 MSamples/s) and spectrum made by FFT with Matlab

Spectrum of multi-bunch data from an unstable beam

Zoomed spectrum: side-bands of the revolution harmonics corresponding to the excited modes

Ex. 2: acquisition of turn-by-turn samples of a given bunch and FFT with Matlab

Diagnostic features: excitation of individual bunches

The feedback loop is switched off on single bunches and the excitation signal is injected in place of the correction. Excitation signals can be:

- white (or pink) noise
- sinusoids

![Diagram of diagnostic features](image)

In this example two bunches are vertically excited with pink noise in a range of frequencies centered around the tune, while the feedback is applied on the other bunches. The spectrum of the excited bunch reveals a peak at the tune frequency.

This technique is used to measure the betatron tune with almost no disturbance for machine operations.
Diagnostic features: multi-bunch excitation

Interesting measurements can be performed by adding pre-defined signals to the output of the feedback.

1. By adding a sinusoid at a given frequency, the corresponding beam multi-bunch mode can be excited to test the performance of the feedback.

2. By adding an appropriate output signal and recording the feedback input data with filter coefficients set to zero, the open loop transfer function can be calculated.

3. By adding an appropriate output signal and recording the feedback input data with filter coefficients set to the nominal values, the closed loop transfer function can be calculated.
Diagnostic features: transient generation

A powerful diagnostic application is the generation of transients. Transients can be generated by changing the filter coefficients accordingly to a predefined timing and by concurrently recording the oscillations of the bunches.

Different types of transients can be generated, damping times and growth rates can be determined by exponential fitting of the transients:

1. free oscillations $\rightarrow$ FB on
2. FB on $\rightarrow$ FB off $\rightarrow$ FB on
3. Stable beam $\rightarrow$ positive FB on, anti-damping $\rightarrow$ FB off, natural damping

\[ \ldots \]
Grow-damp transients can be analyzed by recording the oscillation samples, processing the data, i.e. with Matlab, and displaying the results with 3-D graphs.

Evolution of the bunches oscillation during a grow-damp transient

Evolution of the coupled-bunch unstable modes during a grow-damp transient
Grow-damp transients: real movies

'Movie' sequence:
1. Feedback OFF
2. Feedback ON after 5.2 ms
'Camera' view slice is 50 turns (about 43 μs)
Grow-damp transients studies

Transients generation is a useful tool to characterize and optimize the feedback system and also to study coupled-bunch modes and beam dynamics:

- Feedback damping times: can be used to characterize and optimize the feedback performance
- Resistive and reactive response: feedback not perfectly tuned has a reactive behavior (tune shift)
- Modal analysis: coupled-bunch mode complex eigenvalues, analysis of growth rates and tune shifts of the oscillation with exponential fit
- Storage-ring impedance: evaluation of the transverse/longitudinal machine impedance
- Stable modes: modes below the instability threshold can be studied
- Bunch train studies: different behavior of bunches in the train to study the origins of the coupled-bunch modes
- Phase space analysis: analysis of the phase evolution of unstable coupled-bunch modes
Feedback optimization: detector phase

A number of adjustments have to be carried out after the feedback is set up to make the system work with optimized performance.

Detector demodulation phase: the “local oscillator” signal derived from the bunch frequency must be adjusted in order to be in-phase (amplitude demodulation) or in quadrature (phase demodulation) with the bunch signal. The optimal phase can be found for example with the analysis of the data acquired by the ADC from an unstable beam and by maximizing the detected bunch oscillation amplitude.
Feedback optimization: sampling clock

ADC sampling clock: the phase of the clock must be adjusted to sample the “pulse” of every bunch on the top. This task is usually carried out with the machine filled with a single bunch: the optimum clock phase maximizes the amplitude of the samples of the filled bucket and minimizes the amplitude of the samples of the adjacent empty buckets.
Feedback optimization: delay line

Delay line: must be properly set in order to kick each bunch with the correction signal calculated for that bunch. The adjustment can be done in two phases using the feedback system:

1. Coarse adjustment (resolution < $T_{\text{rf}}$): excite a single stable bunch (ex. with white noise or with a sinusoid at the tune frequency) and adjust the delay until the bunch we wanted to excite is really seen excited by analyzing the acquired data.

2. Fine adjustment (resolution < $T_{\text{rf}}/100$): optimizing the feedback performance. Maximum reduction of coupled-bunch modes (natural or artificially excited by the use of the feedback system), minimum damping time in grow/damp transients, ...

![Diagram of feedback system components: Detector, DC Rejection, ADC, Digital Signal Processing, DAC, Modulator, Delay, Power Amplifier, Combiner, BPM, Kicker.]
Digital filter: the digital filter is designed based on given requirements (phase and amplitude response). A fine optimization can be done with the real machine by adjusting the basic parameters (phase at the tune and gain) and trying to maximize the feedback performance (maximum reduction of coupled-bunch modes). A careful analysis of grow/damp transients gives useful information on the feedback behavior with the objective to find optimal working conditions: evaluation of damping time and reactive component of the feedback, phase space analysis, ...
Effects of the feedback: beam spectrum

Spectrum analyzer connected to a stripline pickup: observation of vertical instabilities. The sidebands corresponding to vertical coupled-bunch modes disappear as soon as the transverse feedback is activated.
Effects of the feedback: beam transverse profile

Pin-hole camera images (courtesy of Micha Dehler SLS/PSI)

Synchrotron Radiation Profile Monitor (SRPM) showing the reduction of the vertical beam dimension when the transverse feedback is switched on

(Elettra)
Effects of the feedback: photon beam spectra

Effects on the synchrotron light: spectrum of photons produced by an undulator
The spectrum is noticeably improved when vertical instabilities are damped by the feedback

SuperESCA beamline at Elettra
Conclusions

- Feedback systems are indispensable tools to cure multi-bunch instabilities in storage rings.

- Technology advances in digital electronics allow implementing digital systems.

- Digital signal processing theory widely used to design and implement filters as well as to analyze data acquired by the feedback.

- Feedback systems not only for closed loop control but also as powerful diagnostic tools.

- Full potentialities of digital feedback systems still to be exploited:
  - improvement of feedback performance
  - studies of beam dynamics.
References and acknowledges

- Herman Winick, “Synchrotron Radiation Sources”, World Scientific

- Many papers about coupled-bunch instabilities and multi-bunch feedback systems (KEK, SPring-8, DaΦne, ALS, PEP-II, ESRF, Elettra, SLS, CESR, DESY, PLS, Bessy, SRRC, …)

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