Superconducting RF Cavities
Rama Calaga, CERN, 2016

Superconductivity & SC-RF Basics
Practical Aspects I & II

†Note: For a detailed treatment, see references (slide 2)
Some References

1. RF Superconductivity: Science, Technology, and Applications

2. SRF Conferences & Tutorials (link for SRF2015)

3. CAS (1992), USPAS (2013, 2015), JUAS (2015) ...
Outline

- RF Input, HOM Output
- Cryostat (including thermal/magnetic shields)
- RF Cavity
- Transition 2K/300K
- Beam
- Pumping line
- He input (2-4.5 K)
SC-RF, European XFEL

2.5-20 GeV electron linac, 800 SC-RF Cavities (2.1 km)
Will be one of the largest SC-RF Linear Accelerator in the world

SC-RF is the basis of practically all high energy accelerators
& w/o which ERLs probably cannot be realized
Superconductivity & RF
(Qualitative Look)
Superconductivity

A thermodynamic phase transition below $T = T_c$, a macroscopic quantum phenomena

$$H_c(T) = H_c(T_c) \left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

Can be qualitatively understood using two-fluid model $(J_n, J_s)$, where below $T_c$ electrons pair-up into cooper pairs (condensation)

Type I – Complete flux expulsion

Type II – Mixed states with flux penetration in quantized vortices

$$\phi_0 = \frac{\hbar}{2e}$$
SC-Elements, Evolution

<table>
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<tr>
<th>Element</th>
<th>Tc</th>
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<tr>
<td>Nb</td>
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</tr>
<tr>
<td>Nb3Sn</td>
<td>18</td>
</tr>
<tr>
<td>YBCO</td>
<td>90</td>
</tr>
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</table>

For Nb (type II): $B_{c1} \sim 180$ mT, $B_{c2} \sim 400$ mT
Characteristic Length Scales

**London penetration depth** – length scale over which the B-field decays in SC

**Coherence length** – distance over the which cooper pairs are correlated

\[
B_y(x) = B_0 e^{-\frac{x}{\lambda_L}}
\]

\[
\lambda_L = \sqrt{\frac{m_e}{u_0 n_s e^2}}
\]

\[
\xi_c = \frac{\hbar v_F}{\Delta}
\]

Critical Field:

\[
B_c = \frac{\Phi_0}{2\sqrt{2}\pi \lambda_L \xi}
\]

<table>
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<tr>
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<td>510</td>
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<td>YBCO</td>
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Surface Resistance

Av. power dissipated on the surface: \( P_d = \frac{1}{2} R_s H_0^2 \)

**Normal-Conductor**

\[ R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\mu_0 \omega}{2\sigma}} \]

For Copper:
\( \sigma = 5.85 \times 10^7 \text{ S/m} \) (300 K)
\( \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \)
\( \omega = 2\pi \times 1 \text{ GHz} \)
\( \delta = 2 \mu\text{m} \)

\( R_s = 8.2 \text{ m\Omega} \)

(Don't forget anomalous skin effect)

**Super-Conductor**

\[ R_s = \frac{1}{(\sigma_n - i\sigma_s)\lambda_L} = \frac{1}{2} \sigma_n \omega^2 \mu_0 \lambda^3 \]

For Niobium:
\( \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \)
\( \omega = 2\pi \times 1 \text{ GHz} \)
\( \xi = 39 \text{ nm} \)
\( \lambda_L (T=0K) = 36 \text{ nm} \)

\( R_s \sim n\Omega \)
Surface Resistance from BCS

Mattis-Bardeen approx (BCS theory):

\[ R_{BCS} = A \frac{\omega^2}{T} e^{-\frac{\Delta}{k_B T}} \]

where \( \Delta \) is the energy gap.

For Niobium:

\[ R_{BCS} \approx 2.4 \times 10^{-4} \left( \frac{f \text{ [MHz]}}{1500} \right)^2 \frac{1}{T} e^{-17.67/T} \]

There is also temperature independent residual resistance which is lower limit.

\[ R_s = R_{BCS} + R_{res} \]

Note: Cryogenic to wall plug power
Cylindrical Cavity

Gap Length = L

Standing waves of TM & TE

\[ \omega_{mnp} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{\left( \frac{p_{mn}}{r} \right)^2 + \left( \frac{p \pi}{l} \right)^2} - (TM) \]

\[ \omega_{mnp} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{\left( \frac{p_{mn}'}{r} \right)^2 + \left( \frac{p \pi}{l} \right)^2} - (TE) \]

\[ E_z = E_0 J_0(\omega_0 r / c) \cos(\omega_0 t) \]

\[ H_\phi = -\frac{1}{\mu_0 c} E_0 J_0(\omega_0 r / c) \sin(\omega_0 t) \]

\[ \omega = \frac{2.405 \ c}{R} \]

1\textsuperscript{st} mode (m=0, n=1, p=0) suited for acceleration with field lines uniform over z

The frequency is only dependent on radius and
Mode Spectrum Vs Geometry

Cylindrical Cavity

\[(fa)^2 \times 10^{-18}\] vs \[(a/L)^2\]
Figures of Merit I

Voltage:

\[ V_{acc} = \left| \int_{z=0}^{z=l} E_z e^{i\omega_0 z/c} \, dz \right| \]

Transit Time:

\[ T = \frac{\int_{0}^{l} E_0 e^{i\omega z/c} \, dz}{\int_{0}^{l} E_0 \, dz} \]

\( (\beta c T = \text{distance covered in a RF period}) \)

Stored Energy:

\[ U = \frac{1}{2} \varepsilon_0 \int_{V} |\vec{E}|^2 \, dv = \frac{1}{2} \mu_0 \int_{V} |\vec{H}|^2 \, dv \]
Figures of Merit II

Quality Factor:
\[
Q_0 = \frac{\omega_0 U(t)}{P_d(t)} \quad U(t) = U_0 e^{-t/\tau}
\]

Shunt Impedance:
\[
R_{\text{shunt}} = \frac{V^2}{P_d} \quad \frac{R_a}{Q_0} = \frac{V^2}{\omega U}
\]

Geometric Factor:
\[
G = R_s Q_0 = \frac{\omega_0 \mu_0 \int |\vec{H}|^2 dv}{\int |\vec{H}|^2 ds}
\]

Power dissipated:
\[
P_{\text{walls}} = \frac{V_c^2}{R/Q \cdot G} R_s
\]
Cavity Design, TM-Class

Standard Criteria:

- Minimize peak surface fields (E, B)
- Optimum R/Q based upon application
- Optimum mechanical stiffness (tuning vs. de-tuning)
- Strong cell-to-cell coupling (multi-cell)

Note: No single optimum shape for everything

½-cell parametrization
(P. Pierini et al.)

Example scan vs iris ratio

Note: No single optimum shape for everything
TEM Class Resonators

Another important class are using \textit{uniform} transmission lines, \( \lambda/4 \) being one of the simplest form.

\[ V_z = V_0 \sin(kz) \]
\[ I_z = I_0 \cos(kz) \]

Line Impedance: \( Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a} \quad \rightarrow \quad R = 4 \frac{Z_0}{\pi} \)

Geometric Factor: \( G = \frac{2\pi \eta}{\lambda} \frac{\ln b/a}{a^{-1}+b^{-1}} \)

Widely used in low velocity (protons, ions) applications for compactness.
Cavity Design, Numerical

Almost all practical applications require a deviation from idealized cavity. Therefore, spatial discretization of the structure and solve Maxwell's equation numerically.

Frequency Domain (eigenvalue problem)
Time Domain (transient response)

Different methods: Finite (difference, integration, element)

Generally used (but not comprehensive):
2D: Superfish, SLANS, ABCI
3D: CST, HFSS, ACE3P, GdfidL

Superfish Mesh
400 MHz (½-cavity)
Multi-Cells (mainly for Linacs)

To improve the “real-estate” gradient & ancillary equipment (couplers, flanges, warm-cold transitions, etc…) it is often efficient to go to N-coupled cells

\[
\left(\frac{\omega_n}{\omega_0}\right)^2 = 1 + 2k_c \left[1 - \cos\left(\frac{n\pi}{N}\right)\right]
\]

\[
a = \frac{N^2}{k_c}
\]
Example 5-cell cavity at 700 MHz:
In the above, 8-cavity cryomodule: 0.16 GeV, 16.5 m
For 20 GeV, LINAC-FEL → 2 km

Homework:
Calculate the real estate length assuming only single cells/cavity
Practical Aspects I
(Measurements, Freq. Detuning, HOMs)
Field Measurements

Standard practice to use Cu-models for fabrication trials & RF measurements

Slaters thoerem: $\Delta \omega / \omega \propto \Delta U / U$ (for small perturbations)

Bead inside a cavity:

$$\frac{\delta \omega}{\omega_0} = \begin{cases} 
- \frac{\pi r^3}{U} (\epsilon_0 \frac{\epsilon_r + 2}{\epsilon_r - 1} E_0^2) & : \text{dielectric} \\
- \frac{\pi r^3}{U} (\epsilon_0 E_0^2 - \frac{\mu_0}{2} H^2) & : \text{metal}
\end{cases}$$

Vector Network Analyzer $\rightarrow$ S-parameters

$$S_{21} = \frac{2\sqrt{\beta_1 \beta_2}}{(1 + \beta_1 + \beta_2) + i Q_0 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

$\beta_1, \beta_2$ are coupling factors for antenna's, assuming they are small:

$$\frac{\delta \omega}{\omega_0} \approx - \frac{1}{2 Q_L} tan(\phi)$$

Example setup of 700 MHz, 5-Cell Cavity

VNA

Labview to control motor
Detuning from Lorentz Force

SC-Cavities are operated typically with narrow bandwidth. It implies careful control/tracking of frequency.

Radiation pressure from the very high electro-magnetic fields will distort the cavity shape and therefore the frequency.

\[ \delta f \propto \frac{f_0}{4U} \int \frac{(\varepsilon_0 \vec{E}^2 - \mu_0 \vec{H}^2)}{\delta V} \, dV \]

- **Inductive**
- **Capacitive**
Detuning From External Forces

External noise can be transferred to cavity via the cryostat (Microphonics)

Mitigation

Tuning system: Mechanical - slow and/or electro-mechanical - moderate
RF feedback - fast, BW limited ($\Delta\omega$)
RF Power

\[ P_f = \frac{R_s}{4\beta} \left[ \frac{(1 + \beta)^2 V^2}{R_s^2} + \left( \frac{V}{X} - I_b \right)^2 \right] \]

\[
Z(\omega) = \frac{R/Q \cdot Q_L}{1 + i \tan \psi} \quad Q_L = \frac{Q_0}{1 + \beta} \quad X = \frac{R}{Q} \frac{\omega}{\Delta \omega} \quad \tan(\psi) = 2 Q_L \frac{\Delta \omega}{\omega}
\]

Assuming no beam-loading (ERL), one can show

\[ P_f = \frac{V^2}{R/Q} \cdot \frac{\Delta \omega}{\omega} \]

To maintain a constant gap voltage, the input power scales linearly with detuning

\[ Q_{opt} = \frac{1}{2} \cdot \frac{\omega}{\Delta \omega} \]

\[ f_{rf} = 800 \text{ MHz} \]
Higher Order Modes

Beyond the fundamental (accelerating) mode, there exists infinite eigenmodes

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{\left(\frac{p_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} - \text{(TM)}$$

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{\left(\frac{p'_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} - \text{(TE)}$$

They can be excited by the beam which typically has a wide frequency range ($1/\sigma_z$) depending on the synchronism condition.

Multi-Cells have N-times the number of HOMs (passbands)

Strong damping of the HOMs is often key to aspect to reach high currents and beam quality.
Dispersion Curves

Approximation of an infinitely periodic structure
Modes with phase velocity $= \beta c$ are strongly excited (also high $R/Q$)

$\nu_g = \frac{d\omega}{dk}$

$\omega = \omega_k = \frac{\phi c}{2\pi L}$
Loss Factor

Bunch traversing the structure losses energy and leaves behind a wakefield into parasitic modes, which can be characterized by loss factor ($k$).

For pulsed linacs (FELs), resulting energy spread & emittance growth.

High current CW (storage rings, ERLs) limited by power.

Energy loss:  $\Delta U = k_n q^2$

$k_n = \frac{\omega}{2} \cdot \frac{R}{Q}$ (loss factor/mode assuming TM-like)

Analogous transverse loss factor  
$k_t \rightarrow$ emittance growth.
HOM Losses

Non-Resonant Case:

\[ P_{HOM} = (\sum k_n - k_0) \cdot q \cdot I_b \]

Resonant Case:

\[ P_{HOM} = I_b^2 \frac{R}{Q} \cdot Q_L \cdot F_n^2 \]

LHC Filling scheme

Homework:
Calculate energy spread, HOM power/cavity for 6-pass ERL
q=1nC, f_{rev} = 1 MHz, k = 1 V/pC

Damping essential for SC-Cavities
HOM Power Contd.

In reality one integrated numerically the HOM impedance over the bunch spectrum/filling scheme

LHC example with an 800 MHz HOM (1 \( \Omega \))
HOM Power, ERLs

\[ P_{\text{avg}} = k_L Q_b I_a \]

Power to be extracted but **NOT** into 2K
HOM Damping & Extraction

Notch Filters  $\rightarrow$ Narrow-band & targeted damping
Waveguides  $\rightarrow$ Higher frequencies more suitable
Ferrites  $\rightarrow$ Broadband room temp

Main objective is have high impedance for the fundamental mode while high transmission for HOMs
Practical Aspects II
(Surface Treatment & Cold Measurements)
SC Cavity Performance Limitations

Graphic: K. Saito
Multipacting

Resonant electron multiplication of electrons from the cavity surface impacting back in integer RF cycles with a surface emission coefficient (SEY) > 1

Consequence
An electron avalanche of electrons absorbing all RF power, leading a thermal breakdown

Mitigation
It is field, phase and SEY dependent.
RF conditioning and/or geometrical shaping to suppress the resonant behavior (ex: elliptical shape)
Field Emission

An electron emitting site on the cavity surface (due to impurities, surface defects etc..) with a sufficient local field enhancement ($10^2 - 10^3$).

They get accelerated/bent by the strong RF field and impact elsewhere on the surface with the typical signature of strong x-rays leading to vacuum and/or thermal breakdown ("hot zones")

Explained by modified F-N theory ($\beta$-enhancement factor)

$$j = \frac{A \cdot \beta^2 \cdot E^2 \cdot e^{-B \Phi^{3/2}}}{\Phi}$$

Mitigation by surface smoothness, cleanliness (HPR + Cleanroom) and RF conditioning
Surface Treatment(s)

Cavity surfaces are typically formed by mechanical means which leave a damaged cortical layer (impurities, inclusions, hydroxides, oxides...)

Standard practice (after degreasing) is to remove 100-200 $\mu$m by oxidation & reduction:

- Buffer Chemical Polishing ($\text{HNO}_3$, $\text{HF}$, $\text{H}_3\text{PO}_4$)
- Electro-Polishing ($\text{HF}$, $\text{H}_2\text{SO}_4$) – roughness ~micron level
- Mechanical Barrel Polishing

\[6 \text{Nb} + 10 \text{HNO}_3 \rightarrow 3 \text{Nb}_2\text{O}_5 + 10 \text{NO} + 5 \text{H}_2\text{O}\]
\[\text{Nb}_2\text{O}_5 + 10 \text{HF} \Leftrightarrow 2 \text{NbF}_3 + 5 \text{H}_2\text{O}\]
\[\text{Nb} \rightarrow \text{Nb}^{5+} + 5 \text{e}^- \rightarrow \text{Nb}_2\text{O}_5\]
\[2\text{H}^+ + 2\text{e}^- \rightarrow \text{H}_2\]

C. Antoine Arxiv:1501.03343

G. Ciovati, USPAS Lecture, 2015


I-V Curve (EP)
Heat Treatment

Substantial Hydrogen concentration is shown to yield Q-disease due to Hydrogen dissolution into the Niobium bulk during chemistry Heat treatment (UHV) at 600-800 °C (10-24 hrs) – Removal of the H2 Niobium is a strong getter above 250 °C → Requires a light chemistry after + (high pressure) water rinsing

CERN UHV Furnace

5-Cell Cavity

H2

Start chauffage (200 degrés/heures), pression = 2,2.10^6 mbar

Début du palier de 650 degrés

Pression limite à 90 degrés = 9,0.10^-8 mbar

Fin du palier de 650 degrés

Pression = 2,0.10^-7 mbar

2,40
12,28
14,29
15,28
16,78
18,00
18,28
26,10
28,10
30,10
32,10
39,10
40,10
41,10
43,10
55,05
57,05
HPR & Clean Room

High Pressure Rinsing ($\rho=18 \ \text{M}\Omega\text{cm}$) and clean room assembly (ISO4) have shown great success in suppressing field emission & improve cavity performance.

Additional low temp baking ($120 \ \text{°C}$) shown to improve high field Q-drop
SC-Cavity Measurements

Fun Generator → TX → Directional Coupler(s) → Vertical cryostat → Transmitted Power

Forward/Reflected Power

Estimate Q0 vs field from power & field decay measurements

Example single cell (Rs ~ 0.5 nΩ)
Requires very precise setup/calibration
High Q, High Gradient

XFEL example: W. Singer, STTIN2010
New Paths

Looking beyond state-of-the-art Niobium Nitrogen doping, Nb3Sn, Multi-layer..

Controlled cool-down for better flux expulsion and good magnetic screening
Finally, Some SC-Cavities in Real Life

\[ \beta = \frac{v}{c} \]

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Facility</th>
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<td>98</td>
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</tr>
<tr>
<td>101</td>
<td>CESR</td>
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<td>KEKB</td>
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<tr>
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<td>1.5</td>
<td>CEBAF</td>
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Note: The axes are only qualitative & list is not comprehensive.