RAREFIED GAS DYNAMICS
AND ITS APPLICATIONS
TO VACUUM TECHNOLOGY

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Typical problems

isothermal flows, Poiseuille flow

\[ P_1 > P_2 \]
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\[ \dot{M} \text{ mass flow rate?} \]
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density distribution?
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$Q$ heat flow rate?

density distribution?

over the whole range of $Kn$
Typical problems

non-isothermal flows, thermal creep

$T_1 < T_2$
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over the whole range of Kn
Typical problems

Thermomolecular pressure difference

\[ P_1, T_1 \quad \rightarrow \quad P_2, T_2 \]
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What is the pressure ratio?
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What is the pressure ratio?

\[ \frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^\gamma \]
Typical problems

Thermomolecular pressure difference

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\[ \dot{M} = 0 \quad \text{no mass flow} \]

What is the pressure ratio?

\[ \frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^\gamma \quad 0 \leq \gamma \leq 0.5 \]
Knudsen number

\[ \text{Kn} = \frac{\text{molecular mean free path}}{\text{characteristic size}} \]
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\( Kn \gg 1 \) Free molecular regime.
Every particle moves independently on each other
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$Kn \gg 1$  
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$Kn \ll 1$  
Hydrodynamic regime.  
Continuum mechanics equations are solved
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\( Kn \gg 1 \) Free molecular regime.
Every particle moves independently on each other

\( Kn \ll 1 \) Hydrodynamic regime.
Continuum mechanics equations are solved

\( Kn \sim 1 \) Transition regime.
Kinetic Boltzmann equation is solved
or DSMC method is applied
Rarefaction parameter

equivalent mean free path

\[ \ell = \frac{\mu v_m}{P} \]
Rarefaction parameter

Equivalent mean free path

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\( \mu \) - viscosity
Rarefaction parameter

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\[ v_m = \sqrt{\frac{2kT}{m}} \] most probable molecular vel.
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\( \mu \) - viscosity

\( v_m = \sqrt{\frac{2kT}{m}} \) most probable molecular vel.

\( P \) - pressure
Boltzmann equation

\[ f(t, r, v) - \text{velocity distribution function} \]
Boltzmann equation

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\[ n(t, r) = \int f(t, r, v)dv - \text{density} \]

\[ u(t, r) = \frac{1}{n} \int v f(t, r, v)dv - \text{bulk velocity} \]

\[ P(t, r) = \frac{m}{3} \int V^2 f(t, r, v)dv - \text{pressure} \]

\[ T(t, r) = \frac{m}{3nk} \int V^2 f(t, r, v)dv - \text{temperature} \]

\[ q(t, r) = \frac{m}{2} \int V^2 V f(t, r, v)dv - \text{heat flux vector} \]

\[ V = v - u \]
Boltzmann equation

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} = Q(f f_*) \]
Boltzmann equation

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} = Q(f f_*)
\]

\[
Q(f f_*) = \int (f' f'_* - f f_*) |\mathbf{v} - \mathbf{v}_*| b \, db \, d\varepsilon \, d\mathbf{v}_*
\]

\(v'\) and \(v_*'\) - pre-collision molecular velocities

\(v\) and \(v_*\) - post-collision molecular velocities
Boltzmann equation

Discrete velocity method:

\( v_1, v_2, \ldots, v_N, \)
Boltzmann equation

Discrete velocity method:

\[ v_1, v_2, \ldots, v_N, \]

The BE is split into N differential eqs. coupled via the collisions integral
Kinetic equations

Till now, a numerical solution of the exact Boltz.Eq. requires great computational efforts
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BGK model

\[ Q(f f_*) = \nu (f^M - f) \]
Kinetic equations

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BGK model

\[ Q(f f_*) = \nu \left( f^M - f \right) \]

S model

\[ Q(f f_*) = \nu \left\{ f^M \left[ 1 + \frac{2m(q \cdot V)}{15n(kT)^2} \left( \frac{mV^2}{2kT} - \frac{5}{2} \right) \right] - f \right\} \]
Kinetic equations

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BGK model

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\[ f^M = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m(v - u)^2}{2kT} \right] \]

\[ \nu = \frac{P}{\mu} - \text{frequency of intermolecular collisions} \]
Viscous slip coefficient

The most used formula

\[ u_y = \frac{\partial u}{\partial y} \]

accommodation coefficient

mean free path
Viscous slip coefficient

The most used formula

\[ u_y = \frac{2 - \alpha}{\alpha} \lambda \frac{du_y}{du_x} \]

\( \alpha \) - accommodation coefficient
\( \lambda \) - mean free path
Viscous slip coefficient

All disagreements with experiments are eliminated by fitting $\alpha$:

$$0.1 \leq \alpha \leq 2,$$

while in reality

$$0.9 \leq \alpha \leq 1,$$
Viscous slip coefficient

\[ u_y = \frac{P}{\Delta u_y \Delta x} \]

\( \Delta x \) equivalent mean free path
Viscous slip coefficient

\[ u_y = \sigma_P \ell \frac{d u_y}{d x} \] at \( x = 0 \)

\( \sigma_P \) - viscous slip coefficient

\( \ell \) equivalent mean free path
Viscous slip coefficient

Diffuse scattering

\[ \sigma_p = \sqrt{\pi}/2 = 0.886 \quad \text{estimation by Maxwell} \]
Viscous slip coefficient

Diffuse scattering

\[ \sigma_p = \sqrt{\pi}/2 = 0.886 \] estimation by Maxwell

\[ \sigma_p = 1.016 \] solution of BGK model

\[ \sigma_p = 1.018 \] solution of S model

\[ \sigma_p = 0.985 \] solution of Boltzmann Eq.
Viscous slip coefficient

Non-diffuse scattering

Estimation by Maxwell

\[ \sigma_p = 0.886 \frac{2 - \alpha}{\alpha} \]
Viscous slip coefficient

Non-diffuse scattering

Estimation by Maxwell

\[ \sigma_p = 0.886 \frac{2 - \alpha}{\alpha} \]

S model with CL bound.cond., (Sharipov-2003)

\[ \sigma_p = 1.018 \frac{2 - \alpha_t}{\alpha_t} - 0.264 \frac{1 - \alpha_t}{\alpha_t} \]

\( \sigma_p \) is sensitive to the gas-surface interaction.
Gas-surface interaction law

Experiment by Porodnov et al. (1974)
technical (contaminated) surface

<table>
<thead>
<tr>
<th>gas</th>
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<th>Ne</th>
<th>Ar</th>
<th>Kr</th>
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<th>H$_2$</th>
<th>N$_2$</th>
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<td>$\alpha_t$</td>
<td>0.88</td>
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For a technical surface αₜ is very close to unity for the most of gases
Thermal slip coefficient

\[ u_y = \sigma_T \frac{\mu}{\varrho} \frac{d \ln T}{dy} \quad \text{at} \quad x = 0 \]

Diffuse scattering

\( \sigma_T = 0.75 \) \quad \text{estimation by Maxwell}

\( \sigma_T = 1.175 \) \quad \text{solution of S model}

\( \sigma_T = 1.01 \) \quad \text{solution of Boltzmann Eq.}
Temperature jump coefficient

\[ T_g = T_w + \zeta_T \ell \frac{dT}{dx} \]

Diffuse scattering

\[ \zeta_T = 1.662 \quad \text{estimation by Maxwell} \]
\[ \zeta_T = 1.954 \quad \text{solution of S model} \]
Flow through a tube

\[ \dot{M} = \frac{\pi a^2 P}{\nu_m} \left( -G_P \frac{a}{P} \frac{dP}{dx} + G_T \frac{a}{T} \frac{dT}{dx} \right) \]

\[ G_P = G_P(\delta) \quad G_T = G_T(\delta) \]

\[ \delta = \frac{a}{\ell} \]
Flow through a tube

Free molecular regime $\delta = 0$

$$G_P = \frac{8}{3\sqrt{\pi}}, \quad G_T = \frac{1}{2}G_P$$

Hydrodynamic regime $\delta \to \infty$

$$G_P = \frac{\delta}{4} + \sigma_P, \quad G_T = \frac{\sigma_T}{\delta}$$
Flow through a tube

Transitional regime $G_P$

Cercignani et al. (1966)
Flow through a tube

Transitional regime $G_P$

Cercignani et al. (1966)
Sharipov, 1996
Flow through a tube

Transitional regime $G_P$

Cercignani et al. (1966)
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Loyalka & Hamoodi (1991)
Flow through a tube

Transitional regime $G_P$

BGK and S model provide reliable results for isothermal flows

Cercignani et al. (1966)
Sharipov, 1996
Loyalka & Hamoodi (1991)
Flow through a tube

Transitional regime $G_T$

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Sharipov, 1999
Loyalka & Hickey, 1991
Flow through a tube

Transitional regime $G_T$

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BGK does not, while S provides reliable results for non-isothermal flows.
Flow through a tube

Numerical data on $G_P$ and $G_T$ can be found in


Numerical calculations of $\dot{M}$ can be carried out on-line

http://fisica.ufpr.br/sharipov
Direct Simulation Monte Carlo

$M$ particles are considered simultaneously

$M \sim 10^7 \quad – \quad 10^8$
Direct Simulation Monte Carlo

$M$ particles are considered simultaneously

$M \sim 10^7 - 10^8$

- Free motion of particles
- Interaction with solid surface, Elimination and Generation of particles
- Simulation of collisions
- Calculation of macroscopic quantities
Orifice flow

\[ W = \frac{\dot{M}}{\dot{M}_0}, \quad \dot{M}_0 = \frac{\sqrt{\pi a^2}}{v_m} P_0 \]
Orifice flow into vacuum \( P_1 = 0 \)
Orifice flow into vacuum \( P_1 = 0 \)

\[ W \]

\[ \delta \]

DSMC

Liepmann, 1961
Orifice flow into vacuum $P_1 = 0$

\[ W \]

\[ \delta \]

DSMC

Liepmann, 1961
Barashkin, 1977
Orifice flow into vacuum \( P_1 = 0 \)

\[
W = \frac{1}{\delta}
\]

- DSMC
- Liepmann, 1961
- Barashkin, 1977
- Fujimoto & Usami, 1984
Orifice flow into vacuum $P_1 = 0$

- DSMC
- Liepmann, 1961
- Barashkin, 1977
- Fujimoto & Usami, 1984
- Jitschin, 1999
Orifice flow at $P_1 > 0$

Graph showing the relationship between $W$ and $\delta$ for different values of $P_1/P_0$.

- $P_1/P_0=0$
- $P_1/P_0=0.1$
- $P_1/P_0=0.5$
- $P_1/P_0=0.9$

References:
- Sreekanth, 1965
- Porodnov et al., 1974
Orifice flow at \( P_1 > 0 \)

Flowfield at \( P_0/P_1 = 100 \) and \( \delta = 1000 \)

- \( \rho/\rho_0 \) density
- \( T/T_0 \) temperature
- Mach number
Orifice flow at $P_1 > 0$

Flowfield at $P_0/P_1 = 10$ and $\delta = 1000$

\(\rho/\rho_0\) density

\(T/T_0\) temperature

Mach number
Holweck pump

![Scheme of pump](image)

*Scheme of pump*
Holweck pump

Scheme of pump

Scheme of groove
Holweck pump. First stage

Four problems are solved

- Poiseuille flow in $x$ direction $G_x^{(P)}(\delta)$
- Poiseuille flow in $z$ direction $G_z^{(P)}(\delta)$
- Couette flow in $x$ direction $G_x^{(C)}(\delta)$
- Couette flow in $z$ direction $G_z^{(C)}(\delta)$

It takes long CPU time
Holweck pump. Second stage

$S$ - pumping speed

$$G_\eta = \frac{S}{u_m \ell^2}$$

$$\ell \frac{dP}{d\eta} = \sin \alpha \frac{U}{u_m} \frac{P \cos \alpha [G_z^{(C)} - \frac{\ell_z}{\ell} G_x^{(C)}]}{G_z^{(P)} \sin^2 \alpha + \frac{\ell_z}{\ell} G_x^{(P)} \cos^2 \alpha} - G_\eta P_h$$

It takes short CPU time
Holweck pump. Second stage

It allows us easily to change many parameters such as:

- groove inclination
- fore vacuum and high vacuum pressures
- angular velocity of rotating cylinder,
- species of gas
- temperature of the gas
Holweck pump. Results

Limit compression pressure ratio

![Graph showing the limit compression pressure ratio](image)

- $K_0$ vs $\delta_f$ for different gases (Ar and N$_2$).
Holweck pump. Results

Dimensional pumping speed

![Graph showing dimensional pumping speed vs. \( \delta_h \)]
Recent results

Slip and jump boundary conditions

- Velocity slip coefficients of single gas for different gas-surface interaction laws
Recent results

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over the whole range of the gas rarefaction

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- Modelling of vacuum pumps
Numerical programs

Some calculations of flow rate through tubes, channels and orifices can be carried out in dimensional quantities on line

http://fisica.ufpr.br/sharipov/
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Thank you for your attention