Seeding Schemes I & II

L. Giannessi

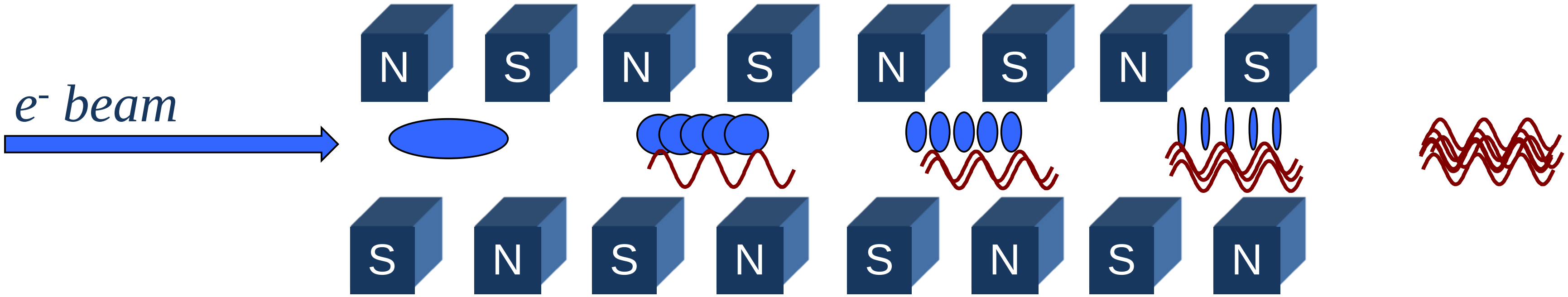
ENEA C.R. Frascati and ELETTRA Sincrotrone Trieste

Free Electron Lasers and Energy Recovery Linacs (FELs and ERLs), 31 May – 10 June, 2016

Summary

- Introduction on high gain and coherence in FEL amplifiers
- Conditions for seeding an FEL amplifier
- Direct Seeding: seeding with high harmonics generated in gas
- High gain harmonic generation
- Pulse properties and pulse control
- Saturation effects – Pulse splitting and superradiance
- The fresh bunch injection technique
- Echo Enhanced Harmonic Generation
- Self-Seeding

High Gain Single pass FELs



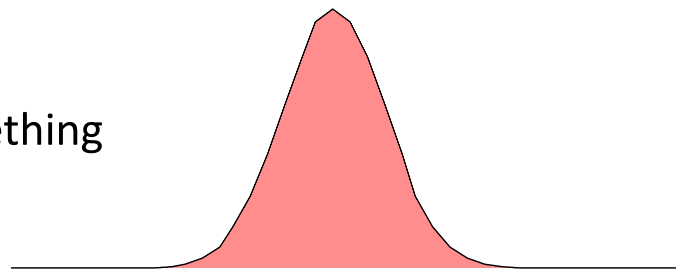
- Tunable in wavelength - **mirrorless configuration**:

  minimize interaction with matter ➔ VUV- X-Rays
- Coherence (Transverse, single TEM 00 mode)
- Narrow spectral bandwidth (typical 10^{-3} relative bandwidth)
- Ultra-short pulses (100 fs – 1 fs)
- High Peak power (Multi GW to TW)

Many applications: Ultra-fast coherent diffractive imaging and time-resolved scattering processes in chemical and biological systems, non-linear processes in ultra-intense X-ray radiation fields, matter in extreme states, phase transitions, population inversion & X-ray atomic lasers, low density systems, i.e. unperturbed atoms, molecules, and clusters.

The FEL is an amplifier

Let's play something



Δt (fwhm) 60 fs
$\lambda = 10$ nm
$\nu = 3\ 10^{16}$ Hz

$c = 3\times10^{8}$ m/s
FTL: 1 10^{-4}

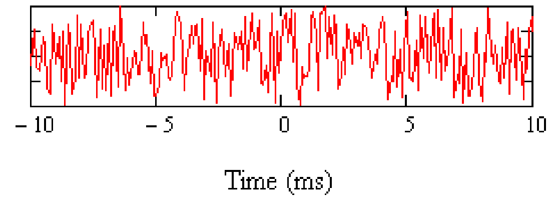
SCALED
x 2 10^{13}

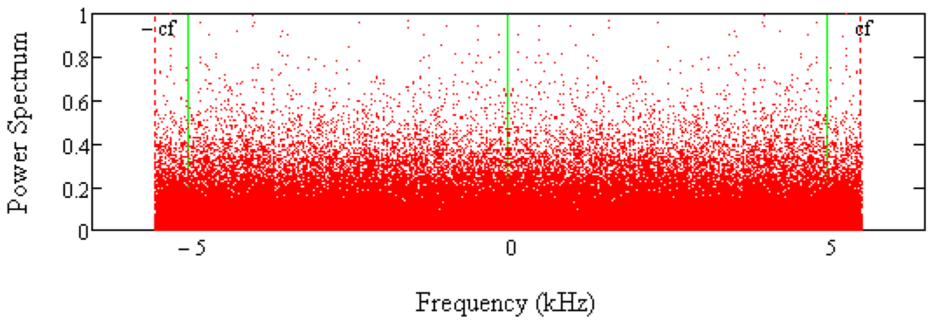
Duration (rms) 0.71 s
$\lambda = 21.6$ cm
$\nu = 1.499\ 10^{3}$ Hz

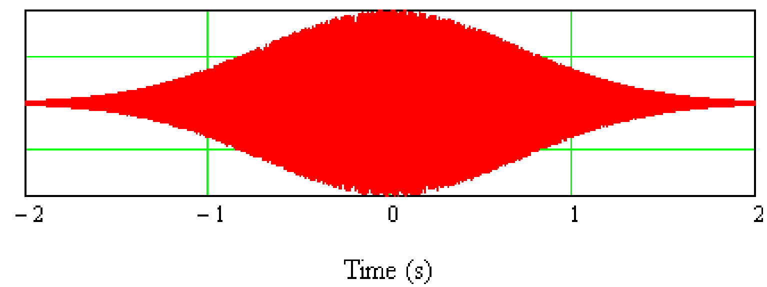
$v_s = 324$ m/s
FTL: 1 10^{-4}

Waveforms

Electron beam shot noise $\Delta\omega/\omega = \infty$

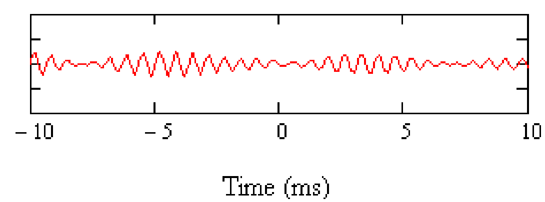


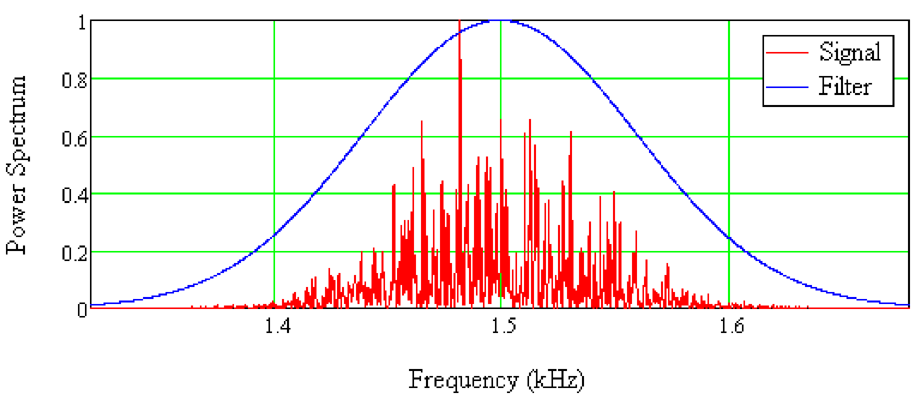


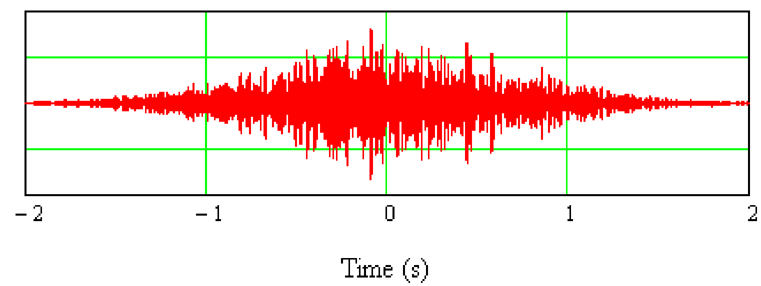


Bending Magnet Radiation $\Delta\omega/\omega \approx 100\%$

Undulator Radiation $\Delta\omega/\omega \approx 4\%$

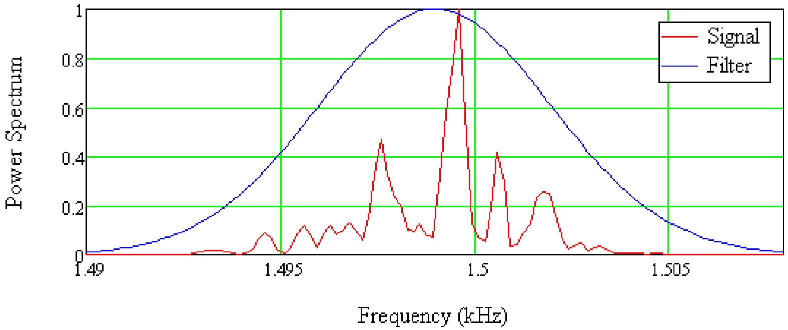


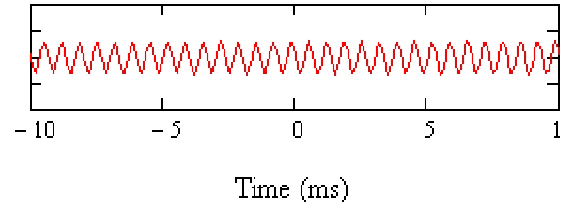


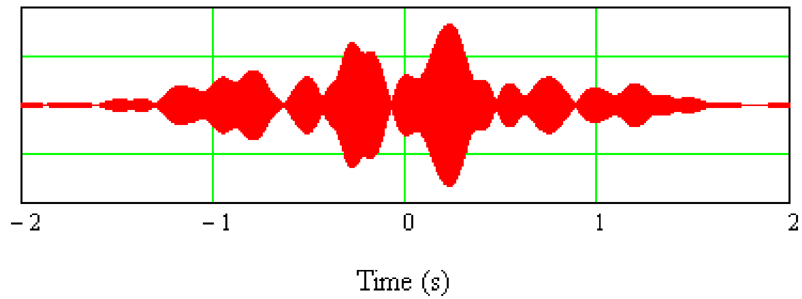


Waveforms

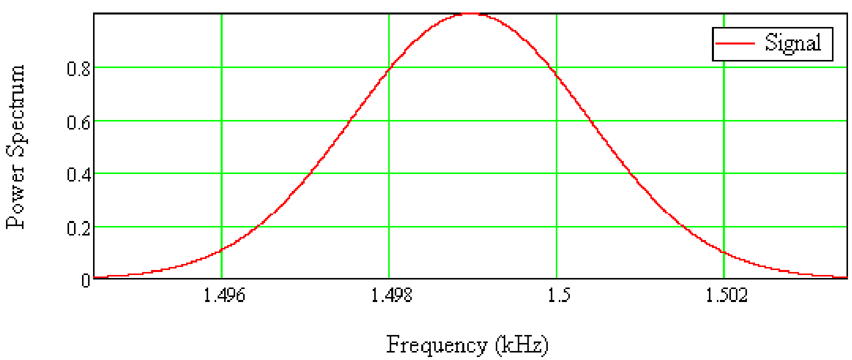
SASE FEL $\Delta\omega/\omega \approx 2\ 10^{-3}$

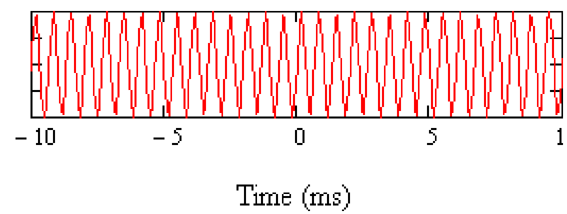


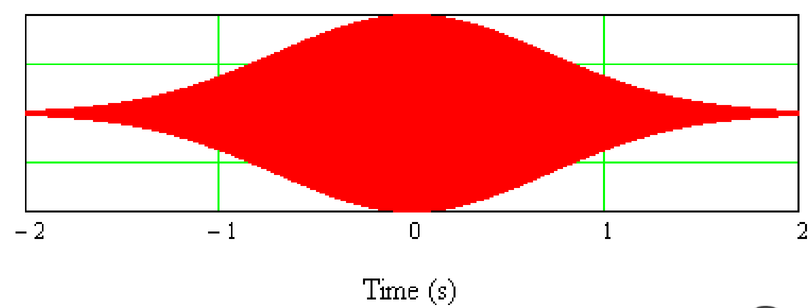




Seeded FTL $\Delta\omega/\omega \approx 1\ 10^{-4}$

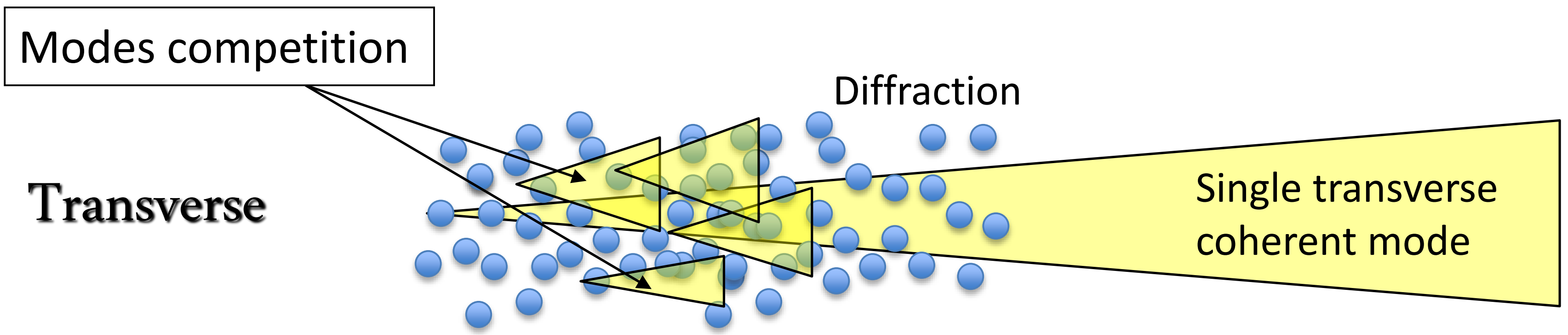






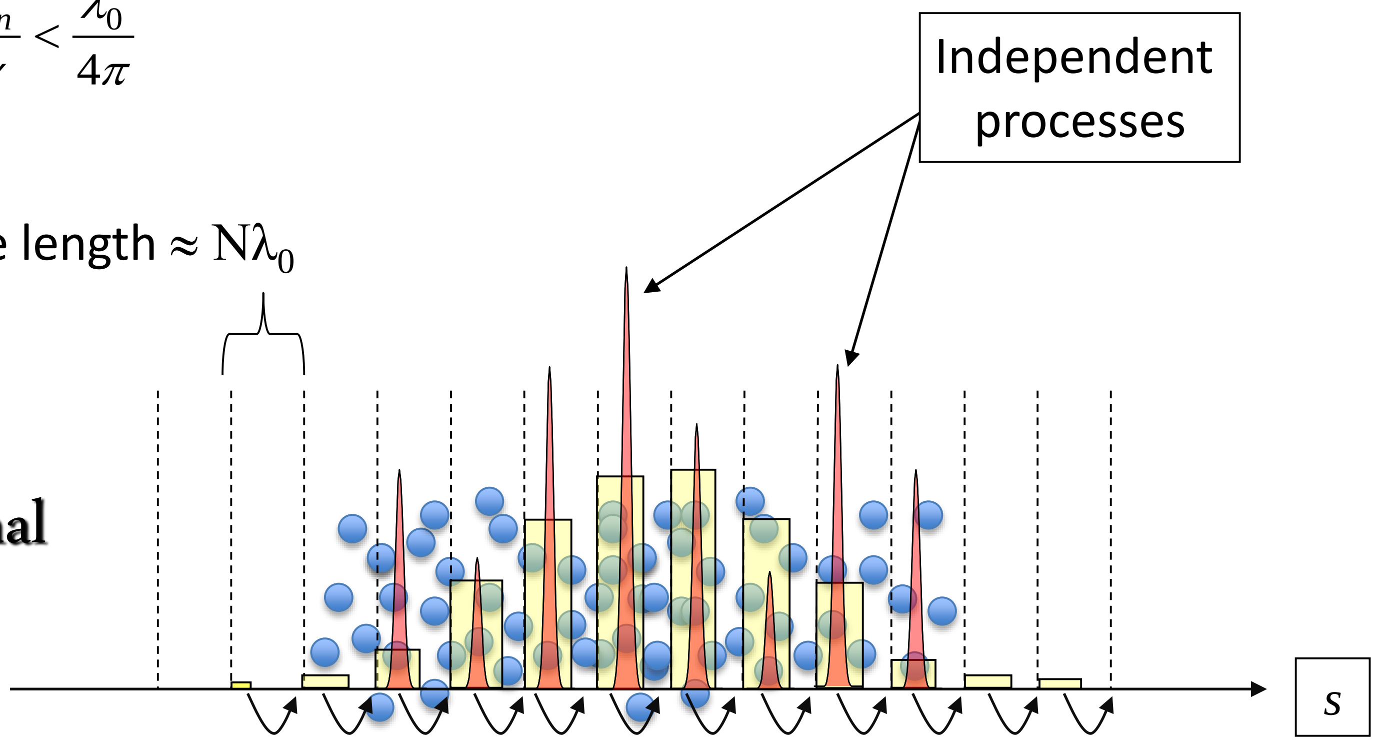
Coherence in SASE FELs

Transverse



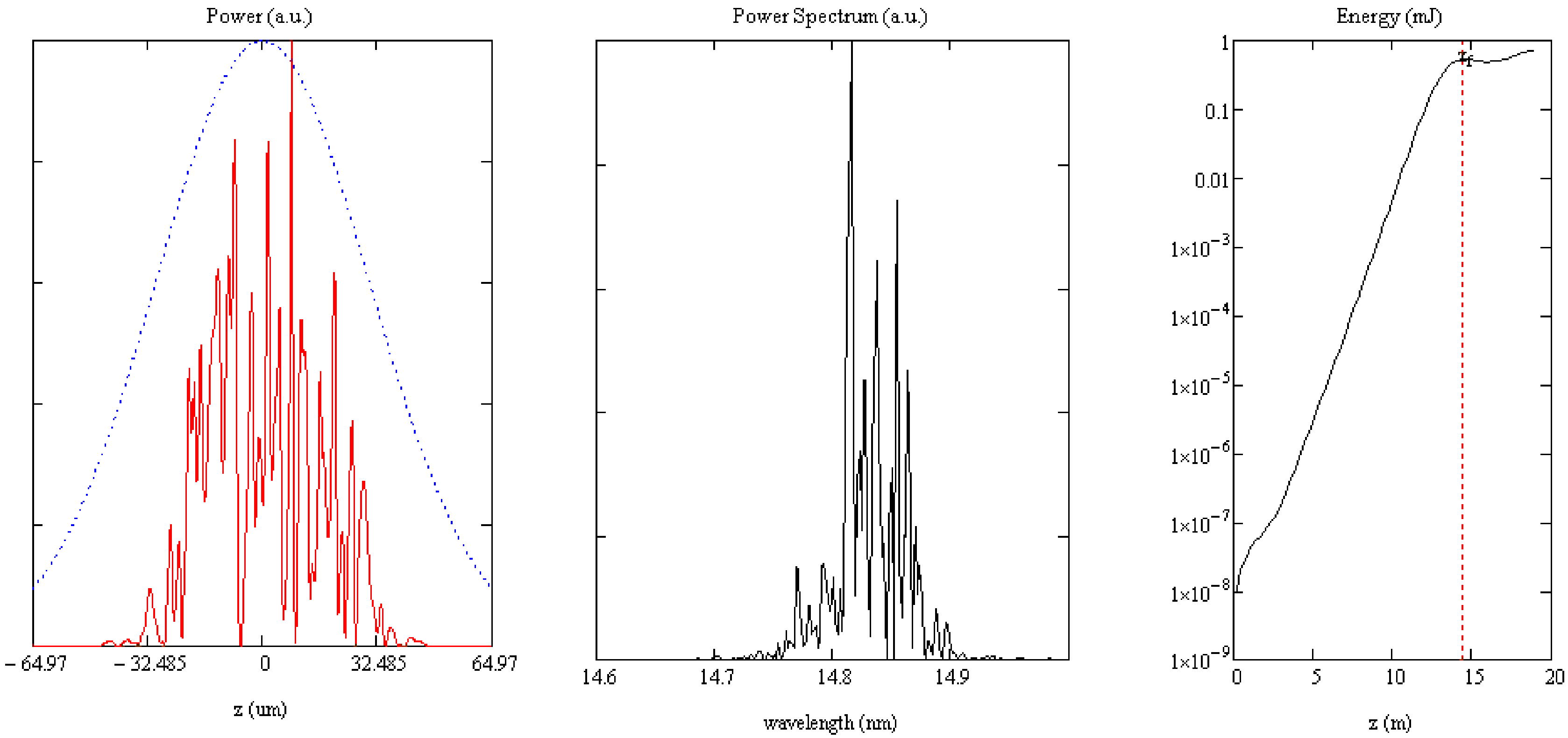
$$\varepsilon = \frac{\varepsilon_n}{\gamma} < \frac{\lambda_0}{4\pi}$$

Longitudinal

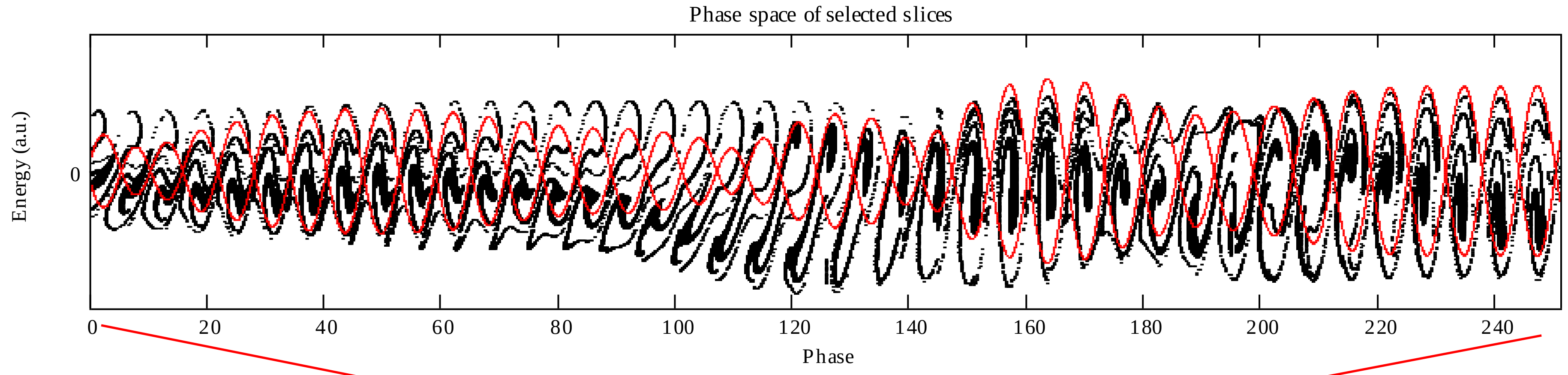


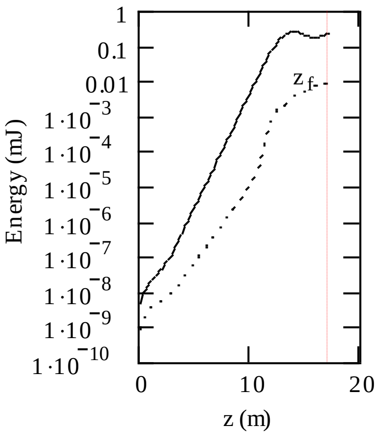
The radiation "slips" over the electrons of a distance $N\lambda_0$

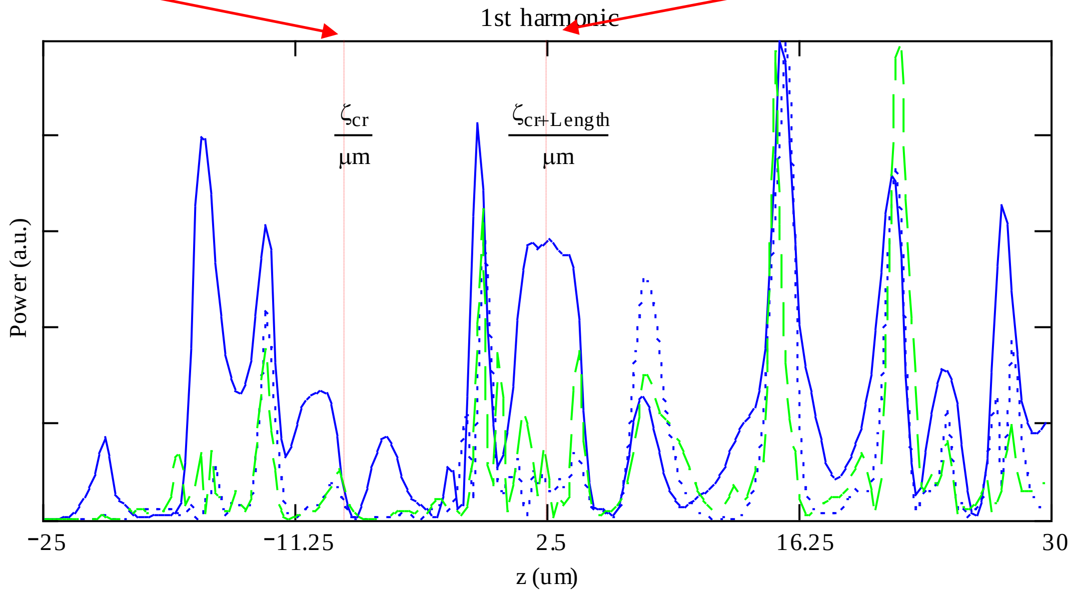
SASE FEL pulse evolution



Phase space at saturation

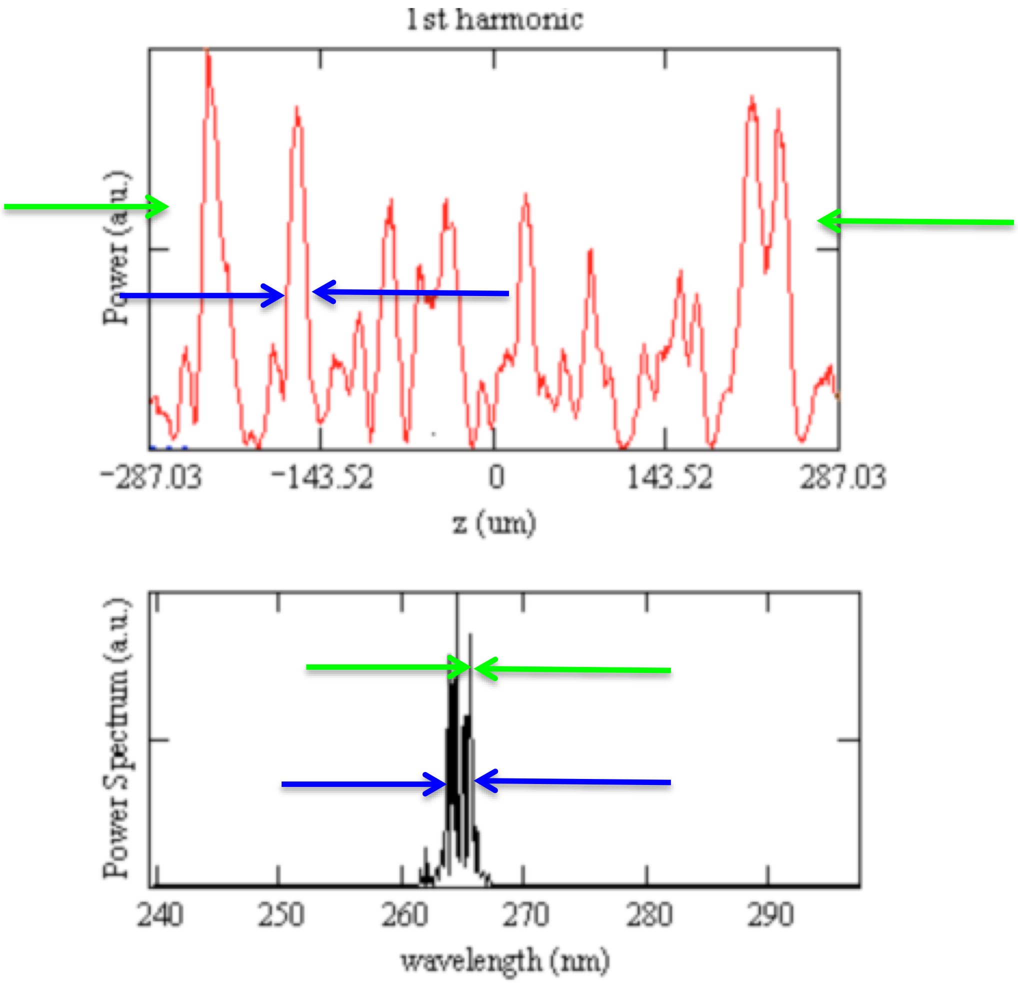






SASE time-spectral structure

- Spectral width is inversely proportional to the temporal spike duration.
- The width of a spike in the radiation spectrum is inversely proportional to the pulse duration (… broader than the window in this example)



Longitudinal Coherence

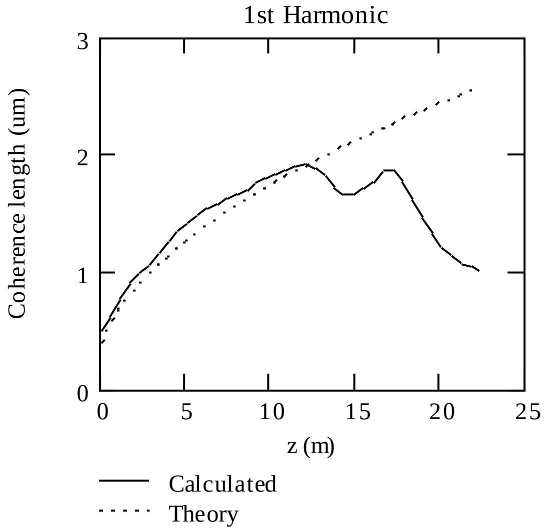
- Coherence length $L_c(z) = \frac{1}{6}\frac{\lambda_0}{\rho_{fel}}\sqrt{\frac{z}{2\rho L_G}}$

  R. Bonifacio et al. PRL **73** (1994) 70
  E. Saldin et al. Opt. Comm. **148** (1998) 383

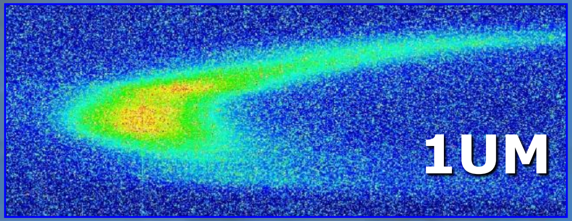
- Saturation length ~ $20L_G$

- Number of Spikes M~ σ_z/L_C ~ 10^2-10^3

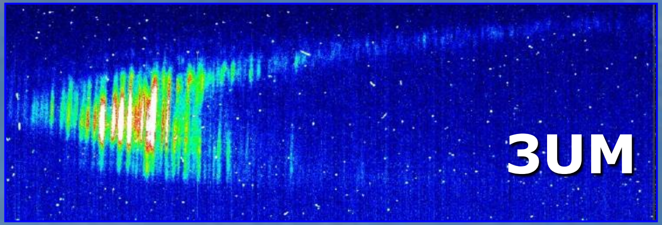
- Energy fluctuations ~$1/M^{-1/2}$

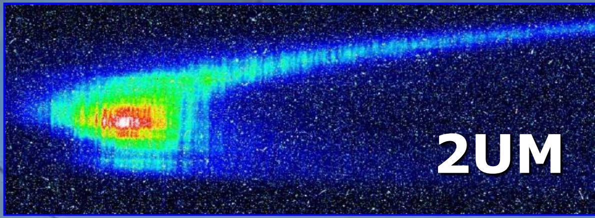


Self Amplified Spontaneous Emission Spectra measurements Summer 2009



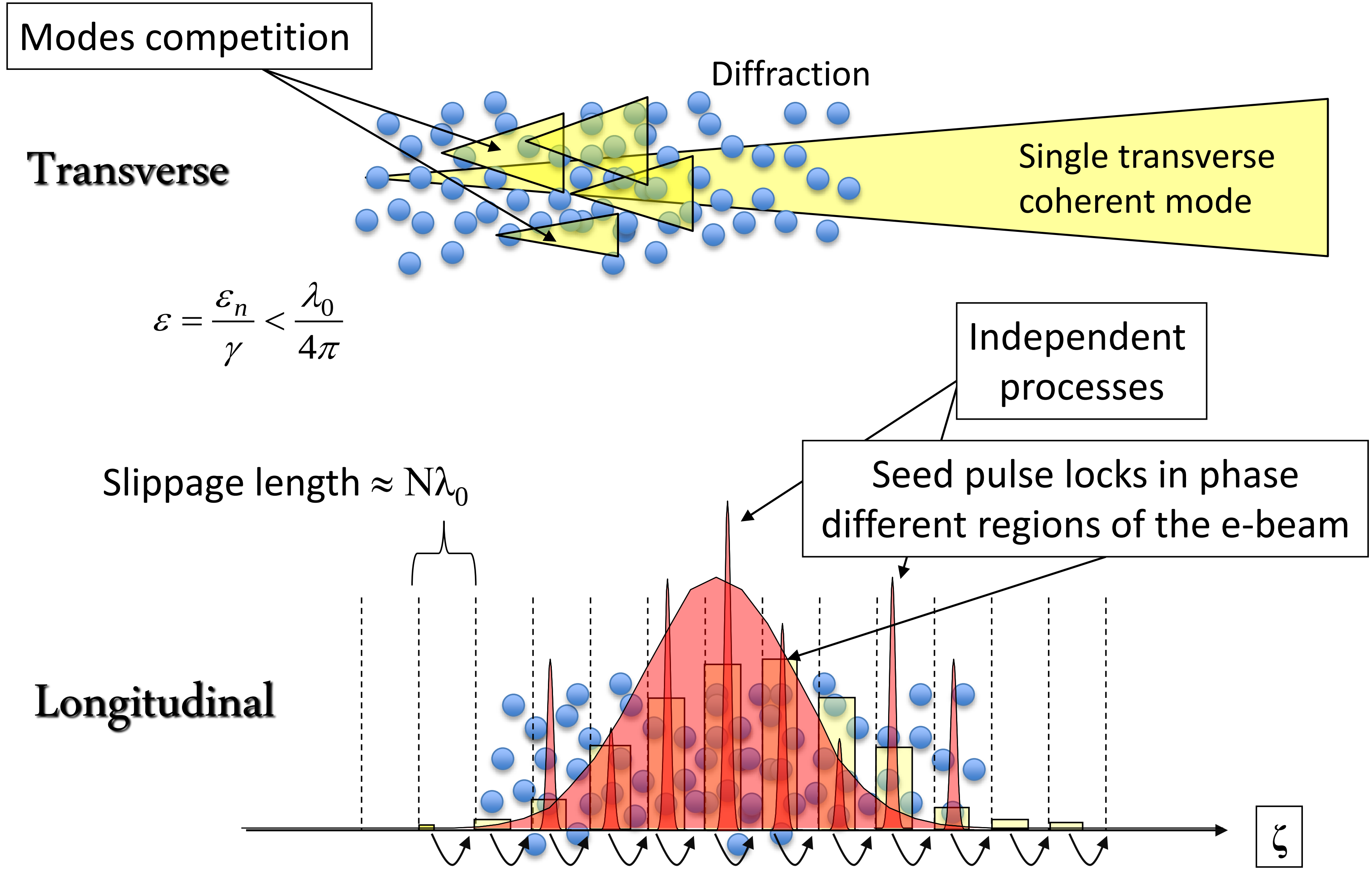
Orbit kicks to selectively inhibit SASE in the first undulators





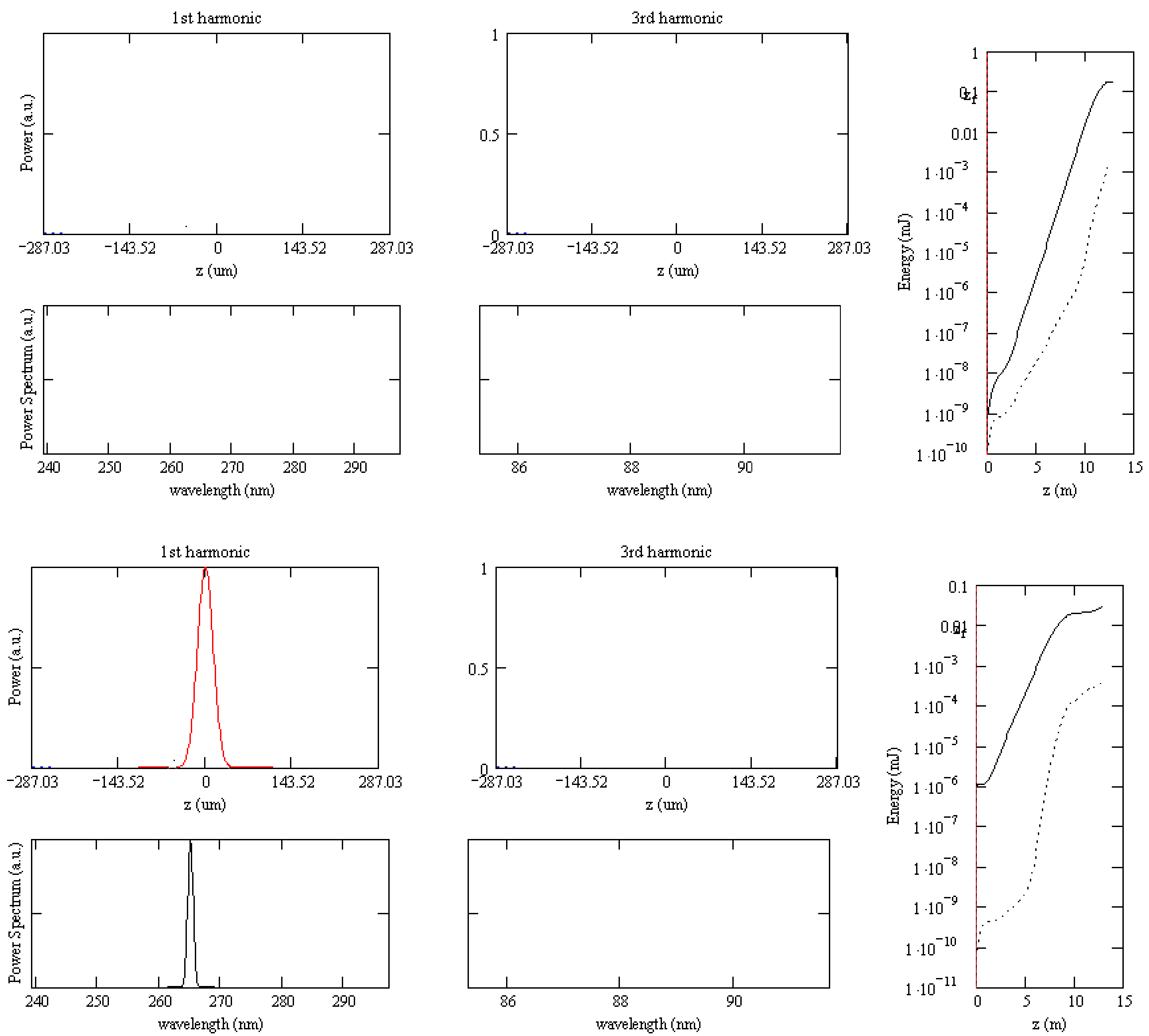
Spectrometer

Coherence in SASE FELs

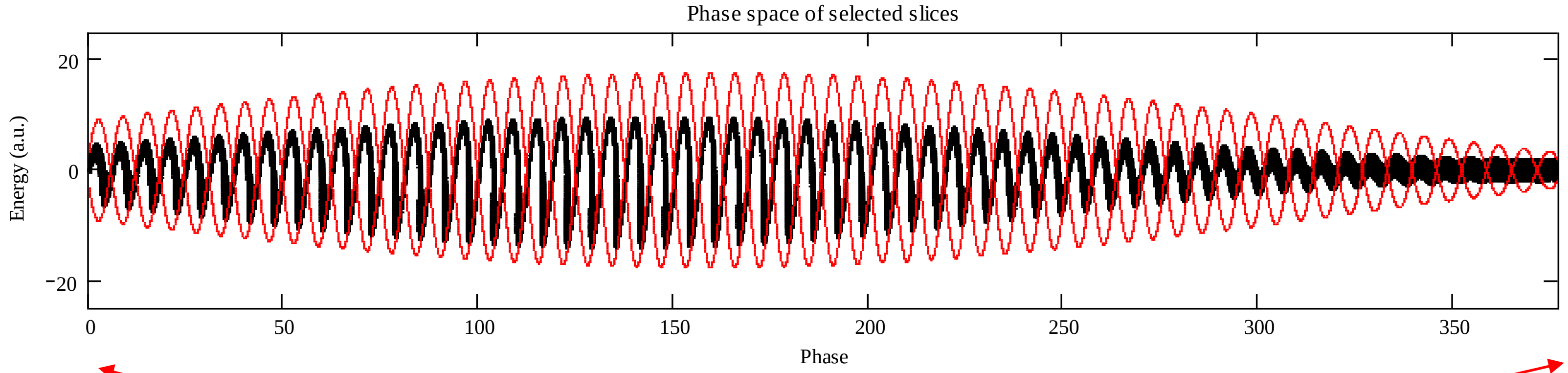


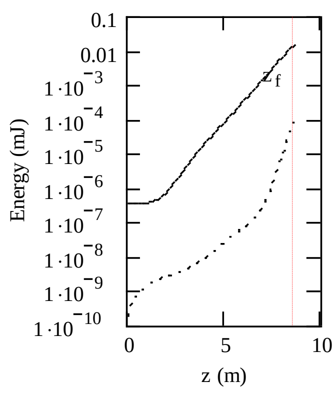
The radiation “slips” over the electrons of a distance $N\lambda_0$

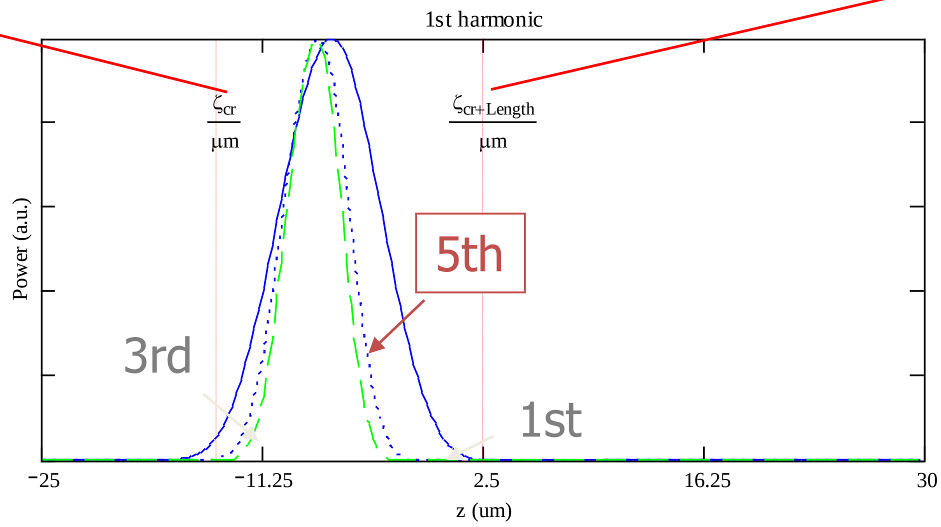
SASE & Seeded pulse & spectra



Seeded FEL –
Phase space @ exit of modulator

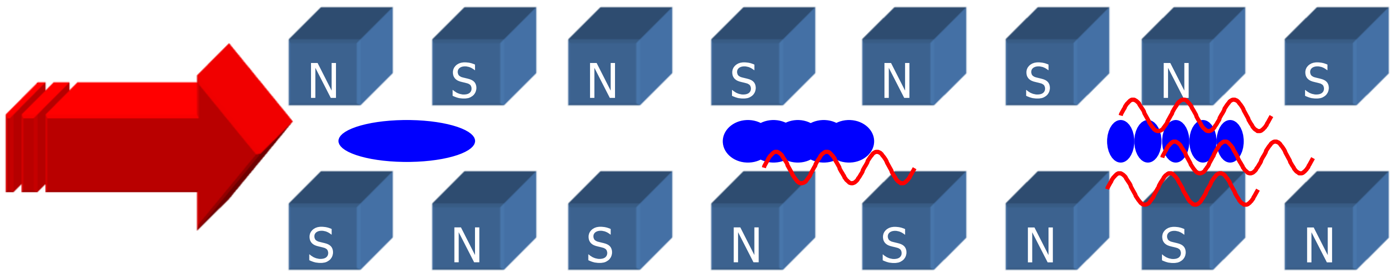






Coherence length determined by the seed

Seeded FELs: The **FEL process** is stimulated by the presence of an **external input source**



$P \sim n_e$ $\qquad$ $P \sim n_e^2$

- **Constrained by the availability of a suitable source**

- Coherence properties (transverse/longitudinal) determined by the seed
- Higher input power → Shorter saturation length
- Deterministic system: fluctuations induced by changes of machine parameters
- Synchronization with external source determined by seed

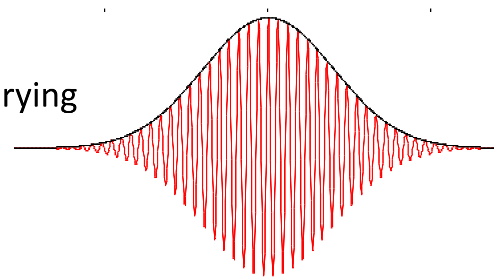
STARTUP IN FEL AMPLIFIERS

FEL dynamics equations (1)

Field vector potential

$$\vec{A}(\vec{x},t)=\frac{1}{2k_R}\left(\vec{\varepsilon}(\vec{r},z,t)\exp(i\psi)+cc\right)$$

$$y = k_R z - w_R t = k_R(z - ct)$$



$\vec{\varepsilon}(\vec{r},z,t)$ is the slowly varying envelope

Field Equations: Wave equation for the EM field in the **slow varying envelope approximation**, with the source term averaged over the undulator period have the following simple form

$$\left[-\frac{i}{2}\vec{\nabla}_{\perp}^{2}\, a(\vec{r},s,\tau)+\frac{\partial}{\partial\tau}a(\vec{r},s,\tau)+N\lambda_0\frac{\partial}{\partial s}a(\vec{r},s,\tau)\right]=-2\pi g_0 j(s)\left\langle e^{-i\theta_l}\right\rangle_l\Big|_s$$

Coordinates

$$\tau=\frac{\beta_z c}{L_u}t$$

$$s=z-\beta_z ct$$

Wave equation | Source term

Definitions of a and g_0

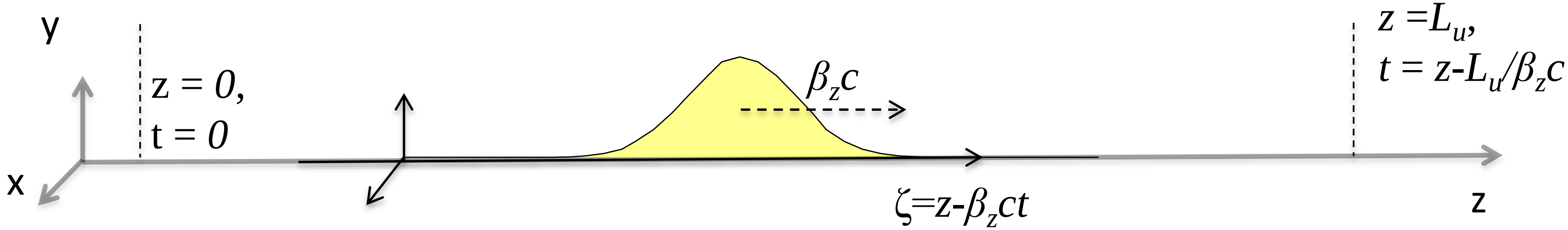
$$g_0=2\pi\frac{N^3}{\gamma^3}\frac{\left(\lambda_u\ K\left[J_0(\xi)-J_1(\xi)\right]\right)^2}{\Sigma_e}\frac{I}{I_A}$$

$$|a|=2\pi\sqrt{\frac{2I}{I_s}}\propto\left|\vec{\varepsilon}(\vec{r},z,t)\right|,$$

$$\rho_{fel}=\frac{(\pi g_0)^{\frac{1}{3}}}{4\pi\ N}$$

$$I_s=\frac{1}{4\pi}\left(\frac{m_0c^3}{r_0}\right)\left(\frac{\gamma}{N}\right)^4\frac{1}{\left(\lambda_u K\left[J_0(\xi)-J_1(\xi)\right]\right)^2}$$

Coordinates



$$\begin{cases} \tau = \dfrac{\beta_z c}{L_u} t \\ s = z - \beta_z ct \end{cases}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial s}\frac{\partial s}{\partial t} + \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial t} = \frac{\beta_z c}{L_u}\frac{\partial}{\partial \tau} - \beta_z c\frac{\partial}{\partial s}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial z} = \frac{\partial}{\partial s}$$

List of definitions

e_0 *electron charge*

m_0 *electron rest mass*

c *speed of light in vacuum*

r_0 *electron classical radius,* $r_0 = \dfrac{e_0^2}{m_0 c^2}$

E *electron beam energy*

γ *electron relativistic factor,* $\gamma = \dfrac{E}{m_0 c^2}$

I_A *Alfven current,* $I_A = \dfrac{e_0 c}{r_0}$

$j(s)$ *normalized current distribution*

I_{peak} *electron beam peak current*

λ_u *undulator period*

λ_r *radiation field period*

$K = \dfrac{e_0 B_y^{peak} \lambda_u}{2\pi m_0 c}$ *undulator strength*

N *undulator periods*

$\lambda_0 = \dfrac{\lambda_u}{2\gamma^2}\left(1 + \dfrac{K^2}{2}\right)$ *resonant wavelength*

$$k_i = \frac{2\pi}{\lambda_i}, \quad ck_i = \omega_i, \quad i = u, r, 0$$

$$\xi = \frac{1}{4}\frac{K^2}{\left(1 + \frac{K^2}{2}\right)}$$

$J_n(\xi)$ *Bessel function of the first kind*

FEL dynamics equations (2)

Particles Equation: Lorentz force equation averaged over the fast electron motion over one undulator period, in the field of the undulator and of the co-propagating em field takes the form of **a pendulum-like equation**

$$q_l(t) = (k_u + k_r) s_l(t) + 2pN\left(\frac{w_0(g_l(t)) - w_r}{w_0(g_l(t))}\right) t$$

$$\frac{dn_l(t)}{dt} = |a(s,t)| \cos(q_l(t) + F(s,t))$$

$$n_l(t) = \frac{dq'_l(t)}{dt} = 2pN\left(\frac{w_0(g_l(t)) - w_r}{w_0(g_l(t))}\right)$$

$l = 1..\, n_e$

l_u *undulator period*

l_r *radiation field period*

$l_0 = \frac{l_u}{2g^2}\left(1 + \frac{K^2}{2}\right)$ *resonant wavelength*

$k_i = \frac{2p}{l_i}, \quad w_i = ck_i, \quad i = u, r, 0$

Detuning parameter: derivative of phase vs τ

ζ, ν canonical variables of a “quasi” periodic Hamiltonian (a is a slowly varying function)

$$H = \frac{1}{2}\left(\frac{dq}{dt}\right)^2 - |a(s,t)| \sin(q + F(s,t))$$

Source Term: The bunching factor b_1

$$b_1(s,t) = \left\langle e^{-i\theta_l} \right\rangle_l \Big|_s = \frac{1}{n_e} \sum_{l=1}^{n_e} e^{-i\theta_l}$$

Sum over n_e electrons "around" the position s

Equivalent to (for $n=1$)

$$b_n(s,t) = \frac{1}{\lambda} \int_s^{\lambda+s} ds' \, \rho_e(s') \, e^{-in\theta(s',t)}$$

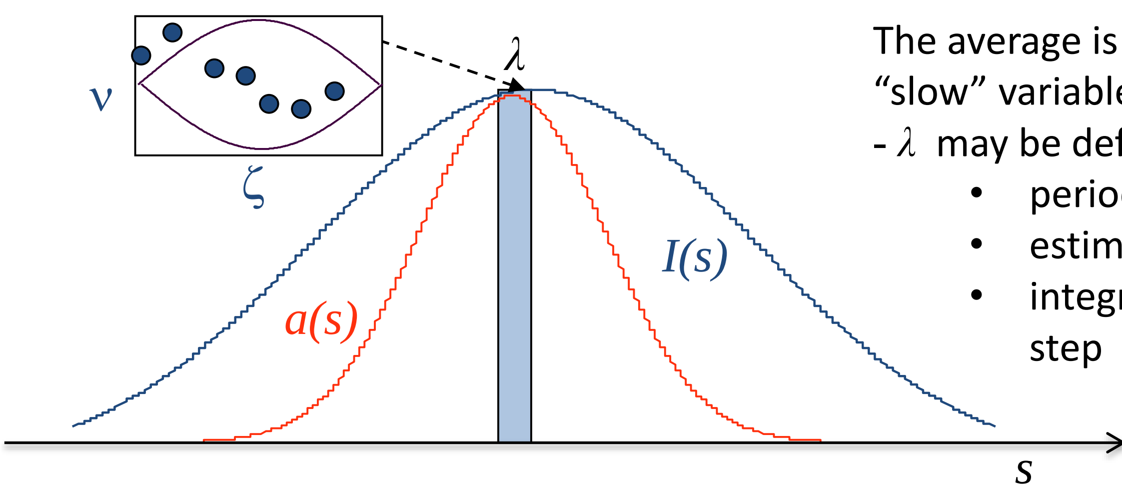
with

$$\rho_e(s) = \sum_{l=1}^{n_e} \delta(s - s_l)$$

For a periodic distribution $\rho_e(s)$ of period λ the coefficient b_n is the n^{th} Fourier coefficient of the distribution

Uniform distribution

$$b_n(s,t) = \frac{1}{\lambda} \int_s^{\lambda+s} ds' \, e^{-in\theta(s',t)}$$



Important

The average is calculated over a region (range λ) where "slow" variables may be considered constant.

- λ may be defined depending on the specific context, e.g.:
  - periodic distribution == it can be the period
  - estimate of shot noise == cooperation length
  - integration of FEL equations == linked to the time step

Study the "startup dynamics", i.e. in the limit : $a<<1$

Simplified problem: assume transversally and longitudinally uniform field

$$a(\vec{r},s,\tau)=a(\tau)$$

The wave equation becomes

$$\frac{\P}{\P t}a(t)=-2pg_0\left\langle e^{-iq_l}\right\rangle_l$$

Expand the pendulum equation to the lowest order in a

$$q_l(t)\,@\,q_l(0)+n_l(0)\;t\;+\;dq_l(t)$$

$$\frac{d^2dq_l(t)}{dt^2}=|a(t)|\cos\left(q_l(t)+F(t)\right)=\frac{1}{2}\left(a(t)e^{iq_l(t)}+cc\right)$$

$$dq_l(t)=\frac{1}{2}\int_0^t\int_0^{t'}a(t'')e^{i\left(q_l(0)+n_l(0)\;t''+dq_l(t)\right)}dt''+cc$$

$$\int_a^b\int_a^{t'}f(t')dt'dt=t\int_a^t f(t')dt'\Big|_a^b-\int_a^b t'\;f(t')dt'=\int_a^b(b-t')\,f(t')dt'$$

$$dq_l(t)=\frac{1}{2}\int_0^t(t-t')a(t')e^{i\left(q_l(0)+n_l(0)\;t'+dq_l(t)\right)}dt'+cc$$

"startup dynamics", limit : *a<<1*

$$q_l(t) @ q_l(0) + n_l(0)\, t \;+\; \frac{1}{2}\int_0^t (t - t')\,a(t')\,e^{i\left(q_l(0)+n_l(0)\, t' + dq_l(t')\right)}\,dt' + cc$$

At the lowest order in ***a*** we ignore the term $dz_l\,(t')$
in the exponent and we substitute the result in the field equation

$$\frac{\P}{\P t}a(t) = -2pg_0 \left\langle e^{-iq_l} \right\rangle_l$$

$$= -2pg_0 \left\langle e^{-i\left(q_l(0)+n_l(0)\, t \;+\; \frac{1}{2}\int_0^t (t - t')a(t')e^{i\left(q_l(0)+n_l(0)\, t'\right)}dt' + cc\right)} \right\rangle_l$$

Expand the exponential in powers of ***a*** and retain only the lowest order

$$\frac{\partial}{\partial t}a(t) @ -2pg_0 \left\langle e^{-i\left(q_l(0)+n_l(0)\, t\right)} \left(1 - \frac{i}{2}\int_0^t (t - t')\,a(t')\,e^{i\left(q_l(0)+n_l(0)\, t'\right)}dt' + cc\right) \right\rangle_l$$

“startup dynamics”, limit : *a<<1*

$$\frac{\partial}{\partial t}a(t) @ -2pg_0\left\langle e^{-i\left(q_l(0)+n_l(0)\,t\right)}\left(1-\frac{i}{2}\int_0^t\left(t-t'\right)a(t')e^{i\left(q_l(0)+n_l(0)\,t'\right)}dt'+cc\right)\right\rangle_l$$

$$\frac{\partial}{\partial t}a(t) @ -2pg_0\left\{\left\langle e^{-i\left(q_l(0)+n_l(0)\,t\right)}\right\rangle_l + \right.$$

$$\left. -\frac{i}{2}\left\langle e^{-i\left(q_l(0)+n_l(0)\,t\right)}\int_0^t\left(t-t'\right)a(t')e^{i\left(q_l(0)+n_l(0)\,t'\right)}dt'\right\rangle_l + \frac{i}{2}\left\langle e^{-i\left(q_l(0)+n_l(0)\,t\right)}\int_0^t\left(t-t'\right)a^*(t')e^{-i\left(q_l(0)+n_l(0)\,t'\right)}dt'\right\rangle_l\right\}$$

$$\left\langle e^{-i\left(q_l(0)+n_l(0)\,t\right)}\right\rangle_l = b_1(t)$$

$$\left\langle e^{-2i\left(q_l(0)+n_l(0)\,t\right)}\right\rangle_l = b_2(t)$$

Monoenergetic beam, $v_l = v_0$, $l=1..n_e$

$$\frac{\partial}{\partial t}a(t) @ -2pg_0 b_1(0)e^{-in_0 t} + ipg_0 b_2(0)e^{-2in_0 t}\int_0^t dt't'e^{in_0 t'}\,a^*\left(t-t'\right) + ipg_0\int_0^t dt't'e^{-in_0 t'}\,a\left(t-t'\right)$$

Startup – Seeded FEL amplifier

We have derived the FEL integral equation starting from a pre-modulated beam

$$\frac{\partial}{\partial t}a(t) \cong -2\pi\gamma_0 b_1 e^{-i\nu_0 t} + i\pi\gamma_0 b_2 e^{-2i\nu_0 t}\int_0^t d\tau'\tau' e^{i\nu_0\tau'} a^*(t-\tau') + i\pi\gamma_0 \int_0^t d\tau'\tau' e^{-i\nu_0\tau'} a(t-\tau')$$

Proportional to b_1
Shot noise = spontaneous emission or emission from a pre bunched beam

Proportional to a & b_2
At startup a=0,
Starting from a uniform beam also b_2=0

Feedback term:
Derivative of the field prop. to the input field.
Exponential growth.

Solution for a uniform beam, $b_1, b_2 = 0$ (and $b_n=0$)

$$\frac{\partial}{\partial t}a(t) @ \cancel{-2pg_0 b_1 e^{-in_0 t}} \cancel{+ipg_0 b_2 e^{-2in_0 t}\int_0^t dt't'e^{in_0 t'} a^*(t-t')} + ipg_0 \int_0^t dt't'e^{-in_0 t'} a(t-t')$$

Deriving in τ three times: third order ODE

$$-\nu_0^2 \frac{\partial}{\partial \tau}a(\tau) + 2i\nu_0 \frac{\partial^2}{\partial \tau^2}a(\tau) + \frac{\partial^3}{\partial \tau^3}a(\tau) = i\pi g_0 a(\tau)$$

Ignoring the dependence on ν_0 (we set $\nu_0=0$)

$$\frac{\partial^3}{\partial \tau^3}a(\tau) = i\pi g_0 a(\tau)$$

Solution:

$$a(\tau) = \sum_{i=1}^{3} a_i e^{-i\alpha_i \tau}$$

Where the α are the solution of the cubic Eq.

$$i\alpha_i^3 = i\pi g_0$$

$$\alpha_1 = (\pi g_0)^{\frac{1}{3}}$$

$$\alpha_2 = (\pi g_0)^{\frac{1}{3}}\left(-1 + \frac{i\sqrt{3}}{2}\right)$$

$$\alpha_3 = (\pi g_0)^{\frac{1}{3}}\left(-1 - \frac{i\sqrt{3}}{2}\right)$$

Solution for a uniform beam, $b_1, b_2 = 0$ (and $b_n=0$)

$$a(t) = \frac{a_0}{3}\left(e^{-i(pg_0)^{\frac{1}{3}}t} + e^{i(pg_0)^{\frac{1}{3}}\left(1-\frac{i\sqrt{3}}{2}\right)t} + e^{i(pg_0)^{\frac{1}{3}}\left(1+\frac{i\sqrt{3}}{2}\right)t} \right)$$

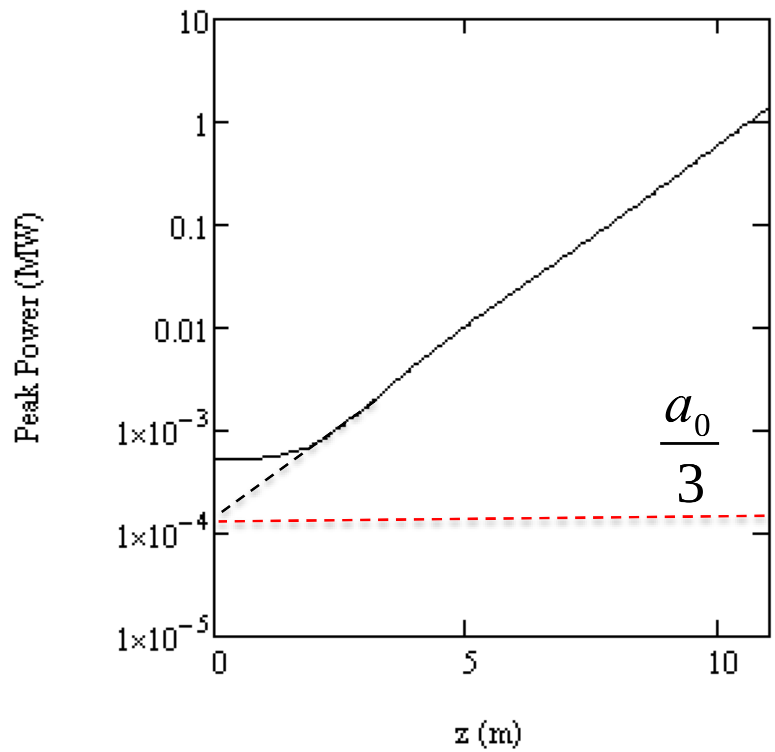
From the condition $a(0) = a_0$

Growing root $$a(z) = \frac{a_0}{3} e^{(pg_0)^{\frac{1}{3}}\left(i+\frac{\sqrt{3}}{2}\right)t} = \frac{a_0}{3} e^{\frac{(pg_0)^{\frac{1}{3}}}{Nl_u}\left(i+\frac{\sqrt{3}}{2}\right)z}$$

$$t @ \frac{z}{Nl_u}$$

Exponential gain $$P(z) = \frac{P(0)}{9} e^{\frac{\sqrt{3}}{l_u N}(pg_0)^{\frac{1}{3}}z} = \frac{P(0)}{9} e^{\frac{z}{L_g}}$$

Gain length $$L_g = \frac{l_u}{4p\sqrt{3}} \frac{4pN}{(pg_0)^{\frac{1}{3}}} = \frac{l_u}{4p\sqrt{3}r_{fel}}$$



Exponential evolution of power

Exponential growth

$$r_{fel} = \frac{(pg_0)^{\frac{1}{3}}}{4p\ N}$$

$$P(z) = \frac{P(0)}{9} e^{\frac{z}{L_g}}, \quad L_g = \frac{l_u}{\sqrt{3}} \frac{N}{(pg_0)^{\frac{1}{3}}} = \frac{l_u}{4p\sqrt{3}r_{fel}}$$

… with real beam, energy spread, emittances, diffraction

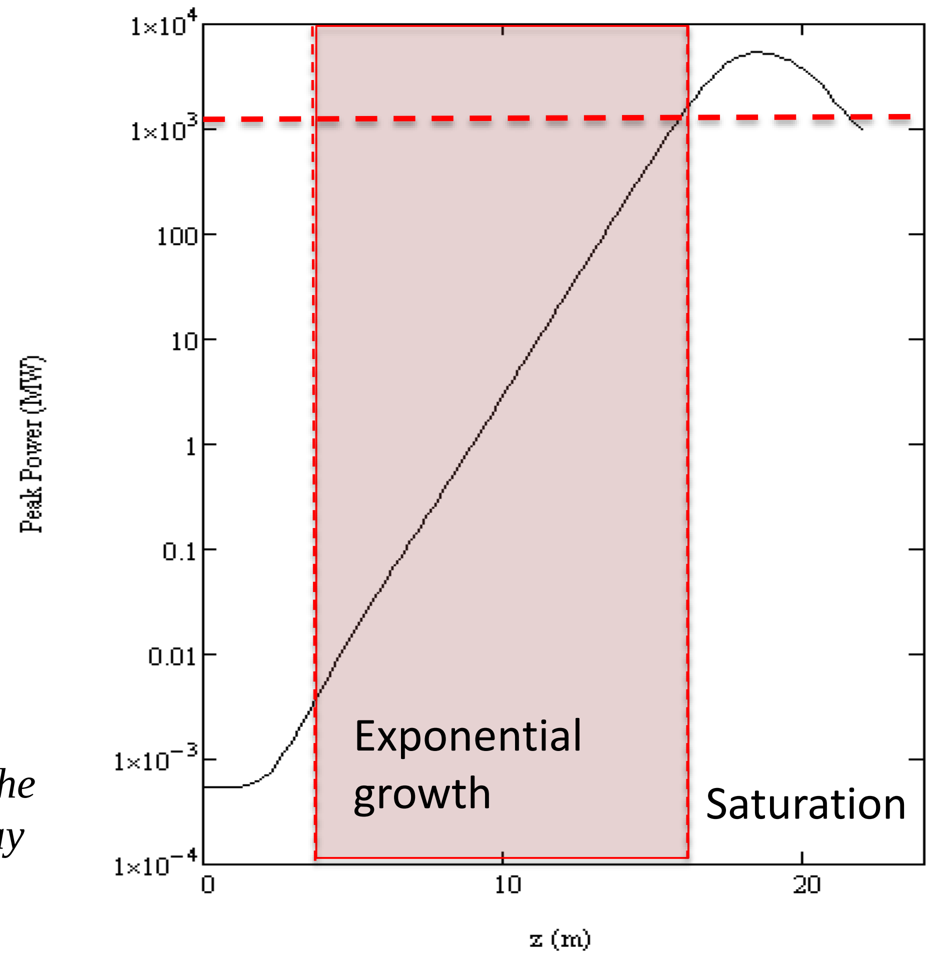
$$L_{gc} = \frac{l_u}{4p\sqrt{3}c_q r_{fel}}, \quad P(z) = \frac{P(0)}{9} e^{\frac{z}{L_{gc}}}$$

$$where \quad c_q = c_q(s_g, e_x, e_y, l, ...)$$

See e.g. M. Xie, Design optimization for an X-ray free electron laser driven by slac linac, in Proceedings of the Particle Accelerator Conference, Knoxville, vol. 1, May 1995, pp. 183–185

Saturation occurs when

$$P_F \text{»} 1.6\, c_q^2 r_{fel} P_{beam}$$



Coherent spontaneous emission: solution for a “pre-bunched” beam, $b_1 \neq 0$, when $|a(0)|=0$

$$\frac{\partial}{\partial t} a(t) @ -2pg_0 b_1 e^{-in_0 t} + ipg_0 b_2 e^{-2in_0 t} \int_0^t dt' t' e^{in_0 t'} a^*(t - t') + ipg_0 \int_0^t dt' t' e^{-in_0 t'} a(t - t')$$

Coherent spontaneous emission growth, quadratic with the position along the undulator

$$a(\tau) = -2\pi g_0 b_1 \frac{\left(1 - e^{-i\nu_0 \tau}\right)}{\nu_0}$$

In the limit $v_0=0$, defining $P_{beam} = m_0 c^2 g \frac{I_{peak}}{e_0}$ we find

$$P_{coh}(z) = \frac{1}{3} \rho_{fel} |b_1|^2 P_{beam} \left(\frac{z}{L_g}\right)^2$$

After some distance in the undulator, when the field $a(\tau) \neq 0$, the homogeneous term

$$-i\pi g_0 \int_0^\tau d\xi \xi e^{-i\nu_0 \xi} a(\tau - \xi)$$

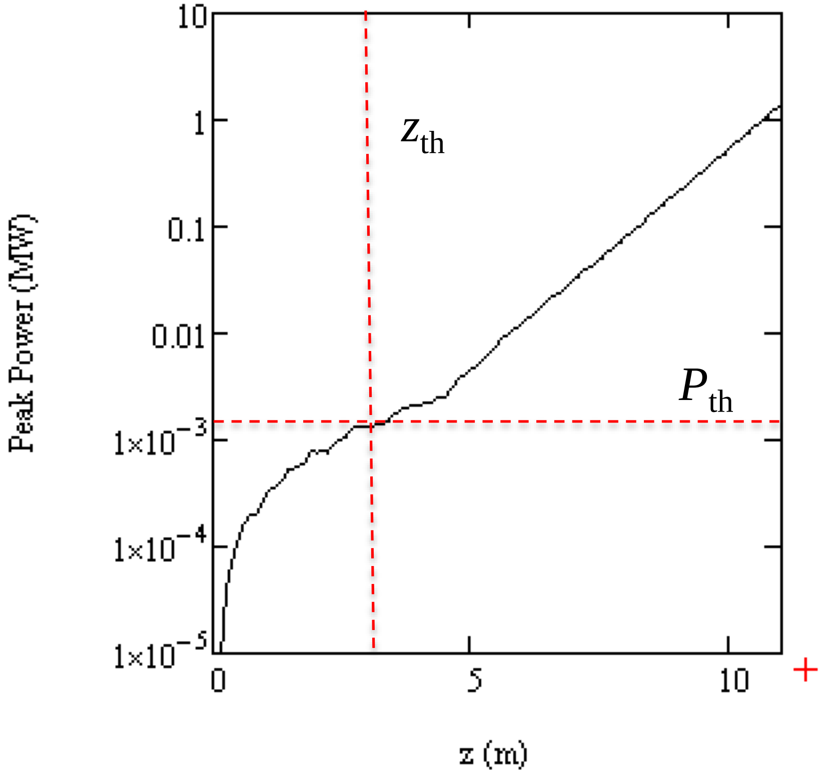
will become larger than the source term

i.e. $\left|-2\pi g_0 b_1 e^{-i\nu_0 \tau}\right| < \left|i\pi g_0 \int_0^\tau d\xi \xi e^{-i\nu_0 \xi} a(\tau - \xi)\right|$

and the growth will turn from quadratic into exponential

Self Amplified Spontaneous Emission

This occurs at $\tau_{th} \simeq \frac{1}{(\pi g_0)^{1/3}}$ where the field is $a(\tau_{th}) = -2(\pi g_0)^{2/3} b_1$



corresponding to

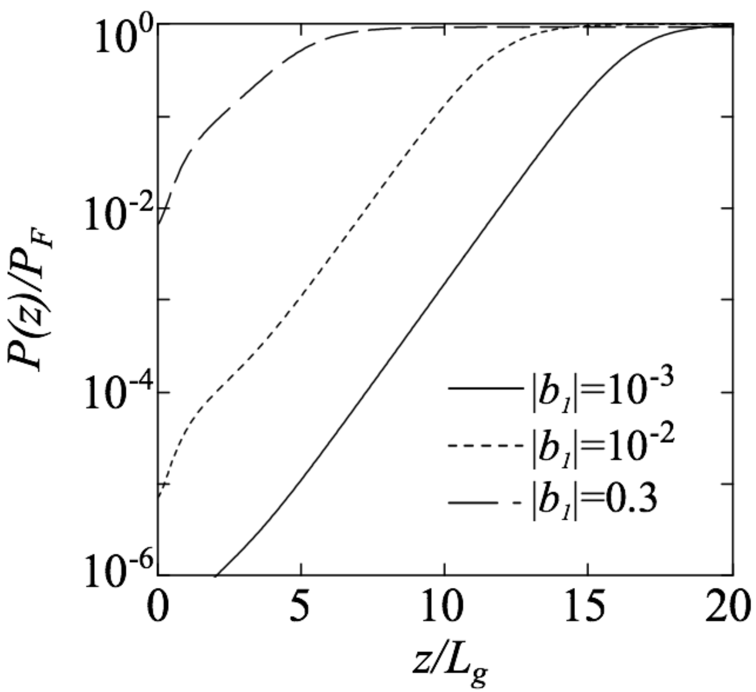
$$z_{th} = \sqrt{3} L_g$$

$$P_{th} = \rho_{fel} |b_1|^2 P_{beam}$$

The two solution can be combined including saturation effects:

$$P(z) = P_{th}\left[\frac{\frac{1}{3}\left(\frac{z}{L_g}\right)^2}{1+\frac{1}{3}\left(\frac{z}{L_g}\right)^2} + \frac{\frac{1}{2}exp\left[\frac{z}{L_g}-\sqrt{3}\right]}{1+\frac{P_{th}}{2P_F^*}\,exp\left[\frac{z}{L_g}-\sqrt{3}\right]}\right]$$

$$P_F^* = P_F - P_{th}$$



Saturation via Logistic function: G. Dattoli, P.L. Ottaviani, Semi-analytical models of free electron laser saturation. Opt. Commun.* ***204**(1), 283–297 (2002)*

L. Giannessi, **Seeding and Harmonic Generation in Free-Electron Lasers** *in* Synchrotron Light Sources and Free-Electron Lasers DOI 10.1007/978-3-319-04507-8_3-1 © Springer International Publishing Switzerland 2015

Pre-bunched beam equivalent power

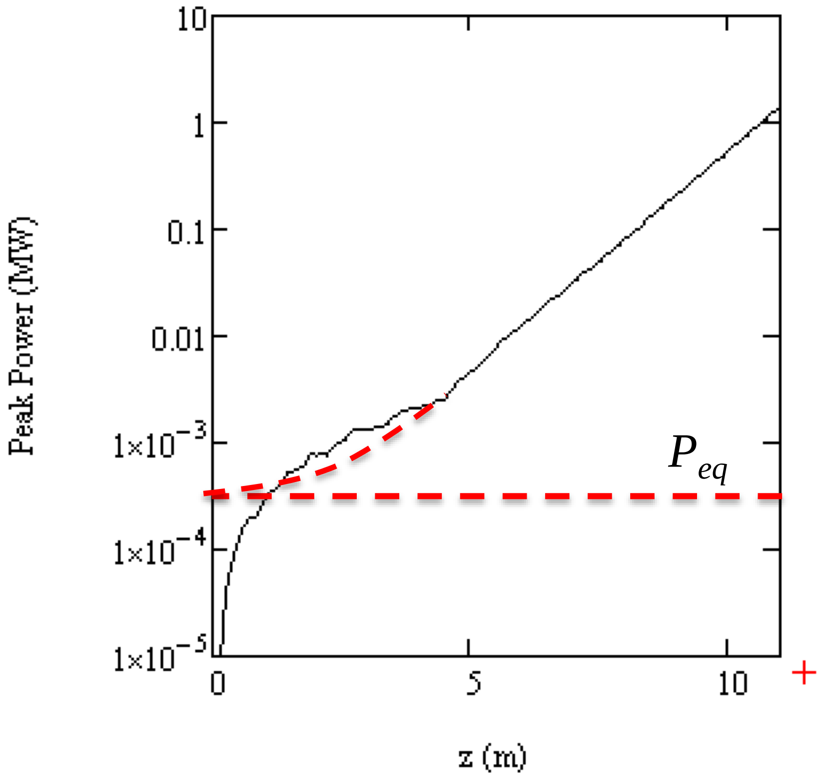
We define an **equivalent input power associated to a given beam pre-bunching** and derive an explicit expression for the **beam "shot noise" equivalent power**.

We impose that **the power associated to the exponentially growing root starting from a virtual seed value P_{eq},** equals the power at the threshold **from a pre-bunched beam**

$$\frac{P_{eq}}{9}e^{\sqrt{3}} = \rho_{fel}|b_1|^2 P_{beam},$$

We get an estimate of the **power required to ensure a growth equivalent to the one induced by an existing pre-bunching**

$$P_{eq} \sim 1.6\rho_{fel}|b_1|^2 P_{beam}$$



Estimate of b_1 for a randomly distributed e-beam

$$b_1 = \frac{1}{n_e}\sum_l e^{-i\theta_l} \cong \frac{1}{\sqrt{n_e}}$$

The distribution is not periodic. We have to account for the interference of the fields emitted by electrons separated by more than a wavelength.

A random, infinitely extended, point-like electron distribution, has a white-noise spectral distribution, but only the components in the FEL gain bandwidth will be then amplified. **The FEL amplifier has as amplification-bandwidth of the order of** $\Delta\omega \cong 2\omega\rho_{fel}$

We need therefore to calculate the fluctuations in a frequency range $\Delta\omega$ and the portion of beam that we have to consider for the, **has to be of the order of the cooperation length**, $L_c = \lambda_0 / (4\pi\rho_{fel})$.

The number of electrons in one cooperation length is: $n_e = I_{peak}\, l_c / e_0 c = I_{peak}\, \lambda_0 / (4\pi e_0 c \rho_{fel})$

Combining it with $P_{eq} \sim 1.6\rho_{fel}|b_1|^2 P_{beam}$ we get a **"shot noise" equivalent intensity**:

$$I_{sn} \sim 18 e^{-\sqrt{3}} \omega \frac{\rho_{fel}^2}{\Sigma_b} \gamma m_0 c^2 \sim 3\,\omega \rho_{fel}^2 \gamma m_0 c^2$$

We introduced $\Sigma_b = \pi\varepsilon\beta_T/\gamma$, the e-beam average cross section

Shot noise scaling with λ

$$I_{sn} \sim 18 e^{-\sqrt{3}} \omega \frac{\rho_{fel}^2}{\Sigma_b} \gamma m_0 c^2 \sim 3\,\omega \rho_{fel}^2 \gamma m_0 c^2$$

A scaling relation of the shot noise equivalent power with the resonant wavelength may be obtained by assuming:

a) $\gamma \propto 1/\sqrt{\lambda_0}$ to preserve the resonant condition
b) An increase of the peak current compensates the increased energy to limit the reduction of the ρ_{fel} parameter.

In these conditions the shot noise power scales as $\lambda_0^{-3/2}$

Example:

	VUV	X-Ray
Wavelength (nm)	20	0.1
Beam Energy (GeV)	1.5	15
I_{peak} (kA)	0.7	3
ρ_{fel}	2.8×10^{-3}	6.7×10^{-4}
e-beam cross s. (μm²)	3.8×10^{4}	3.8×10^{3}
$\|b_1\|$	3×10^{-4}	10^{-3}
I_{sn} (MW/cm²)	1	10^{3}

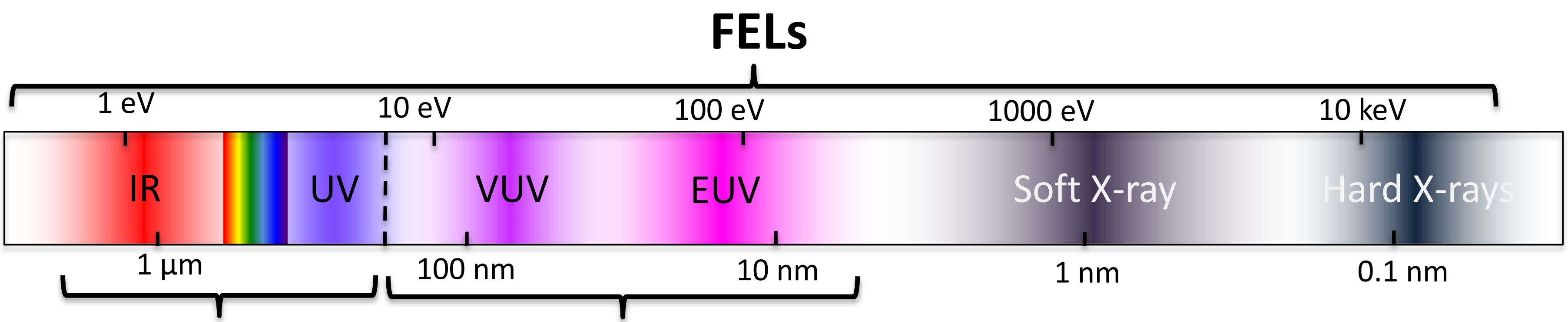
Seeding with high harmonics generated in gas

DIRECT SEEDING

Seed Sources

- Intensity
- Short pulse duration (≈ 100 fs)
- Tunable (to preserve FEL tunability)
- High Spatial quality (Single TEM00 mode)
- Temporal coherence
- Rep. Rate (> 10-100 Hz, or more)

... not much choice !!!



Solid state lasers,
Ti:Sa - OPA - THG

Outperform FELs
in their spectral range.

High harmonics
Generated in gas HHG

Interesting properties,
reasonable for direct seeding

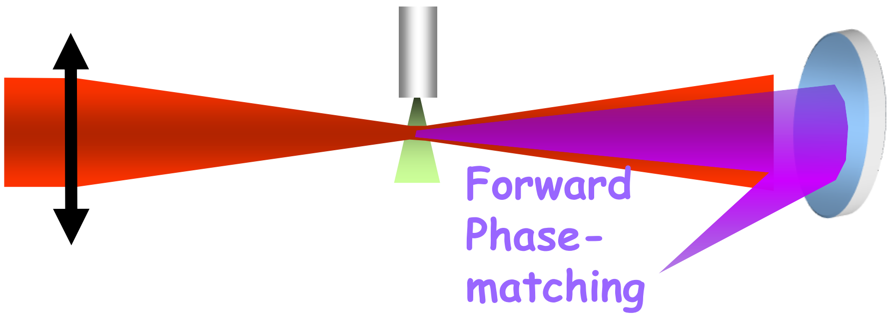
XUV Seed Sources:

Courtesy of B. Carrè

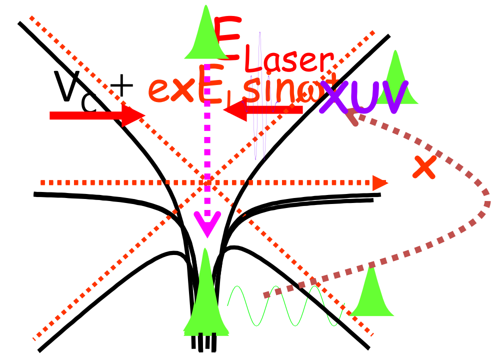
High Harmonics Generation in gases

Rare gas (jet, cell, capillary)

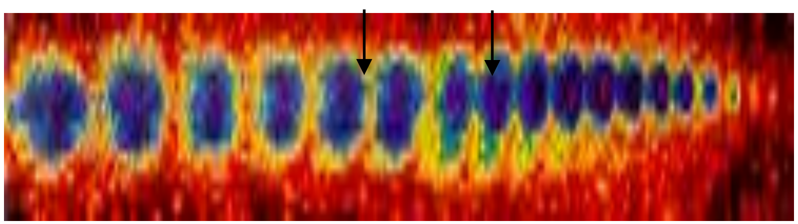
grating



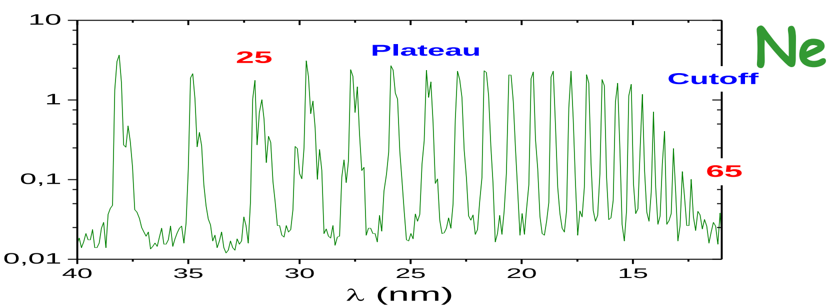
- "Three-step" model



Odd harmonics of Ti:Sa



Frequency →

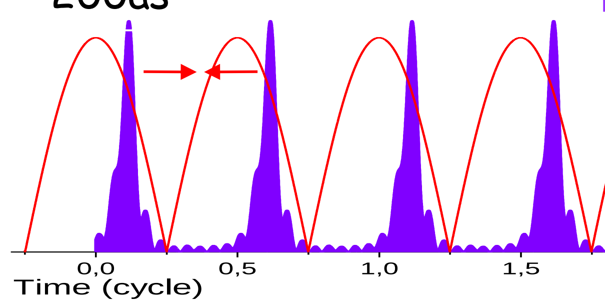


Mairesse et al. SCIENCE (2003)

- Multi-cycle laser pulse

Train of XUV "atto" pulses

~200as

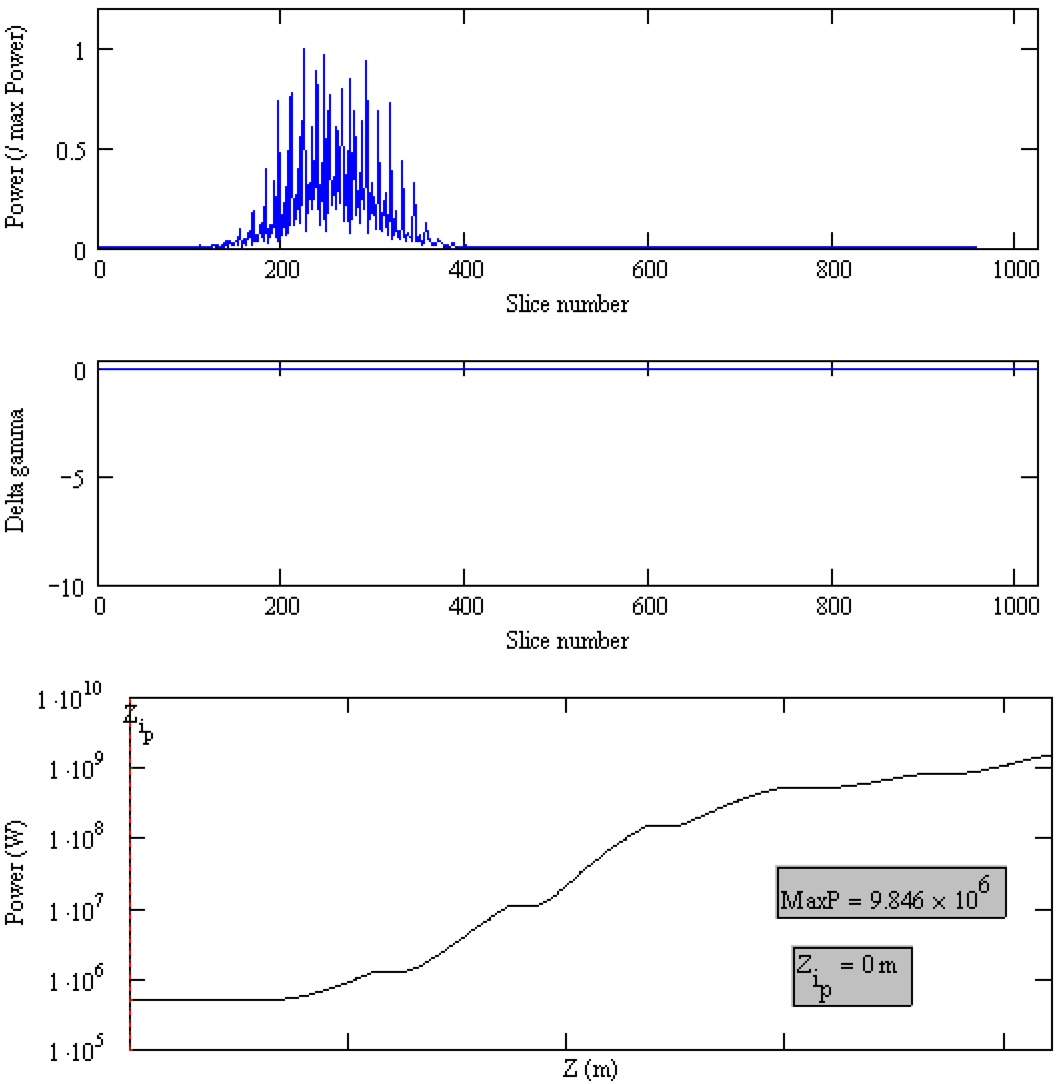


Discrete odd harmonics

Aluminium 0.6 μm, broadband (45 nm)

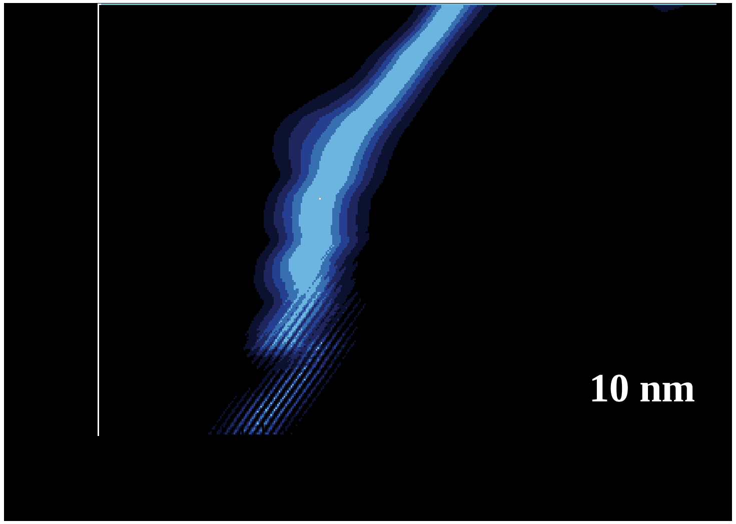
Pulse energy
50 nJ

Energy in 2ρ: 0.8 nJ

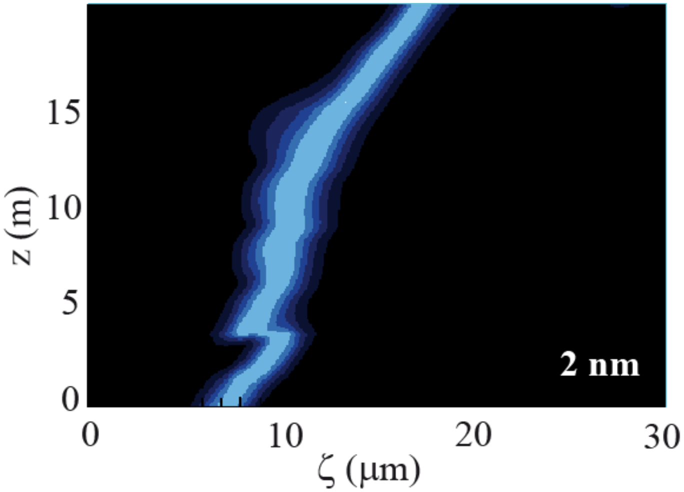


Results at different filter bandwidth

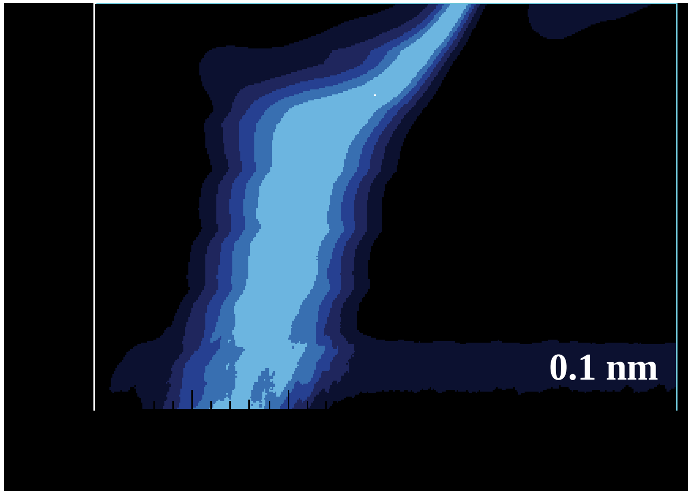
2.5 nJ, ~0.5 nJ in 2ρ



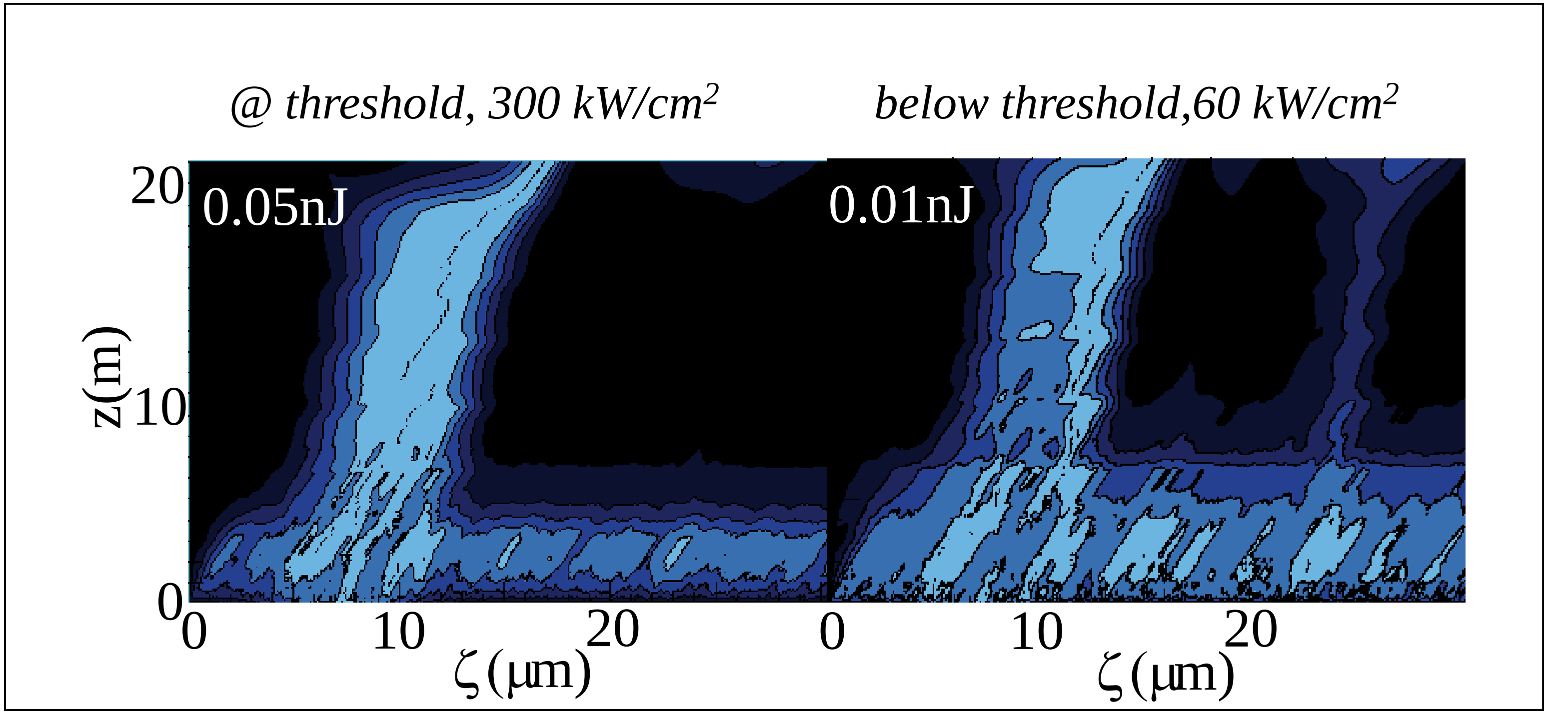
1 nJ, ~0.5 nJ in 2ρ



0.5 nJ, ~0.5 nJ in 2ρ



At threshold for overcoming shot noise



Experiments

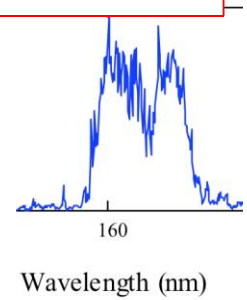
SPARC
(2010) 160nm

TUPB18 Proceedings of FEL2010, Malmö, Sweden

FEL EXPERIMENTS AT SPARC: SEEDING WITH HARMONICS GENERATED IN GAS

L. Giannessi, A. Petralia, G. Dattoli, F. Ciocci, M. Del Franco, M. Quattromini, C. Ronsivalle, E. Sabia, I. Spassovsky, V. Surrenti ENEA C.R. Frascati, IT. D. Filippetto, G. Di Pirro, G. Gatti, M. Bellaveglia, D. Alesini, M. Castellano, E. Chiadroni, L. Cultrera, M. Ferrario, L. Ficcadenti, A. Gallo, A. Ghigo, E. Pace, B. Spataro, C. Vaccarezza, INFN-LNF, IT. A. Bacci, V. Petrillo, A.R. Rossi, L. Serafini INFN-MI, IT. M. Serluca, M. Moreno INFN-Roma I, IT. L. Poletto, F. Frassetto CNR-IFN, IT. J.V. Rau, V. Rossi Albertini ISM-CNR, IT. A. Cianchi, UN-Roma II TV, IT. A.Mostacci, M Migliorati, L. Palumbo, Università di Roma La Sapienza, IT. G. Marcus, P. Musumeci, J.Rosenzweig, UCLA, CA, USA., M. Labat, F. Briquez, M. E. Couprie, SOLEIL, FR. B. Carré, M.Bougeard, D. Garzella CEA Saclay, DSM/DRECAM, FR. G. Lambert LOA, FR. C.Vicario PSI, CH.

SPARC
(2010) 160nm
& 266 nm



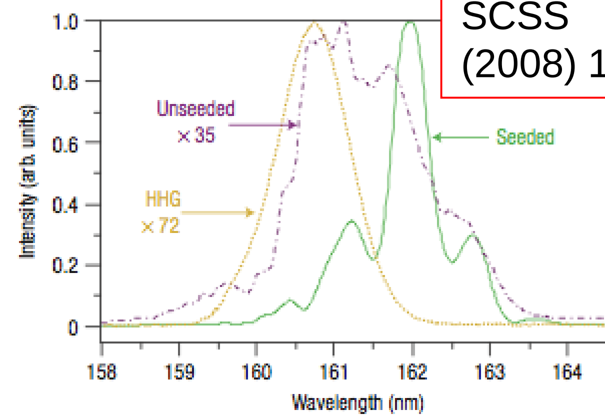
SCSS
(2008) 160nm

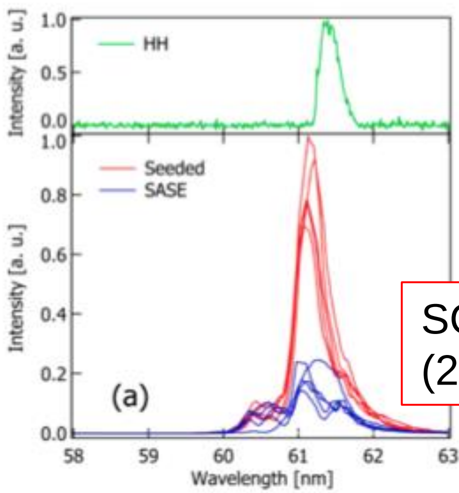
LETTERS

Injection of harmonics generated in gas in a free-electron laser providing intense and coherent extreme-ultraviolet light

G. LAMBERT[1,2,3]*, T. HARA[2,4], D. GARZELLA[1], T. TANIKAWA[2], M. LABAT[1,3], B. CARRE[1], H. KITAMURA[2,4], T. SHINTAKE[2,4], M. BOUGEARD[1], S. INOUE[4], Y. TANAKA[2,4], P. SALIERES[1], H. MERDJI[1], O. CHUBAR[3], O. GOBERT[1], K. TAHARA[2] AND M.-E. COUPRIE[3]
[1]Service des Photons, Atomes et Molécules, DSM/DRECAM, CEA-Saclay, 91191 Gif-sur-Yvette, France
[2]RIKEN SPring-8 Centre, Harima Institute, 1-1-1, Kouto, Sayo-cho, Sayo-gun, Hyogo 679-5148, Japan
[3]Groupe Magnétisme et Insertion, Synchrotron Soleil, L'Orme des Merisiers, Saint Aubin, 91192 Gif-sur-Yvette, France
[4]XFEL Project Head Office/RIKEN, 1-1-1, Kouto, Sayo-cho, Sayo-gun, Hyogo 679-5148, Japan
*e-mail: guillaume.lambert@synchrotron-soleil.fr

SCSS
(2008) 160nm





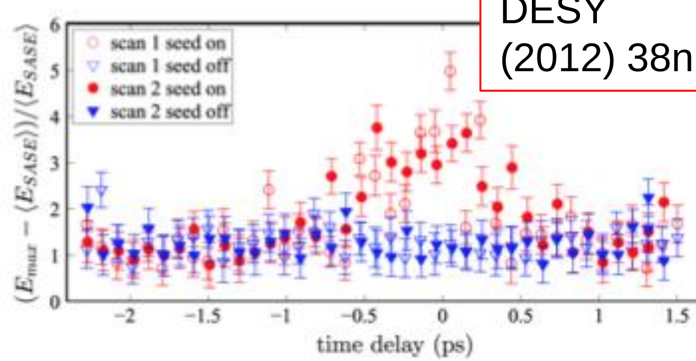
Extreme ultraviolet free electron laser seeded with high-order harmonic of Ti:sapphire laser

Tadashi Togashi,[1,2] Eiji J. Takahashi,[3] Katsumi Midorikawa,[3] Makoto Aoyama,[4] Koichi Yamakawa,[4] Takahiro Sato,[1,5] Atsushi Iwasaki,[5] Shigeki Owada,[5] Tomoya Okino,[5] Kaoru Yamanouchi,[5] Fumihiko Kannari,[6] Akira Yagishita,[7] Hidetoshi Nakano,[8] Marie E. Couprie,[9] Kenji Fukami,[1,2] Takaki Hatsui,[1] Toru Hara,[1] Takashi Kameshima,[1] Hideo Kitamura,[1] Noritaka Kumagai,[1] Shinichi Matsubara,[1,2] Mitsuru Nagasono,[1] Haruhiko Ohashi,[1,2] Takashi Ohshima,[1] Yuji Otake,[1] Tsumoru Shintake,[1] Kenji Tamasaku,[1] Hitoshi Tanaka,[1,2] Takashi Tanaka,[1,2] Kazuaki Togawa,[1] Hiromitsu Tomizawa,[1,2] Takahiro Watanabe,[1,2] Makina Yabashi,[1] and Tetsuya Ishikawa[1]

January 2011 / Vol. 19, No. 1 / OPTICS EXPRESS 317

SCSS
(2011) 61nm

DESY
(2012) 38nm



PRL 111, 114801 (2013) PHYSICAL REVIEW LETTERS week ending 13 SEPTEMBER 2013

Generation of Coherent 19- and 38-nm Radiation at a Free-Electron Laser Directly Seeded at 38 nm

S. Ackermann,[1,2] A. Azima,[1,5,6] S. Bajt,[2] J. Bödewadt,[1,5,*] F. Curbis,[1,†] H. Dachraoui,[2] H. Delsim-Hashemi,[2] M. Drescher,[1,5,6] S. Düsterer,[2] B. Faatz,[2] M. Felber,[2] J. Feldhaus,[2] E. Hass,[1] U. Hipp,[1] K. Honkavaara,[2] R. Ischebeck,[4] S. Khan,[3] T. Laarmann,[2,6] C. Lechner,[1] Th. Maltezopoulos,[1,5] V. Miltchev,[1] M. Mittenzwey,[1] M. Rehders,[1,5] J. Rönsch-Schulenburg,[1,5] J. Rossbach,[1,5] H. Schlarb,[2] S. Schreiber,[2] L. Schroedter,[2] M. Schulz,[1,2] S. Schulz,[2] R. Tarkeshian,[1,‡] M. Tischer,[2] V. Wacker,[1] and M. Wieland[1,5,6]

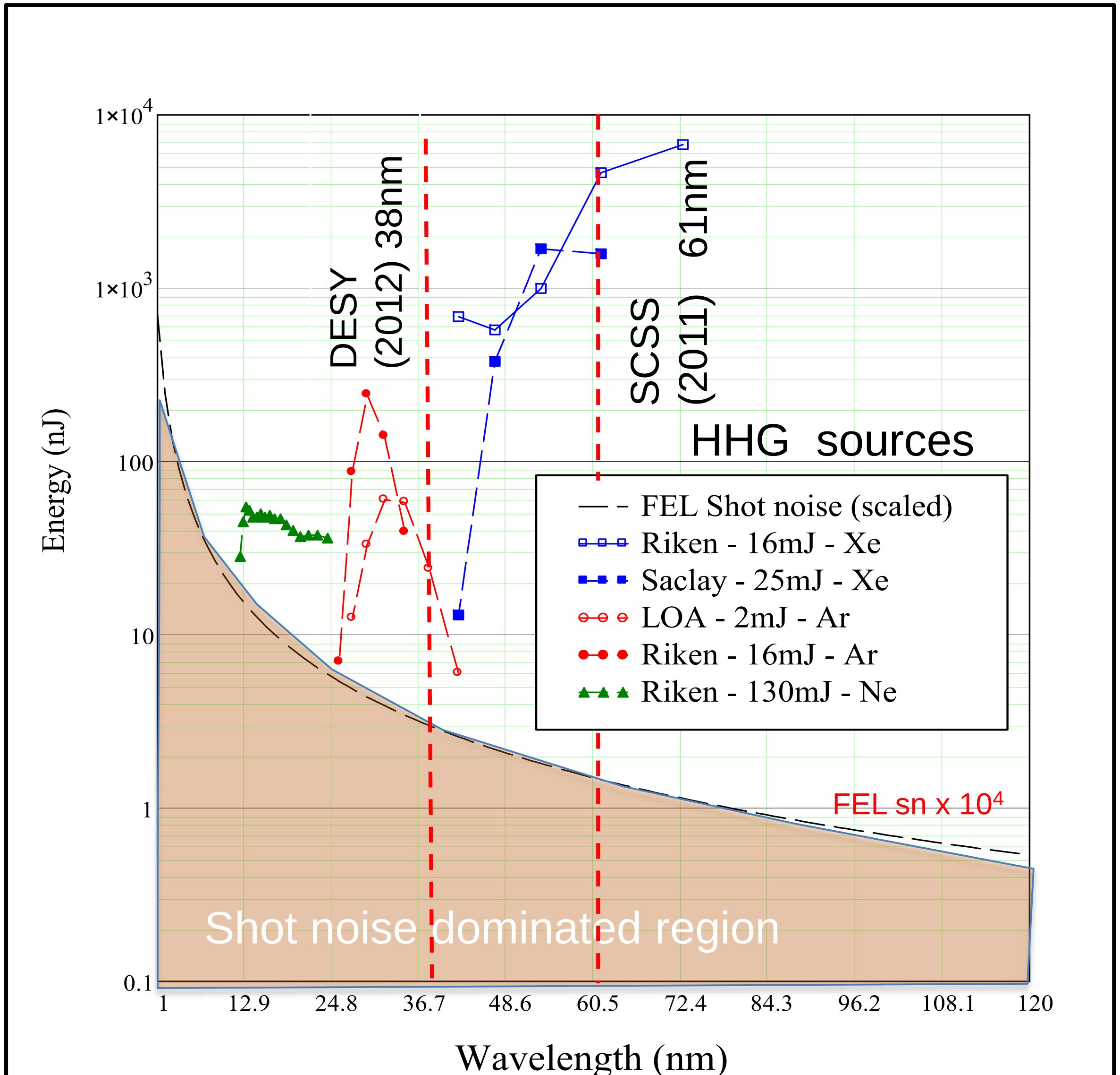
Match the seed to the e-beam

There are other requirements that have to be satisfied to match the seed beam to the FEL amplifier.

- Contrast ratio - S/N ratio between seeded/unseeded beam (x10^2 – x10^3)
- Transport optics & transverse matching: Shot noise calculated in a simplified 1D picture, the power is the fraction really coupled with the electrons (x5 – x10)
- Frequency matching (Harmonics spectra broader than ρ …(x10) or of the desired seed bandwidth (x10^2-10^3)

Factor: x 10^4 (can easily turn into x 10^5 – x 10^6)

Direct seeding an amplifier: the seed power required to overcome the shot noise scale with the inverse of the wavelength



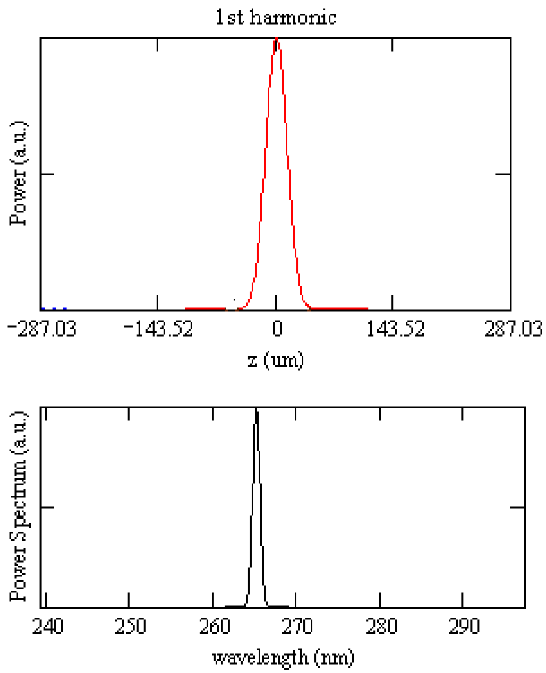
- *data from B. Carré, Colloque AEC - Slicing, Paris 2004*
- *Shot noise estimate includes transport and matching to e-beam – Seeded FELs Workshop, Frascati 10-12 (2008)*

1-W. Boutu M. Ducousso, J.-F. Hergott and H. Merdji on HHG and 2-M.E. Couprie and L. G, on Seeded FELs, both in Springer Series in Optical Sciences 197 (2015) ISBN 978-3-662-47442-6 DOI 10.1007/978-3-662-47443-3

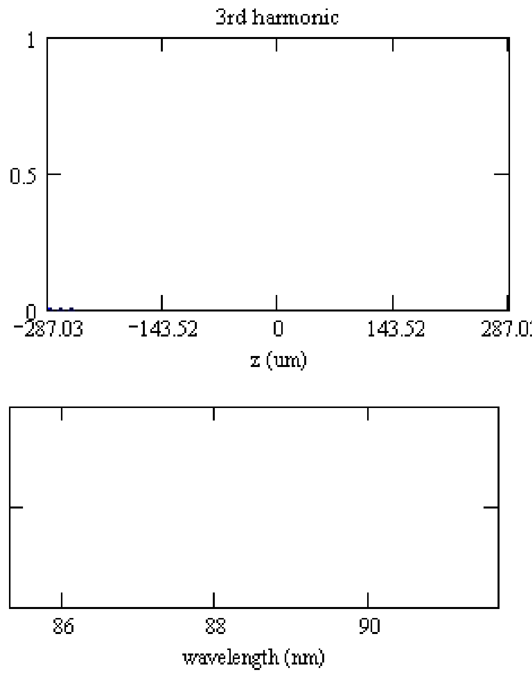
Using harmonic generation to get around the problem

HIGH GAIN HARMONIC GENERATION

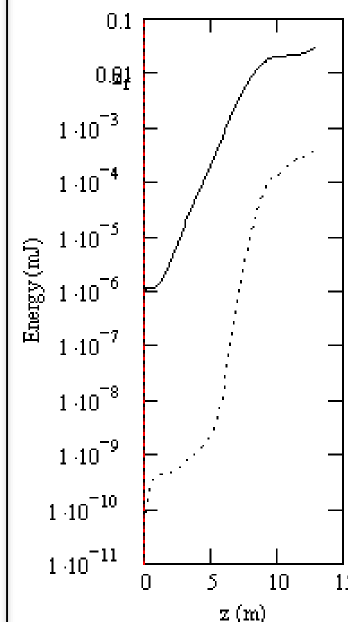
Generation of higher order harmonics



Fundamental harmonic



Third harmonic

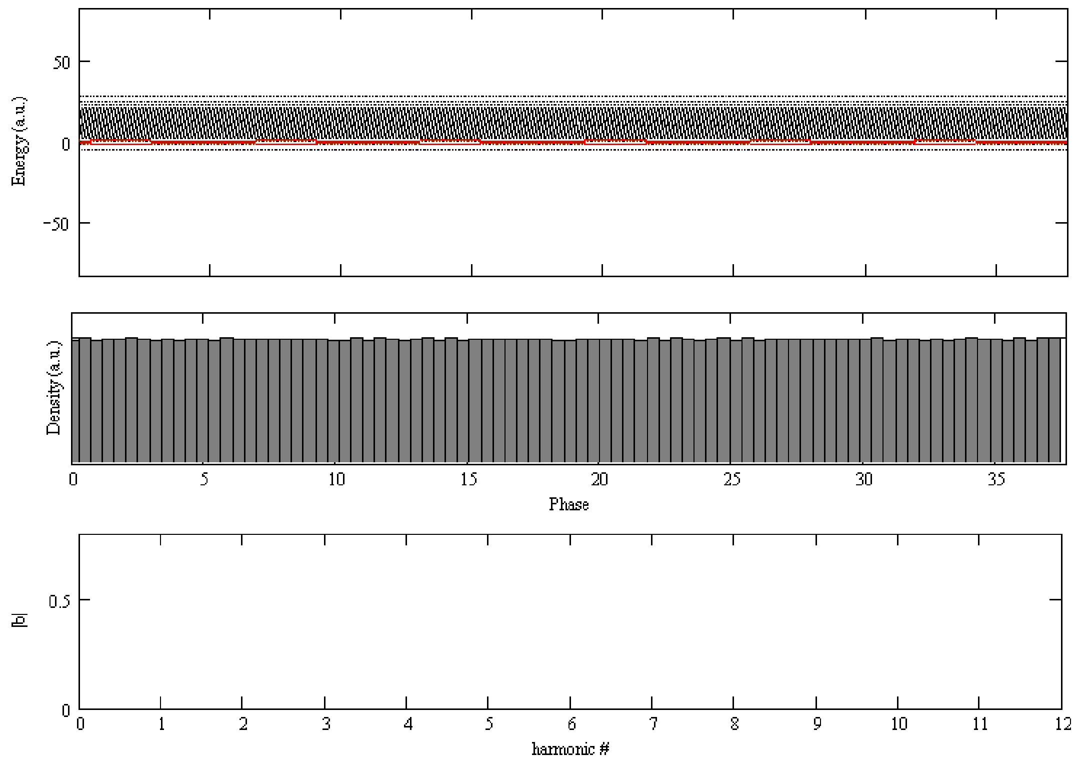


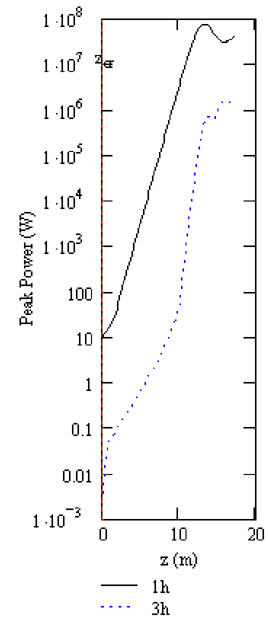
Electrons longitudinal phase space and higher order harmonic emission

Phase space
Energy/position

Longitudinal
density ρ

$b_n(s,t)$ n^{th}
Fourier coefficients
of ρ





Higher order harmonics

The FEL as an “harmonic converter”

I.Boscolo, V. Stagno, Il Nuovo Cimento 58, 271 (1980)

- *R. Barbini e*
  Sag Harbor
  52273 UC-4
  (1990)
- *R. Bonifaci*
- *L. H. Yu PR*

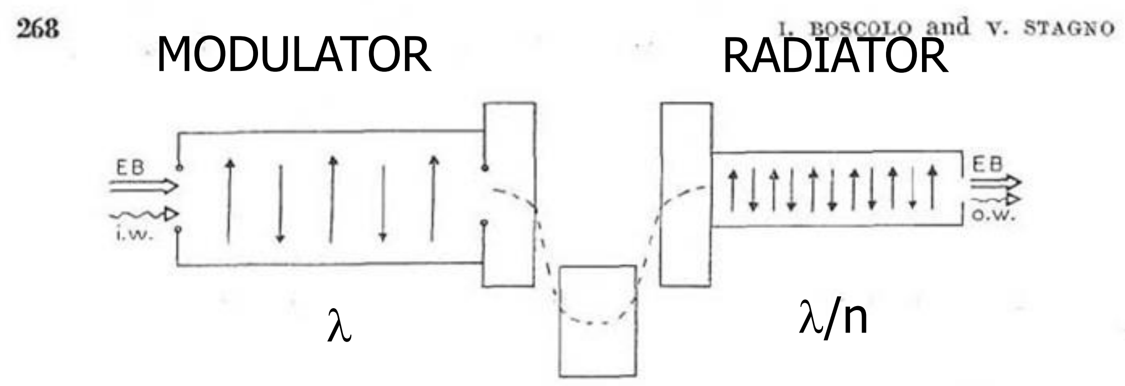
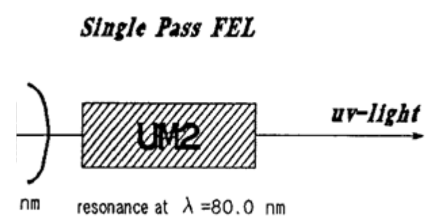


Fig. 1. – Scheme of the converter: EB electron beam, i.w. input wave, o.w. output wave.



Important Milestones

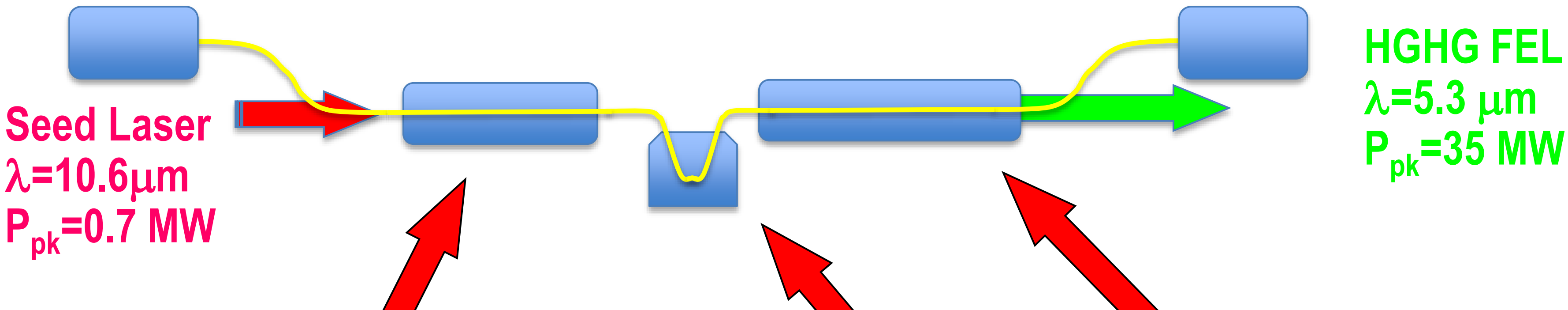
High-Gain Harmonic-Generation Free-Electron Laser
L. H. Yu et al. Science 289 (2000)

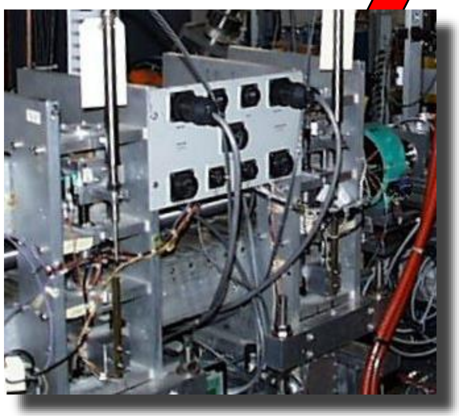
First Ultraviolet High-Gain Harmonic-Generation Free-Electron Laser
L. H. Yu et al. PRL 91 (2003)

Courtesy of I. Ben Zvi

The HGHG Experiment

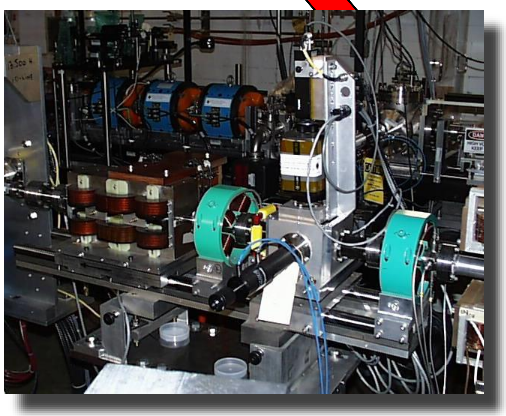
BROOKHAVEN NATIONAL LABORATORY





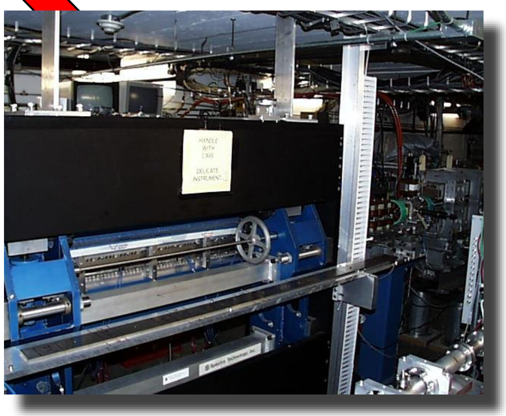
Modulator Section

B_w=0.16T λ_w=8cm L=0.76 m



Dispersion Section

L=0.3 m



Radiator Section

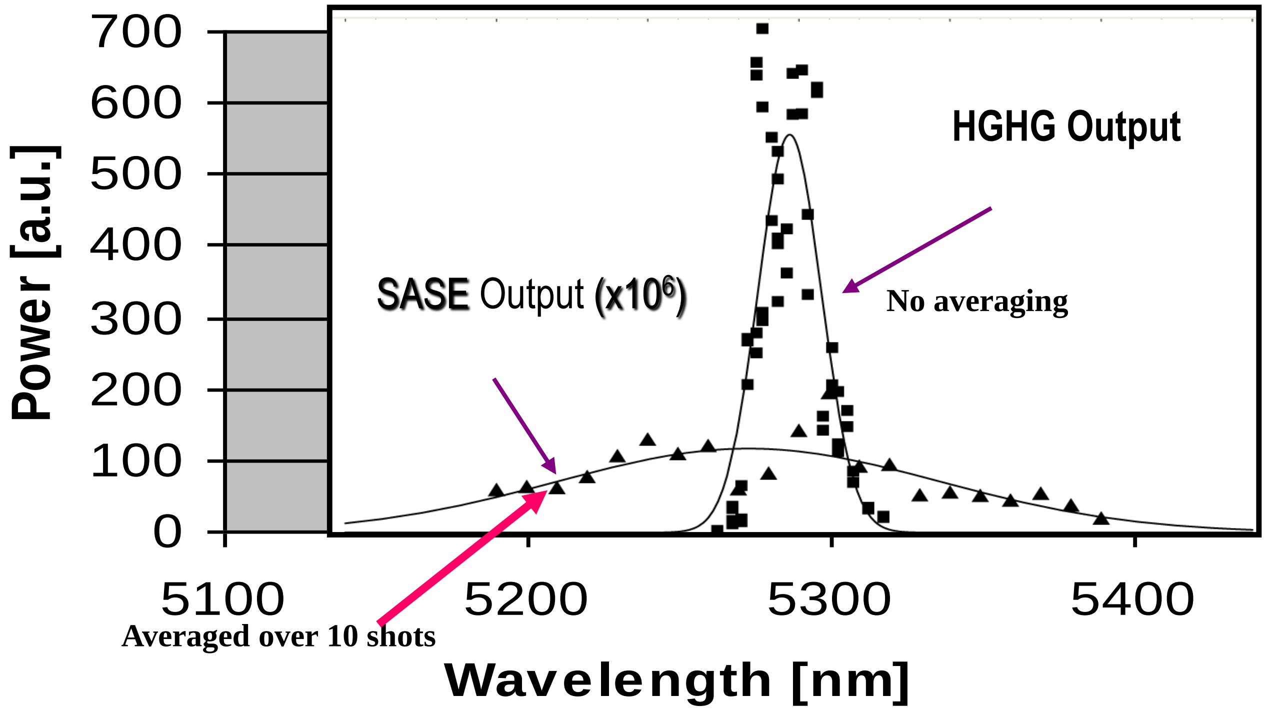
B_w=0.47 λ_w=3.3cm L=2 m

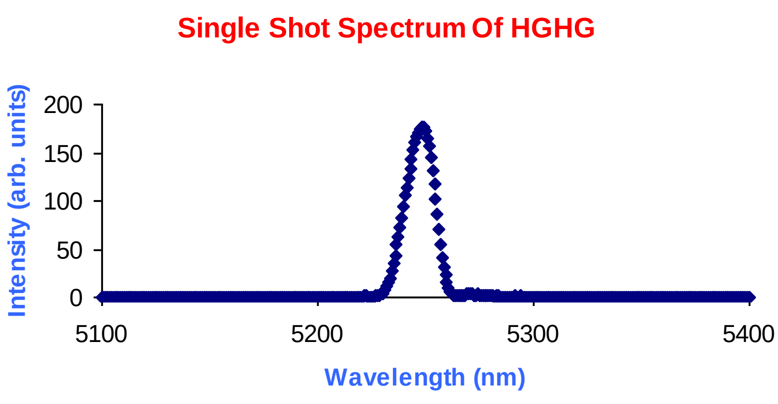
Electron Beam Input Parameters: E= 40 MeV
ε_n= 4π mm-mrad $d\gamma/\gamma$=0.043% I = 110A τ_e= 4 ps

Courtesy of I. Ben Zvi

HGHG multi shot spectrum

HGHG Spectrum





L. H. Yu et al. Science **289** (2000) 932

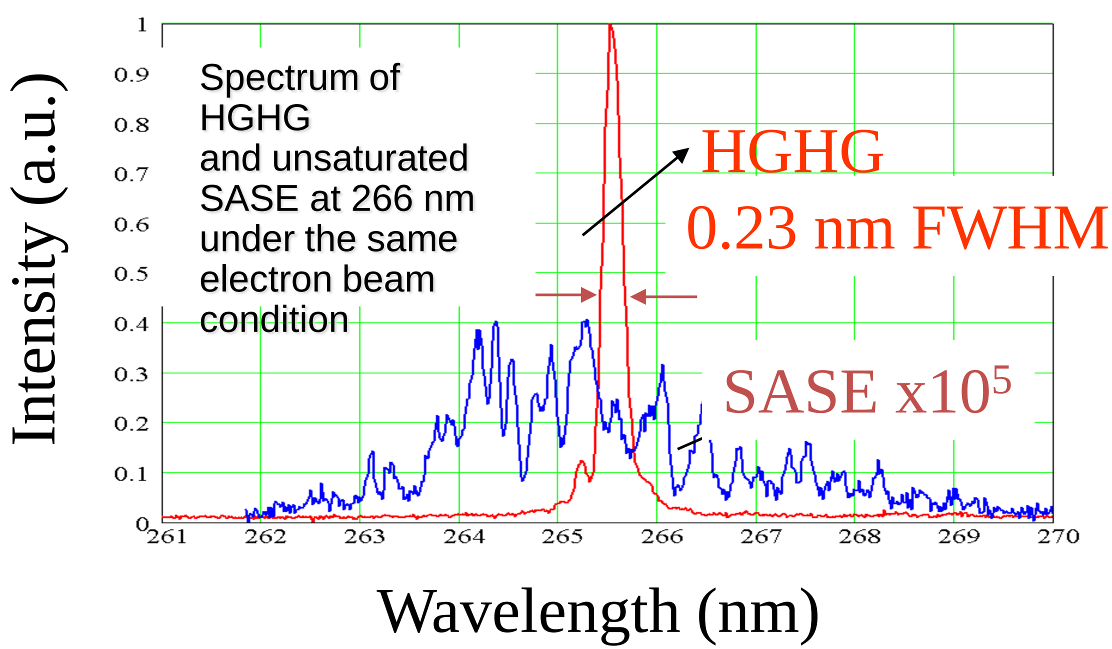
High gain harmonic generation from 800 nm to 266 nm

VOLUME 91, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 15 AUGUST 2003

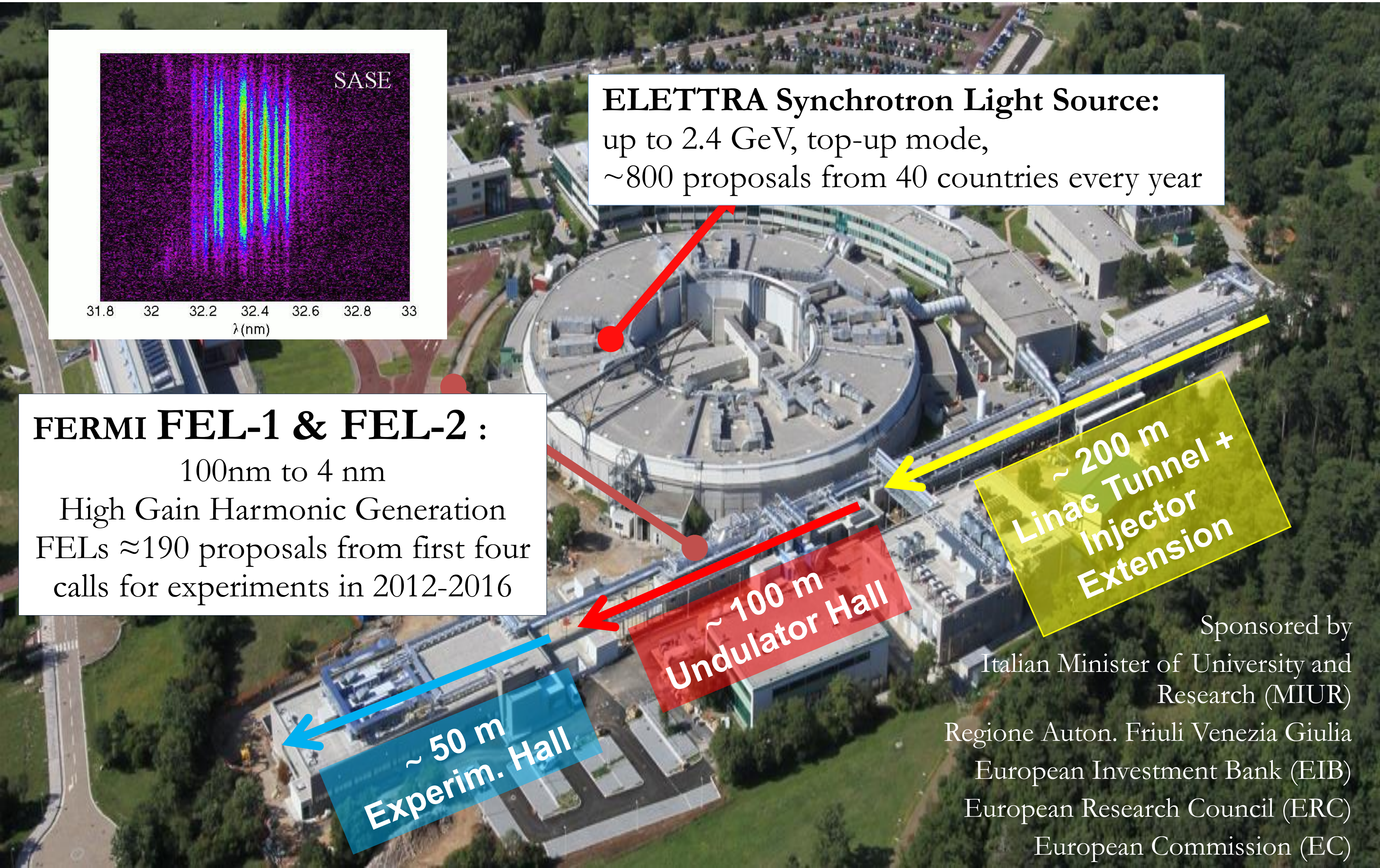
First Ultraviolet High-Gain Harmonic-Generation Free-Electron Laser

L. H. Yu,* L. DiMauro, A. Doyuran, W. S. Graves,† E. D. Johnson, R. Heese, S. Krinsky, H. Loos, J. B. Murphy, G. Rakowsky, J. Rose, T. Shaftan, B. Sheehy, J. Skaritka, X. J. Wang, and Z. Wu
National Synchrotron Light Source, Brookhaven National Laboratory, Upton, New York 11973, USA
(Received 25 March 2003; published 14 August 2003)

Conversion 800nm -> 266nm



FERMI and Elettra

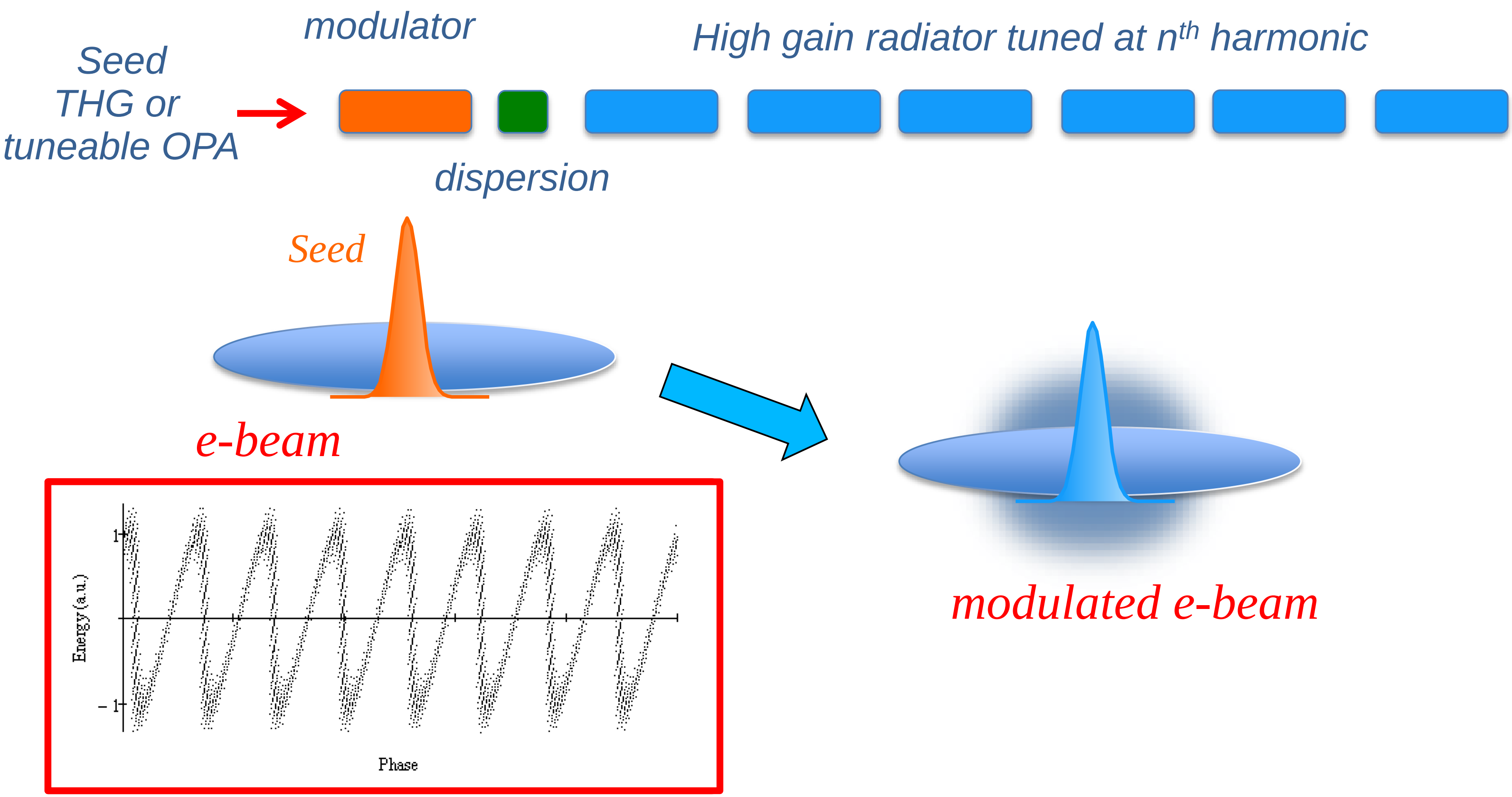


ELETTRA Synchrotron Light Source:
up to 2.4 GeV, top-up mode,
~800 proposals from 40 countries every year

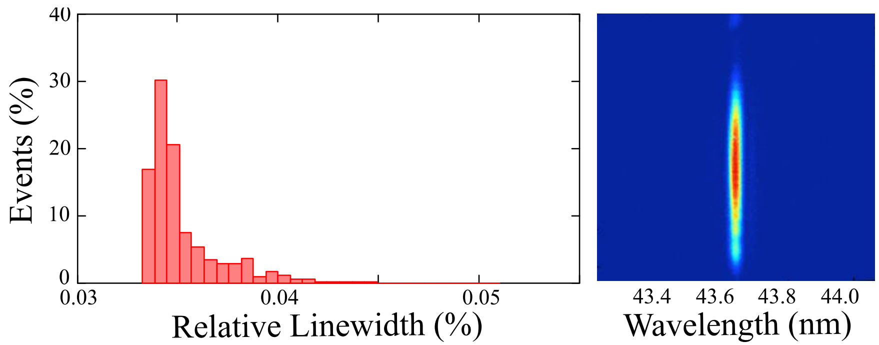
FERMI **FEL-1 & FEL-2** :
100nm to 4 nm
High Gain Harmonic Generation
FELs ≈190 proposals from first four calls for experiments in 2012-2016

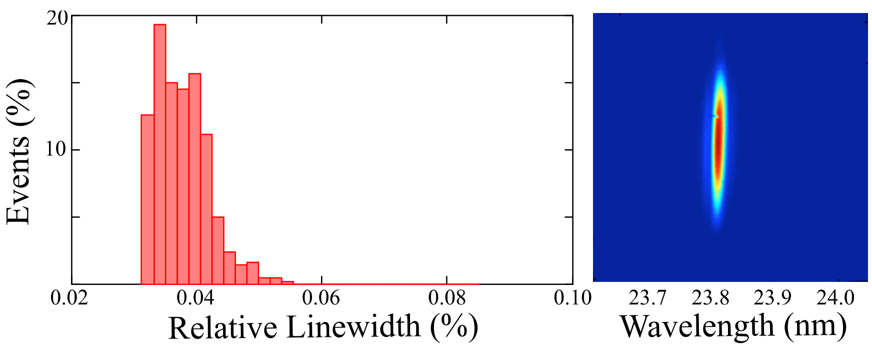
Sponsored by
Italian Minister of University and Research (MIUR)
Regione Auton. Friuli Venezia Giulia
European Investment Bank (EIB)
European Research Council (ERC)
European Commission (EC)

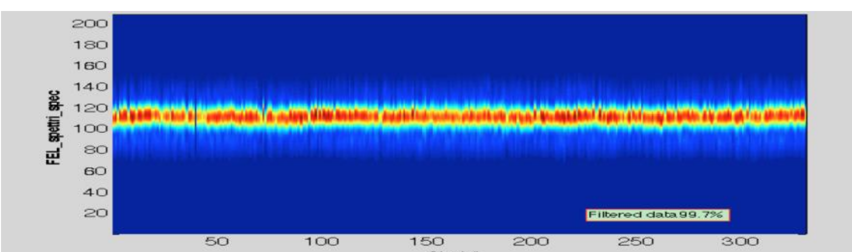
FEL-1 – HGHG FEL at work …



FERMI FEL-1 Spectral properties





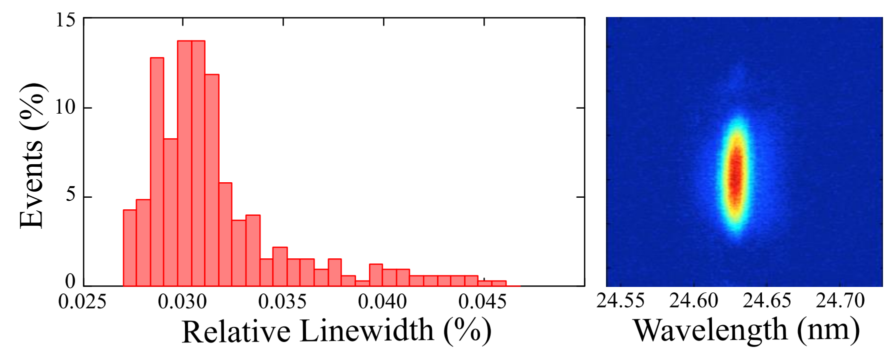


Seeded by the OPA laser at 245nm, the h14 delivers more than 10 uJ with good spectral properties.

Linear and Circular polarization available between 100 and 20 nm

The spectral properties can be preserved up to h13-h15 (h6 and h11 are shown in the pictures)

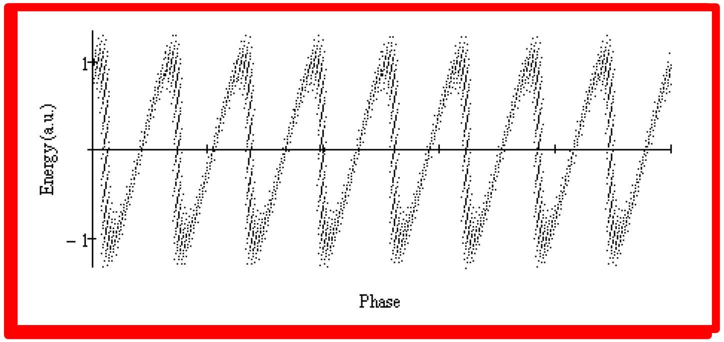
These sequences were acquired with the seed generated as Third harmonic of a Ti-Sa laser system



Continuously tunable with the OPA laser system, with small limitations mainly depending on the optical properties of the mirrors transporting the seed to the undulator.

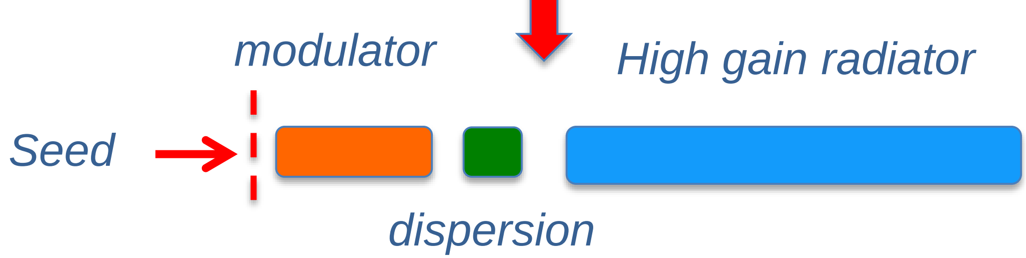
HGHG FEL in detail

Modulation + Dispersion



We start from a uniform distribution $\rho_e(s)=const.$
we assume no gain in the modulator $a(s,\tau)=a(s,0)$

We calculate b_n at this position



At the position s along the e-bunch we have:

Modulator

$$\eta'(s)=\eta(s,\tau=1)=\eta(s,0)+|a(s,0)|\cos\left(\theta(s,0)+\Phi(s,0)\right)$$

$$\theta'(s)=\theta(s,\tau=1)=\theta(s,0)+\eta(s,0)+\frac{1}{2}|a(s,0)|\cos\left(\theta(s,0)+\Phi(s,0)\right)$$

Dispersive section

$$\theta''(s,\tau=1+\tau_{disp})=\theta'+\eta'\tau_{disp}=\theta(s,0)+\eta(s,0)\left(1+\tau_{disp}\right)+|a(s,0)|\cos\left(\theta(s,0)+\Phi(s,0)\right)\left(\frac{1}{2}+\tau_{disp}\right)$$

$$dz=R_{56}\frac{dE}{E}, \quad \textit{in undulator we have} \quad R_{56}=2N\lambda_0 \Rightarrow \quad \tau_{disp}=\frac{R_{56}}{2N\lambda_0}, \quad (N,\lambda_0 \textit{ of the modulator})$$

Bunching factor after the modulator

We assume a uniform electron energy distribution, i.e. indep. of s in the region where we calculate $b_n(s)$

$$f(n)=\frac{1}{\sqrt{2p}s_n}\exp\left[-\frac{(n-n_0)^2}{2s_n^2}\right]$$

Recalling the definition $n=2pN\left(\frac{w_0(g)-w_r}{w_0(g)}\right)$ it holds $s_n=2pNs_{w_0}=4pN\frac{s_g}{g}$

$$b_n(s)=\int_{-\infty}^{+\infty}dn f(n)\frac{1}{l}\int_s^{s+l}ds\exp\left[-ikq''(s)\right]=$$

For simplicity we assume (or re-define) $(1+t_{disp})\approx t_{disp}$

$$=\int_{-\infty}^{+\infty}dn f(n)\frac{1}{l}\int_s^{s+l}ds'\exp\left[-in\left(q(s',0)+n(s',0)t_{disp}+|a(s',0)|\cos\left(q(s',0)+F(s',0)\right)t_{disp}\right)\right]=$$

$$=\frac{1}{\sqrt{2p}s_n}\int_{-\infty}^{+\infty}dn\,e^{-innt_{disp}}e^{\left(-\frac{(n-n_0)^2}{2s_n^2}\right)}\frac{1}{l}\int_s^{s+l}ds'e^{-inq(s',0)}e^{-in\left(|a(s',0)|\cos(q(s',0)+F(s',0))t_{disp}\right)}=$$

$$\frac{1}{\sqrt{2p}s_n}\int_{-\infty}^{+\infty}dn\,e^{-innt_{disp}}e^{\left(-\frac{(n-n_0)^2}{2s_n^2}\right)}=\exp\left[-\frac{1}{2}\left(nt_{disp}s_n\right)^2\right]=\exp\left[-\frac{1}{2}\left(nk_0R_{56}\frac{s_g}{g}\right)^2\right]$$

Bunching factor after the modulator

$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{\sigma_\gamma}{\gamma}\right)^2\right]}\frac{1}{\lambda}\int_s^{s+\lambda} ds' e^{-ink_r s'} e^{-in\left(\left|a(s',0)\right|\cos\left(\theta(s',0)+\Phi(s',0)\right)\tau_{disp}\right)} =$$

$$= e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{\sigma_\gamma}{\gamma}\right)^2\right]}\frac{1}{\lambda}\int_s^{s+\lambda} ds' e^{-ink_r s'} \sum_{m=-\infty}^{\infty}(-i)^m J_m\left(n\tau_{disp}\left|a(s')\right|\right) e^{im\left(k_r s'+\Phi(s')\right)}$$

Using:

$$\theta(s,0) = (k_u + k_r)s \cong k_r s$$

$$(k_u \ll k_r)$$

$$e^{iz\cos(\theta)} = \sum_{m=-\infty}^{\infty} i^m J_m(z) e^{im\theta}$$

We have a periodic modulation. The integral may be calculated over one period at the position *s*. By the SVEA assumption *a(s')* and $\Phi(s')$ do not change over the range λ and can be carried out of the integral

$$b_n(s) = \sum_{m=-\infty}^{\infty}(-i)^m J_m\left(n\tau_{disp}\left|a(s)\right|\right) e^{im\Phi(s)} e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{\sigma_\gamma}{\gamma}\right)^2\right]}\underbrace{\frac{1}{\lambda}\int_s^{s+\lambda} ds' e^{-i(n-m)k_r s'}}_{\delta_{m,n}} =$$

$$= e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{\sigma_\gamma}{\gamma}\right)^2\right]} J_n\left(n\tau_{disp}\left|a(s)\right|\right) e^{in\Phi(s)}$$

$$n\tau_{disp}\left|a(s')\right| = n\left(\frac{R_{56}}{2N\lambda_0}\right)\Delta\nu = n\left(\frac{R_{56}}{2N\lambda_0}\right)4\pi N\left(\frac{\Delta\gamma}{\gamma}\right) = n k_0 R_{56}\left(\frac{\Delta\gamma}{\gamma}\right)$$

Bunching factor after the modulator

L. H. Yu, PRA 1991

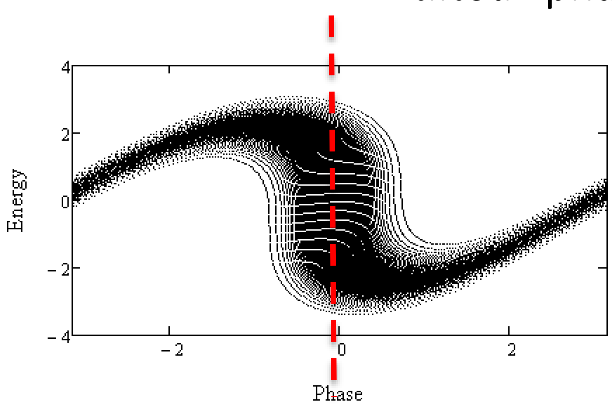
$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56} \frac{s_g}{g}\right)^2\right]} J_n\left(n k_0 R_{56}\left(\frac{\mathrm{D}g}{g}\right)\right) e^{in\mathrm{F}(s)}$$

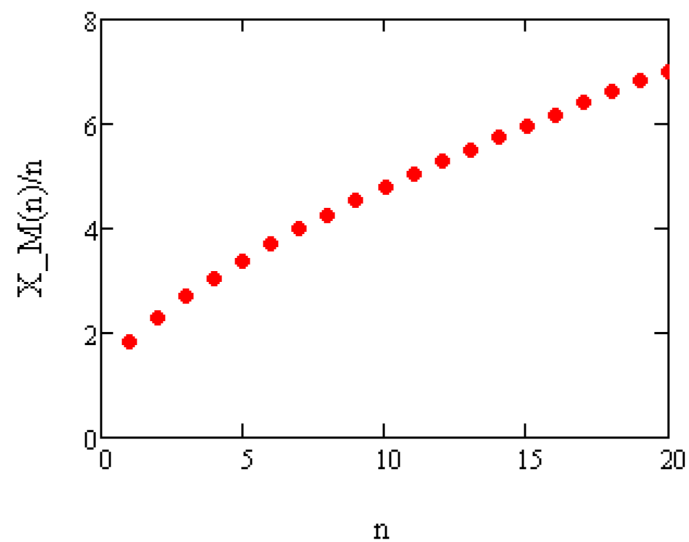
Argument of Bessel function $X(n) = n k_0 R_{56}\left(\frac{\mathrm{D}g}{g}\right)$

$J_n(X)$ *max when* $X = X_M$ *with* X_M *approximated by* $X_M(n) = n\left(1 + \sqrt{\frac{2}{3}}\, n^{-\frac{2}{3}}\right)$

$$k_0 R_{56}\left(\frac{\mathrm{D}g}{g}\right) = \frac{X_M(n)}{n}$$

Corresponds to the condition of rotation in phase space leading to a "tilted" phase space





Bunching factor after the modulator

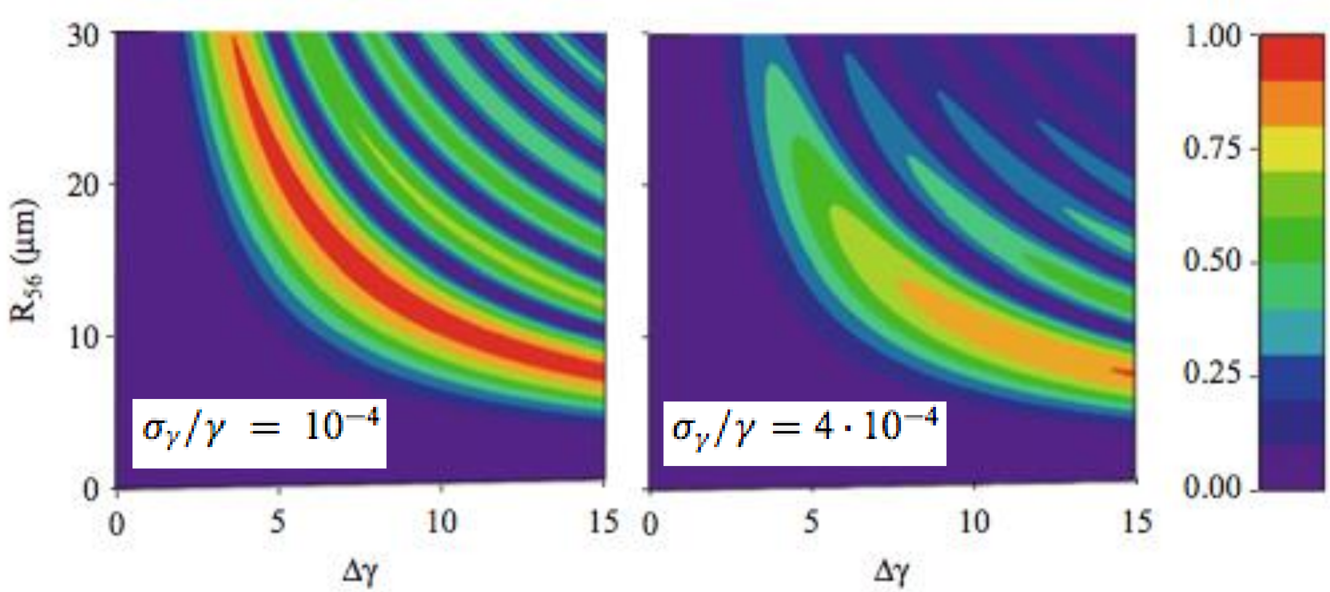
$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56} \frac{\sigma_\gamma}{\gamma}\right)^2\right]} J_n\left(n k_0 R_{56}\left(\frac{\Delta\gamma}{\gamma}\right)\right) e^{in\Phi_{seed}(s)}$$

1. The higher is the harmonic n and the initial beam energy spread σ_γ, the lower has to be the R_{56}

The maximum at a given harmonic
Draws an hyperbole in the space $(R_{56} - \Delta\gamma)$

$$k_0 R_{56}\left(\frac{\Delta\gamma}{\gamma}\right) = \frac{X_M(n)}{n}$$

2. The lower is the R_{56} the higher has to be the induced energy modulation $\Delta\gamma/\gamma$

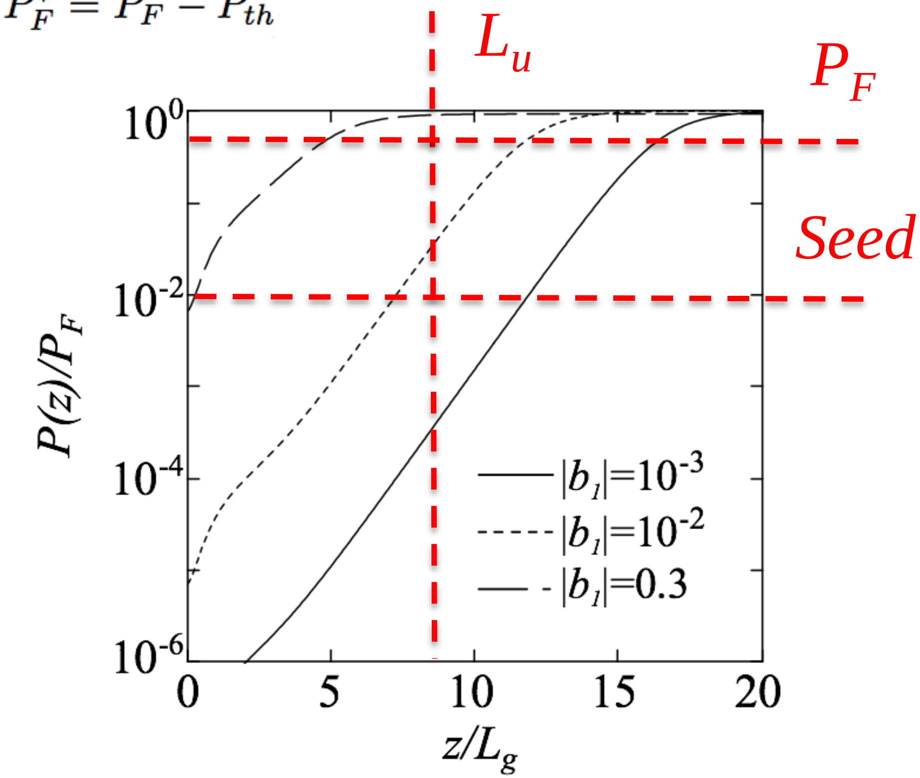


Power growth from a pre-bunched beam:

The seed power/bunching factor can be adjusted to reach saturation with a given gain length (beam quality, peak current, undulator parameters …) and within a defined distance, e.g. the undulator length.

$$P(z) = P_{th}\left[\frac{\frac{1}{3}\left(\frac{z}{L_g}\right)^2}{1+\frac{1}{3}\left(\frac{z}{L_g}\right)^2} + \frac{\frac{1}{2}exp\left[\frac{z}{L_g}-\sqrt{3}\right]}{1+\frac{P_{th}}{2P_F^*}\,exp\left[\frac{z}{L_g}-\sqrt{3}\right]}\right]$$

$$P_F^* = P_F - P_{th}$$



There is a price to pay:

The initial energy spread is not σ_γ any more, but

$$s_{g-tot} = \sqrt{s_g^2 + \frac{1}{2}\left(\frac{Dg}{g}\right)_i^2}$$

$$c_q = c_q(s_{g-tot}, e_x, e_y, l, \ldots)$$

$$L_{gc} = \frac{l_u}{4p\sqrt{3}\,c_q r_{fel\_c}}, \quad P_F \gg 1.6\, c_q^2 r_{fel} P_{beam}$$

… and this affects both gain length and saturation power

Bunching factor after the modulator

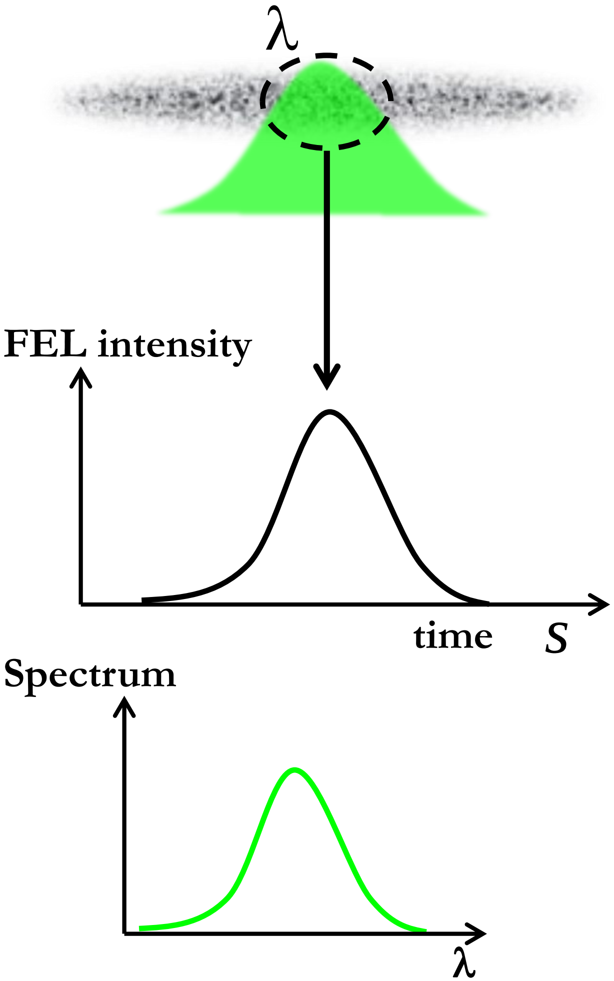
$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56} \frac{\sigma_\gamma}{\gamma}\right)^2\right]} J_n\left(n k_0 R_{56}\left(\frac{\Delta\gamma}{\gamma}\right)\right) e^{in\Phi_{seed}(s)}$$

1. The argument of the Bessel function contains the energy modulation, which is directly proportional to the amplitude of the modulating field. The dependence of the bunching factor on the s coordinate is determined by the seed amplitude

$$\frac{|a_{seed}(s')|}{4\rho N} = \left(\frac{\Delta\gamma}{\gamma}\right)_s$$

2. The bunching factor also carries the phase information of the seed pulse:
   a. i.e, a chirped pulse will produce a chirped bunching factor.
   b. any phase distortion in the seed is magnified by the harmonic order

FEL pulse shape



Standard HGHG

$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56} \frac{\sigma_\gamma}{\gamma}\right)^2\right]} J_n\left(n k_0 R_{56} \frac{\left|a_{seed}(s')\right|}{4\rho N}\right) e^{in\Phi(s)}$$

The FEL pulse shape depends on the seed pulse shape via the Bessel function of argument

$$\frac{X_n}{n} = k_0 R_{56} \frac{\left|a(s')\right|}{4\rho N}$$

At low X (low modulation or low R_{56}) expanding J_n in series we find $b_n(s) \propto \left(a_{seed}(s) e^{i\Phi(s)}\right)^n$

The FEL pulse intensity is proportional to $\left|b_n(s)\right|^2$ and we have $\sigma_{FEL} \approx \frac{\sigma_{seed}}{\sqrt{n}}$

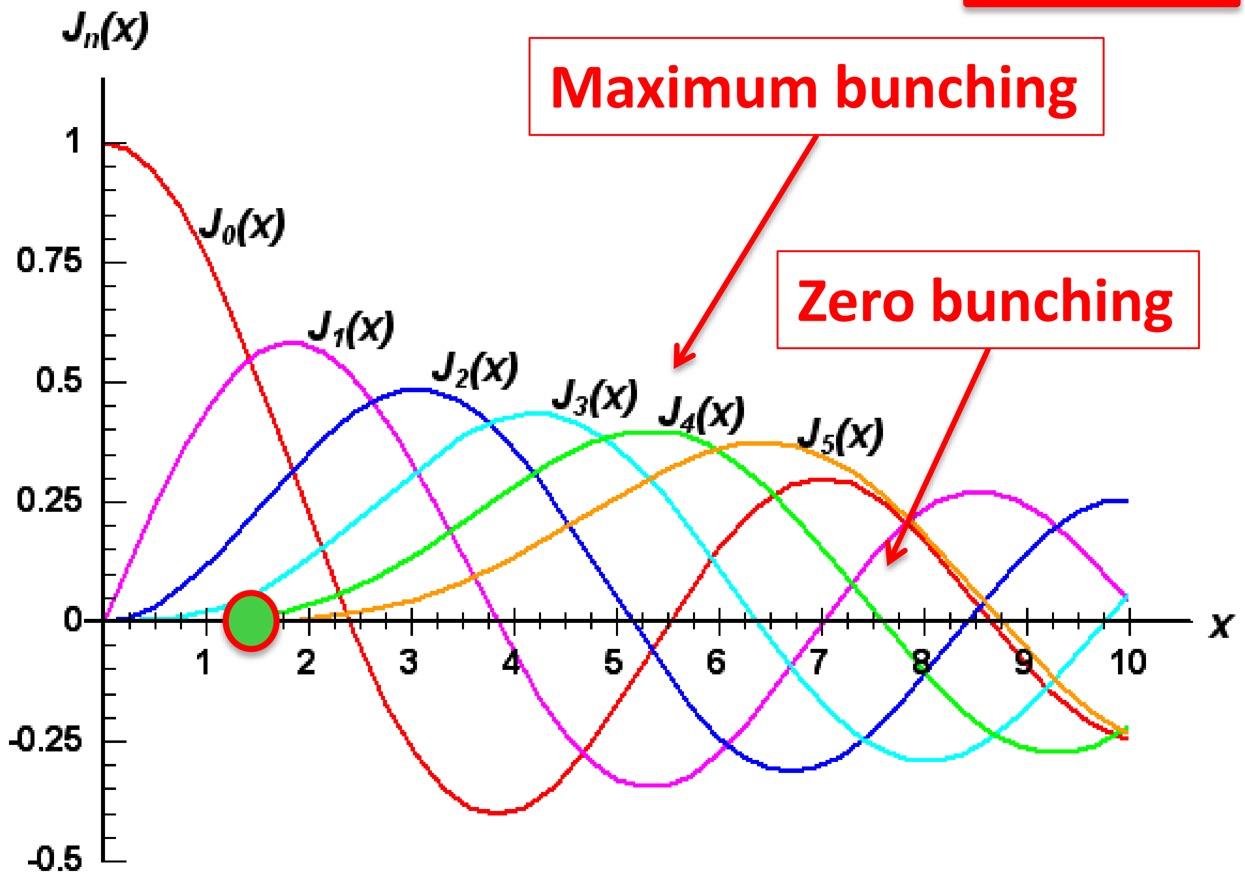
At $X = X_M$ the pulse length scales as*

$$\sigma_{FEL} \approx \frac{7}{6} \frac{\sigma_{seed}}{n^{1/3}}$$

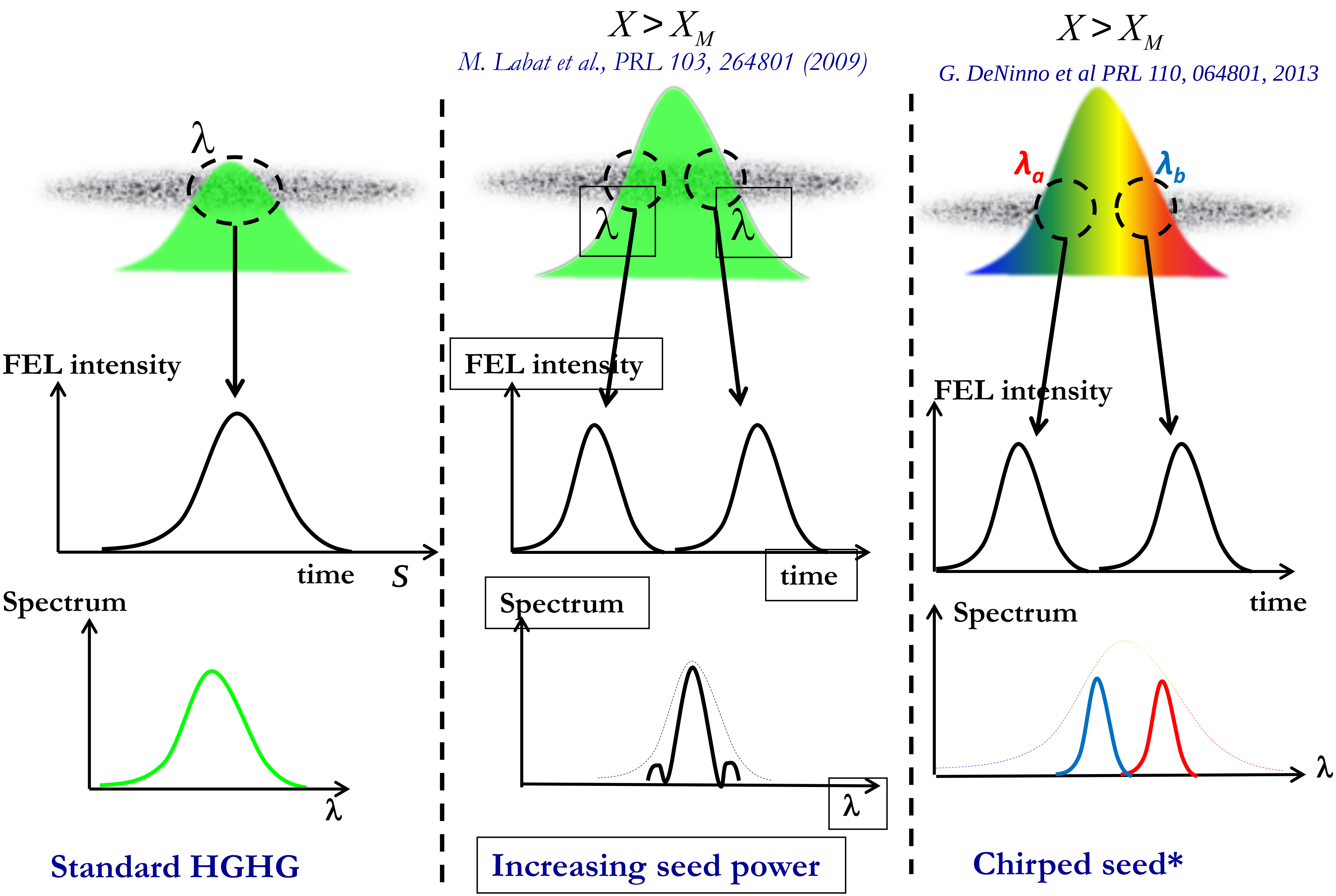
**G. Stupakov. SLAC-PUB-14639, SLAC October 2011.*
**P. Finetti et al. in prep.*

Bessel functions

$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56} \frac{s_g}{g}\right)^2\right]} J_n\left(n k_0 R_{56} \frac{|a(s')|}{4pN}\right) e^{in\Phi(s)}$$



FEL pulse splitting by long. synchrotron oscillation



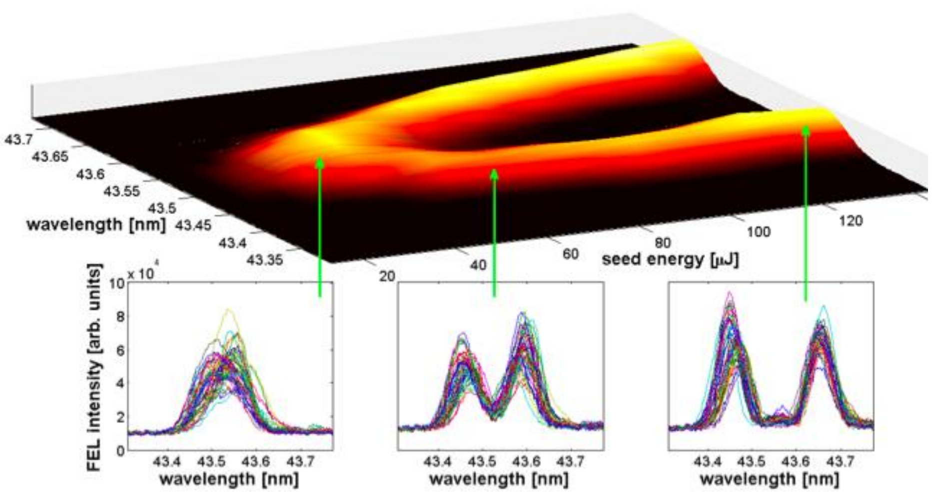
Pulse splitting measured at FERMI

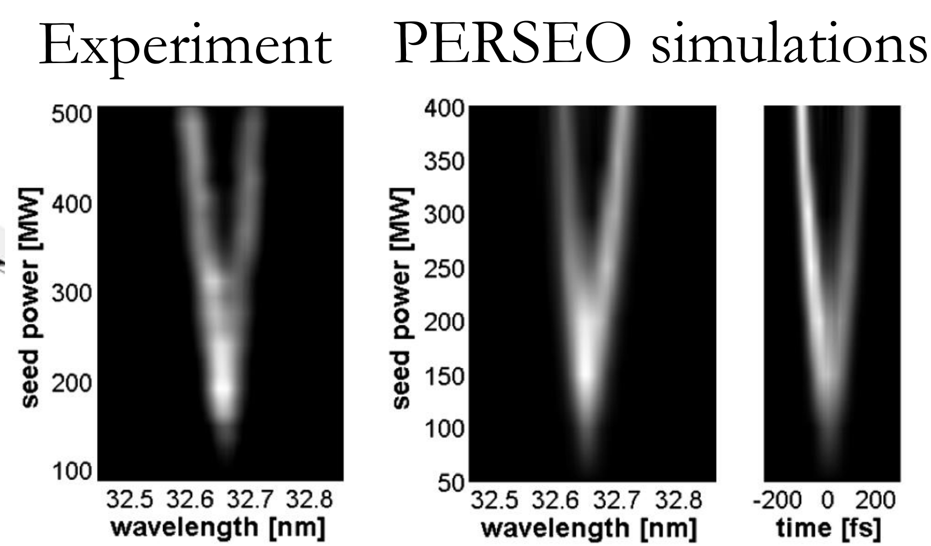
B. Mahieu et al. Optics Express 21, 22728 (2013)

Two-colour generation in a chirped seeded free-electron laser: a close look

Benoît Mahieu,[1,2,3,*] Enrico Allaria,[1] Davide Castronovo,[1] Miltcho B. Danailov,[1] Alexander Demidovich,[1] Giovanni De Ninno,[3,1] Simone Di Mitri,[1] William M. Fawley,[1] Eugenio Ferrari,[1] Lars Fröhlich,[1] David Gauthier,[1,3] Luca Giannessi,[1,4] Nicola Mahne,[1] Giuseppe Penco,[1] Lorenzo Raimondi,[1] Simone Spampinati,[1] Carlo Spezzani,[1] Cristian Svetina,[1,5] Mauro Trovò,[1] and Marco Zangrando[1,6]

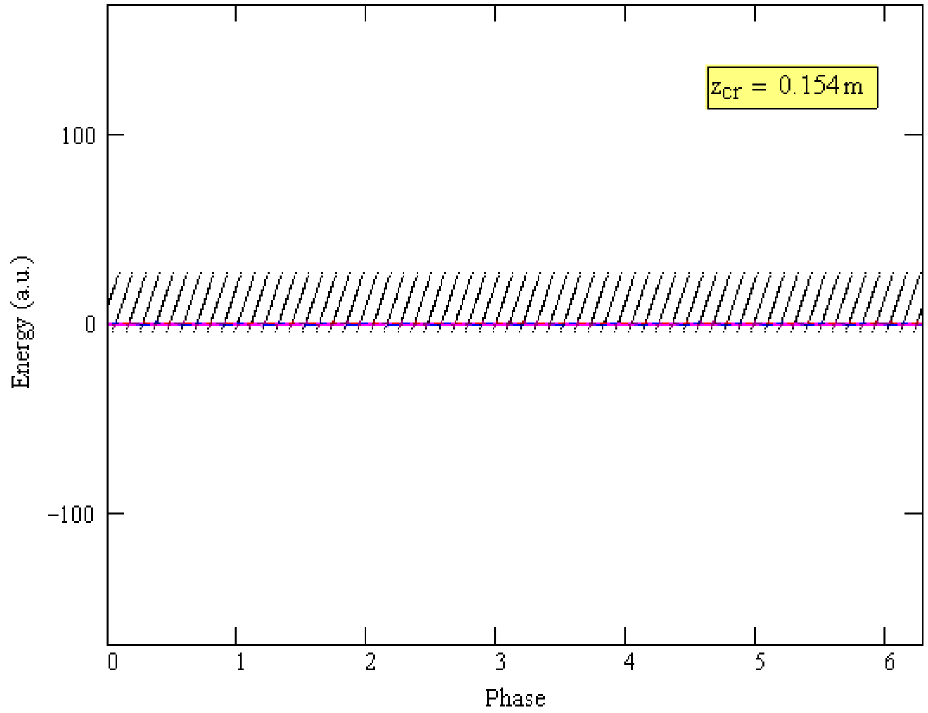
Seed frequency chirp generated by propagating through the different optical components (lenses, windows)

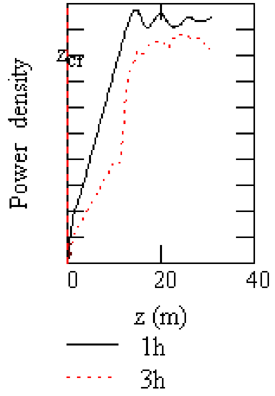




FEL phase space evolution in the amplifier

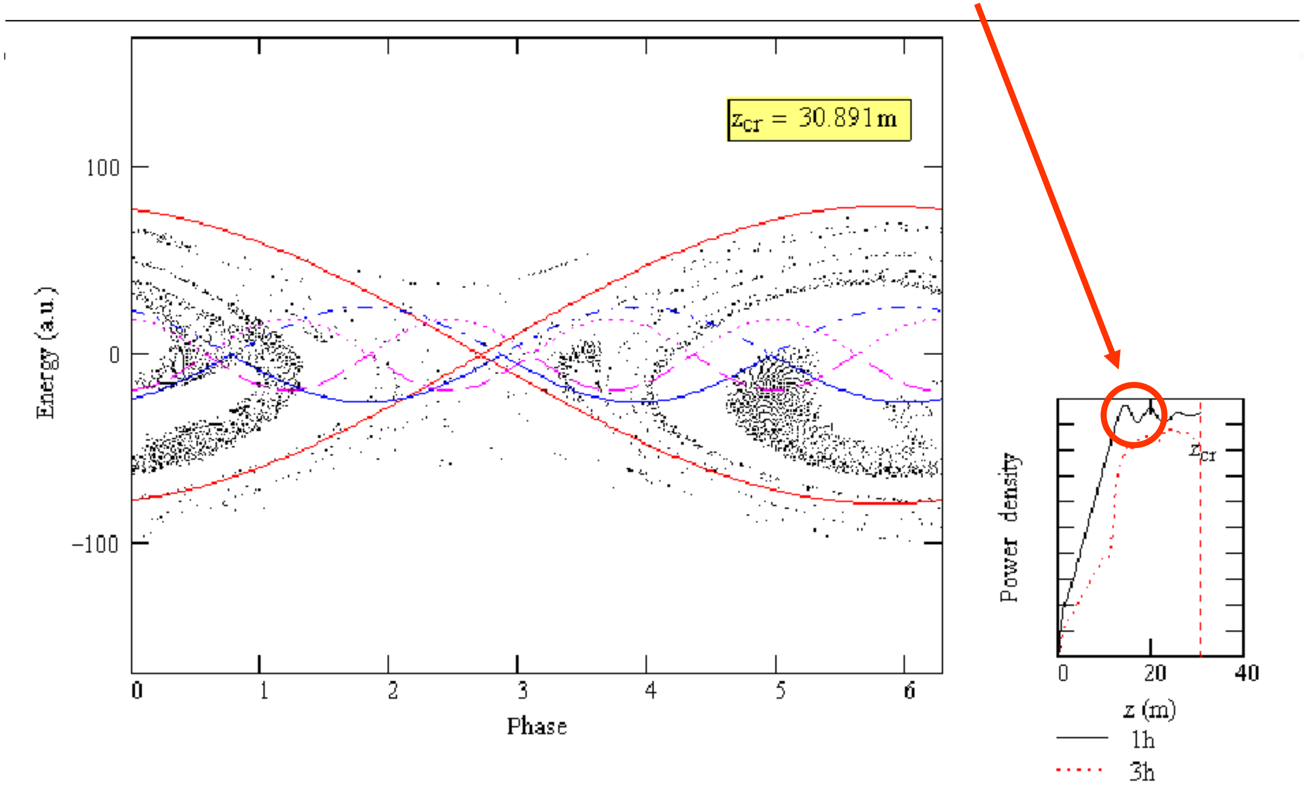
Assumption: uniform current and uniform field $a(\vec{r}, s, \tau) = a(\tau)$





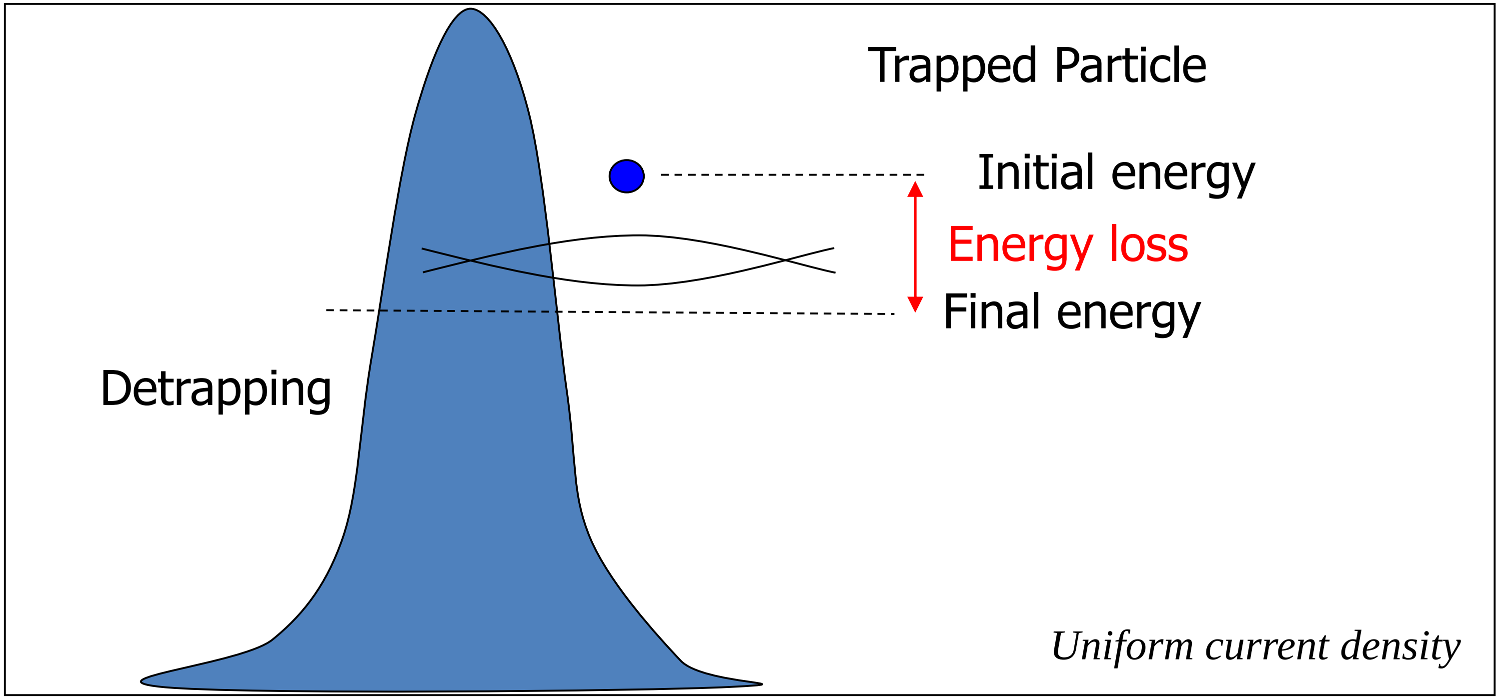
FEL phase space evolution in the amplifier

What happens if we have a short pulse that slips over the electrons in a **time shorter than the synchrotron period** ?



Pulse propagation effects in deep saturation

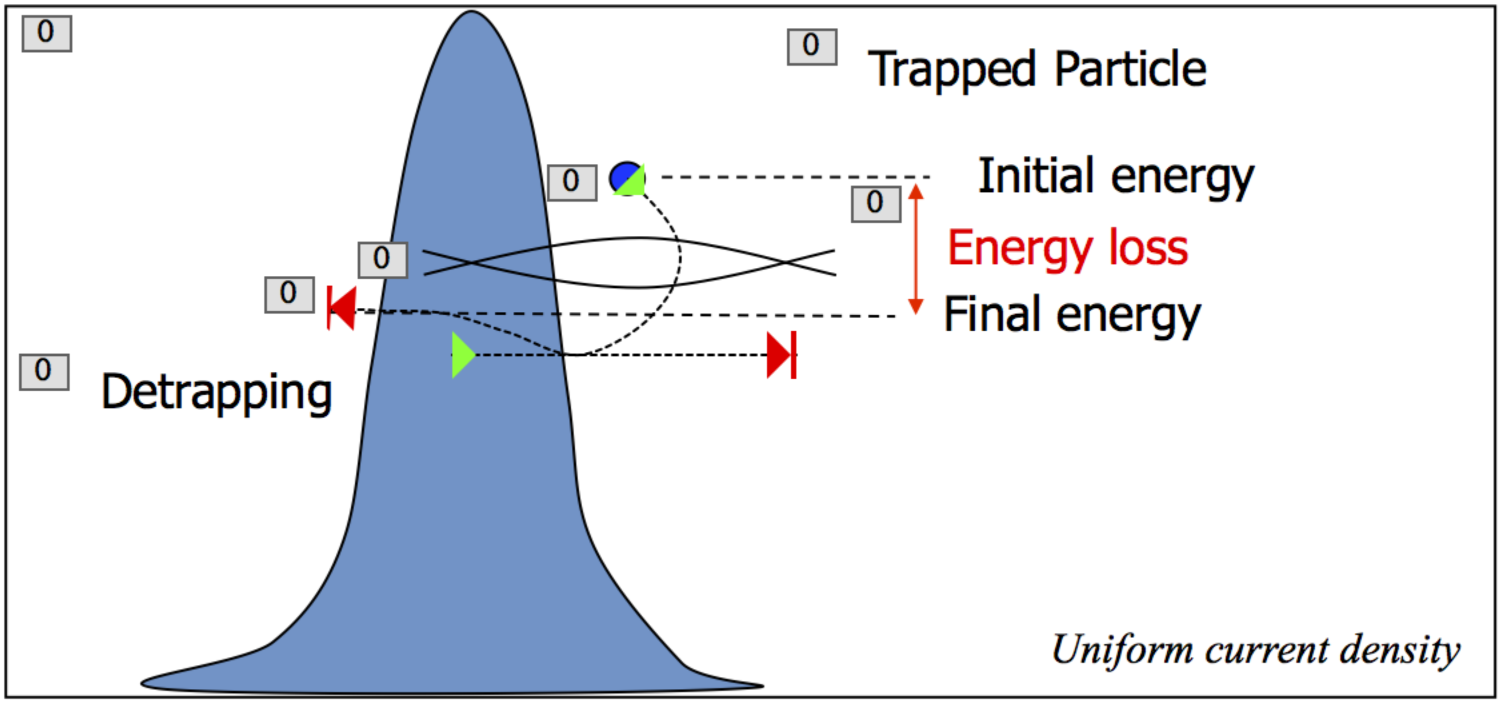
- Saturation: When the FEL laser power reaches ~ $1.6\ \rho P_{beam}$, saturation occurs: there is a cyclic energy exchange between electrons and field (in steady state regime)
- Slippage: The light advances over the electrons of a distance $N\lambda$ in N undulator periods



Superradiance: *Dicke, PR 93, 99 (1954)*
R. Bonifacio, B. W. J. Mc Neil, P. Pierini, PRA 40, 4467 (1989)
R. Bonifacio, L. De Salvo Souza, P. Pierini, N. Piovella, NIM A296, 358 (1990)
L. Giannessi, P. Musumeci, S. Spampinati, J. of Appl. Phys. 98, 043110 2005
T. Watanabe et al. Phys. Rev. Lett. 98, 034802 (2007)

Pulse propagation effects in deep saturation

- **Saturation:** When the FEL laser power reaches $\sim 1.6\,\rho P_{beam}$, saturation occurs: there is a cyclic energy exchange between electrons and field (in steady state regime)
- **Slippage:** The light advances over the electrons of a distance $N\lambda$ in N undulator periods



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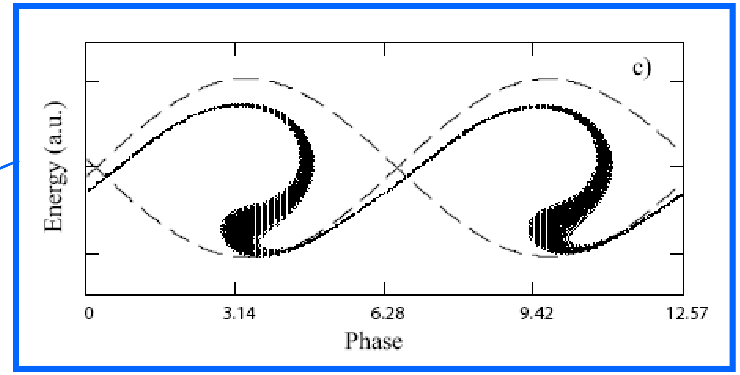
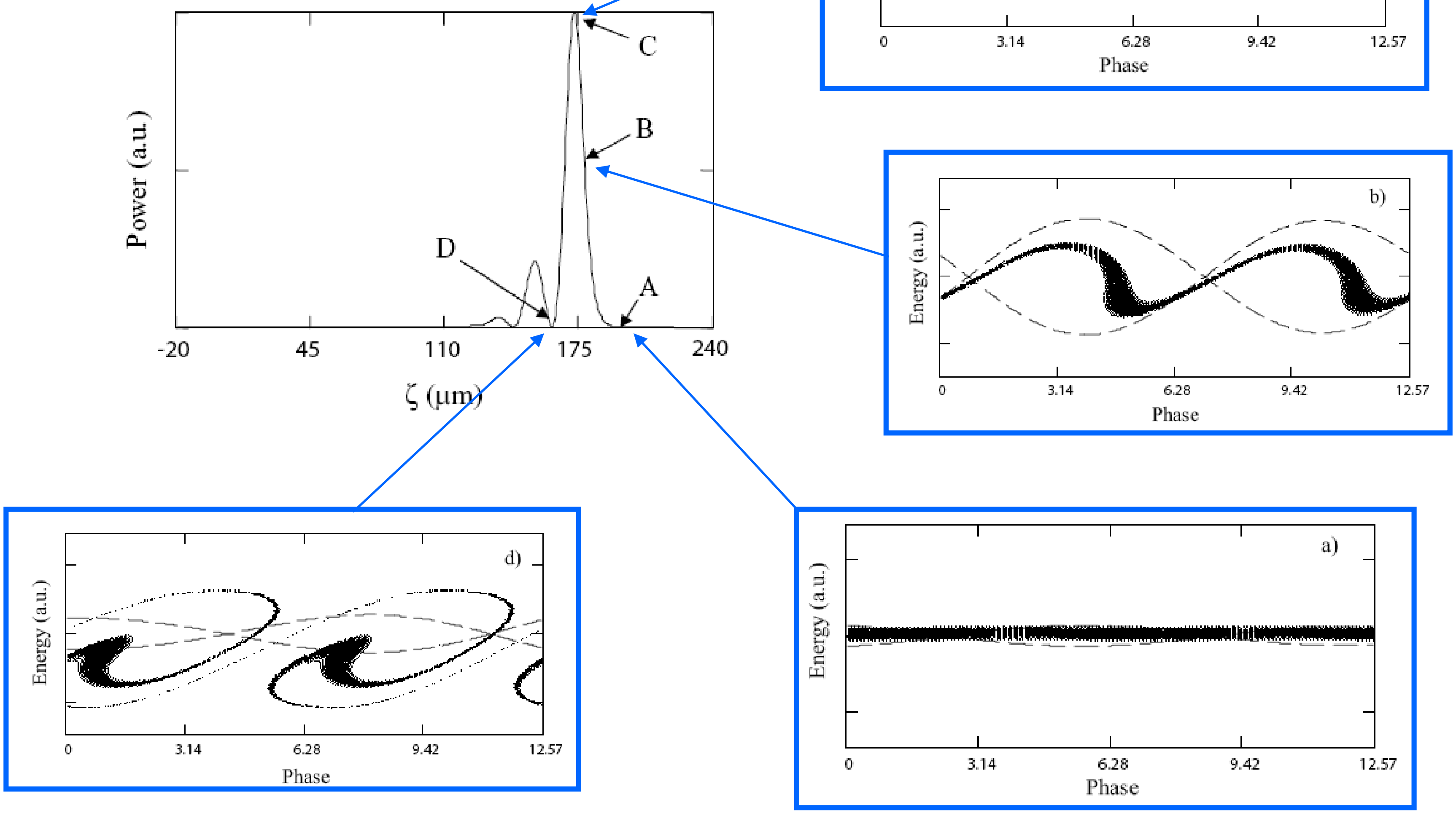
L. Giannessi, P. Musumeci, S. Spampinati, J. of Appl. Phys. 98, 043110 2005
T. Watanabe et al. Phys. Rev. Lett. 98, 034802 (2007)

Solitary wave-like superradiant pulse



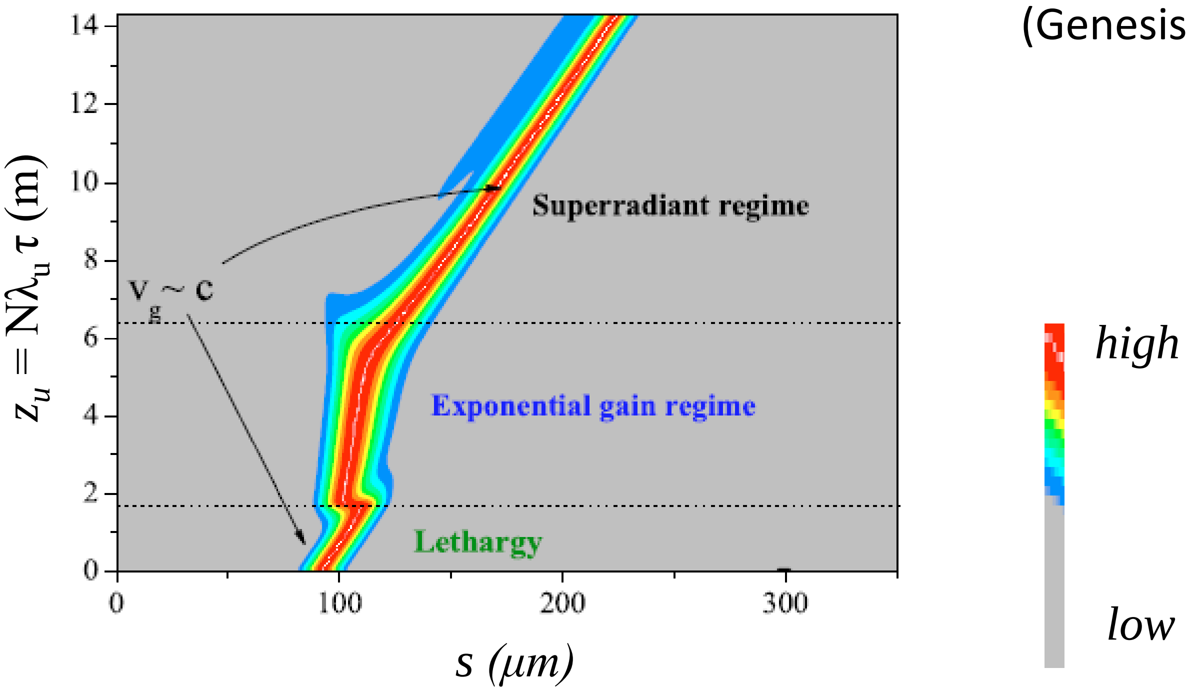


Pulse evolution

Normalized intensity (a.u.)

(Genesis 1.3)

Distance along the undulator



Distance along the e-bunch

Condition: Slippage on a (1/4 of the) synchrotron period comparable to the pulse length.

The synchrotron frequency is $W_R = \sqrt{a}$

The synchrotron period is $dt_s \approx \frac{2p}{W_R}$

During the interval $\delta\tau_s$ the “slippage” distance is $\Delta_s = \delta\tau_s N\lambda_0$

If the pulse length is comparable to the slippage length over ¼ of the synchrotron period we have

$$s_s \gg \frac{pNl_0}{2\sqrt{a}} \mu \frac{1}{P_{FEL}^{1/4}}$$

Scaling law

The pulse energy scale as $E_{FEL} = P_{FEL} s_{s} \mu P_{FEL}^{3/4}$

number of trapped electrons

bucket depth

...but also holds $E_{FEL} \propto s W_{s}$

For a pulse with group velocity ≈ c (z_u is the position in the undulator)

$$s = z - b_z ct = ct(1 - b_z) @ z_u \frac{l_0}{l_u}$$

We have therefore $E_{FEL} \propto s W_{s} \propto z_u P^{1/4}$

and $z_u P_{FEL}^{1/4} \mu P_{FEL}^{3/4}$ Þ $P_{FEL} \mu z_u^2$

... from which follows:

Pulse length

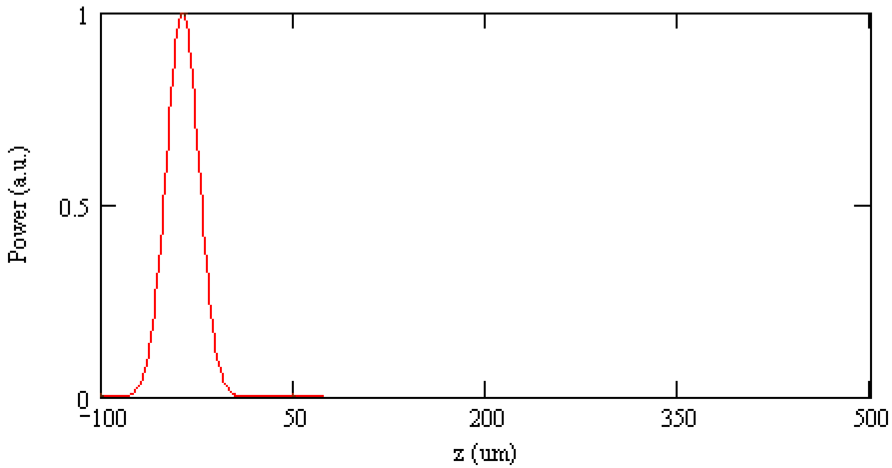
$$s_s \mu z_u^{-1/2}$$

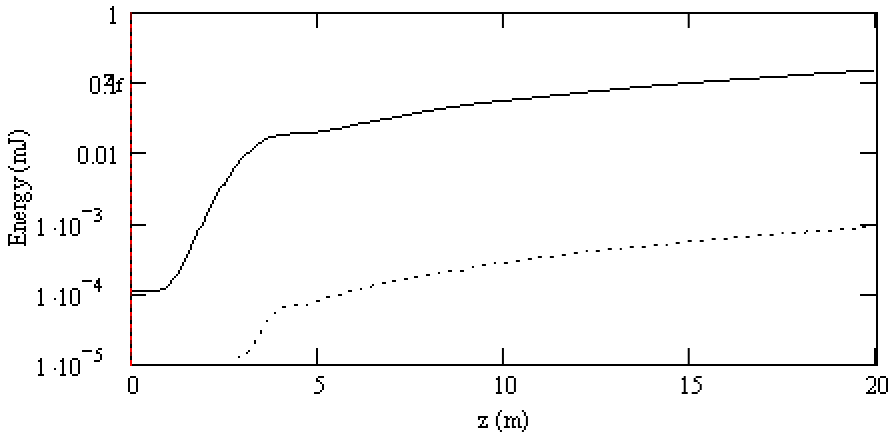
Pulse energy

$$E_{FEL} \mu z_u^{3/2}$$

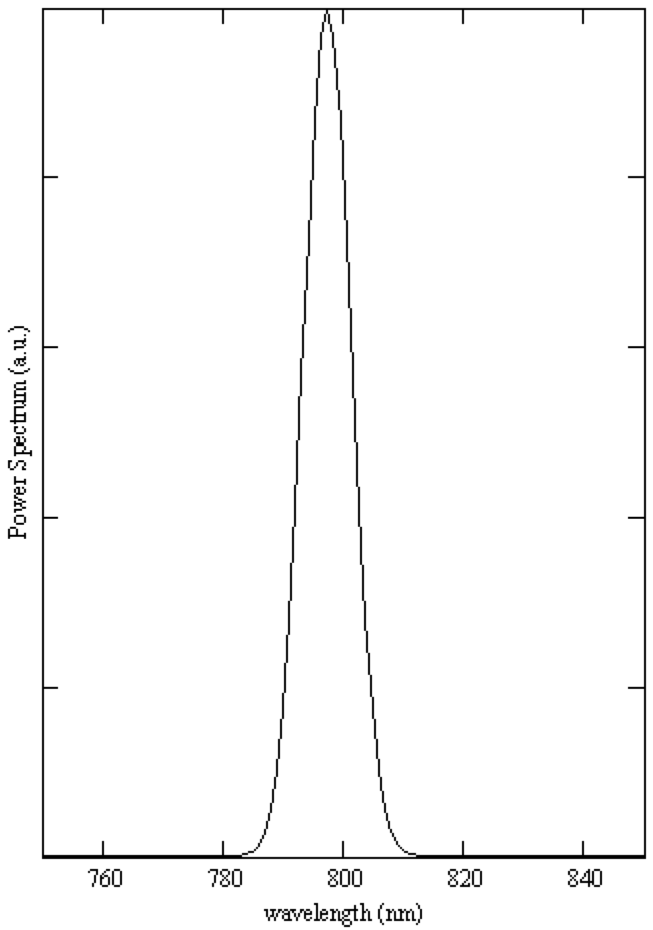
Pulse evolution

(Perseo http:\\www.perseo.enea.it)





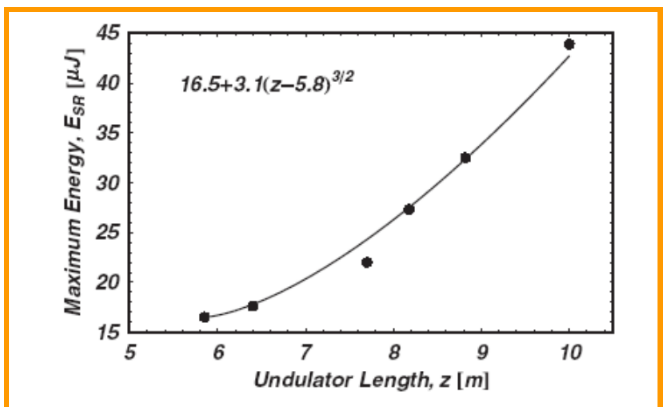
duration = 100.1 fs



Superradiance

- *Solitary wave-like pulse propagation*
- *Peak power exceeding the saturation threshold*
- *Longitudinal self-focusing*
- *Power scaling typical of superradiance*

$$P \propto n_e^2$$

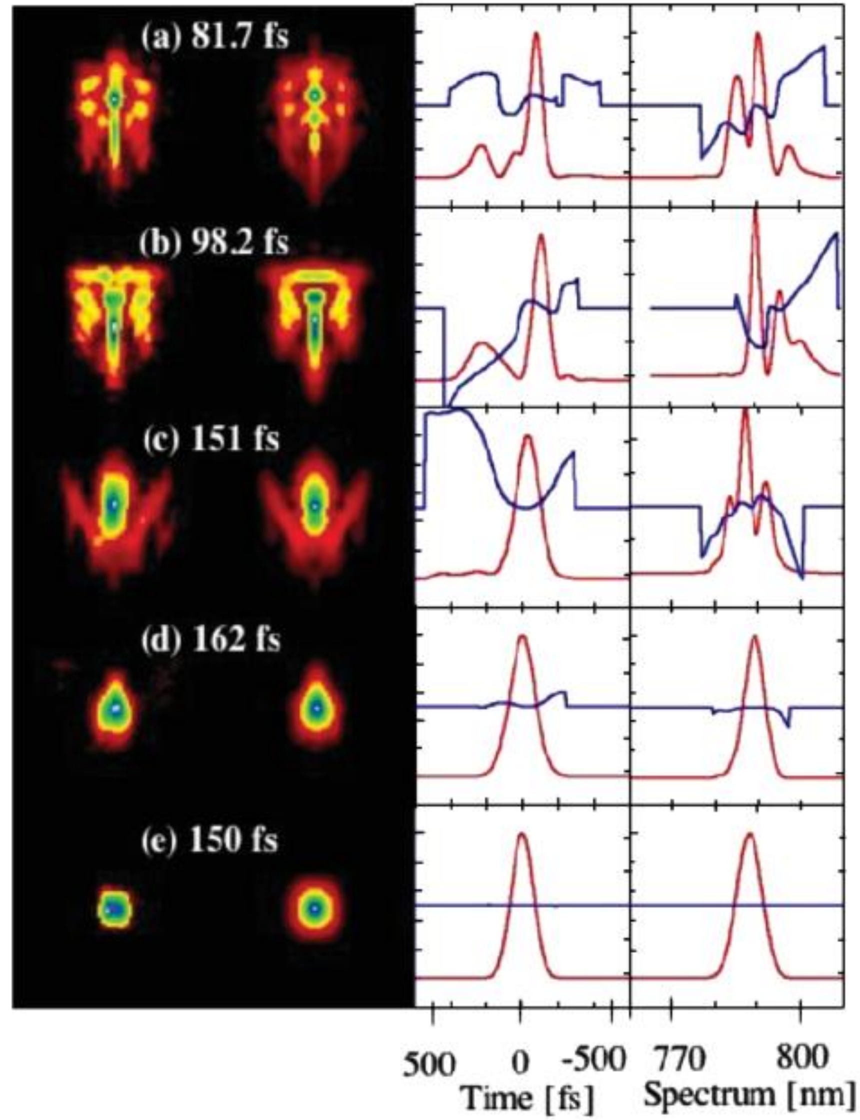


R. Bonifacio, B. W. J. Mc Neil, P. Pierini, PRA 40, 4467 (1989)
R. Bonifacio, L.DeSalvoSouza, P.Pierini, N.Piovella,NIM A296,358 (1990)

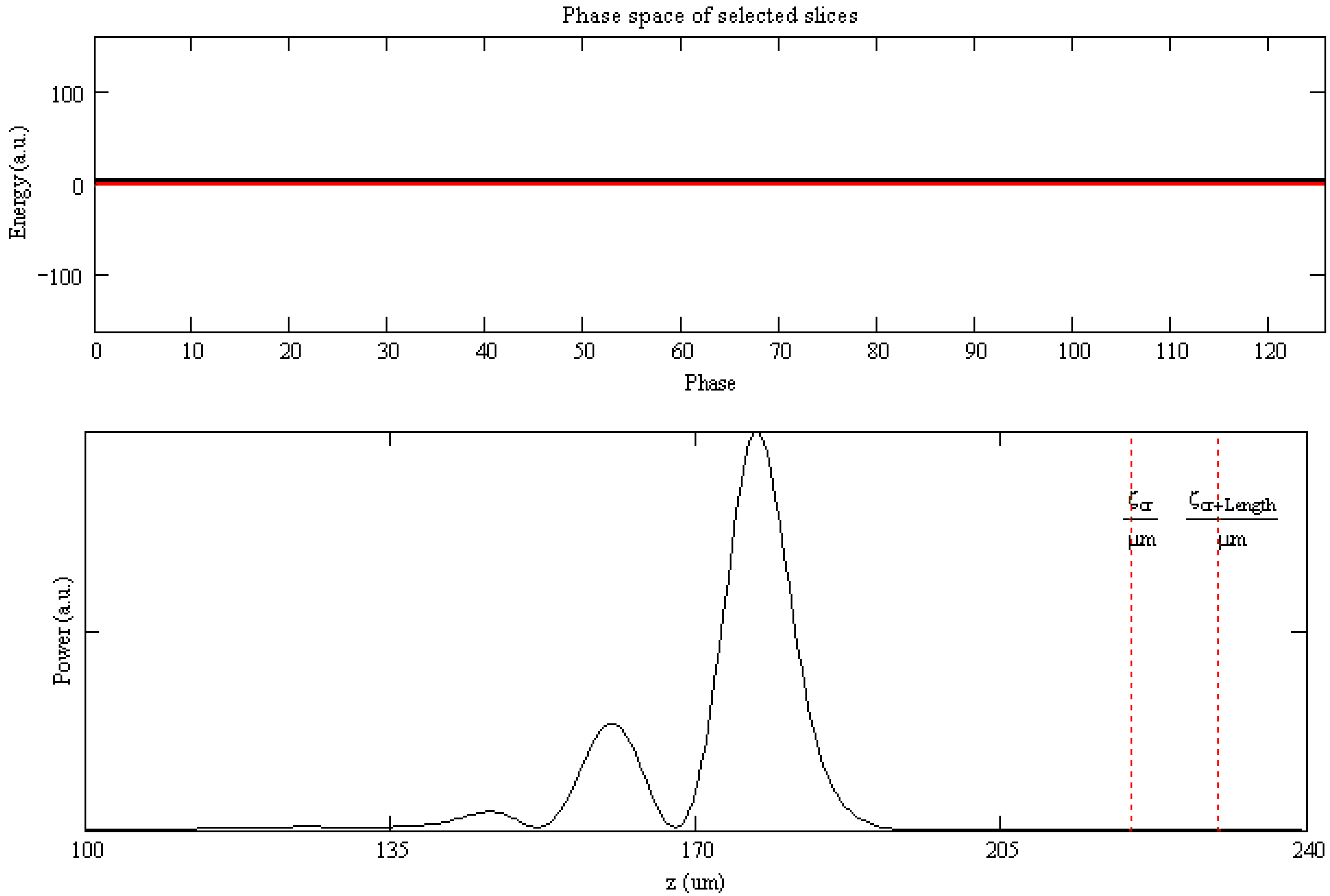
PRL 98, 034802 (2007) PHYSICAL REVIEW LETTERS week ending 19 JANUARY 2007

Experimental Characterization of Superradiance in a Single-Pass High-Gain Laser-Seeded Free-Electron Laser Amplifier

T. Watanabe,[1,*] X. J. Wang,[1] J. B. Murphy,[1] J. Rose,[1] Y. Shen,[1] T. Tsang,[2] L. Giannessi,[3] P. Musumeci,[4] and S. Reiche[5]
[1]*National Synchrotron Light Source, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[2]*Instrumentation Division, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[3]*ENEA C.R. Frascati, Via E. Fermi 45, 00044 Frascati, Italy*
[4]*INFN c/o Dipartimento di Fisica, Università di Roma "La Sapienza", Piazzale Aldo Moro 2, 00185 Roma, Italy*
[5]*Department of Physics and Astronomy, UCLA, Los Angeles, California 90095, USA*
(Received 15 September 2006; published 19 January 2007)



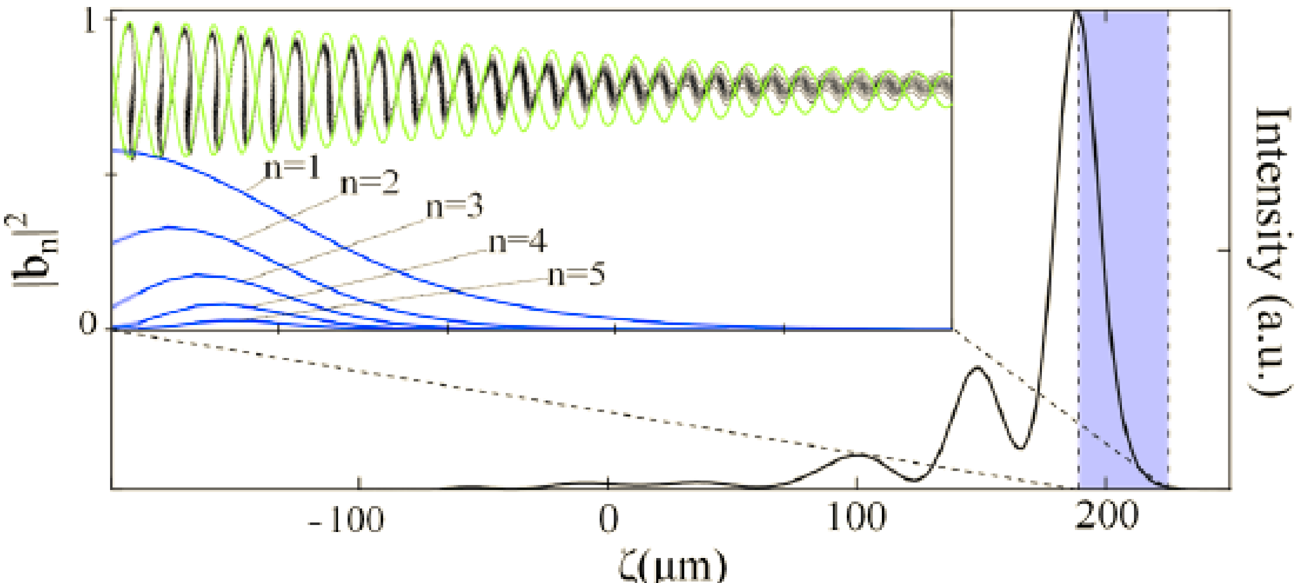
Longitudinal phase space



Simulation with Perseo (www.perseo.enea.it)

Superradiance & higher order harmonics

Modulation (bunching) at high harmonics is preserved by solitary wave-like behavior

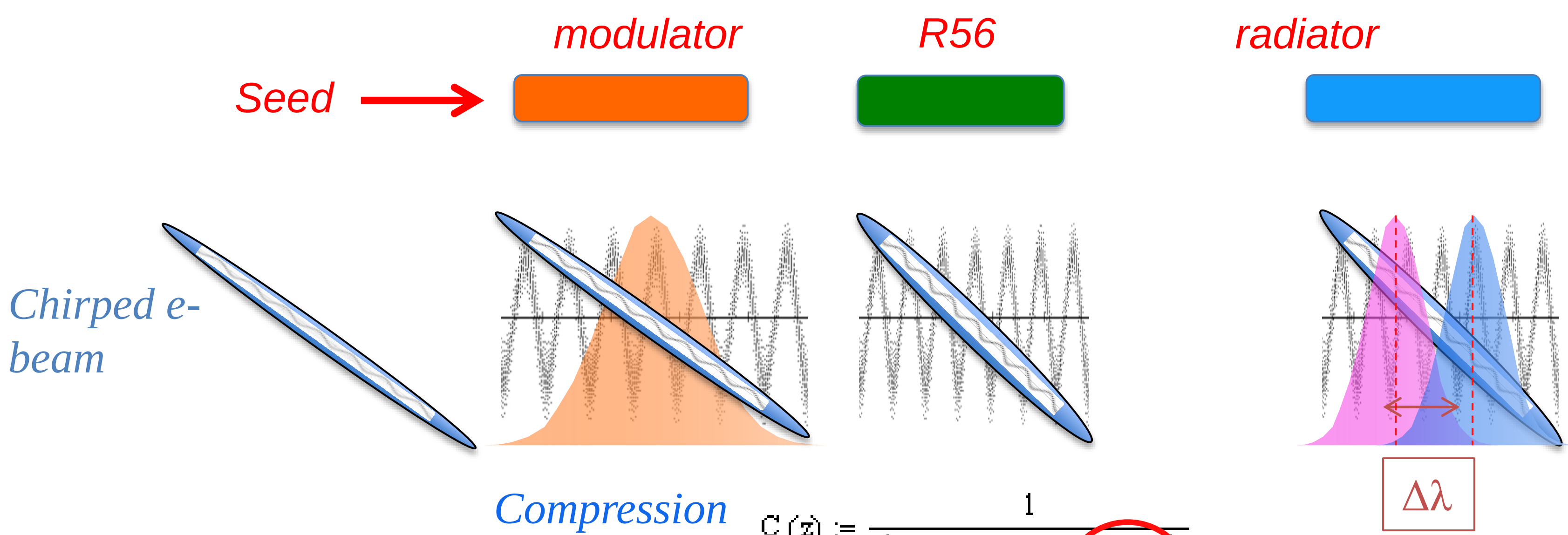


PRL **108**, 164801 (2012) PHYSICAL REVIEW LETTERS week ending 20 APRIL 2012

High-Order-Harmonic Generation and Superradiance in a Seeded Free-Electron Laser

L. Giannessi,[1,*] M. Artioli,[1] M. Bellaveglia,[2] F. Briquez,[9] E. Chiadroni,[2] A. Cianchi,[7] M. E. Couprie,[9] G. Dattoli,[1] E. Di Palma,[1] G. Di Pirro,[2] M. Ferrario,[2] D. Filippetto,[10] F. Frassetto,[5] G. Gatti,[2] M. Labat,[9] G. Marcus,[8] A. Mostacci,[4] A. Petralia,[1] V. Petrillo,[3] L. Poletto,[5] M. Quattromini,[1] J. V. Rau,[6] J. Rosenzweig,[8] E. Sabia,[1] M. Serluca,[4] I. Spassovsky,[1] and V. Surrenti[1]

Electrons Longitudinal Phase Space & High Gain Harmonic Generation



Compression factor due to dispersion

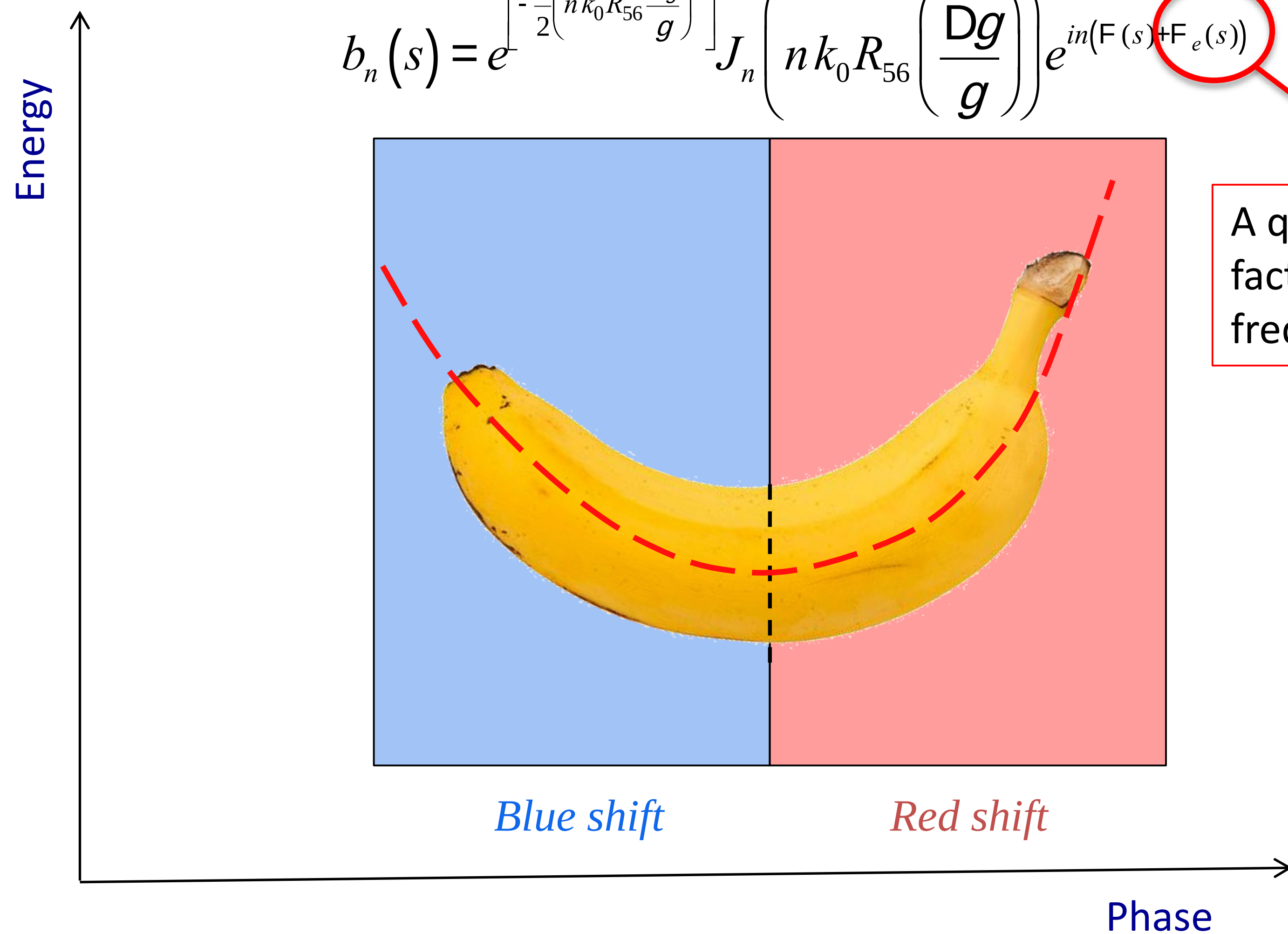
$$C(z) := \frac{1}{\left(1 - R_{56}\,\frac{1}{\gamma(z)}\cdot\frac{d}{dz}\gamma(z)\right)}$$

Wavelength shift

- A e-beam **linear energy chirp** through the dispersive section R_{56} **shifts the FEL wavelength** radiation.
- This effect can be compensated by retuning the undulators/seed wavelength.

For a “generic” distribution

$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{\sigma_\gamma}{\gamma}\right)^2\right]} J_n\left(n k_0 R_{56}\left(\frac{\Delta\gamma}{\gamma}\right)\right) e^{in\left(\Phi(s)+\Phi_e(s)\right)}$$



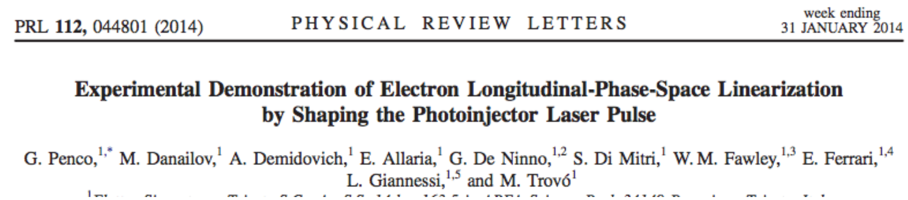
A quadratic phase factor adds a linear frequency chirp.

A quadratic chirp (or higher orders) in the e-beam phase space distribution is one of the causes of frequency chirp & spectral broadening of the FEL pulses

Shaping the photoinjector laser pulse: Linearized phase space

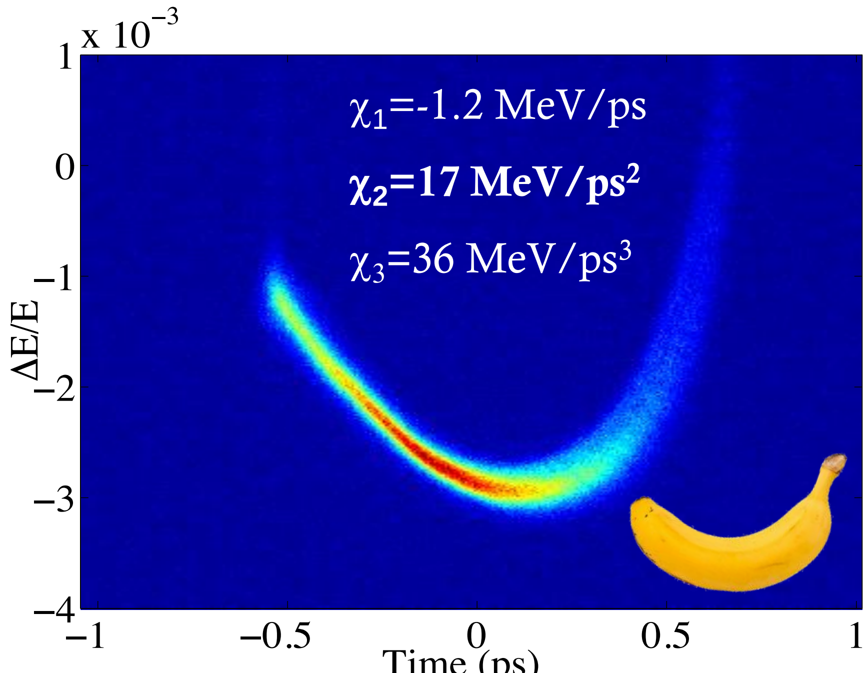
Phase space in nominal conditions

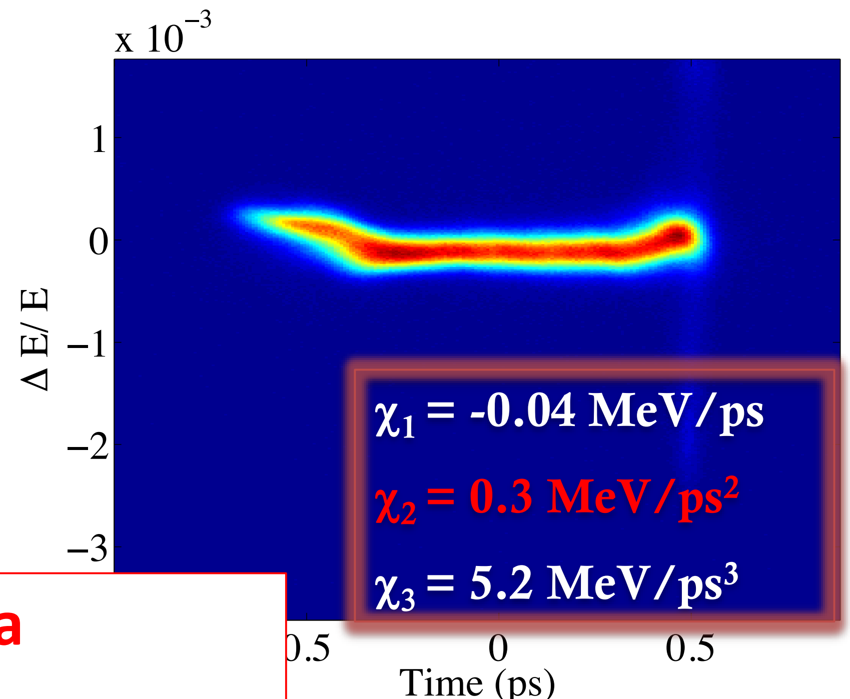
$$E(t) = E_0 + \chi_1 \cdot t + \frac{1}{2}\chi_2 \cdot t^2 + \frac{1}{6}\chi_3 \cdot t^3$$


PRL 112, 044801 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Experimental Demonstration of Electron Longitudinal-Phase-Space Linearization by Shaping the Photoinjector Laser Pulse

G. Penco,[1,*] M. Danailov,[1] A. Demidovich,[1] E. Allaria,[1] G. De Ninno,[1,2] S. Di Mitri,[1] W. M. Fawley,[1,3] E. Ferrari,[1,4] L. Giannessi,[1,5] and M. Trovó[1]



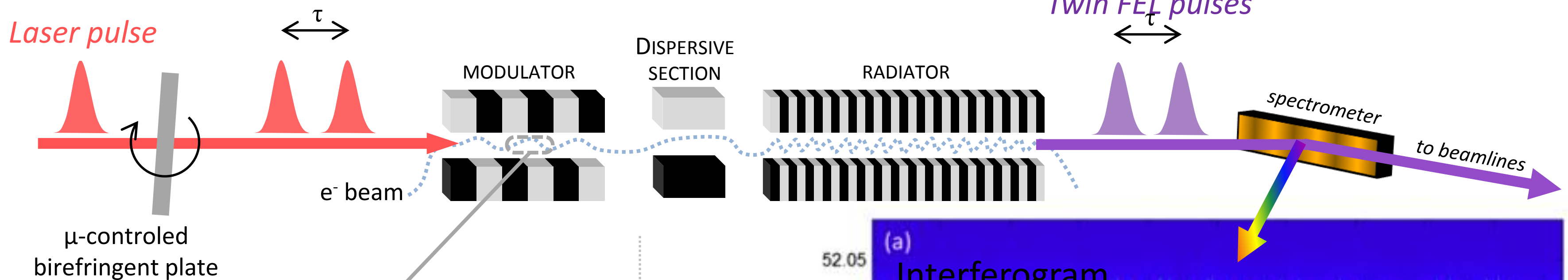


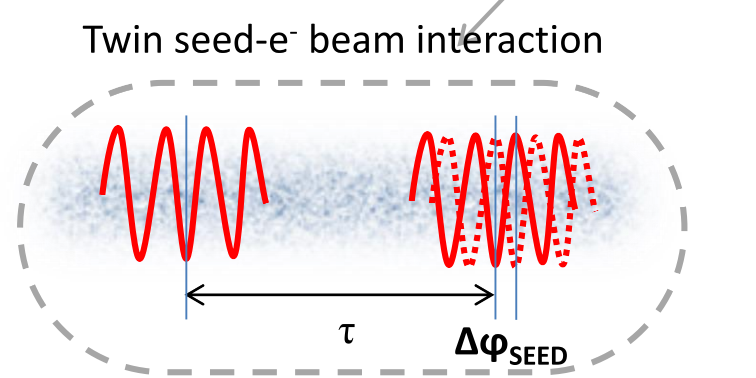
but … how to take advantage of a quadratic chirp in e-beam shape …

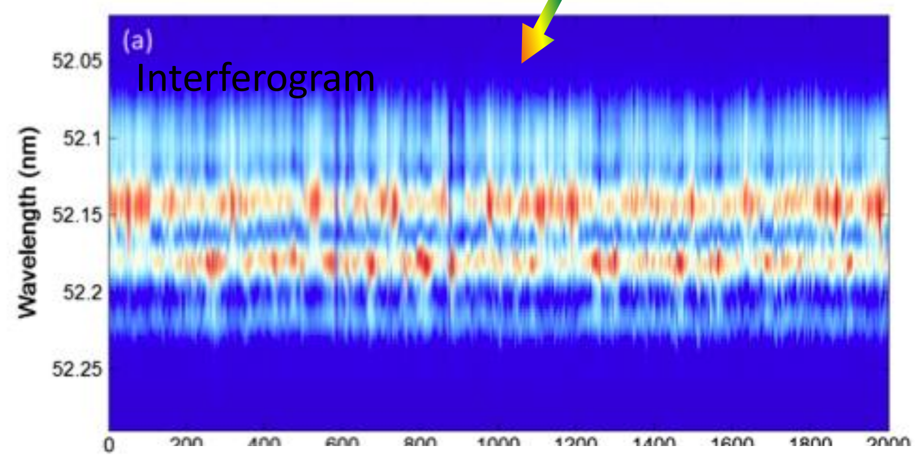
Phase-locked pulses

D. Gauthier et al., PRL 2016

Two phase-locked seed pulses create two FEL pulses locked in phase. The relative phase control and stability between the two FEL pulses is demonstrated with the evolution and stability of the spectral phase



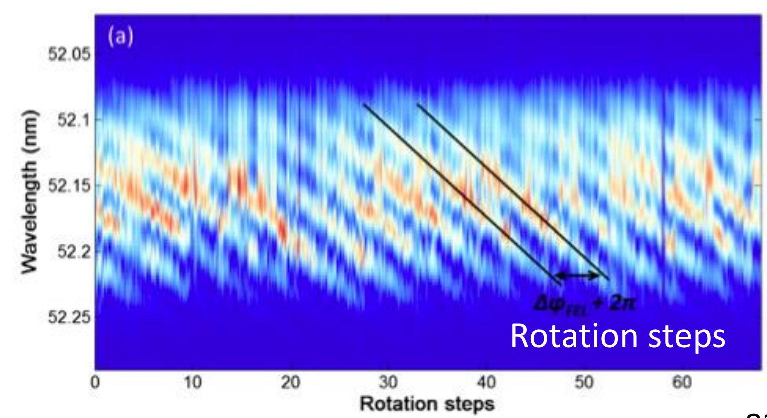




τ = 290fs and duration of the individual FEL pulse is about 70-80fs.

Rotation step: $\Delta\varphi_{SEED} = \lambda_{SEED}/28.33$ => $\Delta\varphi_{FEL} = \lambda_{FEL}/5.67$ (harmonic 5) - Full range of 68 steps => **12** time λ_{FEL} Each step correspond to 20 consecutive single-shot spectra.

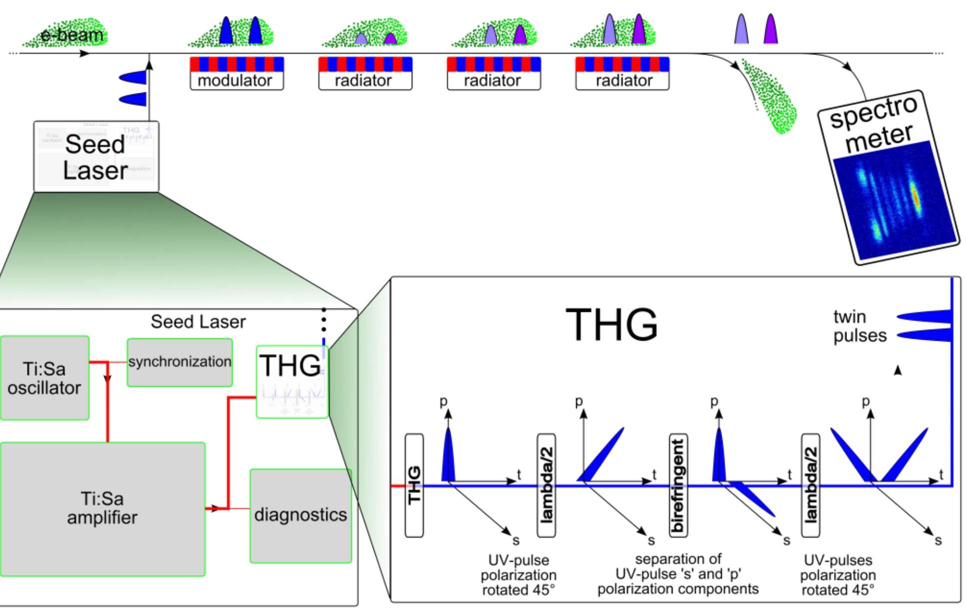
Analysis of fringes gives rms phase stability of $\lambda_{FEL}/10$



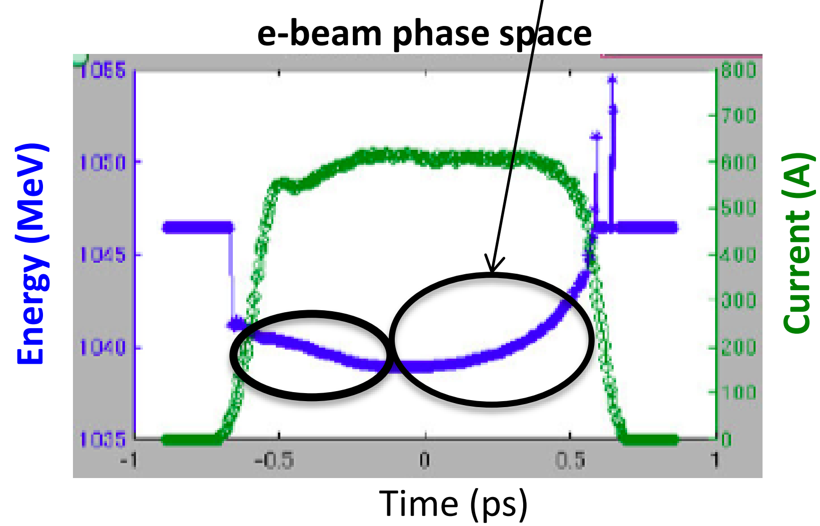
Resolving the FEL pulse properties with Spider technique

- **SPIDER is an interferometric technique allowing a complete single-shot characterization of ultra-short optical pulses**. It relies on the measurement of the interferogram generated by the interaction of two replicas of the pulse to be characterized. The two replicas must be separated in time by a delay τ and shifted in frequency by a shear Ω.

Temporal separation via doubling the seed pulse



Spectral separation via quadratic energy chirp in the electron longitudinal distribution



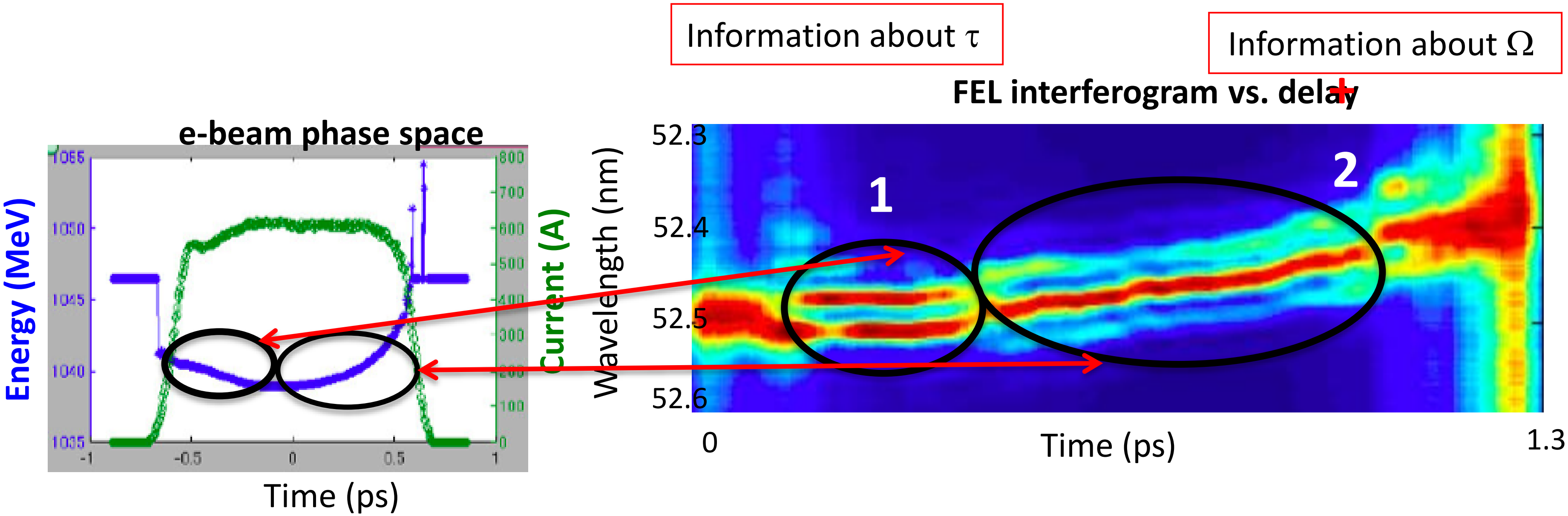
G. De Ninno et al. Nat. Comm. 2015

SPIDER reconstruction - 1

The evolution of the interferogram shows two distinct regions:

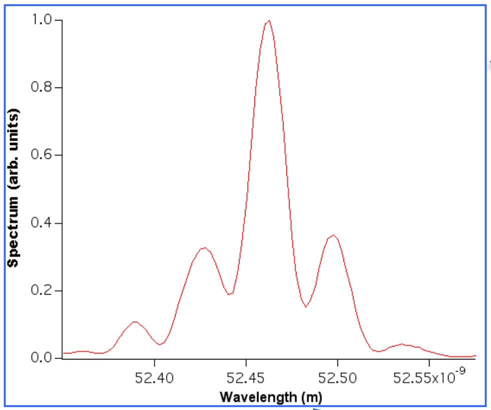
1) one (almost) flat, corresponding to an (almost) linear energy region in the electron-beam phase space (zone 1 in the picture below),
2) and one characterized by an almost linear dependence of the interferogram centroid vs. the seed-electron delay (zone 2).

The analysis of the interferograms in the zone 1 allows one to estimate the delay τ between the two interfering pulses, while zone 2 contains the information relative to spectral shear Ω. The interferograms of zone 2 can be used to carry out the SPIDER reconstruction.

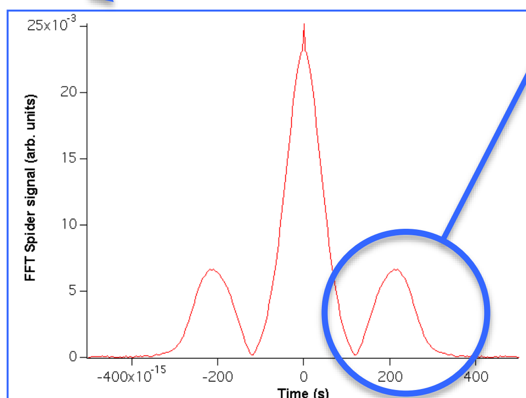


SPIDER reconstruction - 2

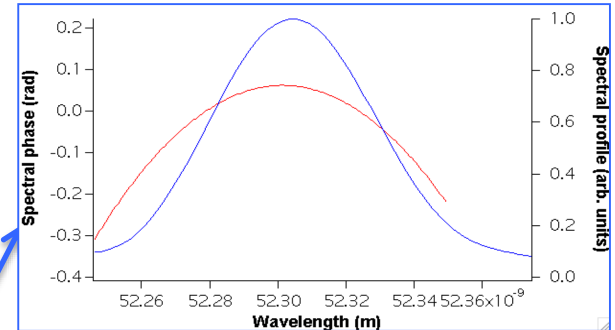
SPIDER signal



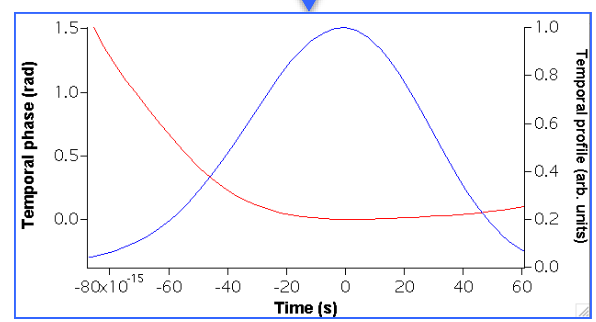
FFT



Spider Inversion Algorithm

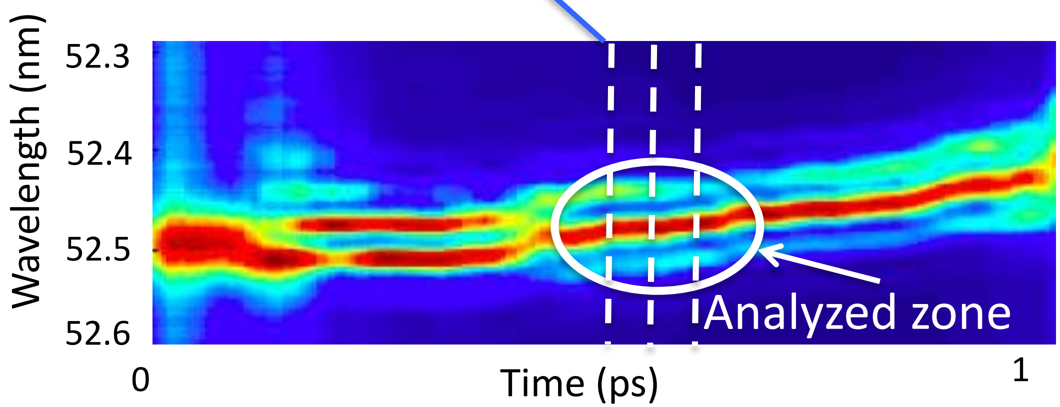


Spectral domain



Temporal domain

Spectral profile



The technique allows the complete reconstruction of the pulse properties as phase and temporal distribution

SPIDER

Reconstruction of the temporal envelope and phase.

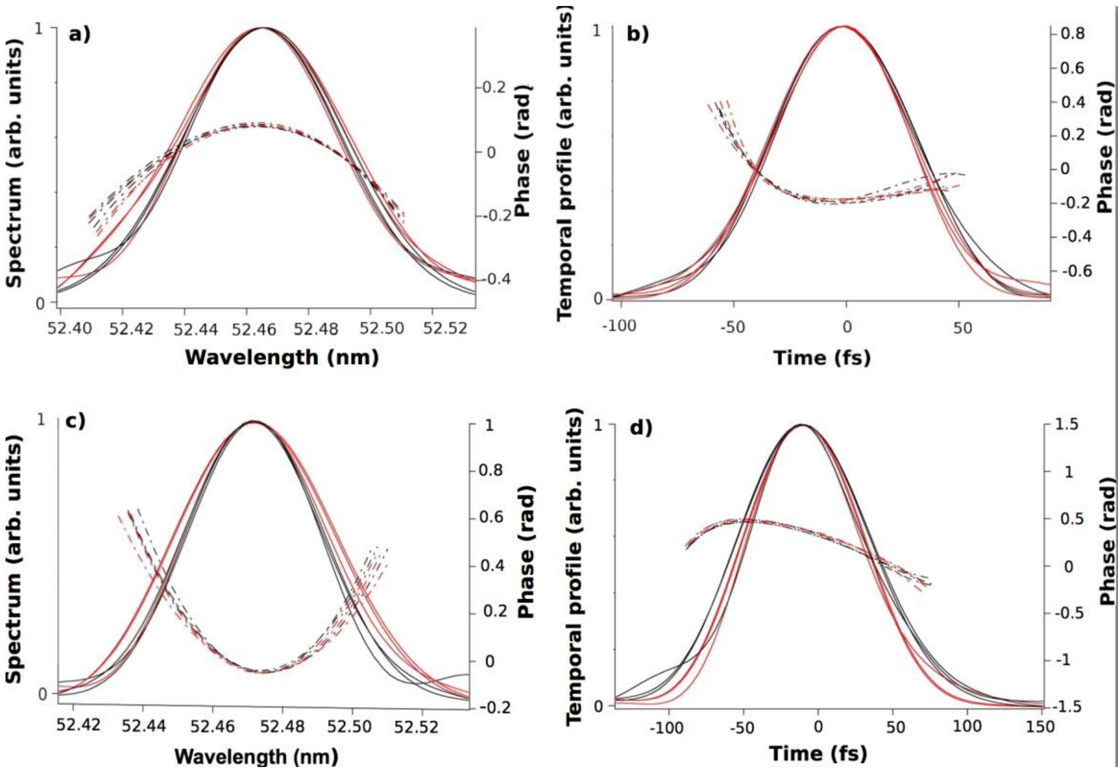
a,b: Positive linear frequency chirp, 125 fs, quadratic temporal phase ≈ 10^{-5} fs^{-2}

≈ 71fs, TBP 1.1 from Fourier Limit

c,d: Negative linear frequency chirp, 180fs, quadratic temporal phase: ≈$-4.5\cdot10^{-5}$ fs^{-2}

≈ 100fs, TBP 1.2 from Fourier Limit

Continuous line: spectral and temporal envelopes; dotted lines: spectral and temporal phases.

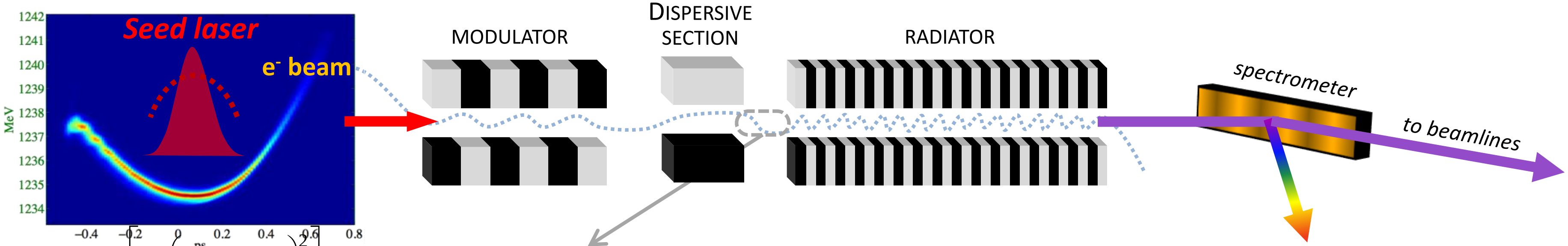


G. De Ninno et al. .

Reconstructions obtained from three consecutive FEL shots (lines of the same colour) at fixed dealy and for two different delays (represented by black and red colours) within the region of interest. For the sake of visualization, spectra were centered at the same wavelength.

We may control the chirp of the output pulse via the laser chirp

FEL pulse characterization & control Spectro-temporal shaping



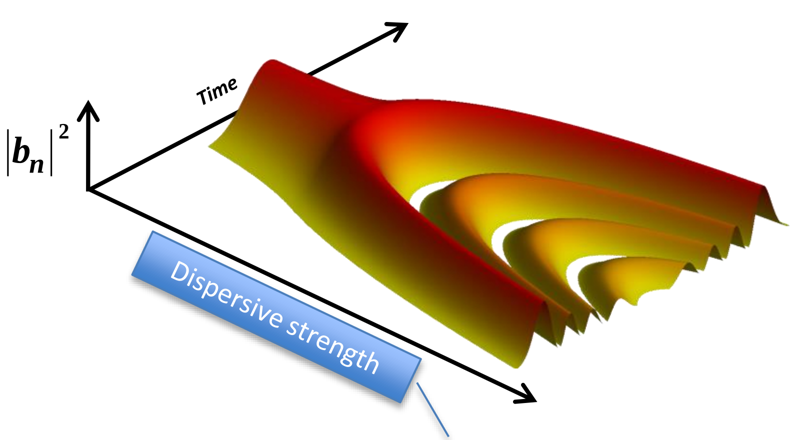
$$b_n(s) = e^{\left[-\frac{1}{2}\left(n k_0 R_{56}\frac{s_g}{g}\right)^2\right]} J_n\left(n k_0 R_{56}\frac{a(s)}{4pN}\right) e^{in(F(s)+F_e(s))}$$

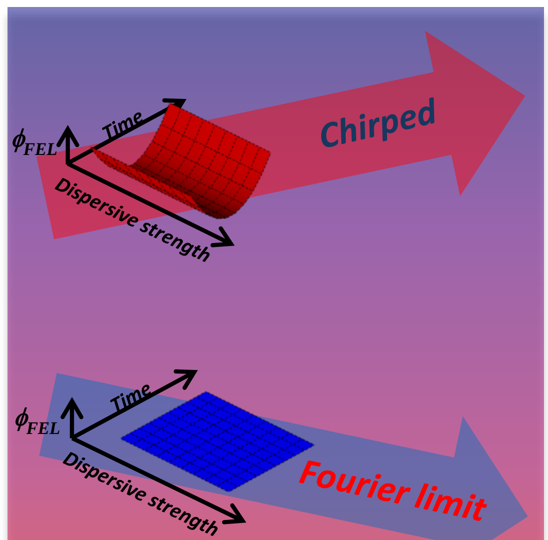
seed envelope

seed slowly varying phase

e- beam phase from time-dependent energy profile

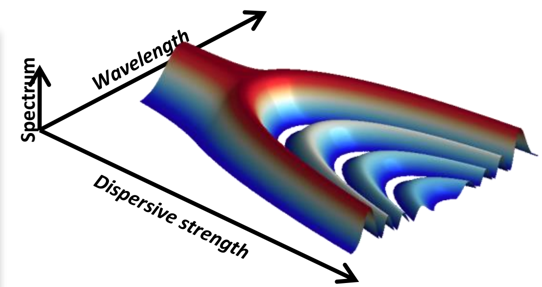
Temporal profile of the microbunching

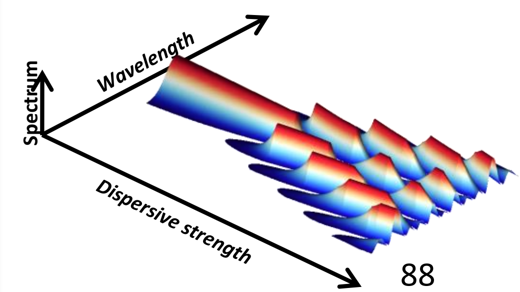




Temporal profile of the phase:
contribution of the microbunching
+ FEL amplification

Spectral profile of the FEL pulse





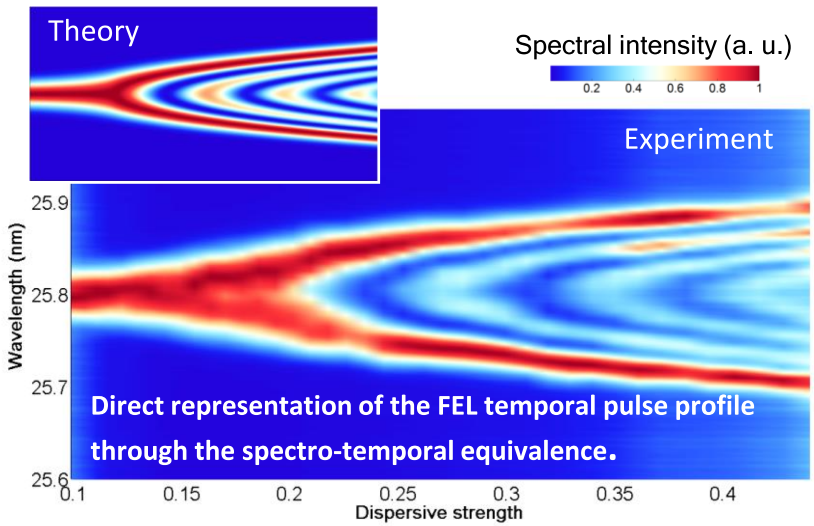
Featured in Physics

Spectrotemporal Shaping of Seeded Free-Electron Laser Pulses

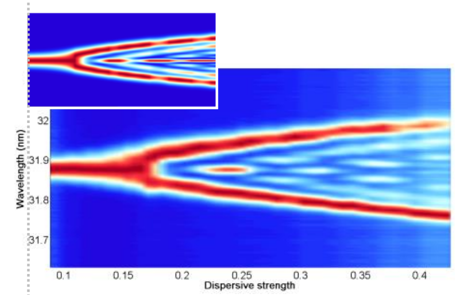
David Gauthier, Primož Rebernik Ribič, Giovanni De Ninno, Enrico Allaria, Paolo Cinquegrana, Miltcho Bojanov Danailov, Alexander Demidovich, Eugenio Ferrari, Luca Giannessi, Benoît Mahieu, and Giuseppe Penco

Phys. Rev. Lett. **115**, 114801 (2015) – Published 8 September 2015

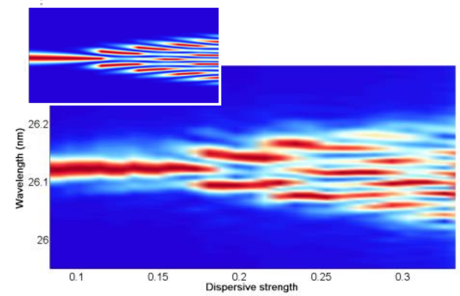
Case 1: seed with strong linear frequency chirp
=> strong chirp on the FEL pulse

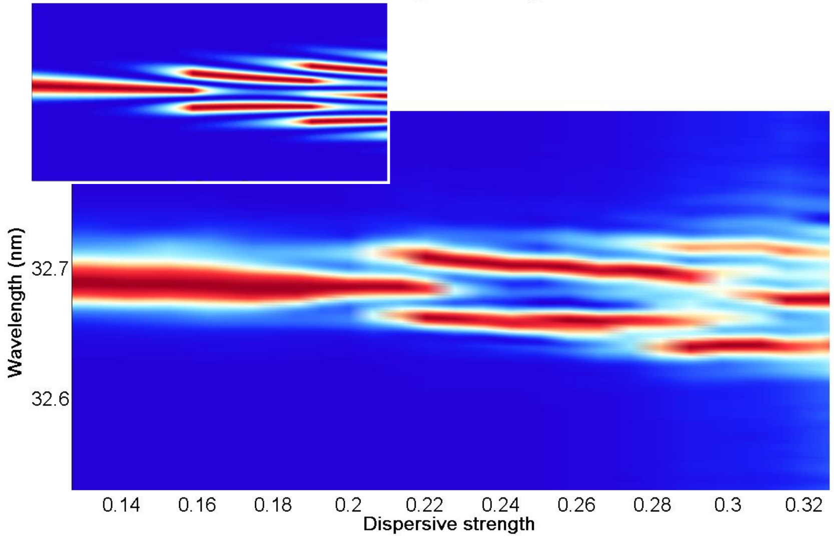


Case 2: intermediate positive chirp on the seed



Case 3: intermediate negative chirp on the seed





Case 4: moderate negative chirp on the seed
<=> chirp compensation

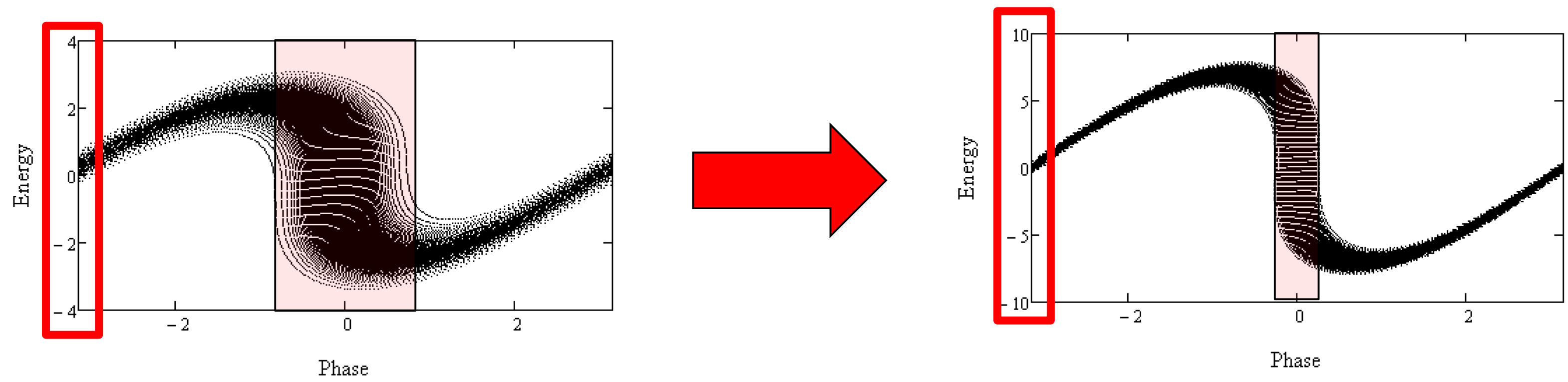
Possibility to compensate chirps from e-beam distribution and seed laser to generate Fourier transform limited pulses.

Fresh bunch, EEHG seeding and self-seeding

SHORTER WAVELENGTHS

High harmonic conversion and the energy spread budget

Virtually any harmonic order can be obtained by increasing the seed power ... at a cost of an increased energy spread



- Required energy spread in order to bunch at the n[th] harmonic (Liouville's theorem)

$$\left(s_g / g\right)_{induced} \approx 2n \left(s_g / g\right)$$

- Condition to ensure high gain growth in final radiator

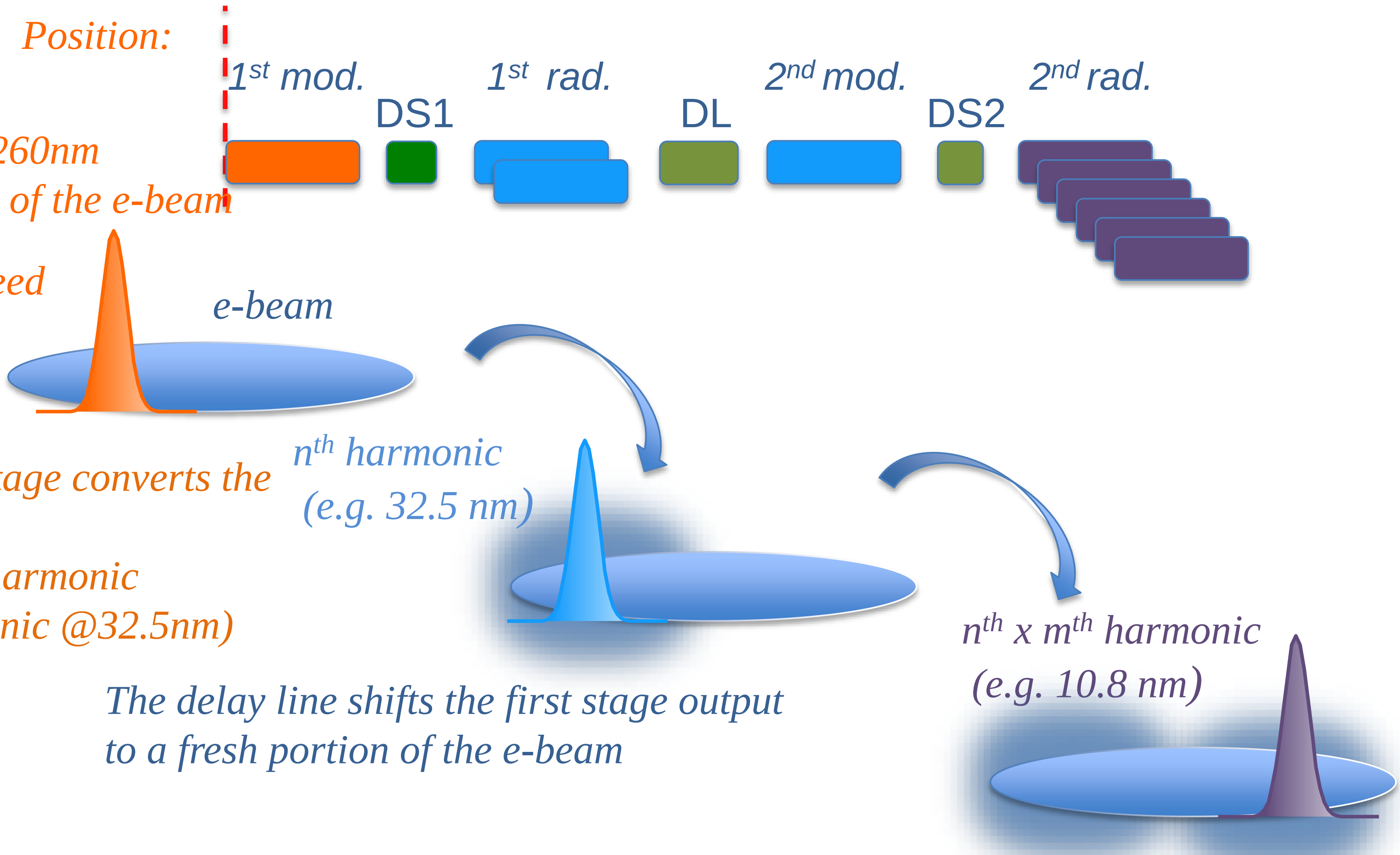
$$n < \frac{r_{fel}}{s_g / g}$$

Ideas:

- Fresh bunch injection technique, *L. H. Yu, I. Ben-Zvi, NIM 1993*
- Echo Enabled harmonic generation, *G. Stupakov, PRL, 2009*
- Non Gaussian energy spread distrib., *E. Ferrari et al., PRL, 2014*
- Energy spread removal by space charge, *E. Hemsing et al., PRL 2014*
- Phase merging in TGU undulator, *H. Deng and C. Feng PRL (2013)*
- ....

*L. H. Yu, I. Ben-Zvi NIM A**393** (1997) 96*

The Fresh Bunch Injection Technique: *



The seed @260nm is on the tail of the e-beam

The first stage converts the seed to the n^{th} harmonic (8^{th} harmonic @32.5nm)

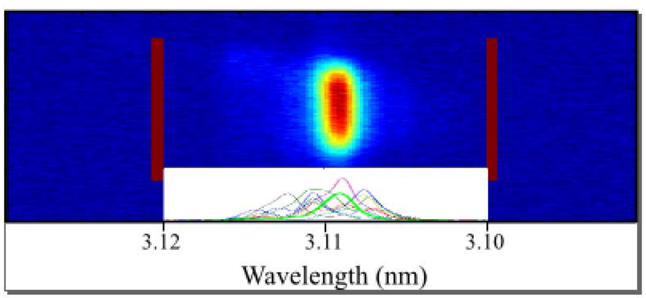
The delay line shifts the first stage output to a fresh portion of the e-beam

The second stage converts the first stage to the n^{th} x m^{th} harmonic of the seed

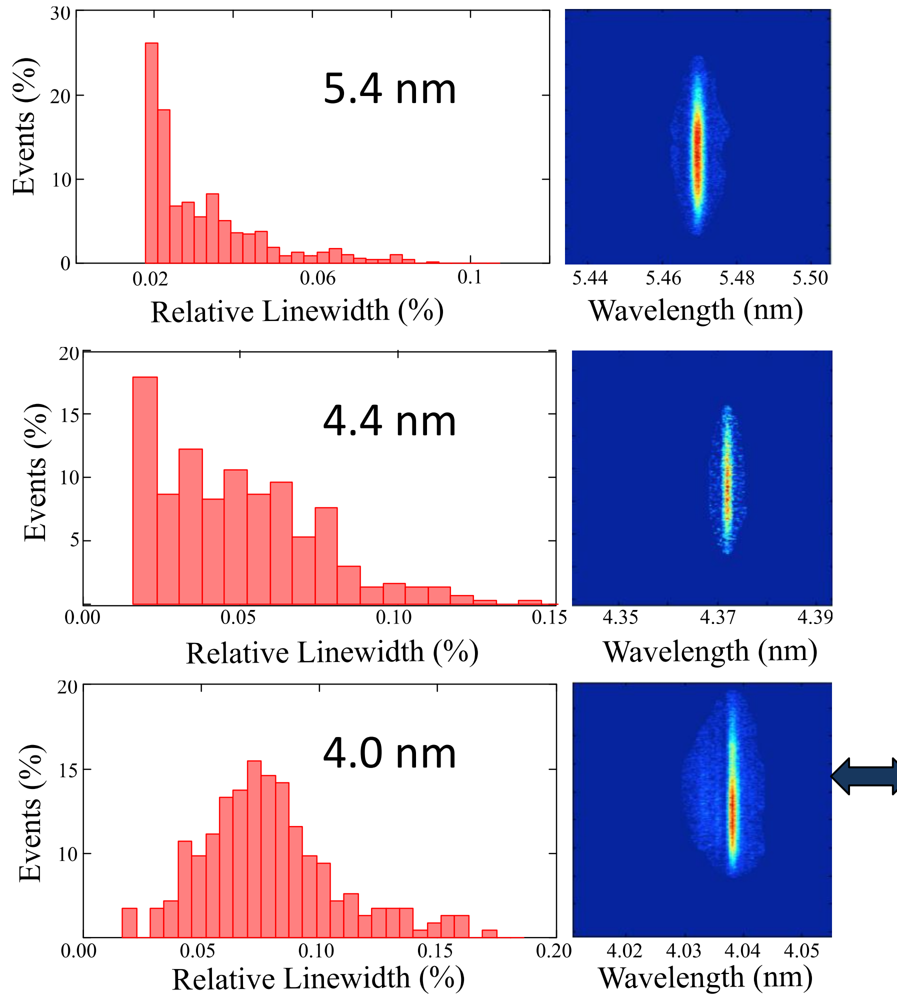
L. H. Yu, I. Ben-Zvi, Nim 1993
FERMI FEL-2 – E. Allaria et al. Nat Phot. 2013

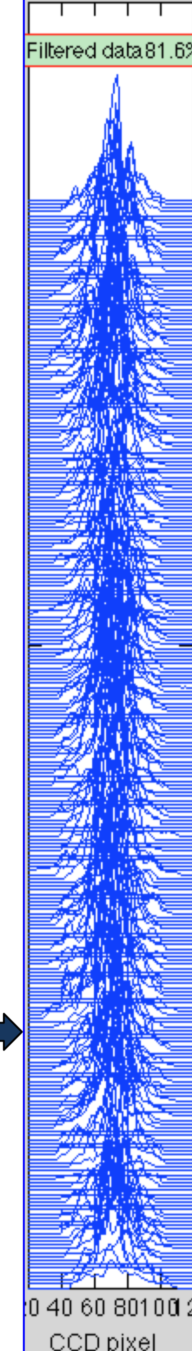
FERMI - FEL-2 spectra vs wavelength

Wavelength range 20 - 4 nm
polarization CR,CL, LV, LH
Single shot spectra measured down to 4 nm and show narrow linewidth with an energy per pulse at shorter wavelengths larger than 15 μJ.



Seed 266nm, h17x5 = h85 3.1 nm
$0.7\text{-}1.0\times10^{-3}$ bw

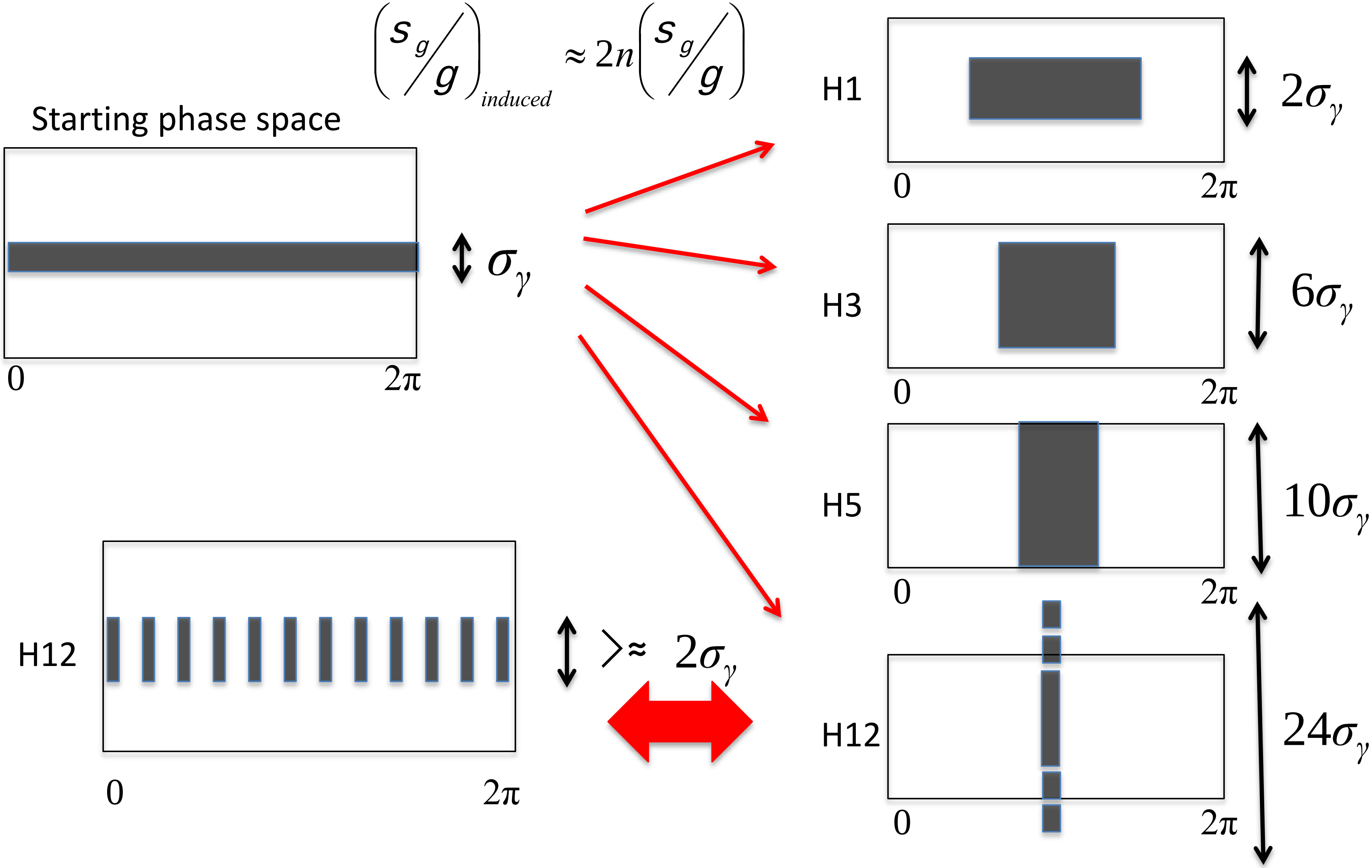




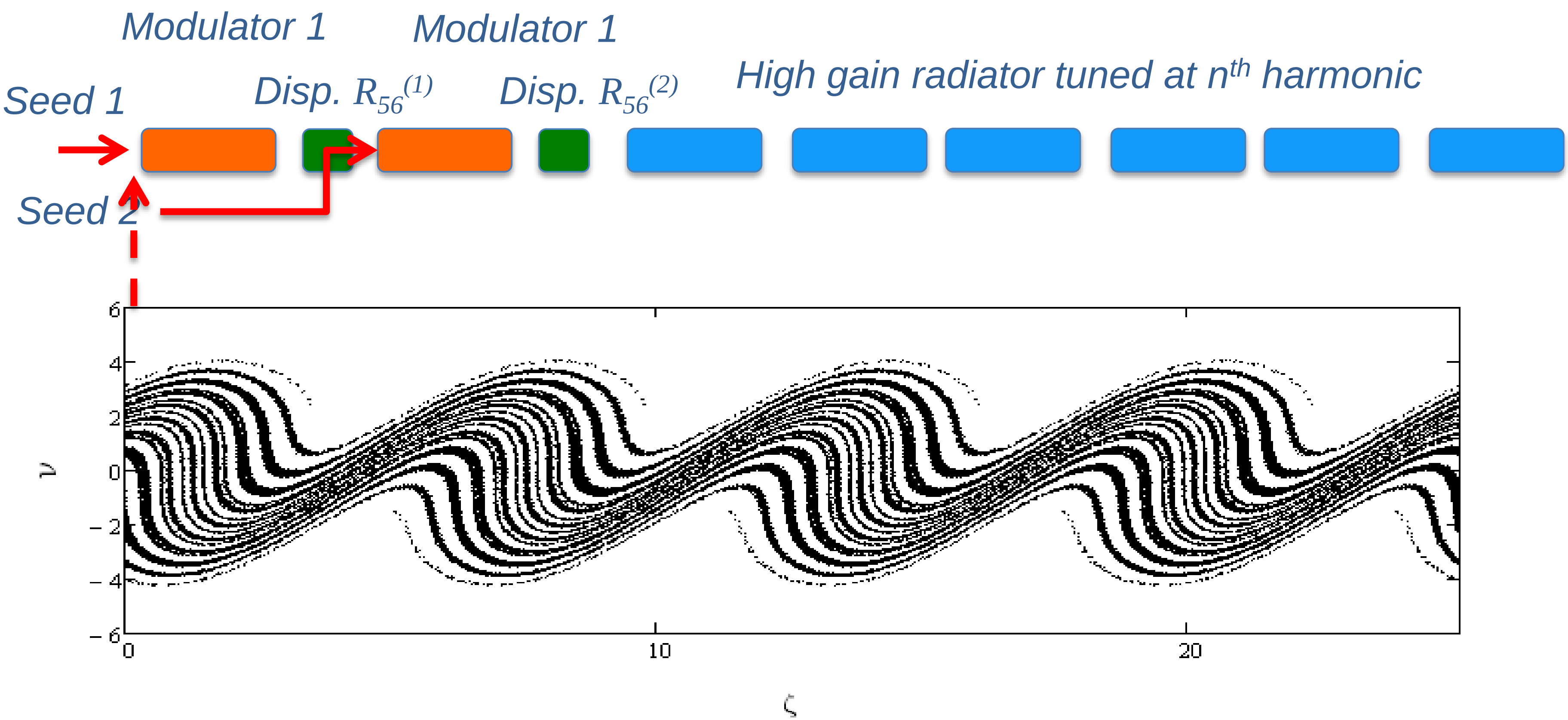
Sequence of spectra @4nm. 20% of data discarded – 10 uJ av. energy

Phase space stretching at high harmonics

$$\left(\frac{s_g}{g}\right)_{induced} \approx 2n\left(\frac{s_g}{g}\right)$$



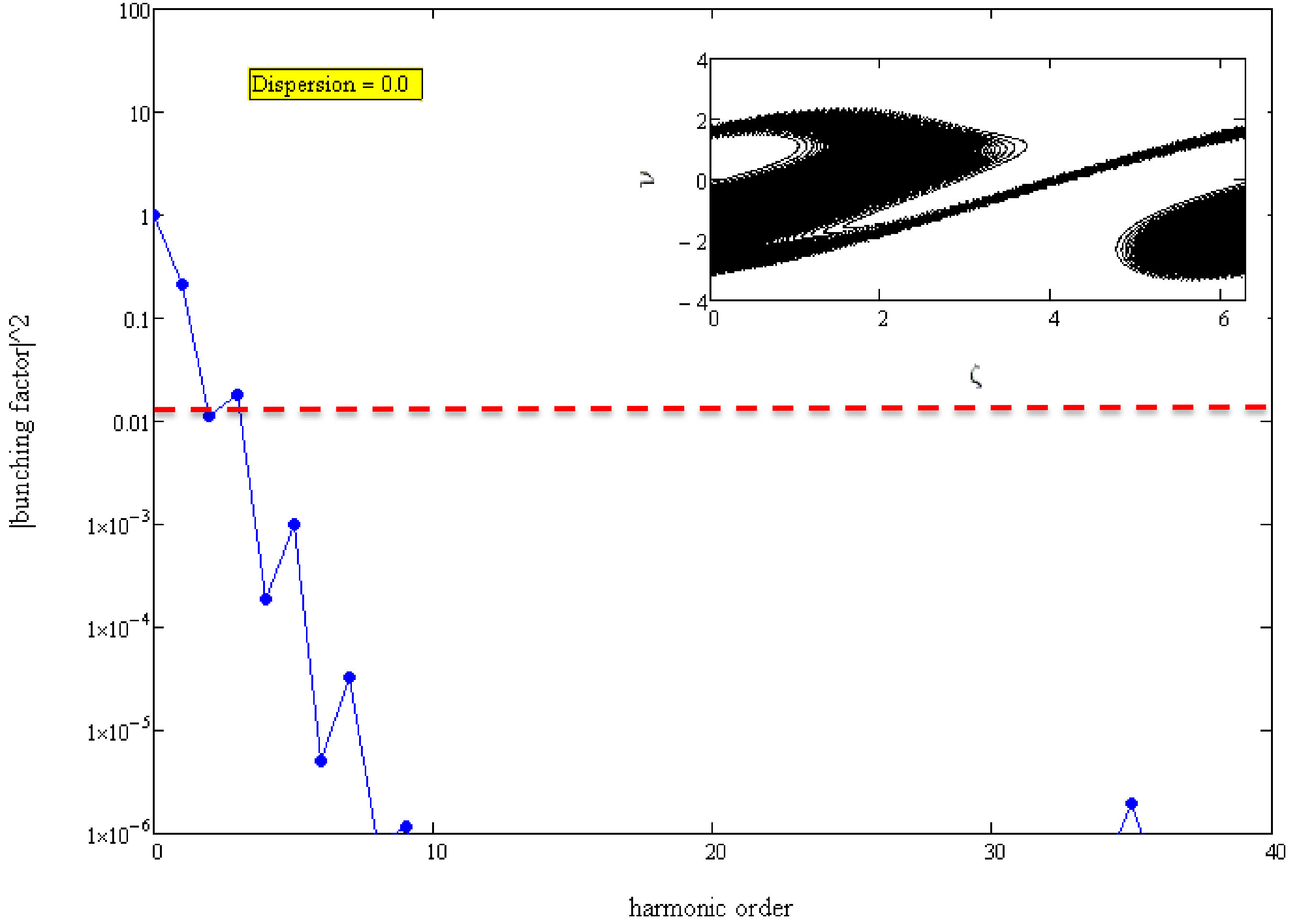
Echo Enabled Harmonic Generation



Frequency mixing,
resonance at $k_h = nk_1 + mk_2$

bunching maximized at harmonic $h \simeq \frac{\lambda_2}{\lambda_1} \frac{nR_{56}^{(1)}}{R_{56}^{(2)}}$

Increasing harmonic order at constant energy spread – only the first dispersion is varied



EEHG Experiments

Demonstrated at harmonic 3 (ECHO 3) at SLAC NLCTA (D. Xiang et al., PRL 2010) and SINAP (Z. T. Zhao Nat. Phot. 2012)

SLAC NLCTA: extension to

ECHO 7 (D. Xiang et al., PRL 2012) demonstrated lower sensitivity to energy spread

ECHO 15 (E. Hemsing et al. PRL 2014), confirmed lower sensitivity to energy spread & improved stability and spectral quality with respect to HGHG

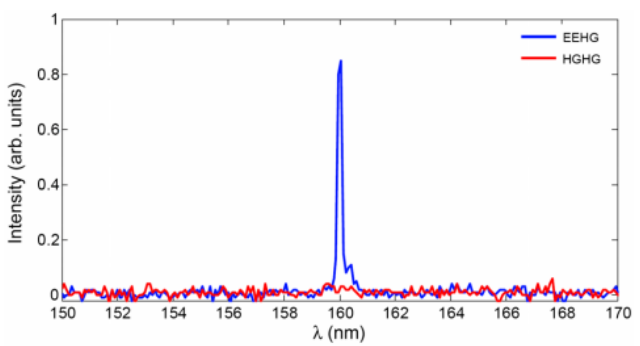
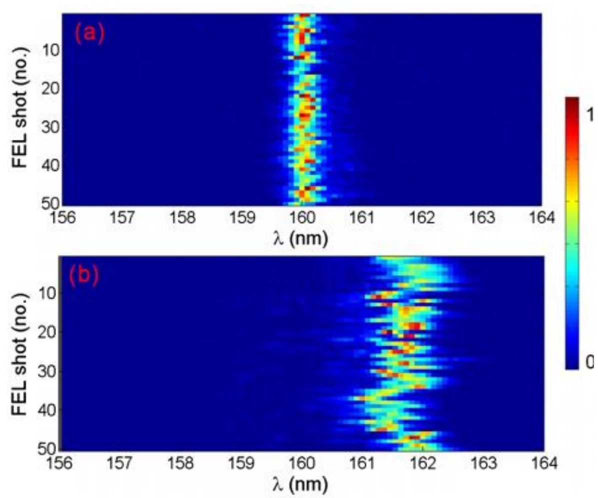


FIG. 8. EEHG and HGHG signals with the beam slice energy spread increased by a TCAV.



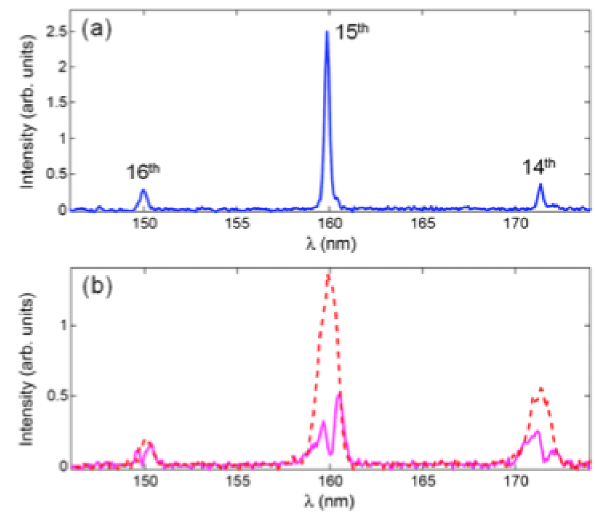


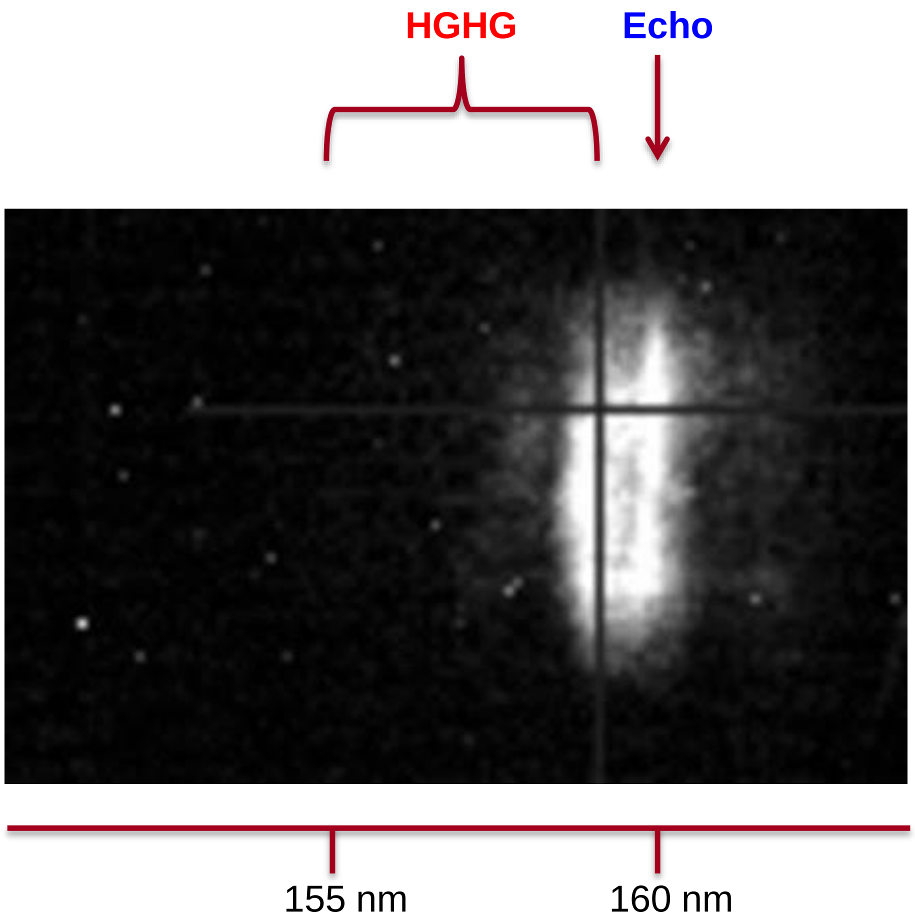
FIG. 4. Representative single-shot radiation spectrum for EEHG (a) and HGHG (b).

ECHO 75 (Successful, work in progress …)

In first semester 2018 experiment at FERMI Single stage EEHG 266nm->5nm

Simultaneous ECHO and HGHG in same beam

- Echo appears insensitive to e-beam phase space distortions leads to more stable central wavelength and narrower bandwidth



Courtesy of E. Hemsing

Hard X-rays via EEHG seeding EEHG …

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DOI 10.1007/s11434-016-1060-8

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Article Physics & Astronomy

Two-beam based two-stage EEHG-FEL for coherent hard X-ray generation

Zhentang Zhao · Chao Feng · Jianhui Chen · Zhen Wang

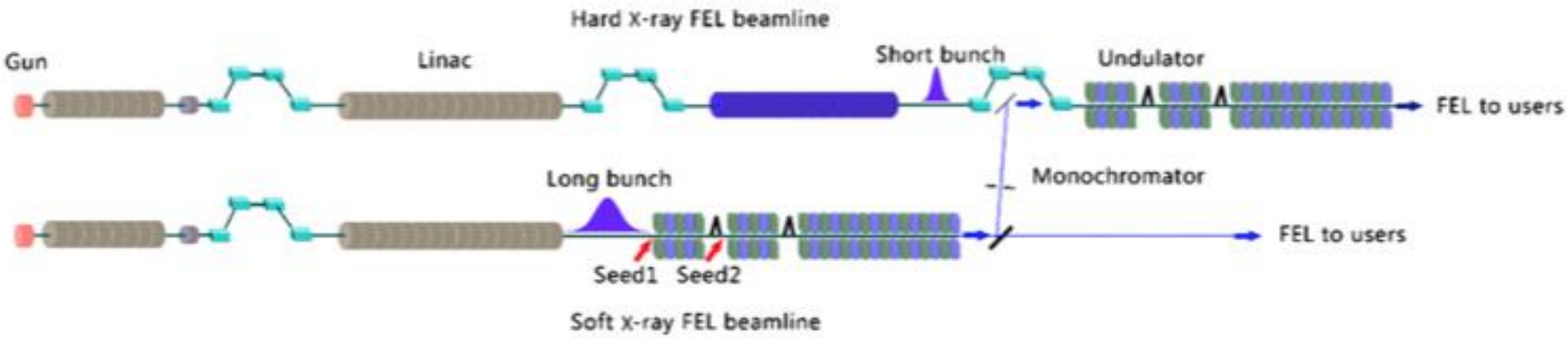
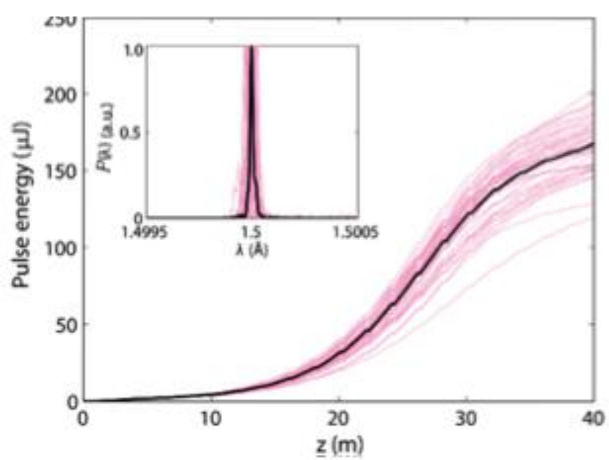


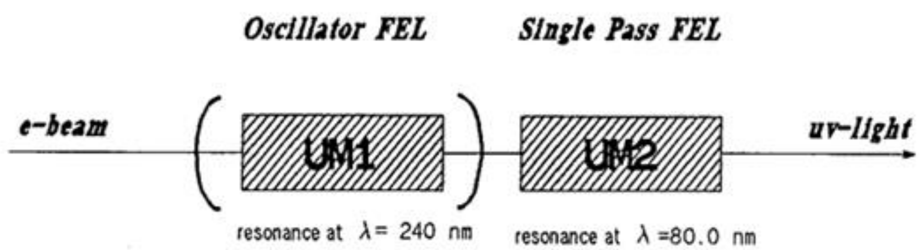
Table 1 Main parameters of our design

Main parameters	The soft X-ray beamline (first stage)	The hard X-ray beamline (second stage)
Beam energy (GeV)	1.6	6
Peak current (kA)	1	3

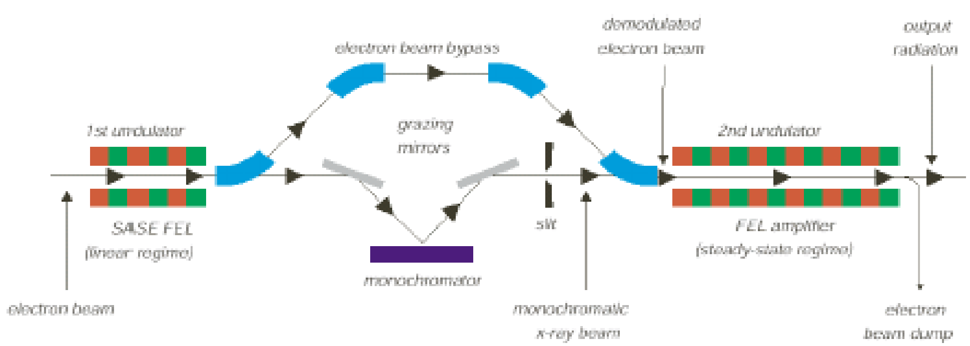


Coherence in the hard X-rays: Self Seeding

- e-beam shot noise energy grows at short wavelengths: the beam itself may produce the seed (and the modulation)
- An oscillator or a “master oscillator power amplifier” scheme is an example, but requires a sequence of e-pulses.



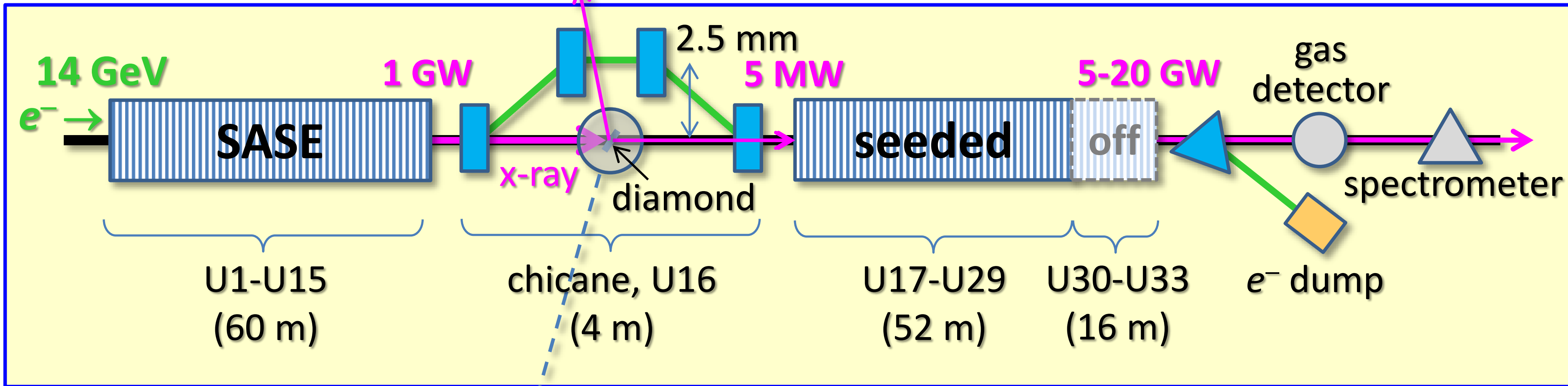
- Can we do the same with a single e-pulse ?



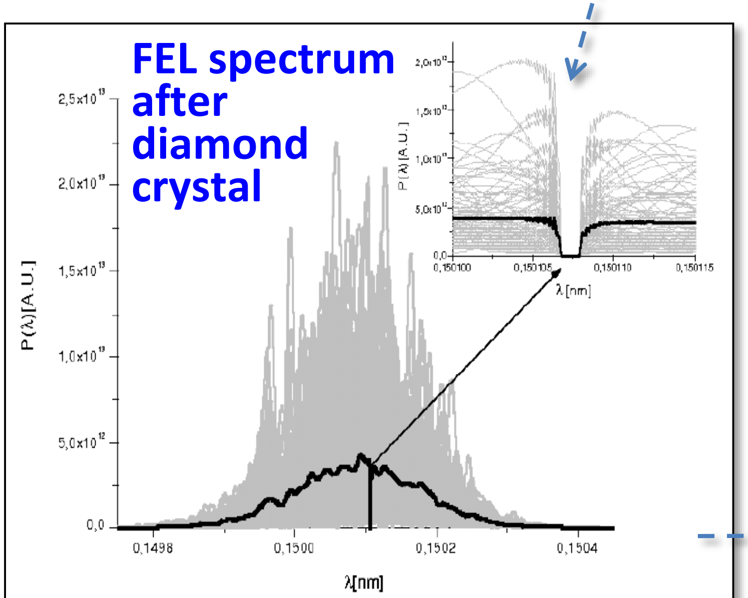
J. Feldhaus et al. / Optics Communications 140(1997) 341-352

Self-Seeding Scheme @ *LCLS*

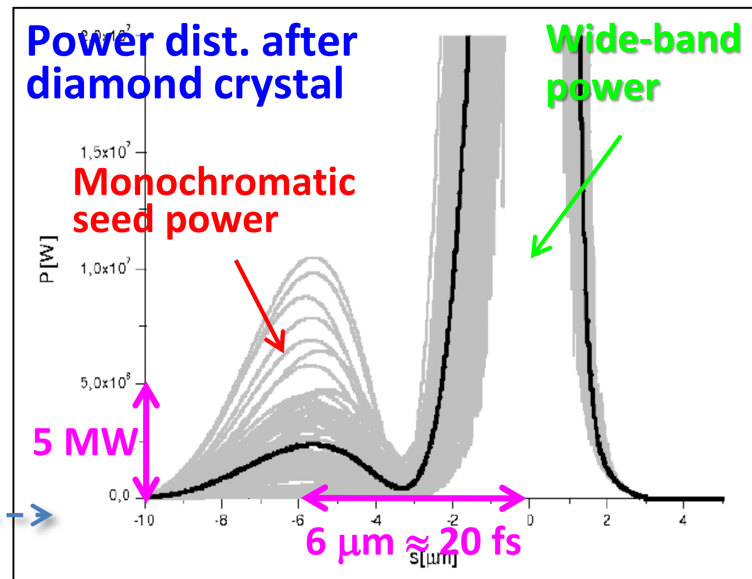
Courtesy of P. Emma



Geloni, Kocharyan, Saldin (*DESY 10-133*)

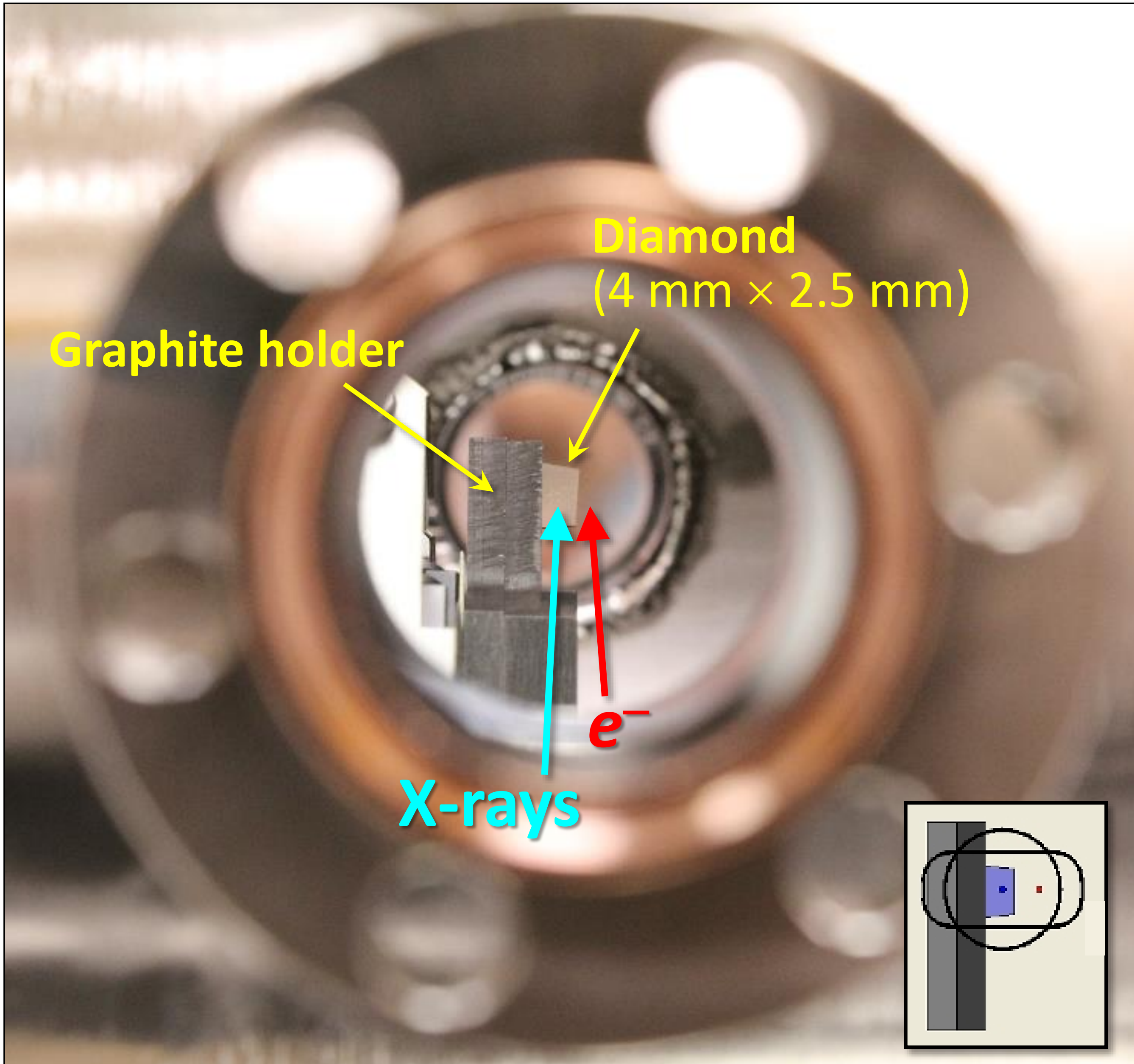


Use short, low-charge bunch to self-seed at 1.5 Å (20-40 pC)



Diamond Seen Through Beam Pipe

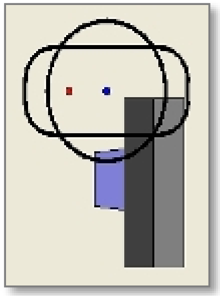
Courtesy of P. Emma



Crystal is high quality 110-μm thick type-IIa diamond crystal plate with (004) lattice orientation.

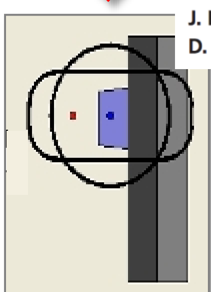
Grown from high-purity (99.9995%) graphite at the Technological Institute for Super-hard and Novel Carbon Materials (TISNCM, Troitsk, Russia) using the temperature gradient method under high-pressure (5 GPa) and high-temperature (~1750 K) conditions.

Courtesy of P. Emma

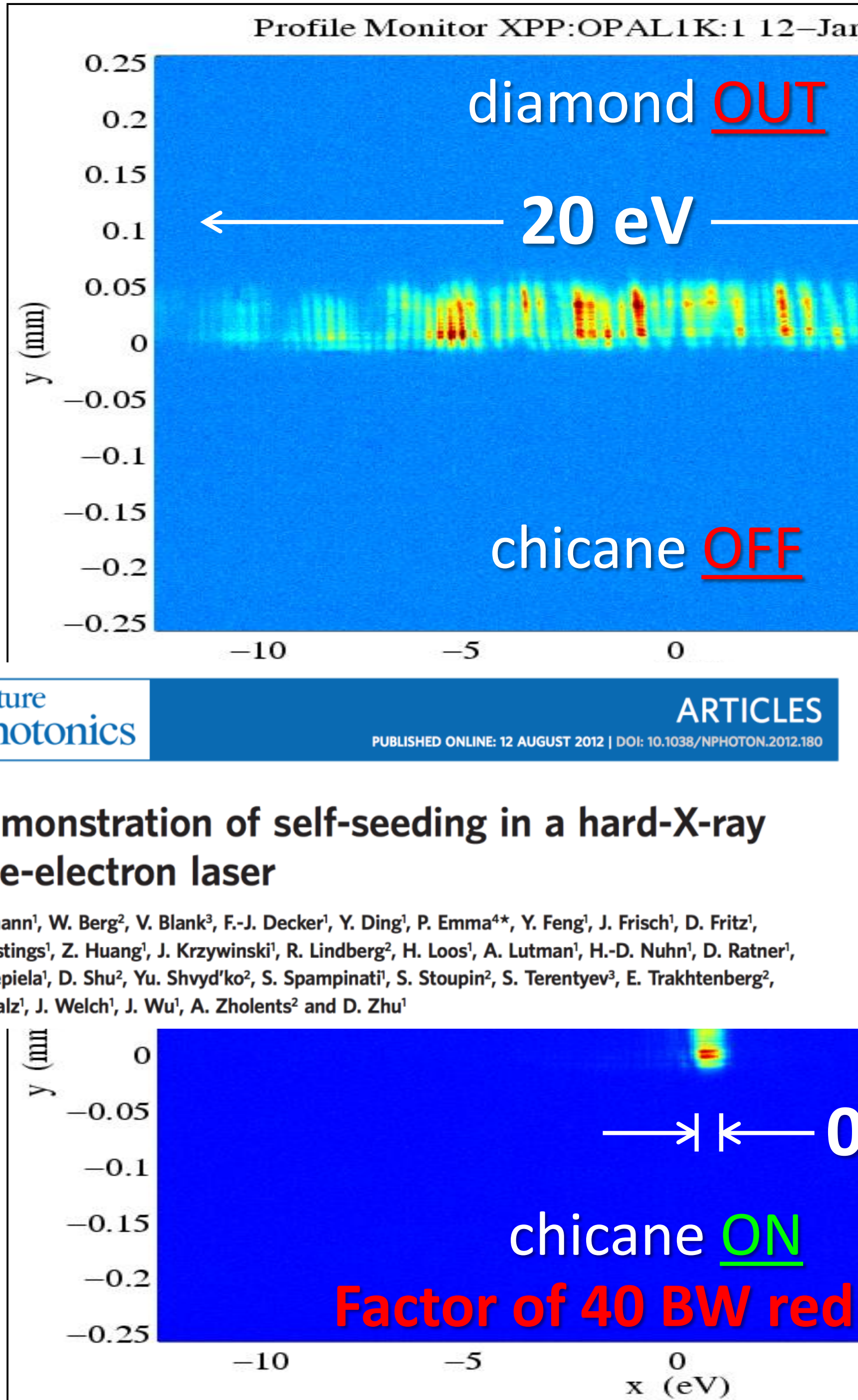


SASE

insert diamond & turn on chicane



seeded



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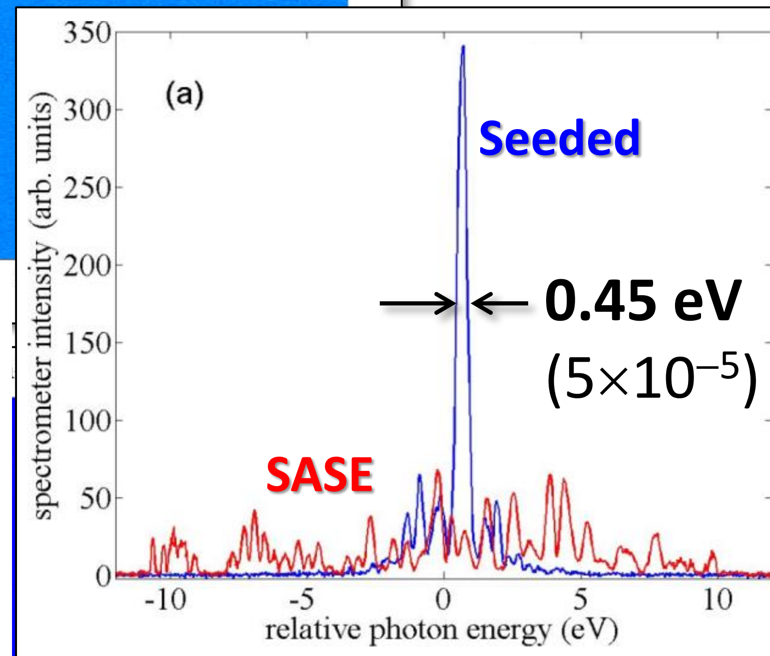
Demonstration of self-seeding in a hard-X-ray free-electron laser

J. Amann[1], W. Berg[2], V. Blank[3], F.-J. Decker[1], Y. Ding[1], P. Emma[4*], Y. Feng[1], J. Frisch[1], D. Fritz[1], J. Hastings[1], Z. Huang[1], J. Krzywinski[1], R. Lindberg[2], H. Loos[1], A. Lutman[1], H.-D. Nuhn[1], D. Ratner[1], J. Rzepiela[1], D. Shu[2], Yu. Shvyd'ko[2], S. Spampinati[1], S. Stoupin[2], S. Terentyev[3], E. Trakhtenberg[2], D. Walz[1], J. Welch[1], J. Wu[1], A. Zholents[2] and D. Zhu[1]



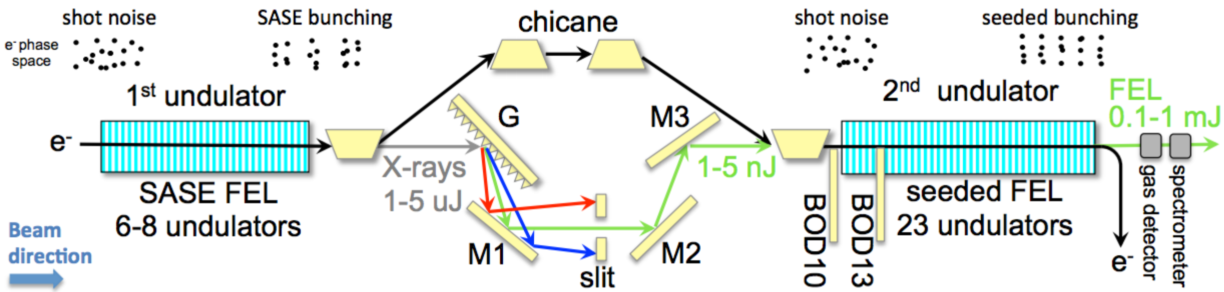
A well seeded pulse (not typical)

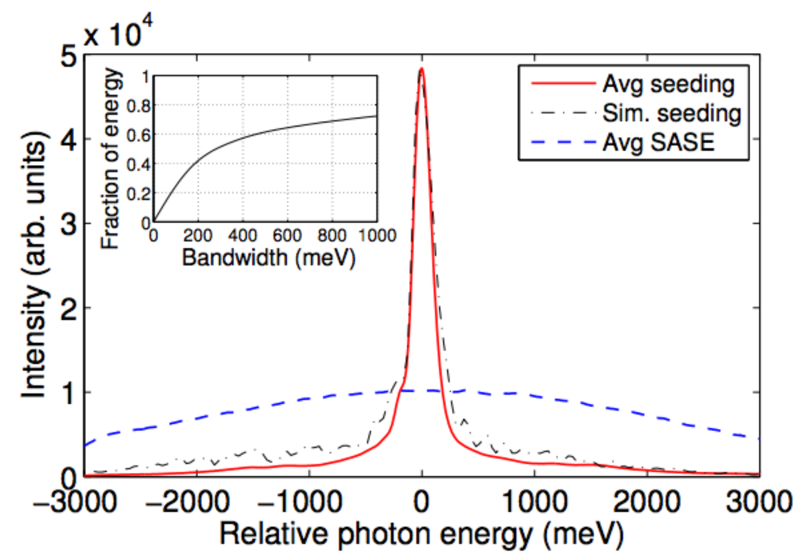
Self seeding: Soft X-ray version

SLAC-PUB-16214

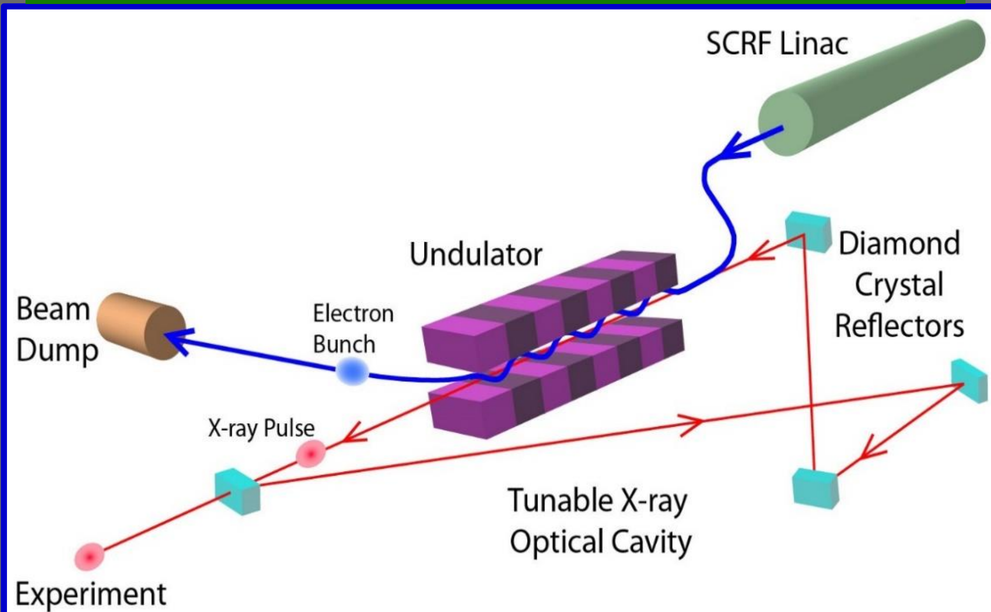
Experimental demonstration of a soft x-ray self-seeded free-electron laser

D. Ratner,[1*] R. Abela,[2] J. Amann,[1] C. Behrens,[1] D. Bohler,[1] G. Bouchard,[1] C. Bostedt,[1] M. Boyes,[1] K. Chow,[3] D. Cocco,[1] F.J. Decker,[1] Y. Ding,[1] C. Eckman,[1] P. Emma,[1] D. Fairley,[1] Y. Feng,[1] C. Field,[1] U. Flechsig,[2] G. Gassner,[1] J. Hastings,[1] P. Heimann,[1] Z. Huang,[1] N Kelez,[1] J. Krzywinski,[1] H. Loos,[1] A. Lutman,[1] A. Marinelli,[1] G. Marcus,[1] T. Maxwell,[1] P. Montanez,[1] S. Moeller,[1] D. Morton,[1] H.D. Nuhn,[1] N. Rodes,[3] W. Schlotter,[1] S. Serkez,[4] T. Stevens,[3] J. Turner,[1] D. Walz,[1] J. Welch,[1] J. Wu[1]
[1]*SLAC, Menlo Park, California*

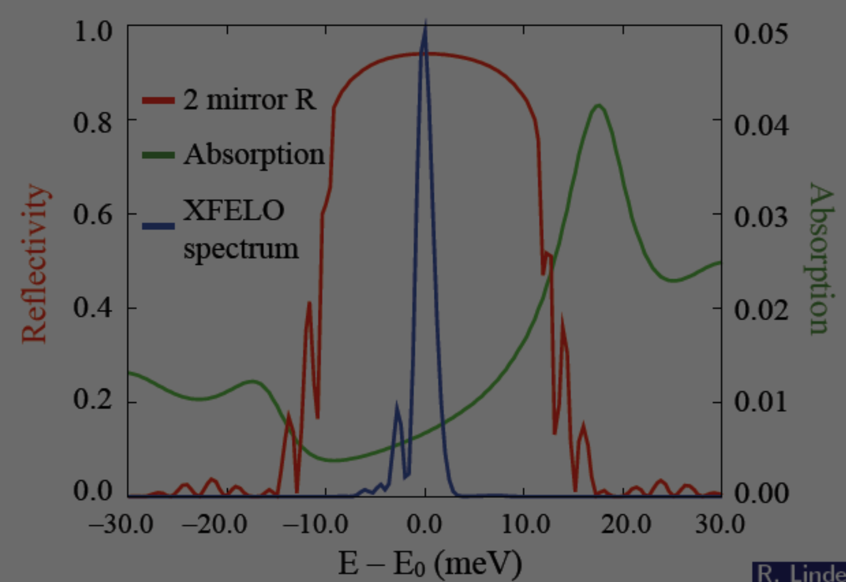




X-Ray FEL Oscillator



Rian Lindberg



Zizag cavity for tuning:

R. M.J. Cotterill (1968, ANL); KJK an Shvyd'ko (2009)

Choice of Bragg crystal based o thermo-mechanical properties→ Diamond

Monochromators

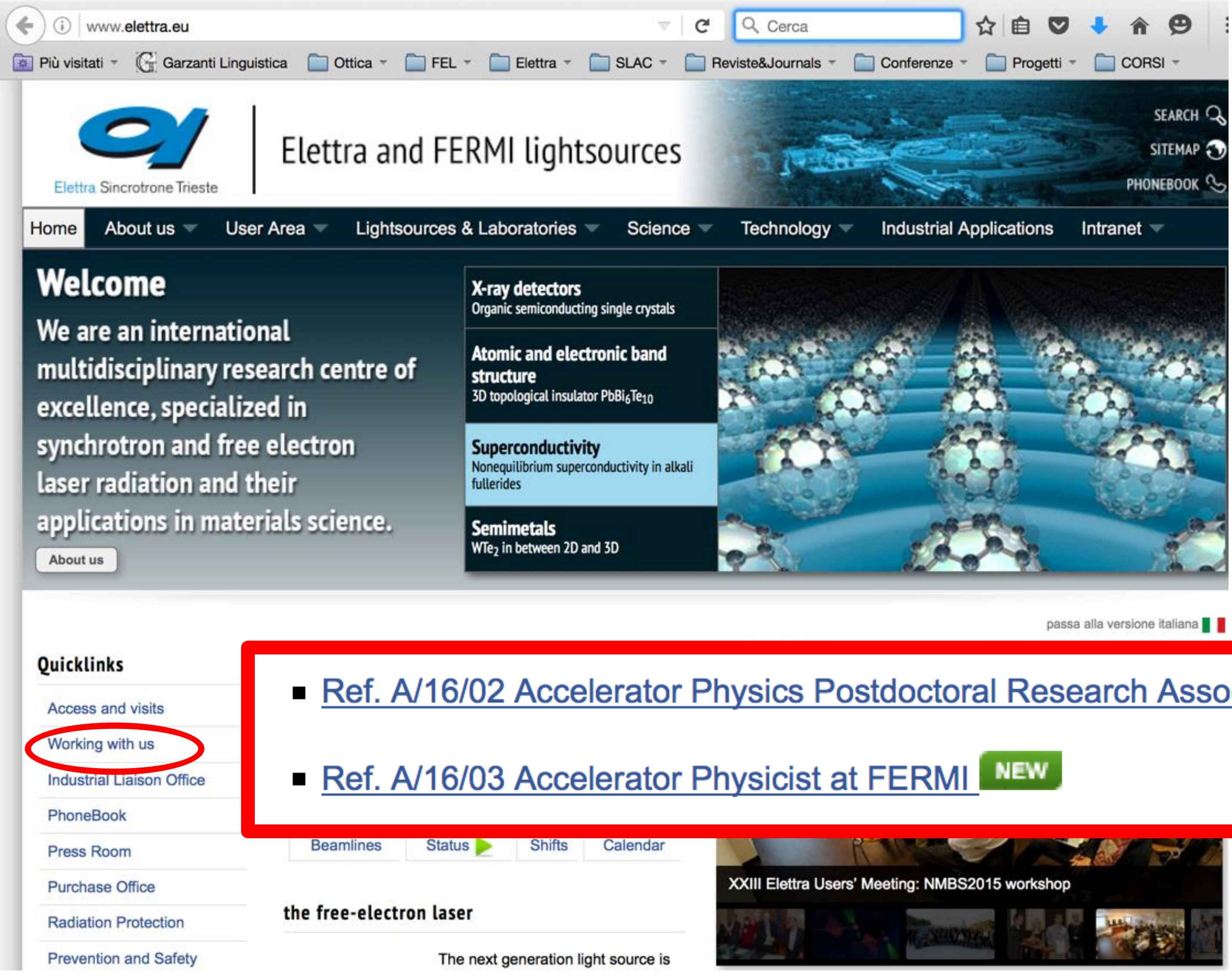
Monday June 6 – XFELO
Kwang Je Kim

Equivalent to an infinite chain of seeded sections

Summary

- Introduction on high gain and coherence in FEL amplifiers
- Conditions for seeding an FEL amplifier
- Direct Seeding: seeding with high harmonics generated in gas
- High gain harmonic generation
- Pulse properties and pulse control
- Saturation effects – Pulse splitting and superradiance
- The fresh bunch injection technique
- Echo Enhanced Harmonic Generation
- Self-Seeding

Open positions at FERMI



- Ref. A/16/02 Accelerator Physics Postdoctoral Research Associate at FERMI NEW
- Ref. A/16/03 Accelerator Physicist at FERMI NEW