RF Systems I

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Definitions & basic concepts

dB

t-domain vs. ω-domain

phasors

Decibel (dB)

- Convenient logarithmic measure of a power ratio.
- A "Bel" ($= 10$ dB) is defined as a power ratio of 10^1. Consequently, 1 dB is a power ratio of $10^{0.1} \approx 1.259$.
- If rdb denotes the measure in dB, we have:

$$rdB = 10\ \mathrm{dB}\log\left(\frac{P_2}{P_1}\right) = 10\ \mathrm{dB}\log\left(\frac{A_2^2}{A_1^2}\right) = 20\ \mathrm{dB}\log\left(\frac{A_2}{A_1}\right)$$

- $\frac{P_2}{P_1} = \frac{A_2^2}{A_1^2} = 10^{rdb/(10\ \mathrm{dB})}$ $\qquad$ $\frac{A_2}{A_1} = 10^{rdb/(20\ \mathrm{dB})}$

rdB	-30 dB	-20 dB	-10 dB	-6 dB	-3 dB	0 dB	3 dB	6 dB	10 dB	20 dB	30 dB
$\frac{P_2}{P_1}$	0.001	0.01	0.1	0.25	.50	1	2	3.98	10	100	1000
$\frac{A_2}{A_1}$	0.0316	0.1	0.316	0.50	.71	1	1.41	2	3.16	10	31.6

- Related: dBm (relative to 1 mW), dBc (relative to carrier)

Time domain – frequency domain (1)

- An arbitrary signal $g(t)$ can be expressed in ω-domain using the ***Fourier transform*** (FT).

$$g(t) \gtrdot G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{j\omega t}\, dt$$

- The inverse transform (IFT) is also referred to as ***Fourier Integral.***

$$G(\omega) \lessdot g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{-j\omega t}\, d\omega$$

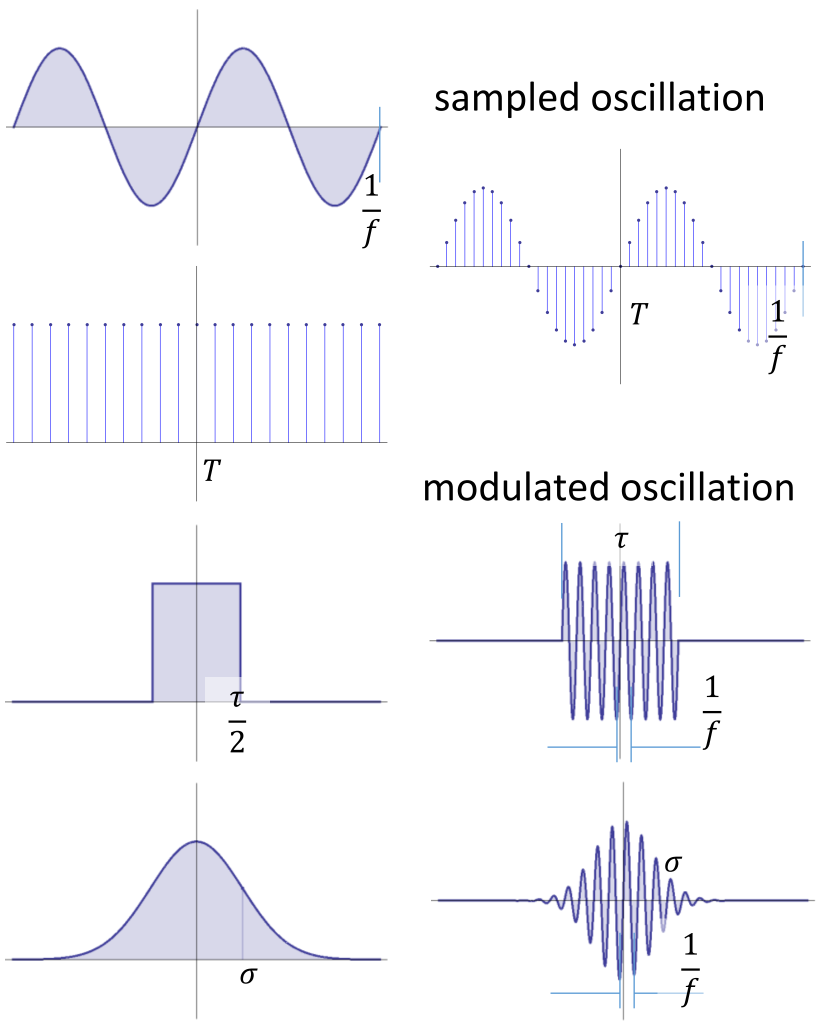
- The advantage of the ω-domain description is that linear time-invariant (LTI) systems are much easier described.
- The mathematics of the FT requires the extension of the definition of a *function* to allow for infinite values and non-converging integrals.
- The FT of the signal can be understood at looking *at "what frequency components it's composed of"*.

Time domain – frequency domain (2)

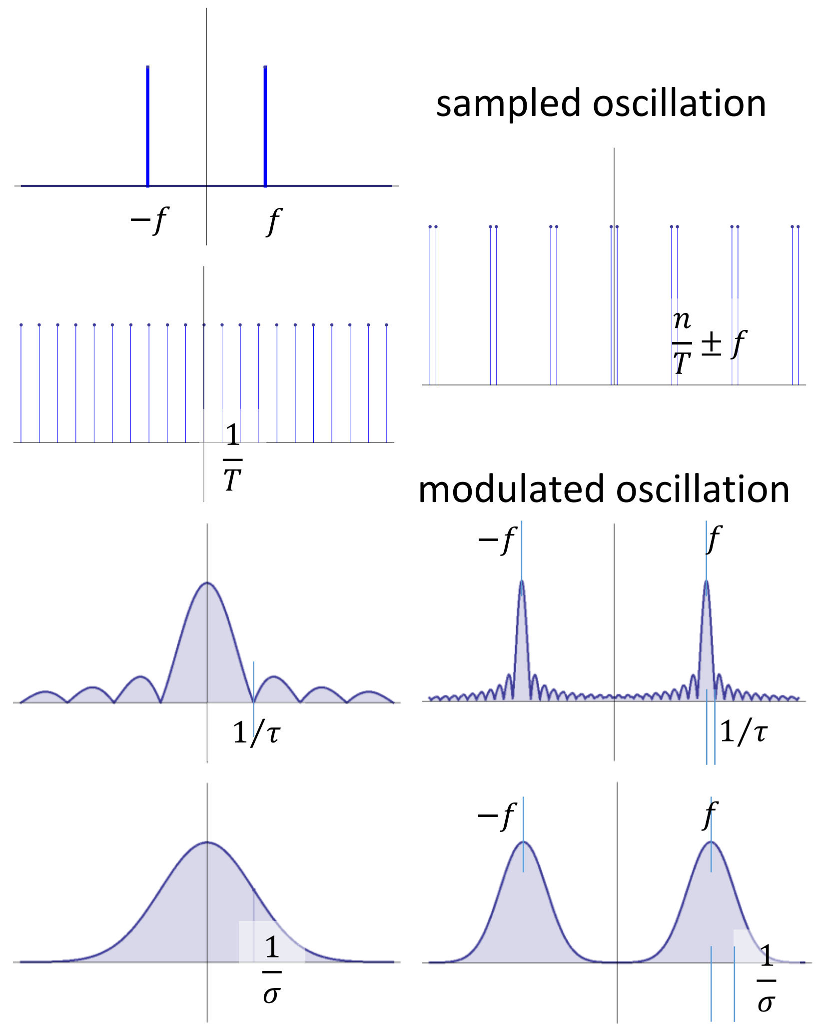
- For T-periodic signals, the FT becomes the Fourier-Series, $d\omega$ becomes $2\pi/T$, $\int$ becomes $\sum$.
- The cousin of the FT is the ***Laplace transform***, which uses a complex variable (often s) instead of $j\omega$; it has generally a better convergence behaviour.
- Numerical implementations of the FT require discretisation in t (sampling) and in ω. There exist very effective algorithms (FFT).
- In digital signal processing, one often uses the related *z*-Transform, which uses the variable $z = e^{j\omega\tau}$, where τ is the sampling period. A delay of $k\tau$ becomes z^{-k}.

Time domain – frequency domain (3)

- Time domain

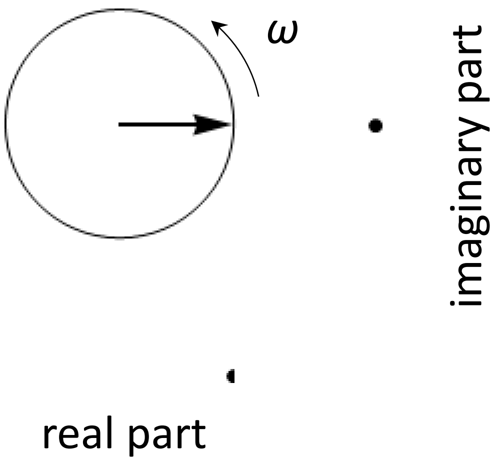


- Frequency domain



Fixed frequency oscillation (steady state, CW) Definition of phasors

- General: $A\cos(\omega t - \varphi) = A\cos\omega t\cos\varphi + A\sin\omega t\sin\varphi$
- This can be interpreted as the projection on the real axis of a rotation in the complex plane.



$$\Re\{A(\cos\varphi + j\sin\varphi)e^{j\omega t}\}$$

- The complex amplitude $\tilde{A}$ is called “phasor”;

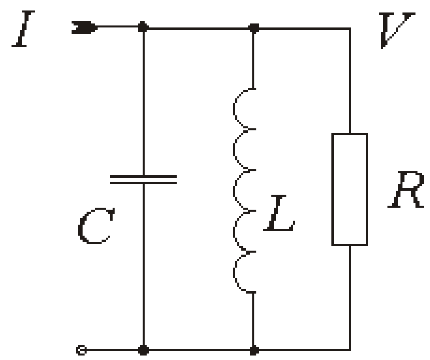
$$\tilde{A} = A(\cos\varphi + j\sin\varphi)$$

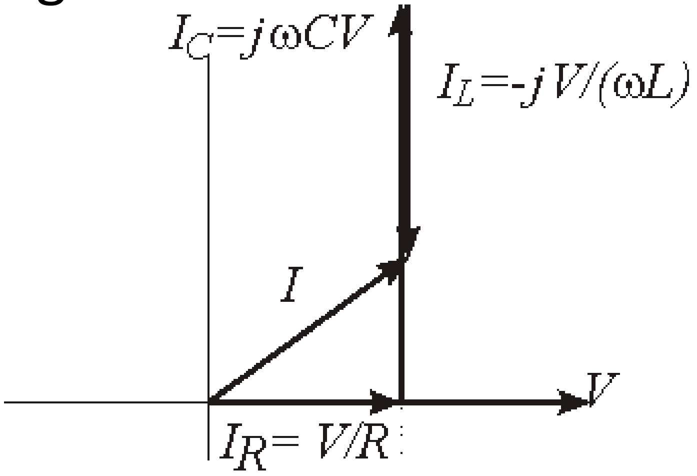
Calculus with phasors

- Why this seeming "complication"?:

  Because things become easier!

- Using $\frac{d}{dt} \equiv j\omega$, one may now forget about the rotation with ω and the projection on the real axis, and do the complete analysis making use of complex algebra!





$$I = V\left(\frac{1}{R} + j\omega C - \frac{j}{\omega L}\right)$$

Slowly varying amplitudes

- For band-limited signals, one may conveniently use “slowly varying” phasors and a fixed frequency RF oscillation.
- So-called in-phase (*I*) and quadrature (*Q*) “baseband envelopes” of a modulated RF carrier are the real and imaginary part of a slowly varying phasor.

On Modulation

AM

PM

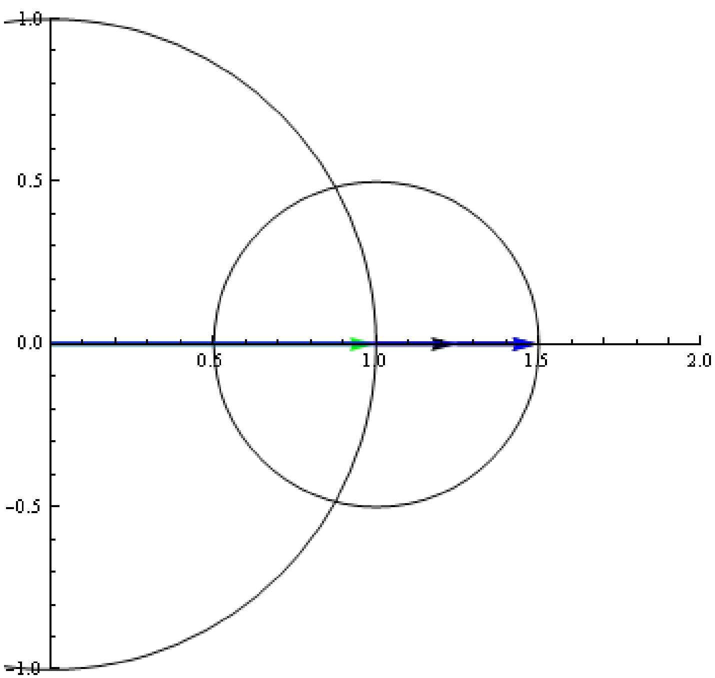
I-Q

Amplitude modulation

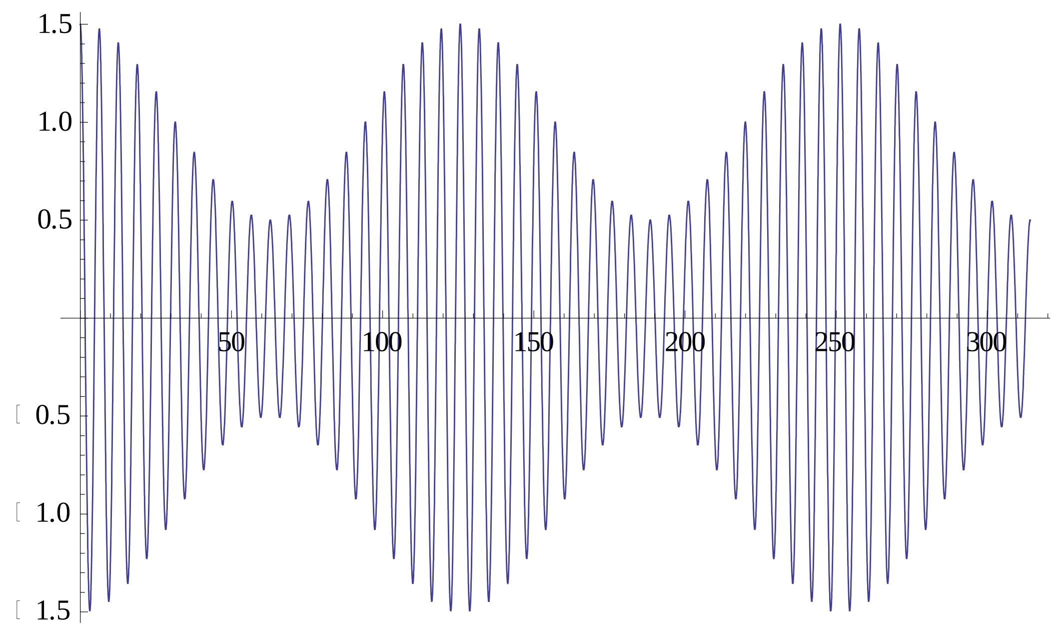
$$(1 + m\cos\varphi)\cdot\cos(\omega_c t) = \Re\left\{\left(1 + \frac{m}{2}e^{j\varphi} + \frac{m}{2}e^{-j\varphi}\right)e^{j\omega_c t}\right\}$$

m: modulation index or modulation depth

example: $\varphi = \omega_m t = 0.05\,\omega_c t$
$m = 0.5$



green: carrier
black: sidebands at $\pm f_m$
blue: sum

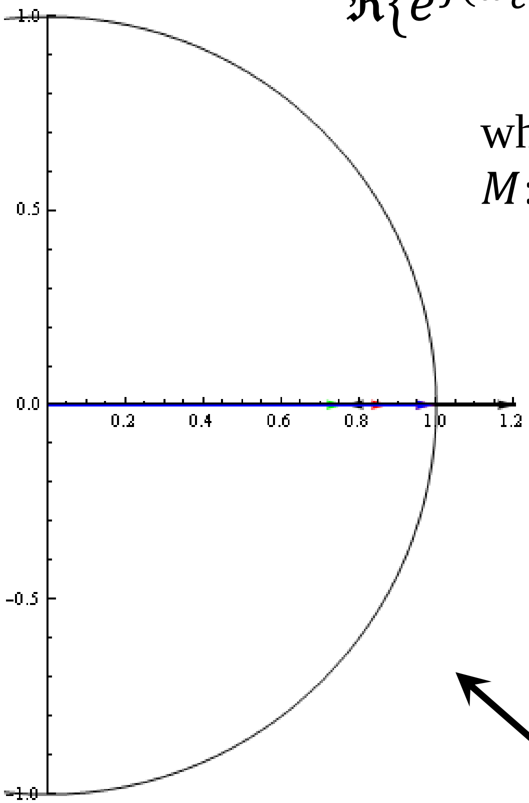


Phase modulation

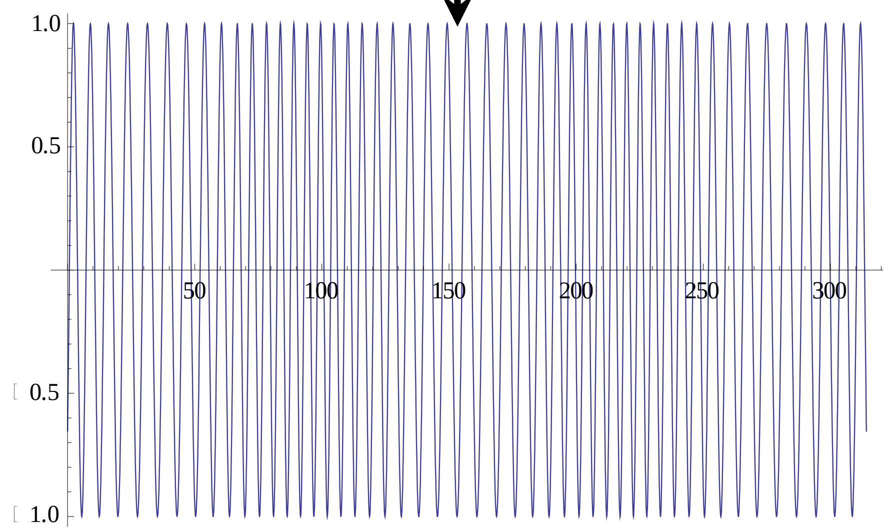
$$\Re\left\{e^{j(\omega_c t + M \sin\varphi)}\right\} = \Re\left\{\sum_{n=-\infty}^{\infty} J_n(M) e^{j(n\varphi + \omega_c t)}\right\}$$

where

M: modulation index (= max. phase deviation)



example: $\varphi = \omega_m t = 0.05\ \omega_c t$
$M = 4$



Green: $n = 0$ (carrier)

black: $n = 1$ sidebands

red: $n = 2$ sidebands

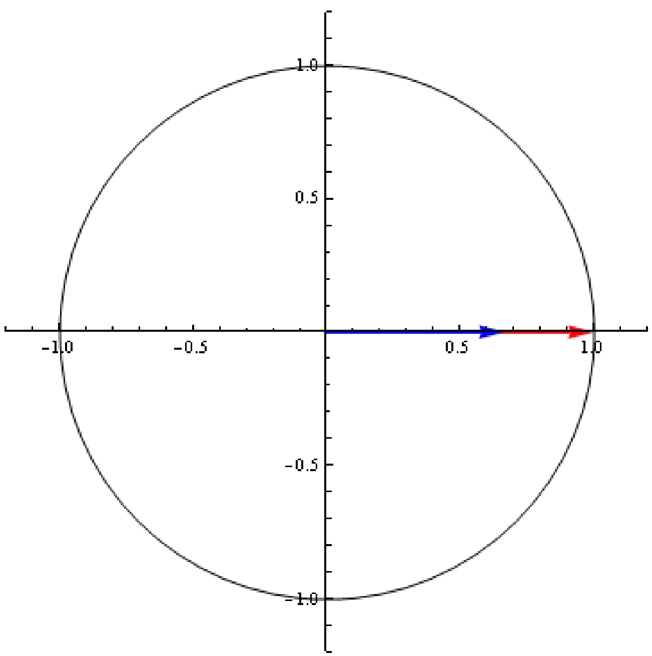
blue: sum

$M = 1$

Spectrum of phase modulation

Plotted: spectral lines for sinusoidal PM at f_m

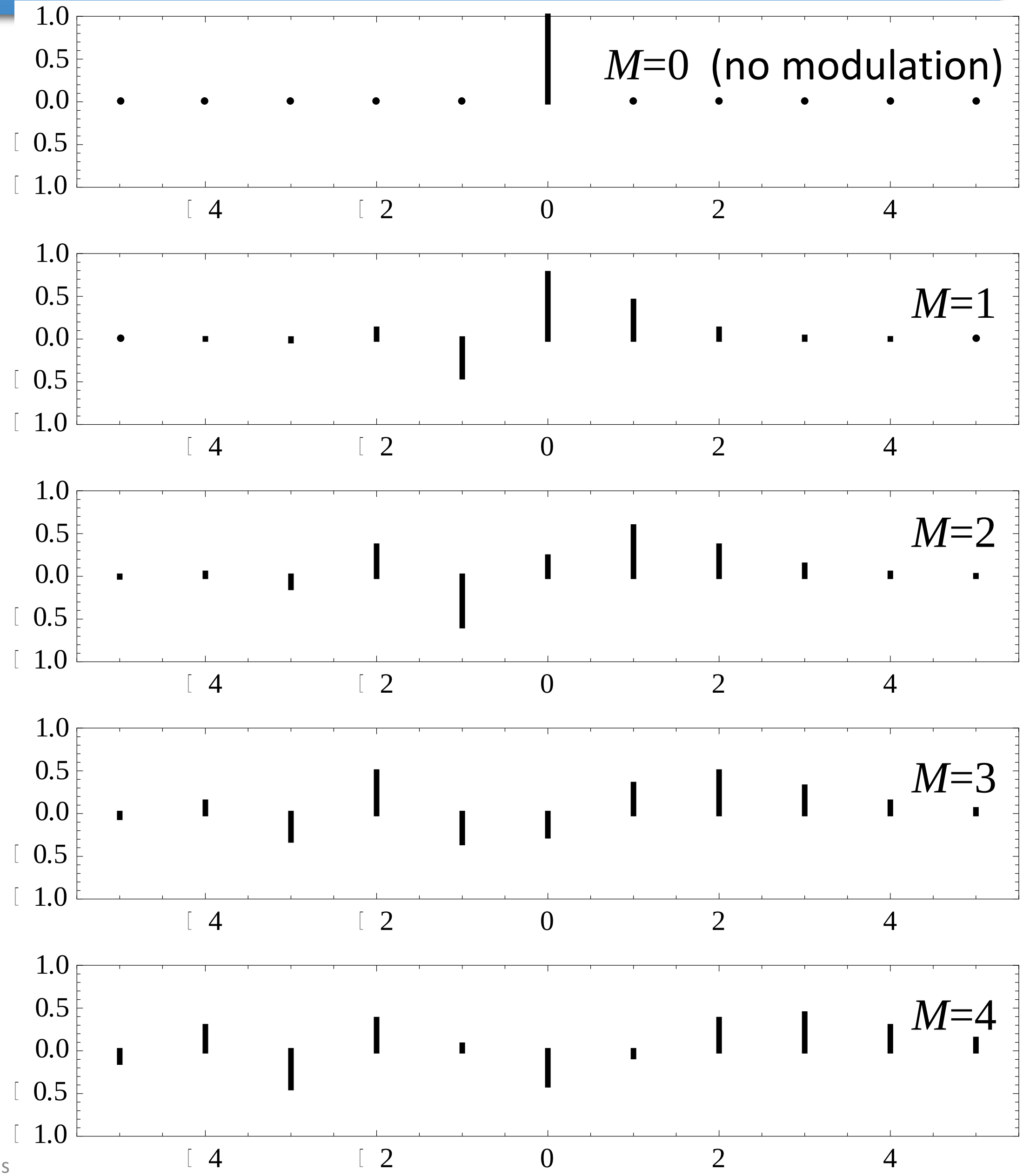
Abscissa: $(f - f_c)/f_m$



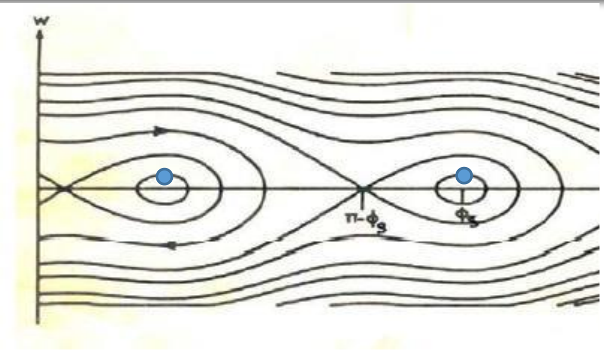
Phase modulation with $M = \pi$:

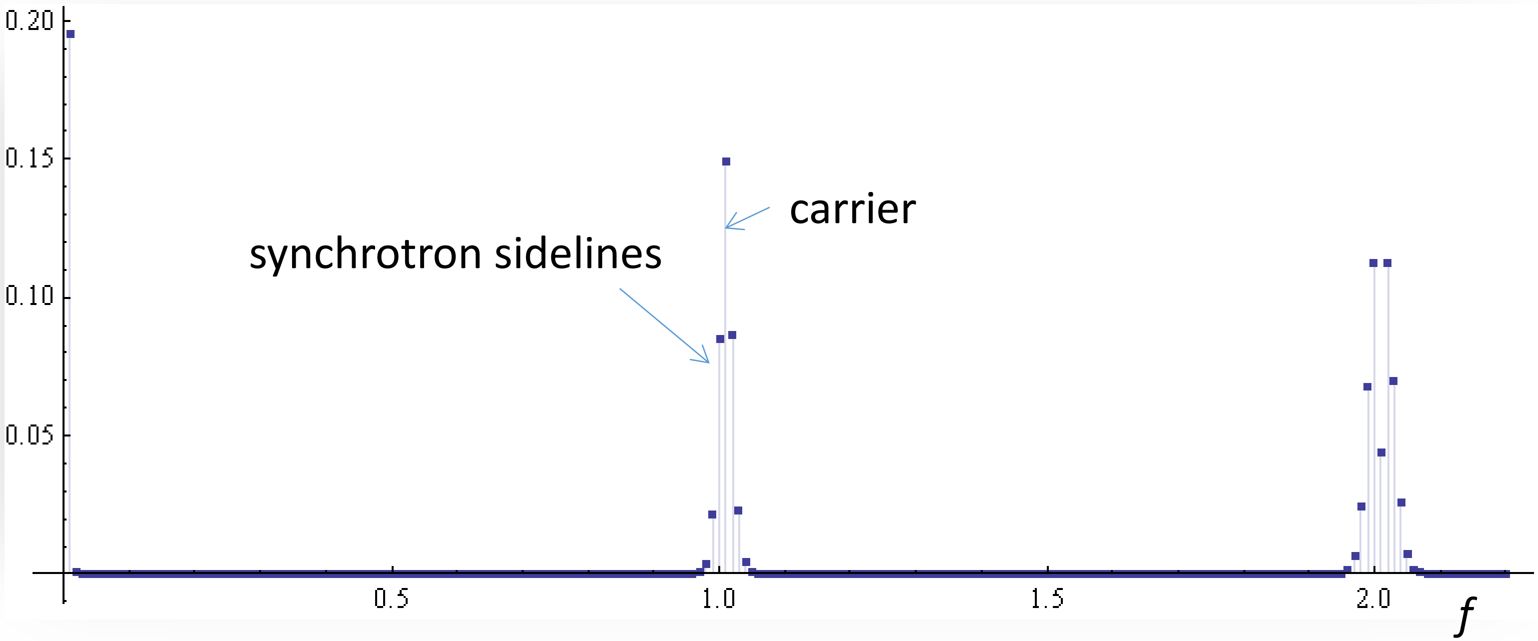
red: real phase modulation

blue: sum of sidebands $n \leq 3$

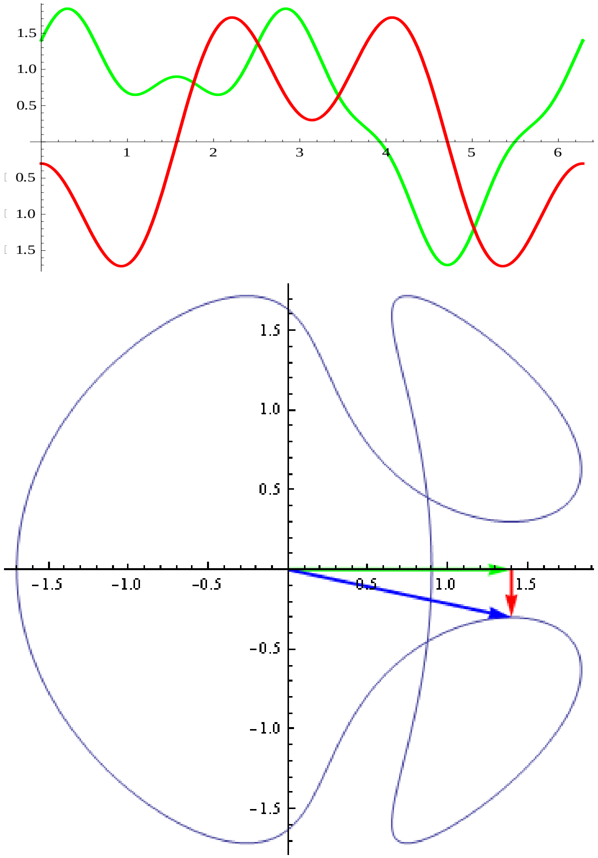


Spectrum of a beam with synchrotron oscillation, $M = 1$ (= 57 °)





Vector (I-Q) modulation



I-Q modulation:
green: *I* component
red: *Q* component
blue: vector-sum

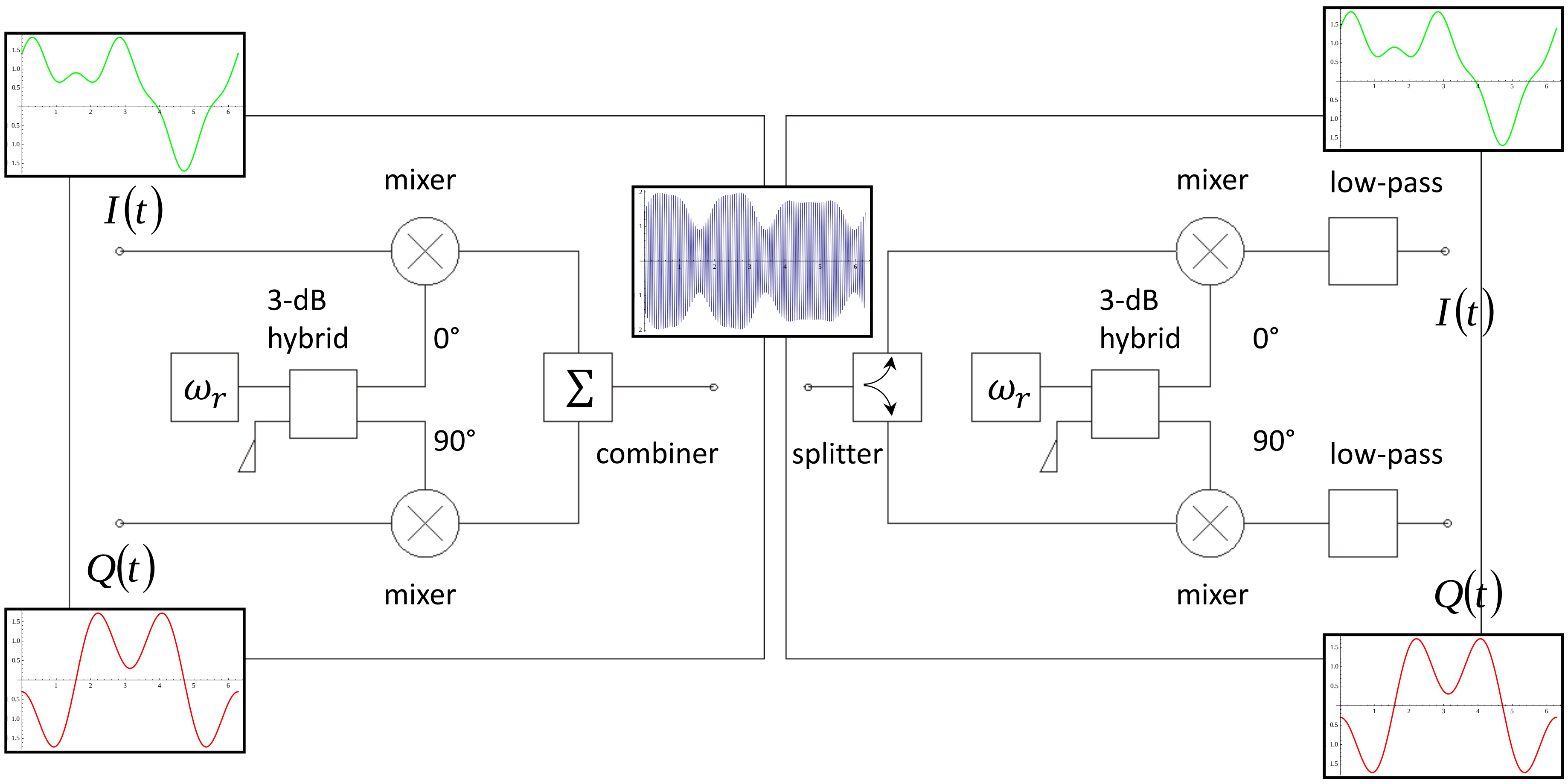
More generally, a modulation can have both amplitude and phase modulating components. They can be described as the in-phase (*I*) and quadrature (*Q*) components in a chosen reference, $\cos(\omega_r t)$. In complex notation, the modulated RF is:

$$\Re\{(I(t)+j\,Q(t))e^{j\,\omega_r t}\} =$$
$$\Re\{(I(t)+j\,Q(t))(\cos(\omega_r t)+j\sin(\omega_r t))\} =$$
$$I(t)\cos(\omega_r t) - Q(t)\sin(\omega_r t)$$

So *I* and *Q* are the Cartesian coordinates in the complex "Phasor" plane, where amplitude and phase are the corresponding polar coordinates.

$$I(t) = A(t)\cos(\varphi)$$
$$Q(t) = A(t)\sin(\varphi)$$

Vector modulator/demodulator

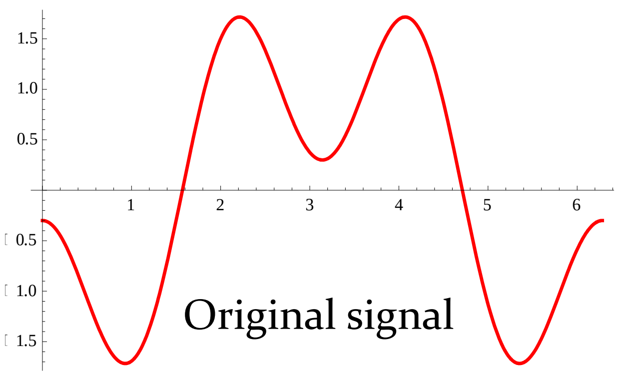


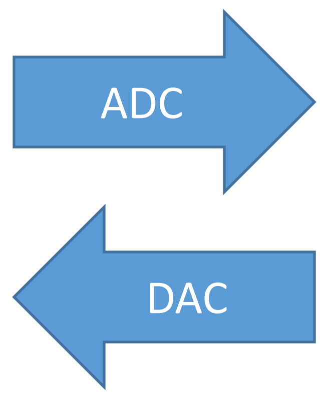
Digital Signal Processing

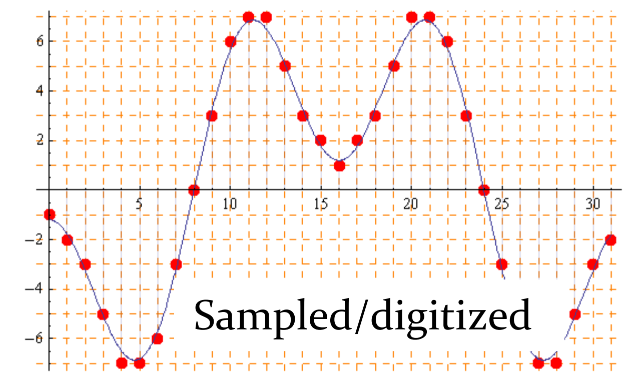
Just some basics

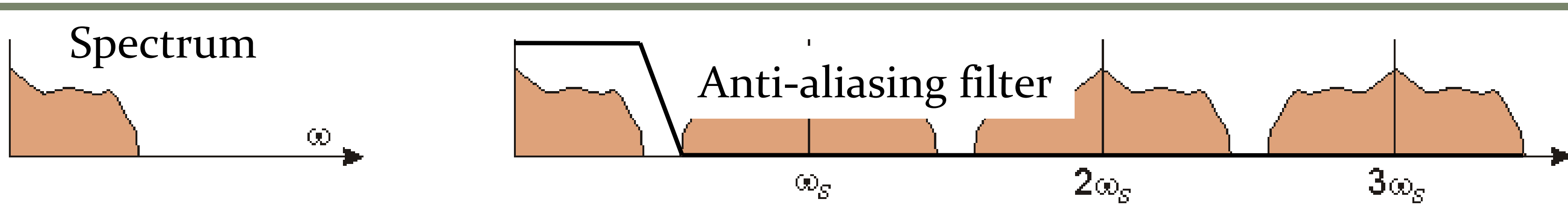
Sampling and quantization

- Digital Signal Processing is very powerful – note recent progress in digital audio, video and communication!
- Concepts and modules developed for a huge market; highly sophisticated modules available "off the shelf".
- The "slowly varying" phasors are ideal to be sampled and quantized as needed for digital signal processing.
- Sampling (at $1/\tau_s$) and quantization (n bit data words – here 4 bit):





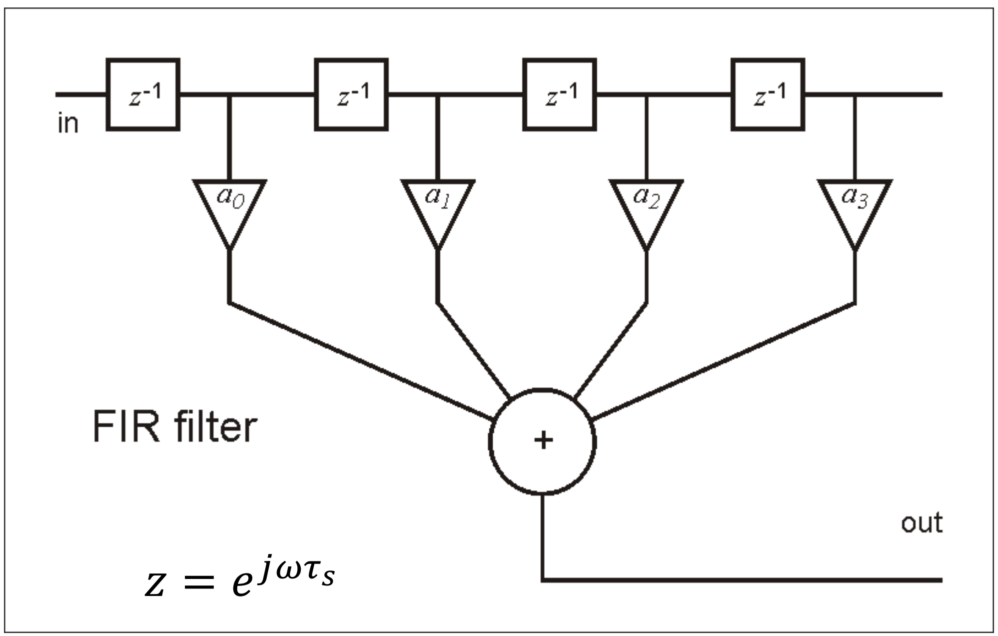


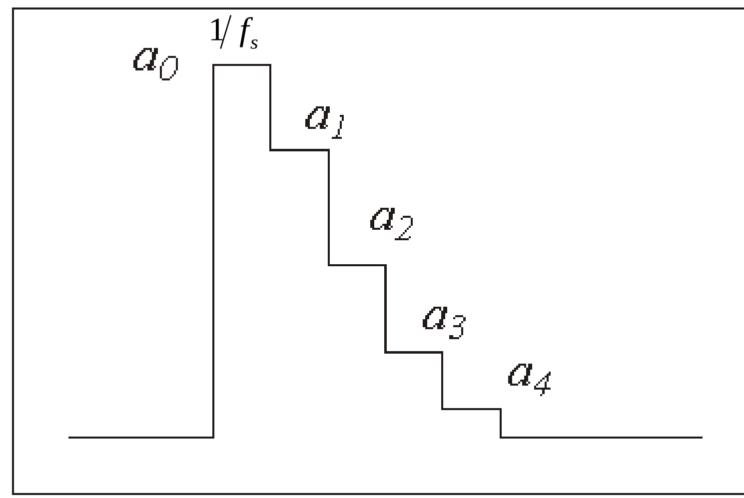


The "baseband" is limited to half the sampling rate!

Digital filters (1)

- Once in the digital realm, signal processing becomes "computing"!
- In a "finite impulse response" (FIR) filter, you directly program the coefficients of the impulse response.



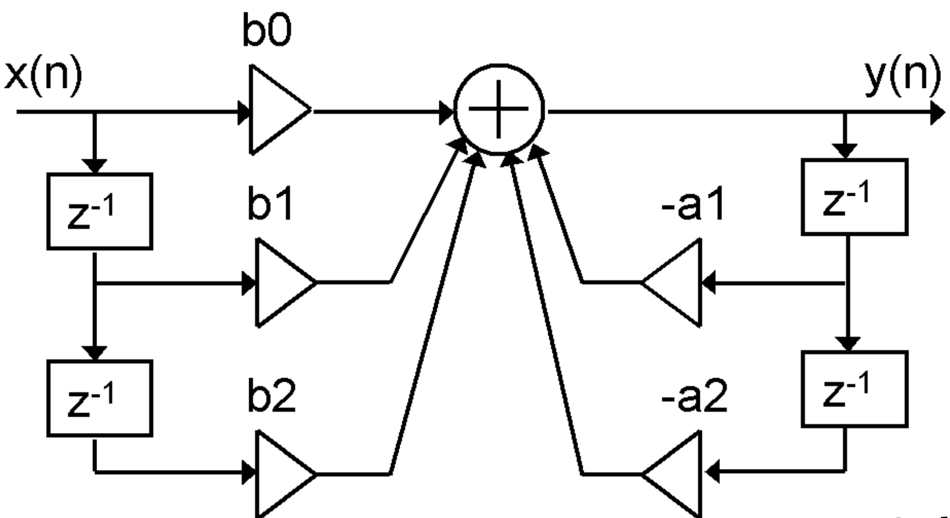


Transfer function:

$$a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}$$

Digital filters (2)

- An "infinite impulse response" (IIR) filter has built-in recursion, e.g. like



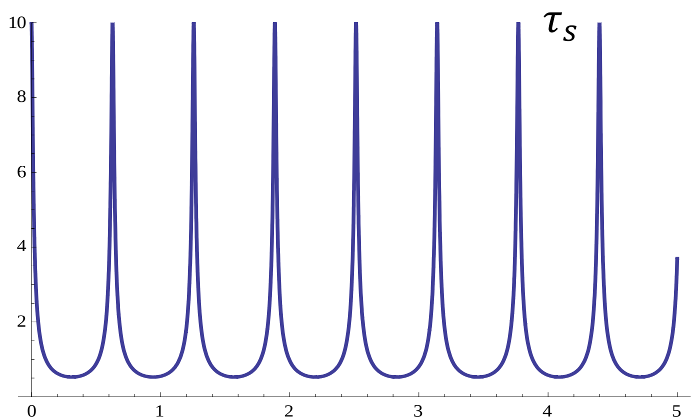
Transfer function:

$$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Example:

$$\frac{b_0}{1 + b_k z^{-k}}$$

$$\frac{2\pi k}{\tau_s}$$



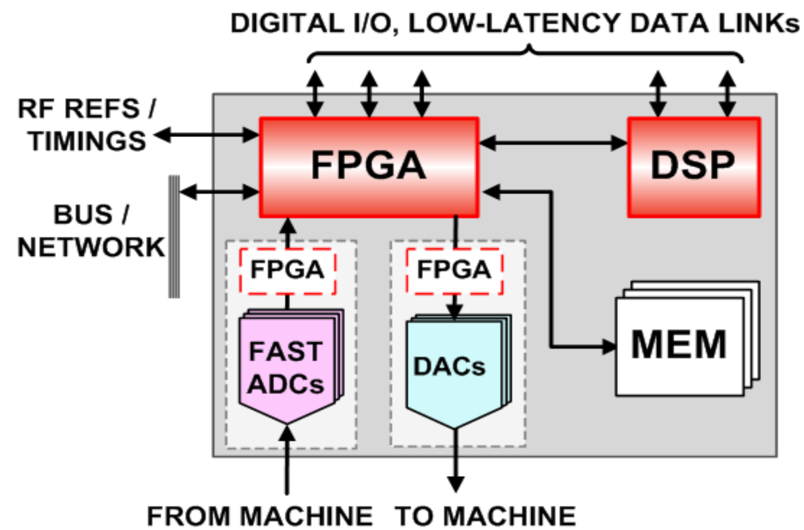
… is a comb filter.

Digital LLRF building blocks – examples

- General D-LLRF board:
  - modular!

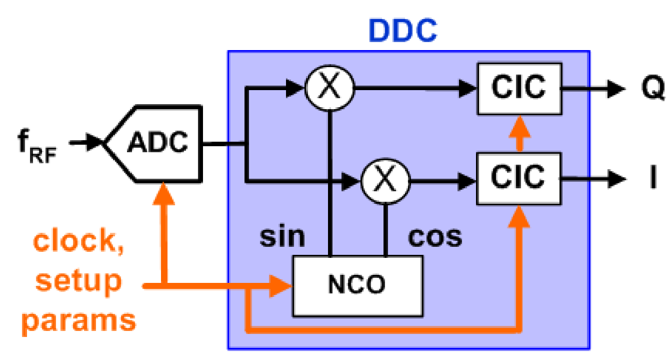
  FPGA: Field-programmable gate array

  DSP: Digital Signal Processor



- DDC (Digital Down Converter)
  - Digital version of the *I-Q* demodulator

  CIC: cascaded integrator-comb (a special low-pass filter)



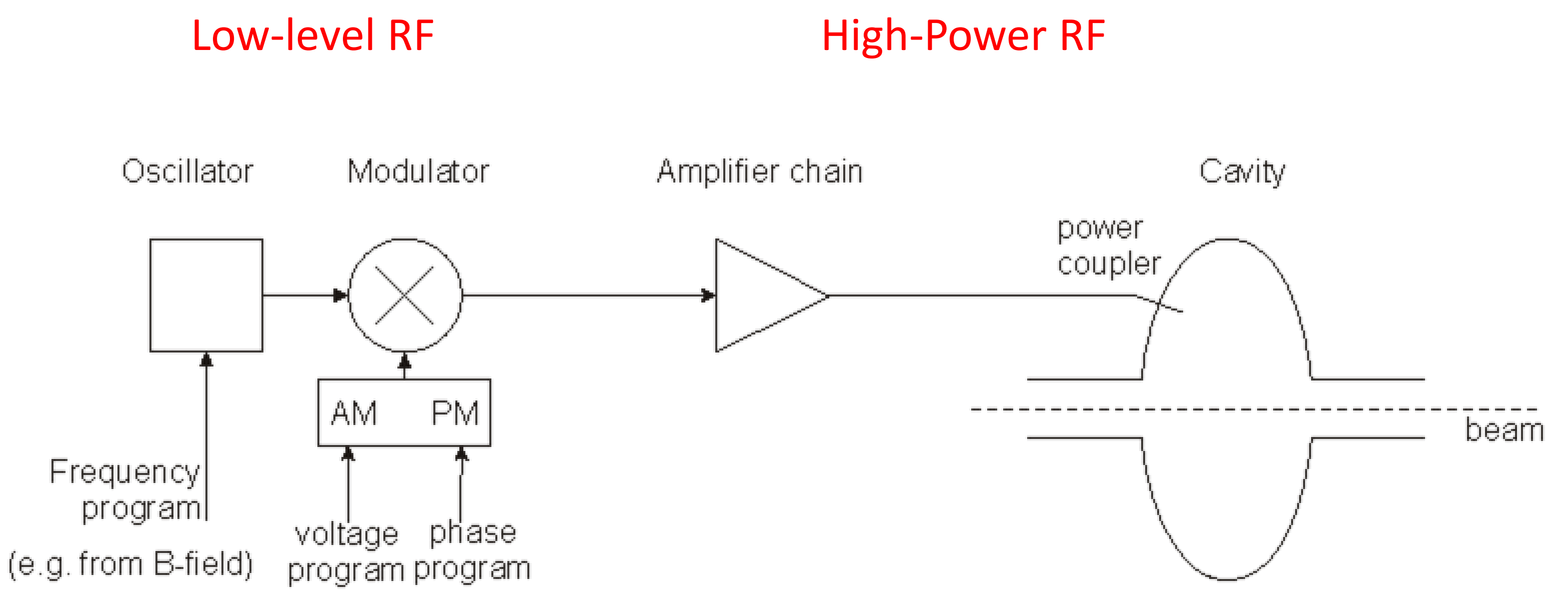
RF system & control loops

e.g.: … for a synchrotron:

Cavity control loops

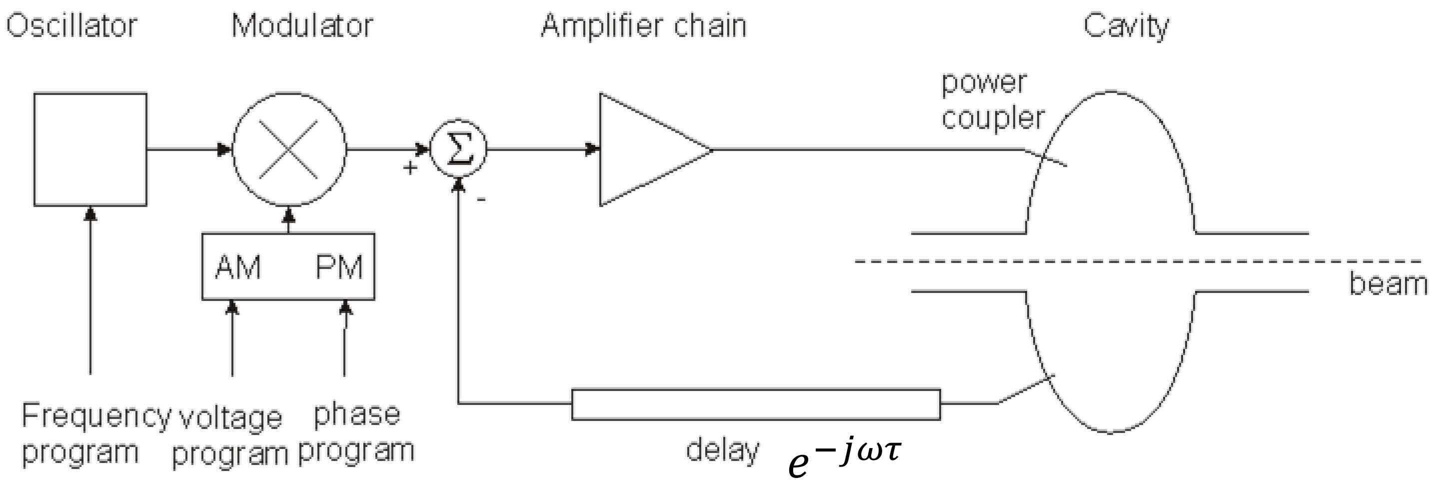
Beam control loops

Minimal RF system (of a synchrotron)



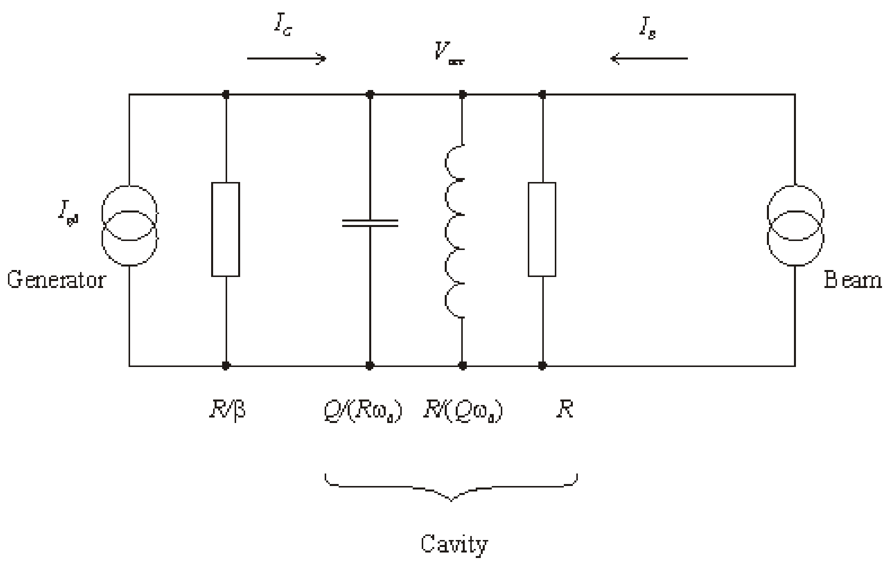
- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

Fast RF Feed-back loop



- Compares actual RF voltage and phase with desired and corrects.
- Rapidity limited by total group delay (path lengths) (some 100 ns).
- Unstable if loop gain $= 1$ with total phase shift $180\,°$ – design requires to stay away from this point (stability margin)
- The group delay limits the gain·bandwidth product.
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.

Fast feedback loop at work



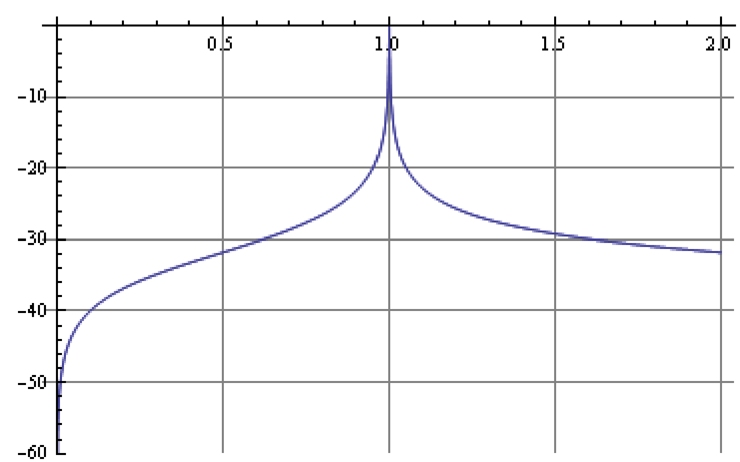
Without feedback, $V_{acc} = (I_{G0} + I_B) \cdot Z(\omega)$, where

$$Z(\omega) = \frac{R/(1+\beta)}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Detect the gap voltage, feed it back to I_{G0} such that $I_{G0} = I_{drive} - G \cdot V_{acc}$, where G is the total loop gain (pick-up, cable, amplifier chain …)

Result:

$$V_{acc} = (I_{drive} + I_B) \cdot \frac{Z(\omega)}{1 + G \cdot Z(\omega)}$$

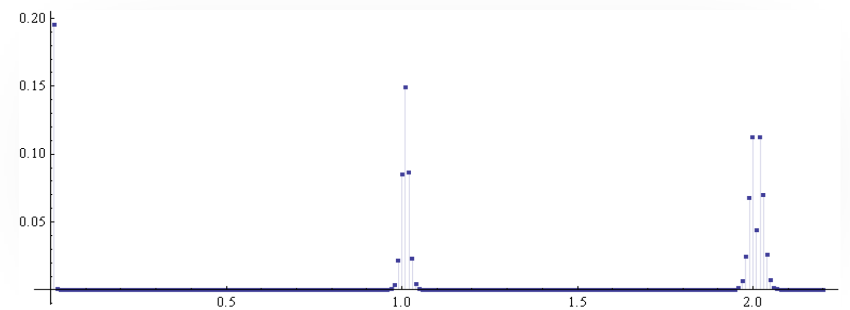


- Gap voltage is stabilised!
- Impedance seen by the beam is reduced by the loop gain!

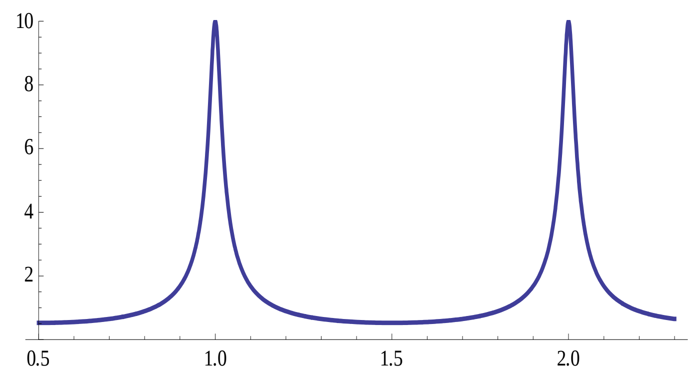
  Plot on the right: $\frac{1+\beta}{R}\left|\frac{Z(\omega)}{1+G\cdot Z(\omega)}\right|$ vs. ω, with the loop gain varying from 0 dB to 50 dB.

1-turn delay feed-back loop

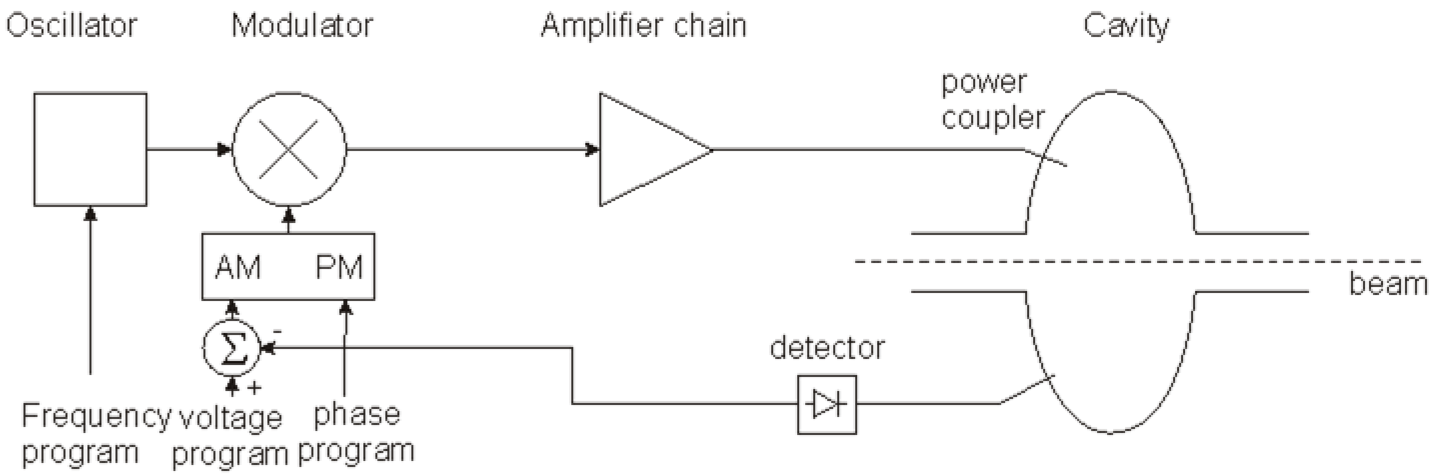
- The speed of the “fast RF feedback” is limited by the group delay – this is typically a significant fraction of the revolution period.
- How to lower the impedance over many harmonics of the revolution frequency?
- Remember: the beam spectrum is limited to relatively narrow bands around the multiples of the revolution frequency!



- Only in these narrow bands the loop gain must be high!
- Install a comb filter! … and extend the group delay to exactly 1 turn – in this case the loop will have the desired effect and remain stable!

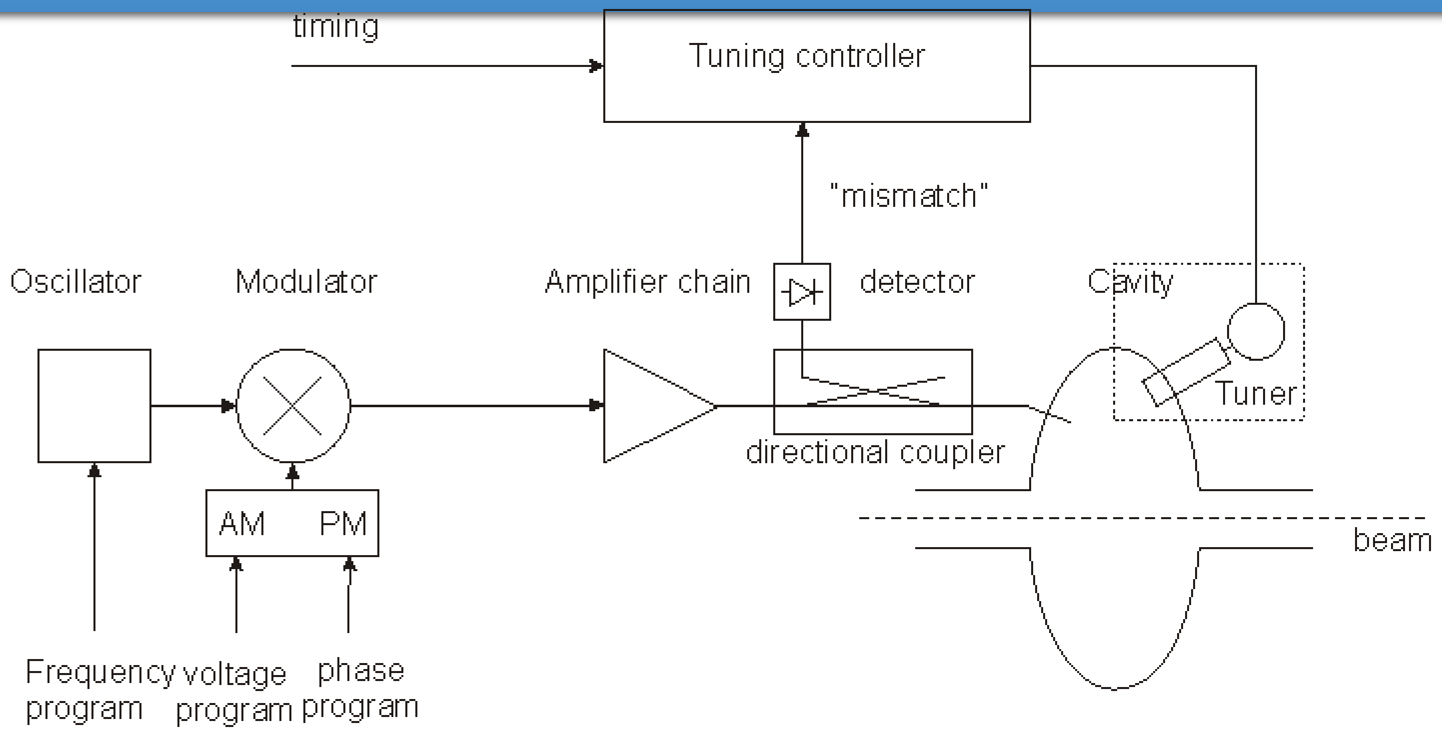


Field amplitude control loop (AVC)



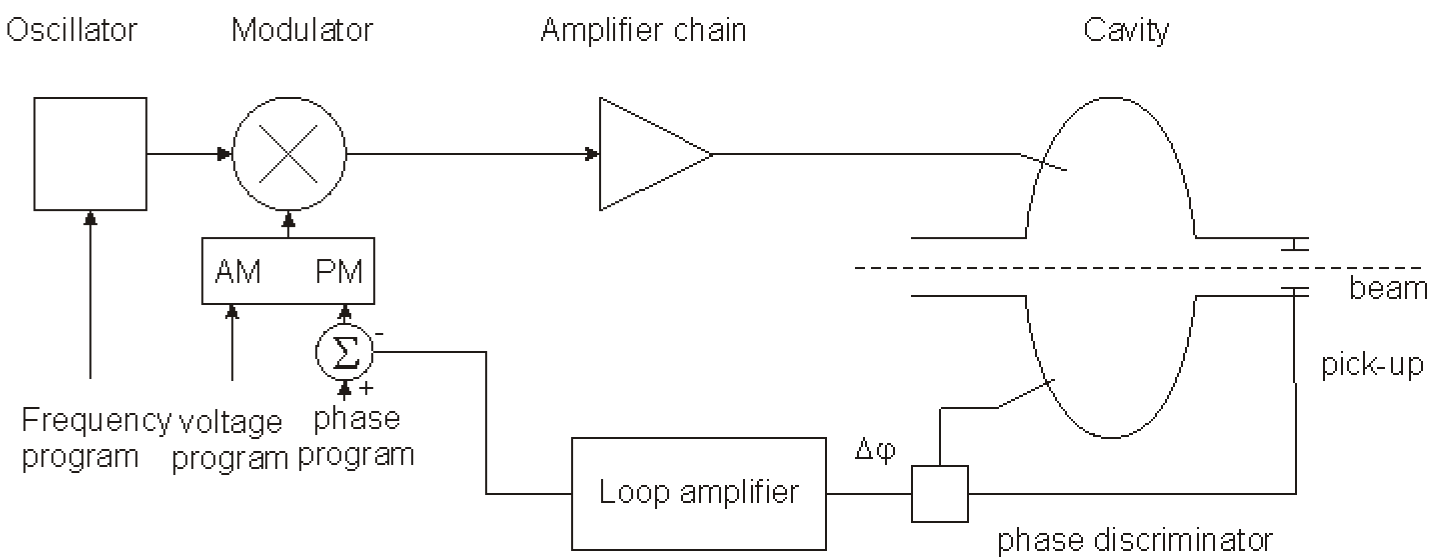
- Compares the detected cavity voltage to the voltage program. The error signal serves to correct the amplitude

Tuning loop



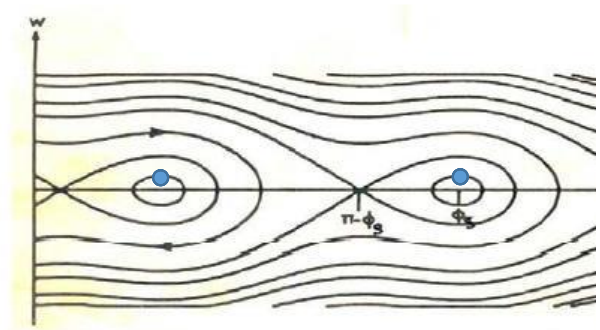
- Tunes the resonance frequency of the cavity f_r to minimize the mismatch of the PA.
- In the presence of beam loading, the optimum f_r may be $f_r \neq f$.
- In an ion ring accelerator, the tuning range might be > octave!
- For fixed f systems, tuners are needed to compensate for slow drifts.
- Examples for tuners:
  - controlled power supply driving ferrite bias (varying µ),
  - stepping motor driven plunger,
  - motorized variable capacitor, …

Beam phase loop



- Longitudinal motion: $\frac{d^2(\Delta\phi)}{dt^2} + \Omega_s^2(\Delta\phi)^2 = 0.$
- Loop amplifier transfer function designed to damp synchrotron oscillation.

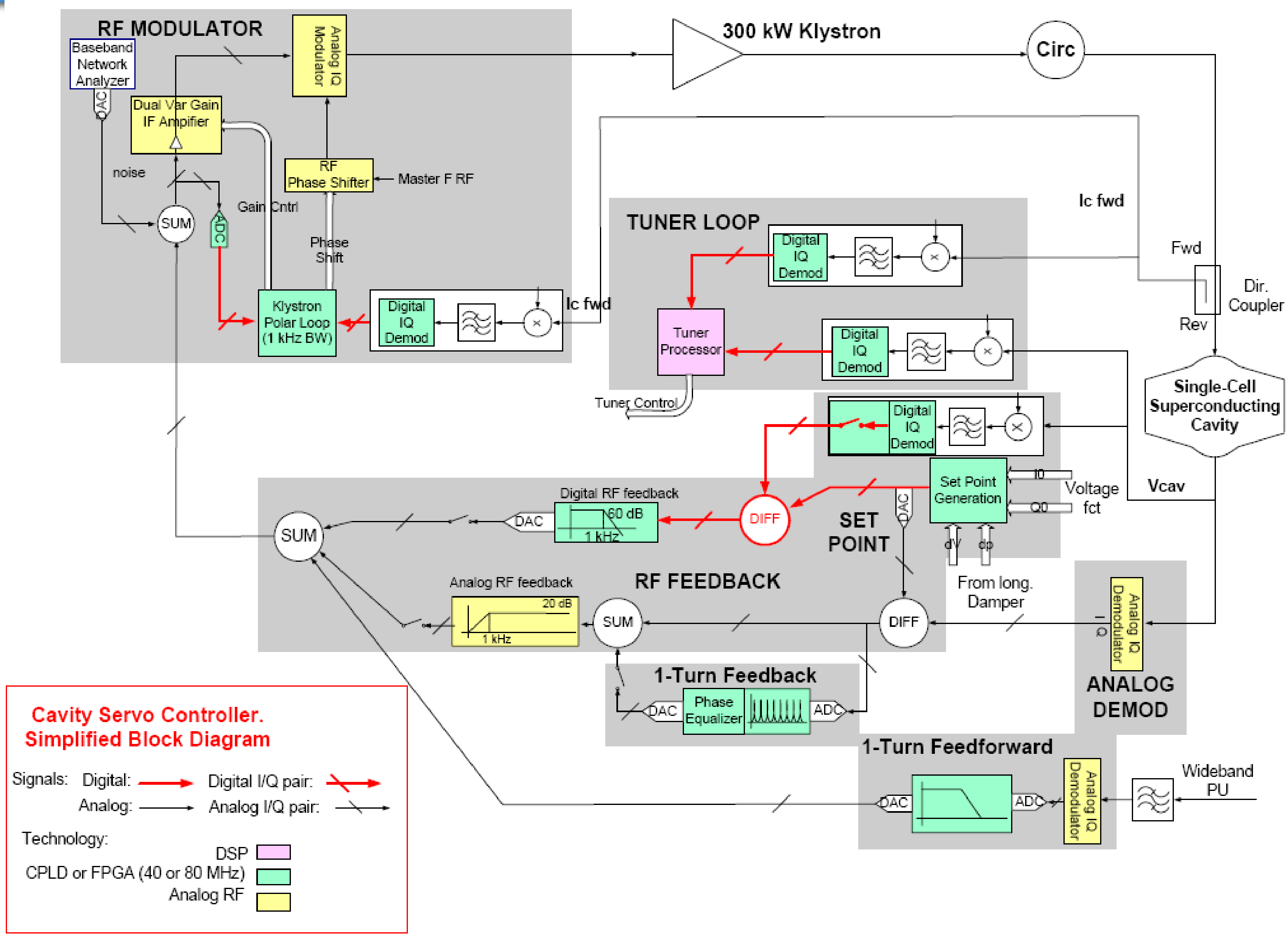
  Modified equation: $\frac{d^2(\Delta\phi)}{dt^2} + \alpha\frac{d(\Delta\phi)}{dt} + \Omega_s^2(\Delta\phi)^2 = 0$



Other loops

- Radial loop:
  - Detect average radial position of the beam,
  - Compare to a programmed radial position,
  - Error signal controls the frequency.
- Synchronisation loop (e.g. before ejection):
  - 1st step: Synchronize f to an external frequency (will also act on radial position!).
  - 2nd step: phase loop brings bunches to correct position.
- …

A real implementation: LHC LLRF

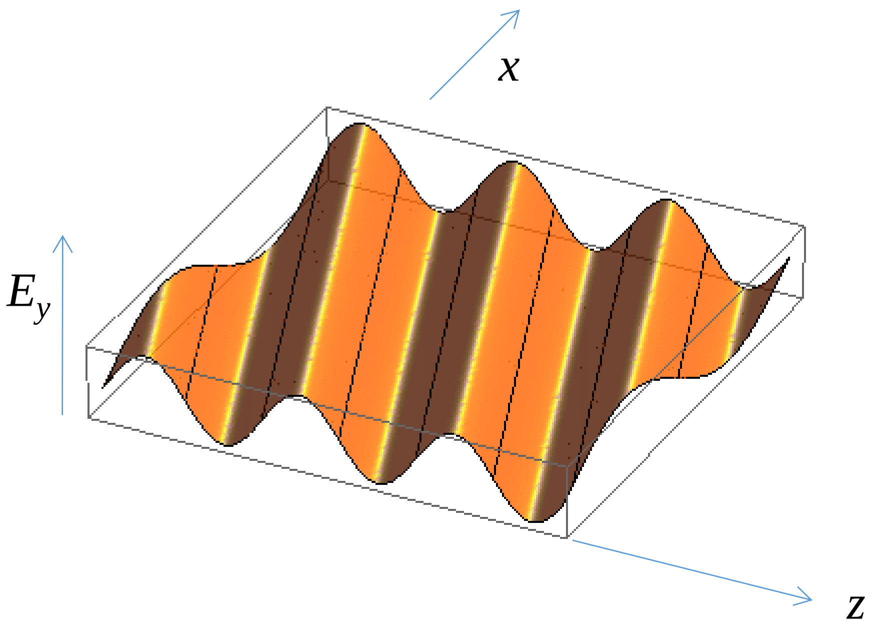


Fields in a waveguide

Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$

$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}(z\cos\varphi + x\sin\varphi)$$



Wave vector $\vec{k}$:

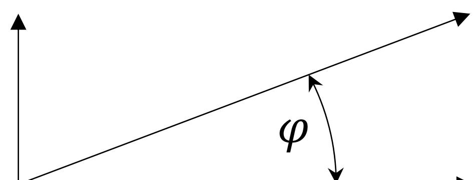
the direction of $\vec{k}$ is the direction of propagation,

the length of $\vec{k}$ is the phase shift per unit length.

$\vec{k}$ behaves like a vector.

$$k_\perp = \frac{\omega_c}{c}$$

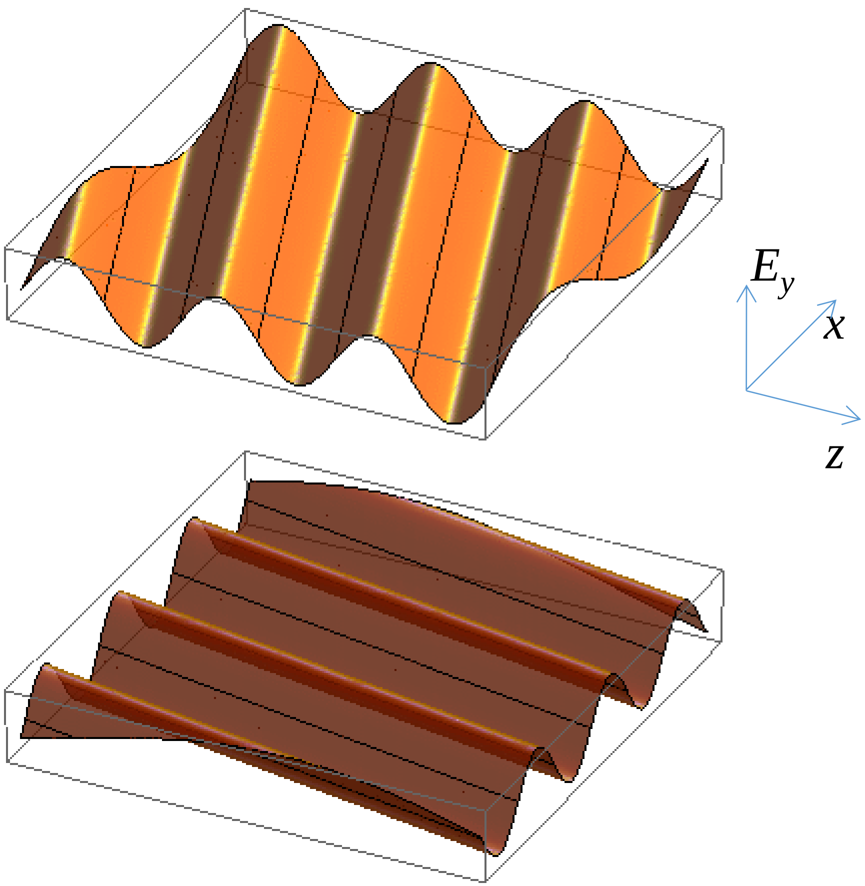
$$k = \frac{\omega}{c}$$

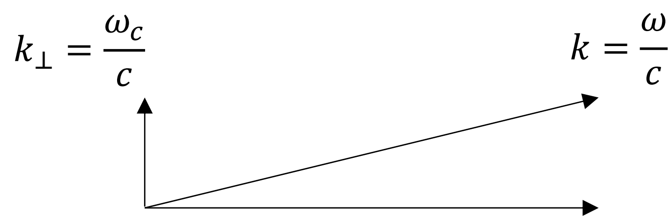


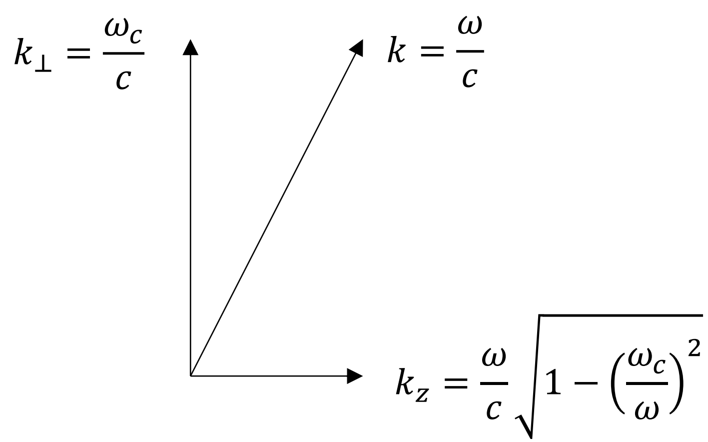
$$k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

Wave length, phase velocity

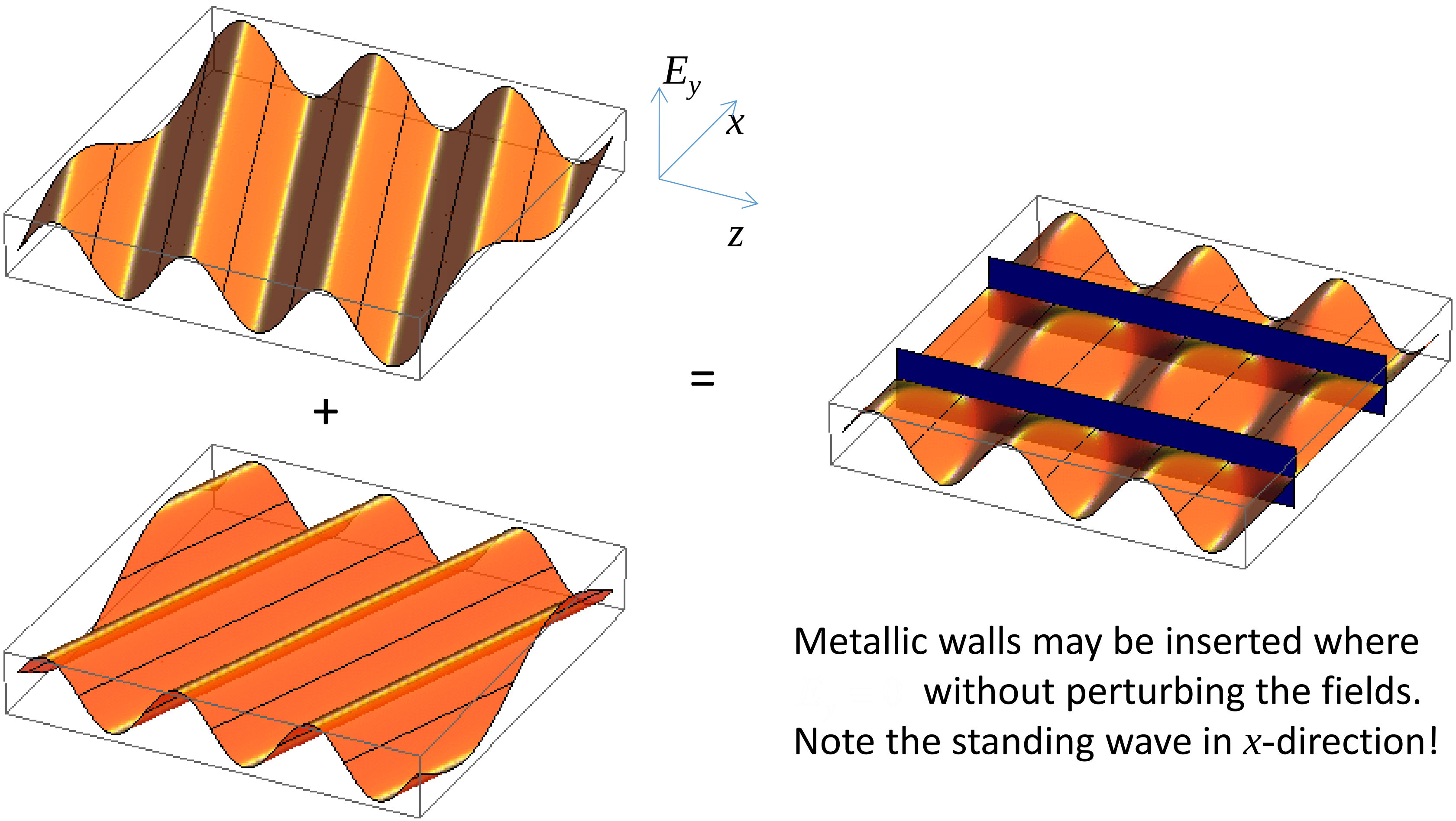
- The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\lambda_z$.







Superposition of 2 homogeneous plane waves



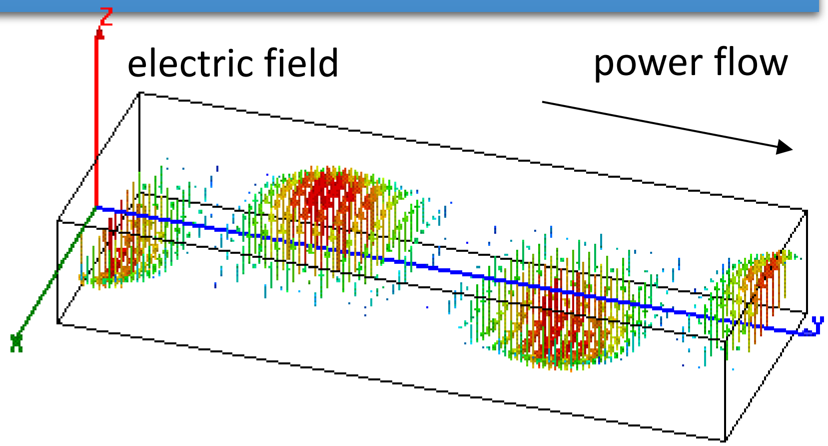
Metallic walls may be inserted where
$E_y = 0$ without perturbing the fields.
Note the standing wave in x-direction!

This way one gets a hollow rectangular waveguide!

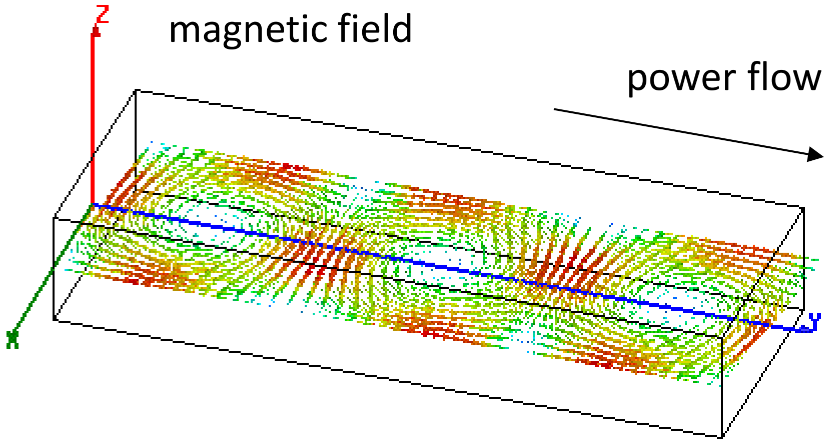
Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode in a standard rectangular waveguide.

E.g. forward wave



power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint \vec{E} \times \vec{H}^{*} dA\right\}$



Waveguide dispersion

What happens with different waveguide dimensions (different width a)?

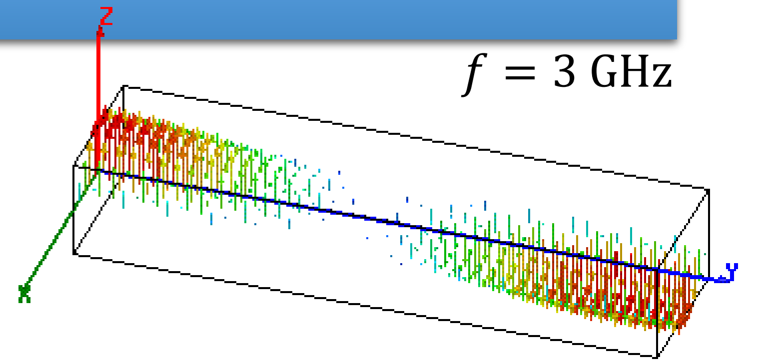
The "guided wavelength" λ_g varies from ∞ at f_c to λ at very high frequencies.

$f = 3$ GHz

1:

$a = 52$ mm

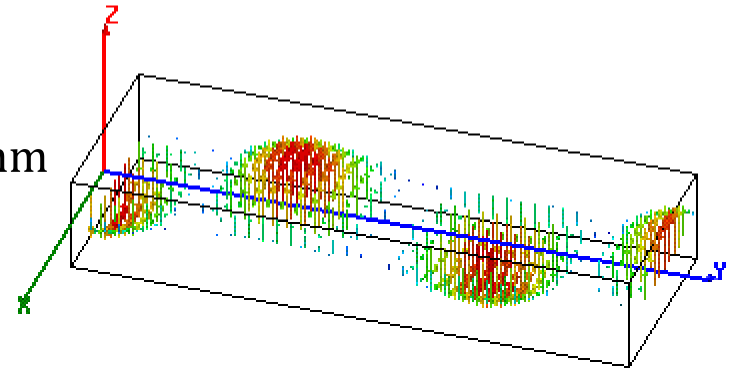
$\frac{f}{f_c} = 1.04$



2:

$a = 72.14$ mm

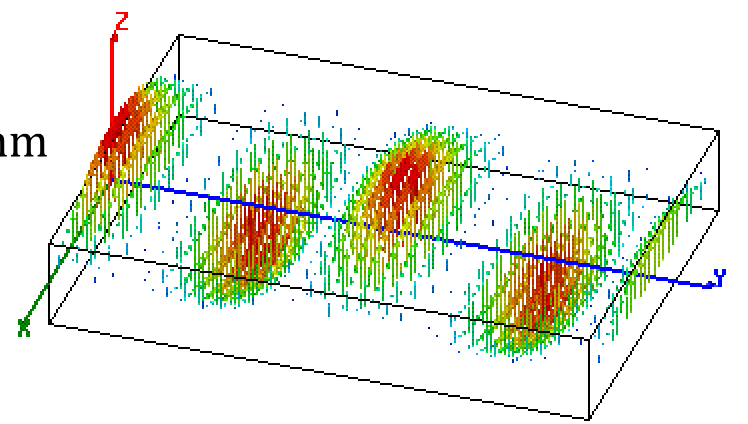
$\frac{f}{f_c} = 1.44$

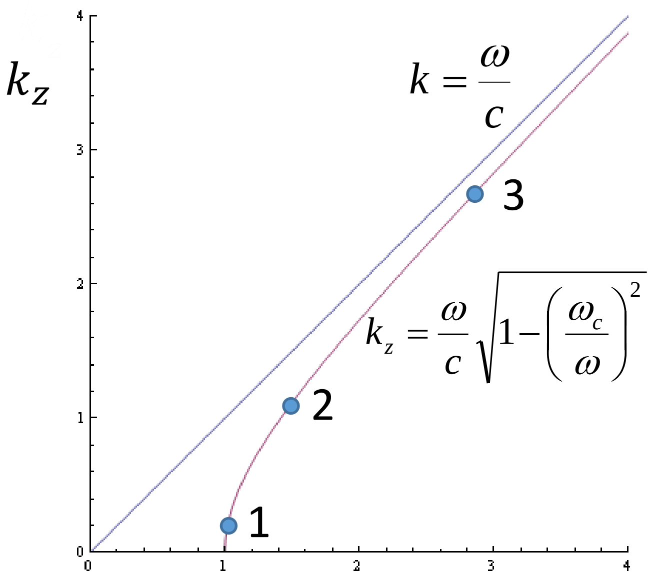


3:

$a = 144.3$ mm

$\frac{f}{f_c} = 2.88$



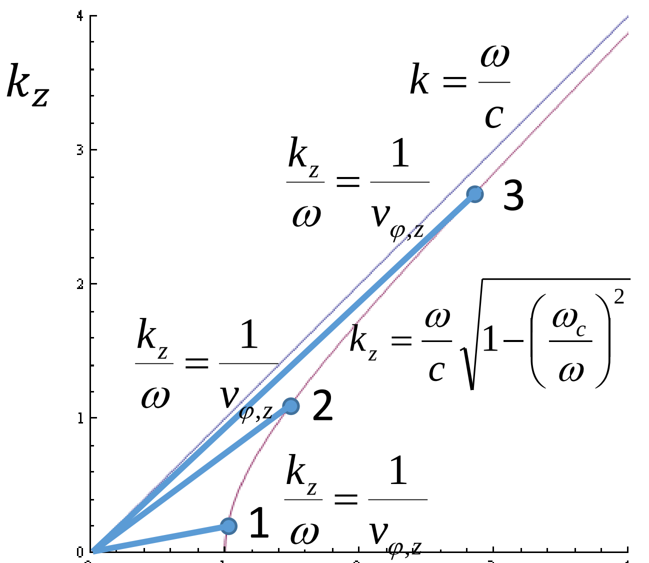


cutoff: $f_c = \frac{c}{2\,a}$

f/f_c

Phase velocity $v_{\varphi,z}$

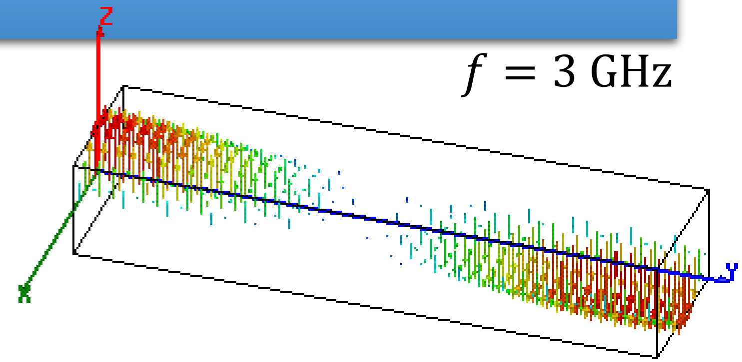
The phase velocity is the speed with which the crest or a zero-crossing travels in z-direction.
Note in the animations on the right that, at constant f, it is $v_{\varphi,z} \propto \lambda_g$.
Note that at $f = f_c$, $v_{\varphi,z} = \infty$!
With $f \to \infty$, $v_{\varphi,z} \to c$!



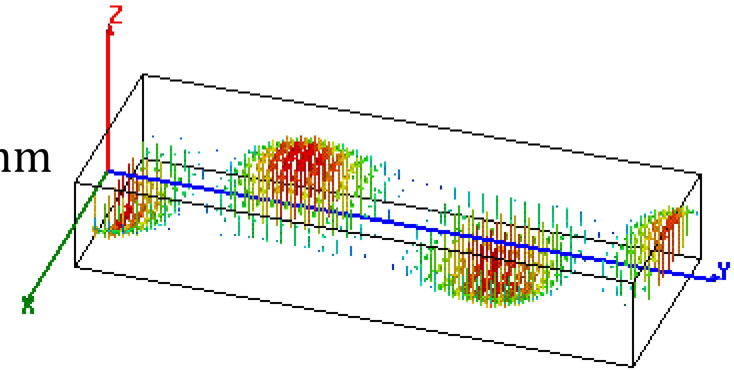
cutoff: $f_c = \frac{c}{2\,a}$ f/f_c

$f = 3$ GHz

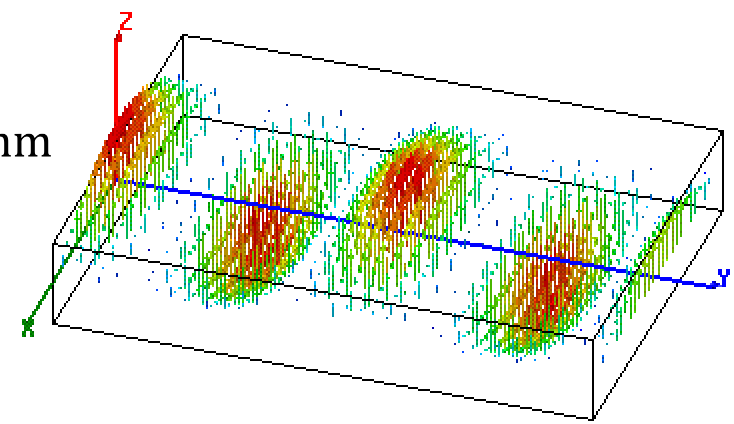
1:
$a = 52$ mm
$\frac{f}{f_c} = 1.04$



2:
$a = 72.14$ mm
$\frac{f}{f_c} = 1.44$

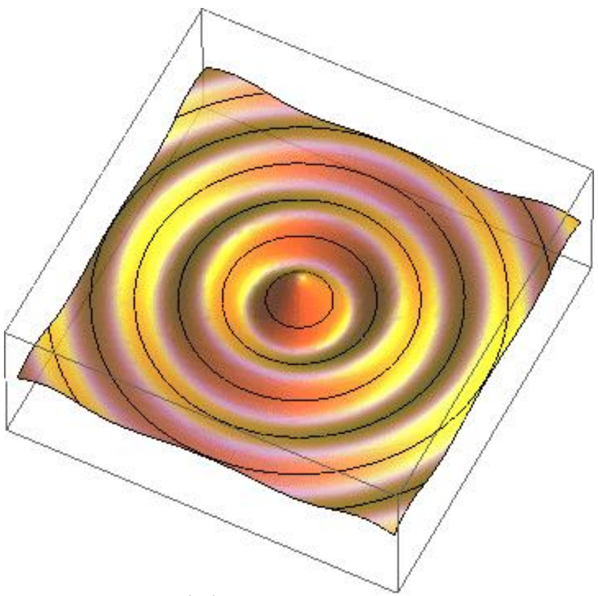


3:
$a = 144.3$ mm
$\frac{f}{f_c} = 2.88$

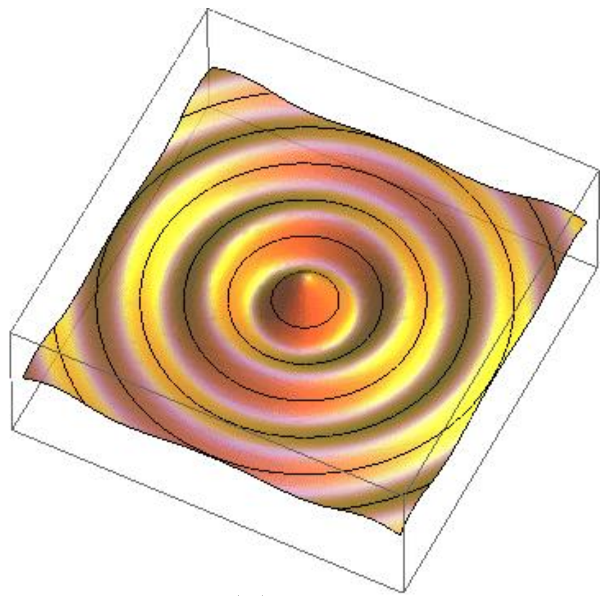


Radial waves

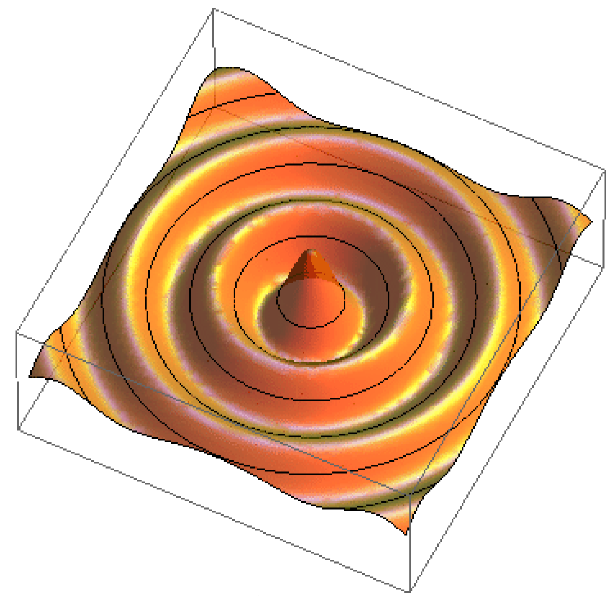
- Also radial waves may be interpreted as superpositions of plane waves.
- The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$



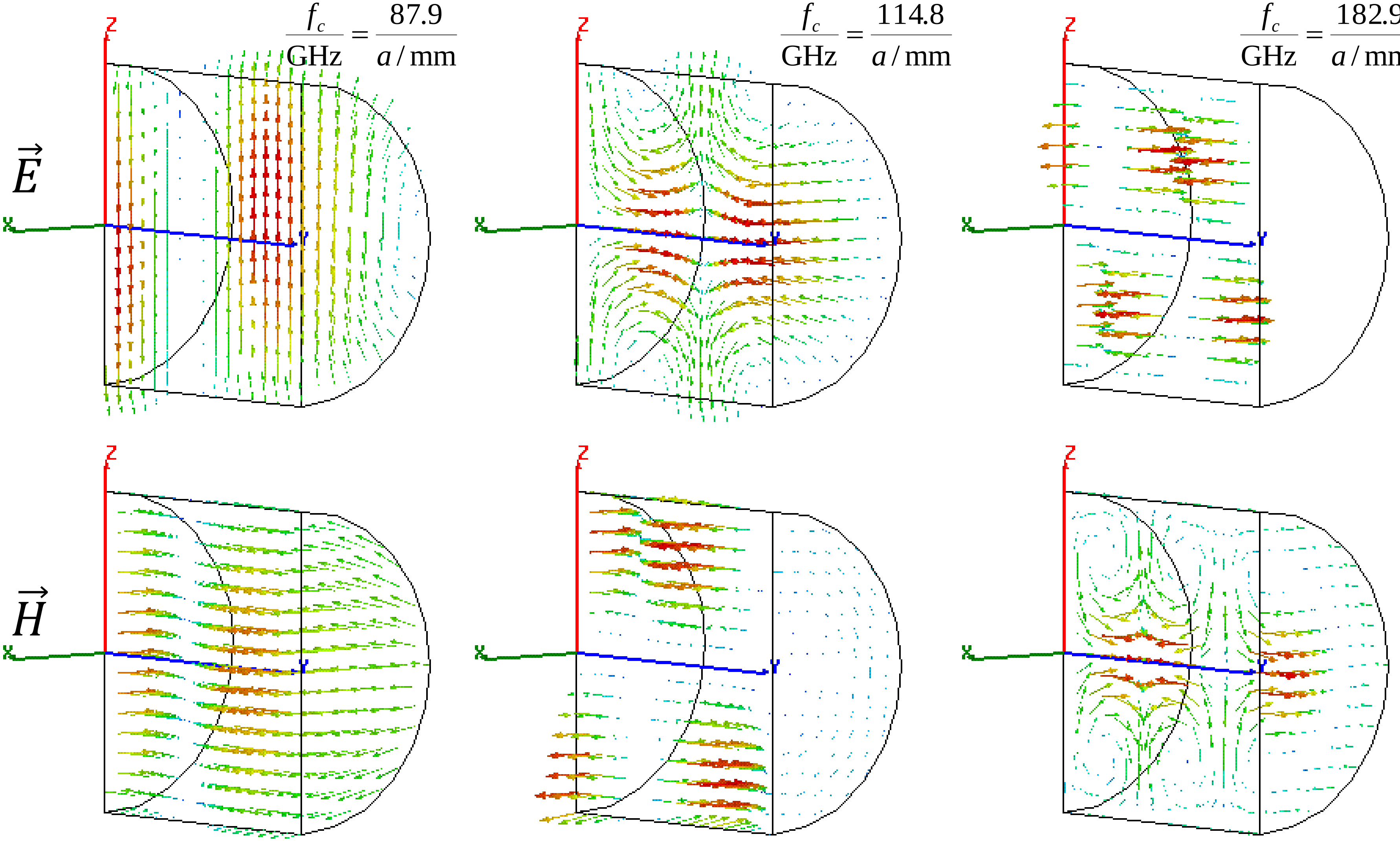
$E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$



$E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$

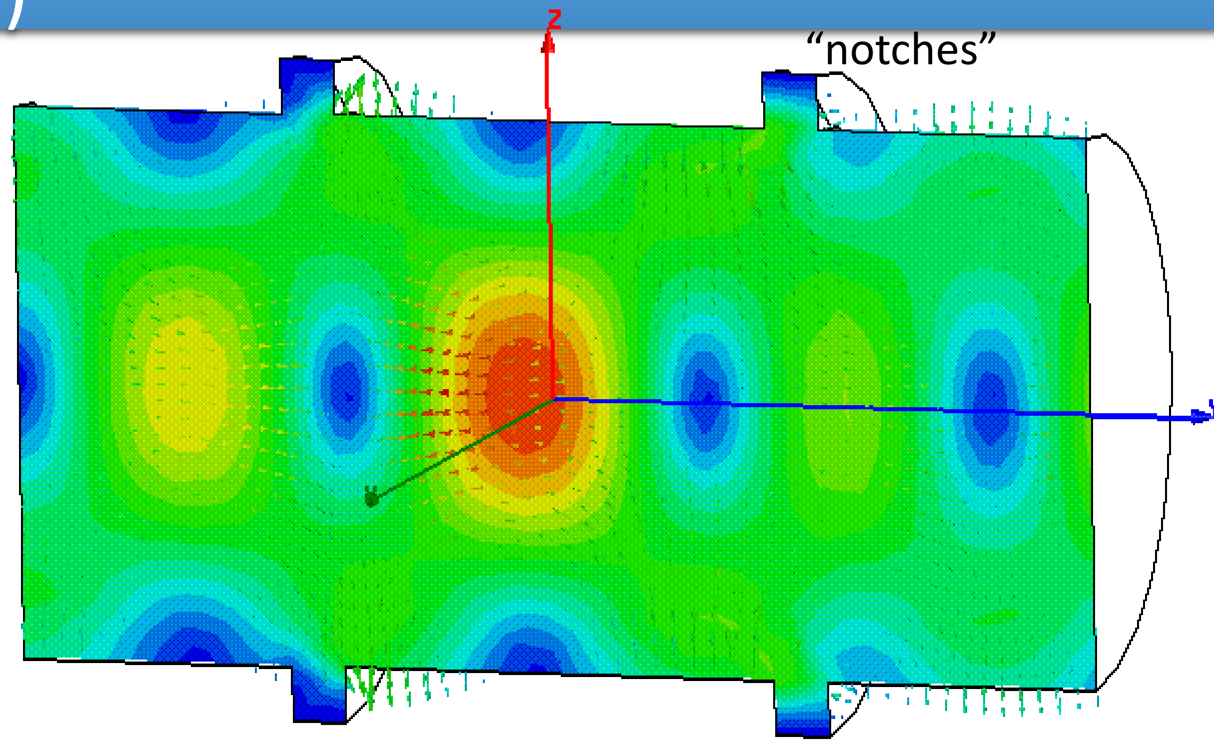
Round waveguide modes

TE_{11} – fundamental	TM_{01} – axial field	TE_{01} – low loss
$\dfrac{f_c}{\text{GHz}} = \dfrac{87.9}{a/\text{mm}}$	$\dfrac{f_c}{\text{GHz}} = \dfrac{114.8}{a/\text{mm}}$	$\dfrac{f_c}{\text{GHz}} = \dfrac{182.9}{a/\text{mm}}$



From waveguide to cavity

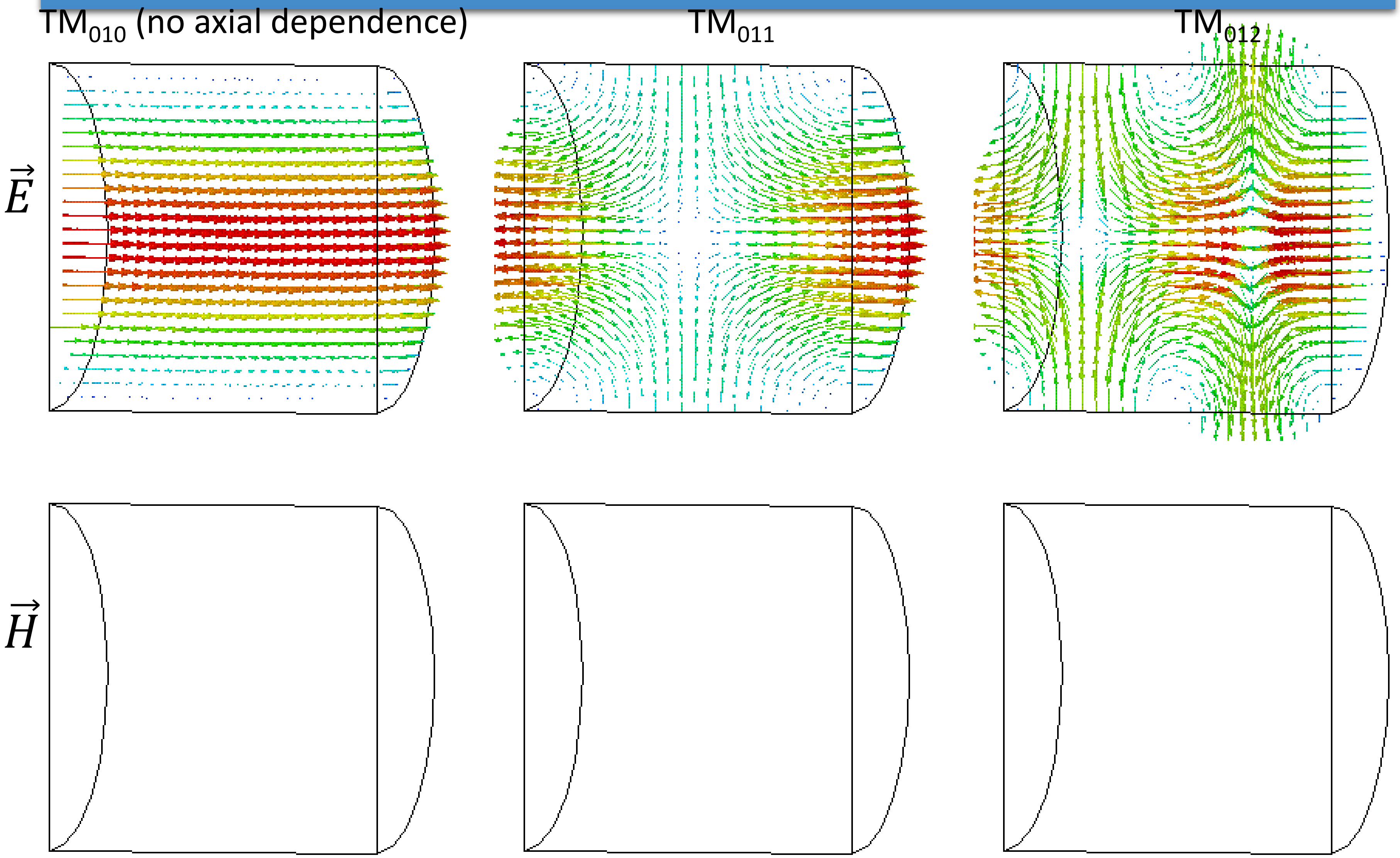
Waveguide perturbed by discontinuities (notches)



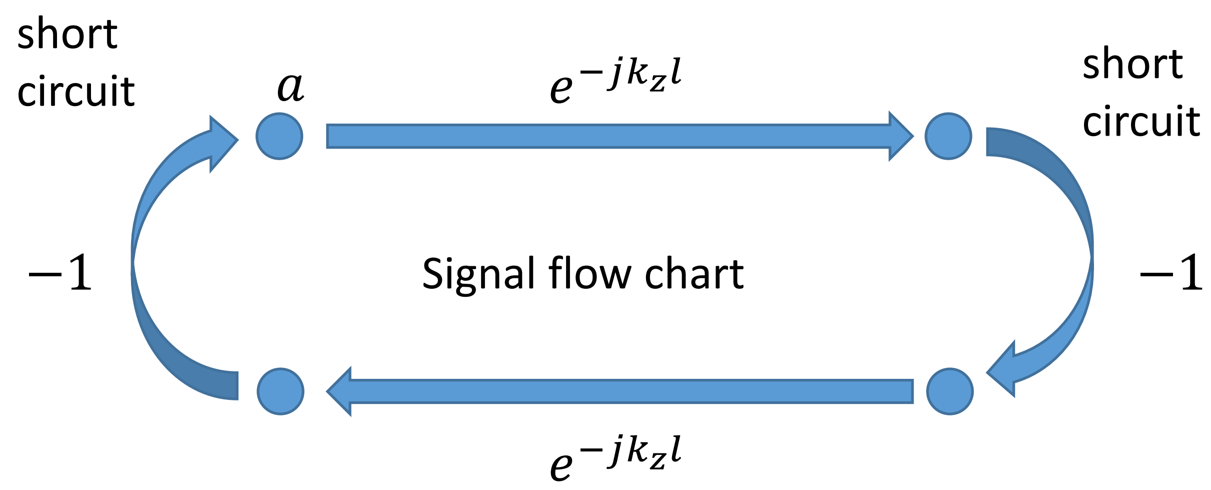
Signal flow chart

Reflections from notches lead to a superimposed standing wave pattern.
“Trapped mode”

Short-circuited waveguide



Single waveguide mode between two shorts



Eigenvalue equation for field amplitude a:

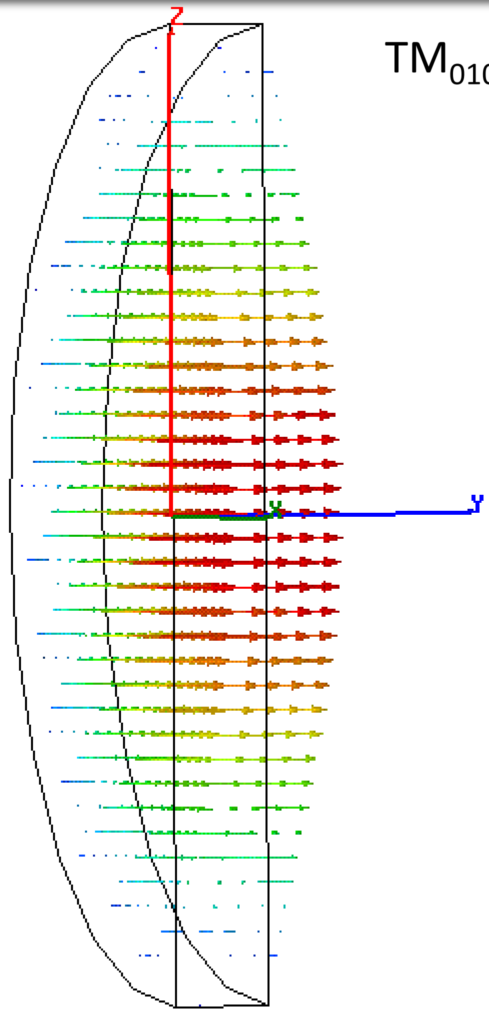
$$a = a\, e^{-jk_z 2l}$$

Non-vanishing solutions exist for $2k_z l = 2\,\pi m$.

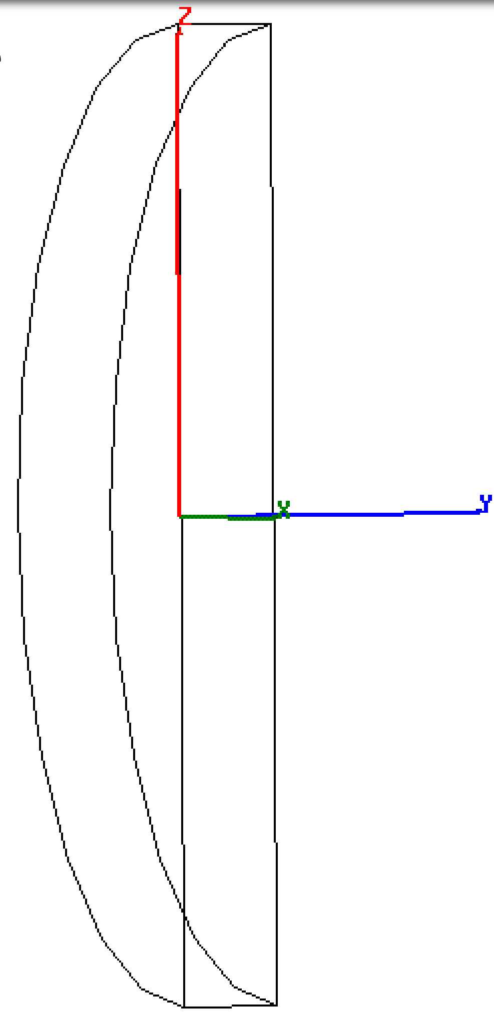
With $k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c\,\frac{m}{2l}\right)^2$.

Simple pillbox cavity

(only 1/2 shown)



TM$_{010}$-mode

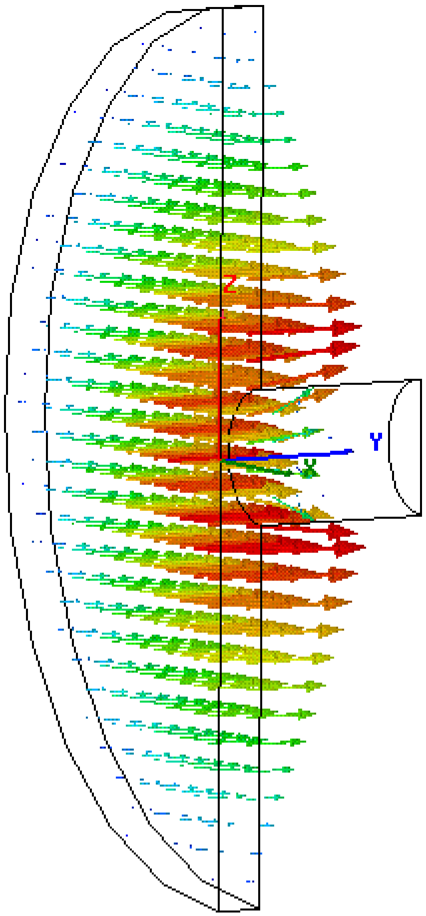


electric field (purely axial) magnetic field (purely azimuthal)

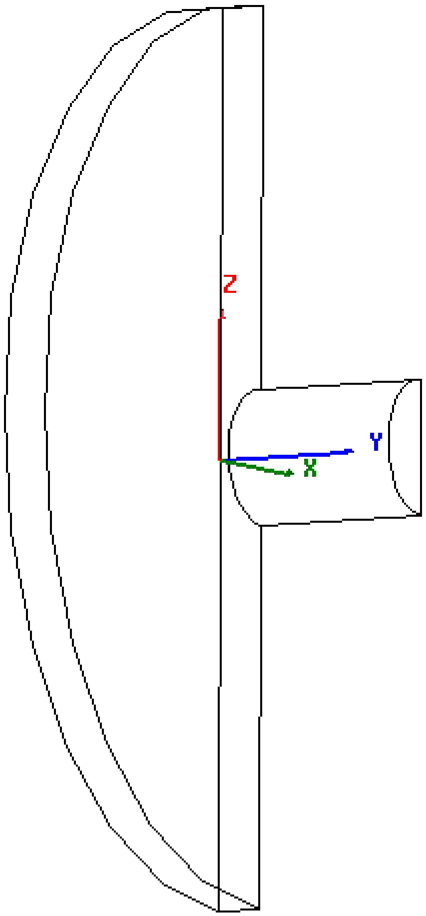
Pillbox with beam pipe

TM_{010}-mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



electric field

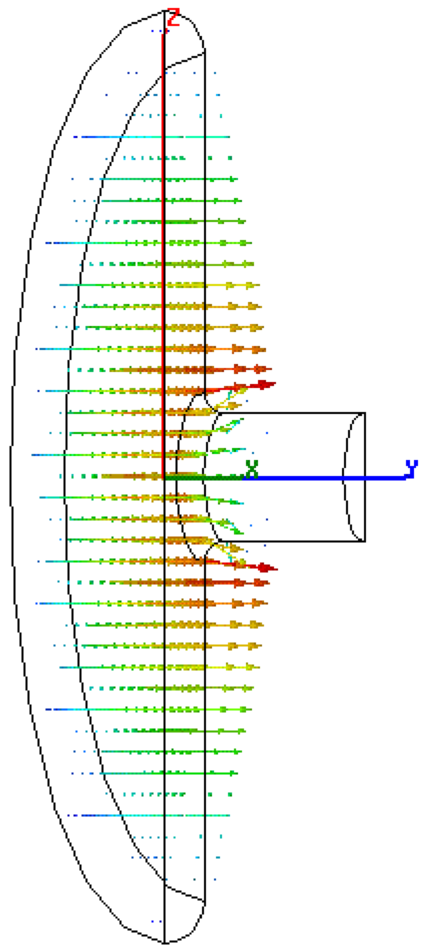


magnetic field

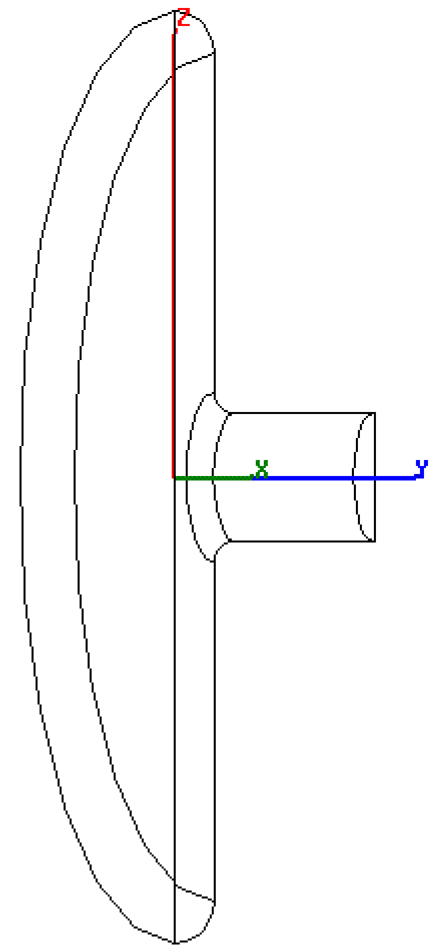
A more practical pillbox cavity

TM_{010}-mode (only 1/4 shown)

Round of sharp edges
(field enhancement!)



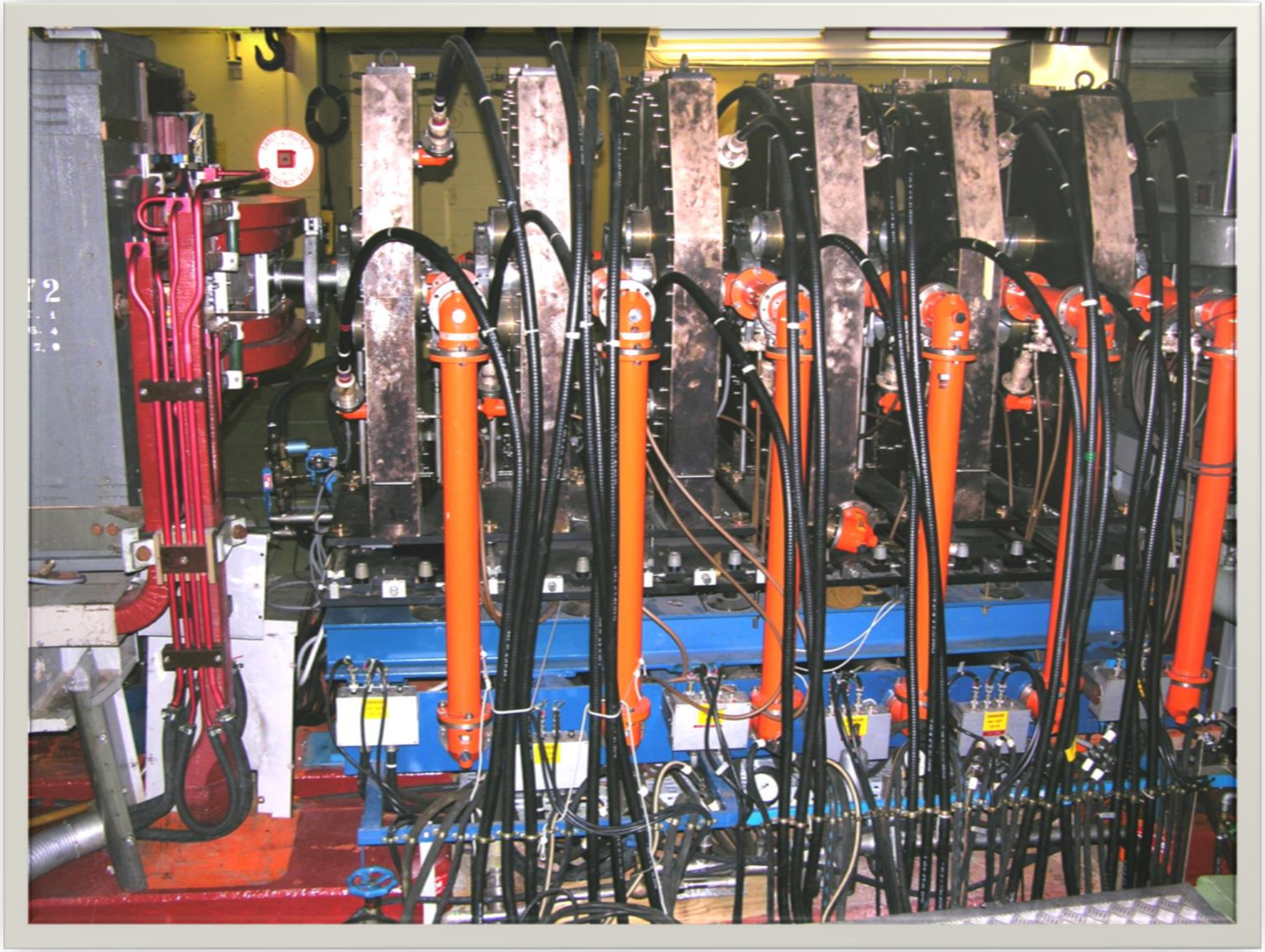
electric field



magnetic field

Some real “pillbox” cavities

CERN PS 200 MHz cavities



End of RF Systems I