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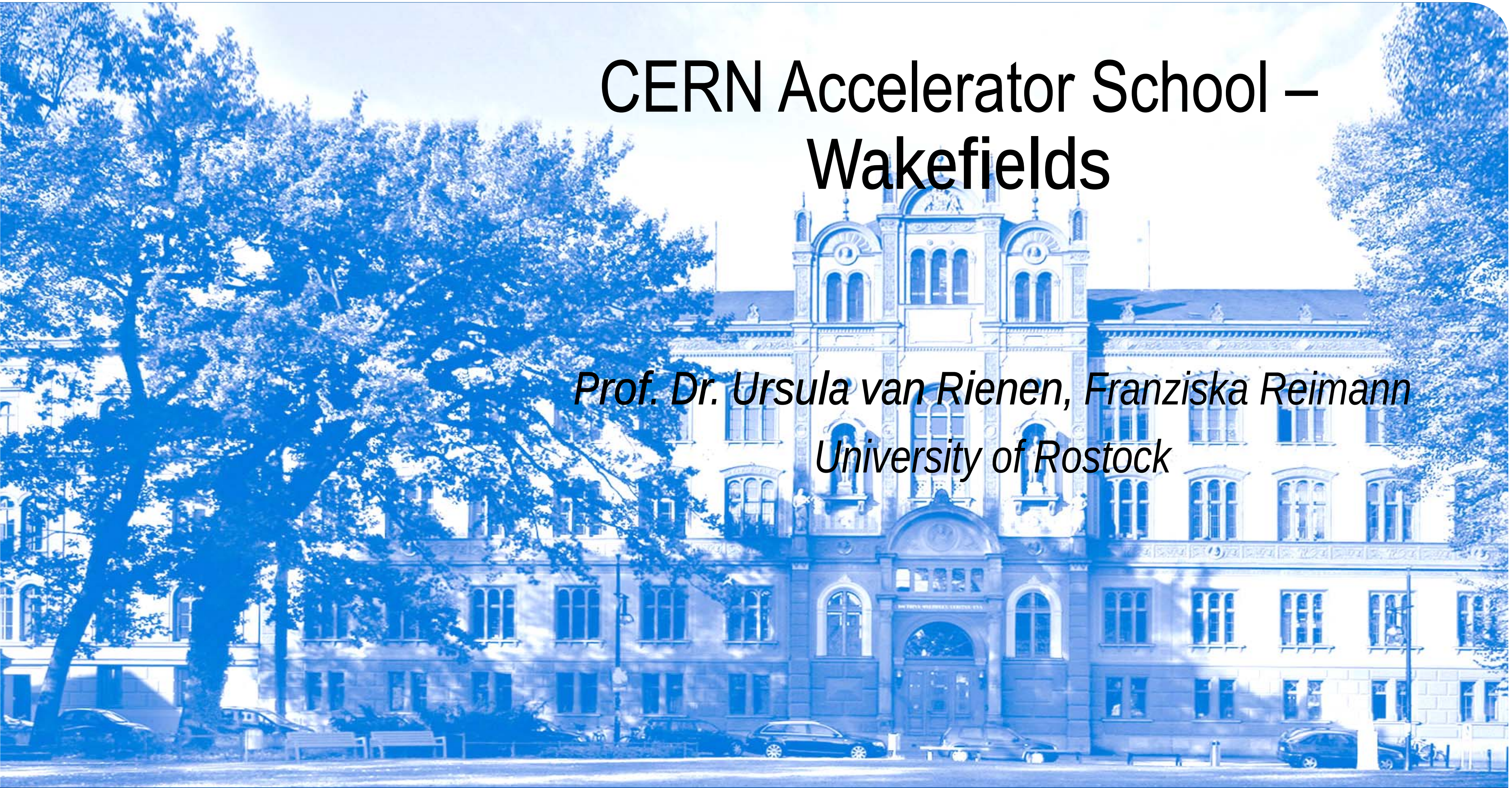

CERN Accelerator School – Wakefields

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University of Rostock

Contents

Follows the lines of: *T. Weiland, R. Wanzenberg: Wake Fields and Impedances.*

The Term „Wakefield“

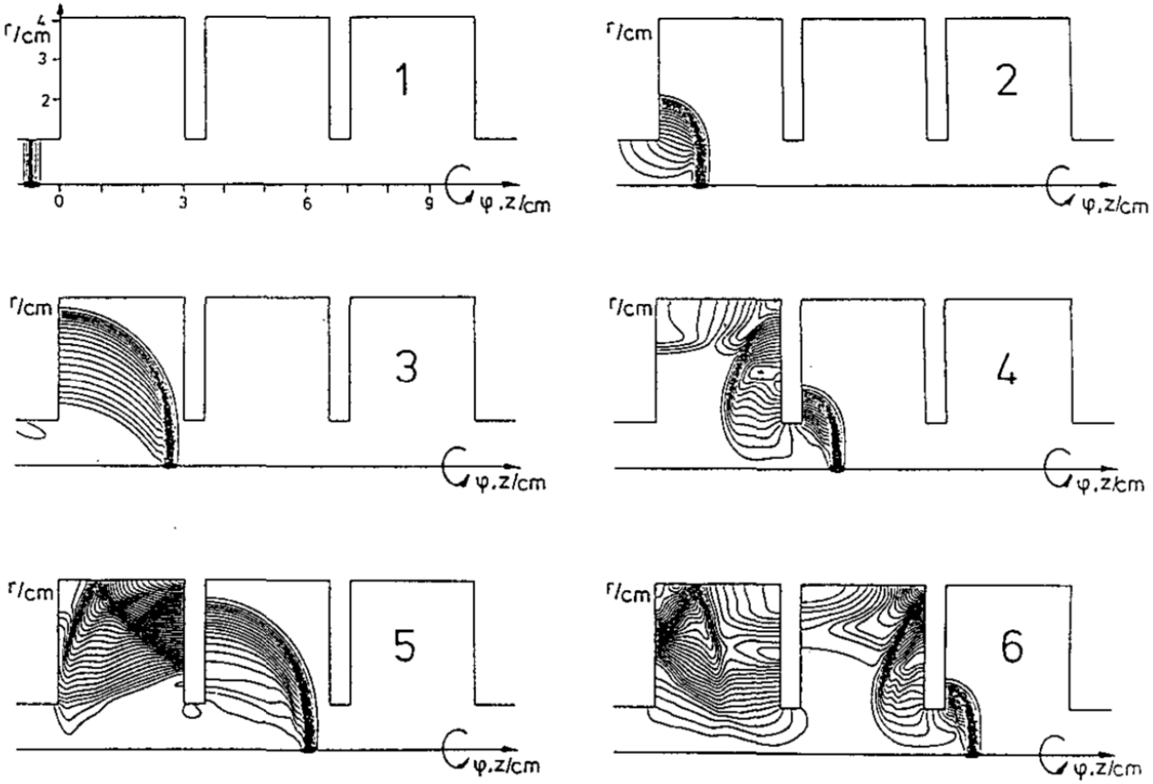
- Outside of accelerator physics:
  - Mostly known from fluid dynamics
  - „Wake“: wave pattern behind objects moving (e.g. ships) in a liquid (e.g. water)

http://i.dailymail.co.uk/i/pix/2009/05/11/article-1180559-04E50EFF000005DC-616_468x286.jpg

https://en.wikipedia.org/wiki/Wake#/media/File:Fjordn_surface_wave_boat.jpg

The Term „Wakefield“

- Gaussian bunch passing a pillbox-like 3-cell cavity:
- In the smooth beam tube (1), the electric field of the ultra-relativistic bunch is purely transversal
- As soon as the cross-section changes, also longitudinal field components occur (2)-(6)
- “Wakefields” are multiply reflected, thus higher order modes can be excited and still oscillate after the bunch left (4)-(6)



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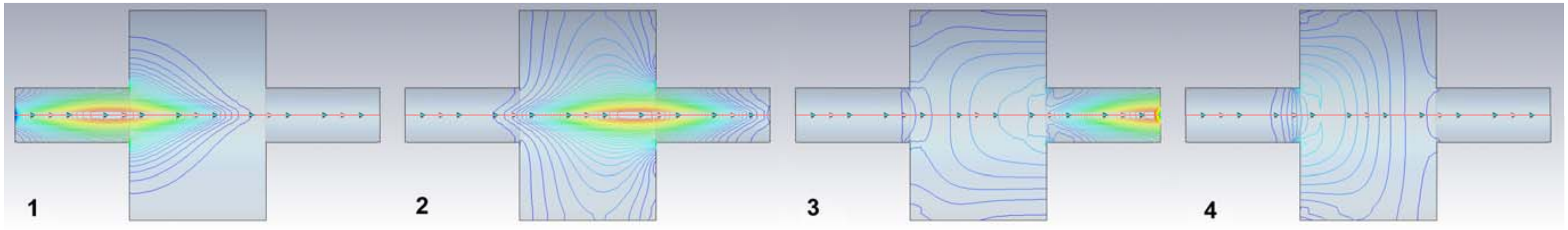
The Term „Wakefield“

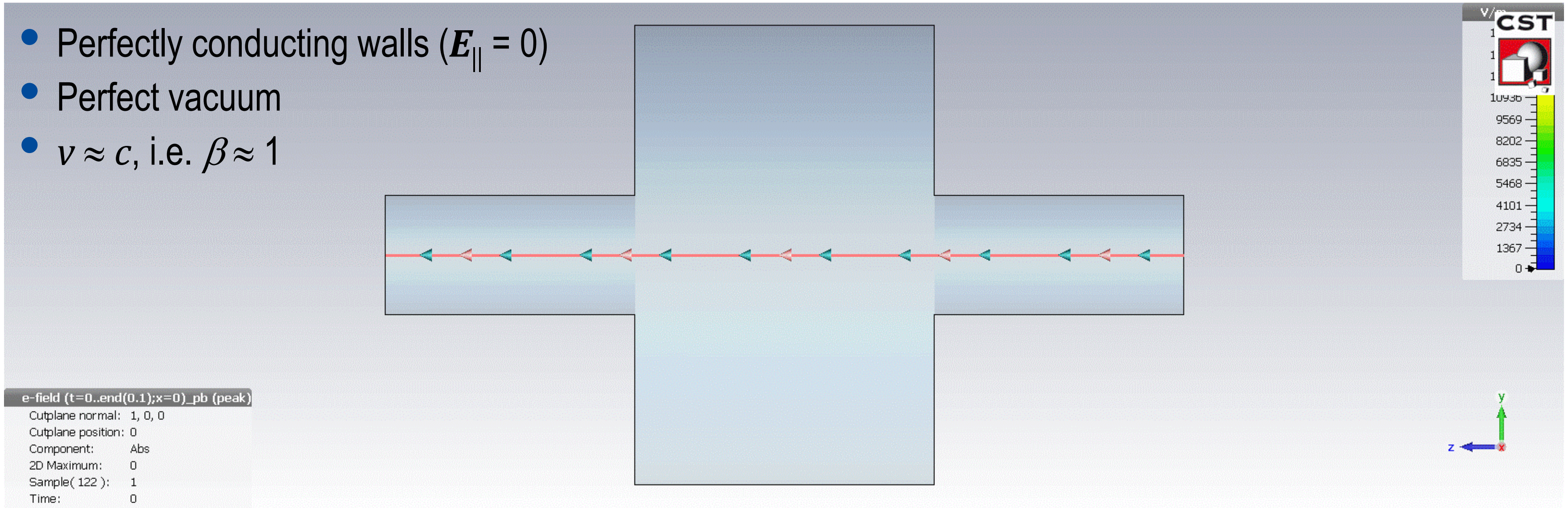
- Bunch of charged particles travels through evacuated beam tube, cavities, etc.
- Induction of surface charges and currents in the enclosing (highly conducting) walls
- The electromagnetic fields have to fulfill the boundary conditions at the metallic walls
- Thus the wakefields are excited at cross-section variations
- Wakefields trail behind the original (bunch of) particles
- Wakefields interact with trailing particles and with following bunches
- Cavity: Wakefields are a superposition of Higher Order Modes



The Term „Wakefield“

- Wakefield of a single bunch:

- Perfectly conducting walls ($\boldsymbol{E}_{||} = 0$)
- Perfect vacuum
- $v \approx c$, i.e. $\beta \approx 1$



(Video)
bunch travels from right to left

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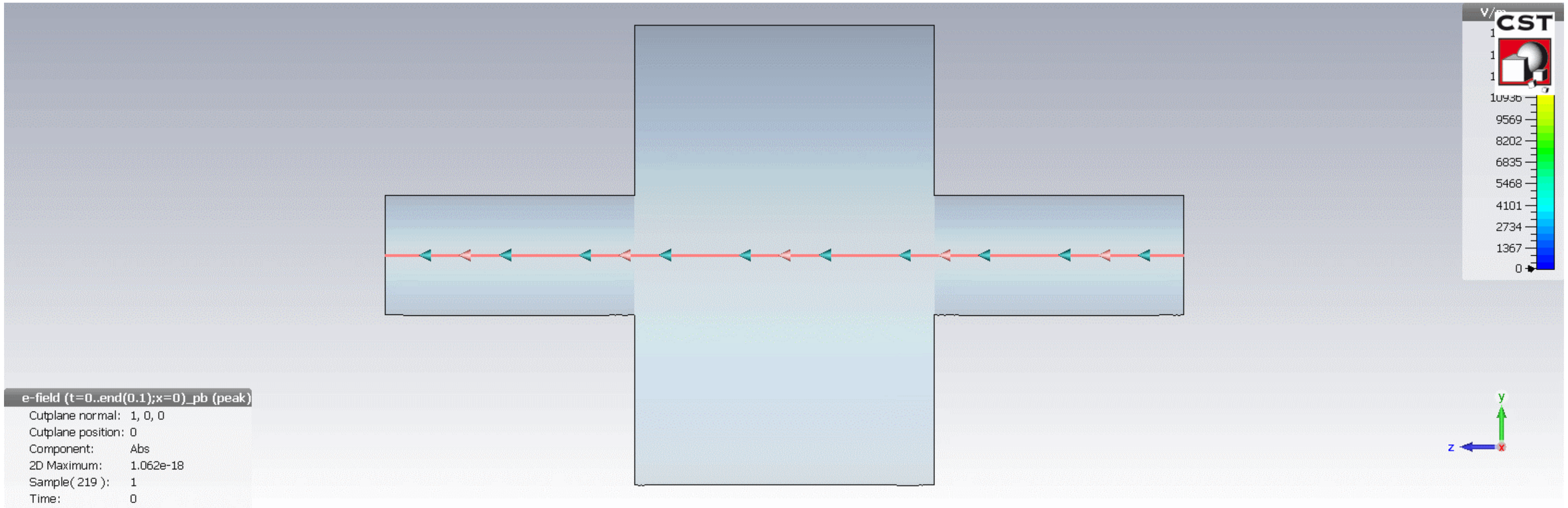
The Term „Wakefield“

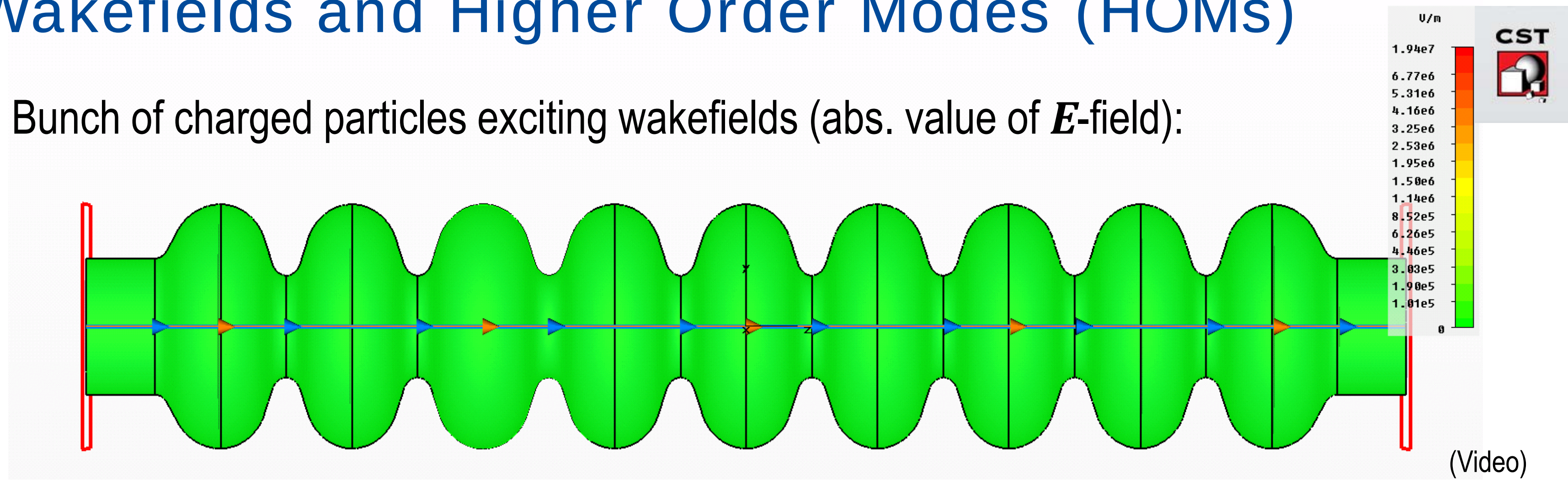
- Wakefield of a bunch followed by another bunch:



(Video)
bunches travel from right to left

Wakefields and Higher Order Modes (HOMs)

- Bunch of charged particles exciting wakefields (abs. value of ***E***-field):



(Video)

- It is possible to decompose the wakefields as superposition of HOMs

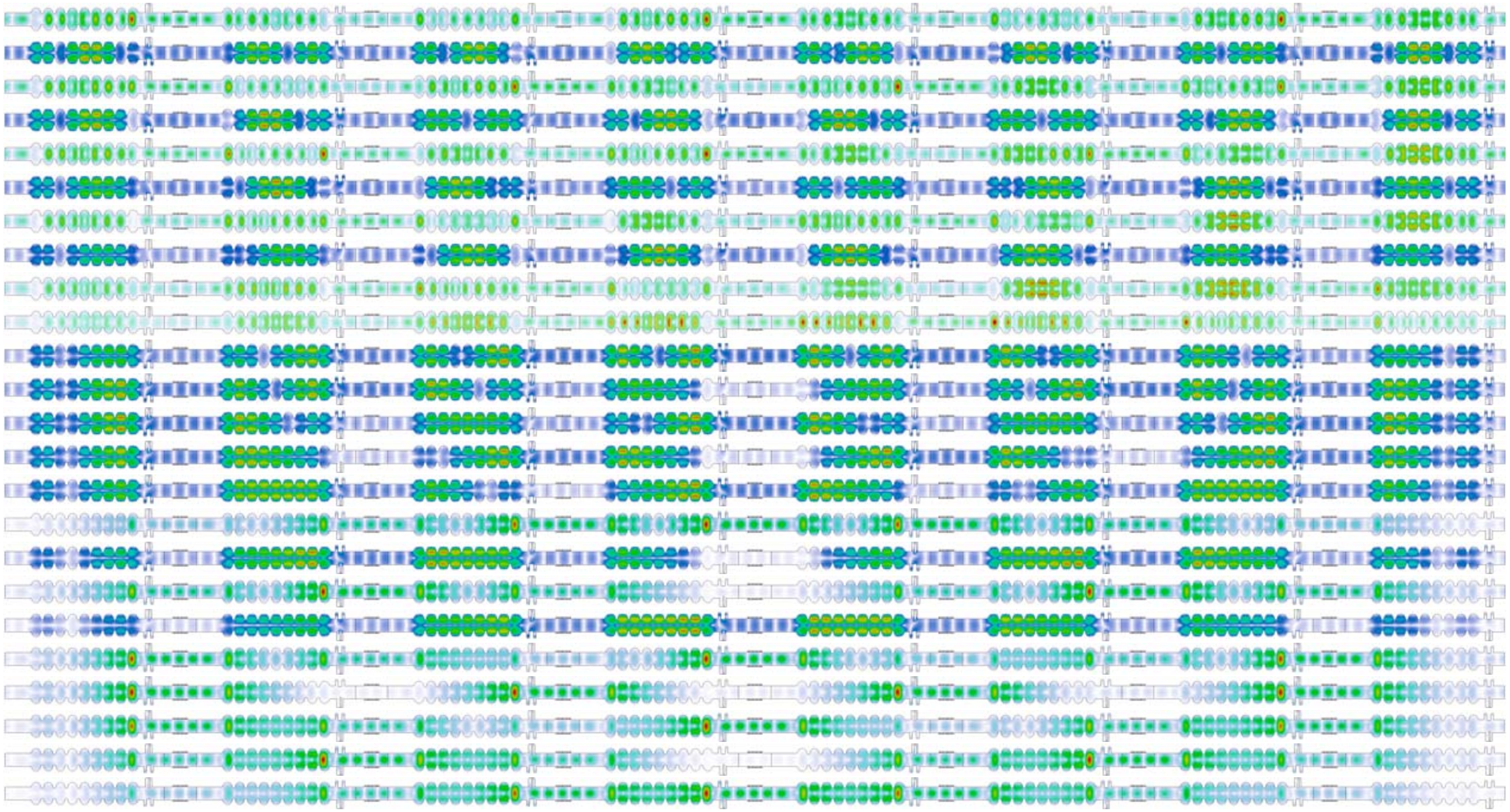
$$\mathbf{E}(\mathbf{r},t) = \sum_{\nu=1}^{\infty} a_\nu(t)\,\tilde{\mathbf{E}}_\nu(\mathbf{r})$$, where $\mathbf{E}(\mathbf{r},t)$ is the wakefield distribution

$a_\nu(t)$ a time-dependent weighting factor

$\tilde{\mathbf{E}}_\nu(\mathbf{r})$ electric field distribution of HOM

Courtesy of T. Flisgen

X-FEL 3rd Harmonic Cavity String: Multi-cavity Modes



24 Dipole modes at about 5.5 GHz

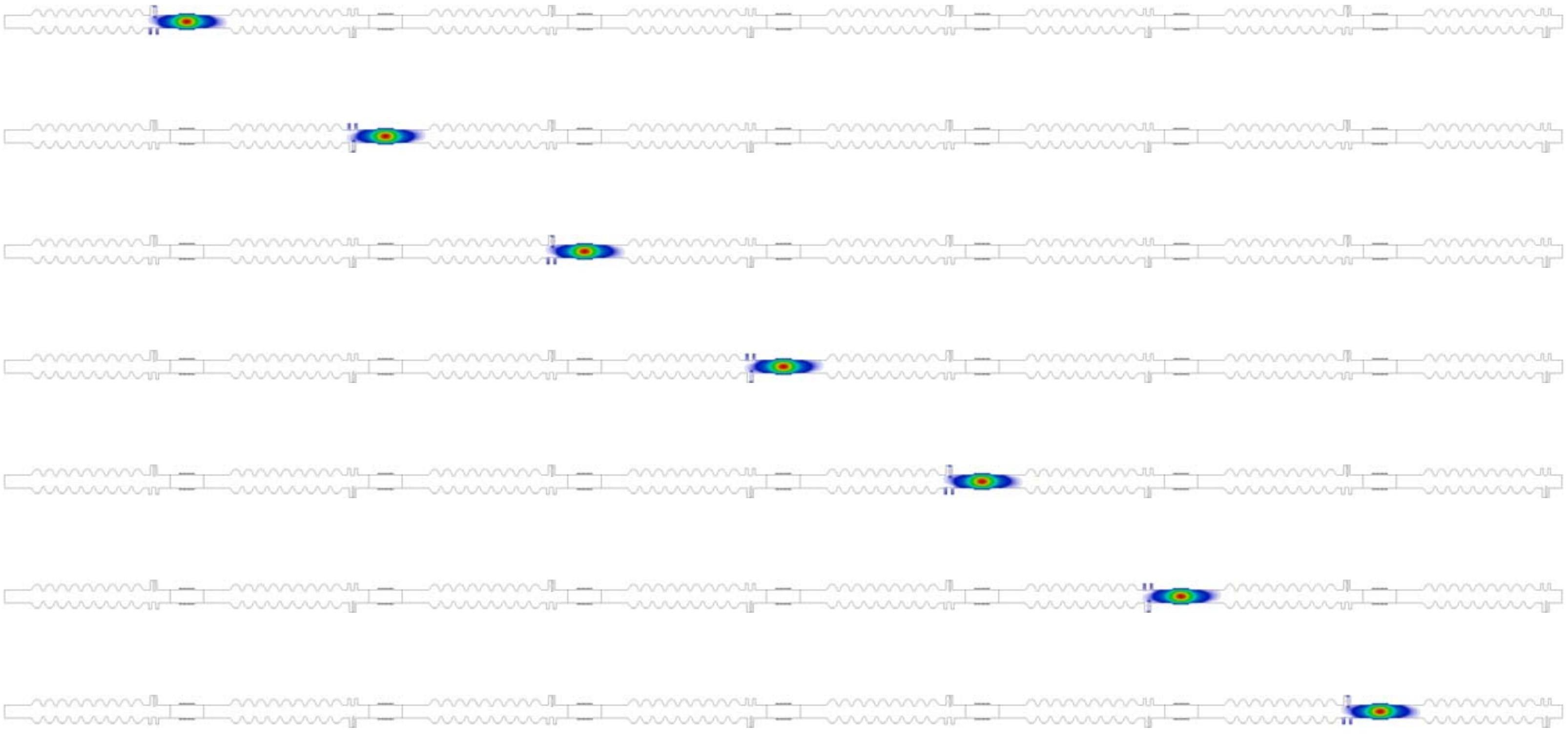
computed by State Space Concatenations (SSC) on a workstation

The plots show the absolute value of the electric field.

Courtesy of T. Flisgen, J. Heller

X-FEL 3rd Harmonic Cavity String: Pure Bellow Modes

at 5.1524 GHz



The plots show the absolute value of the electric field.

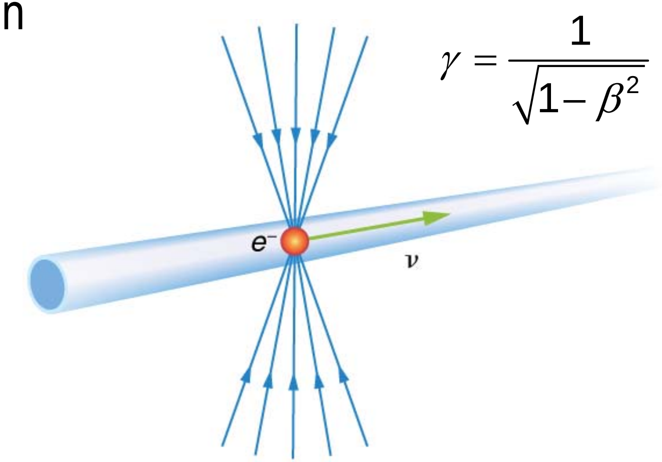
Courtesy of T. Flisgen, J. Heller

Basic Concept of Wakefields

- Point charge q
- Moves at velocity v close to speed of light c in free space: $v = \beta c$
- Lorentz-contracted electromagnetic field: thin disk perpendicular to the moving direction
- If $v \to c$, the thickness of the disk shrinks to a δ- function
- Strictly radial electric field, no components behind or in front of the charge
  - No forces on test particles behind or in front the charge
  - Consequence of the Principle of Causality
  - No wakefield

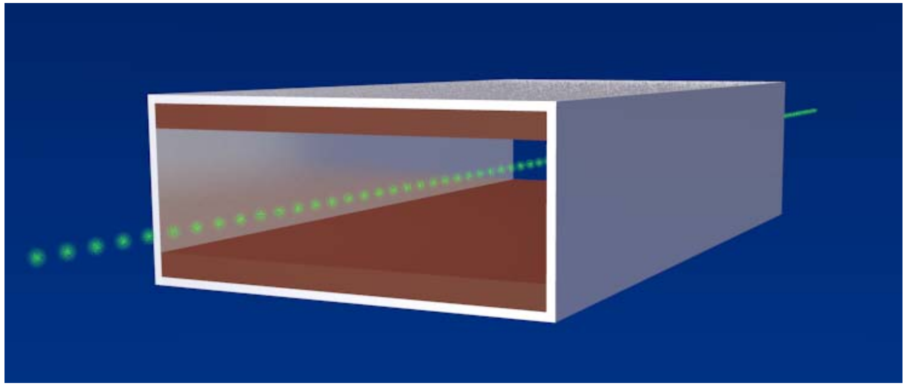
Opening angle

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$



Basic Concept of Wakefields

- To actually generate wakefields behind the charge, special conditions are needed, e.g.
  - Resistive walls, i.e. non-perfect conductors
  - Dielectric walls - due to their lower speed of light that „slows down“ the electric field
  - Obstacles in the beam pipe, geometric irregularities, etc. – mode excitation (see videos above)



Dielectrically lined rectangular waveguide („dechirper“), in which wakefields are generated to counter-act the energy spread of the beam.

Basic Definitions

- Field-generating charge q_1, located at position $\boldsymbol{r} = (x, y, z)$
- Test charge q_2 moving with speed of light c in z-direction (i.e. $\boldsymbol{v} = c\,\boldsymbol{e_z}$)
- Electromagnetic force on q_2: Lorentz force

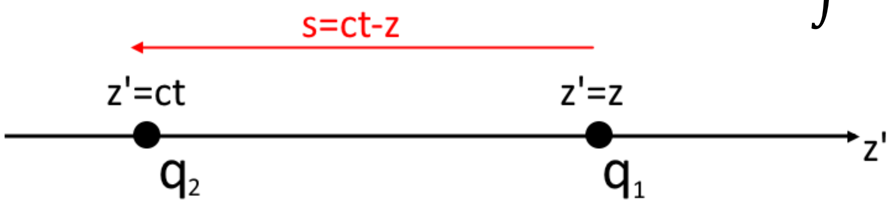
$$\boldsymbol{F}(\boldsymbol{r}, t) = q_2\big(\boldsymbol{E}(\boldsymbol{r}, t) + c\,\boldsymbol{e_z} \times \boldsymbol{B}(\boldsymbol{r}, t)\big)$$

- s: distance between q_2 and q_1 so that $s = ct - z$ ⟹ alternative notation

$$\boldsymbol{F}(s, t) \;=\; \boldsymbol{F}(x, y, z = ct - s, t)$$

- Net momentum change $\Delta\boldsymbol{p}$ of test charge q_2 due to the Lorentz force

$$\Delta\boldsymbol{p} \sim \int \boldsymbol{F}(s, t)\, dt$$

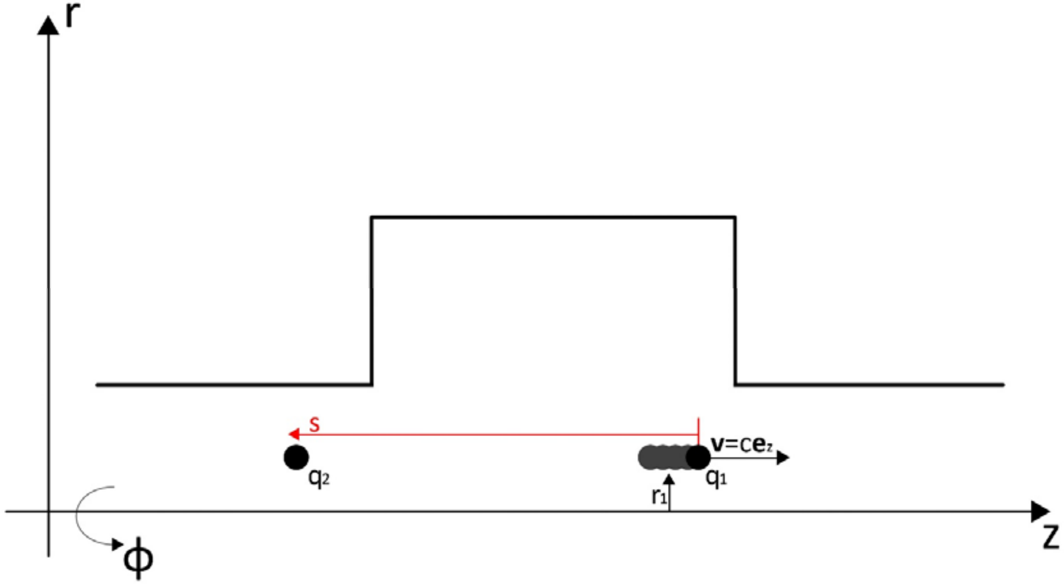


Basic Definitions – Wake Function

Definition: The integral action of the wakefield is described by the wake function

$$\boldsymbol{W}(\boldsymbol{r_1}, s) = \frac{1}{q_1} \int_{-\infty}^{\infty} \left[\boldsymbol{E}(\boldsymbol{r_1}, z, t) + c\boldsymbol{e_z} \times \boldsymbol{B}(\boldsymbol{r_1}, z, t)\right]_{t=(z+s)/c} dz$$

- r_1: (possible) offset of the charges in x- and y- direction (still movement parallel to z-axis)



Basic Definitions – Wake Function

- Momentum change $\Delta\boldsymbol{p}$ of the test charge q_2

$$\Delta\boldsymbol{p} = q_1 q_2 \boldsymbol{W}(s)$$

- The wake function is usually separated into the longitudinal wake function

$$\boldsymbol{W}_{\parallel}(\boldsymbol{r_1}, s) = \frac{1}{\mathrm{q}_1}\int_{-\infty}^{\infty} E_z\left(\boldsymbol{r_1}, z, \frac{z+s}{c}\right) dz$$

where the second term $\boldsymbol{e_z} \cdot (c\boldsymbol{e_z} \times \boldsymbol{B}(\boldsymbol{r_1}, z, t))$ of the Lorentz force vanishes, and the transversal wake function

$$\boldsymbol{W}_{\perp}(\boldsymbol{r_1}, s) = \frac{1}{\mathrm{q}_1}\int_{-\infty}^{\infty} [\boldsymbol{E}_{\perp}(\boldsymbol{r_1}, z, t) + c\boldsymbol{e_z} \times \boldsymbol{B}(\boldsymbol{r_1}, z, t)]_{t=(z+s)/c}\, dz$$

Wake Function and Wake Potential

- Previous definition refers to Dirac-like charge distribution (point charge)
- It can serve as Green’s function to determine the wake potential of an arbitrary charge distribution
- Longitudinal wake potential corresponds to a voltage distribution and describes the energy loss of a single particle as function of the relative position in the beam
- Transverse wake potential describes transversal change in momentum
- Integral over whole passage possible since particle position inside the bunch not influenced for $v \approx c$
- Resonances with high quality factors have a long decay time and thus wakefields can cause collective instabilities

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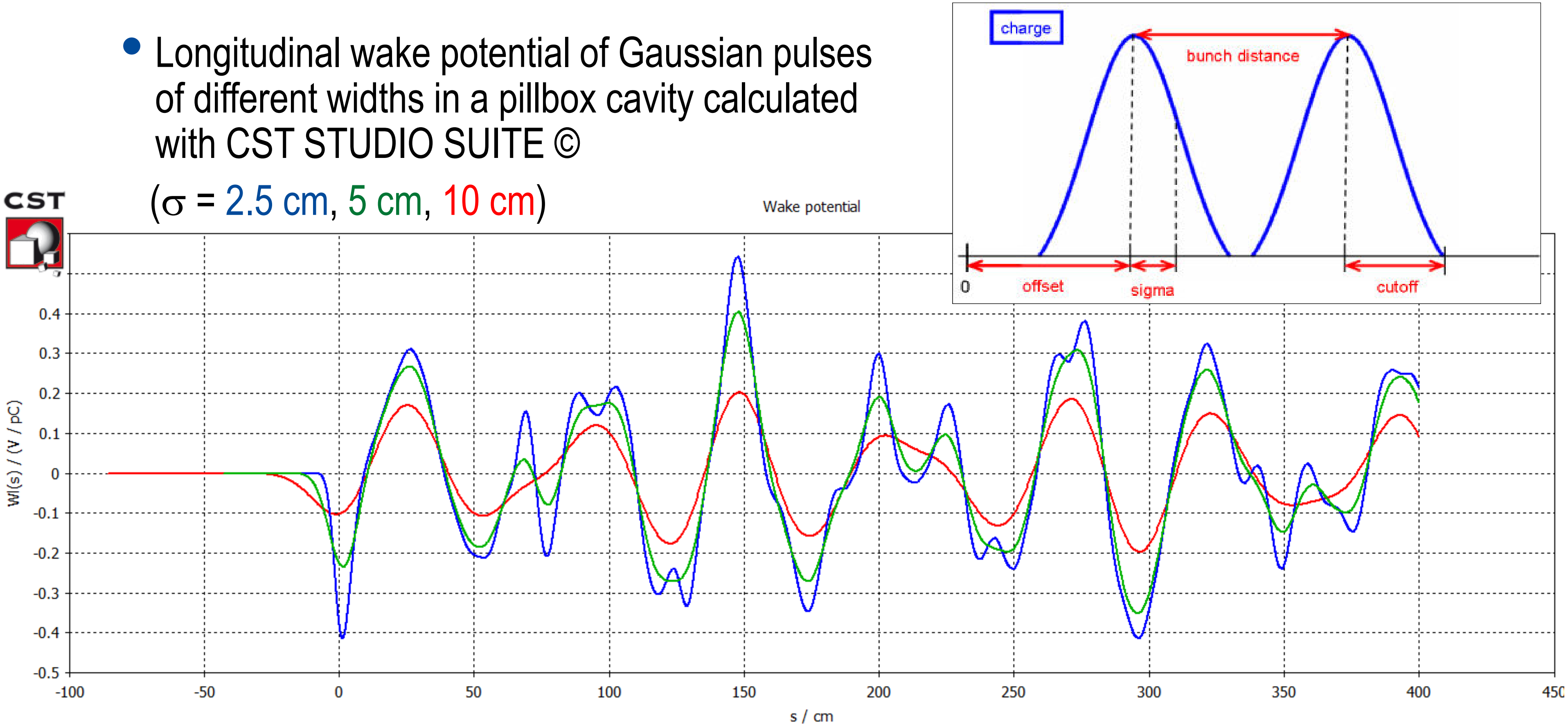
Exemplary Wake Potential

- Longitudinal wake potential of Gaussian pulses of different widths in a pillbox cavity calculated with CST STUDIO SUITE ©

(σ = 2.5 cm, 5 cm, 10 cm)



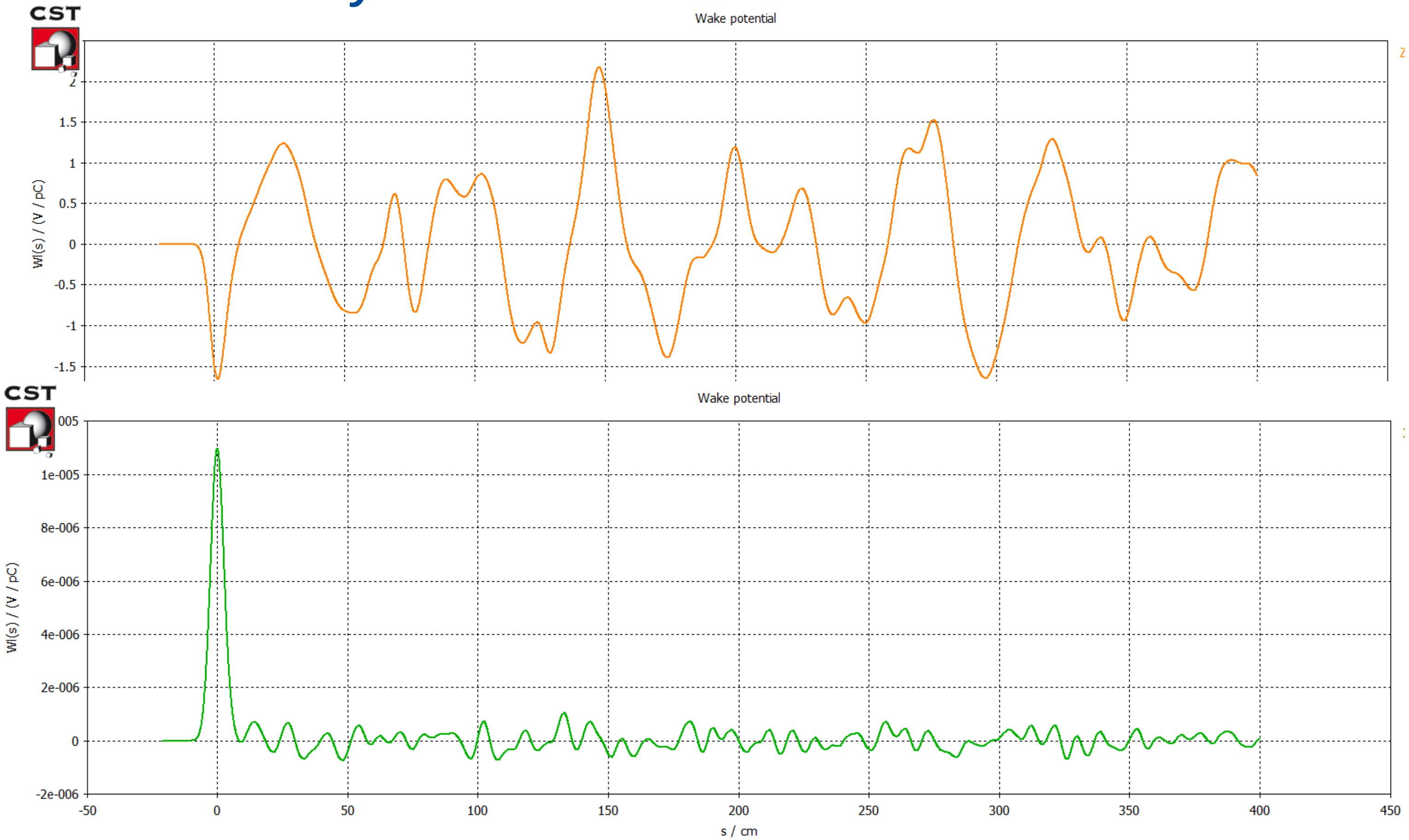
Panofsky-Wenzel-Theorem

- Connects the longitudinal and the transversal wake potential via

$$\boldsymbol{W}_{\perp}(x, y, s) = -\boldsymbol{\nabla}_{\perp} \int_{-\infty}^{s} W_{\parallel}(x, y, s')\, ds'$$

- i.e. the longitudinal derivative of the transverse wake equals the transverse derivative of the longitudinal wake
- Can be derived from the total derivative of the transverse electric field and employing Maxwell‘s equations (see appendix)
- Thus in principle, knowledge of only the longitudinal component of the wake potential suffices to reconstruct the transversal component as well

Panofsky-Wenzel-Theorem



Longitudinal (top) and transversal (bottom) wake potential of a Gaussian pulse in a pillbox cavity

Wakefields and Impedances

Definition: The impedance (often called coupling impedance)

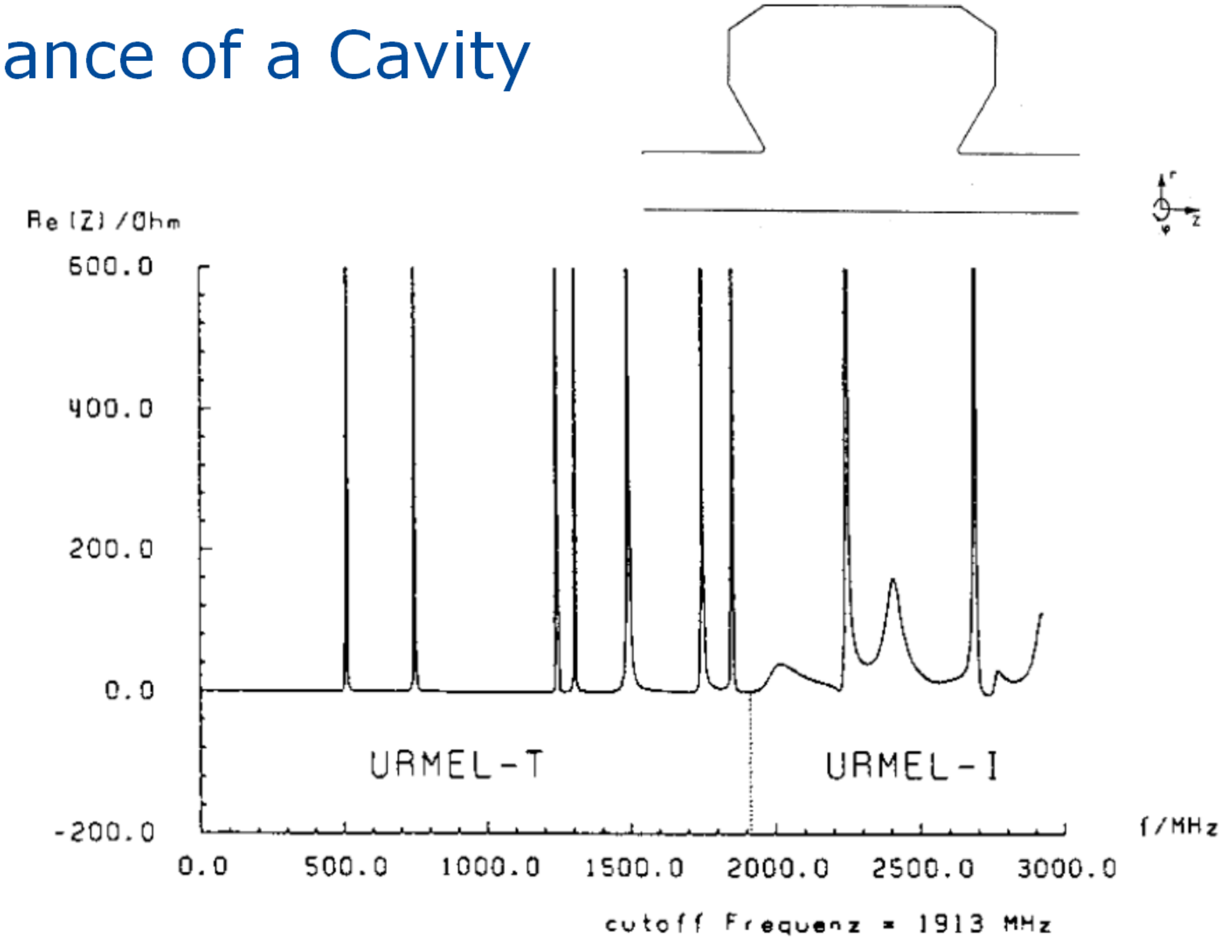
$$Z_{\|}(x,y,\omega) = \frac{1}{c}\int_{-\infty}^{\infty} W_{\|}(x,y,s)\,\exp\left(-i\frac{\omega}{c}s\right)ds$$

is the Fourier transform of the wake potential

- Description of the same fact → coupling between beam and environment
- Wake potential: time domain; impedance: frequency domain
- Impedance ↔ frequency spectrum
  - Shows which of the structure‘s eigenmodes couple with the beam
  - Also indicates the coupling strength

Example: Impedance of a Cavity

- Example: PETRA cavity
- Resonances below cut-off
- "Quasi-resonances" above cut-off
- In general, not only cavities but all insertions in the beam pipe have to be taken into account for the whole impedance budget of an accelerator, e.g. also bellows, collimators, pipe junctions, etc.



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Loss Parameters for Modal Expansion of Wake

- Eigenmode expansion of the electric field in a cavity

$$\boldsymbol{E}(\boldsymbol{r},t)=\sum_{n=0}^{\infty}\chi_n(t)\boldsymbol{E}_{\boldsymbol{n}}(\boldsymbol{r})$$

with spatial eigenmodes $\boldsymbol{E}_{\boldsymbol{n}}(\boldsymbol{r})$

- Energy stored in each mode

$$U_n=\frac{\epsilon_0}{2}\int|\boldsymbol{E}_{\boldsymbol{n}}(\boldsymbol{r})|^2 d^3r$$

- Voltage (per mode) experienced by a point charge at speed of light c

$$V_n=\int_{-\infty}^{\infty}E_{z,n}(z)\exp\left(i\frac{\omega_n z}{c}\right)dz$$

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Loss Parameters

Definition: Loss Parameter k_n

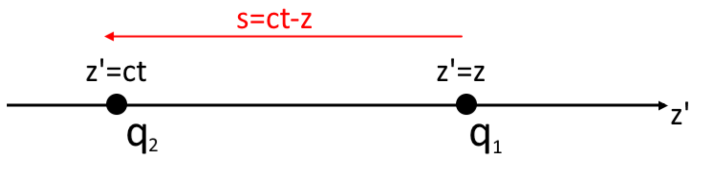
$$k_n = \frac{|V_n|^2}{4U_n}$$

- Wake function of a point charge for $s > 0$

$$W_{\|,0}(s) = \sum_{n=0}^{\infty} 2k_n \cos\left(\frac{\omega_n s}{c}\right)$$

- The wake function of a point charge can be described as a sum of contributions from all modes if the eigenfrequencies and the loss parameters are known
- Energy lost by the point charge q_1 into the mode n

$$\Delta W_n = q_1^2 k_n$$



Loss Parameters - Point Charge Wake Function

- Serves as a Green's Function, i.e. the wake potential of any arbitrary bunch can be derived from it
- Arbitrary bunch shape $\lambda(s)$: distribution of particles over time measured relative to the field-generating particle
- Wake potential of bunch is obtained via convolution of the Green's function with the shape function

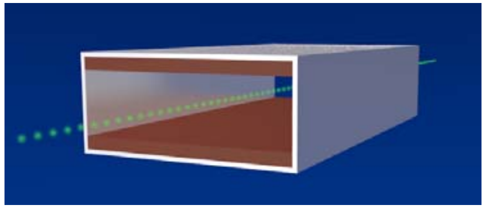
$$W_{\parallel}(s) = \int_0^{\infty} \lambda(s - s')\, W_{\parallel,0}(s')\, ds'$$

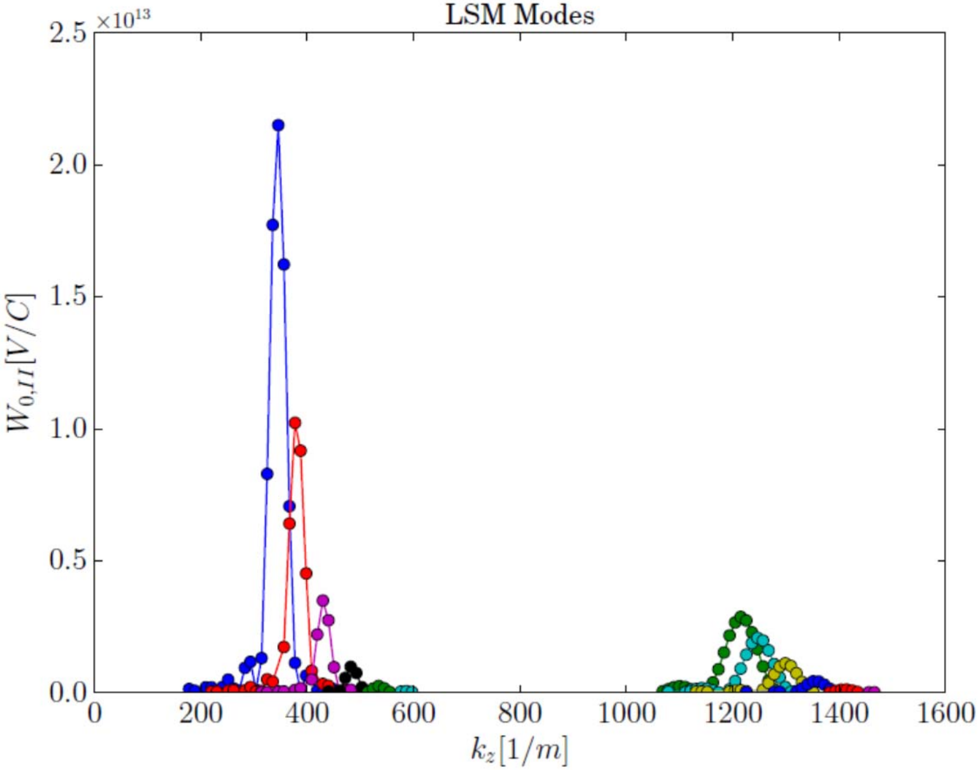
- Total loss parameter

$$k_{tot} = \int_{-\infty}^{\infty} \lambda(s)\, \mathrm{W}_{\parallel}(\mathrm{s})\, ds$$

Loss Parameters - Examples

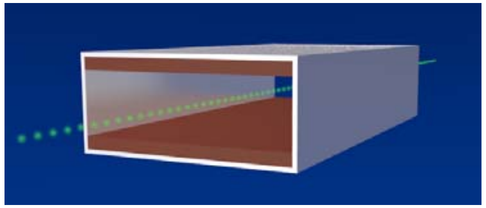
- Loss parameters for different eigenmodes of dielectrically lined rectangular waveguide (dechirper)
- Several modes have higher loss parameters than others
  - High loss parameter ⟶ strong coupling with the beam
  - Some modes (in this case, most of them) have very low loss parameters ⟶ these modes couple only weakly with the beam

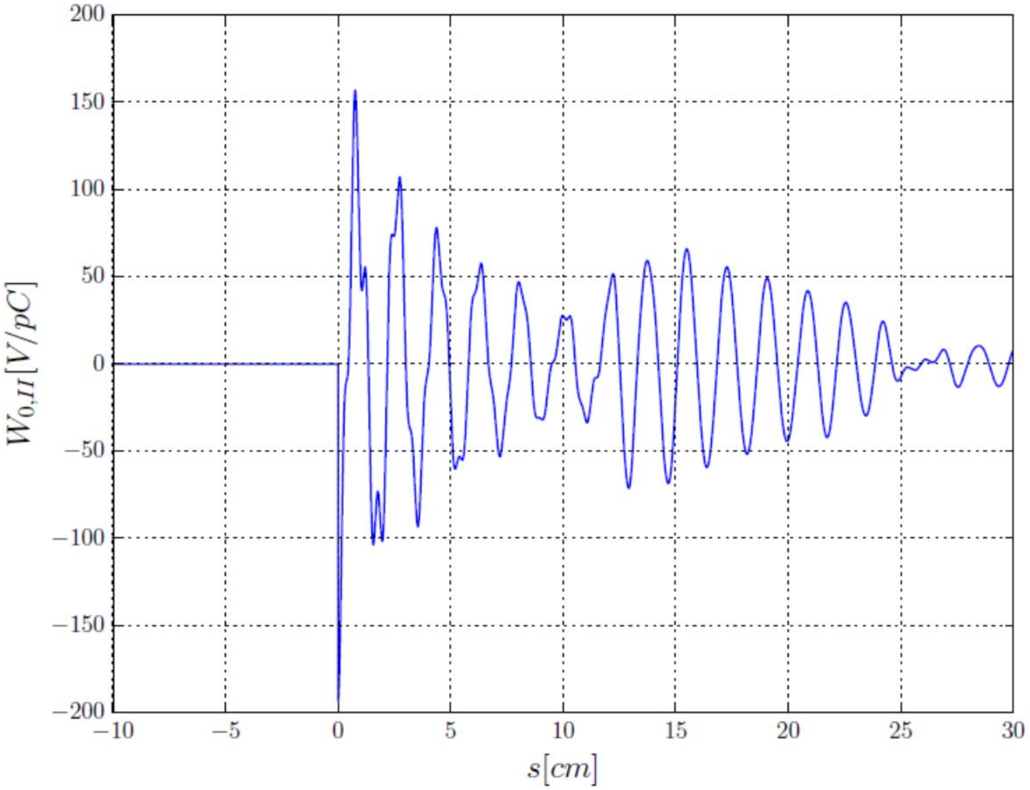




Loss Parameters – Point Charge Wake Function

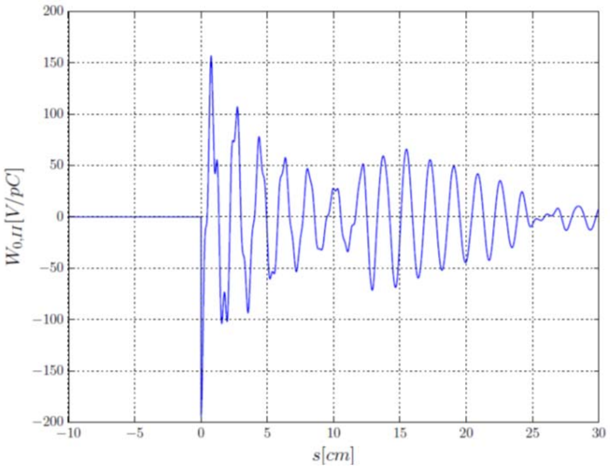
- Point charge wake function for a dielectrically lined rectangular waveguide calculated from eigenmode expansion

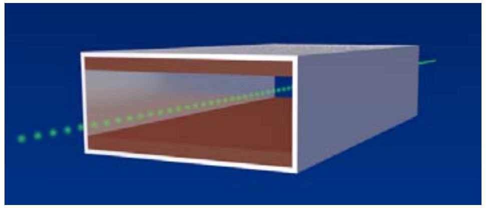


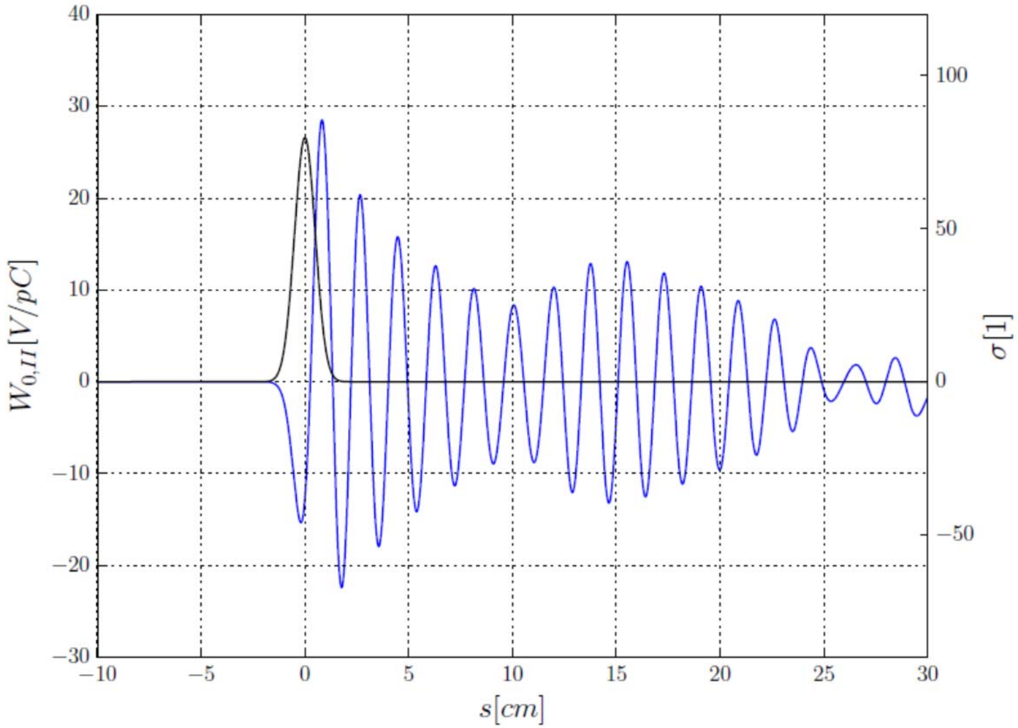


Loss Parameters – Wake Potential for Bunches

- Convolution of the point charge wake function with a Gaussian pulse ($\sigma = 5$ mm) …







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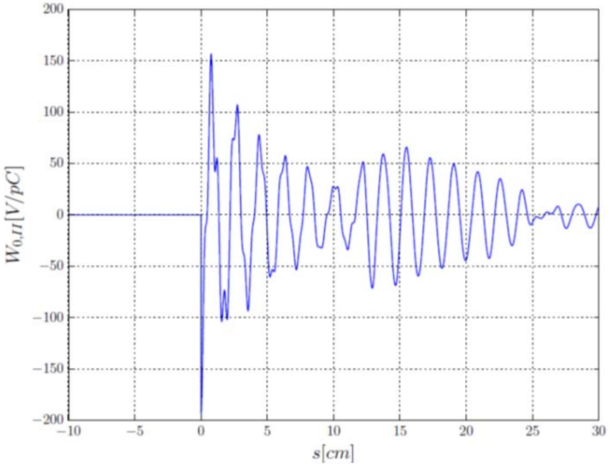
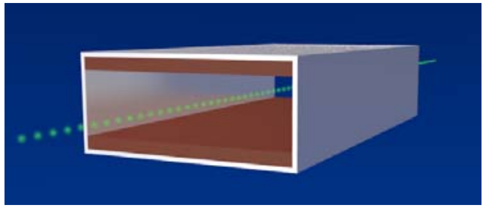
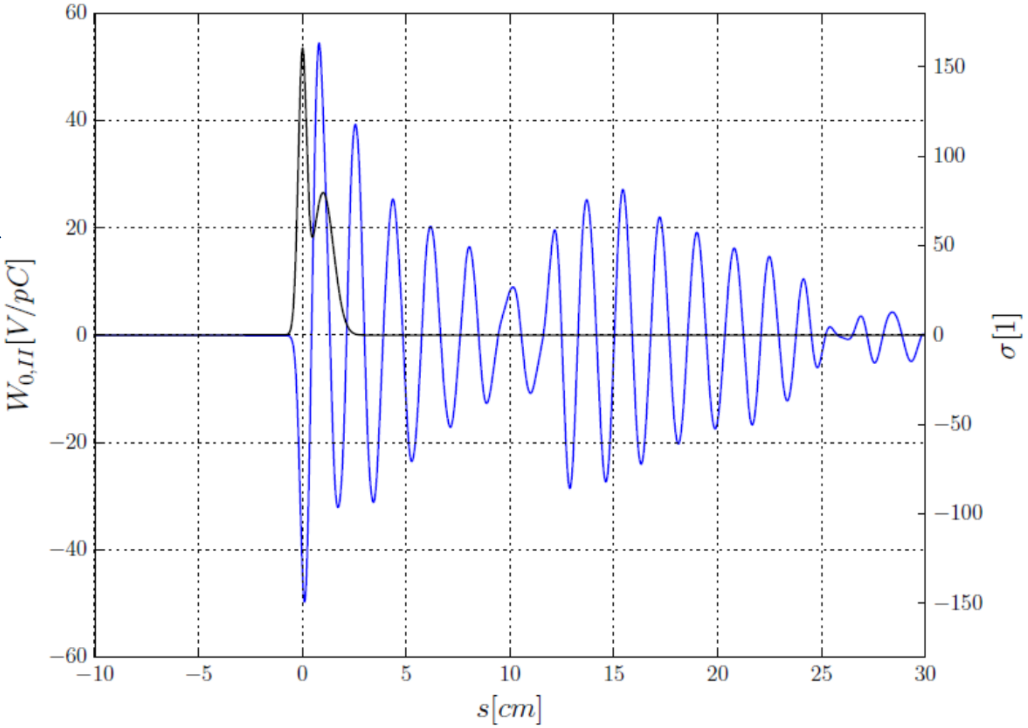
Loss Parameters – Wake Potential for Bunches

- ... or with a Gaussian double pulse ($\sigma_1 = 5$ mm, $\sigma_2 = 2$ mm) ...

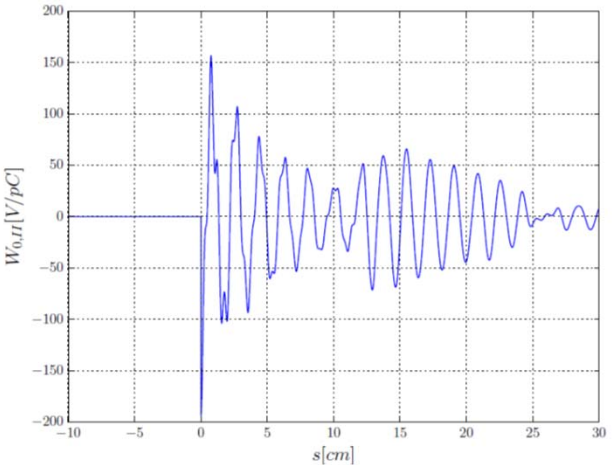


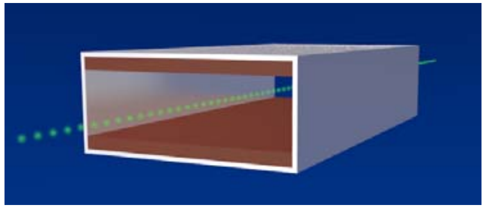


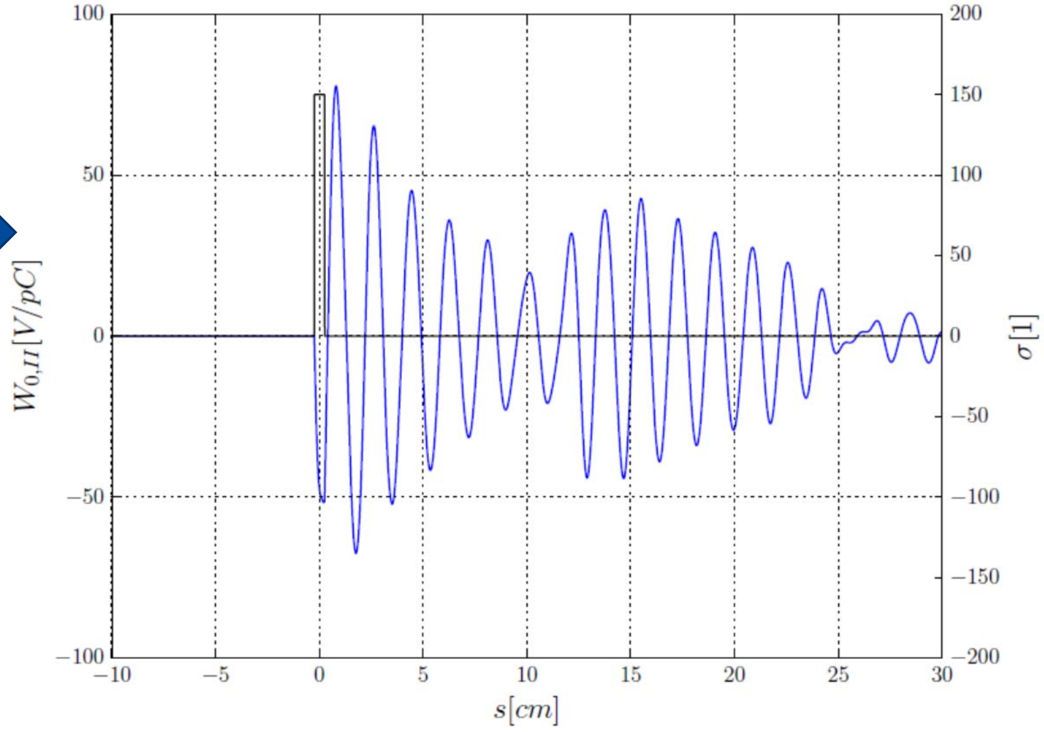


Loss Parameters – Wake Potential for Bunches

- ... or with a flat top pulse ($l = 5$ mm)

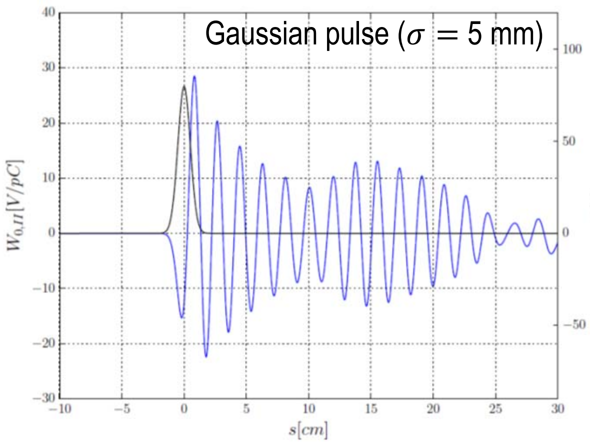


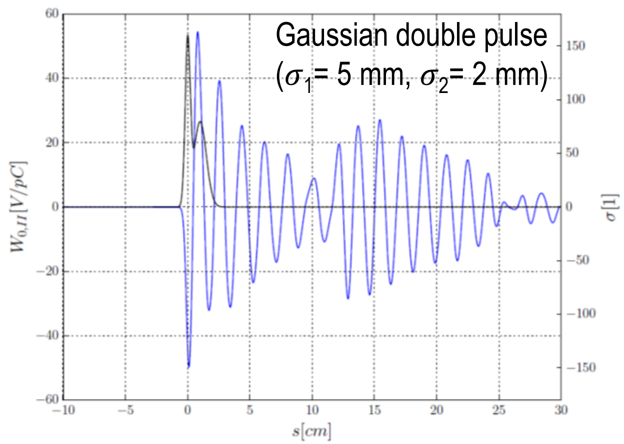


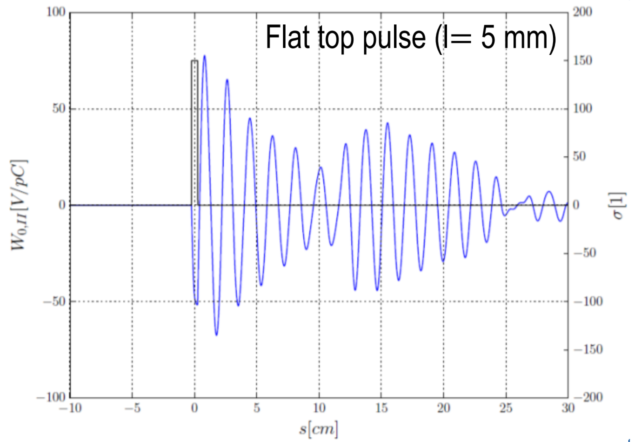


Loss Parameters – Wake Potential for Bunches

- Long range behavior: similar for all three bunch forms
- BUT: very different short range behavior (i.e. in the vicinity of the bunch)







Peak wake potential:

$$\sim 29 \frac{V}{pC} \qquad \sim 55 \frac{V}{pC} \qquad \sim 70 \frac{V}{pC}$$

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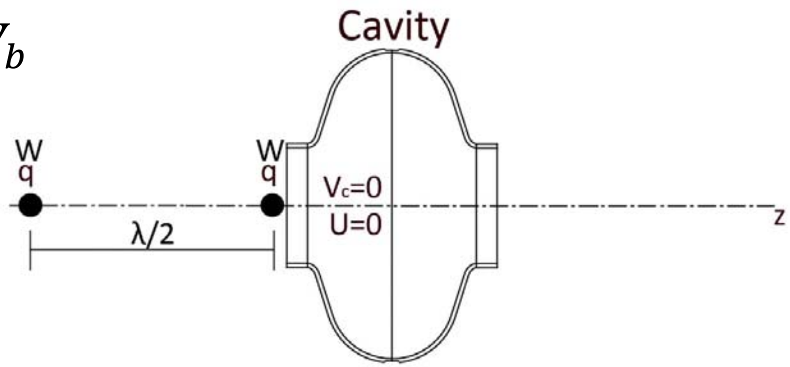
Fundamental Theorem of Beam Loading

- Example: particle 1 with charge q moves through a cavity
  - moving charge $\longrightarrow$ image current on the cavity wall $\longrightarrow$ electric field opposing the particle motion $\longrightarrow$ voltage $-V_b$ remains when q leaves the cavity
  - energy conservation $\longrightarrow$ energy must be left behind in the cavity
- q „sees“ a fraction a of its own induced voltage

$$V_{q1} = -aV_b$$

- Energy loss of particle 1:

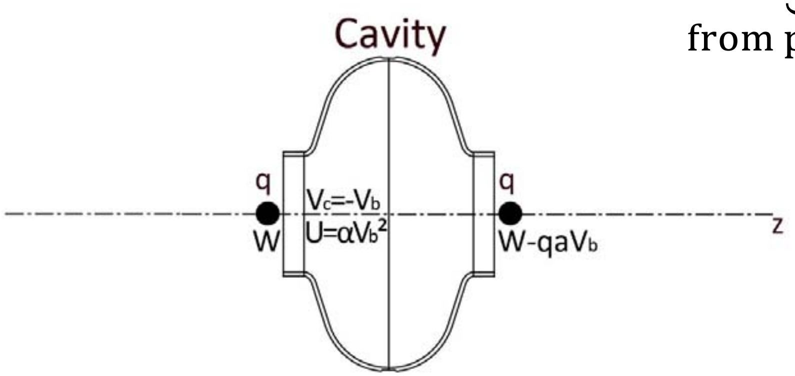
$$\Delta W_1 = -qaV_b$$



Fundamental Theorem of Beam Loading

- 2nd particle with charge q follows in a distance $\frac{\lambda}{2}$ → voltage has changed phase by π
  - Induced voltage from particle 1: $+V_b$
- 2nd particle also induces a voltage – V_b
  - Net cavity voltage $V_c = 0$, net stored energy $U = 0$
- Energy loss of particle 2:

$$\Delta W_2 = \underbrace{qV_b}_{\text{from particle 1}} - \underbrace{qfV_b}_{\text{from own induced charge}}$$



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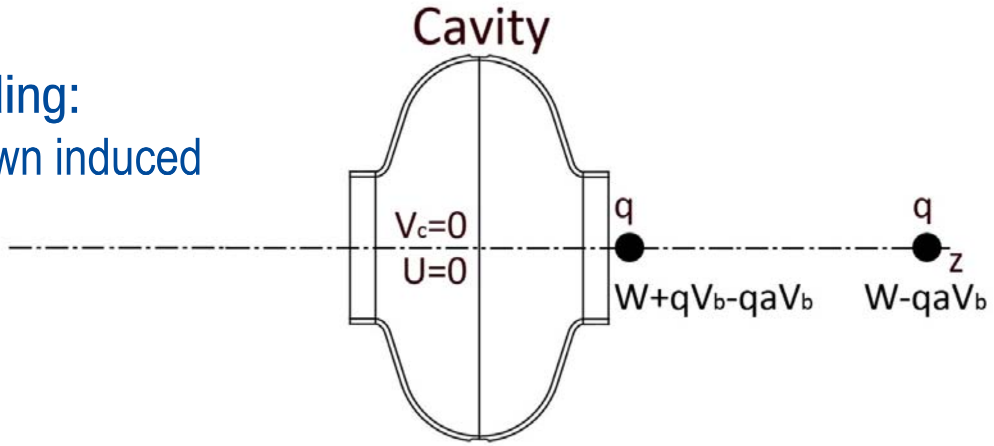
Fundamental Theorem of Beam Loading

- Since the net energy of the cavity stays zero, the energy changes of particle 1 and 2 have to compensate each other:

$$\Delta W_1 + \Delta W_2 = 0$$

$$qV_b - qaV_b - qaV_b = 0$$

- This directly leads to $a = \frac{1}{2}$
- Fundamental Theorem of Beam Loading: A moving charge „sees" the half of its own induced voltage.



Fundamental Theorem of Beam Loading

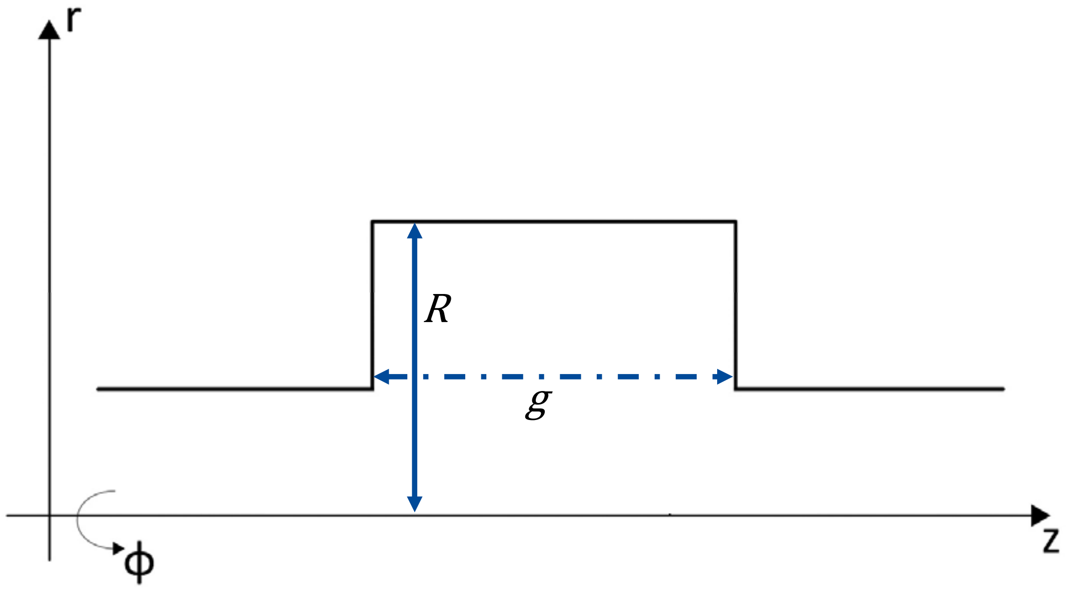
- Reminder: definition of wake function only valid for $s > 0$ (test charge travels behind the field inducing charge)
- Implication of the Fundamental Theorem of Beam Loading for wake function: At $s = 0$, the wake function has to be multiplied with $\frac{1}{2}$
- Case $s < 0$: test charge is *in front* of the field-inducing charge
  - Principle of Causality: There can be no wakefield
- Final definition of the point charge wake function

$$W_{||,0}(s) = \sum_{n=0}^{\infty} 2k_n \cos\left(\frac{\omega_n s}{c}\right) \cdot \begin{cases} 0 & s < 0 \\ 0.5 \text{ for } & s = 0 \\ 1 & s > 0 \end{cases}$$

- Thus: A point charge sees exactly half of its own induced voltage and the center of a bunch sees only the effect of half of the charges in front of it

Example Calculation – Pillbox Cavity

- Loss parameters and wake function can be calculated analytically
- Cylindrical cavity dimensions $r = R, z = g$



Example Calculation – Pillbox Cavity

- For wakefield: only Transverse Magnetic (TM) modes relevant (since for Transverse Electric (TE) modes $E_z = 0$ and thus $V_n = 0$ and $k_n = 0$)
- Relevant field components of lowest monopole modes TM_{0np}

$$E_z^{0,n,p} = \frac{j_{0n}}{R} J_0\left(\frac{j_{0n} r}{R}\right) \cos\left(\frac{\pi p z}{g}\right) \exp\left(i\omega_{0,n,p} t\right)$$

$$E_r^{0,n,p} = \frac{\pi p}{g} J_1\left(\frac{j_{0n} r}{R}\right) \sin\left(\frac{\pi p z}{g}\right) \exp\left(i\omega_{0,n,p} t\right)$$

$$H_\varphi^{0,n,p} = i\omega_{0,n,p} \epsilon_0 \, J_1\left(\frac{j_{0n} r}{R}\right) \cos\left(\frac{\pi p z}{g}\right) \exp\left(i\omega_{0,n,p} t\right)$$

with $(0, n, p)$ – mode indices, $J_0(x)$ and $J_1(x)$ – Bessel functions, j_{0n} – n^th zero of $J_0(x)$

Note: E_φ and $H_r \sim \sin m\varphi$ vanish and $\cos m\varphi = 1$ for $m = 0$, $J_0'(x) = -J_1(x)$

Example Calculation – Pillbox Cavity

- TM_{0np} - eigenfrequencies

$$\omega_{0,n,p} = c\sqrt{\left(\frac{j_{0n}}{R}\right)^2 + \left(\frac{\pi p}{g}\right)^2}$$

- Voltage of TM_{0np} mode

$$V_{0,n,p} = \int_0^g E_z\left(r = 0, z, t = \frac{z}{c}\right) dz$$

$$V_{0,n,p} = \frac{i\omega_{0,n,p}R}{j_{0n}c}\left(1 - (-1)^p \exp\left(\frac{i\omega_{0,n,p}g}{c}\right)\right)$$

Example Calculation – Pillbox Cavity

- Energy stored in TM_{0np} mode:

$$U_{0,n,p} = \frac{\mu_0}{2}\int_0^R r dr \int_0^{2\pi} d\varphi \int_0^g \left(H_\varphi^{0,n,p}\right)\left(H_\varphi^{0,n,p}\right)^* dz$$

$$U_{0,n,p} = \frac{\pi\epsilon_0}{4}\left(\frac{\omega_{0,n,p}}{c}\right)^2 gR^2\, J_1^2(j_{0n})$$

- Loss parameter:

$$k_{0,n,p} = \frac{V_{0,n,p}V_{0,n,p}^*}{4\,U_{0,n,p}}$$

$$k_{0,n,p} = \frac{1}{\pi\epsilon_0 g}\,\frac{2}{1+\delta_{0,p}}\,\frac{1-(-1)^p\cos\left(\frac{\omega_{0,n,p}\,g}{c}\right)}{j_{0n}^2\;J_1^2(j_{0n})}$$

Example Calculation – Pillbox Cavity

- Final wake function is sum of all voltages induced in all modes

$$W_{\parallel,0}(s) = 2q \sum_{n=1}^{\infty} \sum_{p=0}^{\infty} k_{0,n,p} \cos\left(\frac{\omega_{0,n,p}\, s}{c}\right)$$

- Gaussian pulse

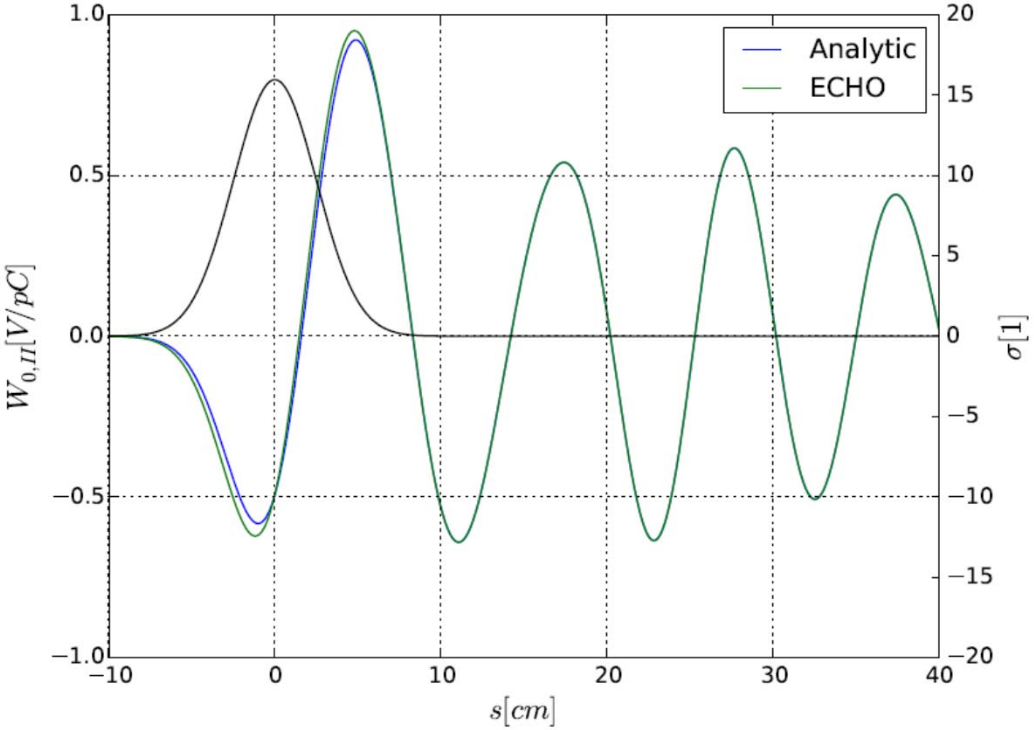
$$\rho(r,t) = q_1 \lambda(z - ct), \qquad \lambda(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{s - s_0}{2\sigma^2}\right)$$

- Gaussian wake potential

$$W_{\parallel}(s) = \int_0^{\infty} \lambda(s - s')\, W_{\parallel,0}(s')\, ds'$$

Example Calculation – Pillbox Cavity

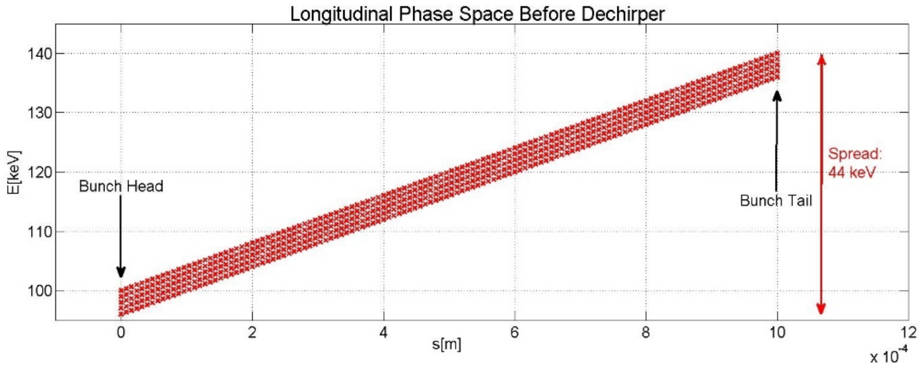
- For $g = 10\ cm$, $R = 5\ cm$ and $\sigma = 2.5\ cm$
- Analytical result (110 modes) vs. numerical result obtained using the 2D code ECHO for rotationally symmetric structures
- Very good agreement



Wakefield Effects

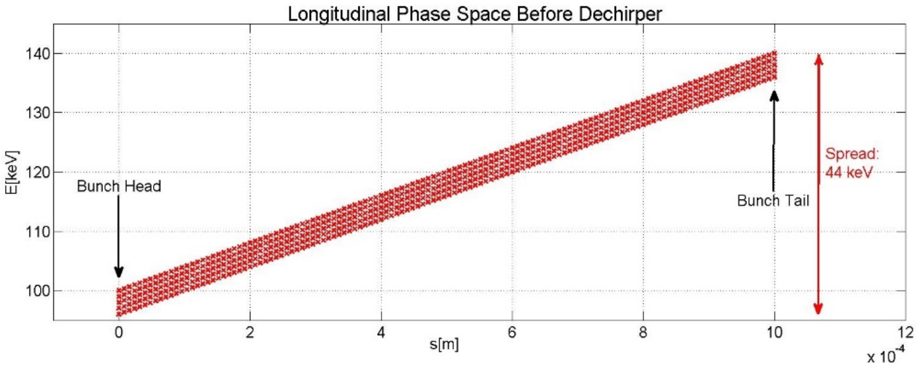
- Seen in the introduction: wakefields remaining in cavities can affect trailing bunches
- Effectively represent energy modulations of the trailing bunches
  - Generally unwanted and unpredictable
  - Inhibition of the accelerator‘s operation
  - Can lead to bunch instabilities / the loss of particle bunches during operation
- Not only affect trailing bunches, but even trailing particles of the same bunch
- On the other hand, effectively *using* the energy modulating nature of wakefields in special accelerator components
  - Wakefield dechirper / wakefield silencer: wakefield generated in a passive accelerator component (dechirper) is used to counteract the energy spread of the particle bunch
  - Wakefield acceleration (e.g. 1st one at DESY, AWA at Argonne, CLIC and, last but not least, the highly topical plasma wakefield acceleration)

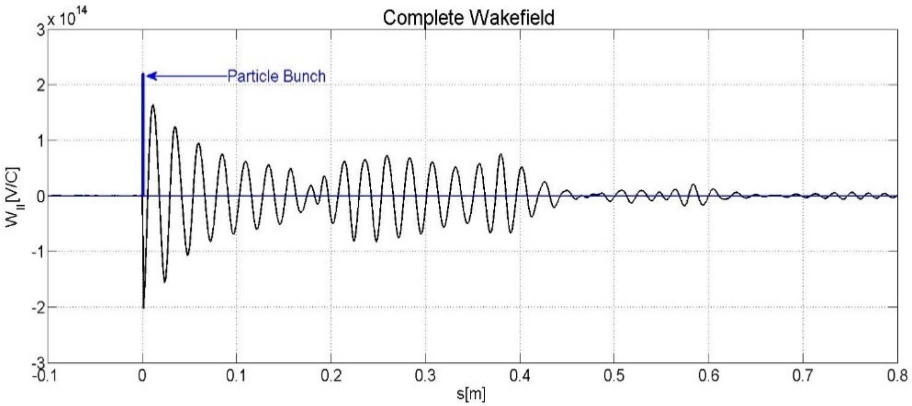
Wakefield Effects - Passive Wakefield Dechirping

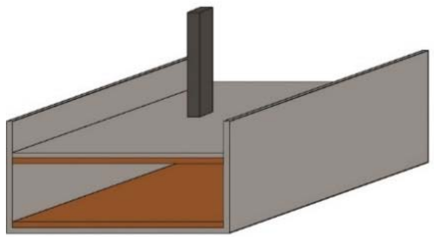


Explanation of the principle with example of a flat top pulse

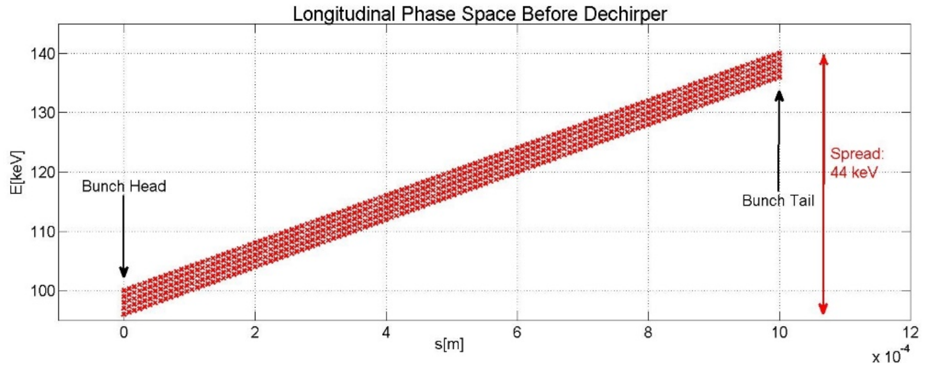
Wakefield Effects - Passive Wakefield Dechirping

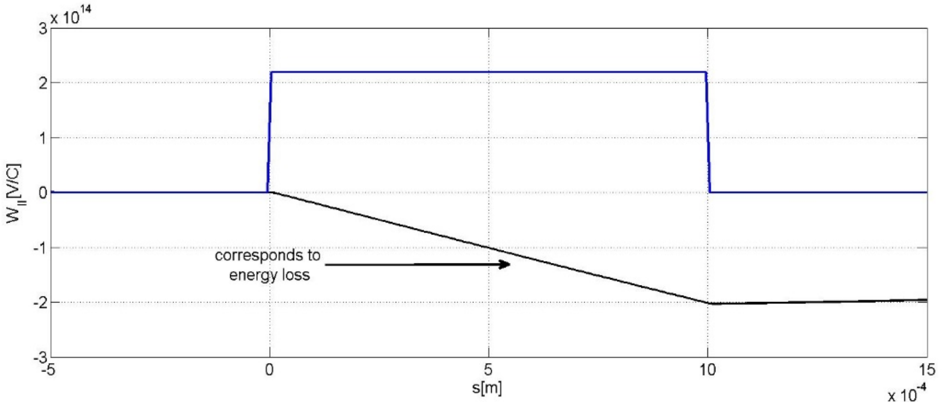


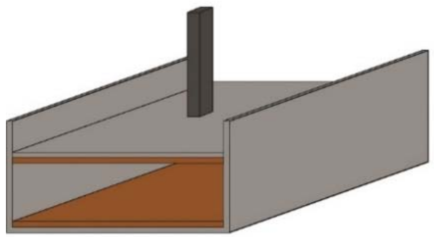




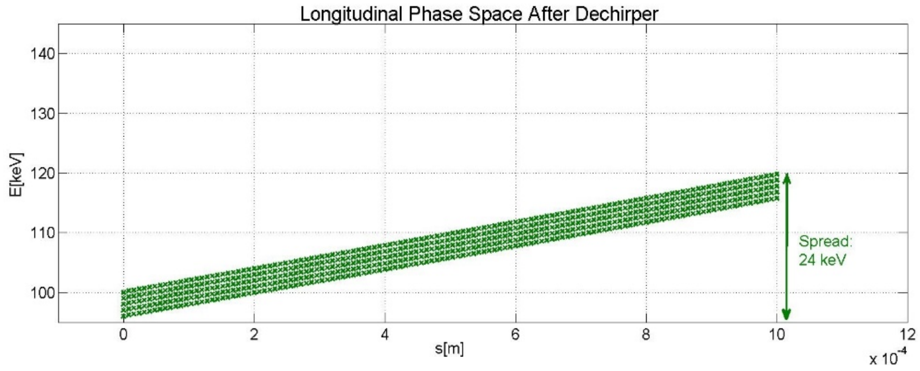
Wakefield Effects - Passive Wakefield Dechirping

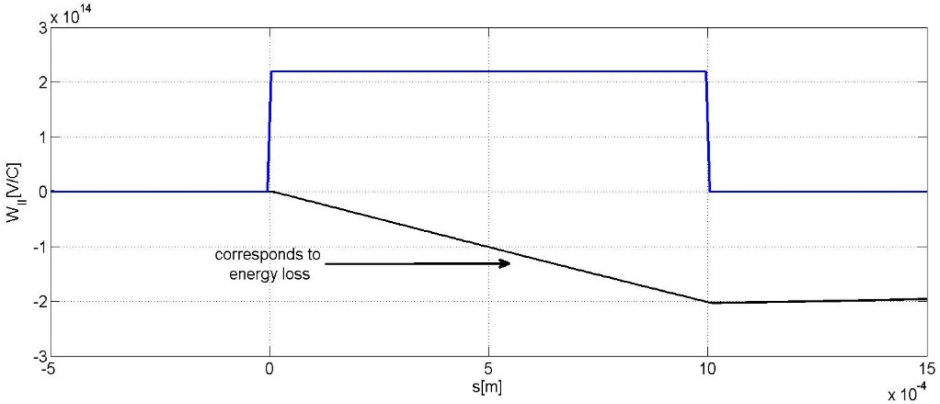


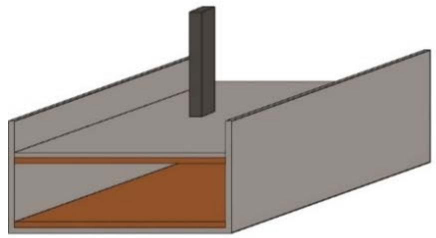




Wakefield Effects - Passive Wakefield Dechirping



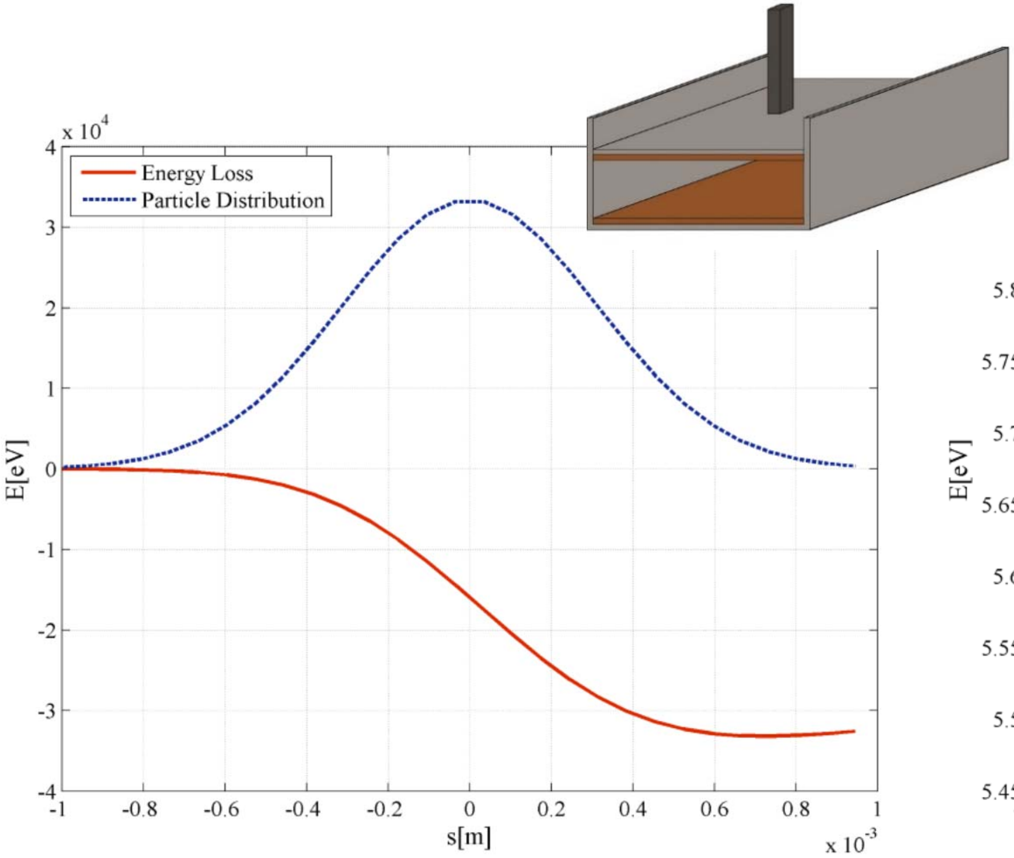


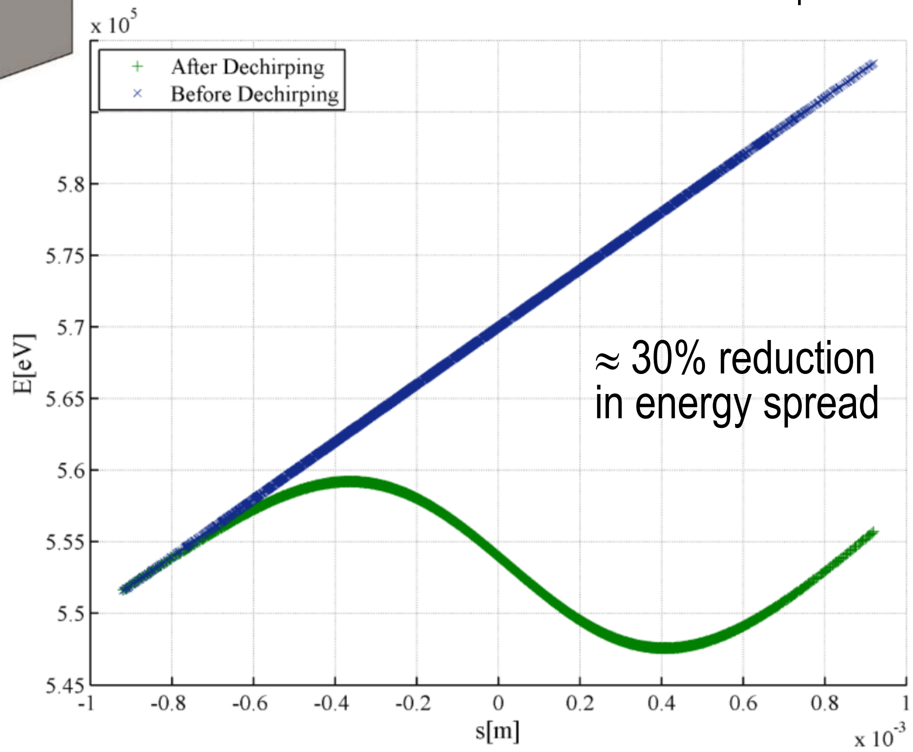


Wakefield Effects - Passive Wakefield Dechirping

gap width 1 cm

F. Reimann et al., ICEAA 2015; since then further improved





Summary

- Wakefields are excited by ultrarelativistic particles as soon as they pass any insertion / obstacle / geometric variation in the beam pipe like e.g. a cavity, bellows, etc.
- Wakefields are also excited if the "beam pipe" wall is dielectrically lined
- Wake potentials / impedances should be kept as small as possible in the general case
- Wakefields are desired on purpose in (plasma) wakefield accelerators and dechirpers
- Point charge wake functions may be computed semi-analytically and then serve as a Green's function to be convoluted with any bunch shape
- Many (commercial and open source) codes exist to compute wake potentials, higher order modes and impedances, e.g. CST MICROWAVE STUDIO, ECHO, which were used here
- For complex modules of cavities, couplers and bellows it may be useful to apply domain decomposition methods combined with model order reduction, see the SSC example