Laser Beam Physics

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Outline

- Cavity modes – longitudinal and transverse
- Gaussian beams & the q parameter
- Ray optics & ABCD matrices
- Beam focusing
Cavity

- Pump gain medium to upper level
- A photon decays spontaneously & stimulates more emission
- The photons bounce back and forth along the cavity – if the number of photons emitted each round trip exceeds losses (mirrors etc.) laser is above threshold
- One of the mirrors allows a small amount of this light out – laser output!
- Laser output controlled by gain of medium and longitudinal & transverse modes of cavity
Longitudinal modes

- Laser oscillator is just a resonator
- Resonant cavity modes exist
- Other frequencies ‘don’t fit’

Cavity mode spacing given by:

\[ \Delta \nu = \frac{c}{2nL} \]

\( n \) is refractive index – may be 1

Form a ‘comb’ of equally spaced modes in frequency space
Longitudinal modes

- Length of cavity determines resonant frequencies and mode spacing
- Laser gain medium has certain bandwidth
- Combination determines what wavelengths can lase
Lasing on longitudinal modes

The oscillator can lase on these modes

Doesn’t mean it will!

These modes will lase

These modes won’t lase

Need to be above threshold – gain > cavity losses for lasing

- Can make the laser run on a single longitudinal mode – SLM
- Very narrow bandwidth
- Spectroscopy etc.
Mode locking

Generating short pulses = Mode-locking

Locking vs. not locking the phases of the laser modes (frequencies)
Short pulse oscillator

Ti:sapphire: how many modes lock?

![Absorption and Fluorescence graph with 200 nm bandwidth.]

Therefore $\Delta v = c/L_{rt} \approx 100 \text{ MHz}$

Q: How many different modes can oscillate simultaneously in a 1.5 meter Ti:sapphire laser?

A: Gain bandwidth $\Delta \lambda = 200 \text{ nm} \implies \Delta v = (c/\lambda^2) \Delta \lambda \sim 10^{14} \text{ Hz}$

$\Delta v_{\text{bandwidth}}/\Delta v_{\text{mode}} = 10^6 \text{ modes}$

Fourier transform of comb of frequencies is train of pulses in time
Duration of individual pulse given by total bandwidth
Pulse train output

- If we can lock all the lasing cavity modes in phase we have a short pulse in the oscillator
- Each round trip a small amount is transmitted through the output coupler
- So laser output is a train of ultrashort pulses
- ‘Front end’ of chirped pulse amplification system
Transverse cavity modes

aka ‘what the laser looks like as you stare into it just before it blinds you’
Transverse cavity modes (safely)

- Not an infinite plane wave – boundary conditions!
- What is the form of that wave that is self consistent after one round trip in cavity?
- Paraxial approximation

Cylindrical symmetry:
Solutions are Laguerre – Gaussian modes

Rectangular symmetry:
Solutions are Hermite – Gaussian modes

HG more common – broken symmetry in oscillator

Lowest order mode is a Gaussian
Gaussian beams

Important as lowest order lasing mode of (most) cavities

- Want to know how this beam propagates through an optical system:
- How does the beam change?
- How does it focus?

This is a simulation. You will never see a real beam this good.
Gaussian beams

A laser beam can be described by this:

\[ E(x, y, z, t) = u(x, y, z) e^{i(kz - \omega t)} \]

where \( u(x,y,z) \) is a Gaussian transverse profile that varies slowly along the propagation direction (the \( z \) axis), and remains Gaussian as it propagates:

\[ u(x, y, z) = \frac{1}{q(z)} \exp \left[ -jk \frac{x^2 + y^2}{2} \cdot \frac{1}{q(z)} \right] \]

The parameter \( q \) is called the “complex beam parameter.” It is defined in terms of \( w \), the beam waist, and \( R \), the radius of curvature of the (spherical) wave fronts:

\[ \frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w^2(z)} \]
Gaussian beams

beam waist at
$z = 0: R = \infty$

• IMPORTANT: ‘spot size’ $w(z)$
  ‘beam waist’ $w(0)$
Gaussian beam size

- **IMPORTANT:** ‘spot size’ \( w(z) \) is \( 1/e^2 \) radius of beam intensity profile

Both of these will be completely different to the way your accelerator colleagues define charged particle beam size
Propagating a Gaussian beam (propagating in empty space, wavelength \( \lambda \)) has an infinite radius of curvature (i.e., phase fronts with no curvature at all) at a particular location (say, \( z = 0 \)).

Suppose, at that location \( (z = 0) \), the beam waist is given by \( w_0 \).

Describe the subsequent evolution of the Gaussian beam, for \( z > 0 \).

We are given that \( R(0) = \infty \) and \( w(0) = w_0 \). So, we can determine \( q(0) \):

\[
\frac{1}{q(0)} = \frac{1}{R(0)} - j \frac{\lambda}{\pi w^2(0)}
\]

\[
= -j \frac{\lambda}{\pi w_0^2}
\]

Thus \( q(0) = j \frac{\pi w_0^2}{\lambda} \)

NOTE: If \( R = \infty \) at a given location, this implies that \( q \) is a pure imaginary number at that location: this is a focal point.
Rayleigh range

\[ z_R = \frac{\pi w_0^2}{\lambda} \]

- Can define new quantity \( z_R \) – the ‘Rayleigh range’
- Distance over which spot size \( w_0 \) goes to \( \sqrt{2}w_0 \)
- Beam area has doubled.

Confocal parameter \( b = 2z_R \)

No such thing as a collimated beam, but \( 2z_R \) is a reasonable approximation
Gaussian beam propagation

A distance $z$ later, the new complex beam parameter is:

$$q(z) = q(0) + z = jz_R + z$$

$$\frac{1}{q(z)} = \frac{1}{z + jz_R} = \frac{z - jz_R}{z^2 + z_R^2}$$

$$R(z) = \frac{z^2 + z_R^2}{z}$$

$$w(z) = w_0 \sqrt{1 + \frac{z^2}{z_R^2}}$$

- At $z = 0$, $R$ is infinite, as we assumed.
- As $z$ increases, $R$ first decreases from infinity, then increases.
- Minimum value of $R$ occurs at $z = z_R$.

$$\text{Reminder:}$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} = j\frac{\lambda}{\pi w^2(z)}$$

- At $z = 0$, $w = w_0$, as we assumed.
- As $z$ increases, $w$ increases.
- At $z = z_R$, $w(z) = \sqrt{2}w_0$. 
ABCD matrices

This just gives free space propagation – how to model beam through optical system?
Back to ray optics

• Define input light ray: position $x_{in}$ and angle $\theta_{in}$
• Propagate through optical system
• Have output light ray: position $x_{out}$ and angle $\theta_{out}$
• Related by 2 x 2 matrix

$$
\begin{bmatrix}
x_{out} \\
\theta_{out}
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x_{in} \\
\theta_{in}
\end{bmatrix}
$$

Exact form of matrix depends on optical system
**ABCD matrices**

- **Common matrices:**
  - Propagation in free space: \[
    \begin{bmatrix}
    1 & L \\
    0 & 1
    \end{bmatrix}
  \]
  - Focusing with thin lens: \[
    \begin{bmatrix}
    1 & 0 \\
    -1/f & 1
    \end{bmatrix}
  \]

  Can multiply several matrices together to model e.g. cavity, telescope.

  **Best bit** – works for Gaussian beams too!

  \[
  q_2 = \frac{Aq_1 + B}{Cq_1 + D}
  \]

  So if you know initial q parameter and ABCD matrix can find spot size and curvature anywhere.

  **Reminder:**

  \[
  \frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w^2(z)}
  \]
Gaussian beam focusing (theory)

- Want high intensity $\rightarrow$ small spot size
- Gaussian beam focus with lens focal length $f$

Small spot:
- Short focal length $f$
- Short wavelength $\lambda$
- Large input spot $w_i$

Beam waist

$W_o = \frac{\lambda f}{\pi W_i}$

Input spot size
Gaussian beam focusing (practice)

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Image Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam on lens</td>
<td>Image of beam on lens at different distances.</td>
</tr>
<tr>
<td>0mm</td>
<td>Image at 0mm.</td>
</tr>
<tr>
<td>50mm</td>
<td>Image at 50mm.</td>
</tr>
<tr>
<td>100mm</td>
<td>Image at 100mm.</td>
</tr>
<tr>
<td>150mm</td>
<td>Image at 150mm.</td>
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<tr>
<td>200mm</td>
<td>Image at 200mm.</td>
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<tr>
<td>250mm</td>
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<td>450mm</td>
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<td>500mm</td>
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</tr>
</tbody>
</table>

What your supervisor thinks the beam should look like:

Beam at focus

v. what the beam actually looks like
Top hat beams

- High power laser beams often top hat spatial profile rather than Gaussian
- More efficiently extract gain from laser amplifier
- Often use concept of f/# (f – number) for focusing
f number and focusing

- ‘f/#’ is a property of focusing (collimating) system e.g. lens, parabola
  
  Given by \( f/# = \frac{f}{D} \)

- We say ‘f 10’ or f 18’
  
  - f – lens/parabola focal length
  - D – diameter

From focusing formula we find

\[ w \approx \lambda \times f/\# \]

- Caution! D is really size of beam on optic here – ‘effective’ f/#
- e.g. f = 150mm, D = 50mm, spot diameter 40 mm

- Smaller f/# for smaller spot
- f = 150mm, D = 100mm, spot diameter 40mm – haven’t changed focus size!
Adaptive optics

Can we make the focus better?
Want to remove aberrations from the laser beam wavefront to give best focus
Use of adaptive optics to correct beam – stolen from astronomy

Use deformable mirror to correct wavefront and produce best focus
Conclusion

• Studied:
  • Longitudinal cavity modes and mode locking
  • Transverse cavity modes
  • Gaussian beam propagation and focusing
  • Adaptive optics