CERN

Materials & Properties: Mechanical Behaviour

C. Garion, CERN TE/VSC

CERN Microcosm exhibition

A dream of engineer…

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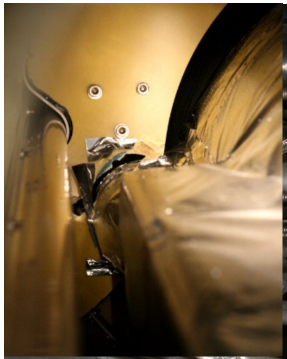

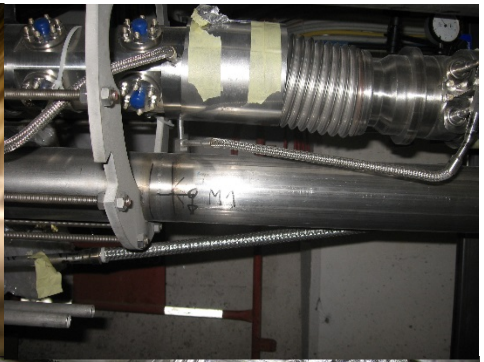

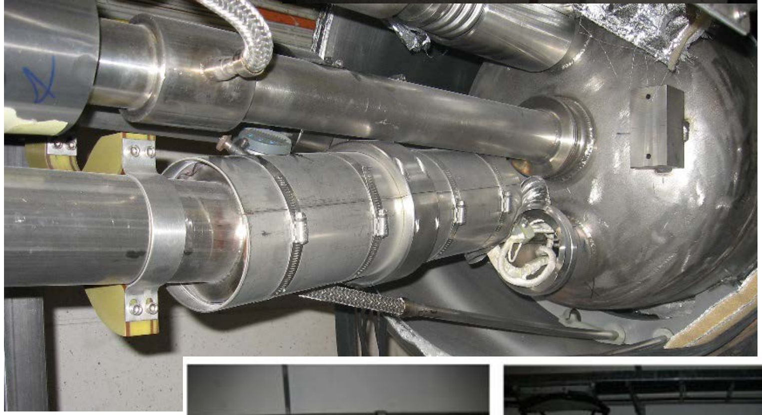

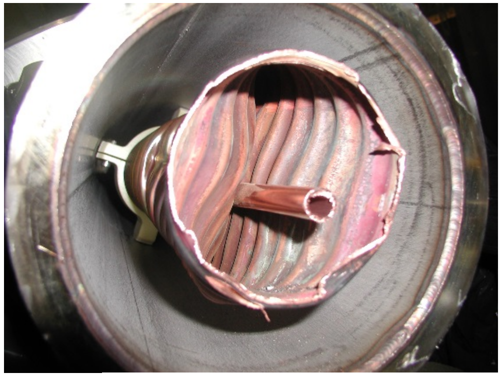

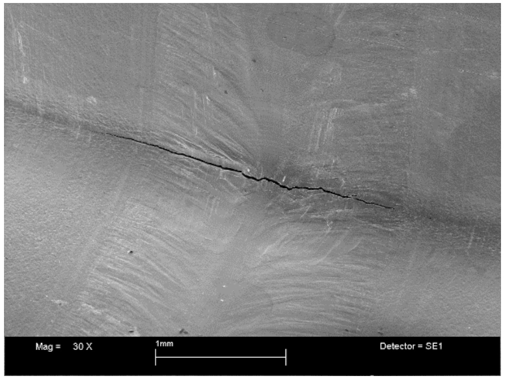

… the hard reality!!!

Plan

1. Material modelling
   a. Basic notions of material behaviours and strength
   b. Stress/strain in continuum mechanics
   c. Linear elasticity
   d. Plasticity
      i. Yield surface
      ii. Hardening
   e. Continuum damage mechanics
   f. Failure mechanics
2. Structural mechanical analysis
   a. Vacuum chamber
      i. Loads on a chamber
      ii. Equilibrium equations
      iii. Stress on tube
      iv. Instability (buckling)
   b. Vacuum system as mechanical system
      i. Support – unbalanced force
      ii. Stability (column buckling)
3. Selection criteria of materials
   a. Figures of merit of materials
   b. Some material properties
   c. Some comparisons based on FoM

4. Conclusion

Plan

1. **Material modelling**
   a. **Basic notions of material behaviours and strength**
   b. **Stress/strain in continuum mechanics**
   c. **Linear elasticity**
   d. **Plasticity**
      i. **Yield surface**
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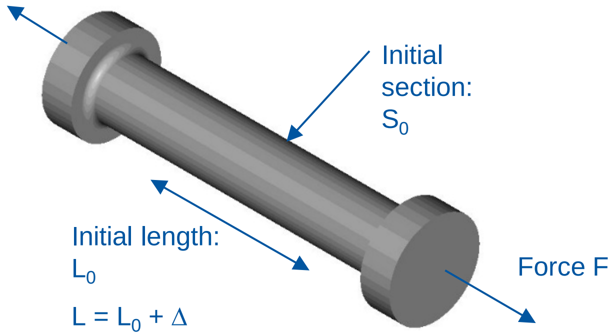
Basic Notions of Material Strength



Stress = Force/Section

σ[MPa] = F[N]/S[mm^2]

Strain = Displacement/length

ε = Δ[mm]/L[mm]

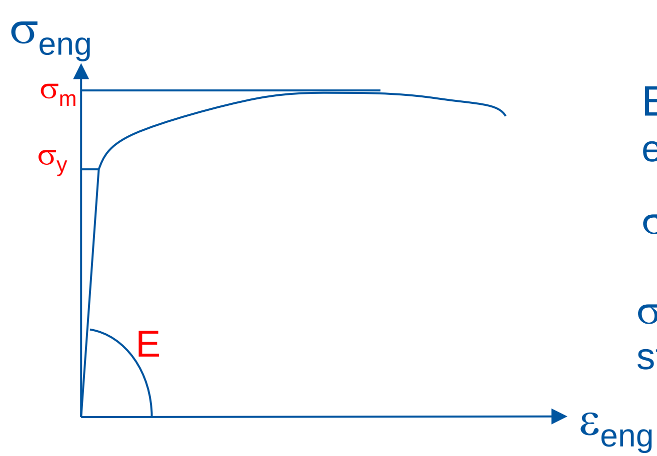
Basic mechanical properties from tensile tests

Nominal or engineering stress:

$$\sigma_{eng} = \frac{F}{S_0}$$

Nominal strain: $\varepsilon_{eng} = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$

Poisson coefficient: $\nu = -\frac{\varepsilon_{trans}}{\varepsilon_{axial}}$

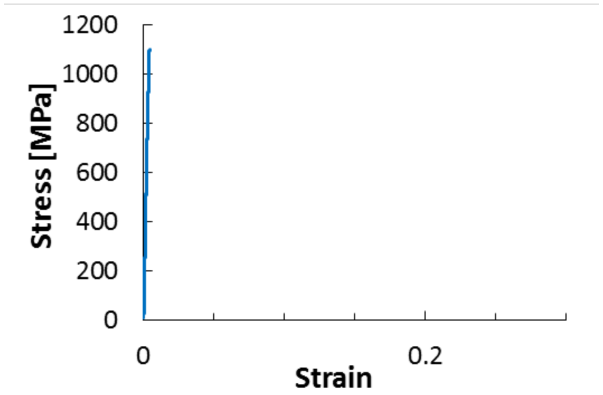


E: Young modulus (linear elastic behavior)

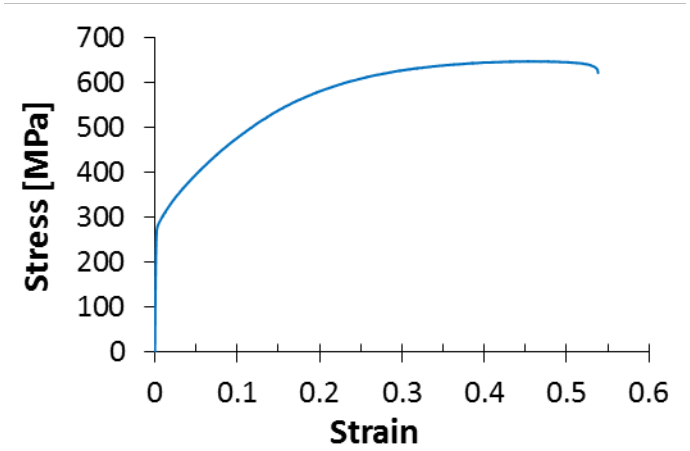
σ_y (R_p): Yield strength

σ_m (R_m): Tensile strength

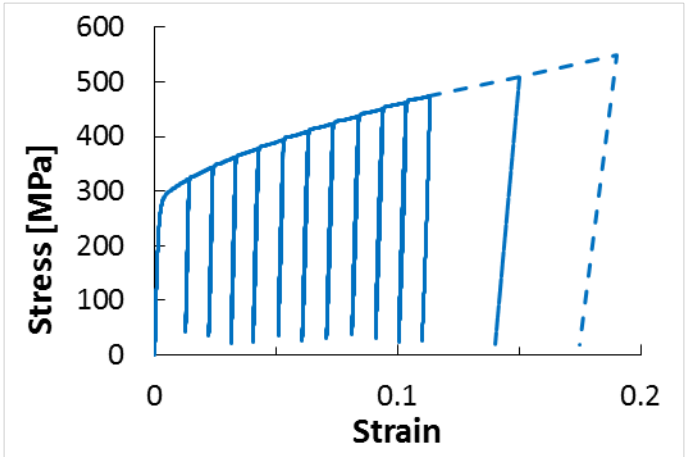
Basic Material Behaviours



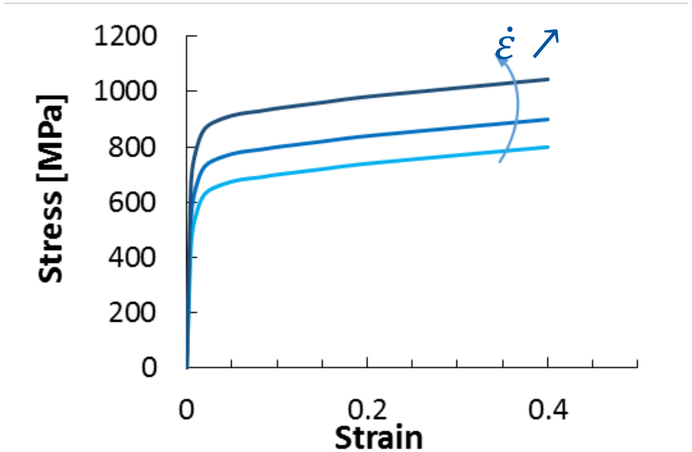
Elastic brittle behaviour



Elastic plastic behaviour



Elastic plastic damageable behaviour

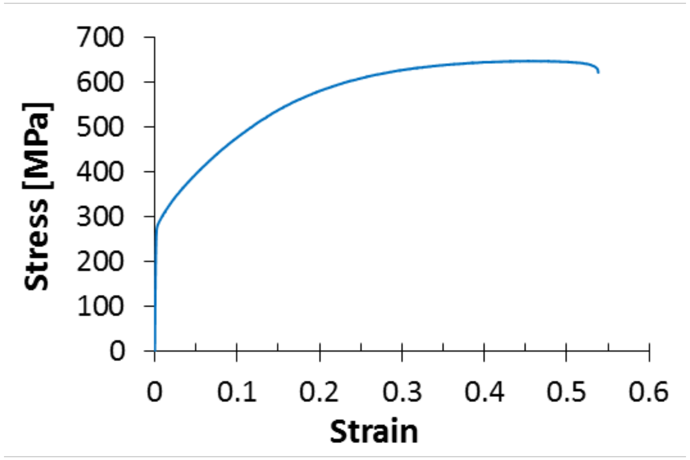


Elastic viscoplastic behaviour

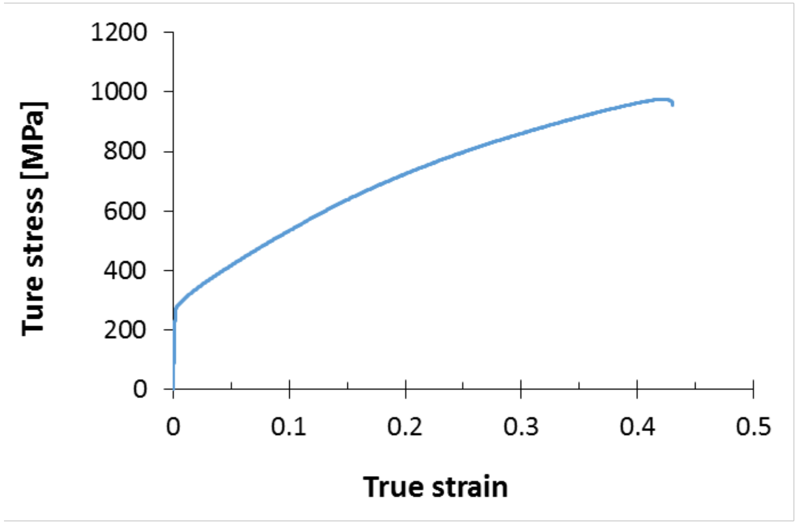
True Strain/Stress vs Engineering Strain/Stress

$$d\varepsilon = \frac{dl}{l} \Rightarrow \varepsilon = ln(1 + \varepsilon_{eng})$$

$$\sigma = \frac{F}{S} \cong \sigma_{eng} \cdot (1 + \varepsilon_{eng})$$

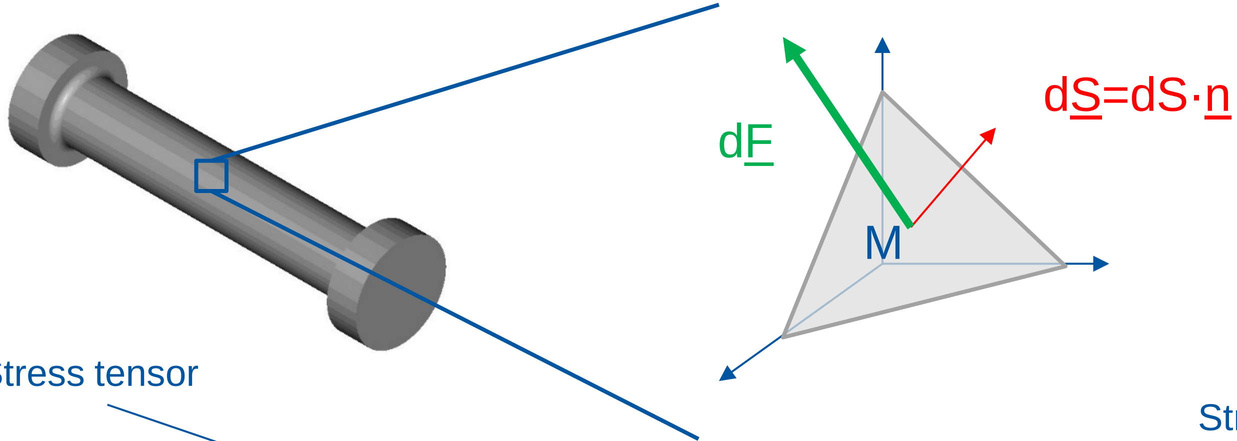


Engineering stress-strain curve



True stress-strain curve

Stress in Continuum Mechanics



$$\underline{T}(\underline{M},\underline{n}) = \lim_{dS\to 0}\frac{d\underline{F}}{dS}$$

Stress vector

Stress tensor

$$\underline{T}(\underline{M},\underline{n}) = \underline{\underline{\sigma}}(\underline{M})\cdot\underline{n}$$

Normal stress: $\sigma_n = \underline{n}\cdot\underline{\underline{\sigma}}(\underline{M})\cdot\underline{n}$

σ_n>0: tensile; σ_n<0: compression

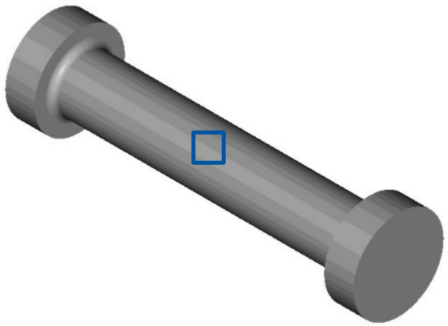
Shear stress: $\tau = \underline{\underline{\sigma}}(\underline{M})\cdot\underline{n} - \sigma_n\cdot\underline{n}$

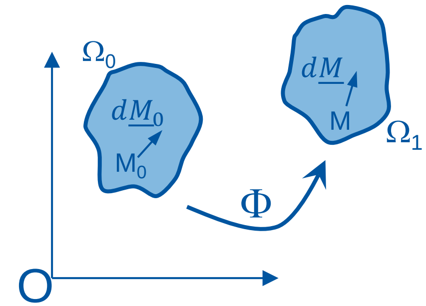
The stress tensor is symmetric: $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$

$$\rightarrow \exists\ (\underline{p}_1,\underline{p}_2,\underline{p}_3),\underline{\underline{\sigma}} = \begin{pmatrix}\sigma_I & 0 & 0\\ 0 & \sigma_{II} & 0\\ 0 & 0 & \sigma_{III}\end{pmatrix}_{(\underline{p}_1,\underline{p}_2,\underline{p}_3)}$$

$\underline{\underline{\sigma}} = \sigma_h\underline{\underline{I}} + \underline{\underline{\sigma}}^D$ with $\sigma_h = \frac{1}{3}tr\left[\underline{\underline{\sigma}}\right]$ the hydrostatic stress, and $\underline{\underline{\sigma}}^D$ the deviatoric stress tensor.

Strain in Continuum Mechanics





$$\underline{M} = \Phi(\underline{M_0})$$

$$d\underline{M} = \frac{\partial \underline{\Phi}(\underline{M_0})}{\partial \underline{M_0}} \cdot d\underline{M_0} = \underline{\underline{F}}(\underline{M_0}) \cdot d\underline{M_0}$$

$\underline{\underline{F}}$ is the deformation gradient.

Green Lagrange deformation: $\underline{\underline{E}}(\underline{M_0}) = \frac{1}{2} \cdot \left[\underline{\underline{F}}^T(\underline{M_0}) \cdot \underline{\underline{F}}(\underline{M_0}) - \underline{\underline{I}}\right]$

Considering the displacements: $(\underline{M} = \underline{M_0} + \underline{u}(\underline{M_0}))$

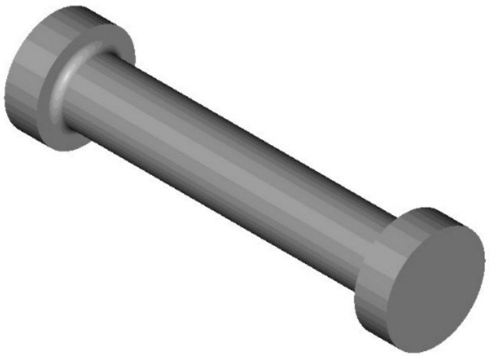
$$\underline{\underline{E}}(\underline{M_0}) = \frac{1}{2} \cdot \left[\frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}} + \left(\frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}}\right)^T + \left(\frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}}\right)^T \cdot \frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}}\right]$$

For small perturbations ($\|\underline{u}\| \ll \|\underline{M_0}\|$):

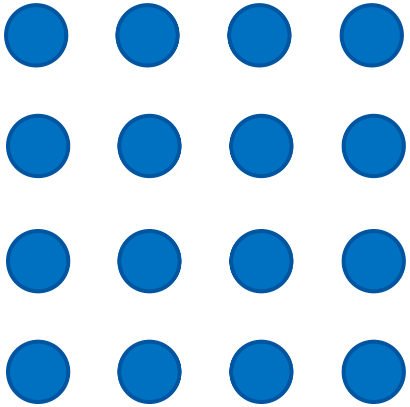
$$\underline{\underline{\varepsilon}}(\underline{u}) = \frac{1}{2} \cdot \left[\frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}} + \left(\frac{\partial \underline{u}(\underline{M_0})}{\partial \underline{M_0}}\right)^T\right]$$

Energy variation: $d\mathbb{E} = \int_\Omega \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}}$

How material behaves?

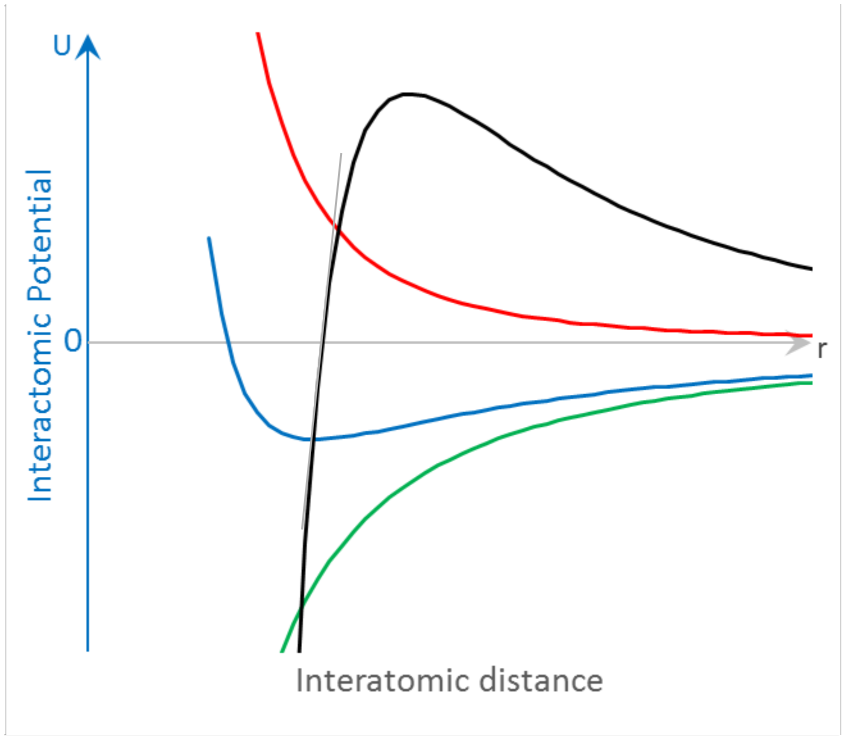


How atoms interact?



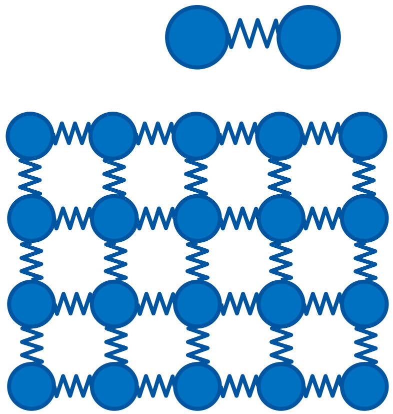
Linear Elasticity

The interatomic potential is the sum of 2 contributions:



It presents a minimum, corresponding to the free equilibrium of the atoms. U0 is the bonding energy.

The interactomic force is defined by $F = \frac{dU}{dr}$ and can be written: $F = k \cdot r$



This can be generalized in the form (Hooke's law): $\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e$ with $\underline{\underline{\underline{\underline{C}}}}$ the stiffness tensor.

Linear Isotropic Elasticity

$\underline{\sigma} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e \quad \Longleftrightarrow \quad \underline{\underline{\varepsilon}}^e = \underline{\underline{\underline{\underline{S}}}} : \underline{\underline{\sigma}}$ with $\underline{\underline{\underline{\underline{S}}}}$ the compliance tensor

In the worst case, $\underline{\underline{\underline{\underline{C}}}}$ has 21 independent parameters!

For homogeneous isotropic material, they can be reduced to two independent parameters !

$$\underline{\underline{\sigma}} = \lambda \cdot tr\left[\underline{\underline{\varepsilon}}^e\right]\underline{\underline{I}} + 2\mu\underline{\underline{\varepsilon}}^e$$

$$\underline{\underline{\varepsilon}}^e = \frac{1+\nu}{E}\underline{\underline{\sigma}} - \frac{\nu}{E} tr\left[\underline{\underline{\sigma}}\right]\underline{\underline{I}}$$

λ and μ are the Lamé coefficients. μ, sometimes denoted G, is the shear modulus.

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

$$E = \mu\frac{3\lambda+2\mu}{(\lambda+\mu)} \qquad \nu = \frac{\lambda}{2(\lambda+\mu)}$$

$$k = \lambda + \frac{2\mu}{3} = \frac{E}{3(1-2\nu)}$$

Bond type	Typical range Young modulus [GPa]
Covalent	1000
Ionic	50
Metallic	30-400
Van der Waals	2

	Aluminium	Copper	Stainless steel	Titanium
E [GPa]	70	130	195	110
ν	0.34	0.34	0.3	0.32

Elastic parameters at room temperature of materials commonly used for UHV applications

Thermal Linear Isotropic Elasticity

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e = \underline{\underline{\underline{\underline{C}}}} : \left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^{th}\right)$$

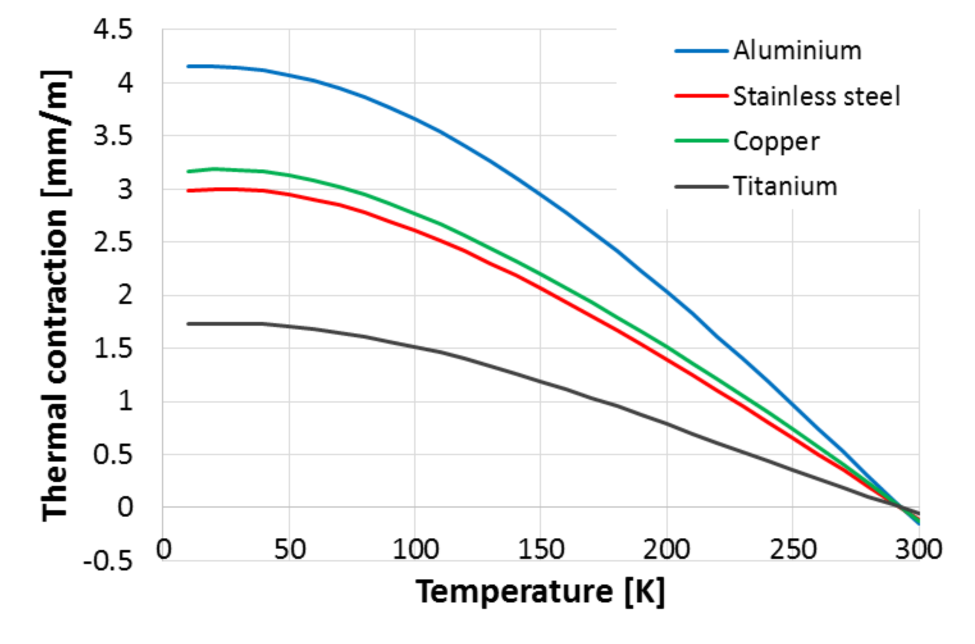
$\underline{\underline{\varepsilon}}^{th}$ is the thermal strain tensor.

$$d\underline{\underline{\varepsilon}}^{th} = \alpha(T) \cdot dT \cdot \underline{\underline{I}}$$

$$\underline{\underline{\varepsilon}}^{th} = \alpha \cdot \left(T - T_{ref}\right) \cdot \underline{\underline{I}}$$

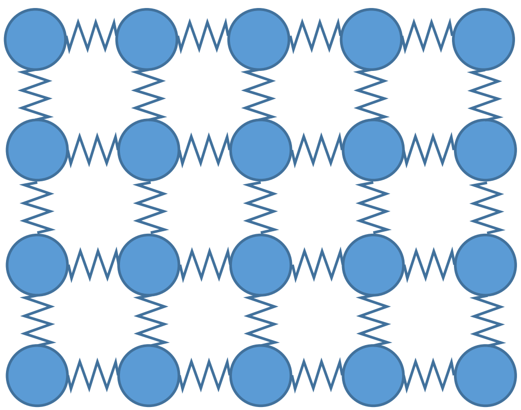
α is the coefficient of thermal expansion (CTE).

In simplified 1D, $\sigma = E \cdot \left(\varepsilon - \varepsilon^{th}\right)$
$= E \cdot \left(\varepsilon - \alpha \cdot \Delta T\right)$

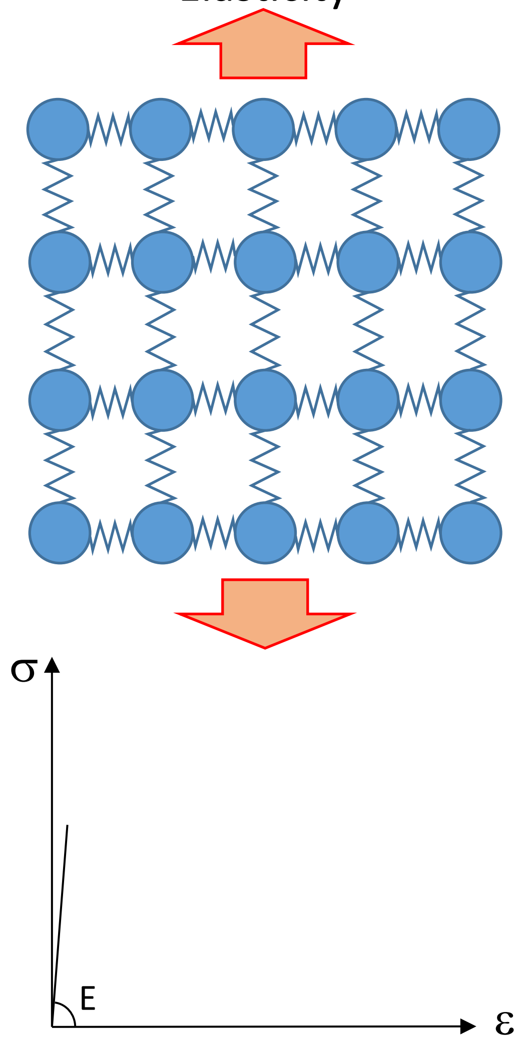


	Aluminium	Copper	Stainless steel	Titanium
α [$10^{-6} \cdot K^{-1}$]	22	17	16	8.9

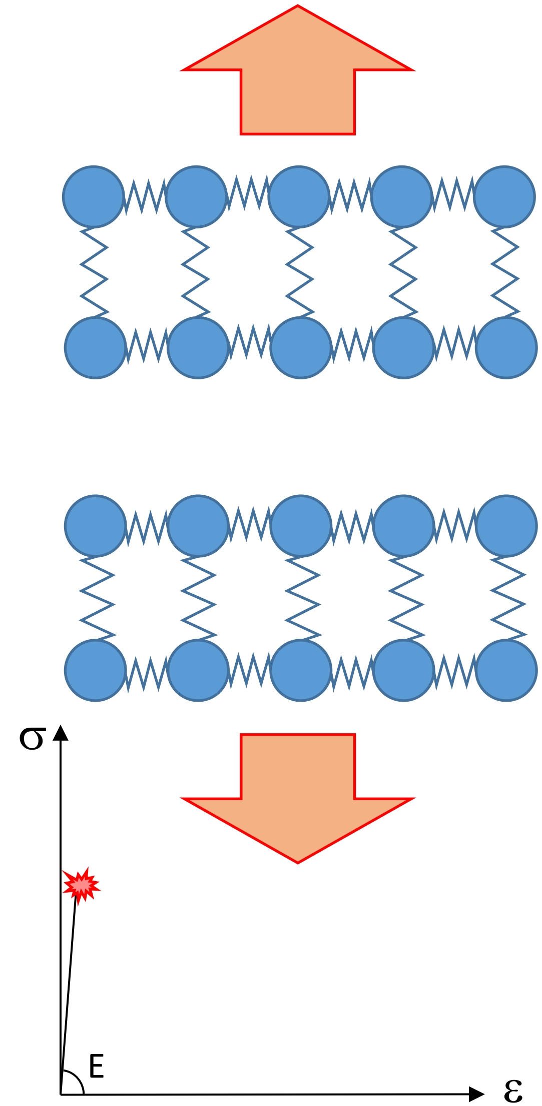
Material in free state



Elasticity

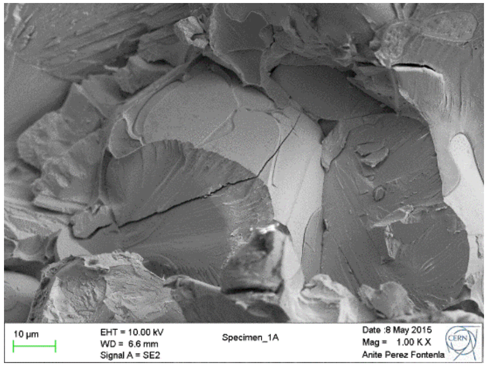


Brittle failure



Strength of Brittle Material

Damage mechanism: Cleavage, intergranular fracture



Fracture surface of heavy tungsten alloy at 77K

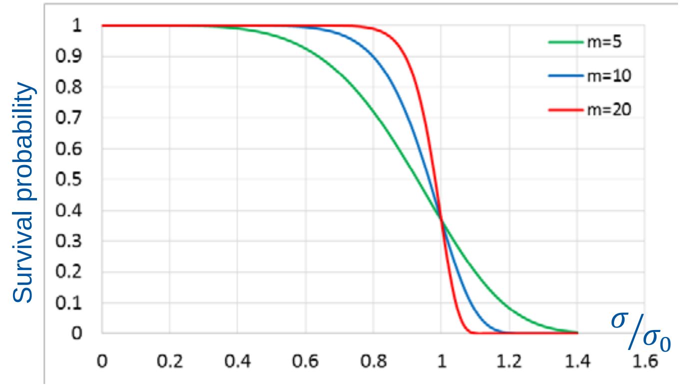
Material is very sensitive to stress concentration and therefore the material strength strongly dependent of the initial defects.

The strength of the material is represented by the **Weibull's law**, defining the survival probability:

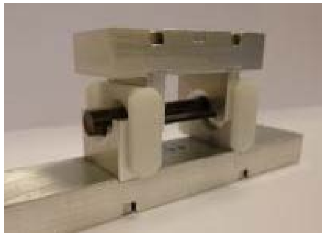
$$P_S^{\Omega} = exp\left(-\frac{1}{V_0}\int_{\Omega}\left(\frac{\sigma_{eq}}{\sigma_0}\right)^m d\Omega\right)$$

m: shape parameter

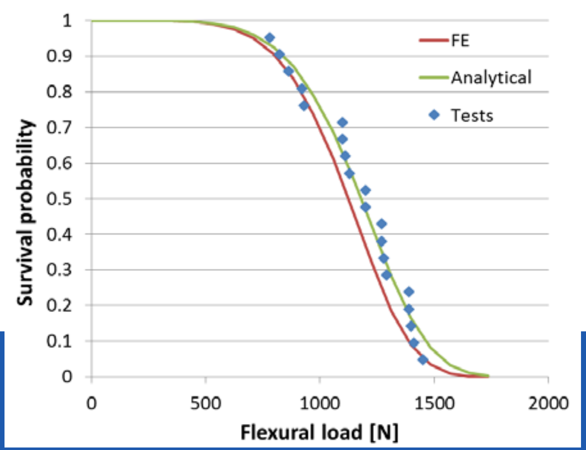
$$\sigma_{eq} = \max(\sigma_I, \sigma_{II}, \sigma_{III}, 0)$$

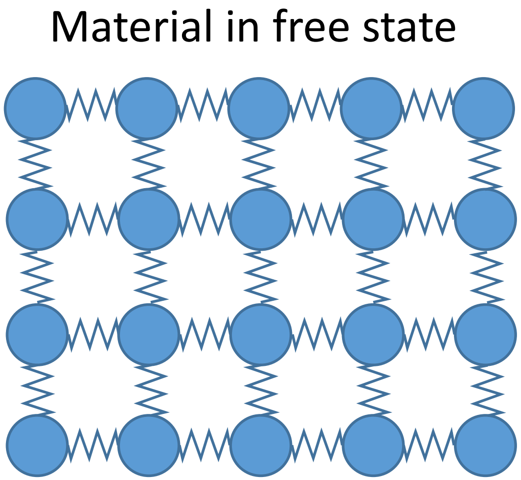


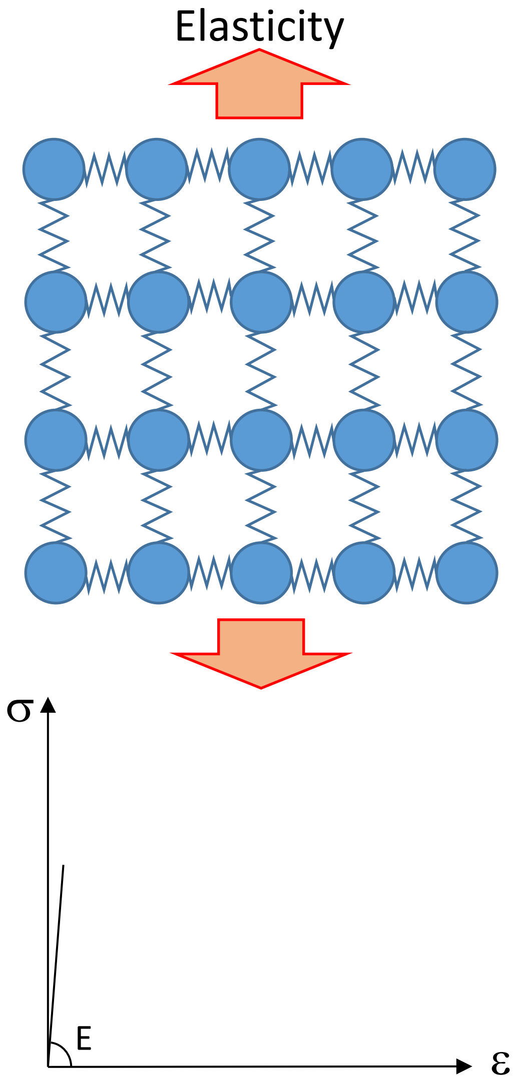
Example on glassy carbon:

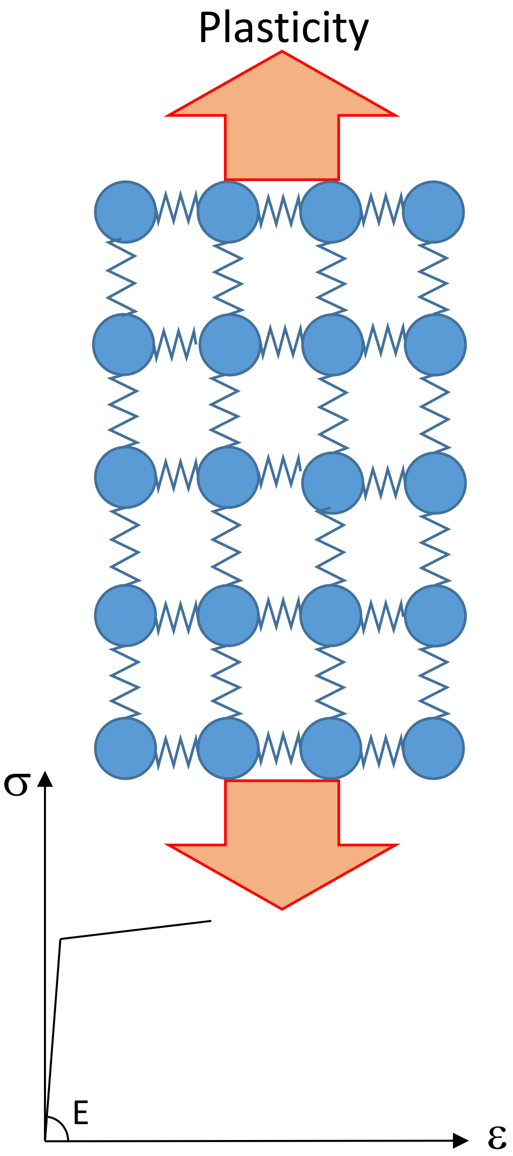

4 points bending test on rods

	Average strength [MPa]	Standard deviation [MPa]	Weibull shape parameter	Weibull scale parameter [MPa]
Flexure	206	37	5.6-6.3	375-416
Compression	1012	73	13.5-14.6	1587-1644




Material in free state

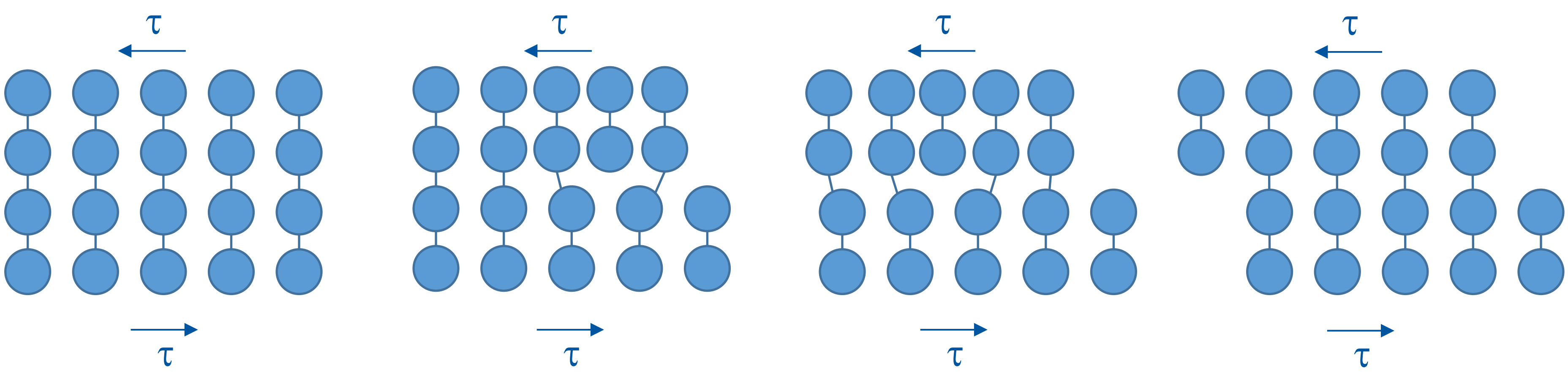

Elasticity
σ
E
ε


Plasticity
σ
E
ε

Plasticity

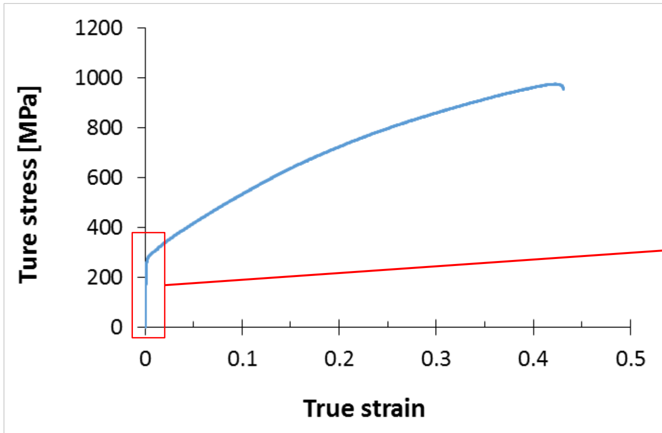
Material with plasticity: irreversible plastic deformation before rupture

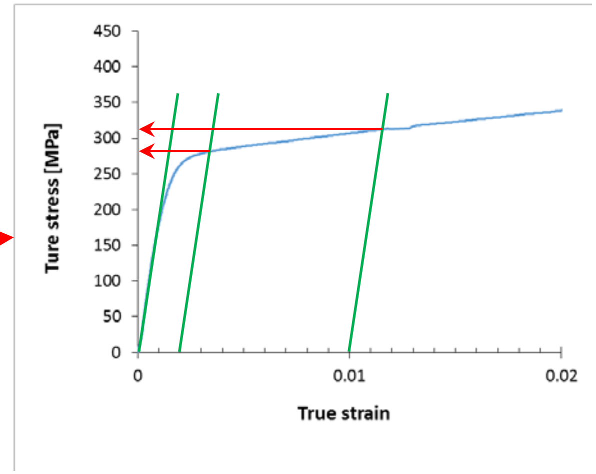
Mechanism:

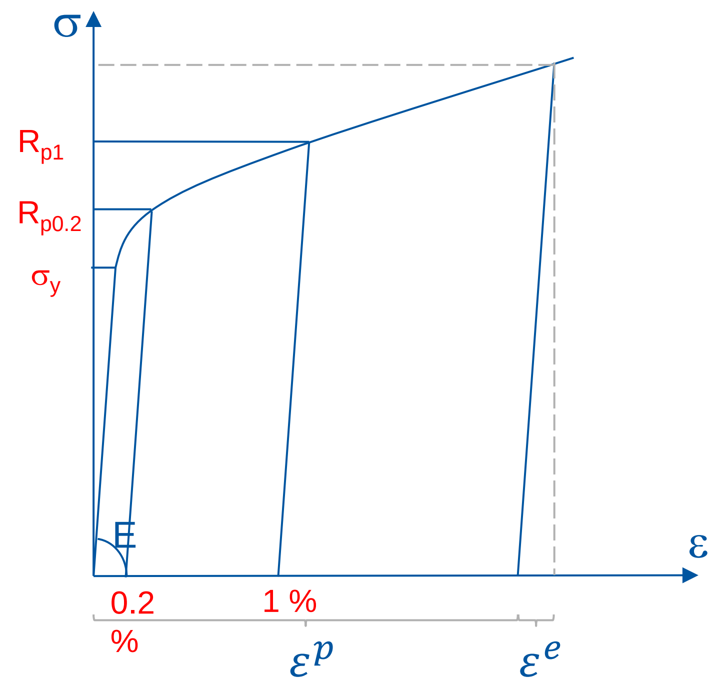


The plastic deformation is associated to the dislocation motions (slip under shear stress).

Plasticity







$R_{p0.2}$: 0,2 % proof strength

R_{p1}: 1 % proof strength

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

$\sigma < \sigma_y$ $(\sigma - \sigma_y < 0)$: Elasticity

In practice, we consider usually that σ_y= $R_{p0.2}$

Plasticity – Yield Surface

Yield surfaces are defined by:

- $f\left(\underline{\underline{\sigma}}, \sigma_y, \dots\right) < 0$: elasticity
- $f\left(\underline{\underline{\sigma}}, \sigma_y, \dots\right) = 0$: plasticity

Isotropic criteria:

- **Tresca**

the maximum shear stress, as a function of the principal stresses, is given by: $\tau_{max} = Max(\sigma_i - \sigma_j)$

Tresca's yield surface: $f\left(\underline{\underline{\sigma}}, \sigma_y\right) = Max(\sigma_i - \sigma_j) - \sigma_y$

Initial Tresca's criteria: $Max(\sigma_i - \sigma_j) - R_p = 0$

- **Von Mises**

Based on the distortional elastic energy $\propto tr\left[\underline{\underline{\sigma}}^D \cdot \underline{\underline{\sigma}}^D\right] = \underline{\underline{\sigma}}^D : \underline{\underline{\sigma}}^D$

$$f\left(\underline{\underline{\sigma}}, \sigma_y\right) = \sqrt{\frac{3}{2}\left(\underline{\underline{\sigma}}^D : \underline{\underline{\sigma}}^D\right)} - \sigma_y$$

In principal stress space: $\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} - \sigma_y$

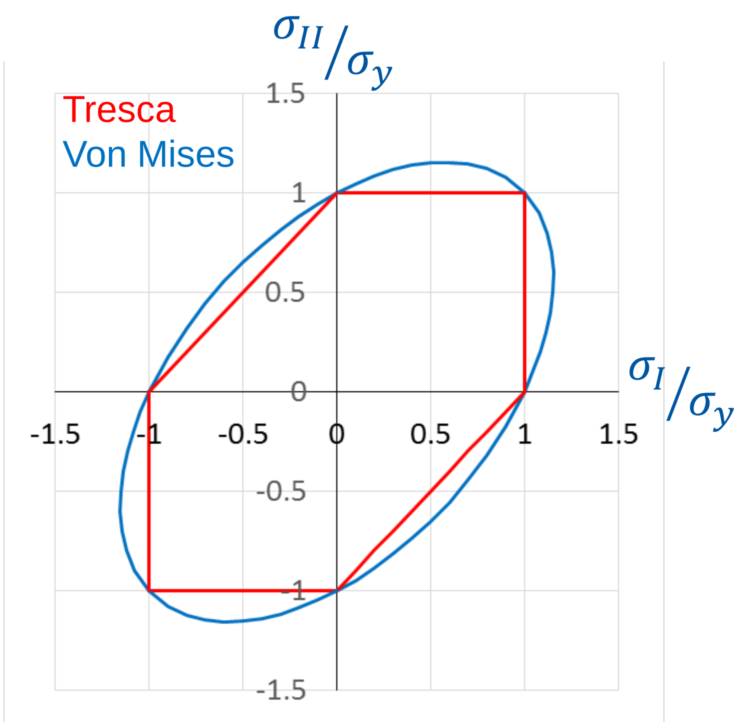
In stress space: $\frac{1}{\sqrt{2}}\sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)} - \sigma_y$

Plasticity – Yield Surface

Comparison of the Tresca's and Von Mises's yield surface for plane stress state:

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_I & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



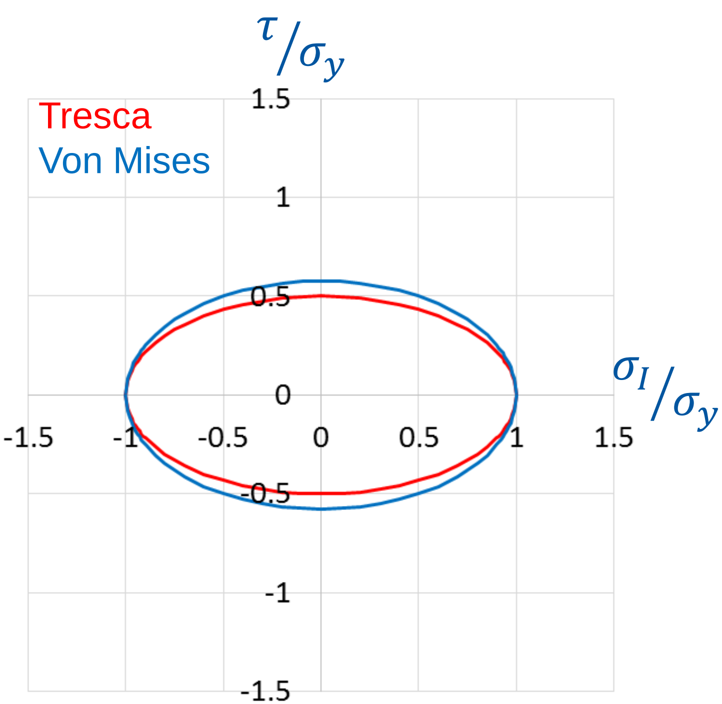
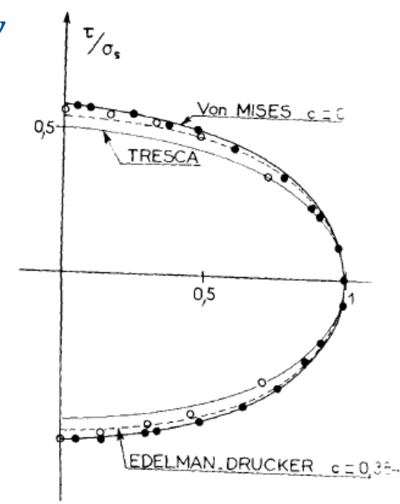


Fig. 15. Frontière de la limite d'élasticité.
○ Alliage Alu. 195 (d'après Ivey).
● Alliage Alu. 24 S-T 4 (d'après Naghdi et al.).



Experimental validity on aluminium alloys

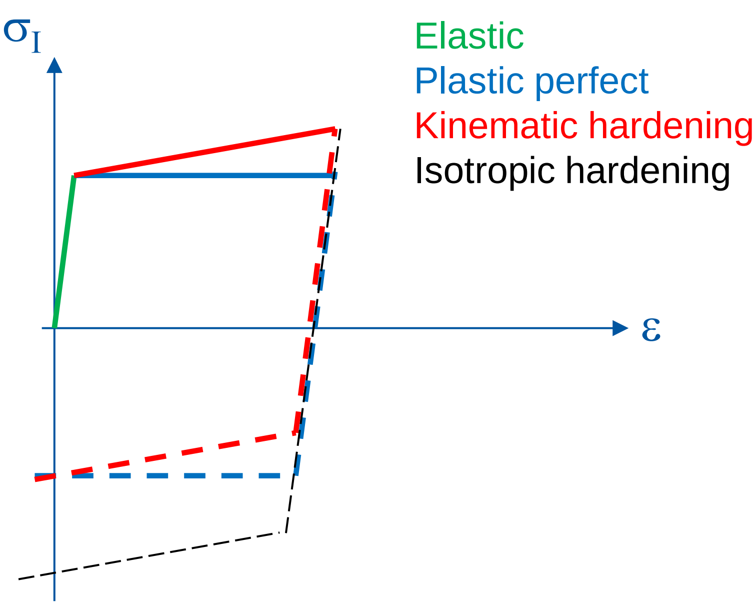
Plasticity - Hardening

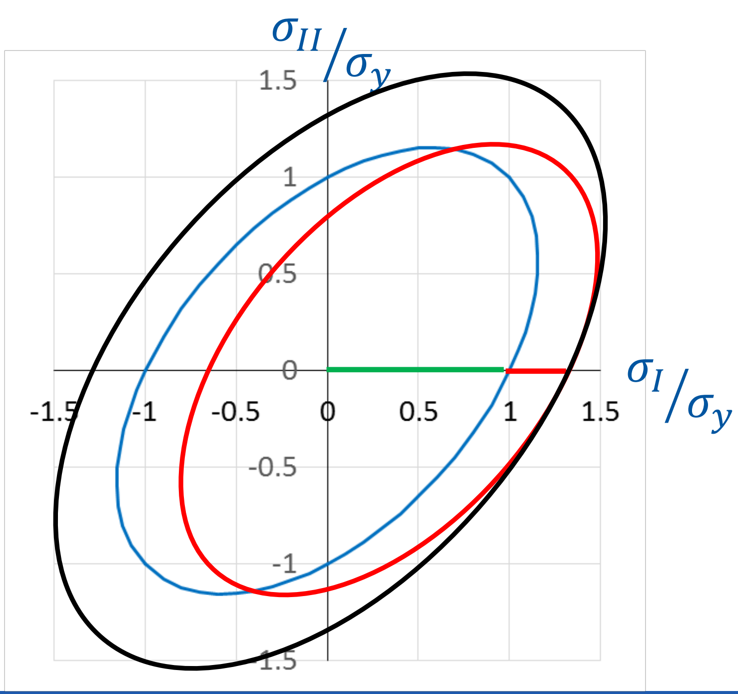
Plastic strain tensor: $\underline{\underline{\varepsilon}}^p = \underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^e$

Accumulated plastic strain, p : $dp = \sqrt{\frac{2}{3} d\underline{\underline{\varepsilon}}^p : d\underline{\underline{\varepsilon}}^p} > 0$

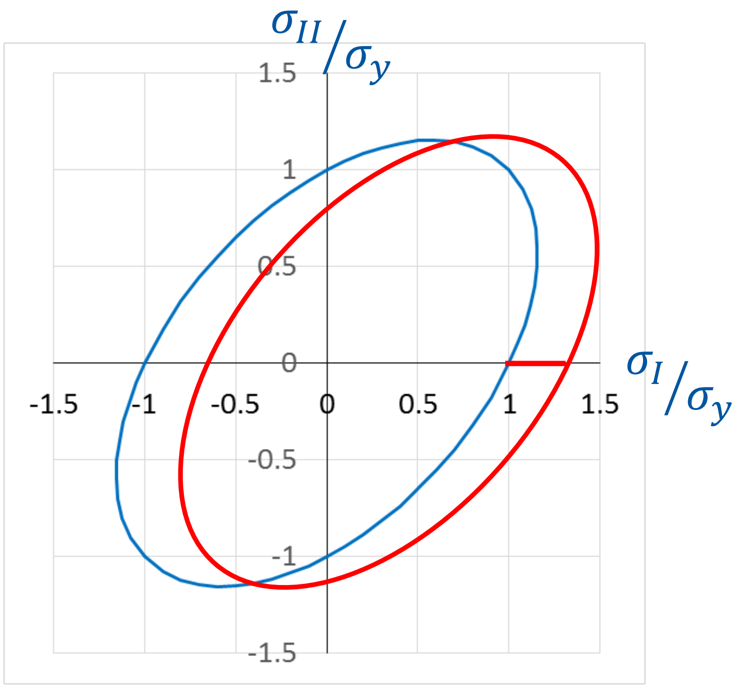
Plastic strain are induced by dislocation motions

→ no volumic change → $\mathrm{tr}\left[\underline{\underline{\varepsilon}}^p\right] = 0$





Plasticity – Kinematic Hardening



$$f\left(\underline{\underline{\sigma}}, \sigma_y, \underline{\underline{X}}\right) = \sqrt{\frac{3}{2}\left(\left(\underline{\underline{\sigma}}^D - \underline{\underline{X}}\right) : \left(\underline{\underline{\sigma}}^D - \underline{\underline{X}}\right)\right)} - \sigma_y$$

$$d\underline{\underline{X}} = \frac{2}{3} H d\underline{\underline{\varepsilon}}^p$$

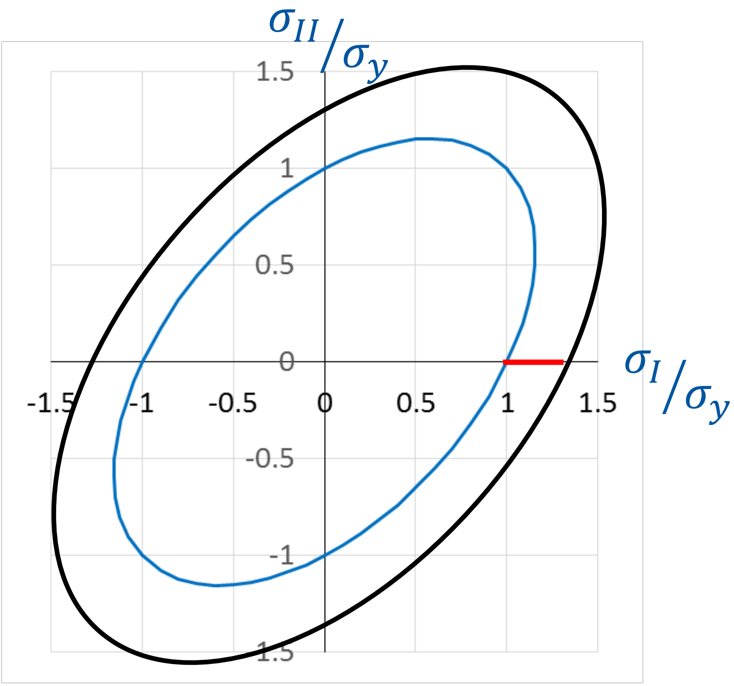
Normality rule:

$$d\underline{\underline{\varepsilon}}^p = d\lambda \frac{\partial f}{\partial \underline{\underline{\sigma}}} = d\lambda \frac{3}{2} \frac{\left(\underline{\underline{\sigma}}^D - \underline{\underline{X}}\right)}{\sqrt{\frac{3}{2}\left(\left(\underline{\underline{\sigma}}^D - \underline{\underline{X}}\right) : \left(\underline{\underline{\sigma}}^D - \underline{\underline{X}}\right)\right)}}$$

Consistency equation: $df = 0$

As a first estimation, the hardening modulus H is estimated by: $H \sim \frac{\sigma_u - R_{p0.2}}{\varepsilon_u}$

Plasticity – Isotropic Hardening



$$f\left(\underline{\underline{\sigma}}, \sigma_y, R\right) = \sqrt{\frac{3}{2}\left(\underline{\underline{\sigma}}^D : \underline{\underline{\sigma}}^D\right)} - \sigma_y - R$$

$$dR = g \cdot dp$$

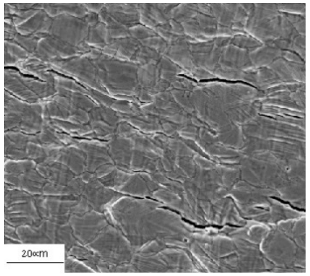
Normality rule:

$$d\underline{\underline{\varepsilon}}^p = d\lambda \frac{\partial f}{\partial \underline{\underline{\sigma}}} = d\lambda \frac{3}{2} \frac{\underline{\underline{\sigma}}^D}{\sqrt{\frac{3}{2}\left(\underline{\underline{\sigma}}^D : \underline{\underline{\sigma}}^D\right)}}$$

Consistency equation: $df = 0$

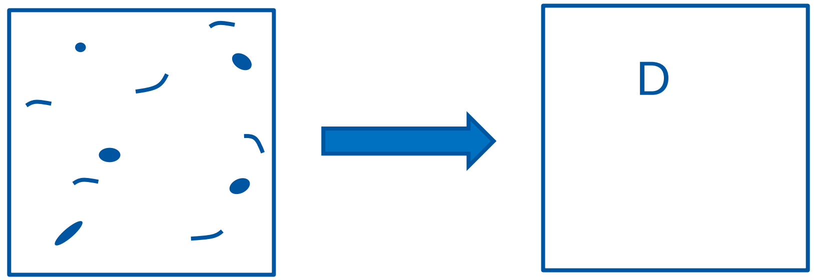
The isotropic hardening is non linear and reaches a saturation level.
Isotropic and kinematic hardening can be mixed.

Continuum Damage Mechanics



Damaged state:
microvoids and/or microcracks (dislocation stack, decohesion or cleavage of precipitates or inclusions,…)

Material representative element — Continuum damaged element

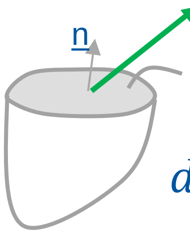


The degradation is characterized by a damage variable D (a scalar for isotropic damage)

Microdefect size << Element size << Structure size

Micro << Meso << Macro scale

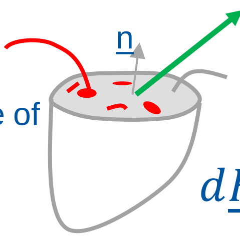
Effective stress and damage variable



$d\underline{F}$

$$d\underline{F} = \underline{\underline{\sigma}}\,\underline{n}\,dS$$

dS_d:
Surface of defects



$d\underline{F}$

$$d\underline{F} = \underline{\underline{\sigma}}'\underline{n}(dS - dS_d)$$

The damage parameter is defined by $D = \frac{dS_d}{dS}$

→ D=0 : Virgin structure; D=1: crack initiation

$\underline{\underline{\sigma}}'$: effective stress tensor

$$\underline{\underline{\sigma}}' = \frac{\underline{\underline{\sigma}}}{1 - D}$$

Continuum Damage Mechanics

Strain equivalence principle: $\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}(1-D):\underline{\underline{\varepsilon}}^e$

$$E' = E \cdot (1-D)$$

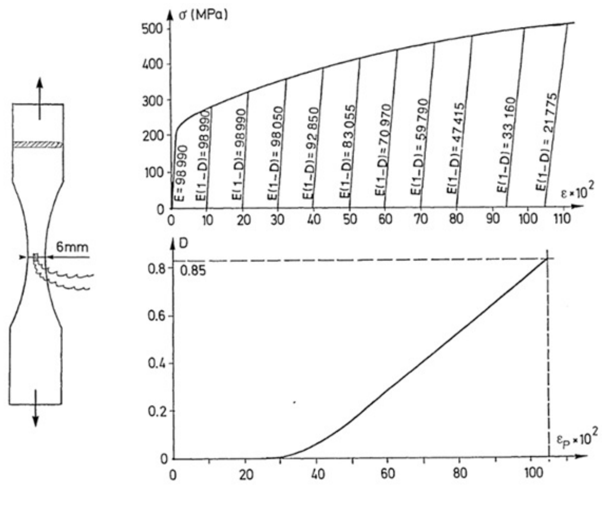


Fig. 1.16. Measurement of ductile damage on 99.9% copper at room temperature (after J. Dufailly)

Continuum Damage Mechanics

Damage evolution

Y: Strain energy density release rate $Y = \frac{\partial \psi}{\partial D} = \frac{1}{2} \underline{\underline{\varepsilon}}^e : \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e$

For ductile material: $dD = \frac{Y}{S} dp$

$$Y = \frac{1}{2E} \frac{\sigma_{VM}^2}{(1-D)^2} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_h}{\sigma_{VM}}\right)^2 \right]$$

Effective Von Mises stress

Triaxiality ratio

This relation can be generalized to:

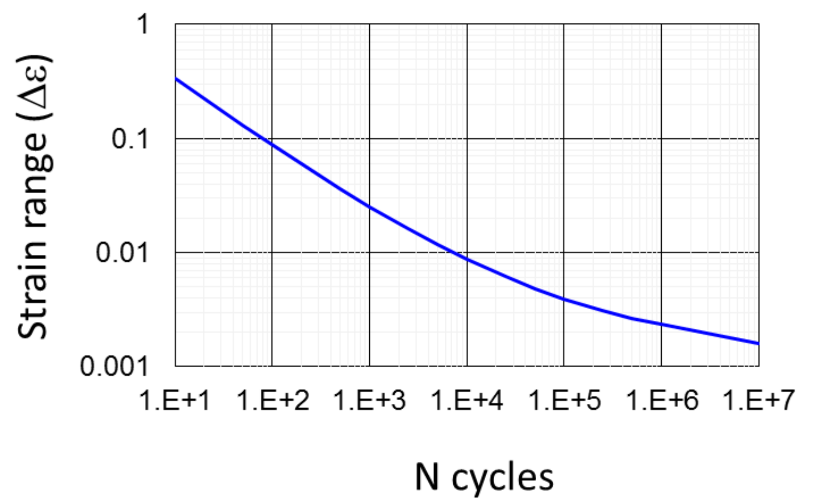
$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^p = \frac{C_2}{E} N_f^{-1/\gamma_2} + C_1 N_f^{-1/\gamma_1}$$

Leading to the Manson-Coffin law used in Low cycle fatigue:

$$N_f (\Delta\varepsilon^p)^\beta = Cst$$

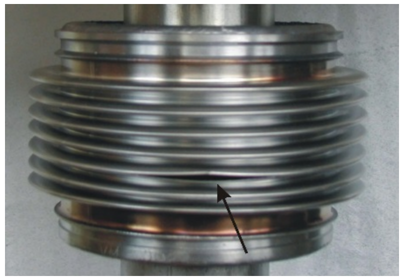
Universal slope equation:

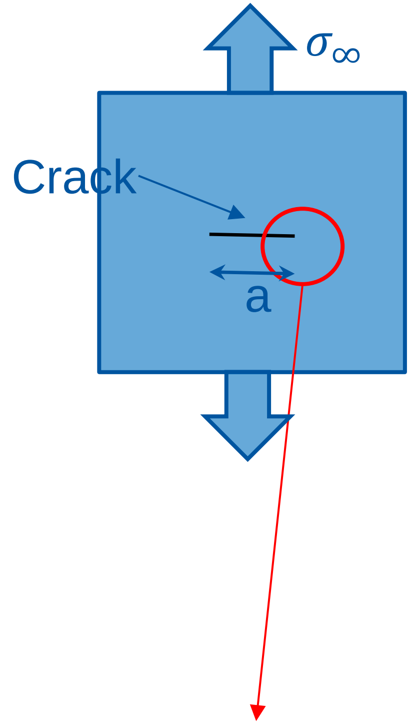
$$\Delta\varepsilon = 3.5 \frac{\sigma_m}{E} N_f^{-0.12} + D_u^{0.6} N_f^{-0.6}$$



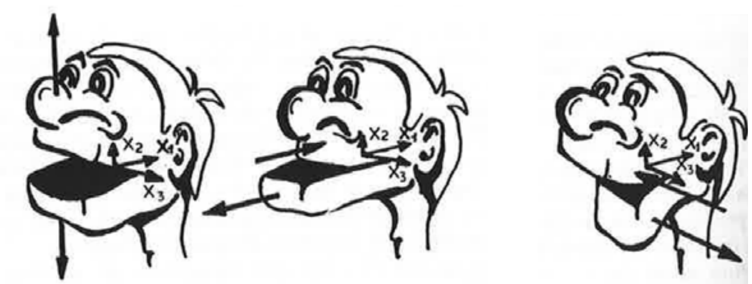
Typical curve for stainless steel

Fracture Mechanics





Solicitation modes:



Mode I: Opening   Mode II: In plane shear   Mode III: Out-of-plane shear

(after J. Lemaitre, J.L. Chaboche, Mécanique des matériaux solides, Dunod)

K: Stress intensity factor

For elastic material, the stress singularity reads:

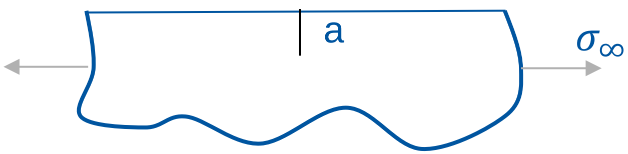
$$\sigma \propto \frac{K(\sigma_\infty, a)}{\sqrt{2\pi r}}$$

r: Distance from the crack tip

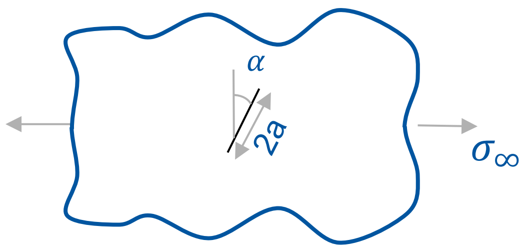
Fracture Mechanics

For elastic material, the stress singularity reads: $$\sigma \propto \frac{K(\sigma_\infty, a)}{\sqrt{2\pi r}}$$

K is in the form $K = f \cdot \sigma_\infty \sqrt{\pi a}$.



$$K_I = 1.122 \cdot \sigma_\infty \sqrt{\pi a}$$



$$K_I = \sigma_\infty \sqrt{\pi a} \cos^2 \alpha$$
$$K_{II} = \sigma_\infty \sqrt{\pi a} \cos \alpha \sin \alpha$$

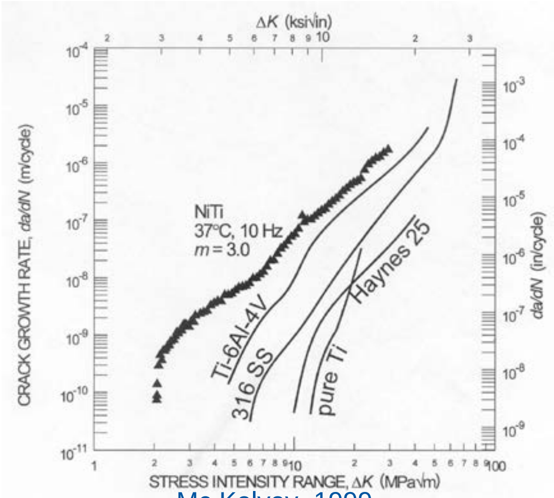
In a more global approach [Griffith], the energy release rate is defined as: $G = -\frac{\partial \wp}{\partial a}$

Criterion: $G \geq G_c$: crack propagation

For elastic material, $G = \frac{K^2}{E}$

Fracture Mechanics

Fatigue crack propagation:



Mc Kelvey, 1999

Paris' law, used for stable crack propagation:

$$\frac{da}{dN} = C_1 \Delta K^{n_1}$$

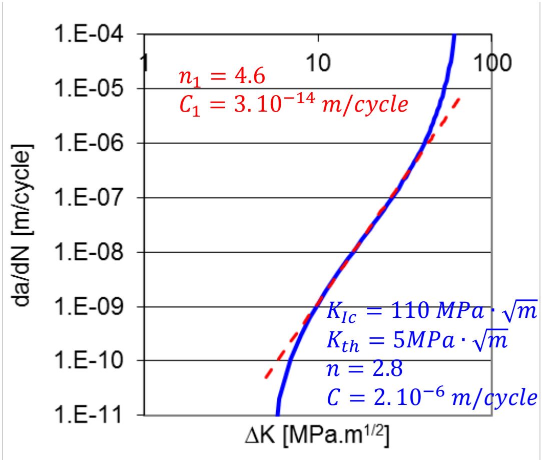
More general law:

$$\frac{da}{dN} = C\left(\frac{K_{max}\frac{1-R}{1-mR} - K_{th}}{K_{Ic} - K_{max}}\right)^n$$

K_{Ic}, K_{th}: fracture parameters

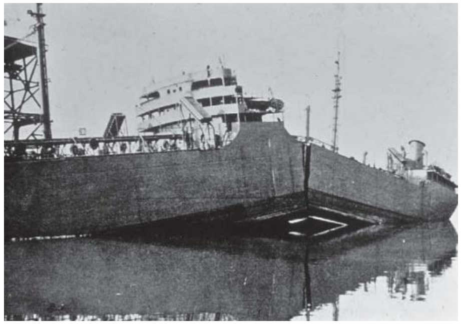
$R = \frac{K_{min}}{K_{max}}$

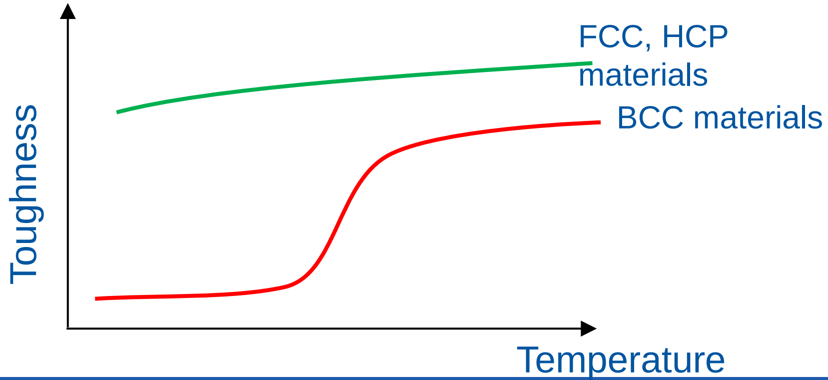
m: average load parameter; usually 0.5



Extrapolated curves for 316

Material may exhibit ductile to brittle transition


https://metallurgyandmaterials.files.wordpress.com/2015/12/liberty-ship-failure.jpg?w=640



Plan

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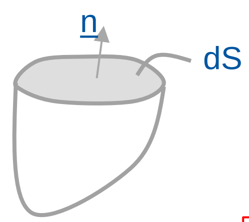
   c. Linear elasticity
   d. Plasticity
      i. Yield surface
      ii. Hardening
   e. Continuum damage mechanics
   f. Failure mechanics
2. **Structural mechanical analysis**
   a. **Vacuum chamber**
      i. **Loads on a chamber**
      ii. **Equilibrium equations**
      iii. **Stress on tube**
      iv. **Instability (buckling)**
   b. **Vacuum system as mechanical system**
      i. **Support – unbalanced force**
      ii. **Stability (column buckling)**
3. Selection criteria of materials
   a. Figures of merit of materials
   b. Some material properties
   c. Some comparisons based on FoM
4. Conclusion

Loads on a Vacuum Chamber

0. **Gravity:** specific force: $\underline{f_v} = \rho \underline{g}$

1. **Vacuum/pressure**



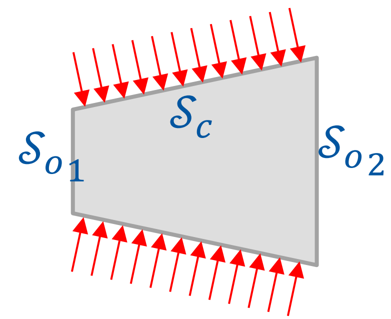
At the interface

$$d\underline{F} = \underline{\underline{\sigma}}\,\underline{n} dS = -p\underline{n} dS$$

$$\boxed{\underline{\underline{\sigma}}\,\underline{n} = -p\underline{n}}$$

Rk #1: if p is uniform: $\oint\!\!\oint p\underline{n} dS = 0$

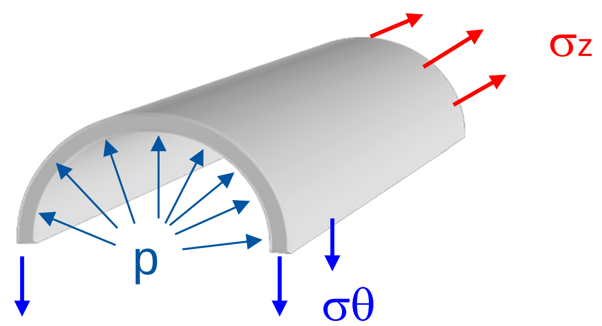
$$\rightarrow \iint_{\mathcal{S}_c} p\underline{n} dS = -\iint_{\mathcal{S}_o} p\underline{n} dS = -p\sum_i \mathcal{S}_{o_i}\underline{n_i}$$



Rk #2 : For a simple tube (radius R, thickness t):

Hoop stress: $\sigma_{\theta\theta} = \dfrac{pR}{t}$

Longitudinal stress (for a close tube): $\sigma_{zz} = \dfrac{pR}{2t}$



2. Electro magnetic forces

Foucault's currents are governed by Maxwell's equation:

$$\mathbf{rot\ E} = -\partial \mathbf{B}/\partial t$$

$$\mathbf{j} = \mathbf{E}/\rho$$

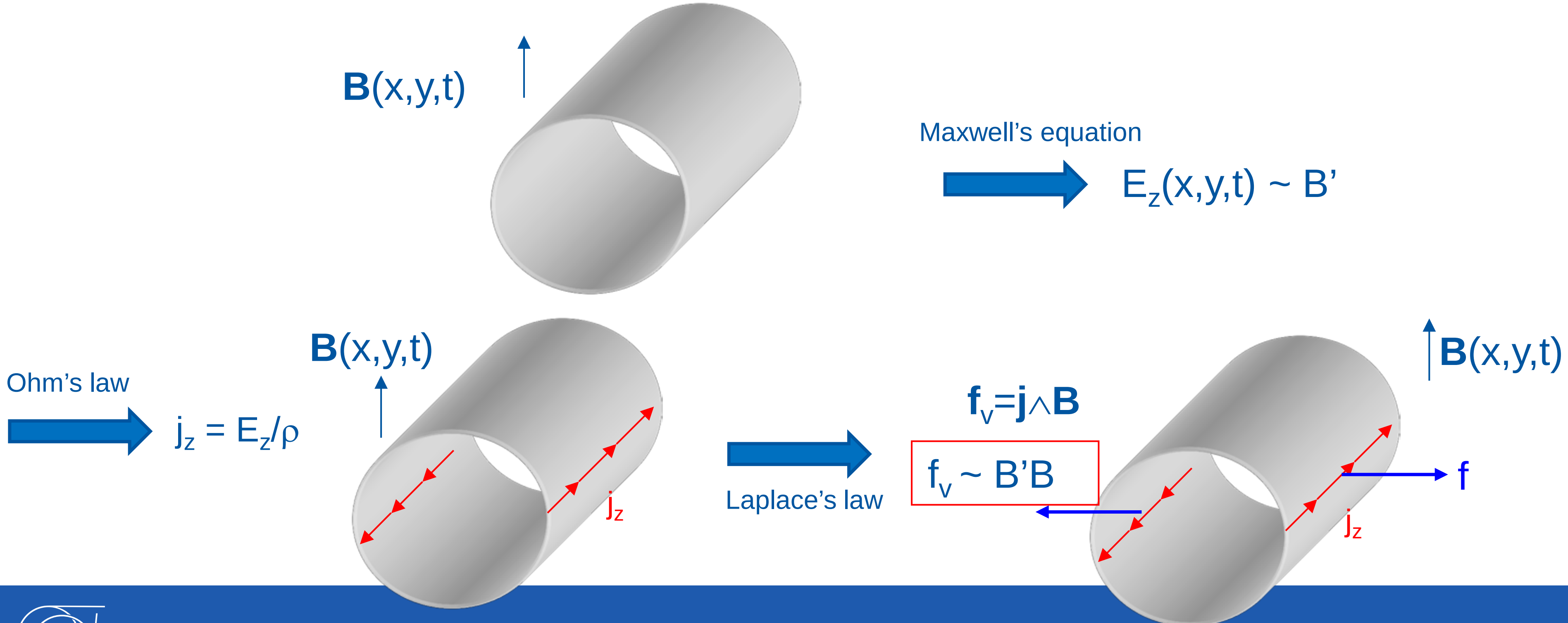
B: magnetic field

E: electric field

j: current density

ρ: electrical resistivity

In a long structure, subjected to the magnetic field B:



Orders of magnitude for cryogenic applications (beam screen)

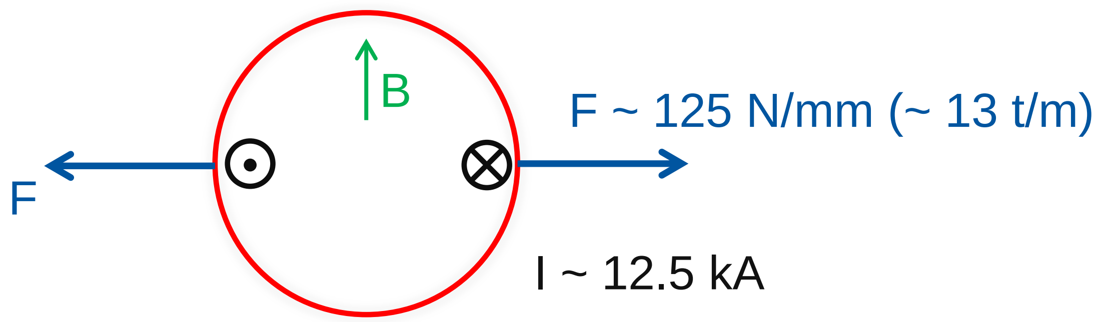
For a given magnetic configuration, force intensity ~ t/ρ

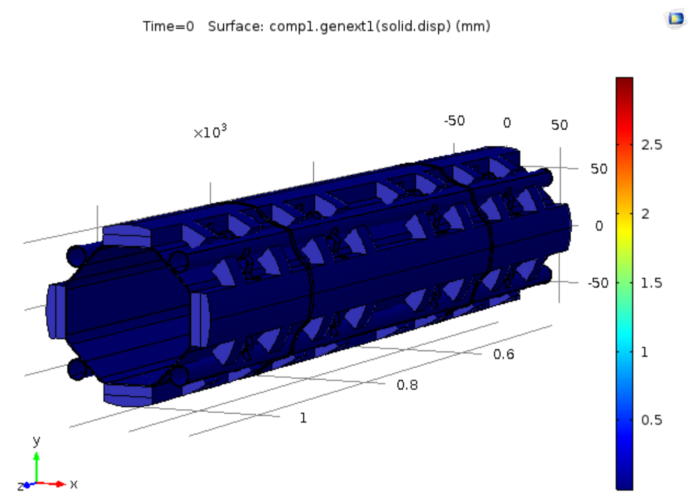
For a (colaminated) copper/stainless steel beam screen at cryogenic temperature:

$$(t/\rho)_{st.st.} / (t/\rho)_{Cu} \sim (1/5E\text{-}7)/(0.1/1E\text{-}9) \sim 0.02$$

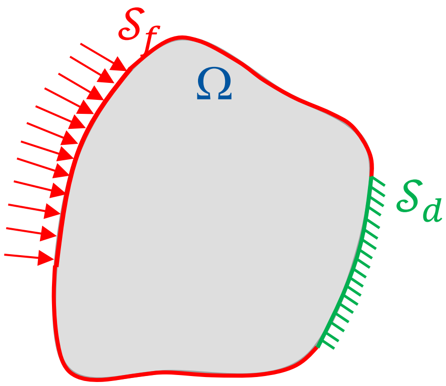
→ Lorentz' force are driven by copper

In a copper tube, 0.1 mm thick, radius of 25 mm, subjected to a magnetic field of 10 T with a decay of 100 T/s:





Mechanical Problem Formulation:



Equilibrium equations in static conditions:

$$\underline{div}\ \underline{\underline{\sigma}} + \underline{f_v} = \underline{0} \text{ in } \Omega$$

$$\underline{\underline{\sigma}}\ \underline{n} = \underline{F_s} \text{ on } \mathcal{S}_f$$

Kinematic:

boundary conditions: $\underline{u} = \underline{u}_{imposed}$ on $\mathcal{S}_d$

$$\underline{\underline{\varepsilon}}(\underline{u}) = \frac{1}{2} \cdot \left[\underline{\underline{grad}}(\underline{u}) + \underline{\underline{grad}}^T(\underline{u}) \right] \text{ in } \Omega$$

Constitutive model:

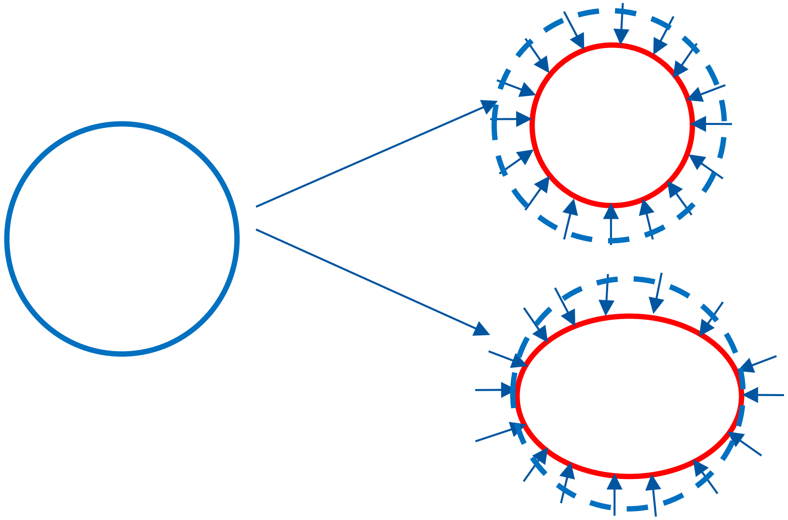
- Constitutive law: $\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e$ with $\underline{\underline{\varepsilon}}^e = \underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^{th} - \underline{\underline{\varepsilon}}^p - \ ...$
- Plasticity or other

→ Mechanical solution: $\underline{u}$, $\underline{\underline{\sigma}}$

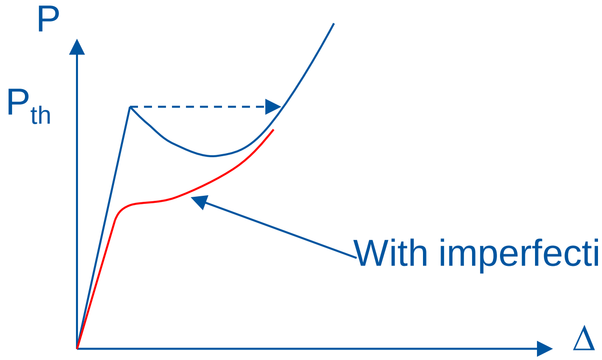
Design Criteria

→ **Material** criterion:
- Maximum stress,
- Elastic regime,
- fatigue, …

→ **Structural** criterion:
- Maximum deformation,
- Stability



Tube under external pressure



Structural Stability

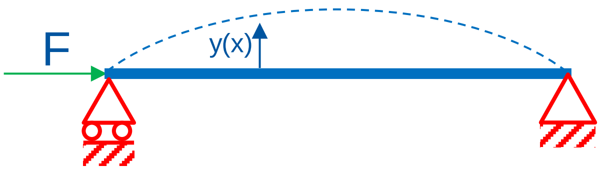
I. For an infinite elastic tube subjected to external pressure:

$$P_{cr} = \frac{E}{4 \cdot (1 - \nu^2)} \left(\frac{t}{R}\right)^3$$

Safety factor of 3 is usually applied.

Design rule for stainless steel: $t \geq \frac{D}{100}$

II. For beam subjected to axial force:



Euler's critical load: $F_{cr} = \left(\frac{\pi}{L_r}\right)^2 EI$

Equilibrium equation: $M = -Fy$

Bending equation: $M = EIy''$

I: Area moment of inertia

~$\pi d^3 t/8$ for a tube

$wh^3/12$ for a rectangular cross section

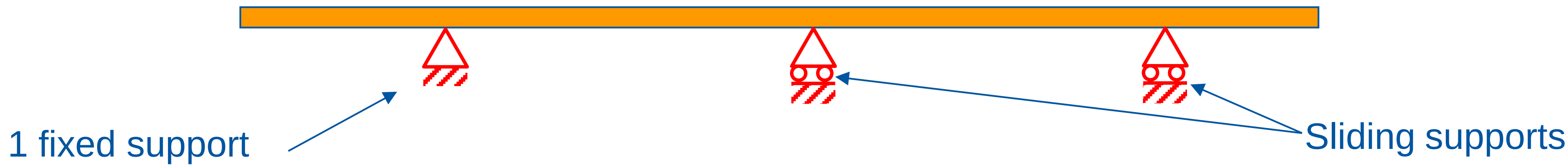
→ Differential equation: $EIy'' + Fy = 0$

Solution depends on boundary conditions.

Boundary conditions	L_r
	$L_r = L$
	$L_r = \sqrt{2} \cdot L$
	$L_r = 0.5 \cdot L$
	$L_r = 2 \cdot L$

Mechanical System - Supports

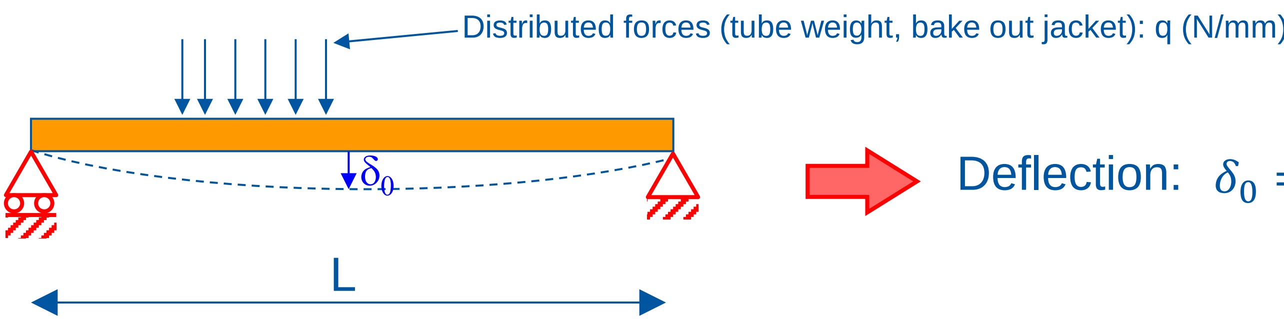
Kinematic (longitudinally):



Only 1 longitudinally-fixed support has to be used.

Static:

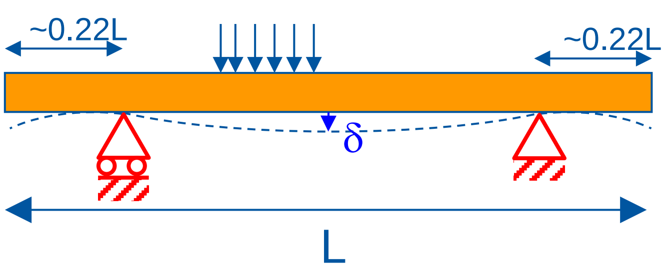
Beam pipe simply supported at its extremities



Deflection: $\delta_0 = \dfrac{5qL^4}{384EI}$

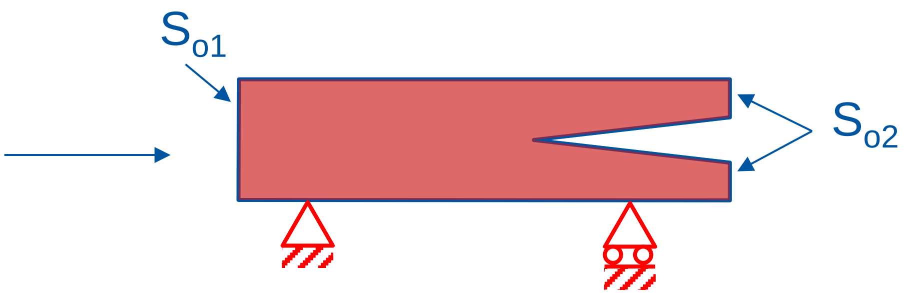
Beam pipe deflection can be minimized by supporting at Gaussian points:

Deflection: $\delta = \dfrac{\delta_0}{50}$



Mechanical System - Supports

Static : Pressure thrust force

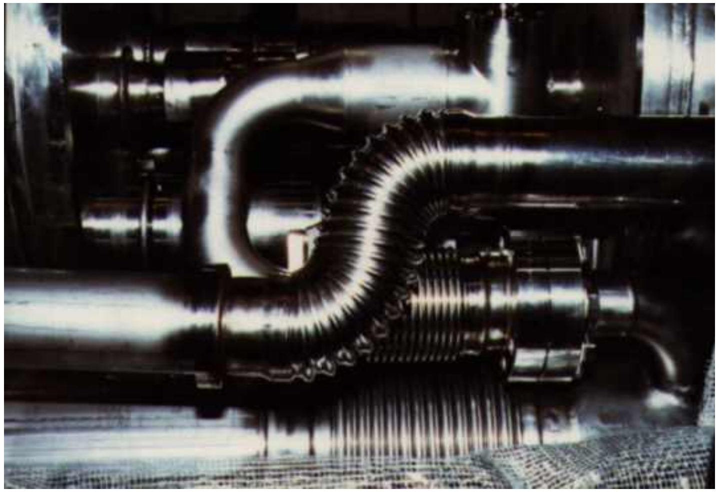


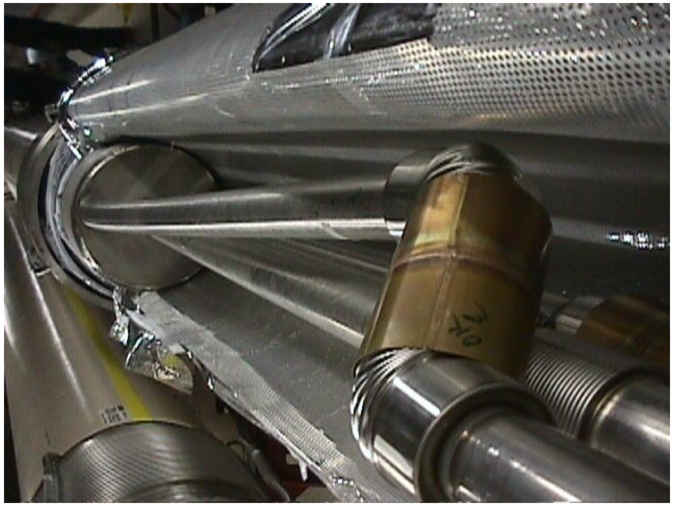
$$\underline{F}_{\rightarrow supports} = p \sum_i S_{o_i} \underline{n}_i$$

P = 0.1 MPa

Be careful to the moment and support anchoring

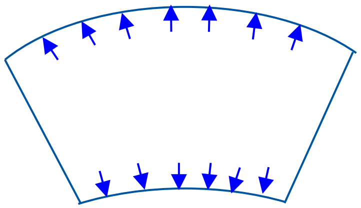
Mechanical System – Global stability



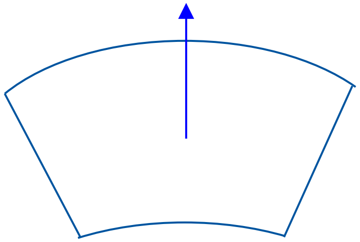


Global stability: the bellows and adjacent lines are unstable

This phenomenon can occur for a line under internal pressure (not necessary with a bellows).

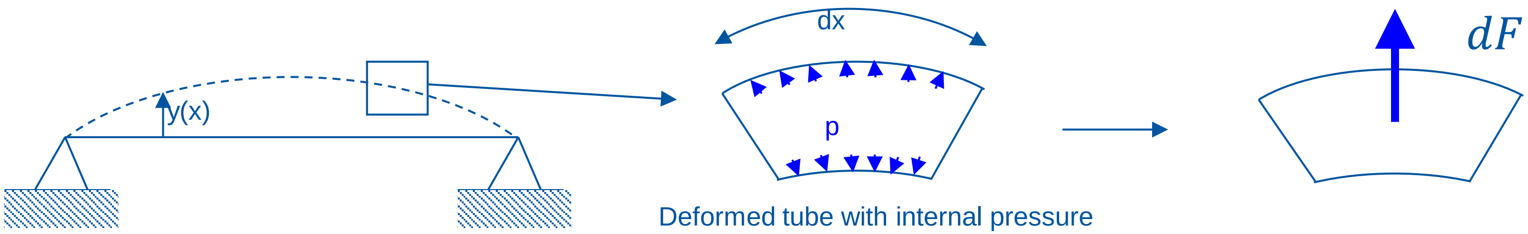


Deformed tube with internal pressure



The resultant of the pressure forces tend to increase the initial defect → instability

Mechanical System – Global stability



Deformed tube with internal pressure

$$dF = -pSy''dx$$

$$f = \frac{dF}{dx}$$

Equilibrium equation (for a tube slice): $M'' - f = 0$

Bending equation: $M = EIy''$

→ Differential equation: $EIy^{(4)} + pSy'' = 0$

Solutions depends on boundary conditions

Buckling pressure can read, in a general formulation (similar to Euler's formula):

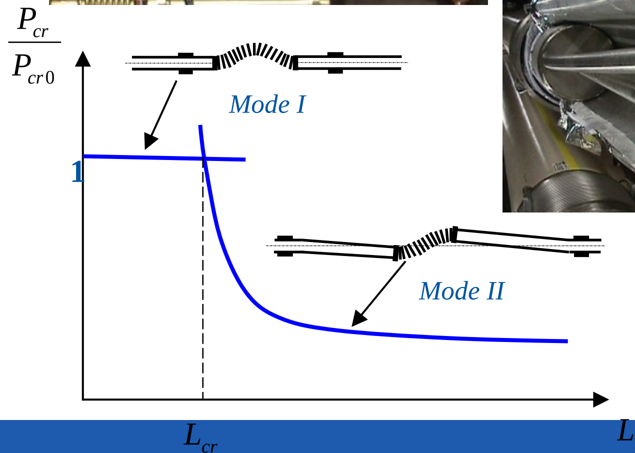
$$P_{cr} = \frac{\pi^2 C_b}{L_r^2 R_m^2}$$

Bending stiffness

Radius

Reduced length (depends on boundary conditions)

→



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A few Selection Criteria

Figures of merit:

Several figures of merit, characterizing the material, can be used depending on the final application.

- Mechanical Stability for transparent vacuum chamber: $X_0 E^{1/3}$

- Mechanical Stability for vacuum chamber subjected to fast magnetic field variation: $\rho E^{1/3}$

For beam-material interaction induced heating:

- Temperature rise in transient regime: $X_0 . \rho . C . T_f$

- Thermal fatigue: $\frac{X_0 . \rho . C . \sigma_y}{E . \alpha}$

- Temperature rise in steady state: $X_0 . \lambda . T_f$

Some Material Properties

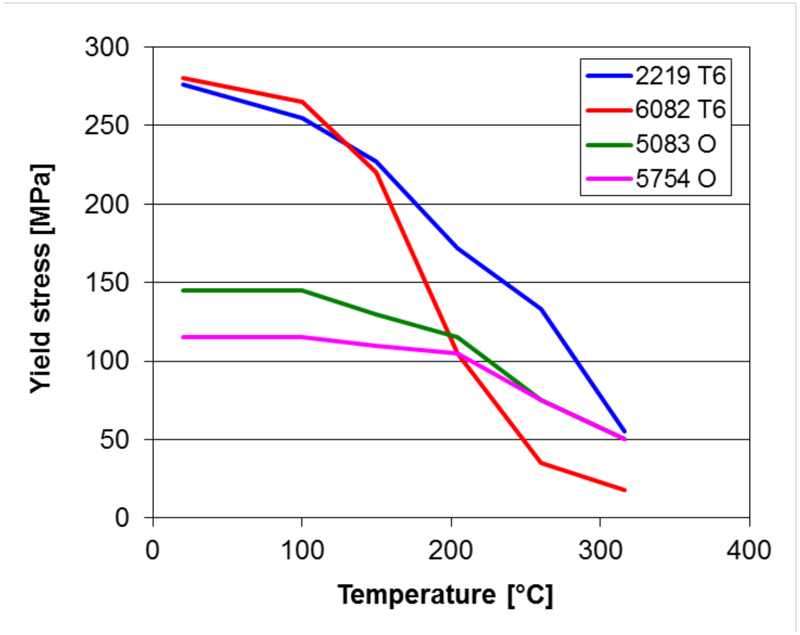
		Beryllium	Aluminium	Titanium G5	316L	Copper	Inconel
Density	$g{\cdot}cm^{-3}$	1.85	2.8	4.4	8	9	8.2
Heat capacity	$J{\cdot}K^{-1}{\cdot}Kg^{-1}$	1830	870	560	500	385	435
Thermal conductivity	$W{\cdot}K^{-1}{\cdot}m^{-1}$	200	217	16.7	26	400	11.4
Coefficient of thermal expansion	$10^{-6}{\cdot}K^{-1}$	12	22	8.9	16	17	13
Radiation length	cm	35	9	3.7	1.8	1.47	1.7
Melting temperature	K	1560	930	1820	1650	1360	1530
Yield strength	MPa	345	275	830	300	200	1100
Young modulus	GPa	230	73	115	195	115	208
Electrical resistivity	$10^{-9}{\cdot}\Omega{\cdot}m$	36	28	1700	750	17	1250

Indicative values at room temperature

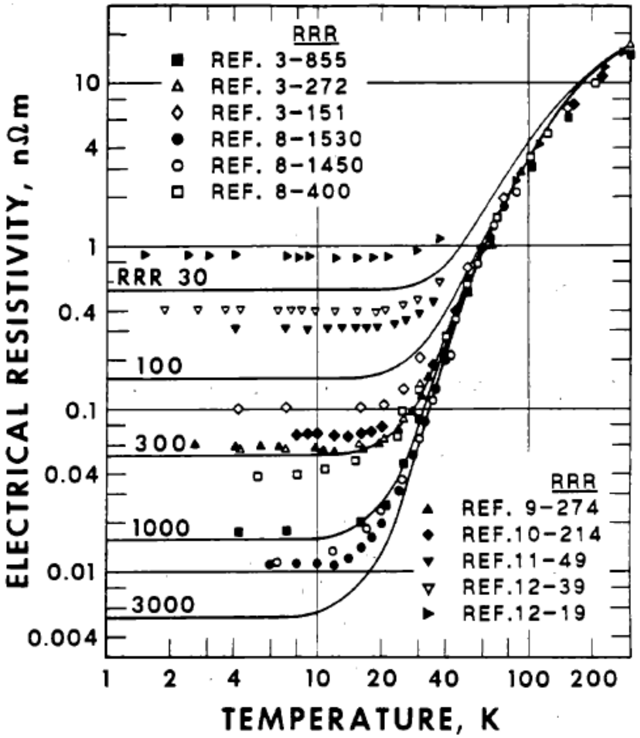
Some Material Properties

Some properties depend strongly on temperature or grade or delivery state (annealed, hard,…).

Just two examples:

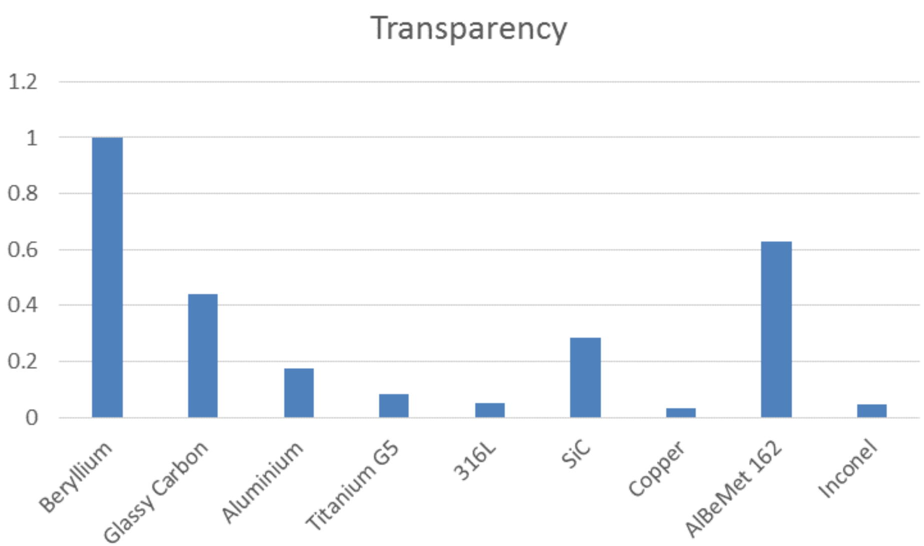


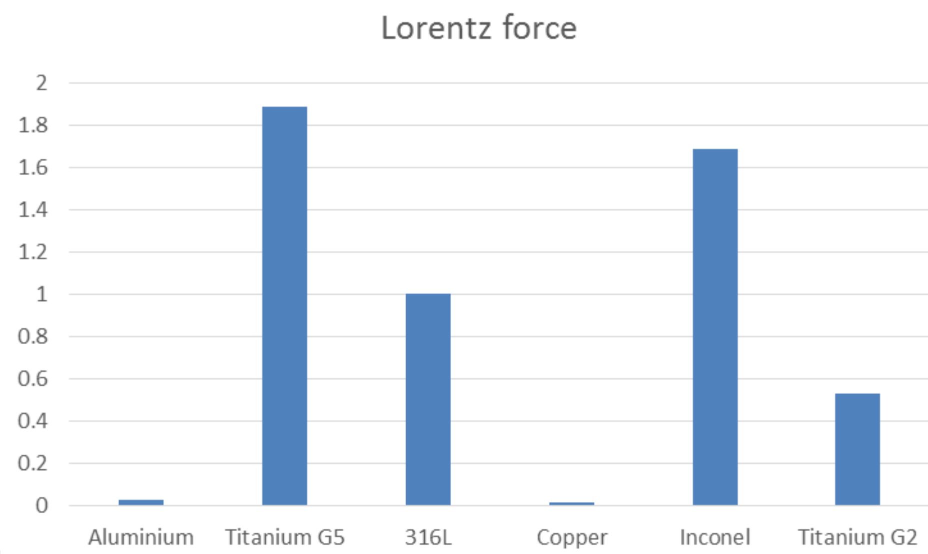
Yield strength of aluminium alloys as a function of temperature

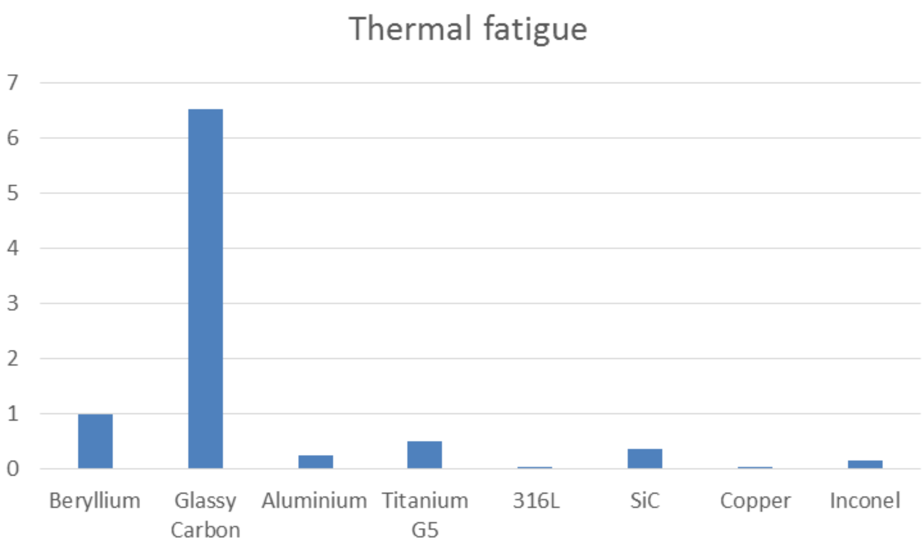


Properties of copper and copper alloys at cryogenic temperatures, Simon et al.

Few Figures of Merit







→ For each application, the correct material.

Conclusion

Most of the time, the mechanical design of a vacuum system is not really complex.

The choice and the knowledge the materials are important to get a robust and reliable mechanical system at the right price.