Bunch Length Compressors

S. Di Mitri, Elettra Sincrotrone Trieste
Why do we need bunch length compressors?
  - FELs

What kind of compressors?
  - Magnetic insertions

Longitudinal beam dynamics in a LINAC
  - Energy chirp

Longitudinal beam dynamics in a CHICANE
  - Transport matrix
Credits and References

• Basics:

• Lectures:
  S. Di Mitri & M. Venturini, USPAS Course (2013, 2015)

• Technical Notes:
  Beam Dynamics Newletter No. 38 (2005)
  P. Emma, LCLS-TN-01-1 (2001)
  S. Di Mitri & M. Cornacchia, Physics Reports 539 (2014)

• Acknowledgment:
  M. Venturini, for valuable support and figures
FEL Brilliance

- FEL radiation is generated in undulators.
- Far higher degree of coherence and peak intensity than synchrotrons, at same wavelength.

SLSs

\[ \sim N_e = \text{number of electrons in a bunch} = 10^7 - 10^9 \]

FELs

1. rf photo-cathode gun:
   - reduction space-charge effects
2. booster accelerator:
   - reaching the target energy
3. bunch compressor:
   - increase of the current
4. accelerator:
5. FEL

Peak brilliance [Photons/(s mrad^2 mm^2 0.1% BW)]

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FEL Gain

Radiation power grows exponentially along the undulator, until saturation:

\[ P(s) = P_0 e^{\frac{4\pi\sqrt{3}}{\lambda_u} \rho s} \]

Radiation power at saturation is proportional to the e-beam power \( P_b = E_b I / e \) (SASE):

\[ P_{sat} \sim \rho P_b \]

FEL power saturation length: this sets the scale for the undulator length (SASE):

\[ L_{sat} \sim \frac{\lambda_u}{\rho} \]

A high peak current makes \( \rho \) large:

\[ \Rightarrow \text{large FEL gain}, \ \Rightarrow \text{high saturation power}, \ \Rightarrow \text{short saturation length.} \]
The 6D e-beam brightness measures the charge density in the 6D phase space (energy-normalized). For a “diffraction limited” \([4\pi \varepsilon_{x,y} = \lambda]\), “cold” \([\sigma_\delta < \rho]\) e-beam, the higher the brightness, the shorter the FEL wavelength achievable with a decent power.

\[ B_{n,0}(\lambda) = \frac{Q}{\varepsilon_{n,x} \varepsilon_{n,y} \varepsilon_{n,z}} = \frac{1}{c \sigma_\varepsilon \gamma_0^2 \varepsilon_0^2} \approx \frac{32\pi^2}{c} \frac{1}{\sigma_\varepsilon \lambda_u (1 + K^2/2)} \frac{1}{\lambda} \]

A high peak current makes the e-beam 6D brightness large ⇒ short FEL wavelengths with reasonable amount of power.
Very short bunches at low energy ($K \approx m_e c^2$) are diluted in the 6-D phase space by "space charge" forces (inter-particle Coulomb interaction).

- The intra-bunch repulsive force is stronger at lower beam energies ($\sim 1/\gamma^2$), and at higher charge density ($\sim I$).

$$\varepsilon_{tot} = \sqrt{\left(\varepsilon_{cathode} \sigma_r^2 \right)^2 + \left(F \frac{Q}{\sigma_r^2 \sigma_z^2} \right)^2}$$

Transverse Emittance $\sim 1/\sigma_z$

Bunch length compressor(s) are needed at beam energies higher than 100s of MeV to reach 100s A to kA peak current level.
RF Structure (Standing-Wave)

- Dynamics is driven by the longitudinal component of electric field, $E_z$ [MV/m].
- Consider standing-wave structures (traveling-wave structures have similar treatment).

1.3GHz, Super Conducting 9-Cell Tesla RF cavities are operated as Standing-Wave structures.

On-axis Longitudinal E-field for TESLA Cavity

- **Design structures so that, as the electron moves from cell to cell, it sees the same $E_z$:**
  - the electron travels through one cell in half rf period,
  - cell length is half the rf wavelength: $\lambda_{rf} = \frac{c}{f_{rf}}$ ("$\pi$-mode").
Longitudinal Dynamics

- On axis (x=y=0) electric field in a cell [-g/2, g/2]:
  \[ E_z(s) = E_0(s) \cos(\omega_{rf} t(s) + \phi_{rf}) \approx E_{z,0} \cos(k_{rf} s) \cos(\omega_{rf} t_{syn} + \omega_{rf} \Delta t + \phi_{rf}) = E_{z,0} \cos(k_{rf} s) \cos(k_{rf} z + \phi_{rf}) \]

- Energy change by an electron with coordinate z:
  \[ \Delta E(g,z) = -e \int_{-g/2}^{g/2} E_z(s) ds \approx -e E_{z,0} \int_{-g/2}^{g/2} ds \cos^2(k_{rf} s) \cos(k_{rf} z + \phi_{rf}) \approx e \Delta V(g) \cos(k_{rf} z + \phi_{rf}) \]

Approximations and Notes.

Fundamental mode of E-field.

Time of arrival of any particle relatively to the reference ("synchronous") particle:
\[ \Delta t(s) = t(s) - t_r(s) \]
\[ z = \Delta t/c \]
\[ \Delta t(s) < 0 \text{ or } z < 0, \text{ means particle is ahead of reference particle (it arrives earlier at } s) \]

Ultra-relativistic particles:
\[ \frac{ds}{dt} \approx c \]

What's the meaning? \( t(s) \) is the arrival time of the electron measured by an observer at longitudinal position \( s \).
How to Choose the RF Phase

For maximum acceleration, the cavities should be operated on crest...

Q: Why do we ever want to operate the cavities off-crest?

A: To control the beam "energy chirp", i.e. the correlation between a particle position $z$ within the bunch and its energy $E$
  - The ability to put an energy chirp on a beam is needed to do bunch compression through a magnetic insertion.
Energy Chirp

Electron beam longitudinal phase space

Beam without energy chirp

Beam with energy chirp

Taylor expand the energy through first order in $z$:

$$E(z) = E_i + eV \cos(k_{rf} z + \varphi_{rf}) \approx E_i + eV \cos \varphi_{rf} - eVk_{rf} z \sin \varphi_{rf} + O(z^2)$$

Linear chirp at exit of structure:

$$h_1 = \frac{1}{E(0)} \frac{dE(0)}{dz} = -\frac{eV k_{rf} \sin \varphi_{rf}}{(E_i + eV \cos \varphi_{rf})} \approx \frac{\sigma_{\delta corr}}{\sigma_z}$$

Beam @ entrance of structure

Beam @ exit of structure

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Off-crest acceleration:

$$f_{rf} = 1.3 \text{ GHz}$$
$$\lambda_{rf} = 23 \text{ cm}$$
$$V_0 = 129 \text{ MV}$$
$$\varphi_{rf} = -30.3^\circ$$

$$h_1 \approx \frac{0.034}{0.004 \text{m}} = 8.5 \text{m}^{-1}$$
Slippage of Ultra-Relativistic Particles

- Compress the bunch length: we need to change the electrons' longitudinal coordinate $z$ (inside the bunch).
- We have problem: equation of motion of ultra-relativistic electron (through an accelerating structure or transport line) gives:

\[ \frac{dz}{ds} \approx 0 \]

Relative longitudinal position of particles in the bunch does not change (the beam is ‘frozen’).

- However, \( \frac{dz}{dE} \neq 0 \)

\[ E(z) \approx E_i + eV \cos \varphi_{rf} - eV k_{rf} z \sin \varphi_{rf} \equiv \left[ (m_e c^2)^2 + (p_z(z)c)^2 \right]^{1/2} \]

And the Lorentz force depends on the particle momentum:

1. Establish a \((z, E)\) correlation [energy chirp]
2. Pass through a magnetic field [magnetic insertion]
3. Particles with different energy will follow different paths.
4. For same velocity \((v \approx c)\), different path lengths will lead to different arrival time.
5. The bunch is ‘time-compressed’!

\[ p_z(z)c = eB_y R(z) \]

\[ p_z [\text{GeV/c}] = 0.2998 \cdot B_y [\text{T}] \cdot R [\text{m}] \]
4-Dipoles C-shape Chicane

- Bend angle for on-momentum (reference) particle: \( \theta_0 \approx \frac{l_B}{R} = \frac{eB}{p_{z,0}} l_B \)

- Bend angle for a particle off-momentum: \( \theta = \frac{\theta_0}{1 + \delta} \quad \delta \equiv \frac{\Delta p}{p_0} \approx \frac{\Delta E}{E_0} \) (ultra-relativistic approx.)

- The system is an achromat by design (barring magnet errors/imperfections): \( \theta_1 + \theta_2 + \theta_3 + \theta_4 = 0 \)

- Energy-dispersion function:
  \( \eta_s(s) = \frac{\Delta x(s)}{\Delta p_z / p_{z,0}} \rightarrow R(1 - \cos \theta) \) for a single dipole
  \( \eta'_s(s) = \frac{\Delta x'(s)}{\Delta p_z / p_{z,0}} \rightarrow \sin \theta \) for a single dipole

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Simone Di Mitri – simone.dimitri@elettra.eu
Momentum Compaction

- Thin lens approximation for the dipoles (finite bend angle resulting from infinitesimally short dipole and infinitely large magnetic field): \( \theta = \frac{\theta_0}{1 + \delta} \)
- Path-length of off-momentum electron: \( s = \frac{2L_1}{\cos \theta} + L_2 \)
- Path-length of on-momentum (reference-particle) electron: \( s_0 = \frac{2L_1}{\cos \theta_0} + L_2 \)
- Path-length difference:

\[ R_{56} \approx -2L_1 \theta_0^2 \]

For finite dipoles' length \( L_b \):

\[ R_{56} = -2\theta_0^2 \left( L_1 + \frac{2}{3} L_b \right) \]

General expression:

\[ R_{56}(0 \to s) = \frac{s}{\int_0^s \eta_x(s') R(s')} \]

"Momentum Compaction": \( \alpha_c := \frac{R_{56}}{L_{tot}} \)
Compression Factor

• Longitudinal action through the chicane:
  \[ z_f = z_i + R_{56} \delta_i \]
  
  - energy spread correlated with \( z \)
  
• Differentiate (pass from local position to bunch length):
  \[ dz_f = dz_i + R_{56} d \delta = dz_i + R_{56} \frac{dE}{E_0} = dz_i \left( 1 + R_{56} \frac{1}{E_0} \frac{dE(z)}{dz_i} \right) + R_{56} \frac{dE_{unc}}{E_0} = \]
  
  \[ = dz_i (1 + hR_{56}) + R_{56} \delta_{unc} \equiv dz_i / C + R_{56} \delta_{unc} \]

  \[ C \equiv \left| \frac{dz_0}{dz_1(z_0)} \right| = \frac{l_{b,i}}{l_{b,f}} = \frac{1}{1 + R_{56} h_1} \]

- If \( E(z) \) - the energy chirp - is nonlinear, then \( C \) depends on \( z \) (compression will vary along bunch). Generally, we refer to \( C(z = 0) \) as the nominal (linear) compression factor.

- When \( C \to \infty \), the minimum bunch length is set by the "uncorrelated" energy spread:
  \[ \sigma_{z,\text{min}} = R_{56} \sigma_{\delta,\text{unc}} \]
Various Types of Compressors…

- Sign of $R_{56}$ sets the sign of the incoming energy chirp (thus RF phase in upstream linac).
- Compactness is usually important.
- In single-pass linacs, usually preferred a net zero-deflection from straight path.
- Arcs are a natural choice for ERLs.

- $R_{56} < 0$
- $R_{56} > 0$

- FLASH
- LCLS
- FERMI
- X-FEL
- SACLA

- SLC arcs
- NLC BC2
- ERLs?
Double S-Chicane:

Proposed to counteract disrupting interaction of electron beam with its own emitted synchrotron radiation.

Four arcs form a “FODO”-Compressor:

Proposed for recirculating accelerators.

Arc based on “double-bend achromat” cell:

MAX-IV SPF has two compressors, each one is half of this.
Correlated, Uncorrelated, Slice Energy Spread

1 + \( R_{56} h_1 > 0 \)

1 + \( R_{56} h_1 = 0 \)

1 + \( R_{56} h_1 < 0 \)

"Under-compression" (prevalent mode)

"Full-compression" (min. bunch length set by uncorrelated energy spread)

"Over-compression" (sign of energy chirp is reversed)

Uncorrelated \( \delta \)

Correlated \( \delta \)

\[
\sigma_{\delta,\text{tot}}^2 = \sigma_{u,i}^2 + \sigma_{c,i}^2 = \sigma_{u,i}^2 + h_i^2 \sigma_{z,i}^2 = \sigma_{u,f}^2 + h_f^2 \sigma_{z,f}^2 = \ldots = \sigma_{u,f}^2 + h_i^2 \sigma_{z,i}^2 + C^2 R_{56}^2 \sigma_{u,i}^2 + o(\sigma_{z,i}^4, \sigma_{u,i}^4)
\]

\[
\sigma_{z,f}^2 = \frac{\sigma_{z,i}^2}{C^2 + R_{56}^2 \sigma_{u,i}^2 + 9T_{56}^2 h_i^4 \sigma_{z,i}^4}
\]

\[
h_f = Ch_i
\]

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RF Curvature

\[ E_I = E_i + eV \cos(kz + \varphi) \approx E_i + eV \cos \varphi - kzeV_0 \sin \varphi - \frac{eV^2}{2} z^2 \cos \varphi + O(z^3) \]

Energy of particle at exit of accelerating structure

\[ \Delta E \]

\[ Z \]

0-order term >0 (acceleration)

Quadratic term <0

\[ V \]

\[ k \]

\[ k_H \]

\[ H \]

\[ E_{II} = E_I + eV_H \cos(k_H z + \varphi_H) \approx E_I + eV_H \cos \varphi_H - k_H eV_H \sin \varphi_H - \frac{V_H k^2}{2} z^2 \cos \varphi_H + O(z^3) \]

0-order term <0

\[ \frac{V H^2 k^2}{2} \cos \varphi + \frac{V H^2 k^2}{2} = 0 \]

\[ V_H = \frac{k^2}{k_H^2} V \cos \varphi \]

To cancel quadratic curvature from accelerating structure this term should be >0; i.e. \( \cos \phi_H < 0 \), say (\( \cos \varphi = -1 \)). This structure is decelerating!

Q: How can we win? (i.e. compensate 2\textsuperscript{nd} order term and still have overall acceleration?)

A: Choose \( k_H > k \)

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Nonlinear Compression…

(Idealized) beam out of the injector
E=100MeV

Beam accelerated
off-crest to E=210MeV

Beam @ exit of compressor

- Current spike and/or nonlinear energy chirp might not be good for FELs.
- In practice, it really depends on scientific target and application.

Not apparent on this scale there is a small quadratic term in the chirp

Compression magnifies the curvature

Current spike results

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Simone Di Mitri – simone.dimitri@elettra.eu
…Compression Linearized

(Idealized) beam out of the injector
\( E=100\text{MeV} \)

Beam accelerated off-crest to \( E=210\text{MeV} \)

Beam @ exit of compressor

- “Linear compression” = linear transformation applied to the longitudinal phase space.
- Current shape is “preserved” through the compression process.

\[ C = \frac{I_f}{I_i} = \frac{0.145A}{0.45A} \approx 3.2 \]

\[ C = \frac{\sigma_{zi}}{\sigma_{zf}} = \frac{450\mu\text{m}}{144\mu\text{m}} \approx 3.1 \]
Operationally, linearizer rf frequency is best chosen to be a harmonic number of rf frequency of main linac (FLASH uses 3.9 GHz vs. 1.3 GHz SC linac; FERMI uses 11.4 GHz vs. 3.0 GHz NC linac; ...).
Compression at Second Order in $\delta$

- Nonlinear momentum compaction in chicane is usually non-negligible and has to be compensated:

$$z_1 = z_0 + R_{56} \delta_0 + T_{566} \delta^2_0$$

For C-type chicanes: $T_{566} \simeq -\frac{3}{2} R_{56} > 0$

- Modified setting of harmonic cavity when accounting for the 2nd order term $T_{566}$ in momentum compaction:

$$eV_H = \frac{1}{(k_H^2/k^2 - 1)} \left( E_{BC} \left[ 1 + \frac{2}{k^2} \frac{T_{566}}{R_{56}} \left( 1 - \frac{1}{C} \right)^3 \right] - E_l \right)$$

Energy of beam entering Linac section

Beam energy at compressor (minimizing $V_H$ may imply compression at lower energy)

Linear compression factor $C = \frac{1}{|1 + R_{56} h_1|}$

- Formula valid for $\phi_H = -180^\circ$ and one-stage (single chicane) compression.
- If multiple compressors are present, $V_H$ setting varies somewhat but typically not too much (after first BC, the bunch is shorter and less vulnerable to rf nonlinearities)
- Alternate method to linearize: sextupole magnets within magnetic compressor (works well in arc-like compressors, where large dispersion and separation between magnets is allowed).
Sextupole magnets can be used as an alternative to a harmonic cavity.

- Usually included in “long” compressors such as dog-legs and arcs, in order to cumulate betatron phase advance to cancel out 2nd order optical aberrations, and eventually avoid emittance growth.

Double-Bend Achromat Cell

- Usually included in “long” compressors such as dog-legs and arcs, in order to cumulate betatron phase advance to cancel out 2nd order optical aberrations, and eventually avoid emittance growth.

Horizontal Emittance through Arc

Green line depicts emittance value oscillation due to aberrations induced by sextupoles. Those eventually (almost) cancel.
Consider the short-term variation of RF phase in a linac upstream of a magnetic compressor. - Evaluate how $C$ varies in the presence of jitter on $\phi_{rf}$:

$$\Delta \left( \frac{1}{C} \right) = \Delta (1 + h R_{s6}); \text{ from the definition of } C$$

$$\frac{\Delta C}{C^2} = -R_{s6} \frac{\Delta h}{h} = -R_{s6} \frac{\Delta (\sin \phi_{rf})}{\sin \phi_{rf}} h = -h R_{s6} \frac{\Delta \phi_{rf}}{\tan \phi_{rf}}; \text{ from the definition of } h, \text{ for } E_z \sim \cos \phi_{rf}$$

$$\frac{\Delta C}{C} = -Ch R_{s6} \frac{\Delta \phi_{rf}}{\tan \phi_{rf}} = \left( C - 1 \right) \frac{\Delta \phi_{rf}}{\tan \phi_{rf}}. \text{ at 1st order in } \Delta \phi_{rf}$$

- Fluctuations of rf structure parameters (voltage, phase) around set values are unavoidable. - They cause undesirable “jitters” in beam energy, arrival time, peak current.

- Aggressive but not unreasonable targets for max. RF fluctuations are:
  - rf phase: 0.1 deg (NC) - 0.01 deg (SC)
  - rf voltage: 0.1% (NC) - 0.01% (SC)
Jitter: Arrival Time

- Consider the short-term variation of: RF phase, RF voltage, dipole field, and arrival time at the linac entrance.

- Evaluate how $t_{\text{syn}}$ at the exit of the chicane varies in the presence of the aforementioned jitters (not derived here):

$$
\sigma_{t,f}^2 \equiv \left( \frac{\sigma_{t,i}}{C} \right)^2 + \left( \frac{R_{56}}{c} \right)^2 \left( \frac{\sigma_B}{B} \right)^2 + \left( \frac{R_{56}}{c} \frac{\Delta E_{\text{linac}}}{E_{BC}} \right)^2 \left( \frac{\sigma_V}{V} \right)^2 + \left( \frac{C - 1}{C} \right)^2 \left( \frac{\sigma_\phi}{ck} \right)^2
$$

**Initial arrival time is “compressed”**. Typical $\sigma_{t,i} \sim 150$ fs

**Power supply stability. Typical** $\sigma_B / B \sim 0.01\%$

**RF peak voltage. Typical** $\sigma_V / V \sim 0.1\% - 0.01\%$

**RF phase. Typical** $\sigma_\phi \sim 0.1$ - 0.01 deg S-band

- For given $R_{56}$ in single-stage compression, the final arrival time jitter may show up a local minimum as a function of the linac RF phase (in that case, $C$ is varying).

- A multi-stage compression scheme has potentiality for reducing final beam jitters. In practice, tracking runs are used to determine a jitter tolerance budget and perform optimization.
Multipolar field expansion (normal mode):

\[ B_y(x) = \sum_{n=0}^{\infty} b_n \left( \frac{x}{R} \right)^n \]

\[ b_n = \frac{1}{n!} \left( \frac{\partial^n B_y}{\partial x^n} \right) \bigg|_{y=0} \]

“x” is the particle’s distance from the magnetic axis

“R” is the arbitrary distance at which the multipole field is sampled

“n” is the multipole order, e.g., n=0 ’dipole’, n=1 ’quad’, n=2 ’sext’,...

“skew” components (rotated magnets) have similar expressions.

e.g., sextupole component in a dipole magnet.

Beam emittance \( \varepsilon \) in terms of 2nd order momenta of the particle distribution in \((x,x’)\). \(\alpha, \beta, \gamma\) are ‘Twiss parameters’.

The “rms” emittance is NOT invariant under nonlinear motion (field).

1. Consider a nonlinear transport matrix with \( M_{21} \sim b_n \neq 0 \) (nonlinear field component).

2. Beam matrix transforms through \( M \) so that \( \langle x_1'^2 \rangle = \langle x_0'^2 \rangle + Q_x x_0^2(b_n,x_0) = \gamma_x \varepsilon_{x,0} + Q_x^2 \)

From the determinant of the perturbed beam matrix we find:

\[ \frac{\Delta \varepsilon_x}{\varepsilon_{x,0}} \approx \frac{1}{2} \frac{\beta_x}{\varepsilon_{x,0}} Q_x^2(b_n) \]

This relationship sets a spec. on \( b_n \) vs. the maximum tolerated \( \Delta \varepsilon_x \).
At low beam energy, different particles energy means significant difference in velocity.

- Particles travel different distances over the same time lap → compression in straight non-dispersive channels.
- Trailing particles should have larger energy (velocity) than leading ones (same for compression in a chicane).
- “Ballistic compression” is often referred to compression without acceleration. “Velocity bunching” is more generic.

**Before entering cavity**

**Right after cavity**

**Downstream cavity**

\[
\Delta z(s) = \Delta \beta_z \cdot s
\]

**Relativistic equation for longitudinal motion**

\[
\frac{dz}{ds} = \frac{\gamma}{\sqrt{\gamma^2 - 1}}
\]

\[
\frac{dy}{ds} = -\frac{eE_{z,0}(s)}{m_0c^2} \cos(k_{rf}z + \varphi_{rf})
\]

**Compression Factor at ‘zero-crossing’**

\[
C(\varphi_{rf} = \pi / 2) \equiv \frac{\Delta z(s)}{\Delta z(s = 0)} = \left[ 1 - \frac{eV_0k_{rf}}{m_0c^2} \frac{s}{(\gamma^2 - 1)^{\frac{3}{2}}} \right]^{-1}
\]
Which, and How Many Compressors?

- **Velocity Bunching (VB):** max. $C$ is limited by “space charge” (particles repulsive Coulomb interaction), in order to preserve beam quality.
  - Presently operating Normal Conducting Photo-Injectors (LCLS, FERMI) usually do not employ VB at all.

- **Magnetic Compression (MC):** commonly with chicanes, rare dog-legs (MAX-IV SPF), proposed arcs.
  - Usually done at energies high enough to limit adverse impact of “space charge” and emitted radiation...
  - ... but too high energy is bad too (energy at first compression sets requirement for linearizer voltage).

- **Favoring Multi-Stage magnetic compression:**
  - First gentle compression can be done at relatively low energies (100-300 MeV)
  - Further compression at higher energy minimizes synchrotron radiation effects on transverse emittance
  - Potential for larger overall compression
  - Reduced sensitivity to RF jitter.

- **Favoring Single-Stage magnetic compression:**
  - Some collective effects (microbunching instability) are alleviated by single-stage compression
  - Shorter and simpler machine layout (usually not a decisive factor)
Bunch length compressors are fundamental tools for increasing the bunch peak current, e.g. for FELs.

Magnetic compressors are made of a linac (properly rf-phased) + magnetic insertion (proper sign of $R_{56}$).

Control of current profile requires linear compression, thus linearization of the compression process.

Bunch length compression changes the uncorrelated energy spread.

Bunch length compression implies peak current jitter as a function of RF parameters.

Magnetic compressors require magnetic specifications also for the beam transverse emittance.

Velocity bunching (VB) is complementary to magnetic compression (MC).
- The choice of VB + MC, one- or multi-stage MC, depends on many (inter-dependent) parameters such as: emittance, collective effects, stability, infrastructure, final application...
Homework (1/2)

You should be able to work out all of the following ones, looking to the presented slides. You are encouraged to work together, use books, and ask for help if needed (I’ll be around all night).

CAS “policy” adopted: homework are not mandatory, but your efforts in facing them will be appreciated!

1. Show that $E_z$ in a standing-wave structure (assume for simplicity $E_{z,0}$ in the fundamental accelerating mode) can be written as the superposition of two counter-propagating e.m. waves [Note: in fact, the forward-traveling component depicts the accelerating field in a real traveling-wave structure; in a standing-wave structure, the counter-propagating wave does not contribute to acceleration on average].
   -Hint: slide 9

2. Demonstrate that in a standing-wave structure (assume for simplicity $E_z$ in the fundamental accelerating mode) the effective accelerating voltage over a cell of length $[-g/2, g/2]$ is always $< E_0 g$, even if $E_z$ were ideally uniform along the gap.
   -Hint: slide 9

3. Derive the relationship $p_z = eB_y R$.
   -Hint: slide 12 + Lorentz force

4. Estimate the value of $\eta_x'$ right at the exit of a dipole magnet ($\eta_x = \eta_x' = 0$ at its entrance). What is $\eta_x'$ in the middle of a 4-dipoles C-shape chicane?
   -Hint: slide 13
5. Consider a Linac made of S-band structures, an X-band harmonic cavity, and followed by a magnetic chicane: the parameters are (refer to slides for notation): $\lambda_{\text{rf}} = 3 \text{ GHz}$, $\lambda_H = 11.4 \text{ GHz}$, $R_{56} = -41 \text{ mm}$, $E_{\text{BC}} = 280 \text{ MeV}$, $E_i = 100 \text{ MeV}$, $C = 10$. What is the peak voltage of the harmonic cavity required to linearize the compression process (assume $\phi_H$ at the decelerating crest)? What is the peak voltage and the rf phase of the S-band linac?
   - Hint: slide 11, 22

6. Consider a beam entering a 4-dipoles C-shape chicane; the parameters are (refer to slides for notation): $R_{56} = -41 \text{ mm}$, $\sigma_{u,i} = 3 \text{ keV}/280\text{MeV}$, $\sigma_{c,i} = 5.6\text{MeV}/280\text{MeV}$. What is the minimum achievable bunch length? Consider 2nd order terms for the beam transport through the chicane, but 1st order energy chirp only.
   - Hint: slide 24

7. Consider a 3.0 ps rms long Gaussian bunch with 0.1% correlated energy spread at the entrance of a symmetric double-bend achromatic (DBA) cell. Evaluate the dipole length of the DBA for achieving a total linear compression factor of 10, at the beam energy of 1 GeV, for a dipole magnetic field of 0.5 T. What is the minimum bunch length achievable, and therefore the maximum effective compression factor, if the beam initial uncorrelated energy spread is 20 keV rms?
   - Hint: slide 11-15, 24