Theory of Dielectric Laser Acceleration

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**Particle accelerators: from RF to optical/photonic drive?**

**RF cavity (TESLA, DESY)**

<table>
<thead>
<tr>
<th></th>
<th>Conventional linear accelerator (RF)</th>
<th>Laser-based dielectric accelerator (optical)</th>
</tr>
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<tbody>
<tr>
<td><strong>Based on</strong></td>
<td>(Supercond.) RF cavities</td>
<td>Dielectric nano structures</td>
</tr>
<tr>
<td><strong>Peak field limited by</strong></td>
<td><strong>Surface breakdown:</strong> 200 MV/m</td>
<td><strong>Damage threshold:</strong> 30 GV/m</td>
</tr>
<tr>
<td><strong>Max. achievable gradients</strong></td>
<td><strong>100 MeV/m</strong></td>
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# Particle accelerators: from RF to optical/photonic drive?

![Image](image.png)

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Rasmus Ischebeck

- Plasma wakefield & Laser plasma accelerators
- Laser-based dielectric accelerator (optical)

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Preview: where do we want to end up after the lecture

t = 0

t = \pi/2

t = \pi

1. acceleration
2. deceleration
3. deflection
4. deflection
Proposal for an Electron Accelerator Using an Optical Maser

Koichi Shimoda

Fig. 1. Schematic diagram of an electron linear accelerator by optical maser.
An old idea ... II

Electron Acceleration by Light Waves

October 3, 1962

A. Lohmann*

Department 522
Photo-Optics
Technology

GPD Development
Laboratory
San Jose

Aug. 16, 1966

A. W. LOHMANN

PARTICLE ACCELERATOR UTILIZING COHERENT LIGHT

Filed May 27, 1963

3,267,383

2 Sheets-Sheet 2
Gauss’ law for electricity

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

- Electric field \( \vec{E} \)
- Total charge \( \rho \)
- Vacuum permittivity \( \varepsilon_0 \)
- Divergence \( \nabla \cdot \hat{x} \)

The electric flux out of a closed surface is proportional to the enclosed charge
Electromagnetic waves – Maxwell Equations

Gauss‘ law for magnetism

\[ \nabla \star \vec{B} = 0 \]

• Magnetic field \( \vec{B} \)

The magnetic flux out of a closed surface is zero
Electromagnetic waves – Maxwell Equations

Faraday’s law of induction

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \text{Curl } \nabla \times \vec{x} \]

The curl of the electric field is equal to the negative rate of change of the magnetic field.
Electromagnetic waves – Maxwell Equations

Ampere’s law

\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \]

- Current \( \vec{J} \)
- Vacuum permeability \( \mu_0 \)
- Divergence \( \nabla \cdot \vec{\chi} \)

The curl of the magnetic field is proportional to the electric current flowing through a loop and the rate of change of the electric field.
Electromagnetic waves – Maxwell Equations

Gauss’ law for electricity
\[ \nabla \times \vec{E} = \frac{\rho}{\varepsilon_0} \]

Gauss’ law for magnetism
\[ \nabla \times \vec{B} = 0 \]

Faraday’s law of induction
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

Ampere’s law
\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \]

Evaluate in vacuum -> no charges and currents
Maxwell Equations predict waves

Start with an oscillating electric field.

Source: The University of Texas at El Paso: EE 4347 Applied Electromagnetics
Maxwell Equations predict waves

This induces a circulating magnetic field.

Source: The University of Texas at El Paso: EE 4347  Applied Electromagnetics
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Maxwell Equations predict waves

This induces a circulating magnetic field.

Source: The University of Texas at El Paso: EE 4347  Applied Electromagnetics
Electromagnetic waves in vacuum

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

Faraday’s law of induction

\[ \nabla \times (\nabla \times \vec{E}) = \nabla \times -\frac{\partial \vec{B}}{\partial t} \]

Take the curl

\[ \nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t} (\nabla \times \vec{B}) \]

Change RHS order of differentiation

But we know already Ampere’s law

\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]
Electromagnetic waves in vacuum

\[ \nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t} \left( \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \]

Substitute Ampere’s law

\[ \nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \]

Assume \( \mu_0 \varepsilon_0 \) are not time dependent

With identity \( \nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \)

\[ \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \]

But: \( \nabla \cdot \vec{E} = 0 \)

\[ \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \]
Electromagnetic waves in vacuum

Generalized form of the wave equation

\[ \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \]

\[ \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \]

\[ \nabla^2 \vec{A} = \frac{1}{v^2} \frac{\partial^2 \vec{A}}{\partial t^2} \]

Solution: Plane waves

\[ \vec{E}(t, \vec{r}) = E_0 * e^{i \vec{k} \vec{r} - i \omega t} \]

\[ \vec{B}(t, \vec{r}) = B_0 * e^{i \vec{k} \vec{r} - i \omega t} \]

\[ c = \frac{1}{\mu_0 \varepsilon_0} \]
Electron light interaction in free space

no net acceleration
Lawson-Woodward theorem

No net acceleration if all the following are true:

- The interaction takes place in vacuum (unity refractive index)
- No boundaries or surfaces are present, i.e., the distance from any source of field is large compared to the wavelength (far-field)
- The particle is moving in a region without other free charges
- (The particle is highly relativistic)
- No static electric or magnetic fields are present
- The interaction region is infinitely large
- Non-linear forces (e.g., the ponderomotive force) are neglected.


Inelastic ponderomotive scattering of electrons at a high-intensity optical travelling wave in vacuum, M. Kozák et. al., Nature Physics volume14, pages121–125 (2018)
Ponderomotive acceleration

\[ \lambda_1 = 1356 \text{ nm} \]
\[ \lambda_2 = 1958 \text{ nm} \]
\[ \alpha = 41^\circ \]
\[ \beta = 107^\circ \]

\[ \lambda_g = \frac{2\pi c}{(\omega_1 \cos \alpha - \omega_2 \cos \beta)} = 1.41 \mu\text{m} \]
Ponderomotive acceleration

In both pulsed beams:
\[ E_p = 85 \, \mu\text{J} \]
\[ I_p = 3 \times 10^{15} \, \text{W/cm}^2 \]
(rep. rate: 1 kHz)

Gradient: 2.2 GeV/m

- +/- 7 keV broad shoulders
- corresponding to absorption/emission of ~10,000 photons
Electromagnetic waves at interfaces

Boundary conditions:

- \( n_{12} \times (\vec{E}_2 - \vec{E}_1) = 0 \)
- \( (\vec{D}_2 - \vec{D}_1) \times n_{12} = \sigma_s \)
- \( (\vec{B}_2 - \vec{B}_1) \times n_{12} = 0 \)
- \( n_{12} \times (\vec{H}_2 - \vec{H}_1) = \vec{j}_s \)

With:
- \( n_{12} \) the normal vector from medium 1 to 2
- \( \vec{D} = \epsilon_0 \vec{E} + \vec{P} \) the electric displacement field
- \( \sigma_s \) the surface charge
- \( \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \) the magnetic field strength in matter
- \( \vec{j}_s \) the surface current

Dielectrics only \( \Rightarrow \sigma_s = 0 = \vec{j}_s \)
Dielectric – Dielectric interface

From the boundary conditions + plane waves:
\[
(k_i - k_r) \cdot \hat{r} = 0 \\
(k_i - k_t) \cdot \hat{r} = 0
\]

Evaluating the scalar product yields:
\[
k_{i,x} = k_{r,x} = k_{t,x} \\
k_{i,x} = |\vec{k}_i| \sin \phi = \frac{n_i \omega}{c} \sin \phi
\]

Similar for transmitted wave:
\[
|\vec{k}_t| = \frac{n_t \omega}{c} = \sqrt{k_{t,x}^2 + k_{t,y}^2}
\]

Angle of incidence $\phi$
Dispersion relation $k = \frac{n \omega}{c}$
Dielectric – Dielectric interface

Finally: solve for \( k_{t,y} \) with \( k_{t,x}^2 = k_{i,x}^2 \)

\[
k_{t,y}^2 = \left( \frac{n_t \omega}{c} \right)^2 - \left( \frac{n_i \omega}{c} \right)^2 \sin^2 \phi
\]

For \( \phi = \sin^{-1} \frac{n_t}{n_i} \)

\[
k_{t,y} = \pm i k_t \sqrt{\frac{n_i^2}{n_t^2} \sin^2 \phi - 1} = \pm i \beta k_t
\]

Transmitted plane wave:

\[
\vec{E}_t = E_0 e^{-\beta k_{t,y}} e^{i k_{t,x}x} e^{-i \omega t}
\]
Acceleration with evanescent fields in vacuum

Phase matching:

\[ v_{ph} = \frac{c}{n \sin \phi} \quad v_e = c \beta \]

Decay length

\[ \Gamma = \frac{c}{\omega \sqrt{n^2 \sin^2 \phi - 1}} \]

\[ \Gamma = \frac{1}{2\pi} \gamma \beta \lambda \]

Acceleration of sub-relativistic electrons with an evanescent optical wave at a planar interface, M. Kozák et. al., *Optics Express* 25 (2017), S. 19195-19204
Acceleration of sub-relativistic electrons with an evanescent optical wave at a planar interface, M. Kozák et. al., *Optics Express* 25 (2017), S. 19195-19204
Acceleration with evanescent fields in vacuum

Control only via refractive index \( n \) and incidence angle \( \phi \)

Acceleration of sub-relativistic electrons with an evanescent optical wave at a planar interface, M. Kozák et. al., *Optics Express* 25 (2017), S. 19195-19204
Fields at dielectric gratings

Assume infinite plane grating of periodicity $\lambda_p$

Diffracted light creates spatial harmonics $\vec{k}^n = \vec{K} + n\vec{k}_p$

With:
- $\vec{K}_0$ incident wave vector
- $\vec{K}$ component parallel to surface
- $\vec{k}_\parallel$ parallel diffracted component
- $\vec{k}_\perp$ perpendicular diffracted component
Fields at dielectric gratings

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- $\vec{k}_{\perp}$ perpendicular diffracted component
Fields at dielectric gratings

Grating fields can be described as:

$$\mathbf{A}(\mathbf{r}, t) = \sum_{n=-\infty}^{\infty} A_n e^{i(k_{\perp} n z + k_{||} n r - \omega t + \theta)}$$

- total field is comprised of a series of spatial harmonics

For phase matching, electrons ($v = \beta c$) and the grating mode ($v_{ph} = \omega / k_{||} \cos \phi$) have to have the same speed:

$$k_{||} = \frac{\omega}{\beta c \cos \phi} = \frac{k_0}{\beta \cos \phi}$$

with the dispersion relation $k_0 = \omega / c$.

Assuming particle trajectory is parallel to grating vector $k_p$, and laser is incident perpendicular on grating surface $\mathbf{K} = 0$

Synchronicity condition:

$$\lambda_p = n \beta \lambda$$
Electron light interaction

Integration over 2 periods, dist=30nm

kin. energy (keV)

Position (nm)

net acceleration of 1.1 GeV/m
Fields and forces at dielectric gratings

Using \(k_{||}\) and \(k_{\perp}\) in Ampere's and Faraday's laws, we obtain:

\[
\vec{E} = \begin{pmatrix} icB_y / (\tilde{\beta} \tilde{\gamma}) \\ E_y \\ -cB_y / \tilde{\beta} \end{pmatrix} \quad \quad \vec{B} = \begin{pmatrix} icE_y / (\tilde{\beta} \tilde{\gamma}) \\ B_y \\ E_y / (\tilde{\beta} \tilde{\gamma}) \end{pmatrix}
\]

From these fields we can calculate the Lorentz force:

\[
\vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) = q \begin{pmatrix} icB_y / (\tilde{\beta} \tilde{\gamma}) + \tan \phi E_y \\ 0 \\ -cB_y (1 - \tilde{\beta}^2) / \tilde{\beta} + i \tan \phi E_y / \tilde{\gamma} \end{pmatrix}
\]

\[
\vec{F} = q \begin{pmatrix} icB_y / (\beta \gamma) \\ 0 \\ -cB_y / (\beta \gamma^2) \end{pmatrix}
\]
Acceleration at dielectric gratings

Fields of a dielectric laser accelerator based on a one sided grating structure. Depicted are 3 moments in time, $t = 0$ (a, d), $t = \pi/2$ (b, e), $t = \pi$ (c, f).

Electrons injected in different phases experience different fields:
1: Acceleration
2: Deceleration
3: Deflection to structure
4: Deflection to vacuum

left: first spatial harmonic
right: third spatial harmonic
$\rightarrow 1/3$ decay length

$$\lambda_p = n \beta \lambda$$
Acceleration at dielectric gratings

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$$\lambda_p = n\beta\lambda$$
Implications of these forces and fields

• There is a transversal force component
  • At this geometry the transversal position of the electrons is non recoverable, due to the evanescent nature of the fields
• There is no light speed mode, a mode capable of accelerating $\beta = 1$ electrons, in the case of a single sided grating, since the solution would require a linearly increasing electric field extending to infinity
Solution: double sided grating

Adding a second grating, inverted, on the other side, creates a symmetric field with either a cosh or sinh mode. While deflecting forces are not mitigated, the symmetric field profile can be used to confine the electron beam. More later with Alternating Phase Focusing (APF)

Bonus: double sided structures support speed of light mode
A synchronous mode is used for acceleration while an asynchronous mode confines the bunches.
Simulations

We use different simulation tools to compute the characteristics of our accelerating devices:

- Finite Difference Time Domain (FDTD) code to calculate exact fields
- The resulting fields can be broken down into kicks per period to approximate the accelerator
- For special cases we use a PIC implementation to look at wakefields in dielectric accelerators

Electron tracking:

- Integrate computed kicks
- Runge-Kutta motion solver (with spacecharge)
- PIC code for self consistent solution