RAREFIED GAS DYNAMICS AND ITS APPLICATIONS TO VACUUM TECHNOLOGY

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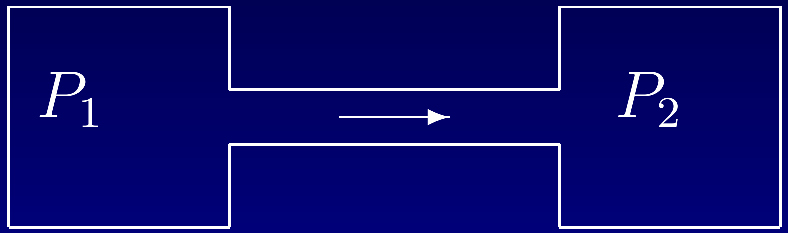
Typical problems

isothermal flows, Poiseuille flow



$$P_1 > P_2$$

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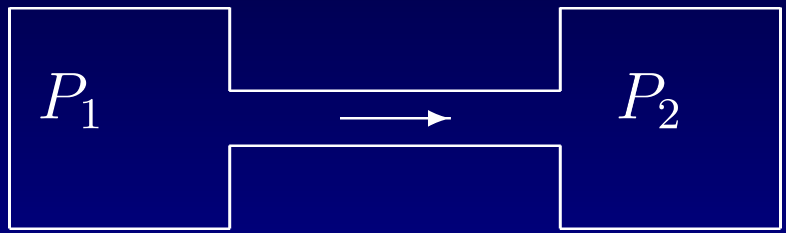

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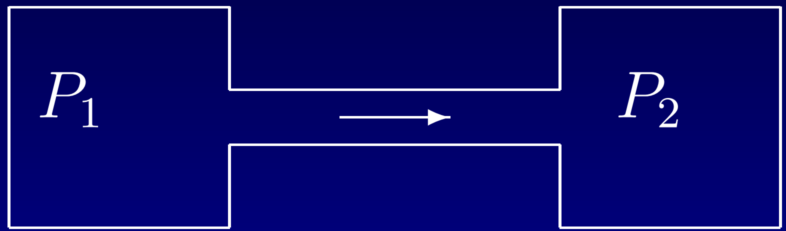

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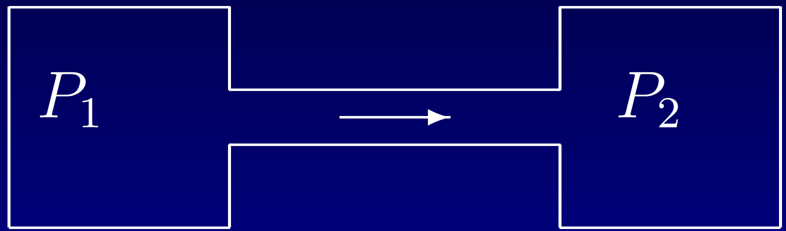

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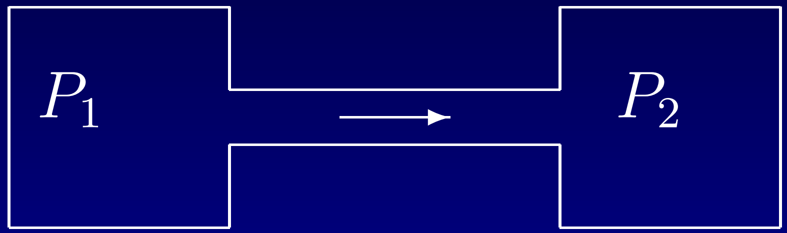

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over the whole range of Kn

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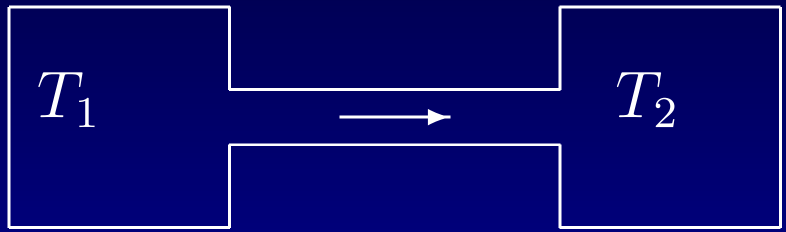
non-isothermal flows, thermal creep



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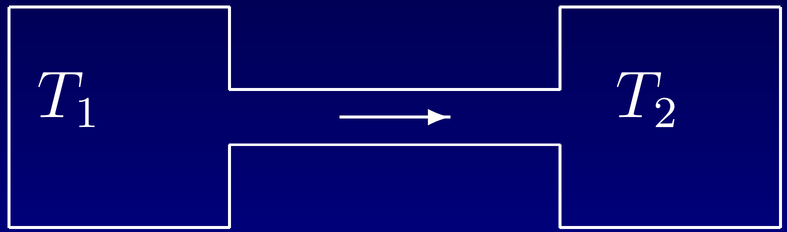

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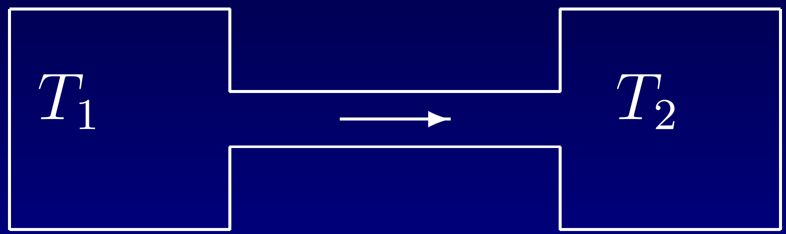

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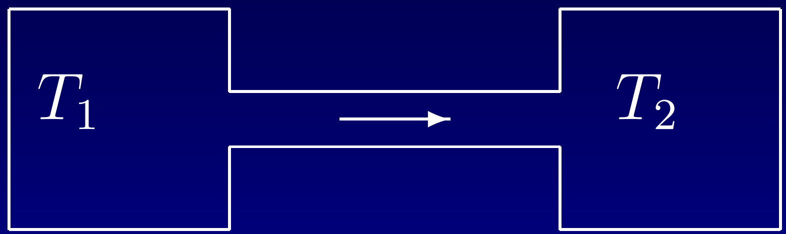

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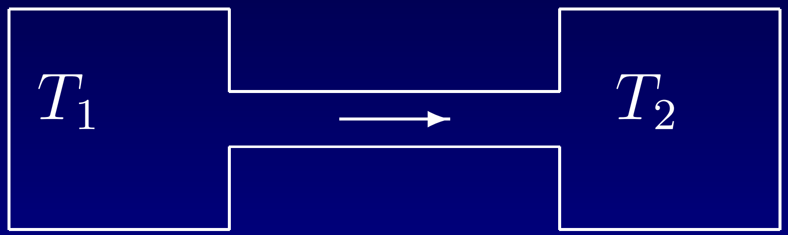

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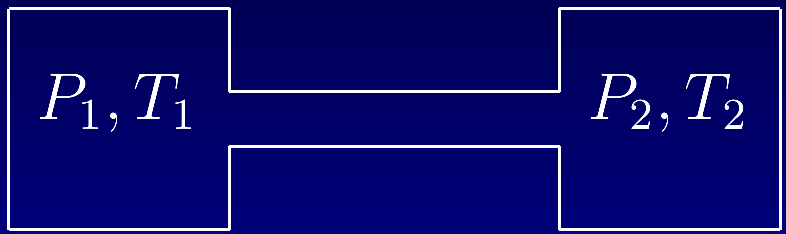
Thermomolecular pressure difference



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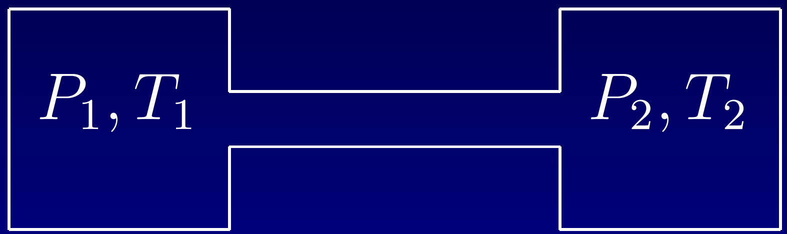

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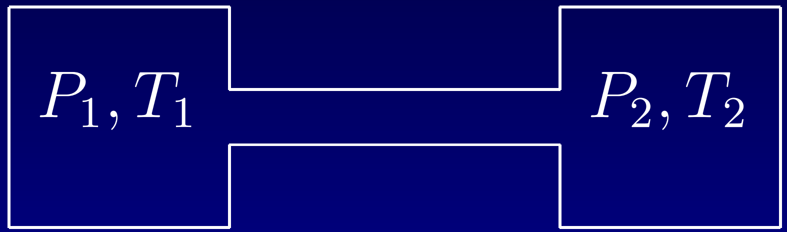

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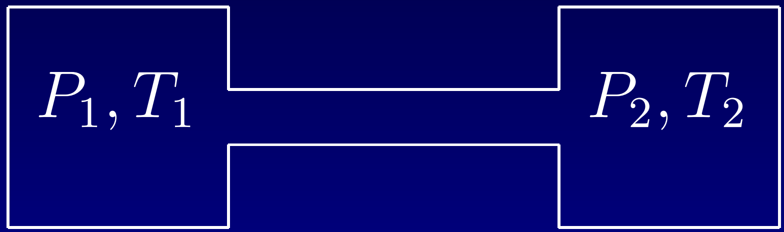

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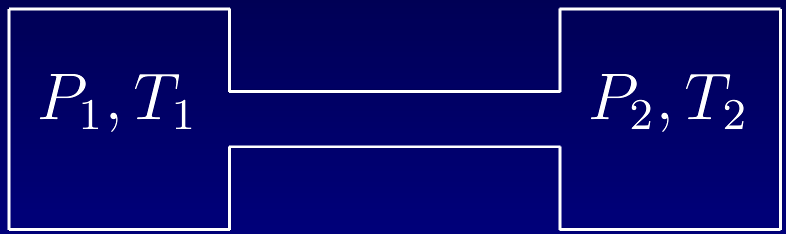

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What is the pressure ratio?

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma} \qquad 0 \le \gamma \le 0.5$$

Knudsen number

$$\mathrm{Kn} = \frac{\textit{molecular mean free path}}{\textit{characteristic size}}$$

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Kn$\gg$1 Free molecular regime.
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Kn$\sim$ 1 Transition regime.
Kinetic Boltzmann equation is solved
or DSMC method is applied

Rarefaction parameter

equivalent mean free path

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P - pressure

Boltzmann equation

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$n(t, \mathbf{r}) = \int f(t, \mathbf{r}, \mathbf{v}) \mathrm{d}\mathbf{v}$ - density

$\mathbf{u}(t, \mathbf{r}) = \frac{1}{n} \int \mathbf{v} f(t, \mathbf{r}, \mathbf{v}) \mathrm{d}\mathbf{v}$ - bulk velocity

$P(t, \mathbf{r}) = \frac{m}{3} \int V^2 f(t, \mathbf{r}, \mathbf{v}) \mathrm{d}\mathbf{v}$ - pressure

$T(t, \mathbf{r}) = \frac{m}{3nk} \int V^2 f(t, \mathbf{r}, \mathbf{v}) \mathrm{d}\mathbf{v}$ - temperature

$\mathbf{q}(t, \mathbf{r}) = \frac{m}{2} \int V^2 \mathbf{V} f(t, \mathbf{r}, \mathbf{v}) \mathrm{d}\mathbf{v}$ - heat flux vector

$$\mathbf{V} = \mathbf{v} - \mathbf{u}$$

Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} = Q(f f_*)$$

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$$Q(f f_*) = \int (f' f'_* - f f_*) \, |\mathbf{v} - \mathbf{v}_*| b \, \mathrm{d}b \, \mathrm{d}\varepsilon \, \mathrm{d}\mathbf{v}_*$$

$\mathbf{v}'$ and $\mathbf{v}_*{}'$ - pre-collision molecular velocities

$\mathbf{v}$ and $\mathbf{v}_*$ - post-collision molecular velocities

Boltzmann equation

Discrete velocity method:

$$\mathbf{v}_1,\ \mathbf{v}_2,\ \dots,\mathbf{v}_N,$$

Boltzmann equation

Discrete velocity method:

$$\mathbf{v}_1,\ \mathbf{v}_2,\ \dots,\mathbf{v}_N,$$

The BE is split into N differential eqs. coupled via the collisions integral

Kinetic equations

Till now, a numerical solution of the exact Boltz.Eq. requires great computational efforts

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BGK model

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S model

$$Q(ff_*) = \nu \left\{ f^M \left[1 + \frac{2m(\mathbf{q} \cdot \mathbf{V})}{15n(kT)^2} \left(\frac{mV^2}{2kT} - \frac{5}{2} \right) \right] - f \right\}$$

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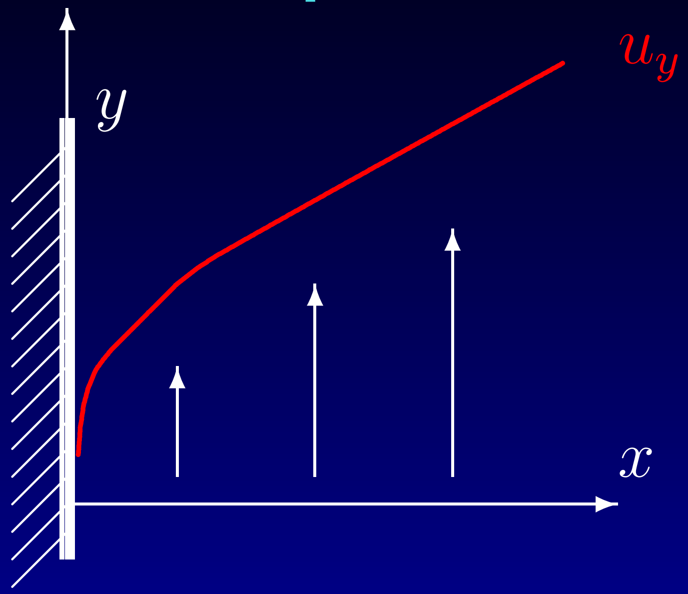
$$f^M = n\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left[-\frac{m(\mathbf{v}-\mathbf{u})^2}{2kT}\right]$$

$\nu = P/\mu$ - frequency of intermolecular collisions

Viscous slip coefficient



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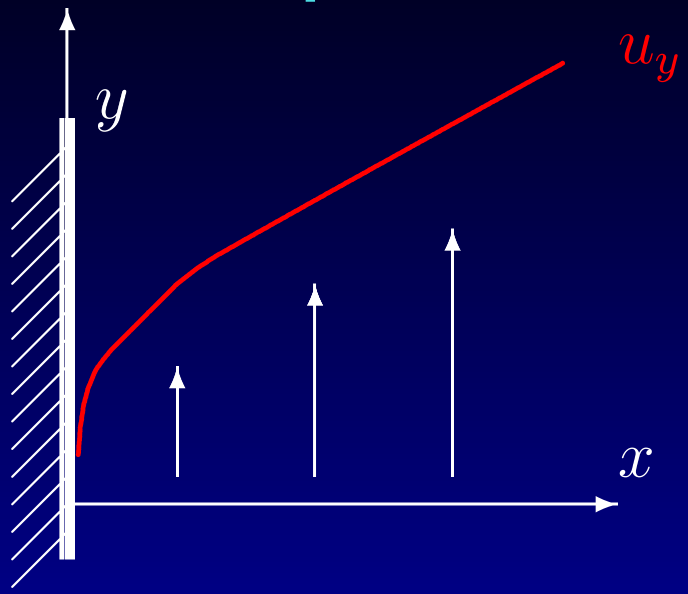


The most used formula

$$u_y = \frac{2-\alpha}{\alpha}\lambda\frac{du_y}{du_x}$$

α -accommodation coefficient

λ - mean free path

Viscous slip coefficient

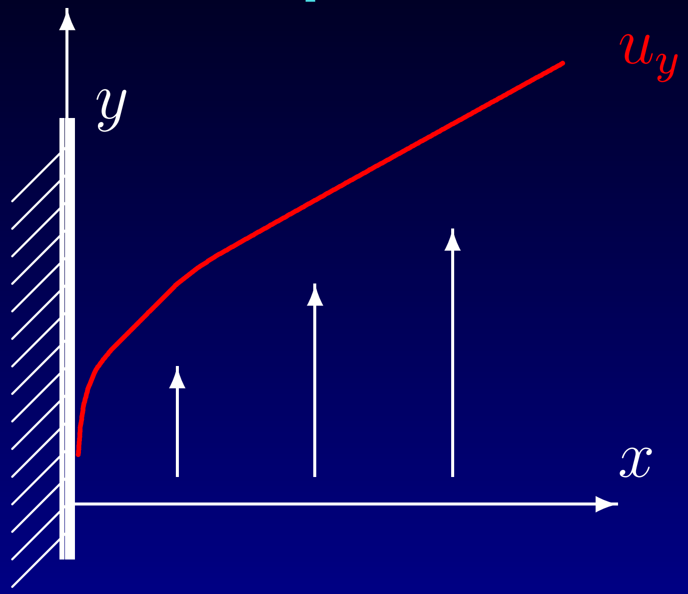
All disagreements with experiments are eliminated by fitting α:

$$0.1 \leq \alpha \leq 2,$$

while in reality

$$0.9 \leq \alpha \leq 1,$$

Viscous slip coefficient



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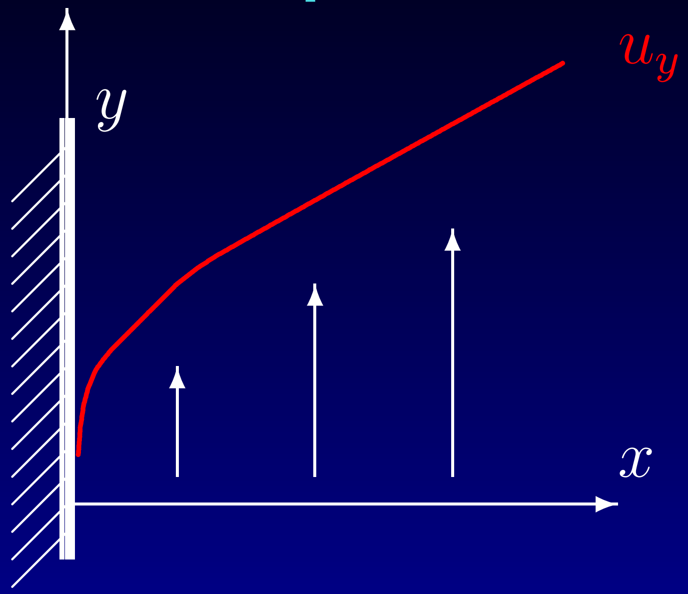

$$u_y = \sigma_\mathrm{P} \ell \frac{\mathrm{d}u_y}{\mathrm{d}x} \quad \text{at} \quad x = 0$$

σ_P - viscous slip coefficient

ℓ equivalent mean free path

Viscous slip coefficient

Diffuse scattering

$\sigma_{\mathsf{P}} = \sqrt{\pi}/2 = 0.886$ estimation by Maxwell

Viscous slip coefficient

Diffuse scattering

$\sigma_{\mathsf{P}} = \sqrt{\pi}/2 = 0.886$ estimation by Maxwell

$\sigma_{\mathsf{P}} = 1.016$ solution of BGK model

$\sigma_{\mathsf{P}} = 1.018$ solution of S model

$\sigma_{\mathsf{P}} = 0.985$ solution of Boltzmann Eq.

Viscous slip coefficient

Non-diffuse scattering

Estimation by Maxwell

$$\sigma_{\mathsf{P}} = 0.886 \frac{2-\alpha}{\alpha}$$

Viscous slip coefficient

Non-diffuse scattering

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$$\sigma_{\mathsf{P}} = 0.886 \frac{2-\alpha}{\alpha}$$

S model with CL bound.cond., (Sharipov-2003)

$$\sigma_{\mathsf{P}} = 1.018 \frac{2-\alpha_t}{\alpha_t} - 0.264 \frac{1-\alpha_t}{\alpha_t}$$

σ_{P} is sensitive to the gas-surface interaction

Gas-surface interaction law

Experiment by Porodnov et al. (1974)

technical (contaminated) surface

gas	He	Ne	Ar	Kr	Xe	H_2	N_2	CO_2
α_t	0.88	0.85	0.92	1.0	1.0	0.95	0.91	0.99

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α_t	0.88	0.85	0.92	1.0	1.0	0.95	0.91	0.99

For a technical surface α_t is very close to unity for the most of gases

Thermal slip coefficient

$$u_y = \sigma_\mathsf{T} \frac{\mu}{\varrho} \frac{\mathsf{d} \ln T}{\mathsf{d} y} \quad \text{at} \quad x = 0$$

Diffuse scattering

$\sigma_\mathsf{T} = 0.75$ estimation by Maxwell

$\sigma_\mathsf{T} = 1.175$ solution of S model

$\sigma_\mathsf{T} = 1.01$ solution of Boltzmann Eq.

Temperature jump coefficient

$$T_g = T_w + \zeta_{\mathsf{T}} \ell \frac{\mathsf{d}T}{\mathsf{d}x}$$

Diffuse scattering

$\zeta_{\mathsf{T}} = 1.662$	estimation by Maxwell
$\zeta_{\mathsf{T}} = 1.954$	solution of S model

Flow through a tube

$$\dot{M} = \frac{\pi a^2 P}{v_m} \left(-G_P \frac{a}{P} \frac{\mathrm{d}P}{\mathrm{d}x} + G_T \frac{a}{T} \frac{\mathrm{d}T}{\mathrm{d}x} \right)$$

$$G_P = G_P(\delta) \qquad G_T = G_T(\delta)$$

$$\delta = \frac{a}{\ell}$$

Flow through a tube

Free molecular regime $\delta = 0$

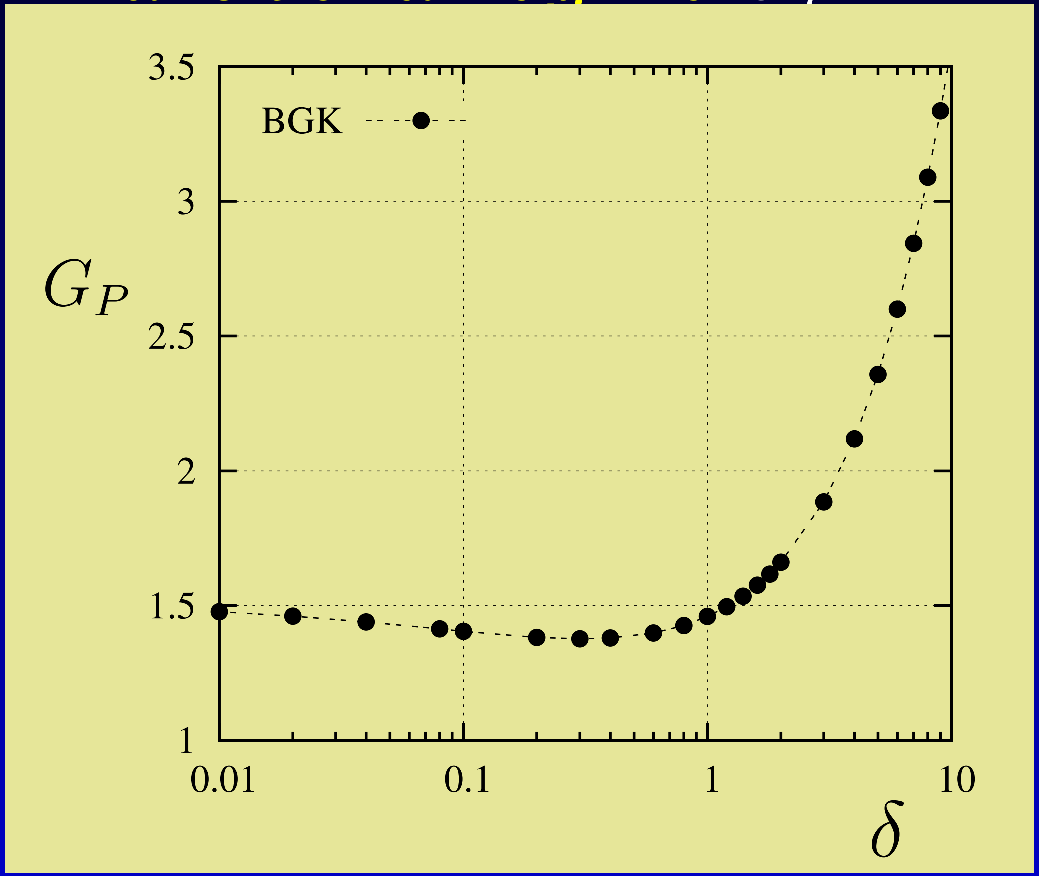
$$G_P = \frac{8}{3\sqrt{\pi}}, \qquad G_T = \frac{1}{2} G_P$$

Hydrodynamic regime $\delta \to \infty$

$$G_P = \frac{\delta}{4} + \sigma_{\mathrm{P}}, \qquad G_T = \frac{\sigma_{\mathrm{T}}}{\delta}$$

Flow through a tube

Transitional regime G_P



Cercignani et al. (1966)

Flow through a tube

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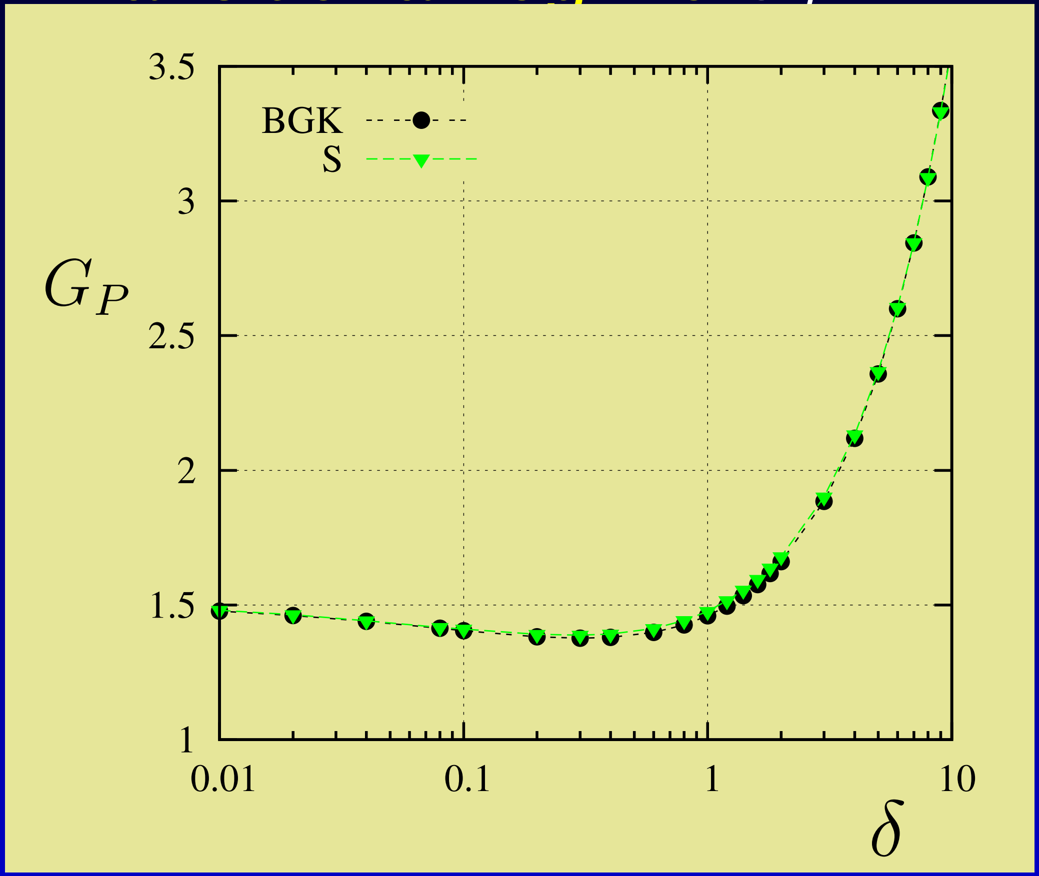

Cercignani et al. (1966)
Sharipov, 1996

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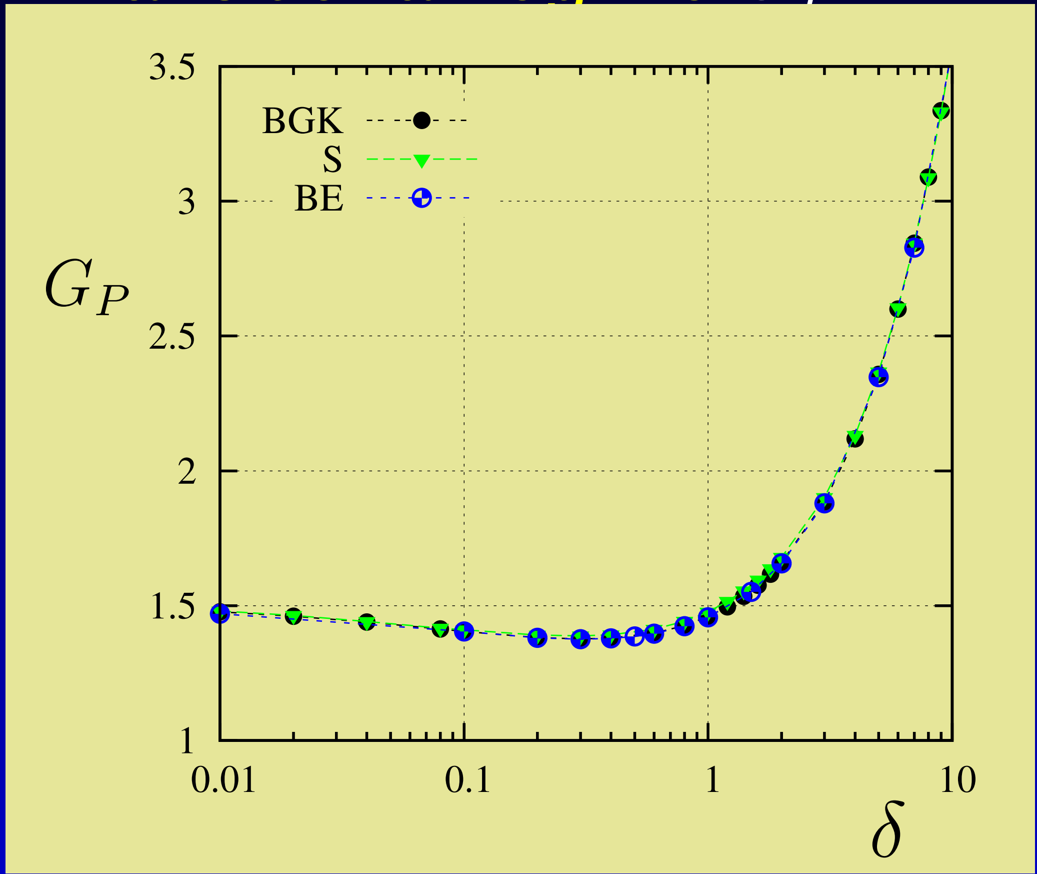

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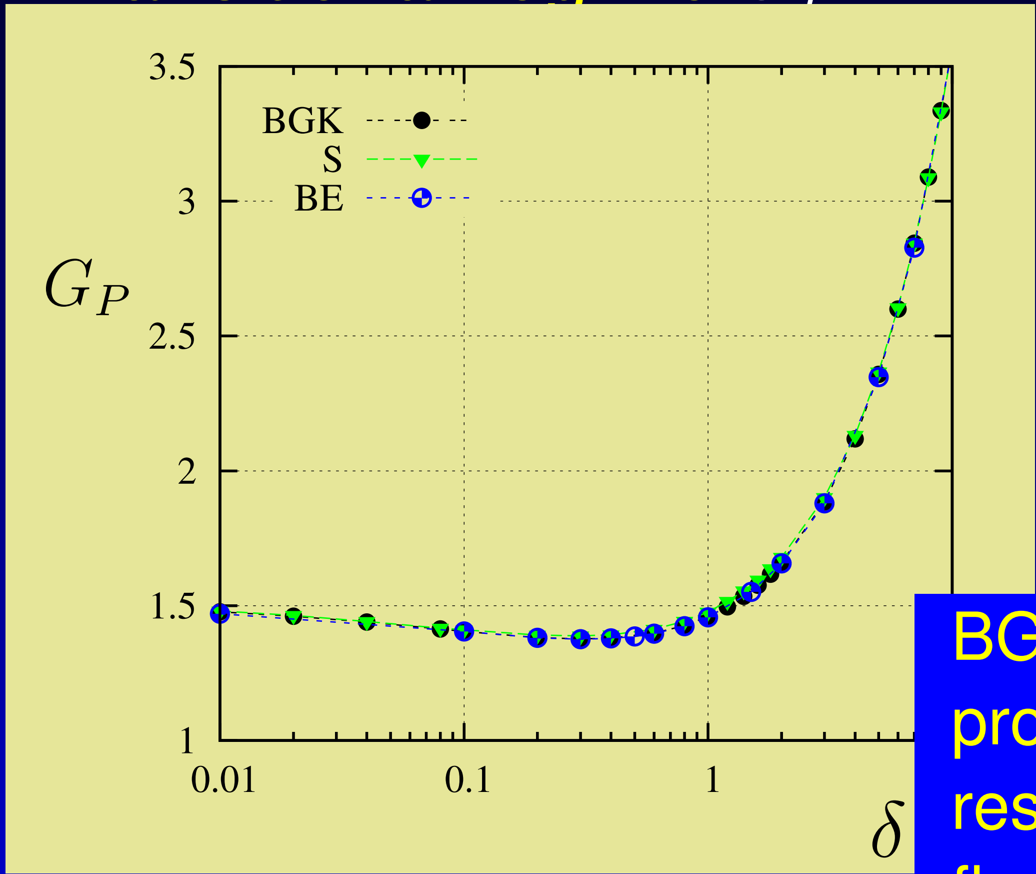

Cercignani et al. (1966)
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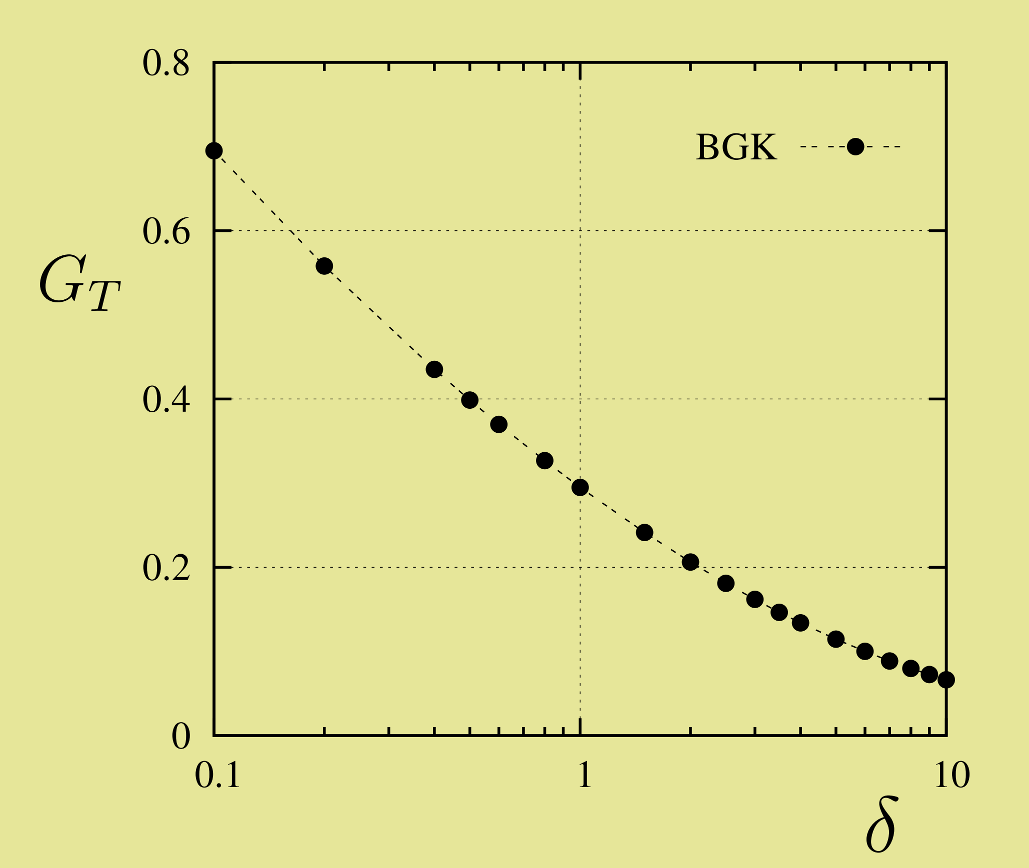
Loyalka & Hamoodi (1991)

BGK and S model provide reliable results for isothermal flows

Flow through a tube

Transitional regime G_T



Loyalka (1994)

Flow through a tube

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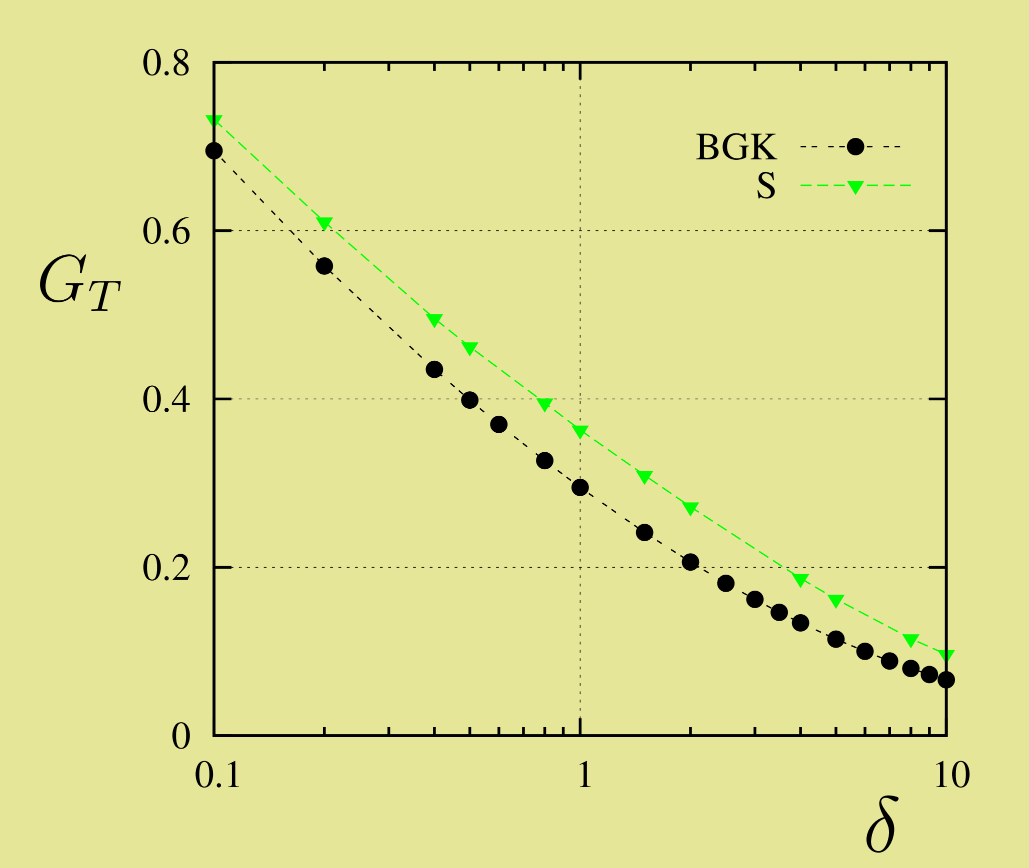

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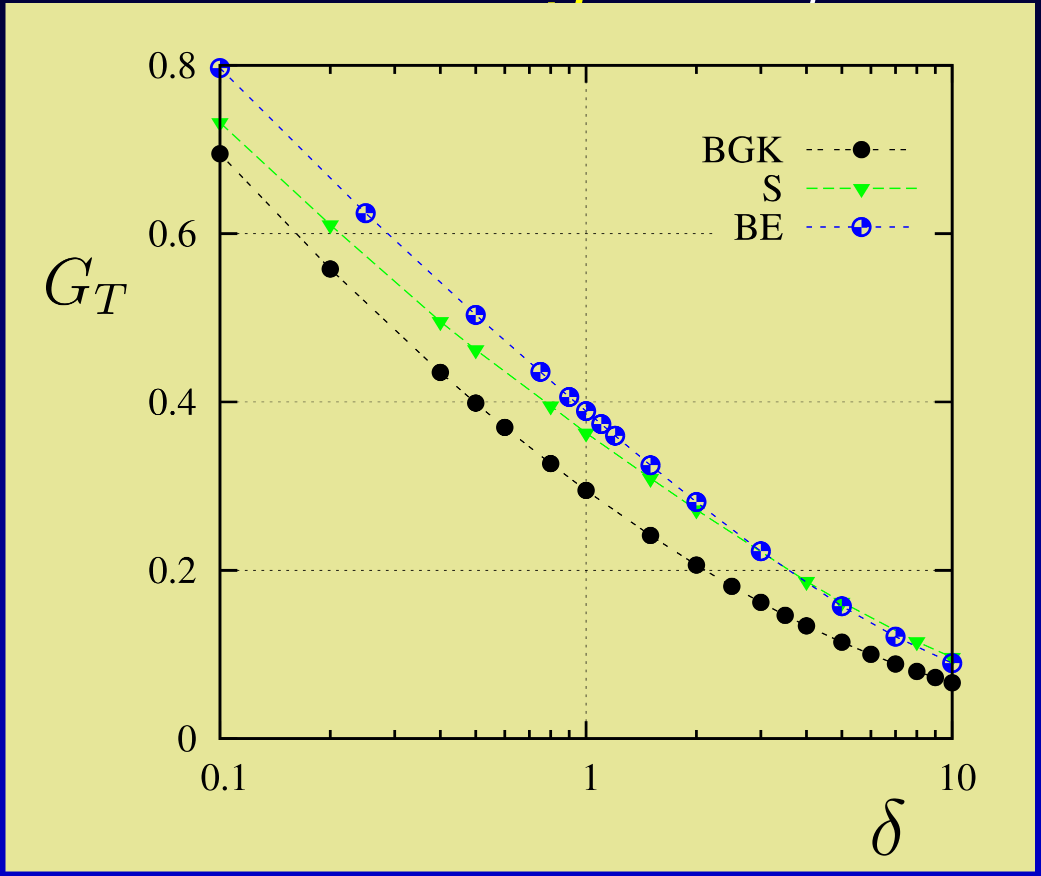
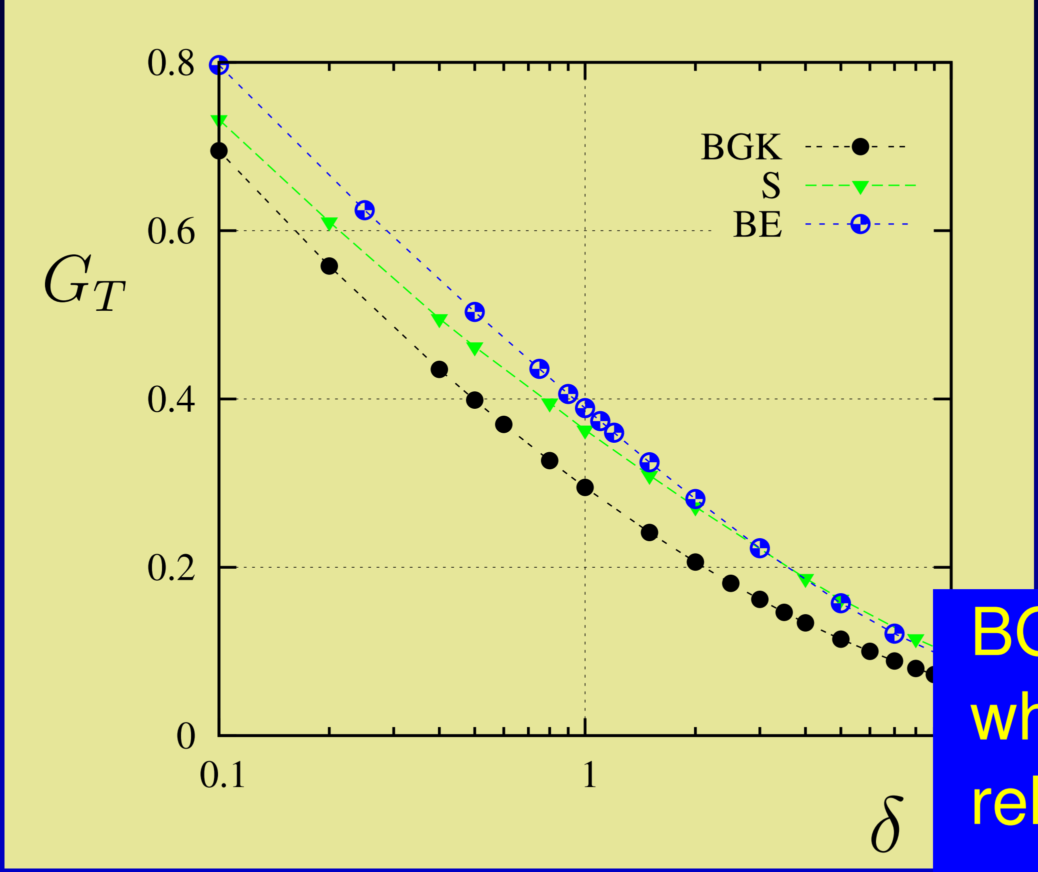


Loyalka (1994)
Sharipov, 1999
Loyalka & Hickey, 1991

Flow through a tube

Transitional regime G_T



Loyalka (1994)
Sharipov, 1999
Loyalka & Hickey, 1991

BGK does not, while S provides reliable results for non-isothermal flows

Flow through a tube

Numerical data on G_P and G_T can be found in

Sharipov & Seleznev, Data on Internal Rarefied gas Flows *J. Phys. Chem. Ref. Data* **27**, 657-706 (1998)

Numerical calculations of $\dot{M}$ can be carried out on-line

http://fisica.ufpr.br/sharipov

Direct Simulation Monte Carlo

M particles are considered simultaneously

$$M \sim 10^7 \ - \ 10^8$$

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M particles are considered simultaneously

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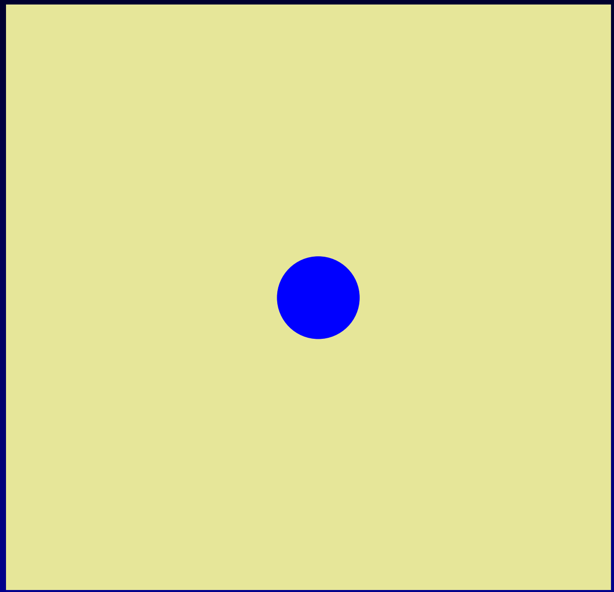
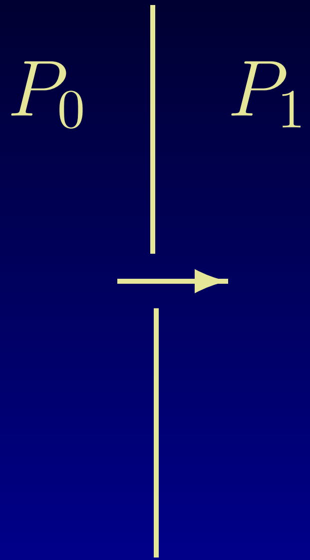
- Free motion of particles
- Interaction with solid surface, Elimination and Generation of particles
- Simulation of collisions
- Calculation of macroscopic quantities

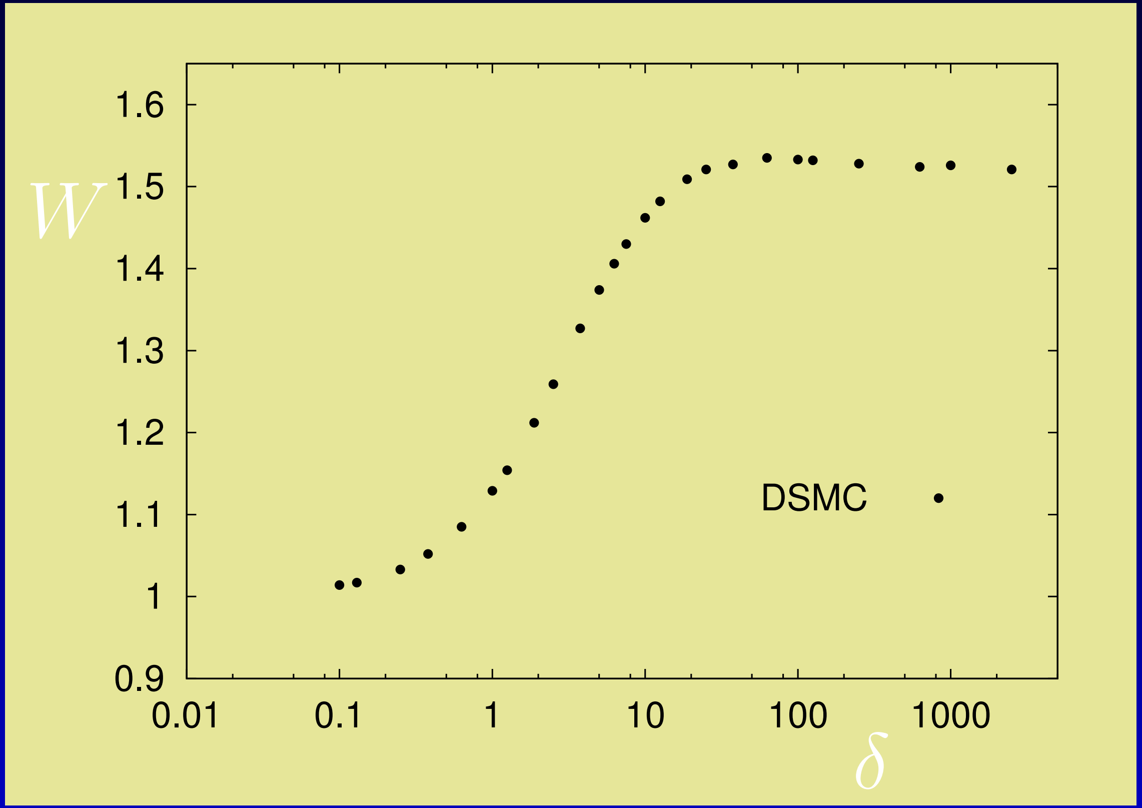
Orifice flow





$$W = \frac{\dot{M}}{\dot{M}_0}, \qquad \dot{M}_0 = \frac{\sqrt{\pi} a^2}{v_m} P_0$$

Orifice flow into vacuum $P_1 = 0$



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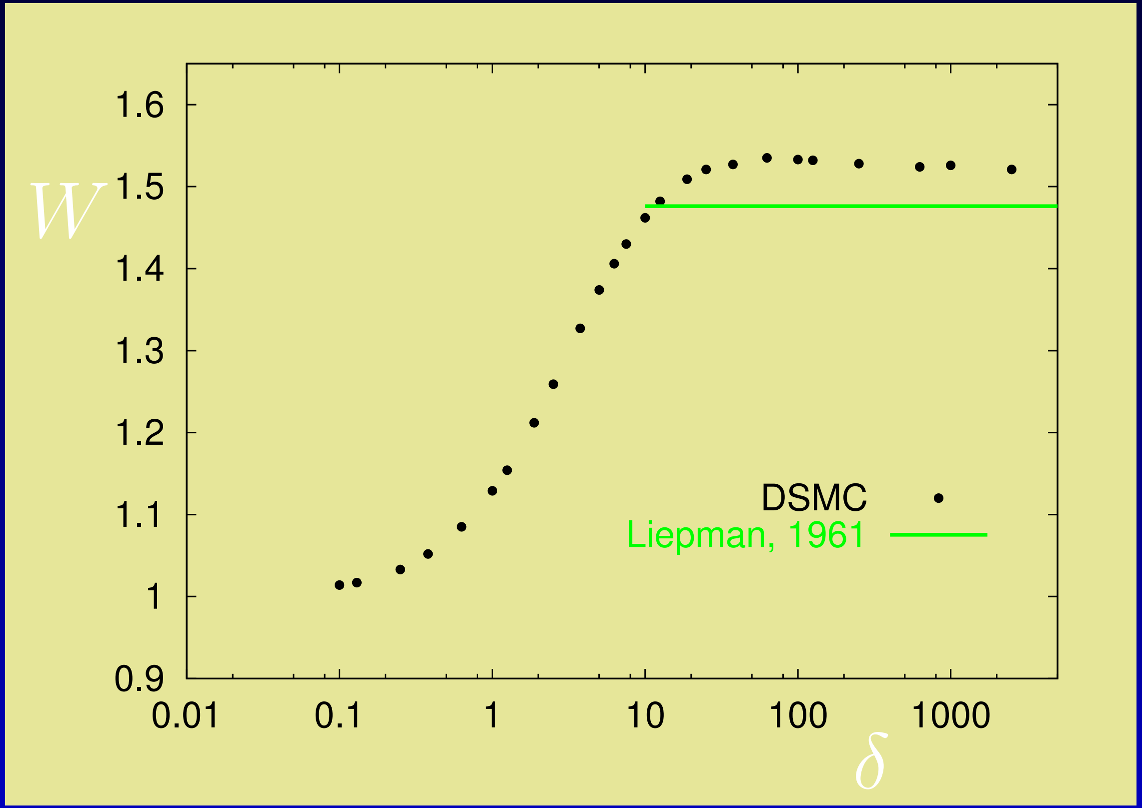

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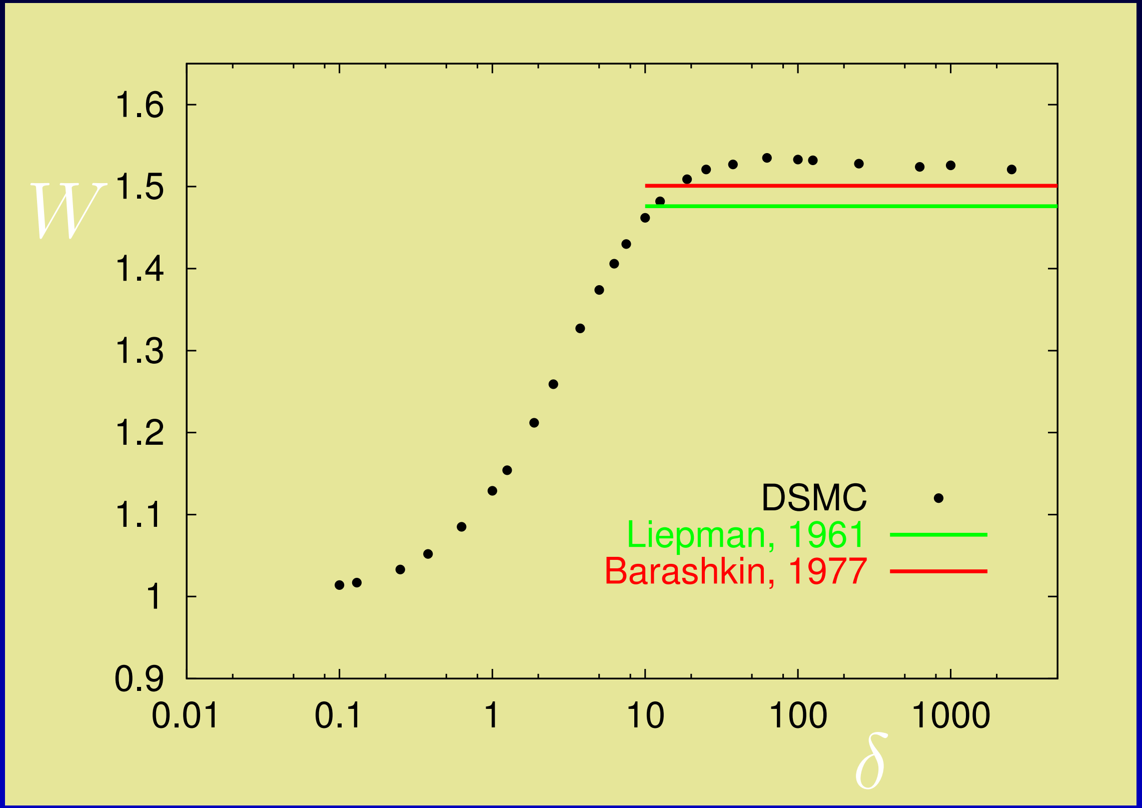

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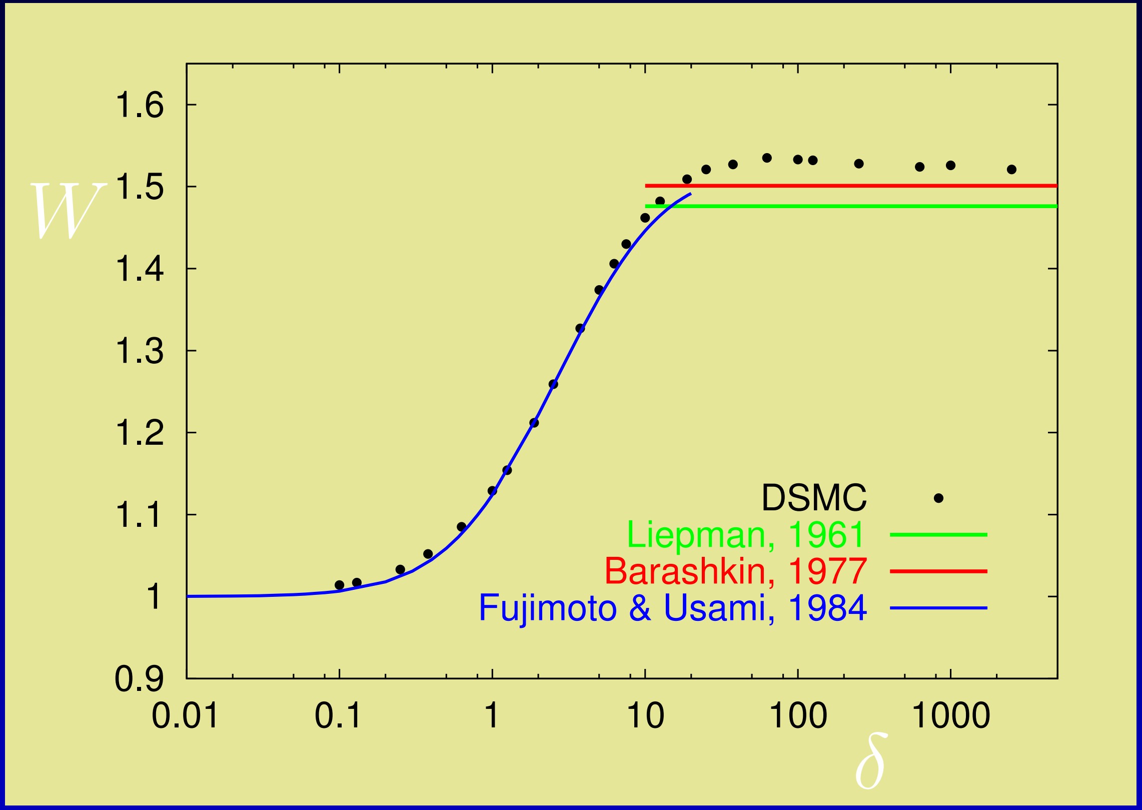

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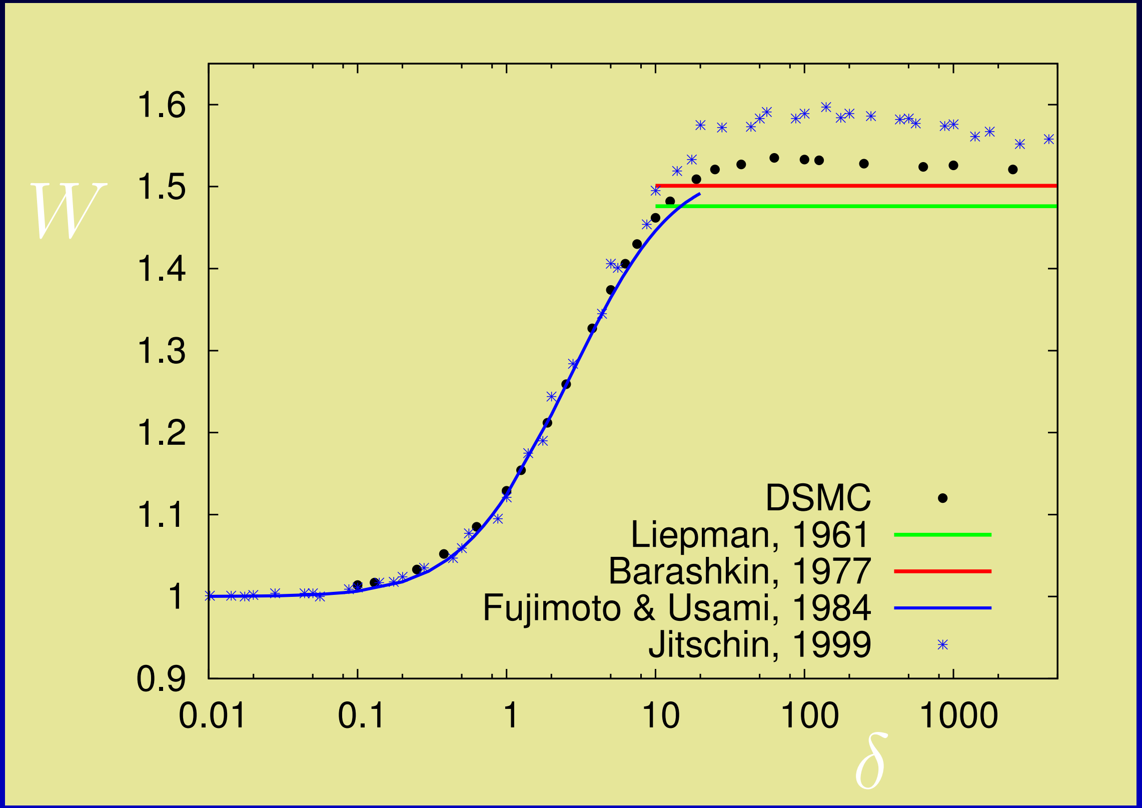
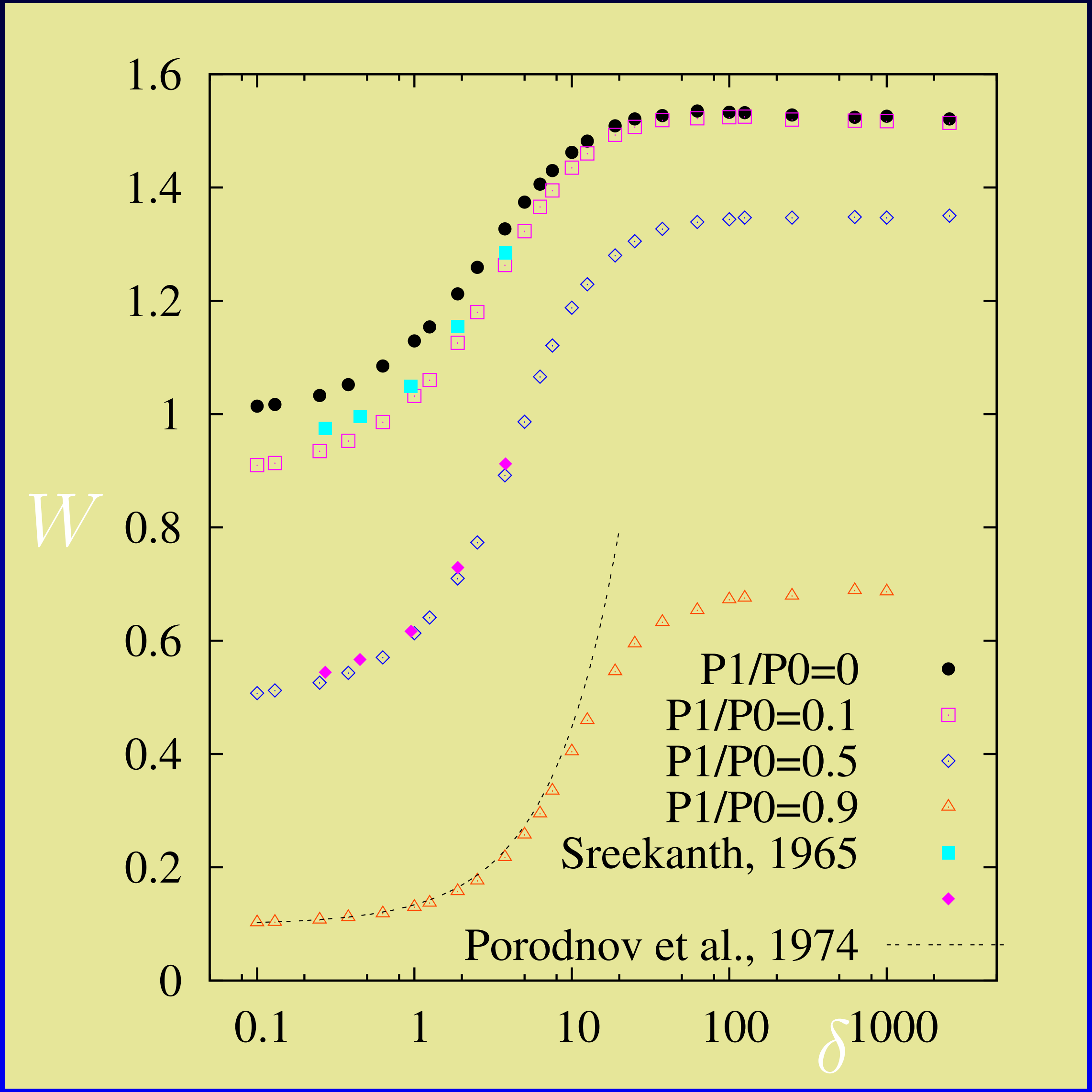
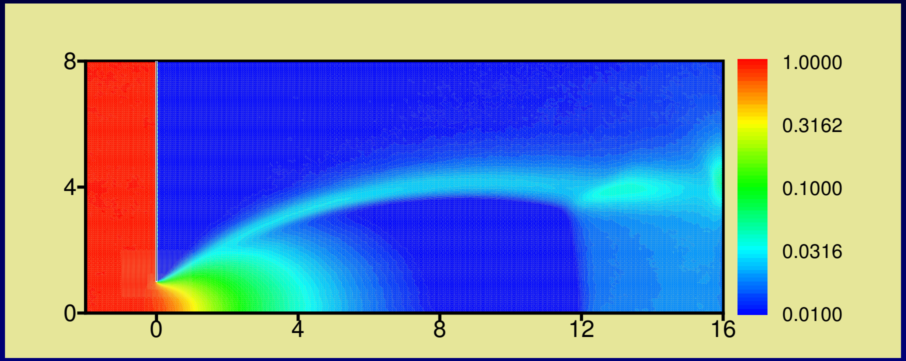


Orifice flow at $P_1 > 0$

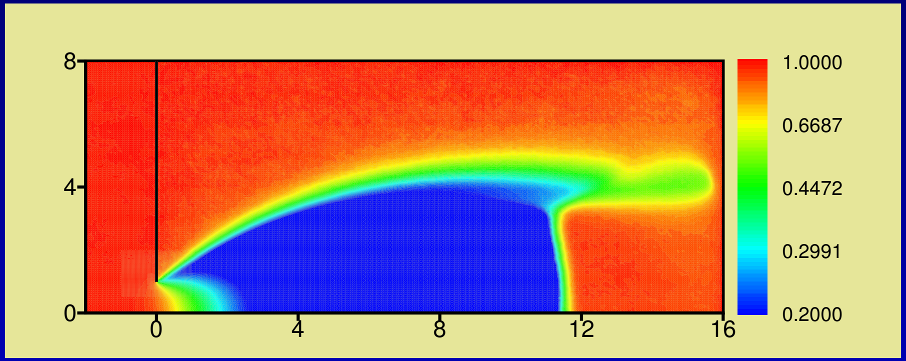


Orifice flow at $P_1 > 0$

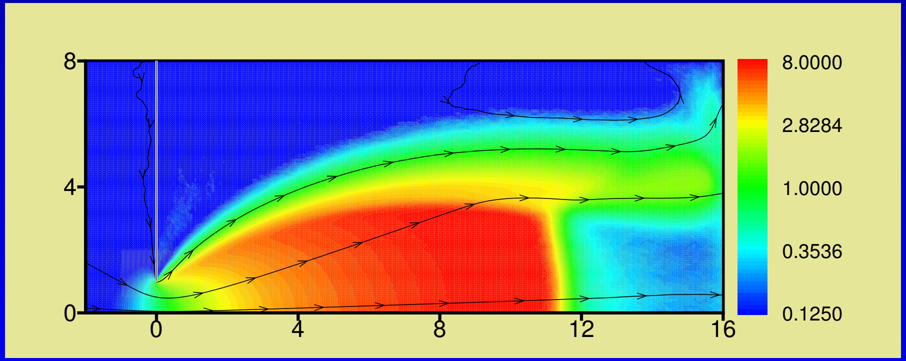
Flowfield at $P_0/P_1 = 100$ and $\delta = 1000$



ϱ/ϱ_0 density



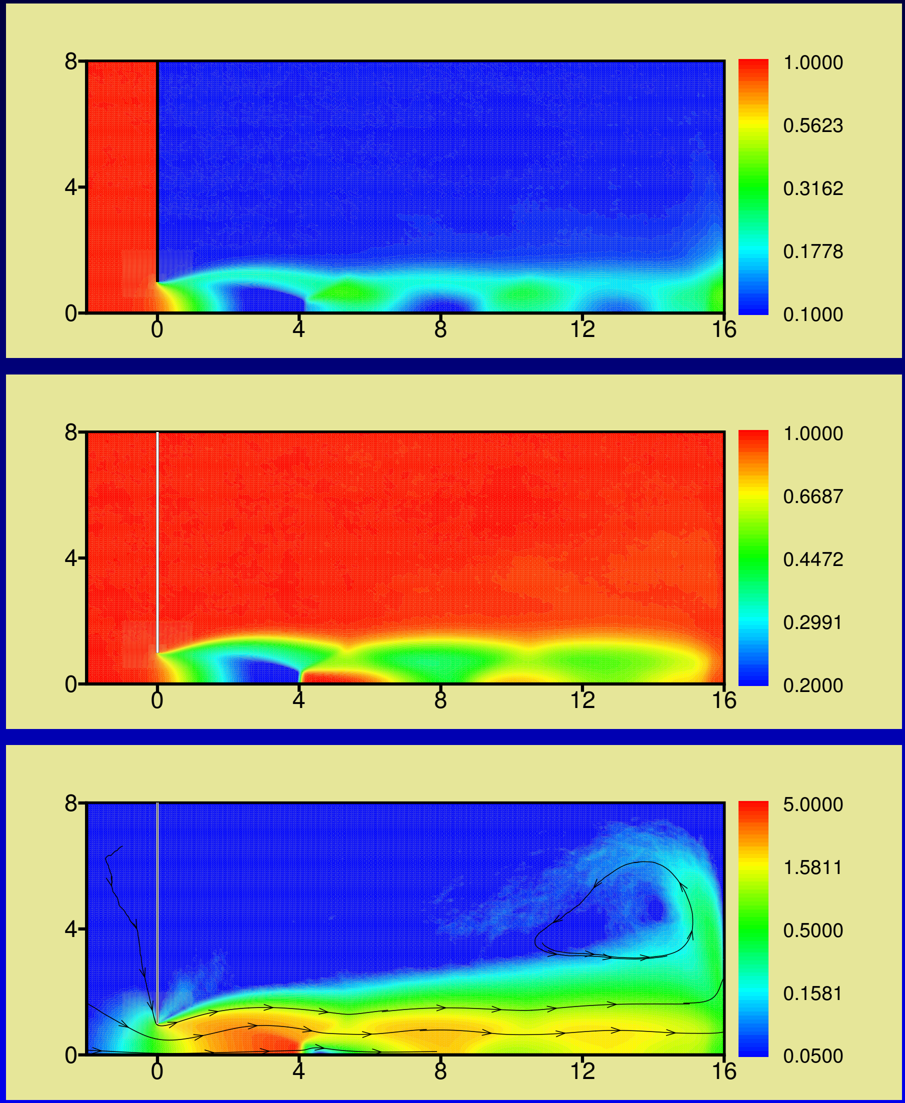
T/T_0 temperature



Mach number

Orifice flow at $P_1 > 0$

Flowfield at $P_0/P_1 = 10$ and $\delta = 1000$

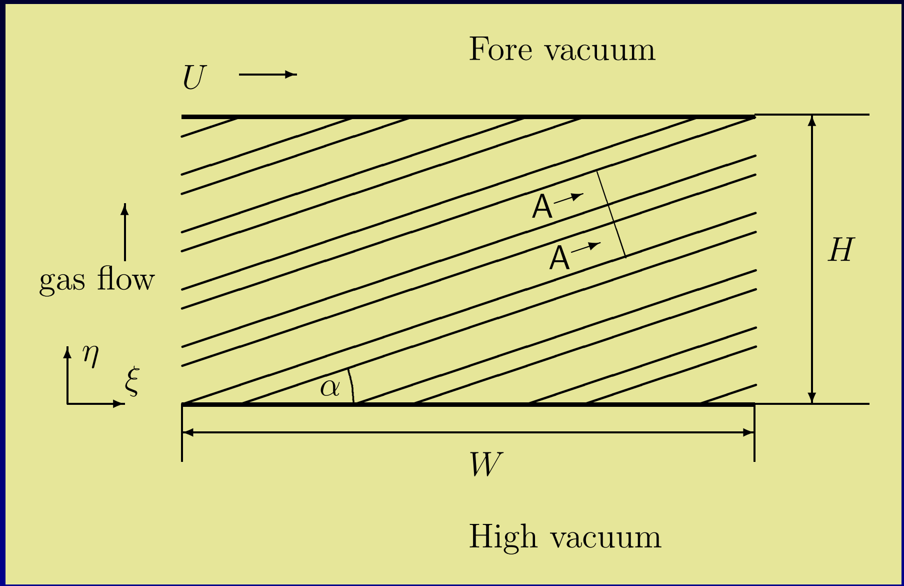


ϱ/ϱ_0 density

T/T_0 temperature

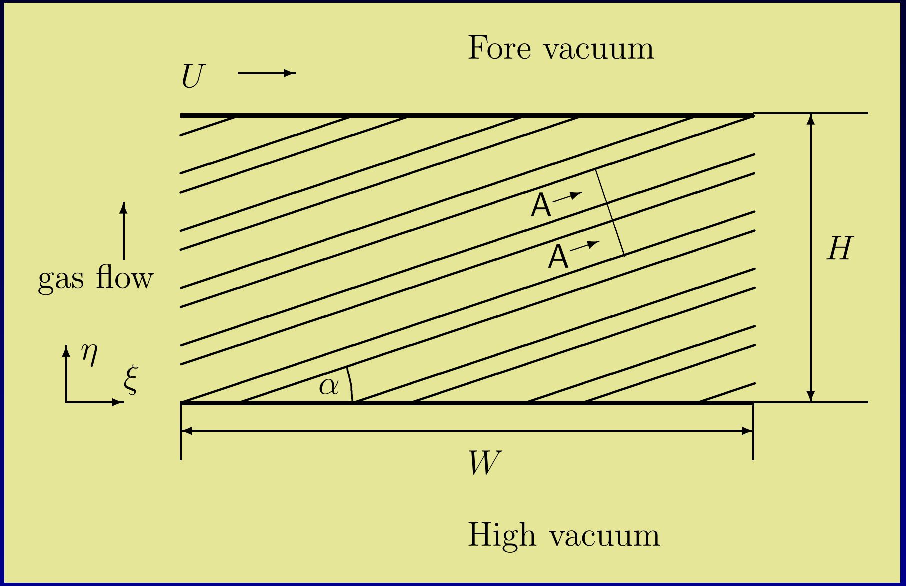
Mach number

Holweck pump

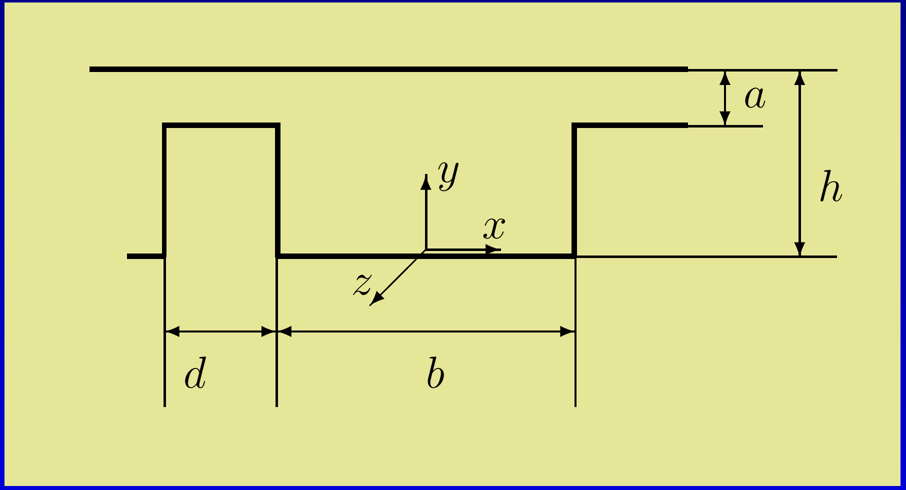


Scheme of pump

Holweck pump



Scheme of pump

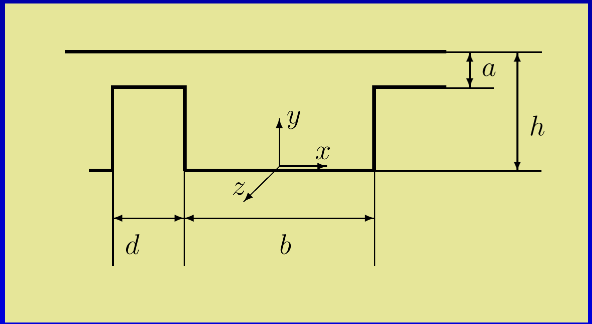


Scheme of groove

Holweck pump. First stage

Four problems are solved

- Poiseuille flow in x direction $G_x^{(P)}(\delta)$
- Poiseuille flow in z direction $G_z^{(P)}(\delta)$
- Couette flow in x direction $G_x^{(C)}(\delta)$
- Couette flow in z direction $G_z^{(C)}(\delta)$



It takes long CPU time

Holweck pump. Second stage

S - pumping speed

$$G_\eta = \frac{S}{v_m \ell^2}$$

$$\ell \frac{dP}{d\eta} = \sin\alpha \frac{\frac{U}{v_m} P \cos\alpha [G_z^{(C)} - \frac{\ell_z}{\ell} G_x^{(C)}] - G_\eta P_h}{G_z^{(P)} \sin^2\alpha + \frac{\ell_z}{\ell} G_x^{(P)} \cos^2\alpha}$$

It takes short CPU time

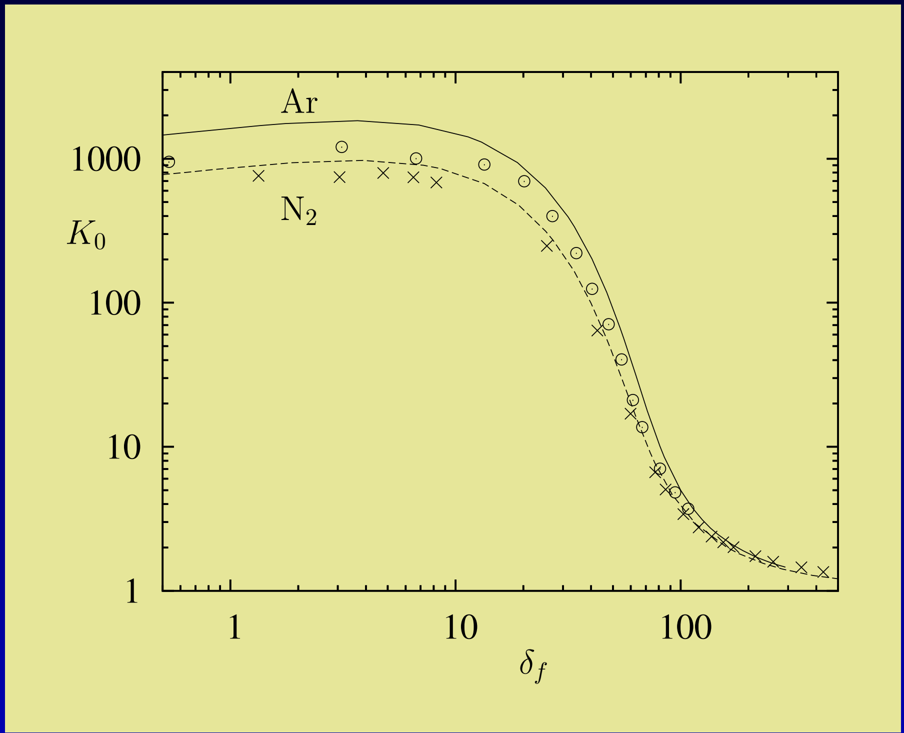
Holweck pump. Second stage

It allows us easily to change many parameters such as:

- groove inclination
- fore vacuum and high vacuum pressures
- angular velocity of rotating cylinder,
- species of gas
- temperature of the gas

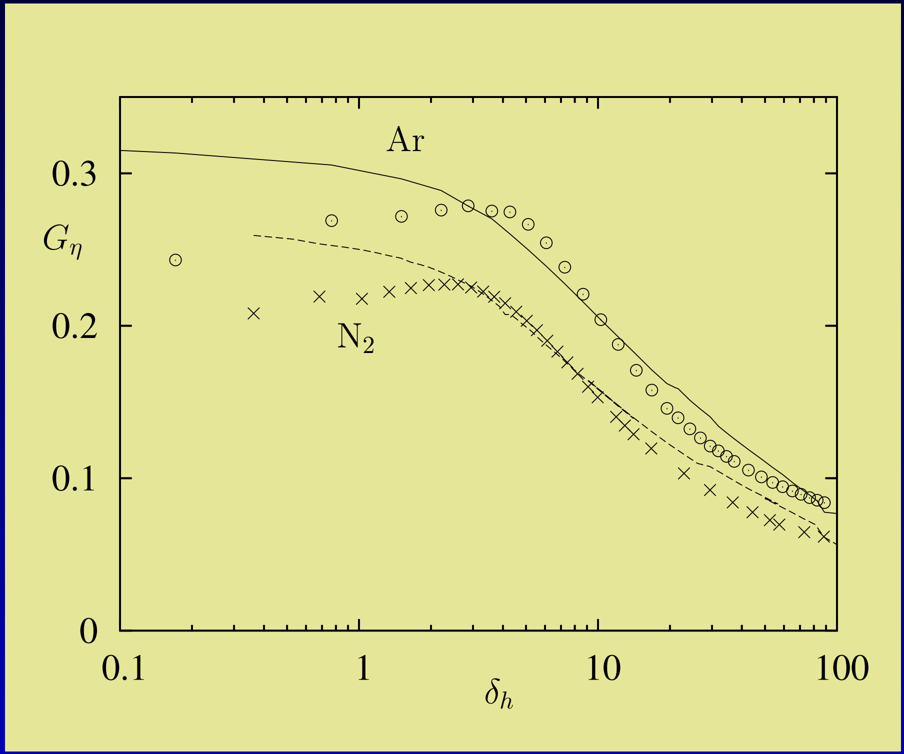
Holweck pump. Results

Limit compression pressure ratio



Holweck pump. Results

Dimensional pumping speed



Recent results

Slip and jump boundary conditions

- Velocity slip coefficients of single gas for different gas-surafce interaction laws

Recent results

Slip and jump boundary conditions

- Velocity slip coefficients of single gas for different gas-surafce interaction laws
- Velocity slip coefficients for gaseous mixtures

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- Temperature jump coefficient of single gas for different gas-surafce interaction laws

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Slip and jump boundary conditions

- Velocity slip coefficients of single gas for different gas-surafce interaction laws
- Velocity slip coefficients for gaseous mixtures
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Recent results

over the whole range of the gas rarefaction

- Single gas flows through long tubes and channels

Recent results

over the whole range of the gas rarefaction

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- Mixture gas flows through long tubes and channels

Recent results

over the whole range of the gas rarefaction

- Single gas flows through long tubes and channels
- Mixture gas flows through long tubes and channels
- Gas flow through orifices and slits

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- Modelling of vacuum pumps

Numerical programs

Some calculations of flow rate through tubes, channels and orifices can be carried out in dimensional quantities on line

http://fisica.ufpr.br/sharipov/

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Thank you for your attention