Bench Measurements and Simulations of Beam Coupling Impedance

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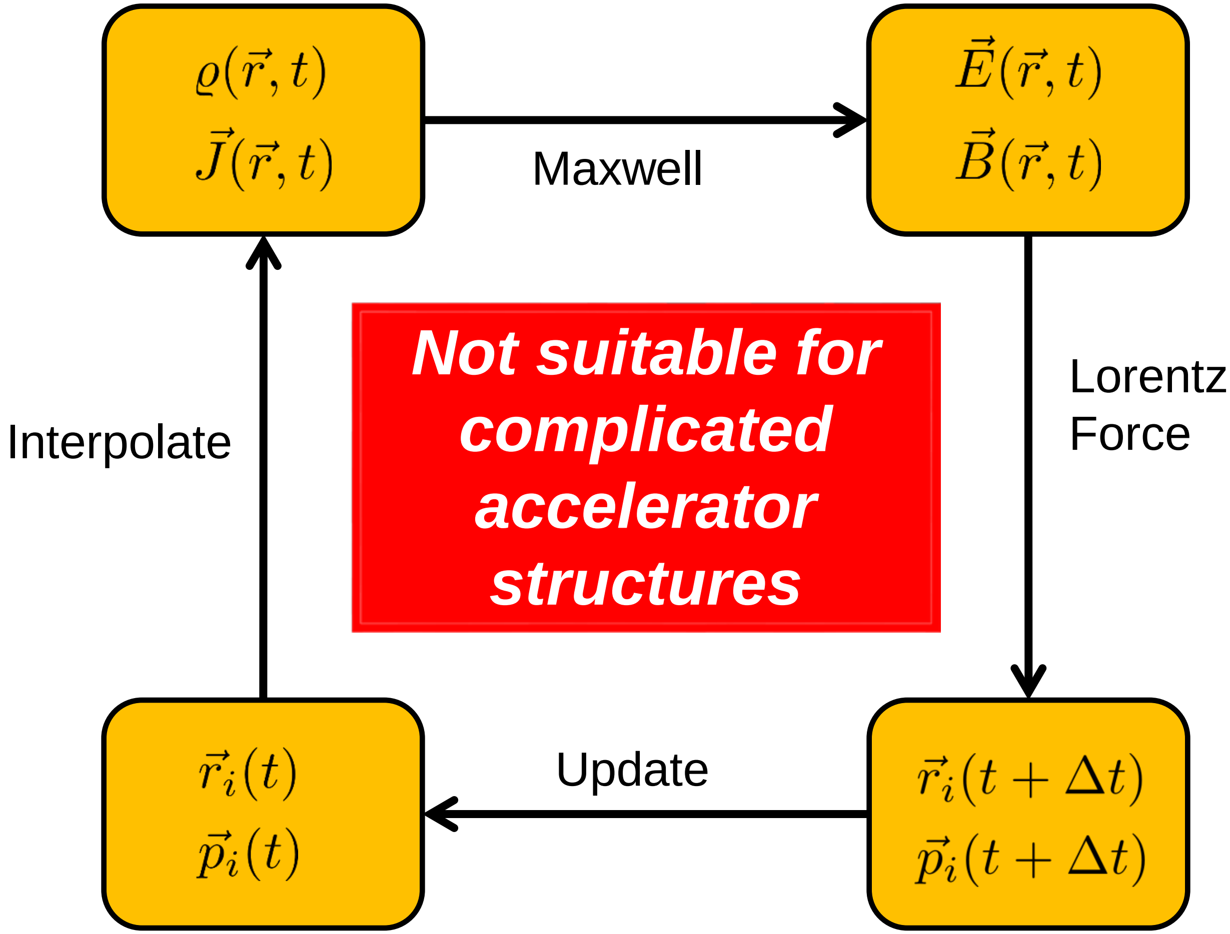
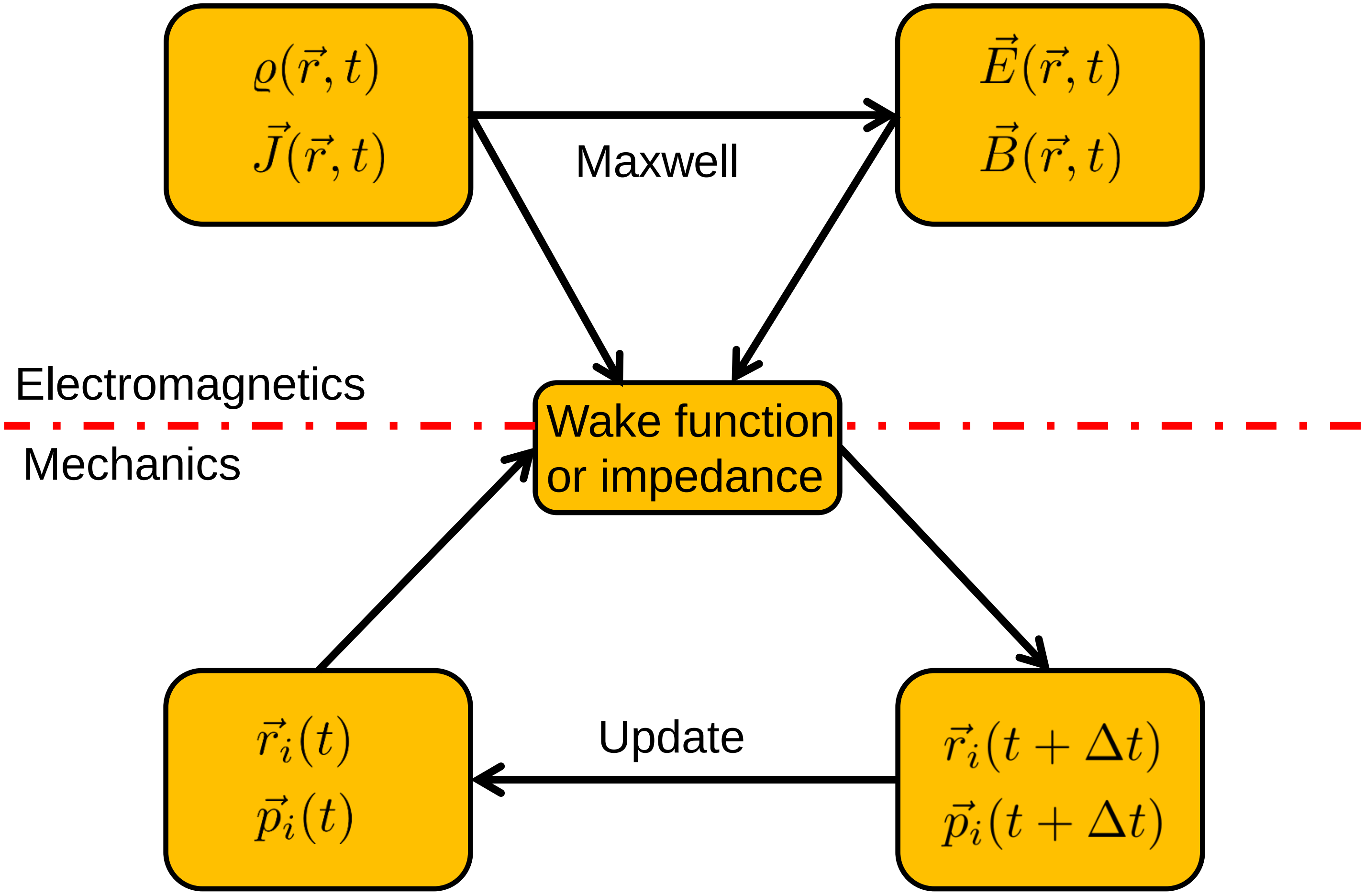
CERN Accelerator School on Intensity Limitations in Particle Beams

Maxwell's Equations and Particle Beams



Self consistent for $\Delta t \rightarrow 0$

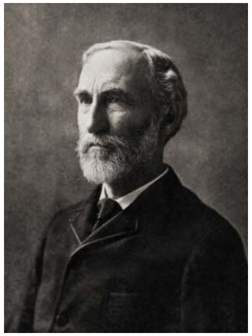
Wake fields and Beam Coupling Impedance: *Divide and Conquer*



Maxwell's equations

Pictures: Wikipedia

J.C. Maxwell
1831-1879



J. W. Gibbs
1839-1903

Transformation by laws of Gauss and Stokes

$\mathrm{rot}\,\vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$	$\oint_{\partial A} \vec{E}(\vec{r},t)\cdot d\vec{s} = -\frac{d}{dt}\int_A \vec{B}(\vec{r},t)\cdot d\vec{A}$
$\mathrm{rot}\,\vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}$	$\oint_{\partial A} \vec{H}(\vec{r},t)\cdot d\vec{s} = \int_A \left(\frac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}(\vec{r},t)\right)\cdot d\vec{A}$
$\mathrm{div}\,\vec{B}(\vec{r},t) = 0$	$\oint_{\partial V} \vec{B}(\vec{r},t)\cdot d\vec{A} = 0$
$\mathrm{div}\,\vec{D}(\vec{r},t) = \rho(\vec{r},t)$	$\oint_{\partial V} \vec{D}(\vec{r},t)\cdot d\vec{A} = \int_V \rho(\vec{r},t)\,dV$
$\forall \vec{r} \in \Omega \subseteq \mathbf{R}^3$	$\forall A, \forall V \subset \Omega \subseteq \mathbf{R}^3$

Material relations: (macroscopic)

$$\vec{D} = \varepsilon\vec{E},\ \vec{B} = \mu\vec{H},\ \vec{J} = \kappa\vec{E}$$

Frequency Domain:

$$\frac{\partial}{\partial t} \to i\omega$$

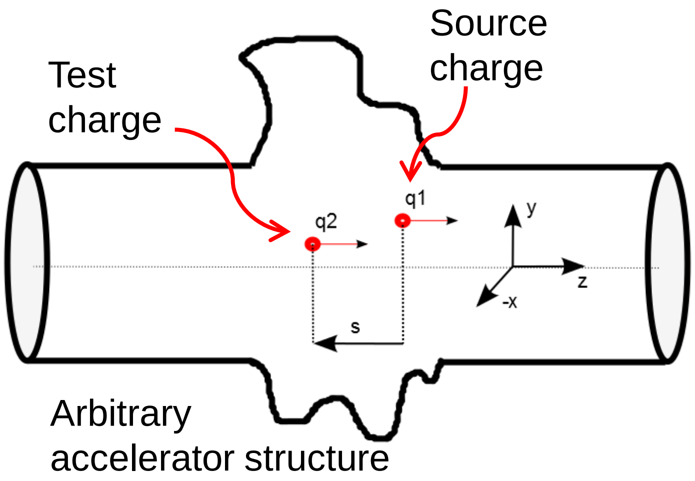
Contents

- Intro: wake functions and beam coupling impedance

- **Electromagnetic field simulations**
  - Examples with the Finite Integration Technique (FIT)
  - Overview of simulation techniques
  - Frequency domain simulations

- **Wire bench measurements**
  - Primer on S-parameters and Vector Network Analysis (VNA)
  - Connection between S-parameters and beam coupling impedance
  - Modeling and simulation of the wire measurement

Wake Function



$$\dot{\vec{p}}_2 = \vec{F} = q_2 \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

$$\vec{W}(\vec{r}_2^{\perp}, \vec{r}_1^{\perp}, s) := \frac{1}{q_2 q_1} \int_{-\infty}^{\infty} \vec{F}\left(\vec{r}_2^{\perp}, z_2, t = \frac{z_2 + s}{v}\right) \mathrm{d}z_2$$

- Characterizes the accelerator structure
- “Kick“ after passage $\Delta\vec{p}_2(s) = \vec{W}(s)\, q_1 q_2 / v$
- Green‘s function
  → Wake potential by convolution
- Beam Coupling Impedance by Fourier Transform

Consequences on Beam Dynamics

- Can lead to beam instabilities (both longitudinal and transverse)
- Describes beam induced component heating

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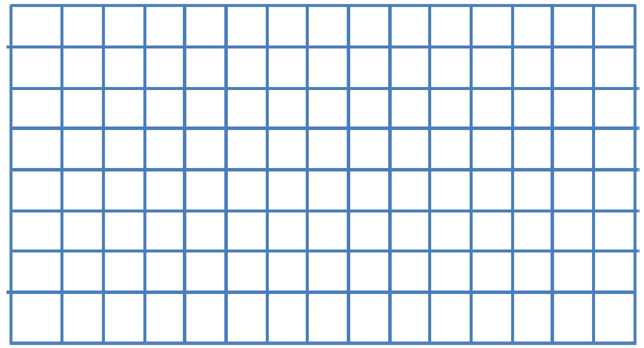
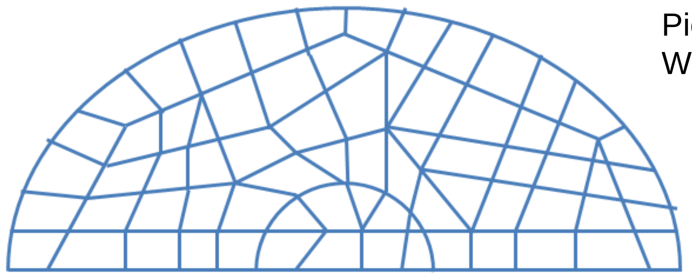
Mesh

- In general ***Mesh*** and ***Method*** are independent.

- Usually FEM on tetrahedral mesh (unstructured)
- Usually FIT/FDTD on hexahedral mesh (structured)



Structured Grid



Unstructured Grid

Pictures: Wikipedia

→ Computational advantages

→ Modeling advantages

Finite Integration Technique (FIT): Grid-Maxwell-Equations

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$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_{A} \vec{B} \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H} \cdot d\vec{s} = \int_{A} \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right) \cdot d\vec{A}$$

$$\oint_{\partial V} \vec{D} \cdot d\vec{A} = \int_{V} \rho \, dV$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

FIT →

$$\mathbf{C}\widehat{\mathbf{e}} = -\frac{d}{dt}\widehat{\widehat{\mathbf{b}}}$$

$$\widetilde{\mathbf{C}}\widehat{\mathbf{h}} = \frac{d}{dt}\widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}}$$

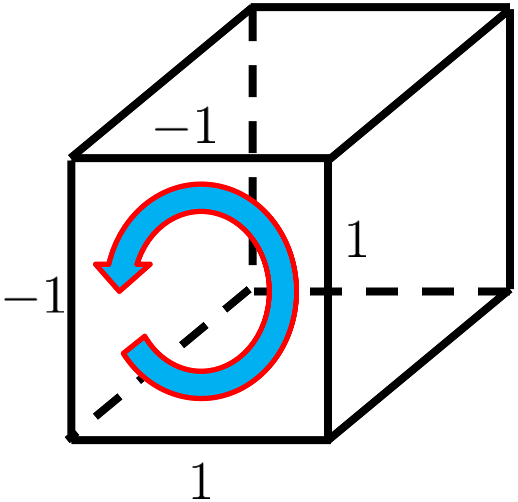
$$\widetilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q}$$

$$\mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}$$

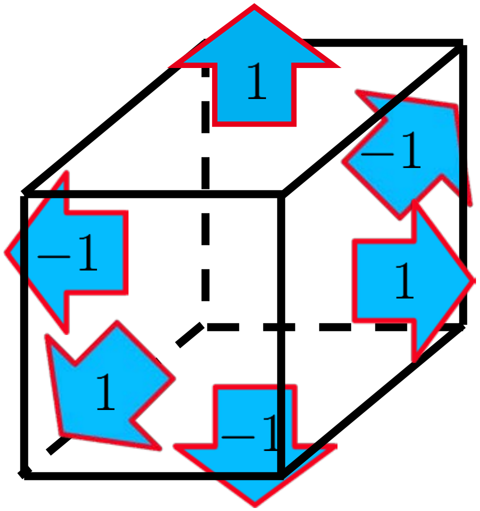
The Grid-Equations represent an EVALUATION of Maxwell‘s equations
→ Therefore they are exact

Lecture: Verfahren und Anwendungen der Feldsimulation, TUD

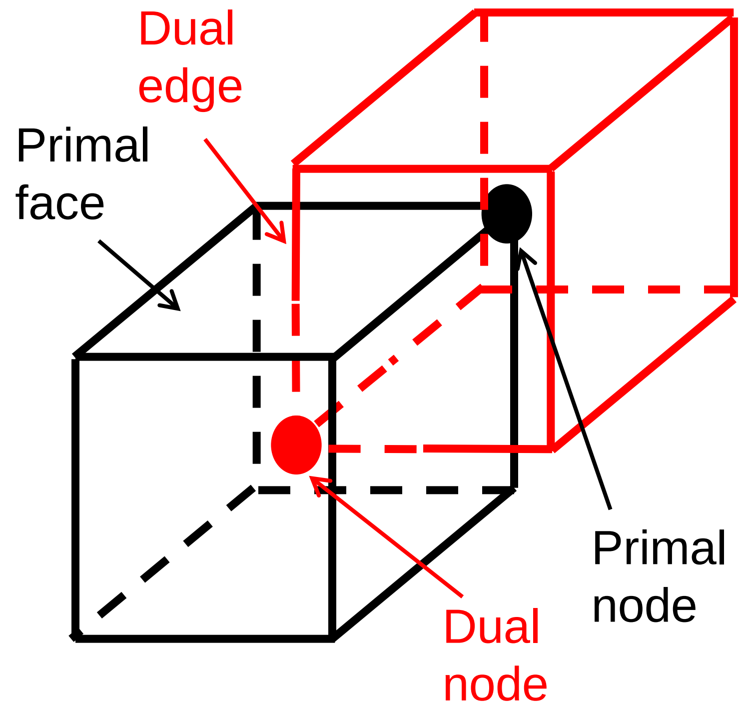
FIT Topological relations



Topological
curl (**C**)



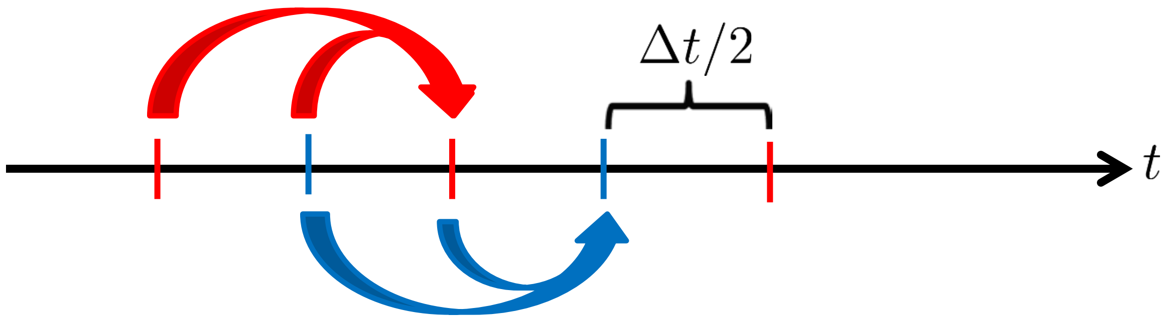
Topological
div (**S**)



Mesh duality

- Dual orthogonal mesh leads to <u>diagonal material matrices</u>

Leapfrog algorithm and stability



$$\widehat{\mathbf{h}}^{n+1/2} = \widehat{\mathbf{h}}^{n-1/2} - \Delta t \mathbf{M}_\mu^{-1} \mathbf{C} \widehat{\mathbf{e}}^n \qquad \widehat{\widehat{\mathbf{b}}} = \mathbf{M}_\mu \widehat{\mathbf{h}}$$

$$\widehat{\mathbf{e}}^{n+1} = \widehat{\mathbf{e}}^n - \Delta t \mathbf{M}_\epsilon^{-1} (\widetilde{\mathbf{C}} \widehat{\mathbf{h}}^{n+1/2} - \widehat{\widehat{\mathbf{j}}}_s^{n+1/2}) \qquad \widehat{\widehat{\mathbf{d}}} = \mathbf{M}_\epsilon \widehat{\mathbf{e}}$$

- Courant Friedrichs Levy (CFL) stability criterion:

$$\Delta t \le \min_i \sqrt{\frac{\mu_i \varepsilon_i}{\frac{1}{\Delta x_i^2} + \frac{1}{\Delta y_i^2} + \frac{1}{\Delta z_i^2}}} = \frac{\Delta x}{c\sqrt{3}}$$

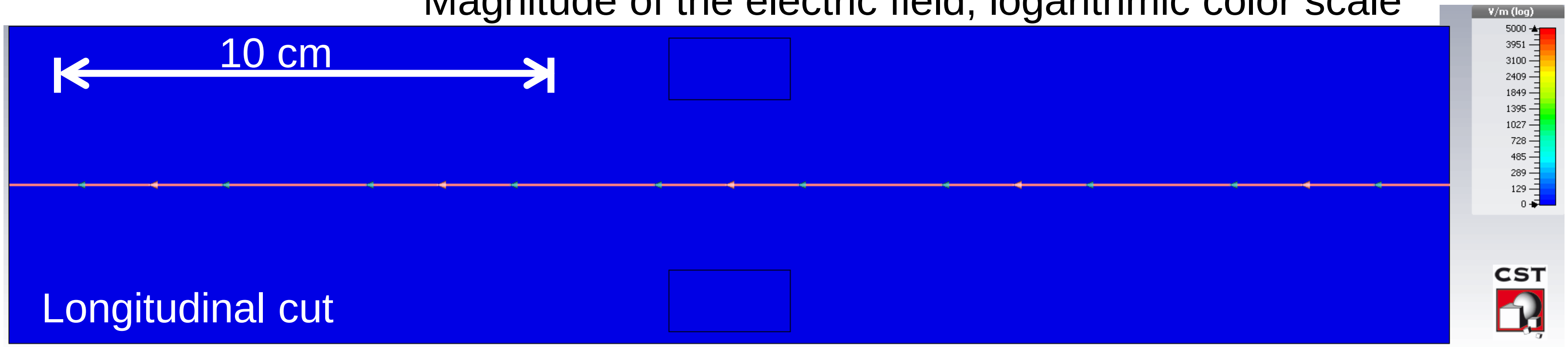
For equidistant mesh and vacuum

- Moreover: grid dispersion, numerical Cherenkov radiation, …

Wake Potential Example (Broadband)

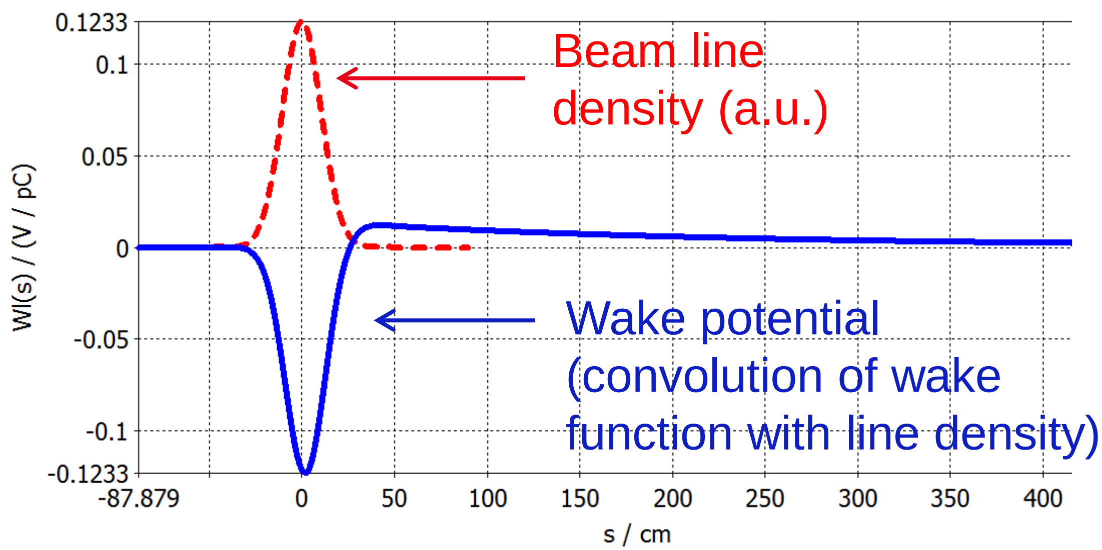
Ferrite Ring in Perfectly Electric Conducting (PEC) Pipe

Magnitude of the electric field, logarithmic color scale



CST Particle Studio®

Dispersively lossy ferrite material



Impedance of the ferrite ring
(normalized DFT of wake potential)

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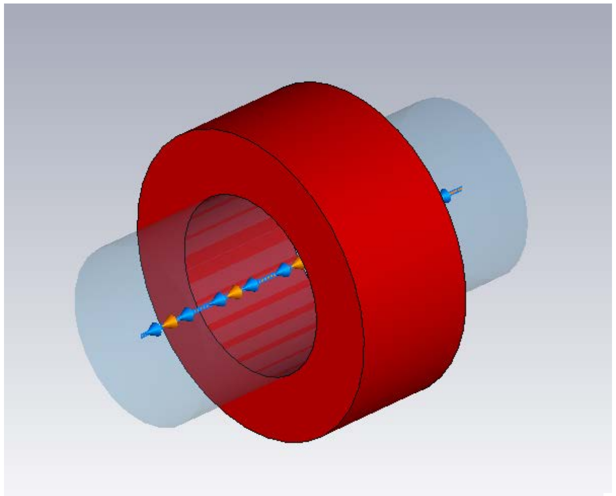
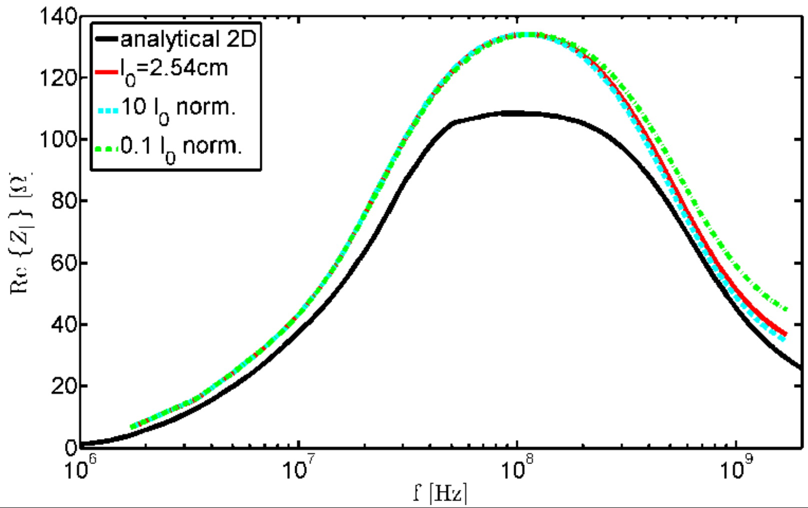
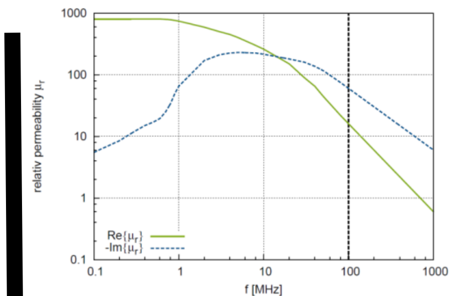
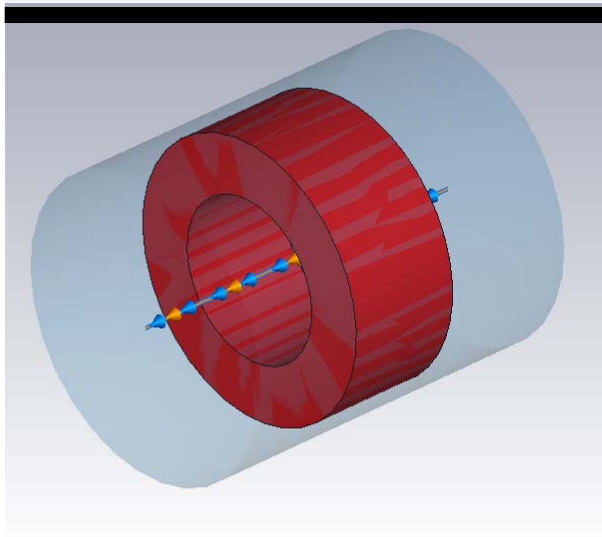
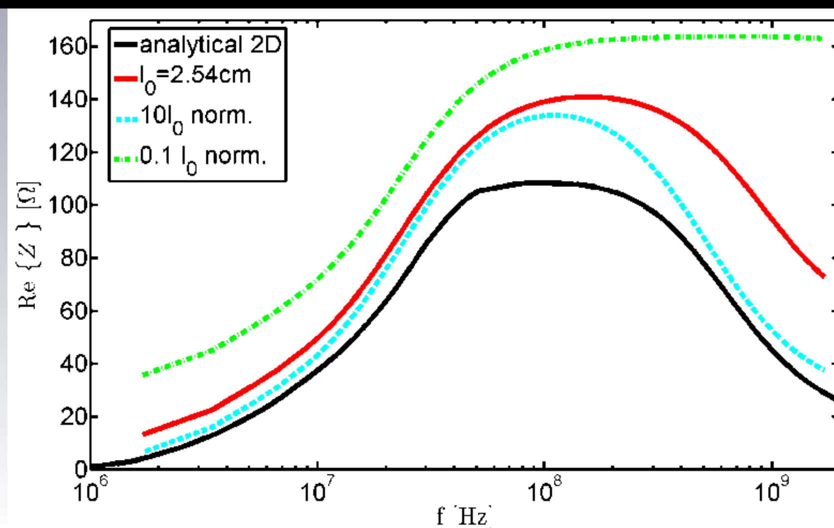






Ferrite Amidon Material 43

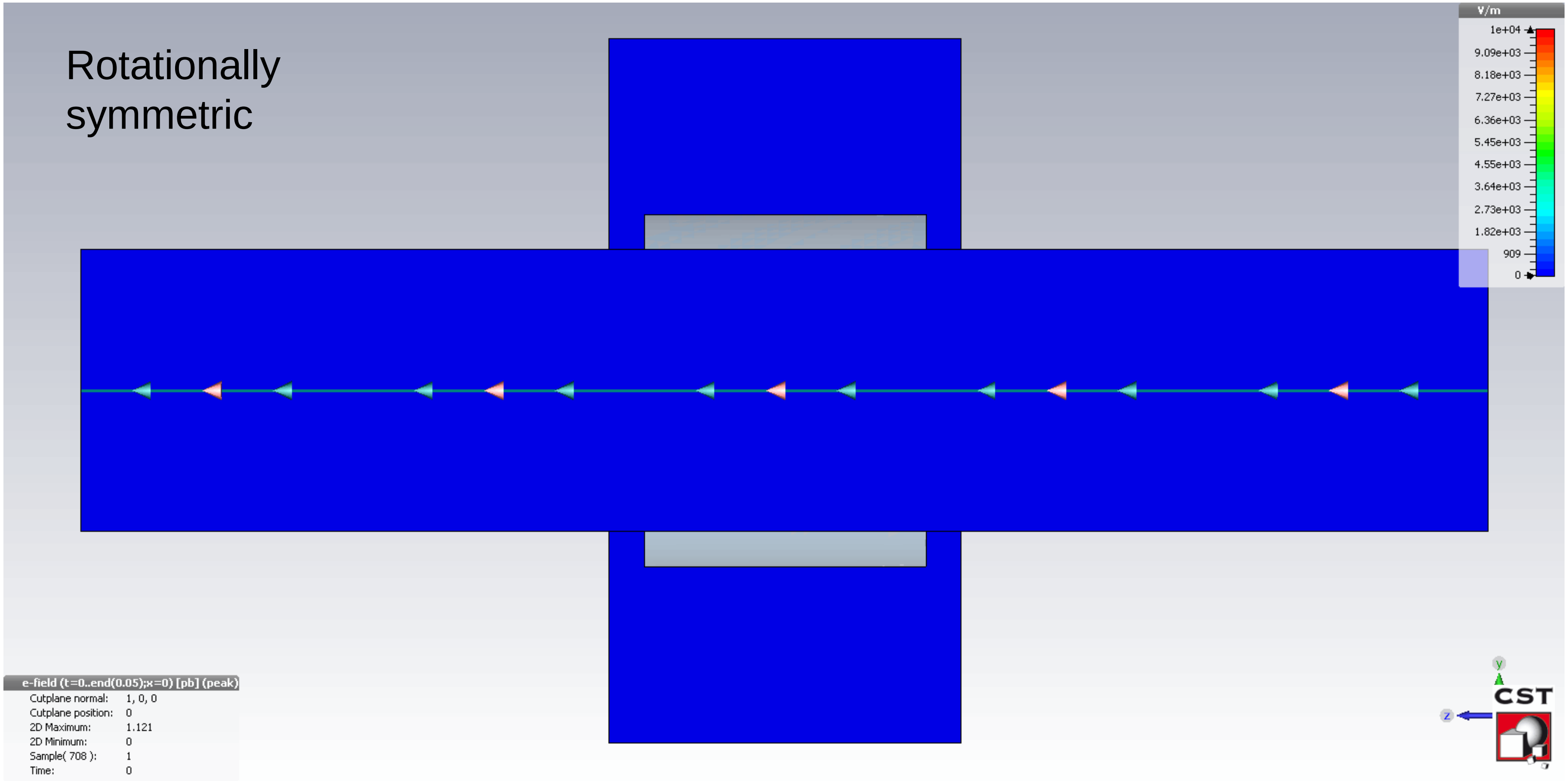




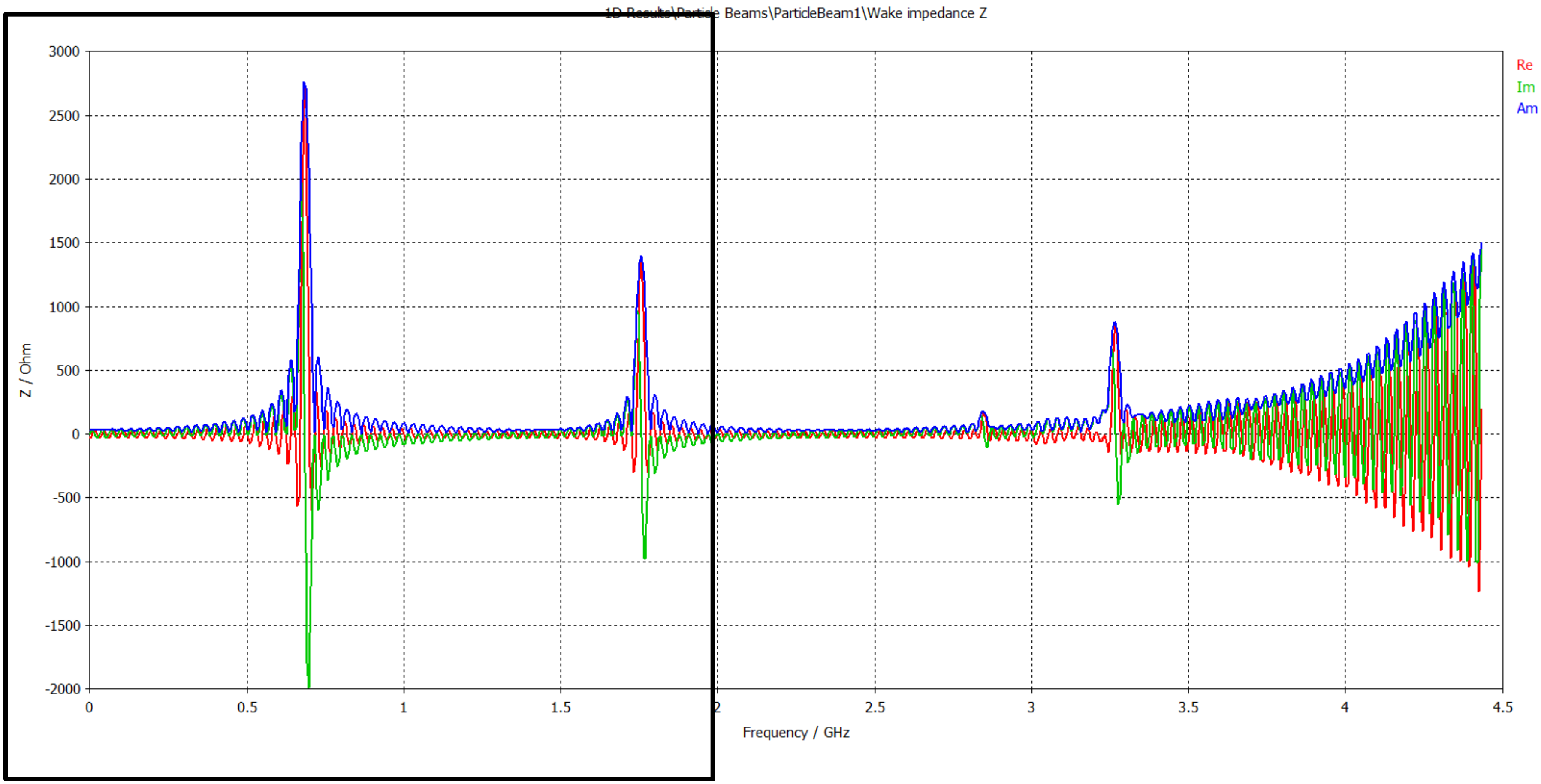
Fitting of material data on impulse response model required!

Wake Potential Example (Narrowband)

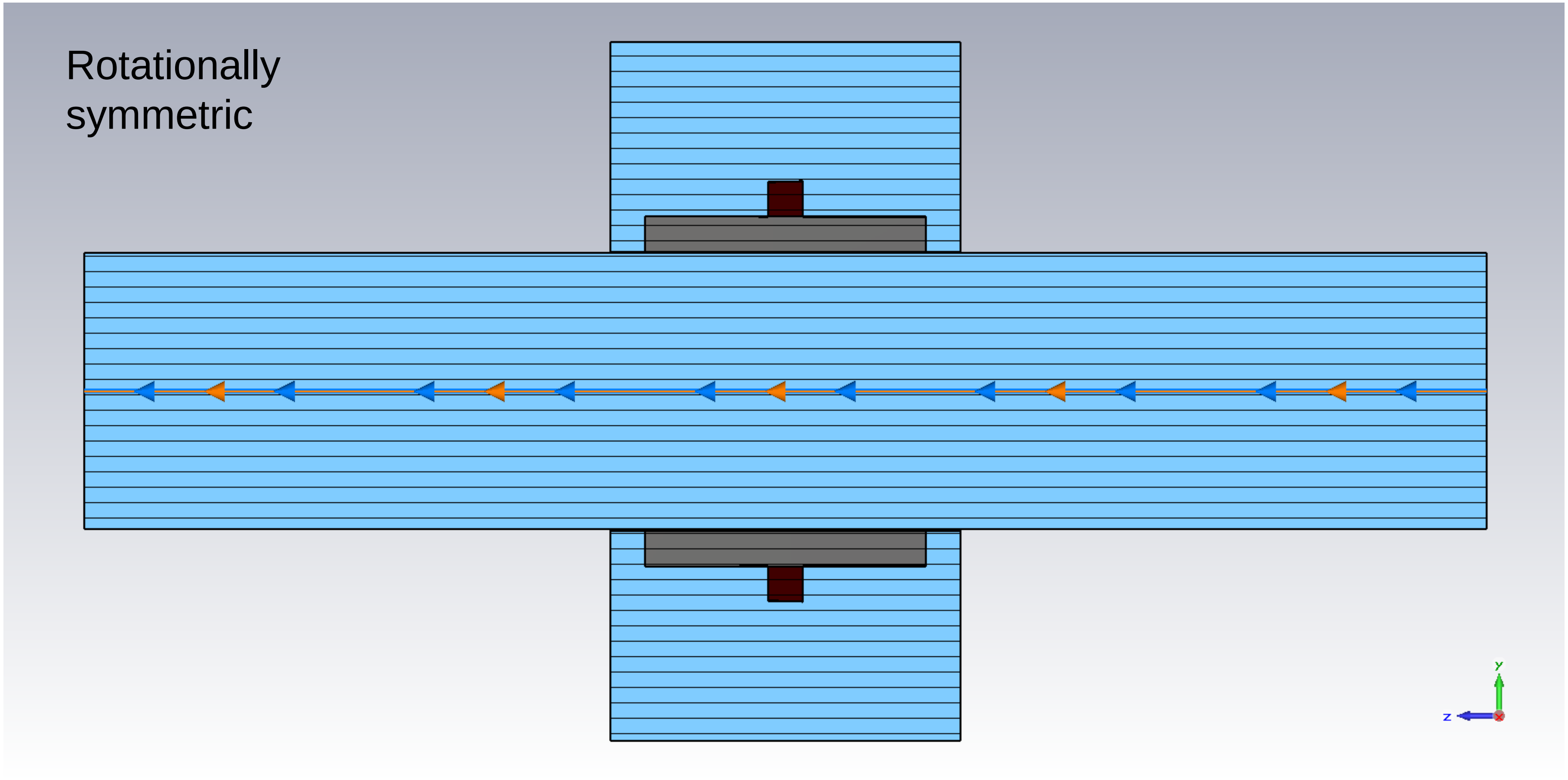
Parasitic cavity with 2 gaps (arbitrary…)



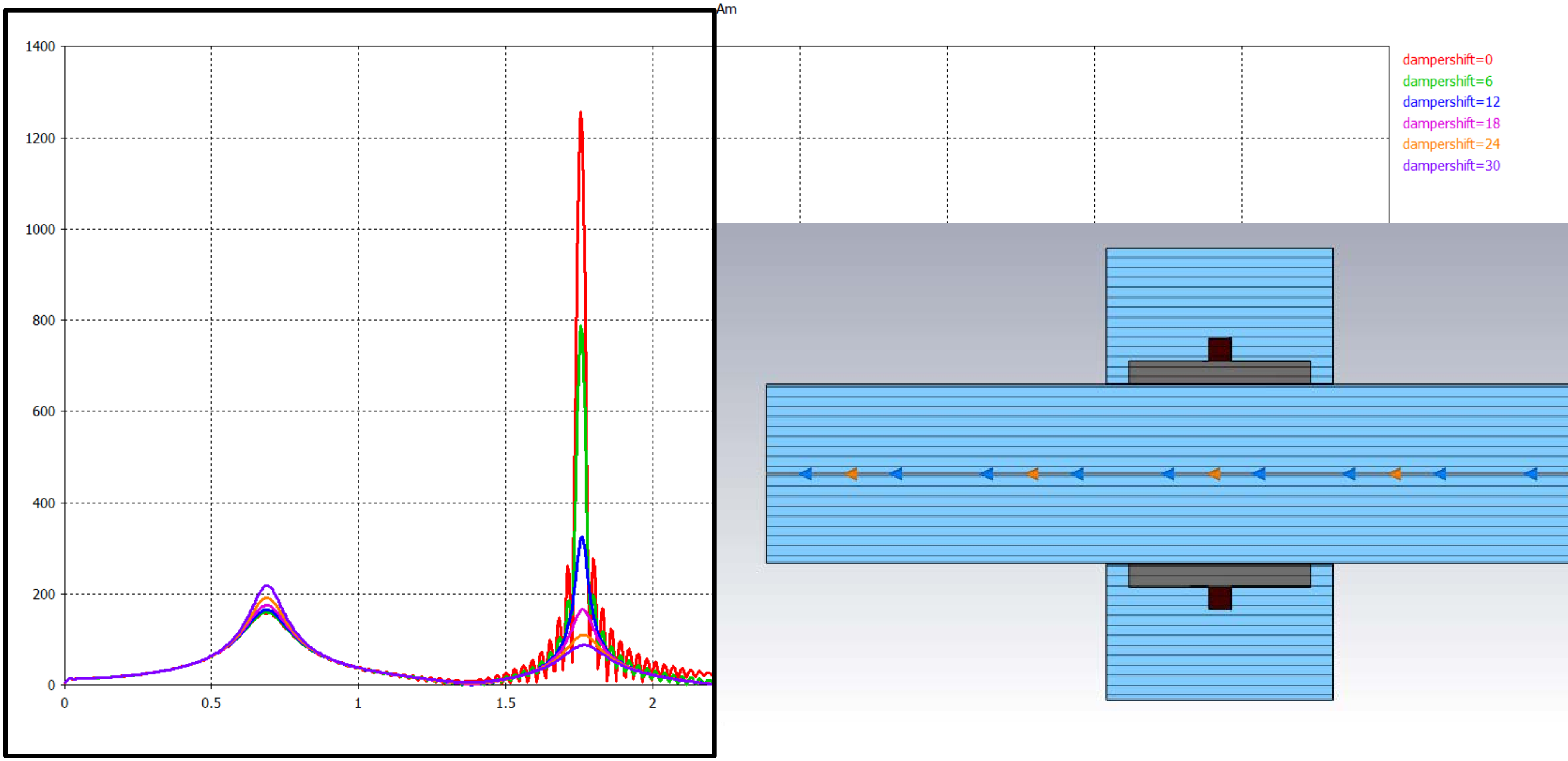
Longitudinal impedance
(unfiltered and coarse mesh)



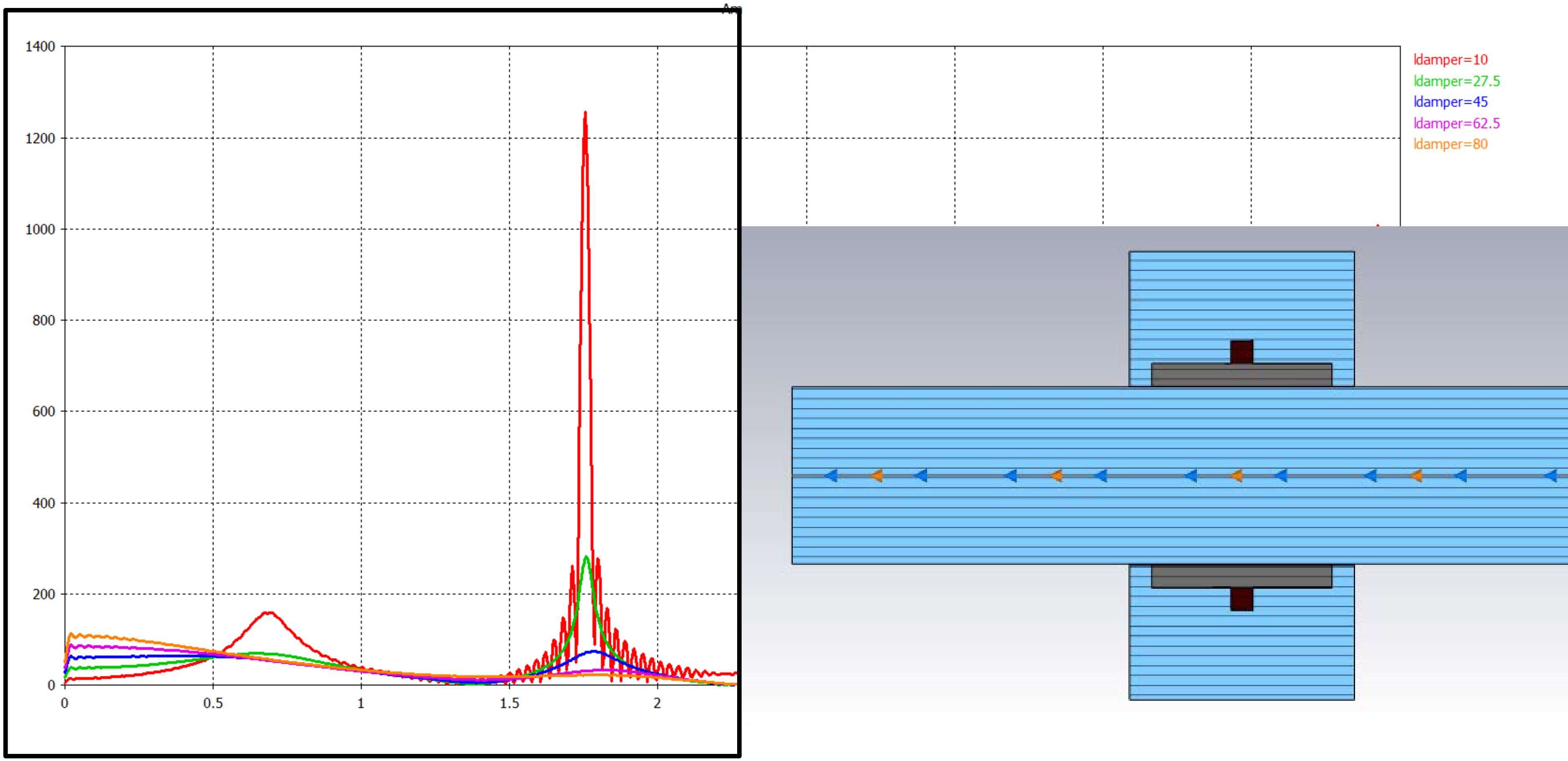
Introducing a Ferrite Ring to damp the modes in the structure



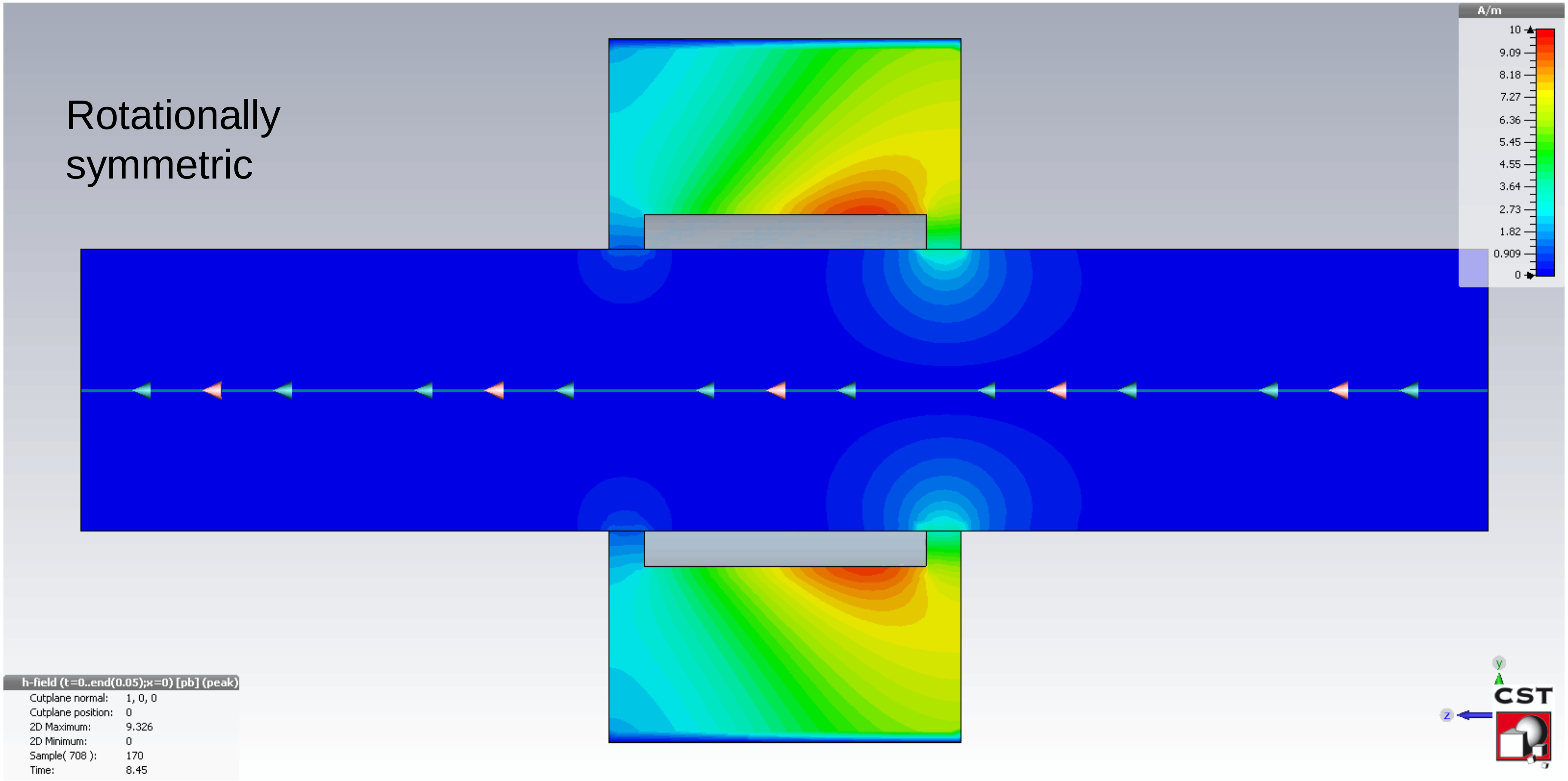
Shifting the ring



Increasing the length of the ring



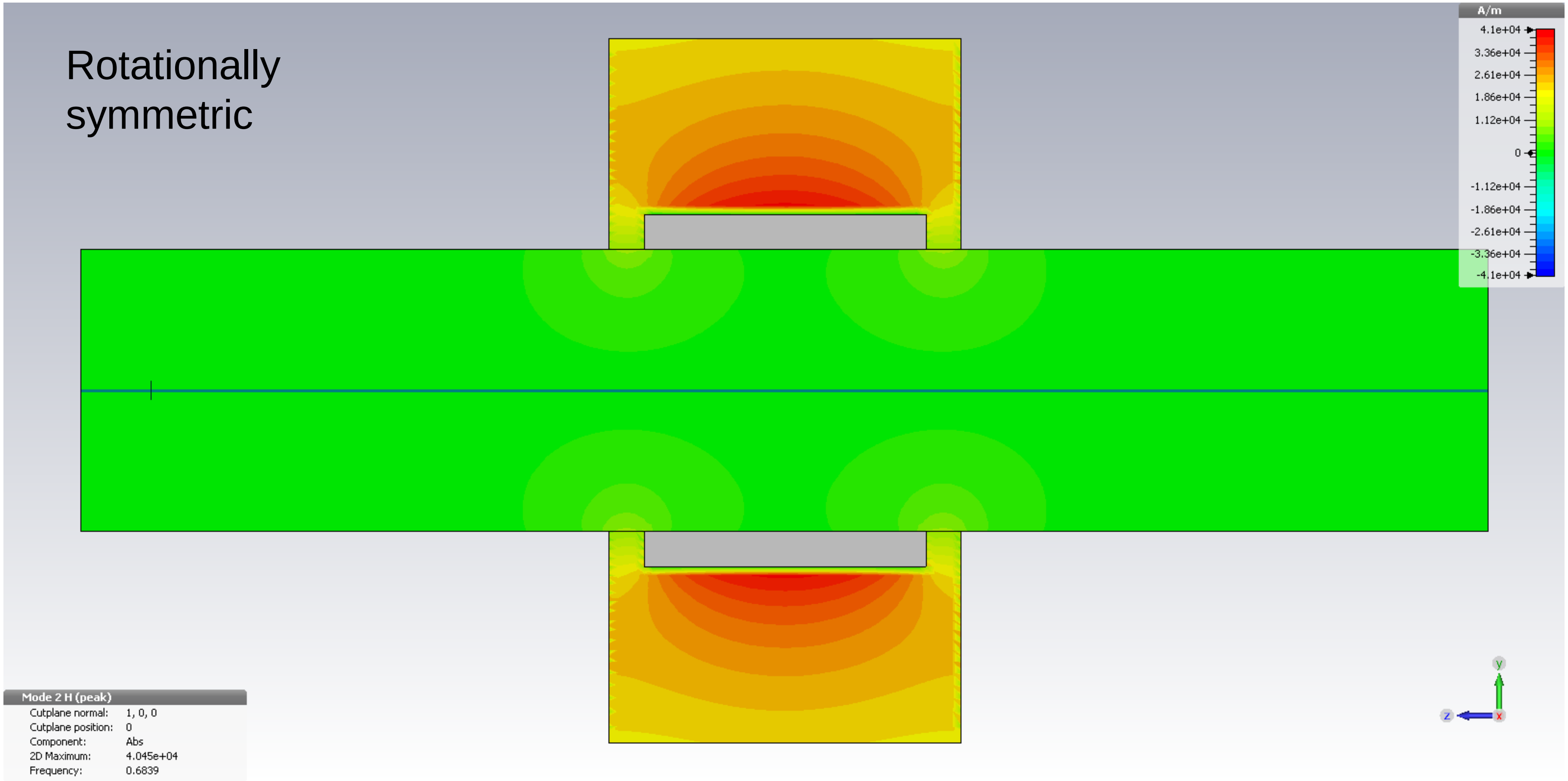
Magnetic field



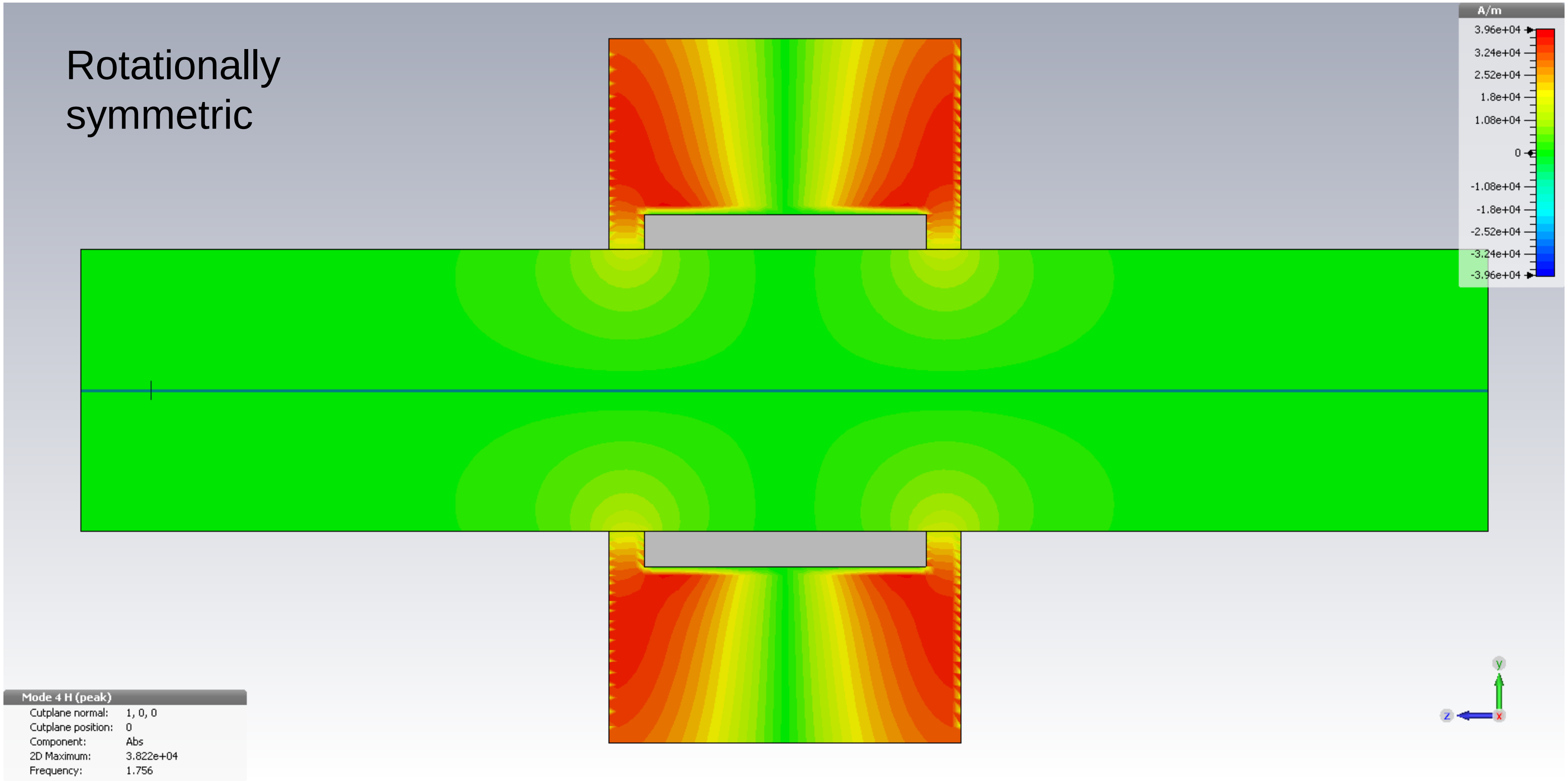
So we have to look at the magnetic field of the Eigenmodes of the structure…

First Mode H-Field

Rotationally symmetric



Second Mode H-Field



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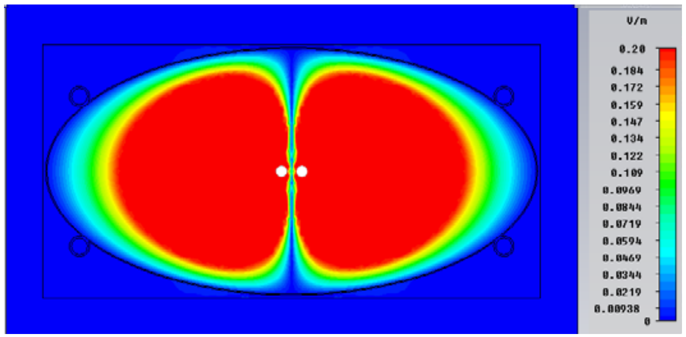
Overview of methods and codes

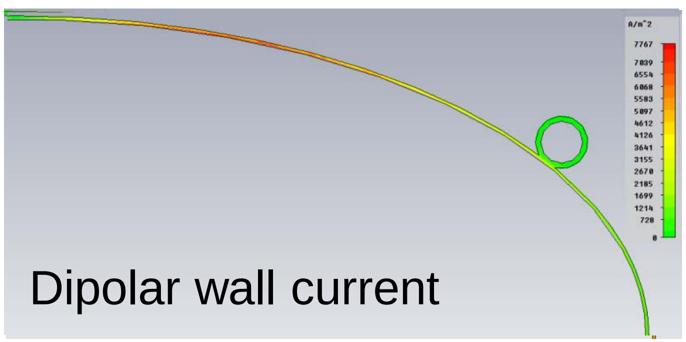
- **Explicit TD**: CST-PS, GdfidL, Echo, ABCI, …
  →extremely fast
- **Implicit TD**: ACE3P (SLAC), …
  → not limited by CFL, computationally expensive
- **FD**: BeamImpedance2D, …
  → nice for 2D (x,y) problems
- Using commercial software tools **without beam** (e.g. HFSS) for the determination of beam coupling impedance

See e.g.

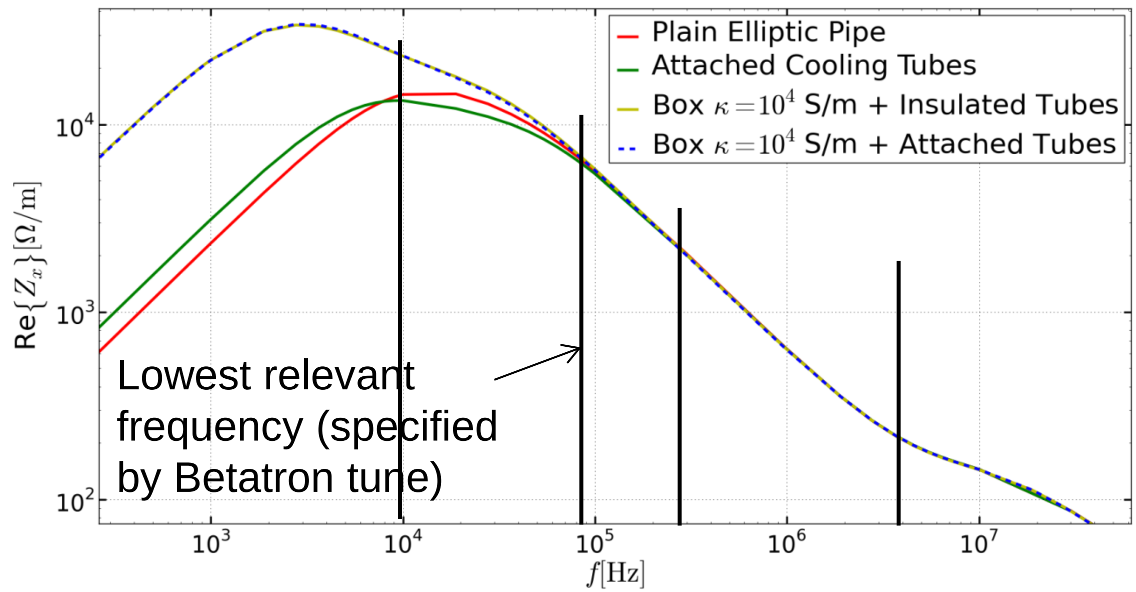
- T. Kroyer, CERN AB-Note 2008-17
- Niedermayer and Boine-Frankenheim NIM A 2012
- Kononenko and Grudiev PRSTAB 2011

Impedance of SIS100 dipole chamber
(FD power loss calculation with CST-EMS)





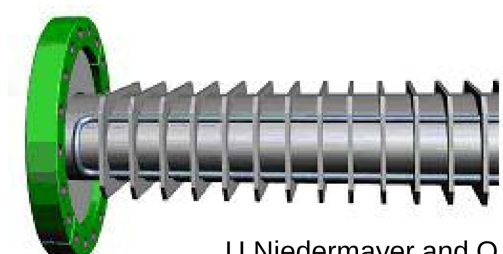
Longitudinal E-field and wall current for $f = 300\,\mathrm{kHz}$



$$\delta P = \frac{1}{2} \int_{pipe} \vec{\underline{E}} \cdot \vec{\underline{J}}^{*} \, \mathrm{d}V$$

$$\frac{\mathrm{Re}\left\{\underline{Z}_{\perp,x}(\omega)\right\}}{l} = \frac{c}{\omega d_x^2} \frac{1}{I^2} \frac{\delta P}{\delta z}$$

Neclect of beam charge
→ valid at LF and high beam velocity



U.Niedermayer and O. Boine-Frankenheim, *Analytical and numerical calculations of resistive wall impedances for thin beam pipe structures at low frequencies*, NIM A, 2012

Properties of Time Domain (TD) and Frequency Domain (FD) Computation

	(Explicit-) **TD** (Impedances obtained by DFT)	**FD** (Impedances obtained directly)
	▪ Broadband ▪ Matrix-vector products (cheap) ▪ Commercial / non-commercial codes available	▪ Arbitrary frequency points ▪ Beam velocity and dispersive material data are just parameters
	▪ **Limitation at Low Frequency** by uncertainty relation and CFL criterion ▪ Difficult for low beam velocity ▪ Difficult for dispersive material	▪ **Computationally expensive** (one *ill conditioned* linear system of equations (**LSE**) for each frequency) ▪ Optimized codes not (yet) available

Contents

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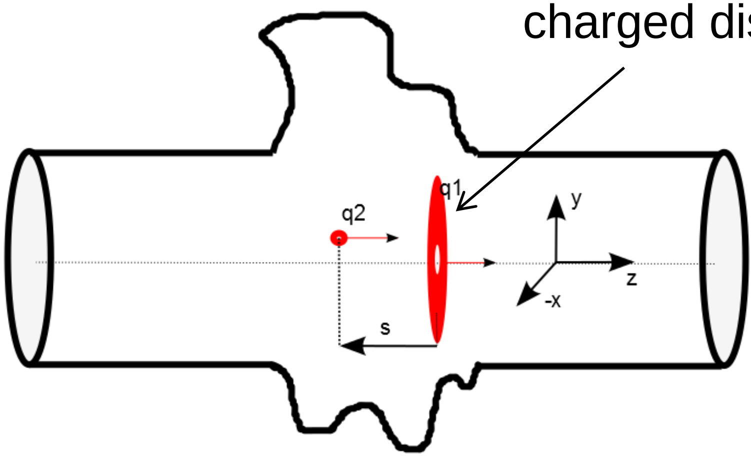
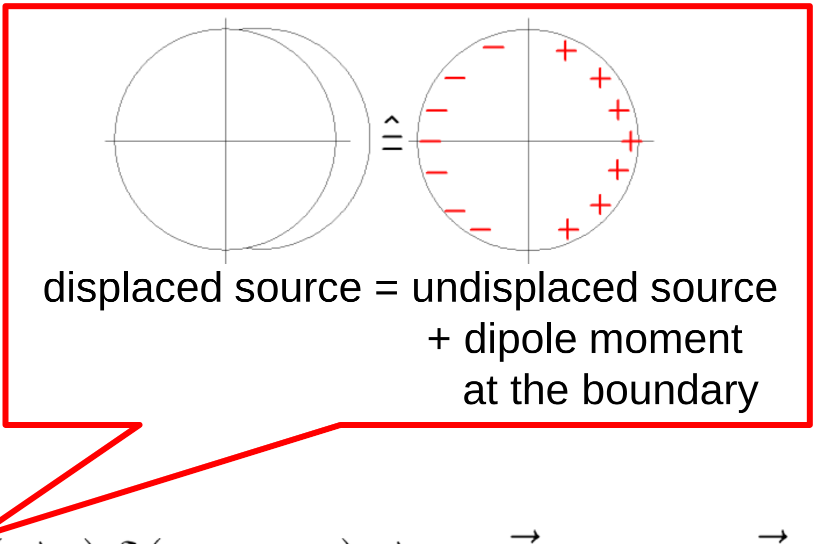
Defining Sources for Impedance Computation in the Frequency Domain





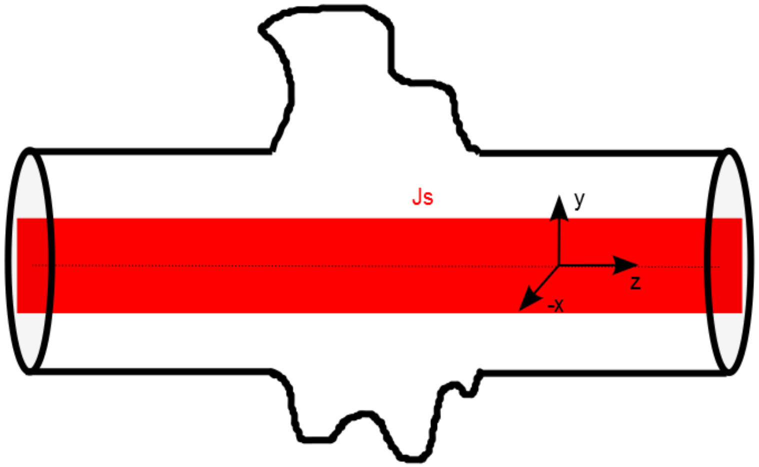
$$\vec{J}_s(\vec{r}_\perp, z, t) = \varrho_s(\vec{r}_\perp, z, t)\vec{v} = \sigma(\vec{r}_\perp)\delta(z - vt)\vec{v} = \vec{J}_{\mathrm{mono}} + \vec{J}_{\mathrm{dip}}$$

$$\sigma(\varrho, \varphi) \approx \frac{q}{\pi a^2}\left(\Theta(a - \varrho) + \delta(a - \varrho) d_x \cos\varphi\right)$$

Monopole moment → Longitudinal wake function / impedance

Dipole moment → Transverse wake function / impedance

Beam Coupling Impedance
→Fourier transform of wake function



$$\vec{J}_s(\vec{r}_\perp, z, t) = \sigma(\vec{r}_\perp)\delta(z - vt)\vec{v}$$

Fourier transform

$$\underline{\vec{J}}_s(\vec{r}_\perp, z, \omega) = \sigma(\vec{r}_\perp)e^{-i\omega z/v}\vec{e}_z$$

Longitudinal monopolar impedance

$$\underline{Z}_{\|}(\omega) = -\frac{1}{q^2}\int_{\text{beam}} \underline{\vec{E}} \cdot \underline{\vec{J}}^{\,*}_{\text{mono}} \mathrm{d}V$$

$$\text{Unit}: [\Omega]$$

$$\underline{Z}_{\|}(\widehat{\underline{\mathbf{e}}}(\omega)) = -\frac{1}{q^2}\widehat{\underline{\mathbf{e}}} \cdot \widehat{\widehat{\underline{\mathbf{j}}}}^{*}_{\text{mono}}$$

Transverse dipolar impedance

$$\underline{Z}_{\perp,x}(\omega) = -\frac{v}{(qd_x)^2\omega}\int_{\text{beam}} \underline{\vec{E}} \cdot \underline{\vec{J}}^{\,*}_{\text{dip}} \mathrm{d}V$$

$$\text{Unit}: [\Omega/\text{m}]$$

$$\underline{Z}_{\perp}(\widehat{\underline{\mathbf{e}}}(\omega)) = -\frac{v}{(qd_x)^2\omega}\widehat{\underline{\mathbf{e}}} \cdot \widehat{\widehat{\underline{\mathbf{j}}}}^{*}_{\text{dip}}$$

→ Volume integral definition well suited for computation on mesh

Frequency Domain Computation: Curl-Curl Equation and FIT

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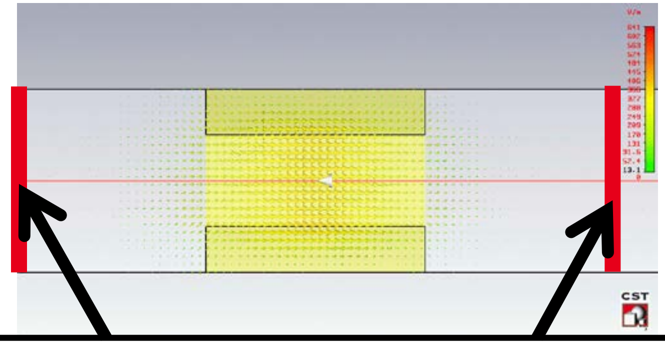
$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}} + i\omega\kappa\underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_s$$

FIT

$$\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C}\underline{\widehat{\mathbf{e}}} + i\omega\mathbf{M}_{\kappa}\underline{\widehat{\mathbf{e}}} - \omega^2\mathbf{M}_{\varepsilon}\underline{\widehat{\mathbf{e}}} = -i\omega\underline{\widehat{\widehat{\mathbf{j}}}}_s$$

Complex linear system of equations (LSE) size $3N_p$

- Dedicated boundary conditions required for the entry and exit of the beam
- Longitudinal Phaseshift given *a priori* → Floquet (periodic) boundary conditions:



Connection of the two boundaries with proper phase advance

U.Niedermayer and O. Boine Frankenheim, Proc. of ICAP 2012 and references therein

Frequency Domain Computation: Curl-Curl Equation and FIT

$$\nabla \times \underline{\mu}^{-1}\nabla \times \underline{\vec{E}} + i\omega\kappa\underline{\vec{E}} - \omega^2\varepsilon\underline{\vec{E}} = -i\omega\underline{\vec{J}}_{s}$$

FIT

$$\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C}\underline{\widehat{\mathbf{e}}} + i\omega\mathbf{M}_{\kappa}\underline{\widehat{\mathbf{e}}} - \omega^2\mathbf{M}_{\varepsilon}\underline{\widehat{\mathbf{e}}} = -i\omega\underline{\widehat{\widehat{\mathbf{j}}}}_{s}$$

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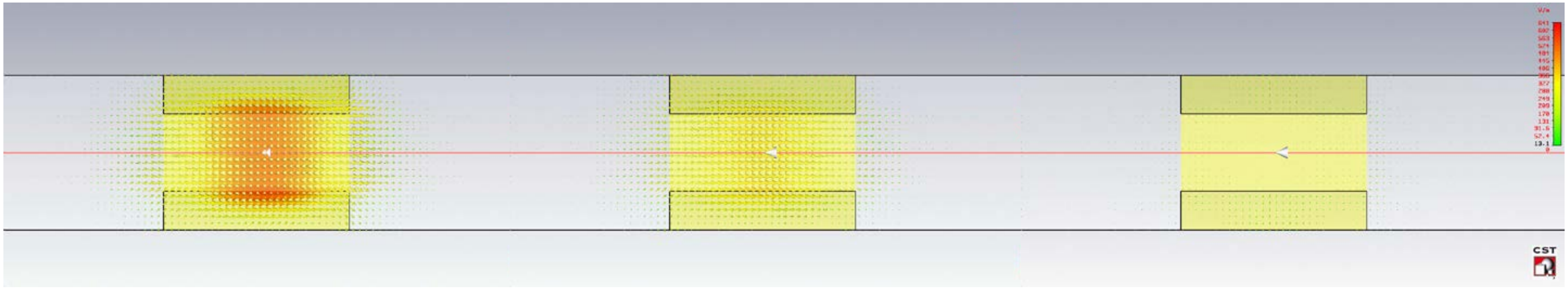
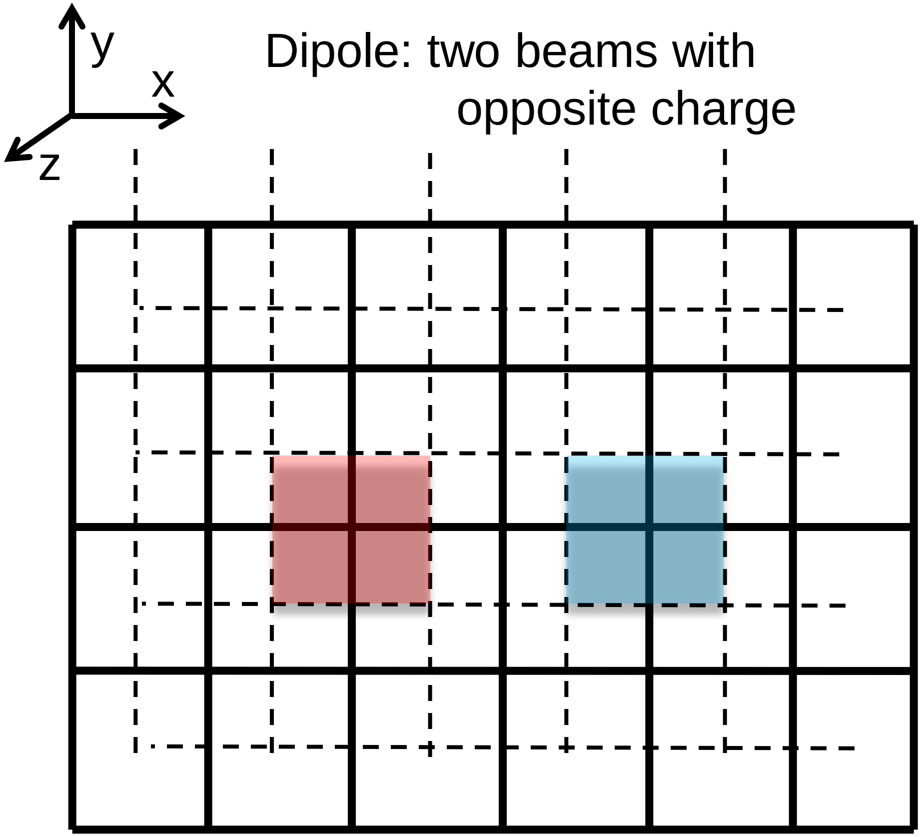


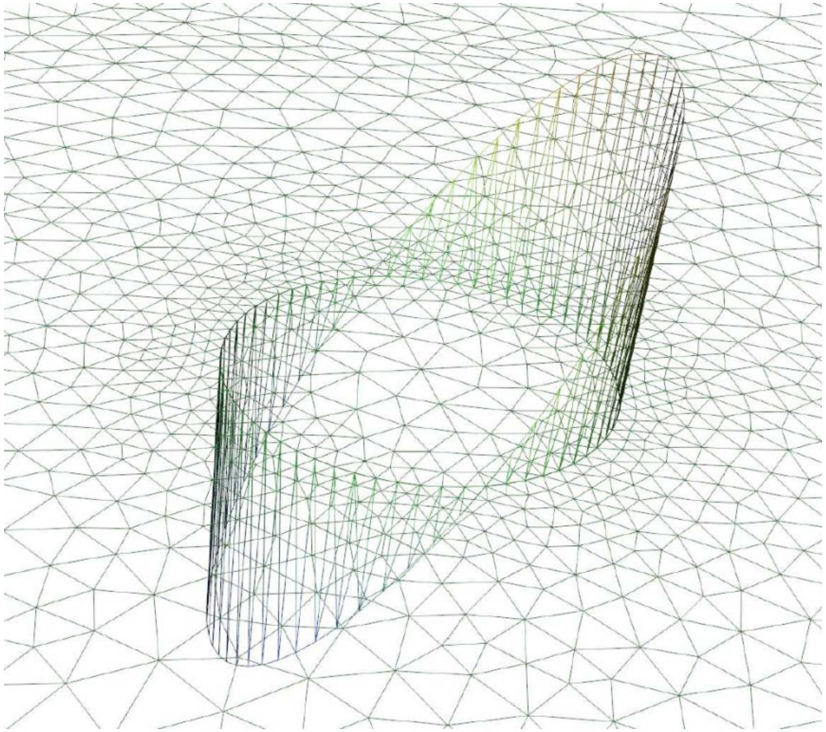
U.Niedermayer and O. Boine Frankenheim, Proc. of ICAP 2012 and references therein

Dipole Source for Transverse Impedance

Dipole: two beams with opposite charge



Dipole: ring shaped beam



Direct space charge fields $(\beta < 1)$ **cannot be properly modeled!**

Direct fields are known analytically and can be subtracted!

Finite Element Method (FEM)

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- Very flexible due to unstructured mesh
- Discretization of “weak formulation“
- Standard Ritz-Galerkin FEM: <u>trial</u> and <u>test</u> functions are identical
- 2D impedance solver implemented
- Open source package FEniCS *(A. Logg, K. Mardal, G. Wells et al.)*
  Mesh from Gmsh *(C. Geuzaine, J. Remacle)*
  → *all open source*
  → Weak formulation of PDE can be interpreted
  → No complex numbers, can be overcome by coupled function spaces

Discretization of the Electromagnetic Problem in 2D

$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}} + i\omega\kappa \underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_{\mathrm{s}}$$

$$\underline{\vec{E}} : \mathbb{R}^2 \to \mathbb{C}^3$$

Split:
Longitudinal / Transverse
Real / Imaginary

$$\underline{\vec{E}} = \begin{pmatrix} \vec{E}^{\mathrm{r}}_{\perp} \\ E^{\mathrm{r}}_{z} \end{pmatrix} + i \begin{pmatrix} \vec{E}^{\mathrm{i}}_{\perp} \\ E^{\mathrm{i}}_{z} \end{pmatrix}$$

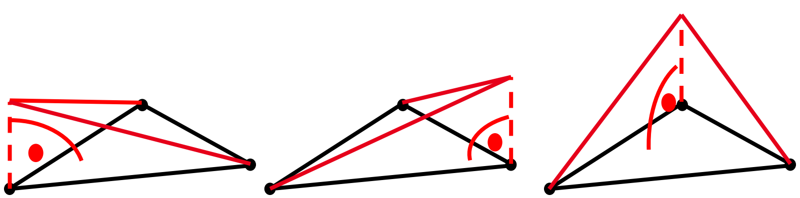
$$\nabla = \begin{pmatrix} \partial_x \\ \partial_y \\ -i\omega/v \end{pmatrix}$$

$$E_z^{\mathrm{r/i}} \in \mathcal{H}^1(\Omega)$$

Discretized by 1st order nodal elements

$$N_i(x,y) = a_i + b_i x + c_i y$$

$$N_i(\vec{x}_j) = \delta_{ij}$$

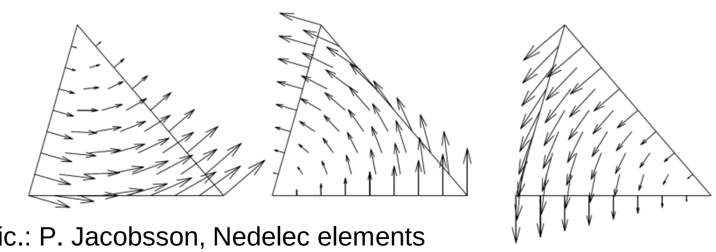


$$\vec{E}_{\perp}^{\mathrm{r/i}} \in \mathcal{H}^{\mathrm{curl}}_{\mathrm{2D}}(\Omega)$$

Discretized by 1st order Nédélec edge elements of the first kind

$$\vec{w}_i(x,y) = N_j \nabla_{\perp} N_k - N_k \nabla_{\perp} N_j$$

$$\frac{1}{|l_j|} \int_{l_j} \vec{w}_i \cdot \vec{t}_j \mathrm{d}s = \delta_{ij}$$



Pic.: P. Jacobsson, Nedelec elements for computational electromagnetics

Helmholtz Split
(Domain needs to be simply connected)

$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}} + i\omega\kappa\underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_{\mathrm{s}}$$

$$\underline{\vec{E}} = \underline{\vec{E}}_{\mathrm{curl}} + \underline{\vec{E}}_{\mathrm{div}}$$

solenoidal irrotational

$$\underline{\vec{E}}_{\mathrm{div}} = -\nabla \underline{\Phi}$$

$$-\nabla \cdot \underline{\varepsilon} \nabla \underline{\Phi} = \underline{\varrho} = \frac{1}{v} \underline{J}_{\mathrm{s,z}}$$

$$\underline{\varepsilon} = \varepsilon + \frac{\kappa}{i\omega}$$

$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}}_{\mathrm{curl}} - \omega^2 \underline{\varepsilon}\, \underline{\vec{E}}_{\mathrm{curl}} = \underline{\vec{R}}$$

$$\underline{\vec{R}} = \omega^2 \underline{\varepsilon}\, \underline{\vec{E}}_{\mathrm{div}} - i\omega \underline{\vec{J}}_{\mathrm{s}}$$

$$\nabla \cdot \underline{\vec{R}} = 0$$

“Continuity equation“

Ritz-Galerkin FEM Discretization

$$-\nabla \cdot \underline{\varepsilon} \nabla \underline{\Phi} = \underline{\varrho}$$

$$\begin{bmatrix} \mathbf{S}_{\varepsilon}^{\mathrm{rr}} + \mathbf{M}_{\varepsilon}^{\mathrm{rr}} & \mathbf{S}_{\kappa}^{\mathrm{ri}} + \mathbf{M}_{\kappa}^{\mathrm{ri}} \\ \mathbf{S}_{\kappa}^{\mathrm{ir}} + \mathbf{M}_{\kappa}^{\mathrm{ir}} & \mathbf{S}_{\varepsilon}^{\mathrm{ii}} + \mathbf{M}_{\varepsilon}^{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varphi}^{\mathrm{r}} \\ \boldsymbol{\varphi}^{\mathrm{i}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varrho}_s^{\mathrm{r}} \\ 0 \end{bmatrix}$$

$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}}_{\mathrm{curl}} - \omega^2 \underline{\varepsilon}\, \underline{\vec{E}}_{\mathrm{curl}} = \underline{\vec{R}}$$

$$\left[\mathbf{S}_{\mathrm{curlcurl}} + \mathbf{M}_{\underline{\varepsilon}} + \mathbf{M}_{\mathrm{SIBC}}\right] \mathbf{e}_{\mathrm{curl}} = \mathbf{r}$$

$$\mathbf{e}_{\mathrm{curl}} = \begin{bmatrix} \mathbf{e}_{\perp}^{\mathrm{r}} \\ \mathbf{e}_{\perp}^{\mathrm{i}} \\ \mathbf{e}_{\mathrm{z}}^{\mathrm{r}} \\ \mathbf{e}_{\mathrm{z}}^{\mathrm{i}} \end{bmatrix}$$

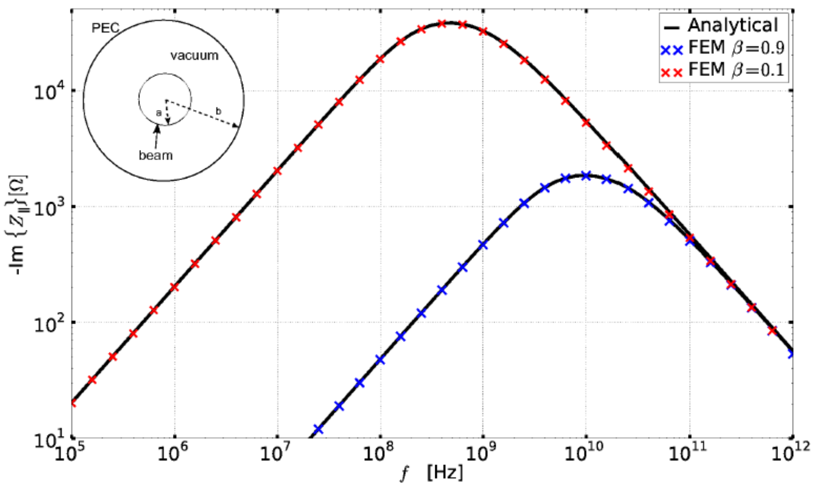
Few (<10^7) degrees of freedom (dof) in 2D → solved with sparse direct solver

U. Niedermayer *et al.*, **Space charge and resistive wall impedance computation in the frequency domain using the finite element method**, Phys. Rev. –STAB 18, 032001, 2015

Code named “BeamImpedance2D“ (in PYTHON)

Lonitudinal Impedance Examples

"Longitudinal space charge impedance"

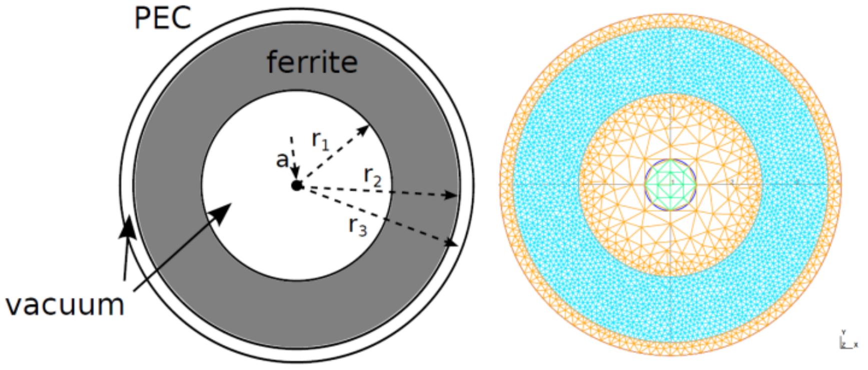


Beam of radius *a=1*cm in perfecly conducting pipe of radius *b=4*cm

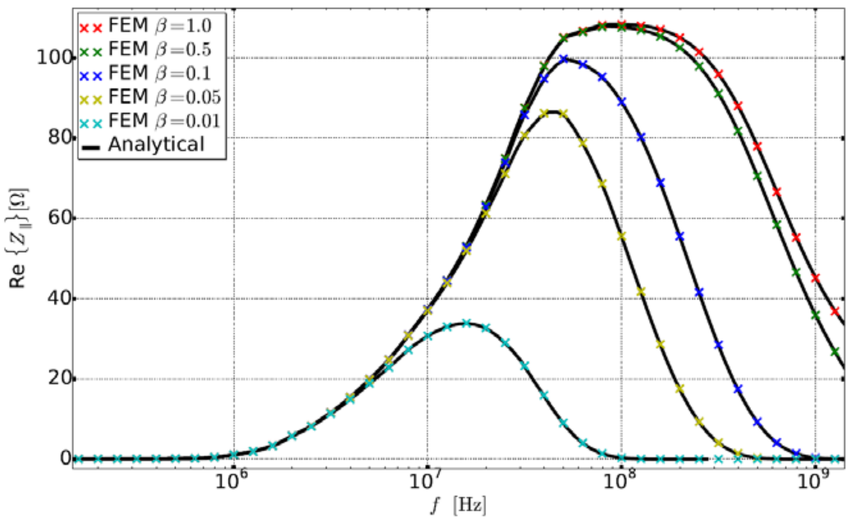
Asymptotes:

$$\underline{Z}_{\parallel,\mathrm{LF}}^{\mathrm{spch}} = \frac{-i\omega\mu_0 l g_0}{2\pi\beta^2\gamma^2}\ , \quad g_0 = \frac{1}{4} + \ln\frac{b}{a}$$

$$\underline{Z}_{\parallel,\mathrm{HF}}^{\mathrm{spch}} = \frac{-il}{\omega\varepsilon_0\pi a^2}$$

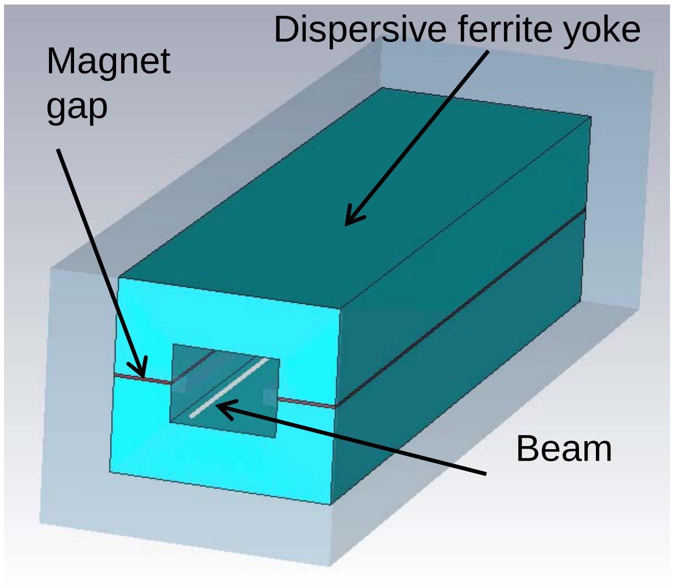


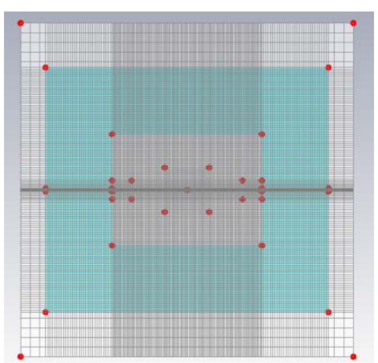
A ferrite ring (dispersively lossy material)

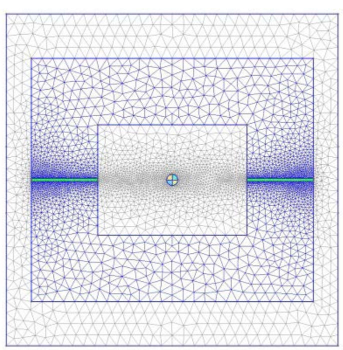


Application: Beam Induced Heat Power

in SIS 100 transfer kicker magnet



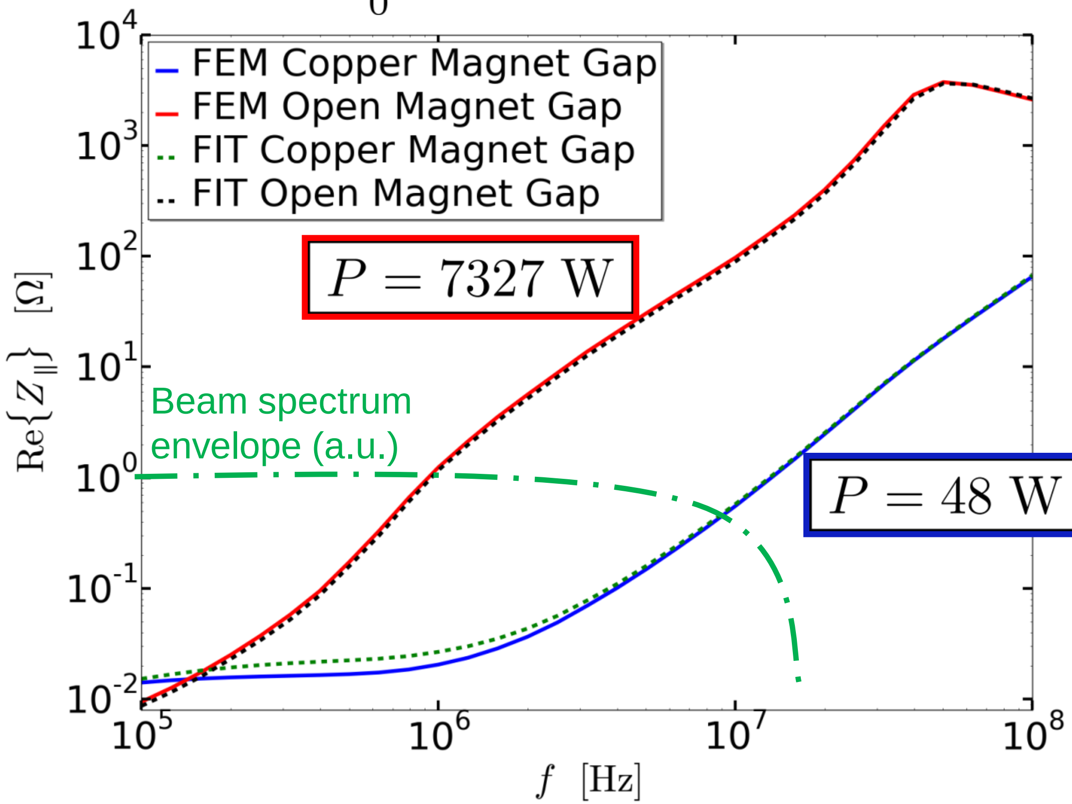




Angular revolution frequency

Beam power spectral density

$$P = \omega_0 \frac{q^2 v^2}{2\pi^2} \int_0^\infty \mathrm{Re}\{\underline{Z}_{\parallel}(\omega)\} |\underline{\lambda}(\omega)|^2 \mathrm{d}\omega$$



Transverse Impedance Example

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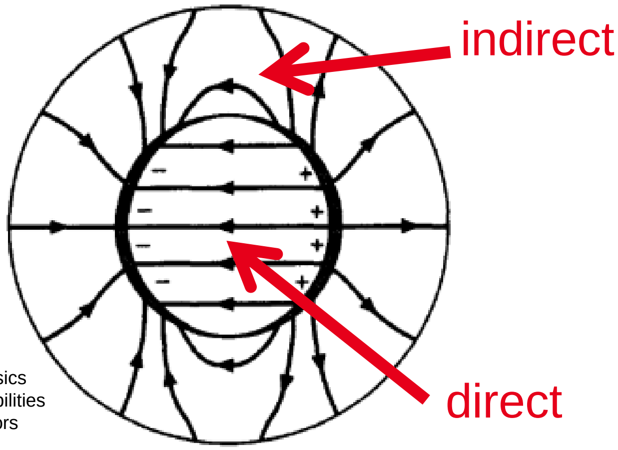
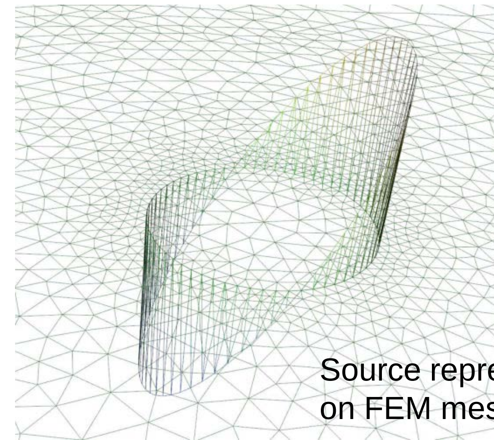
“Transverse space charge impedance“

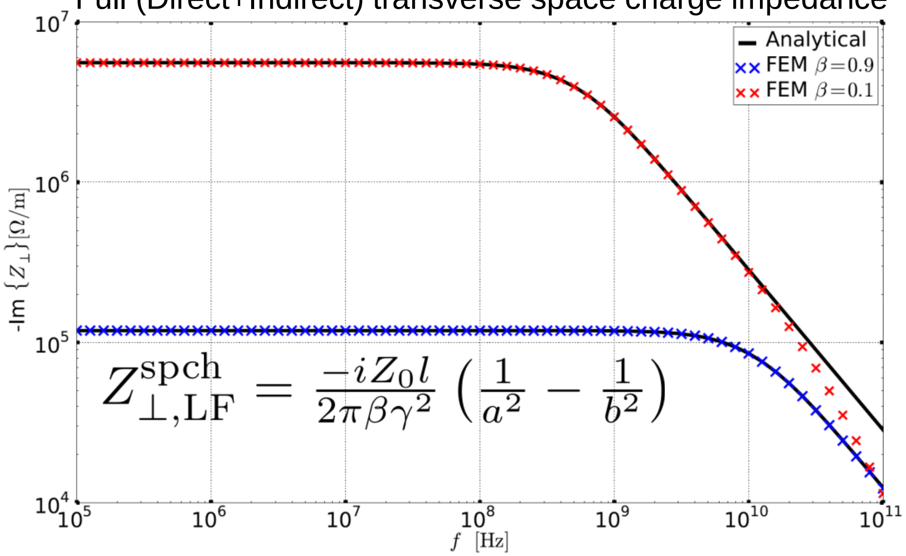


Picture: A. W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators

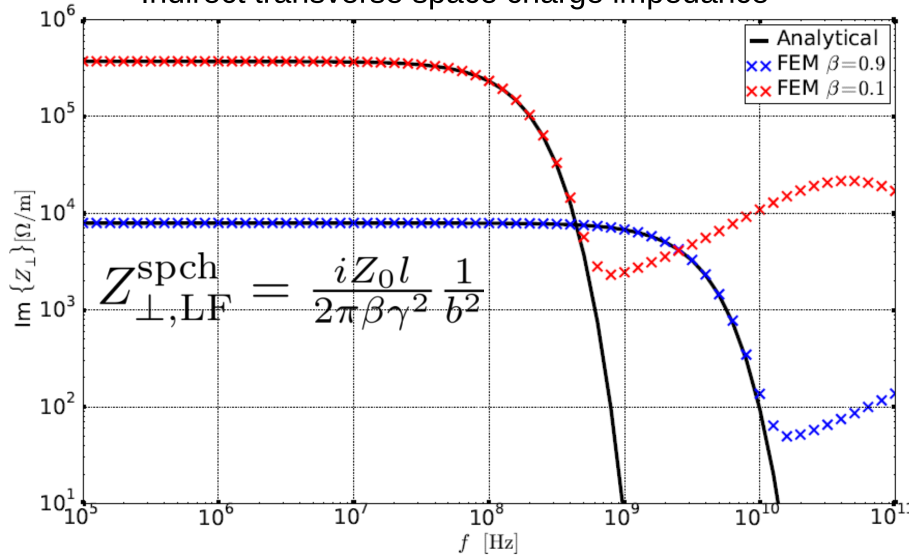


Source representation on FEM mesh (a.u.)

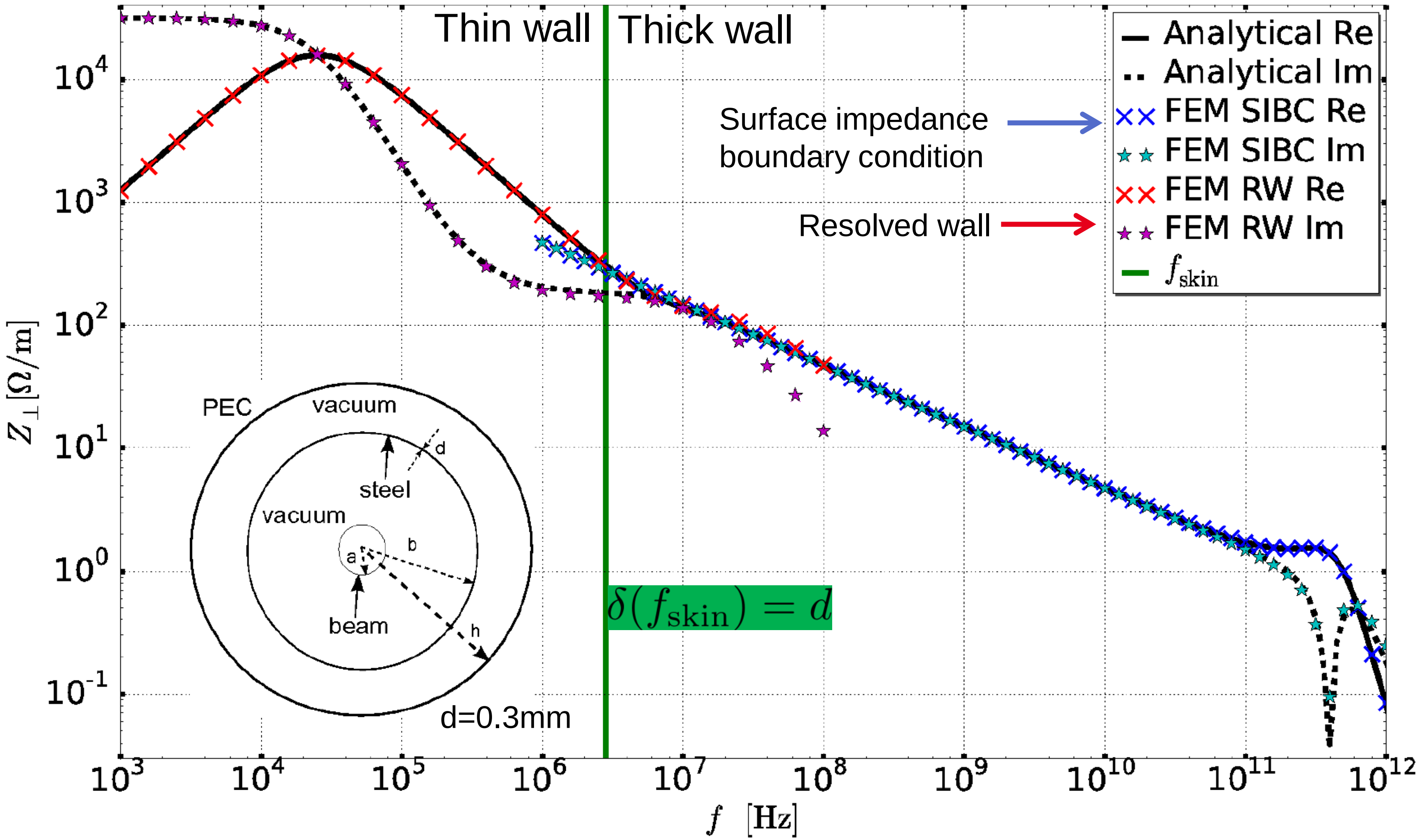
Full (Direct+Indirect) transverse space charge impedance



Indirect transverse space charge impedance



Transverse Resistive Wall Impedance: Thin Steel Beam Pipe (idealized SIS-100 pipe)



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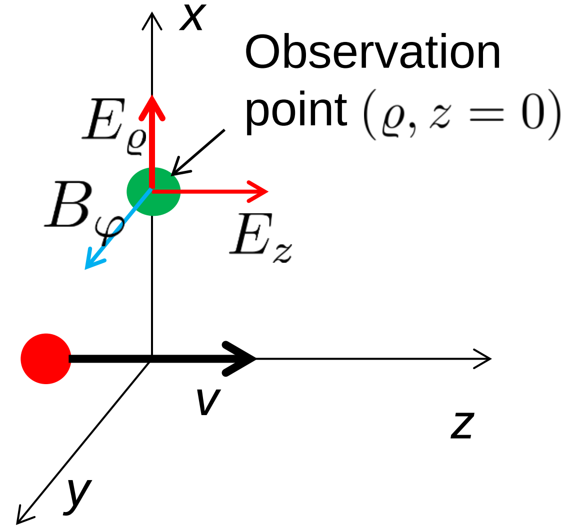
Motivation for Wire Bench Measurements: Source Fields

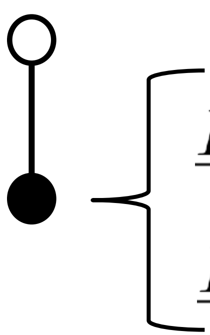
- Coulomb field of point charge

$$\vec{E}' = \frac{q}{4\pi\varepsilon}\left(\frac{\varrho'}{\sqrt{\varrho'^2+z'^2}^3}\vec{e}_\varrho + \frac{z'}{\sqrt{\varrho'^2+z'^2}^3}\vec{e}_z\right)$$



- Lorentz transformation to lab-frame

$$\vec{E} = \frac{q}{4\pi\varepsilon}\left(\frac{\gamma\varrho}{\sqrt{\varrho^2+(\beta\gamma ct)^2}^3}\vec{e}_\varrho + \frac{-\beta\gamma ct}{\sqrt{\varrho^2+(\beta\gamma ct)^2}^3}\vec{e}_z\right), \quad B_\varphi = \frac{v}{c^2}E_\varrho$$



$$\underline{E}_z = iq\frac{\mu_0}{2\pi}\frac{\omega}{\beta^2\gamma^2}\mathrm{K}_0\left(\frac{|\omega|}{\beta\gamma c}\varrho\right) \xrightarrow{\gamma\to\infty} 0$$

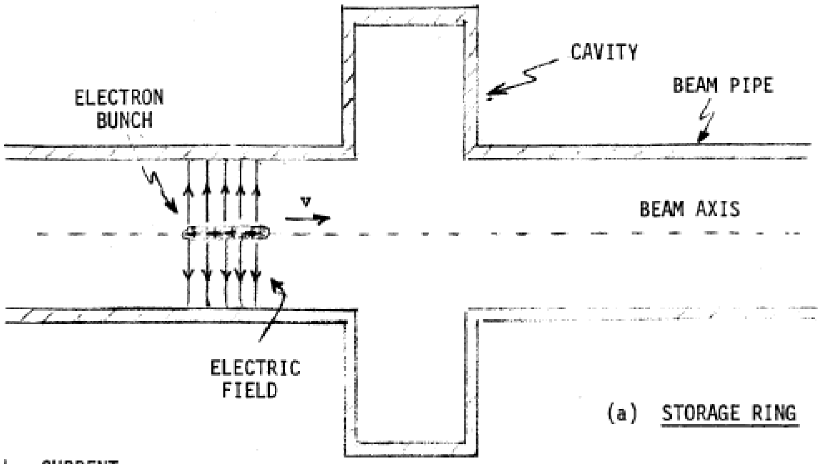
"Source Fields"

$$\underline{E}_\varrho = q\frac{\mu_0}{2\pi}\frac{|\omega|}{\beta^2\gamma}\mathrm{K}_1\left(\frac{|\omega|}{\beta\gamma c}\varrho\right) \xrightarrow{\gamma\to\infty} q\frac{Z_0}{2\pi\varrho}$$

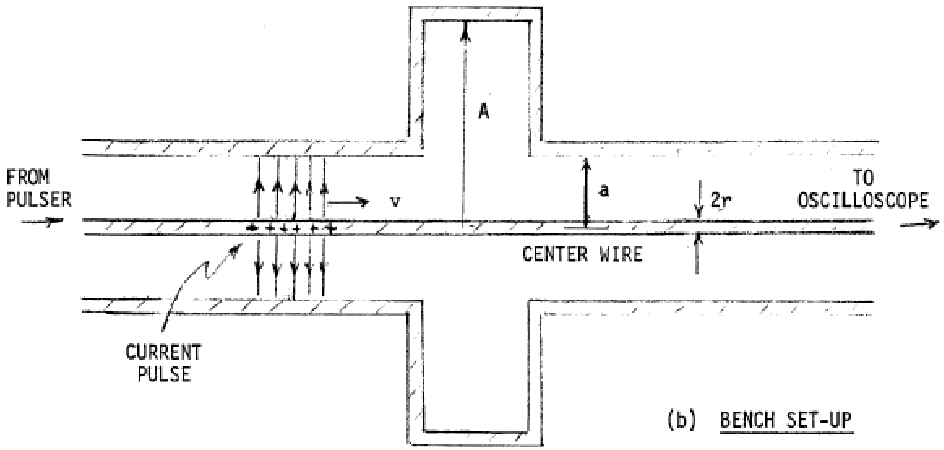
TEM Mode!

Motivation and History of Wire Bench Measurements

Beam



Bench



M. Sands, J. Rees, SLAC Report PEP-95, 1974

- The two setups produce approximately the same image current in the wall
- Measurements of loss factors in time domain

It has evolved since 1974…

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- Read as many papers as possible before you start!
  - F. Caspers, article in the Handbook of Accelerator Physics and Engineering
  - F. Caspers and A. Mostacci, talk given at ICFA mini workshop on wakefields and impedance, Erice 2014
  - G. Nassibian and F. Sacherer, “Methods for Measuring Transverse Coupling Impedances,” Nucl. Instrum. Meth., vol. 159, no. 6, pp. 21–27, 1978
  - H. Hahn and F. Pedersen, “On Coaxial Wire Measurements of the Longitudinal Coupling Impedance,” BNL Report 78-9, 1978
  - T. Kroyer, F. Caspers, and E. Gaxiola, “Longitudinal and Transverse Wire Measurements for the Evaluation of Impedance Reduction Measures on the MKE Extraction Kickers,” CERN Rep., 2007
  - V. Vaccaro, “Coupling Impedance Measurements: An improved wire method,” INFN/TC-94/023, 1994
  - E. Jensen, “An improved log-formula for homogeneously distributed impedance,” PS/RF/2000-001
  - A. Argan, L. Palumbo, M. R. Masullo, and V. G. Vaccaro, “On the Sands and Rees Measurement Method of the Longitudinal Coupling Impedance”, Proc. of PAC 8, 1999
  - F. Caspers, C. Gonzalez, M. D’yachkov, E. Shaposhnikova, H. Tsutsui, Impedance Measurement Of The SPS MKE Kicker By Means Of The Coaxial Wire Method, PS/RF/Note 2000-004
  - E. Métral, F. Caspers, M. Giovannozzi, A. Grudiev, T. Kroyer, and L. Sermeus, “Kicker impedance measurements for the future multiturn extraction of the CERN Proton Synchrotron,” in EPAC, Edinburgh, 2006
  - Many more, see proceedings

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  - Examples with the Finite Integration Technique (FIT)
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  - Frequency domain simulations
- **Wire bench measurements**
  - Primer on S-parameters and Vector Network Analysis (VNA)
  - Connection between S-parameters and beam coupling impedance
  - Modeling and simulation of the wire measurement

Wave Amplitudes and Scattering Parameters

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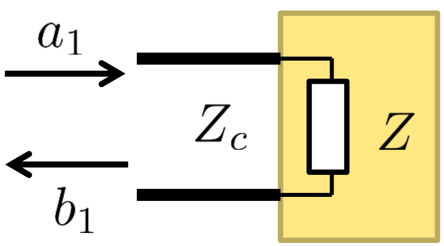
- Voltage cannot be uniquely defined in RF systems
- Thus one defines power flow parameters a_i and b_i (Unit: sqrt(W)) which are related to the voltage and current in a TEM line by

$$a_i := \frac{1}{2\sqrt{Z_c}}(U_i + Z_c I_i)\ , \quad b_i := \frac{1}{2\sqrt{Z_c}}(U_i - Z_c I_i)$$



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{z-1}{z+1} \qquad z = \frac{Z}{Z_c}$$

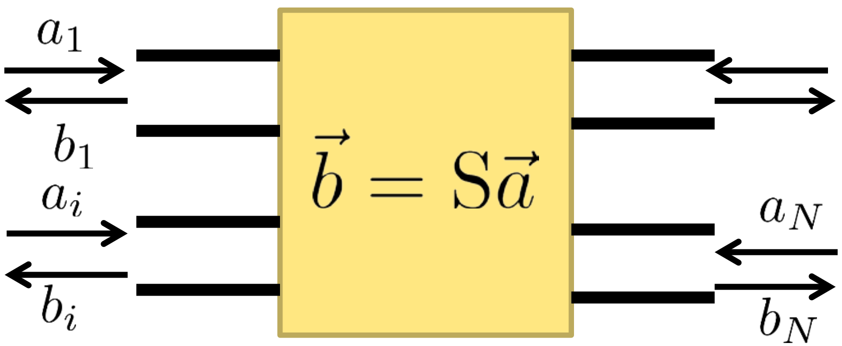
$$\Gamma(1/z) = -\Gamma(z)$$

“Smith Chart“ maps between z and Γ

Scattering parameters measurement

- For simplicity we consider only TEM modes with same Z_c at each port

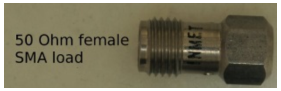
$$S_{ij} = \left. \frac{b_i}{a_j} \right|_{a_k=0 \quad \forall k \neq j}$$

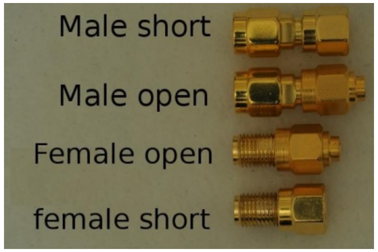


Vector Network Analyzer (VNA)

- Measures S-parameters
  →'vector' means mag and phase
- Stimulus and detection
- Calibration of cables to reference plane required (SOLT)
- Phase stable coaxial cables required
- Can convert S,Z,Y,T paramerters internally
- Can display Bode, Smith, Polar…

Examples of calibration standards

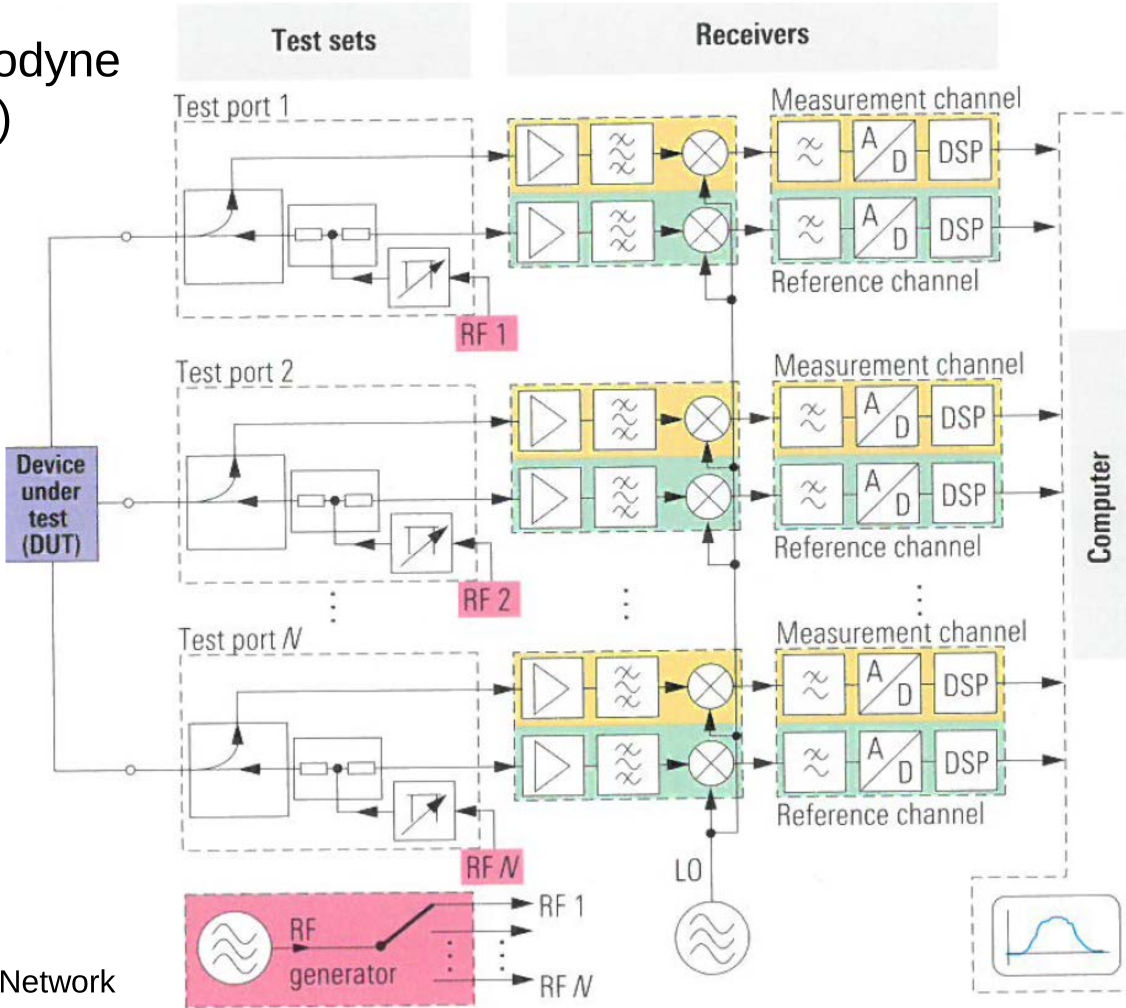




Pics: http://www.kirkbymicrowave.co.uk/support/85033/HP/

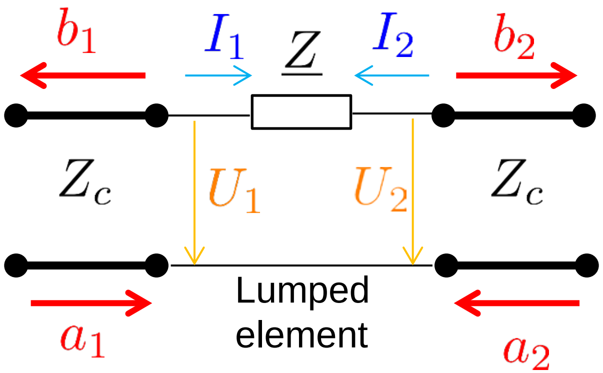
Internal Setup of the VNA

- Most VNA work with heterodyne detection (i.e. 2 oscillators)
- IF filter (digital FIR filter) bandwidth to be chosen → trade-off between noise level and filter filling time (sweep time)
- Averaging
- 4-port VNAs offer conversion to symmetric S-parameters



Picture: M. Hiebel, Fundamentals of Vector Network Analysis, published by Rohde&Schwarz, 2011

Example: Calculation of S-parameters for a resistor within a transmission line



$$a_i := \frac{1}{2\sqrt{Z_c}}(U_i + Z_c I_i)$$

$$b_i := \frac{1}{2\sqrt{Z_c}}(U_i - Z_c I_i)$$

$$a_i + b_i = U_i/\sqrt{Z_c}$$

$$a_i - b_i = I_i\sqrt{Z_c}$$

$$\left.\begin{aligned} a_2 = 0 \Rightarrow U_2 &= -Z_c I_2 \\ I_2 &= -I_1 \\ U_1 &= Z I_1 + U_2 \end{aligned}\right\} \frac{U_1}{I_1} = Z + Z_c$$

$$S_{11} = \frac{b_1}{a_1} = \frac{U_1 - Z_c I_1}{U_1 + Z_c I_1} = \frac{\frac{U_1}{I_1} - Z_c}{\frac{U_1}{I_1} + Z_c} = \frac{Z}{Z + 2Z_c}$$

$$S_{21} = \frac{b_2}{a_1} = \frac{U_2 - Z_c I_2}{U_1 + Z_c I_1} = -\frac{\frac{U_2}{I_2} - Z_c}{\frac{U_1}{I_1} + Z_c} = \frac{2Z_c}{Z + 2Z_c}$$

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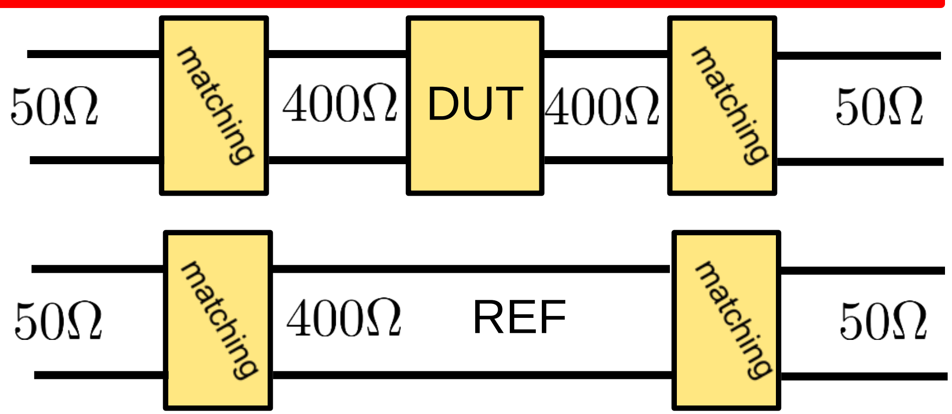
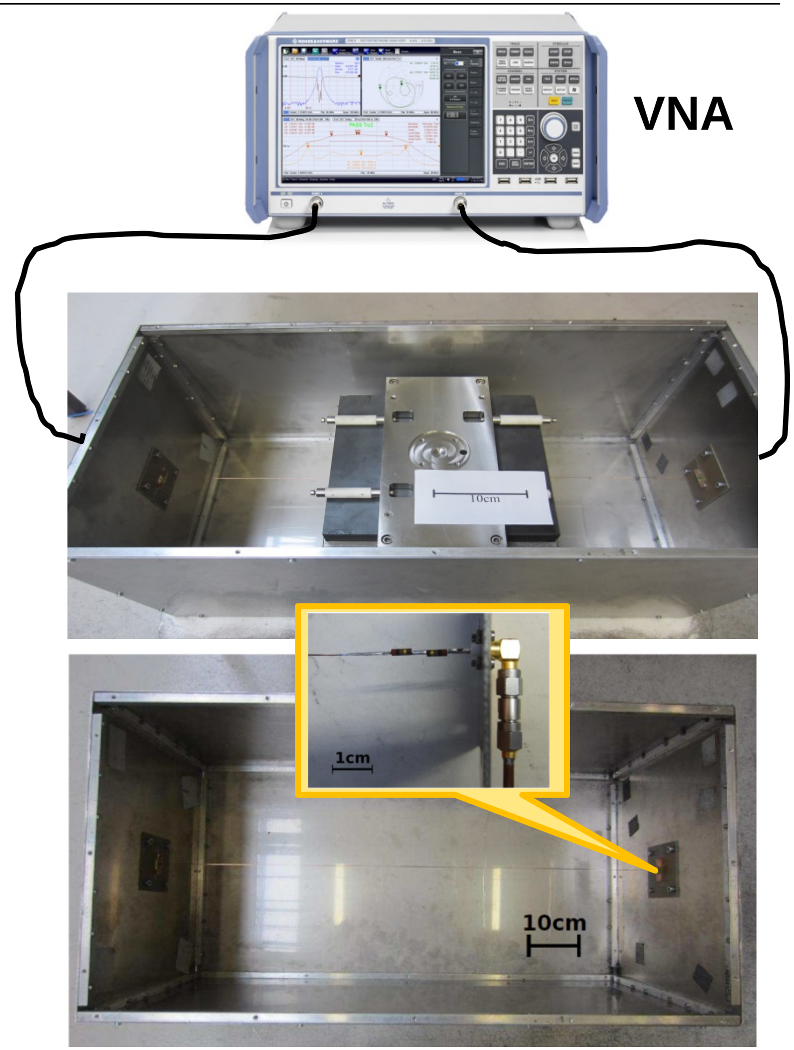
Bench Measurements of Broadband Impedances

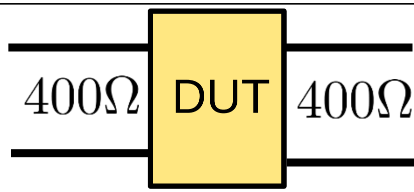
- Measurement in the Frequency Domain
- Measure transmission parameter S_{21}
- Two fundamental issues
  - Corresponds only to $\beta = 1$
    → scaling by FD simulation
  - Wire must be thin
    → high characteristic impedance





A Priori Distinction between Lumped and Distributed Impedance

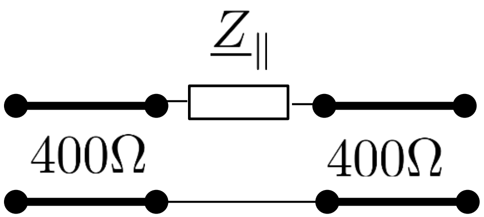
De-embedded structure:



Characteristic Impedance

$$Z_c = 400\Omega$$

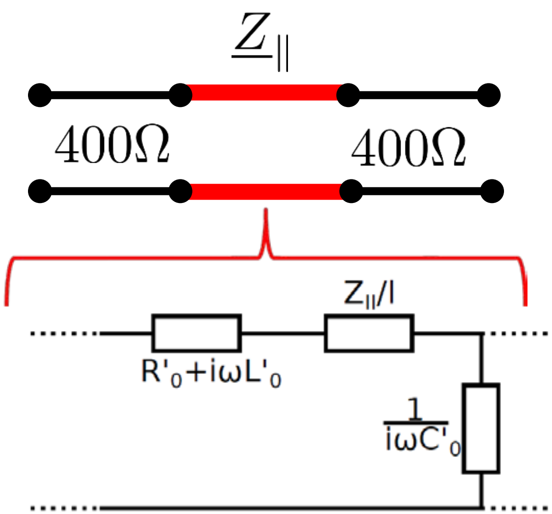
Lumped (concentrated) impedance



$$\underline{Z}_{||,\mathrm{HP}}^{\mathrm{lump}} = 2Z_c \frac{S_{21}^{\mathrm{REF}} - S_{21}^{\mathrm{DUT}}}{S_{21}^{\mathrm{DUT}}}$$

Hahn and Pedersen 1978

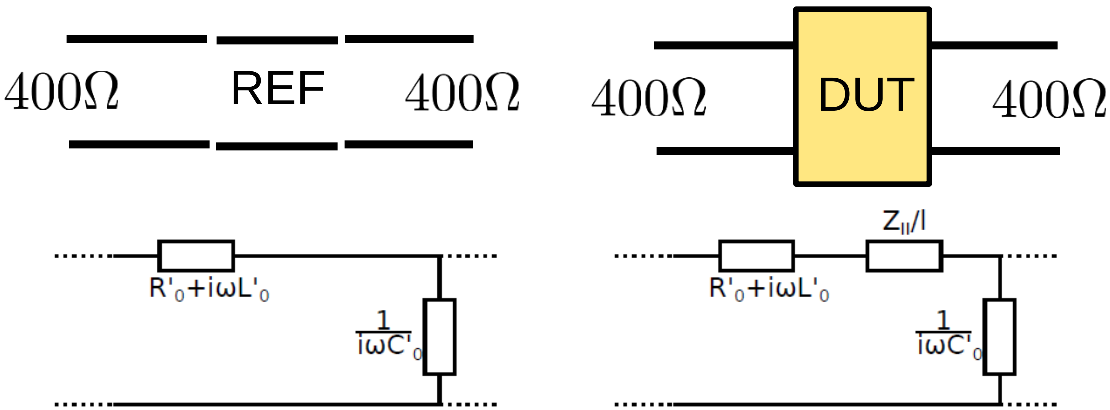
Equally distributed impedance



$$\underline{Z}_c^{\mathrm{DUT}} \approx Z_c$$

$$S_{21}^{\mathrm{DUT}} = e^{-ik_z^{\mathrm{DUT}} l} \qquad S_{11}^{\mathrm{DUT}} = 0$$

Distributed Impedance cont'd



$$Z_{\parallel}^{\mathrm{coax}} = iZ_c^{\mathrm{REF}} l \cdot (k_z^{\mathrm{DUT}} - k_z^{\mathrm{REF}}) \cdot \left(1 + \frac{k_z^{\mathrm{DUT}}}{k_z^{\mathrm{REF}}}\right)$$

$$Z_{\parallel}^{\mathrm{Imp.Log}} = Z_c \ln\left(\frac{S_{21}^{\mathrm{REF}}}{S_{21}^{\mathrm{DUT}}}\right)\left[1 + \frac{\ln(S_{21}^{\mathrm{DUT}})}{\ln(S_{21}^{\mathrm{REF}})}\right]$$

Improved Log Formula, Vaccaro et al. 1994

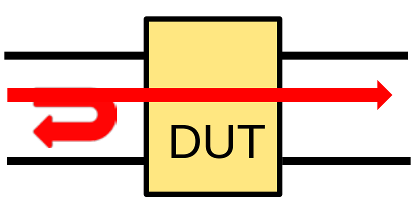
Textbook, e.g. Pozar

$$k_z^{\mathrm{DUT}} = \omega\sqrt{C_0' L_0'}\sqrt{1 - i\frac{R_0' + Z_{\parallel}/l}{\omega L_0'}}$$

$$k_z^{\mathrm{REF}} = \omega\sqrt{C_0' L_0'}\sqrt{1 - i\frac{R_0'}{\omega L_0'}}$$

$$Z_c^{\mathrm{DUT}} = \sqrt{\frac{R_0' + i\omega L_0' + Z_{\parallel}/l}{i\omega C_0'}}$$

$$Z_c^{\mathrm{REF}} = \sqrt{\frac{R_0' + i\omega L_0'}{i\omega C_0'}} \approx \sqrt{\frac{L_0'}{C_0'}} =: Z_c.$$



$$S_{21}^{\mathrm{C}} := \exp(-ik_z^{\mathrm{DUT}} l) = f(S_{21}, S_{11})$$

$$(S_{21}^{\mathrm{C}})^2 + \frac{{S_{11}}^2 - S_{21}^2 - 1}{S_{21}} S_{21}^{\mathrm{C}} + 1 = 0$$

J.Wang and S.Zhang,," NIM A 459, 2001

What if the impedance is neither lumped nor equally distributed?

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Mixed Impedance: Log-Formula

$$Z_{\parallel}^{\log} = 2Z_c \cdot \ln\left(\frac{S_{21}^{\mathrm{REF}}}{S_{21}^{\mathrm{DUT}}}\right)$$

Log-Formula (Walling et al. 1989)

Requirements for lumped impedance:

$$\frac{Z_{\parallel}^{\log}}{Z^{\mathrm{lump}}} = 1 - \frac{1}{2}\frac{Z^{\mathrm{lump}}}{2Z_c} + \frac{1}{3}\left(\frac{Z^{\mathrm{lump}}}{2Z_c}\right)^2 - \ldots$$

$$Z^{\mathrm{lump}} \ll 2Z_c$$

Requirements for distributed impedance:

$$\frac{Z_{\parallel}^{\log}}{Z_{\parallel}^{\mathrm{coax}}} = 1 + \frac{i}{4}\frac{Z_{\parallel}^{\mathrm{coax}}}{\Theta_{\mathrm{z}}^{\mathrm{REF}} Z_c} - \frac{1}{8}\left(\frac{Z_{\parallel}^{\mathrm{coax}}}{\Theta_{\mathrm{z}}^{\mathrm{REF}} Z_c}\right)^2 + \ldots$$

'electrical length' $\Theta_{\mathrm{z}}^{\mathrm{REF}} = k_{\mathrm{z}}^{\mathrm{REF}} l$

$$\frac{k_{\mathrm{z}}^{\mathrm{DUT}}}{k_{\mathrm{z}}^{\mathrm{REF}}} = \frac{Z_c^{\mathrm{DUT}}}{Z_c^{\mathrm{REF}}} \approx 1$$

U. Niedermayer *et al.*, Analytic modeling, simulation and interpretation of broadband beam coupling impedance bench measurements, Nucl. Instrum. Meth. A 776, 2015

H. Hahn, Validity of coupling impedance bench measurements, PRSTAB 3, 122001, 2000

Benchmarking the different $S_{21} \rightarrow Z_{\|}$ formulas in <u>***2D***</u> *(analytical)*

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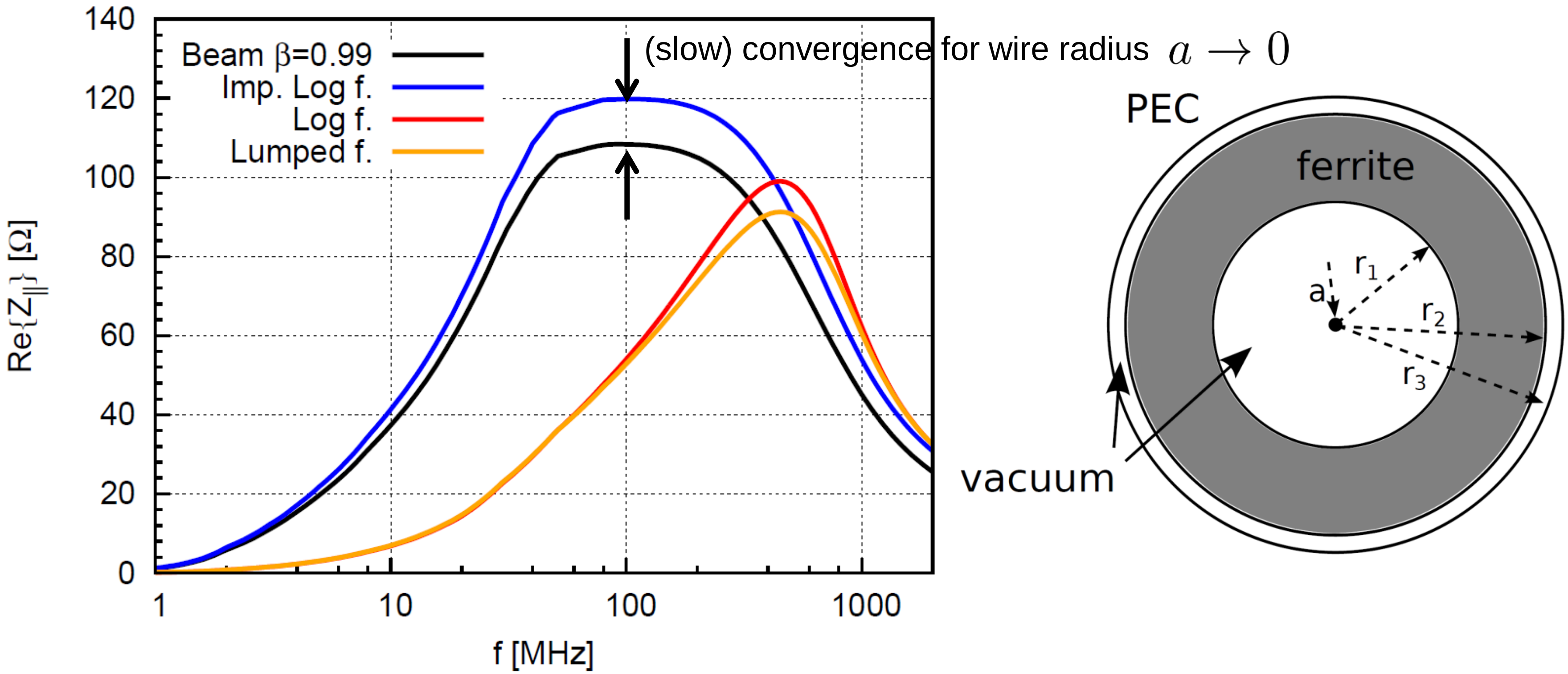
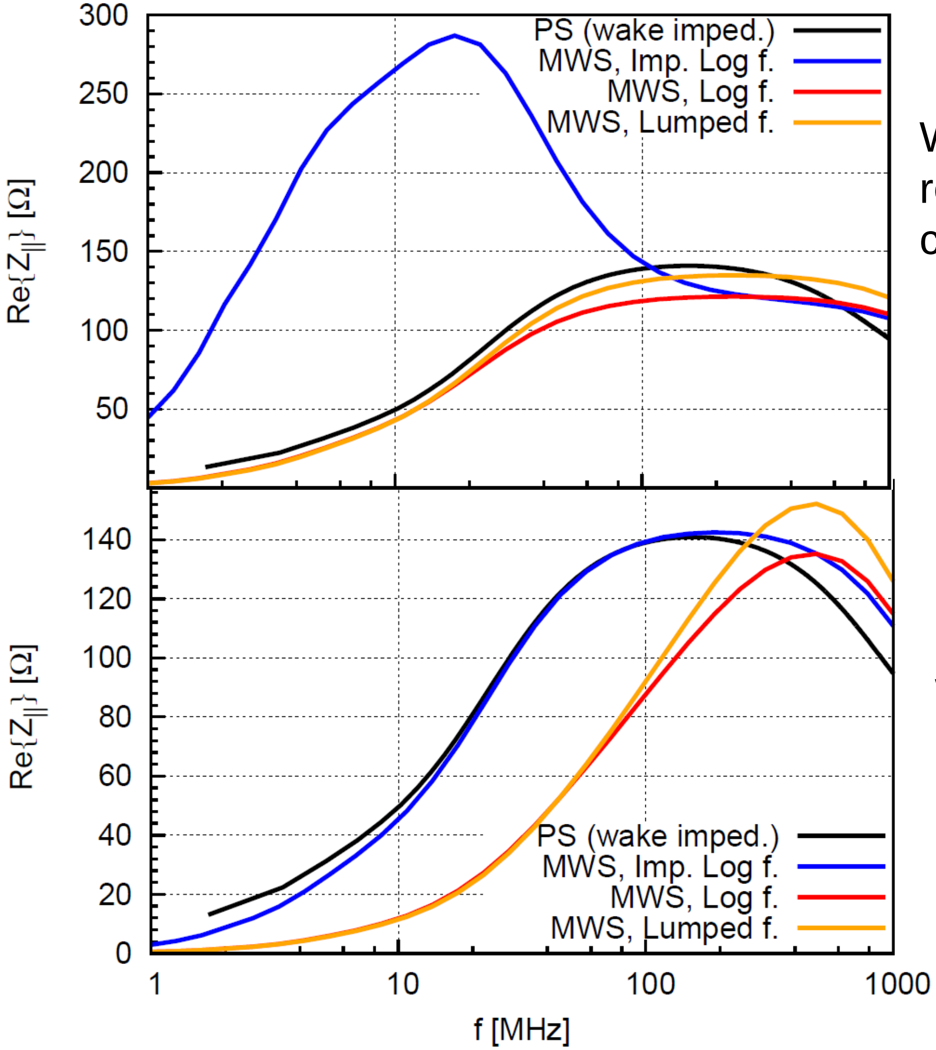


- Beam impedance calculated analytically (multilayer field matching)
- S_{21} calculated from Eigenvalue equation for k_z of Quasi-TEM mode (semi-analytically)

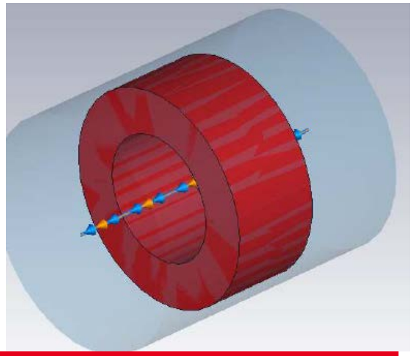
U. Niedermayer *et al.*, Analytic modeling, simulation and interpretation of broadband beam coupling impedance bench measurements, Nucl. Instrum. Meth. A 776, 2015

Benchmarking the different $S_{21} \rightarrow \underline{Z}_{\parallel}$ formulas in ***3D*** *(numerical)*



Without reflection correction

With reflection correction

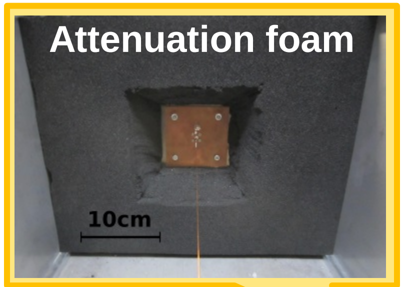


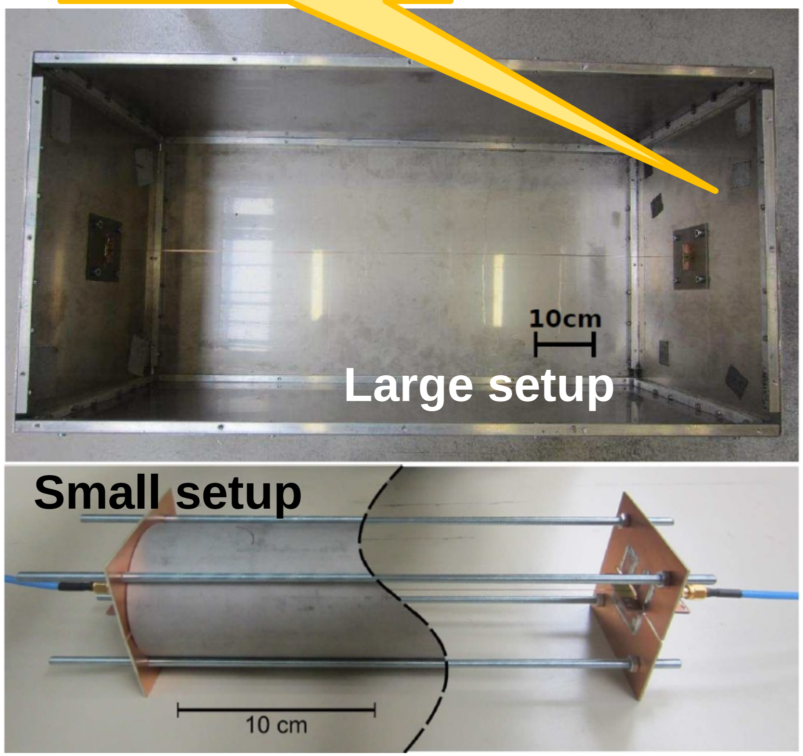
Reflection correction

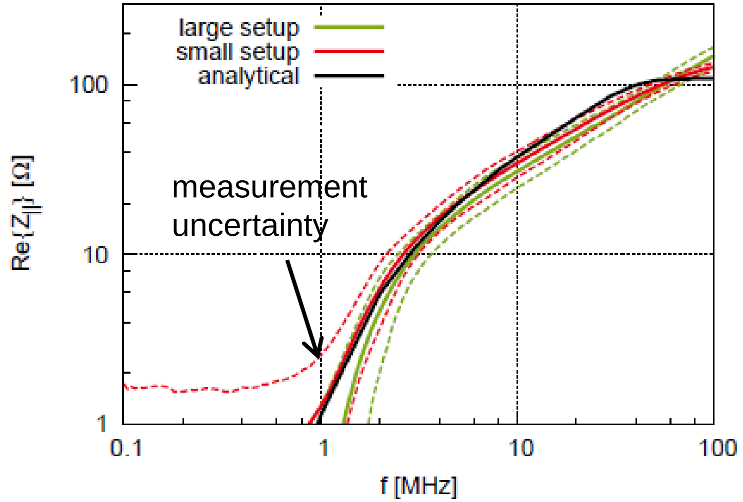
- Easy in simulation
  → waveguide ports
- Difficult in reality!
  → Multiple reflections

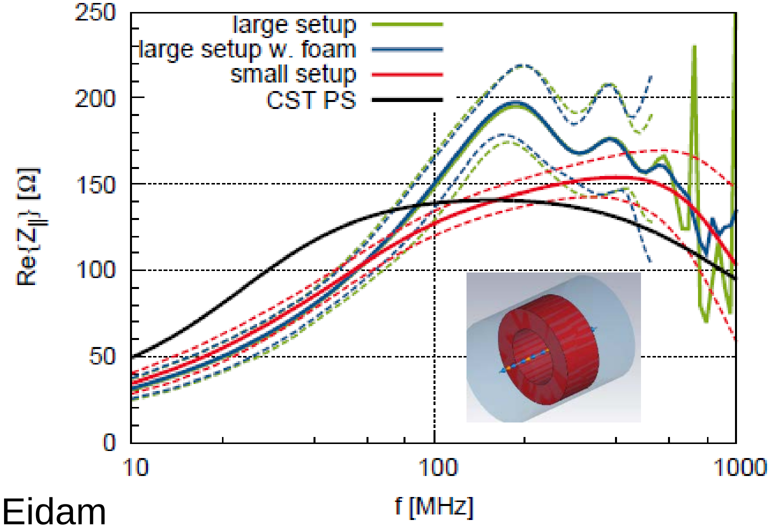
- Beam impedance from CST Particle Studio (PS)
- S-parameters from CST Microwave Studio (MWS)

Measurement of the ferrite ring









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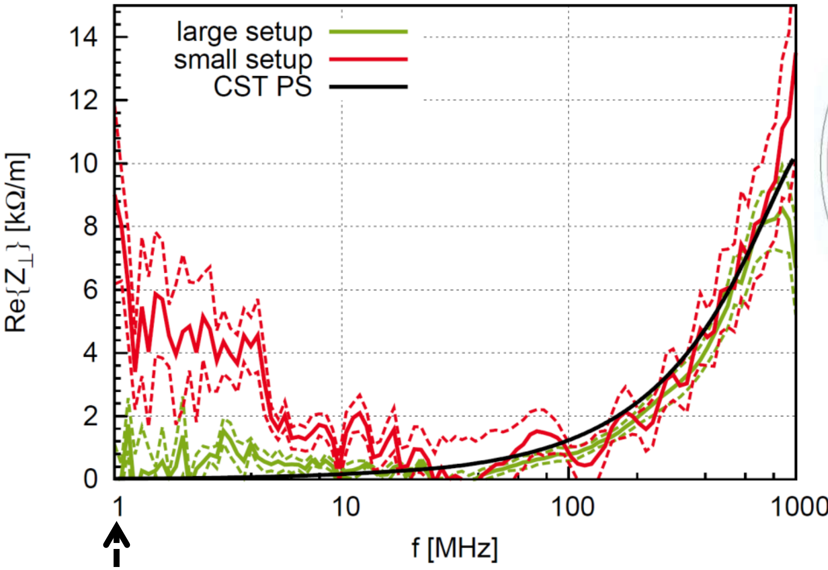
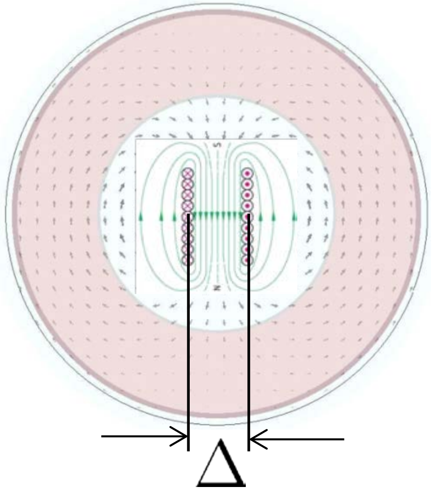
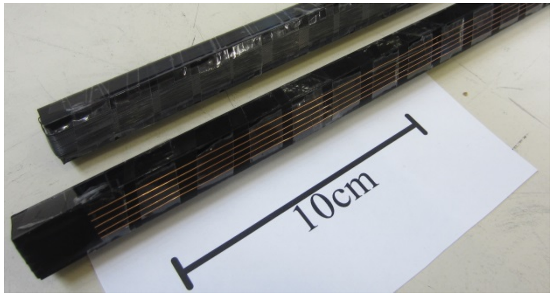
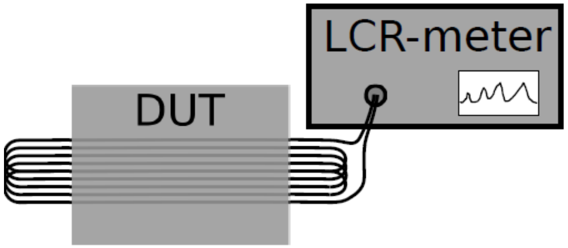
Measurement of Transverse Impedance

Twin wire measurement









Expectation by microwave simulation:

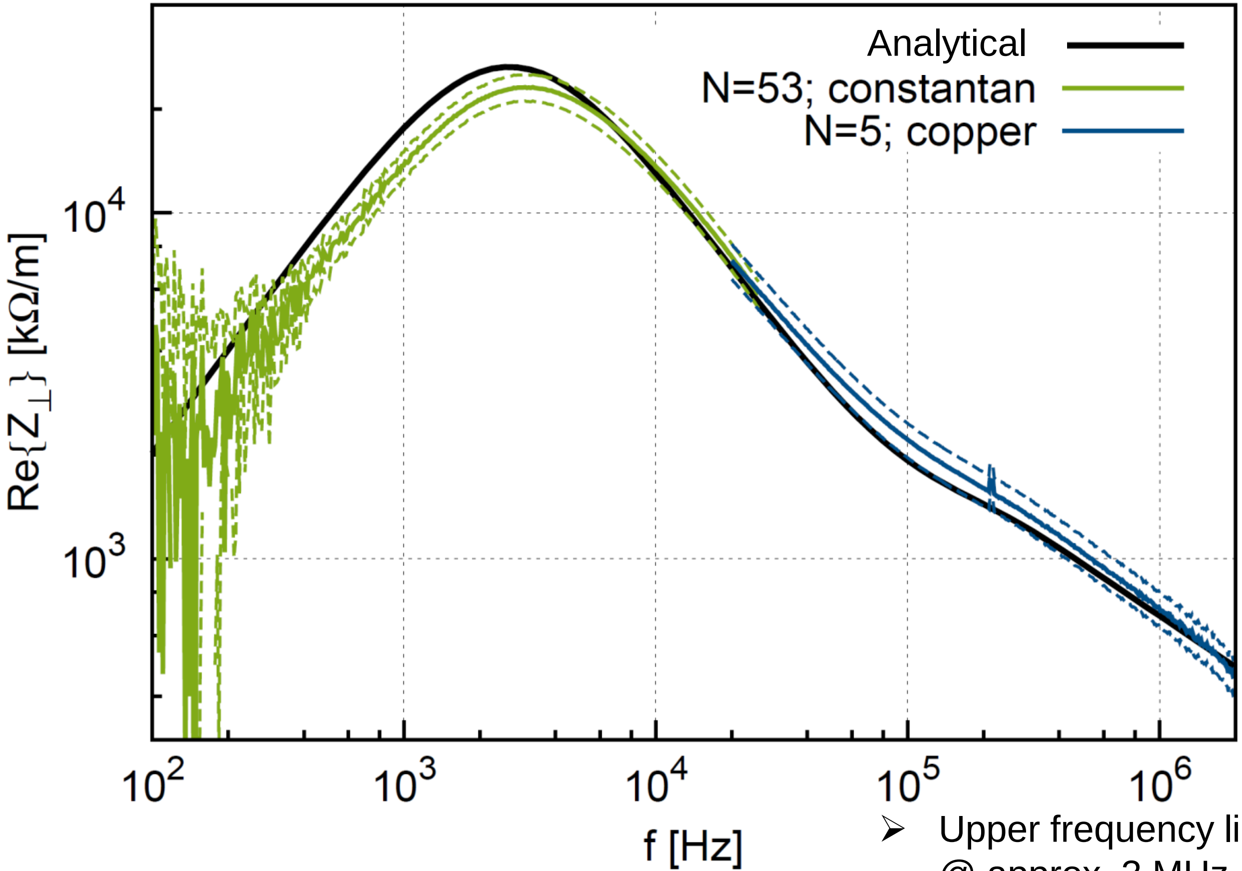
$$\left| \frac{S_{21}^{\mathrm{DUT}}}{S_{21}^{\mathrm{REF}}} \right|_{f=1\,\mathrm{MHz}} = 1 - 1.6 \cdot 10^{-8}$$

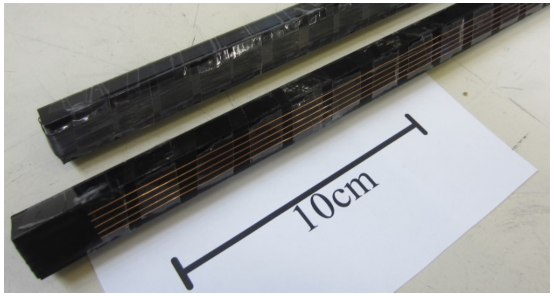
$$\underline{Z}_{\perp} \approx \frac{c\, \delta \underline{Z}_{\parallel}}{\omega \Delta^2} \times \frac{1}{N^2}$$

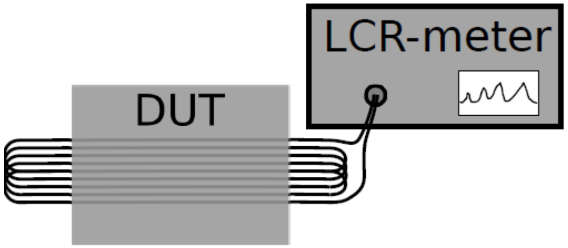
- Upper frequency limit due to coil resonance @ approx. 3 MHz
- **Reasonable measurement results down to 1 kHz**
- Quasi-stationary interpretation required

L. Eidam

Low frequency measurement of beam pipe transverse impedance







- Upper frequency limit due to coil resonance @ approx. 3 MHz
- **Reasonable measurement results down to 1 kHz**
- Quasi-stationary interpretation required

L. Eidam

Summary

- **Frequency domain beam coupling impedance computation**
  - Advantageous for LF, dispersive material, low beam velocity
  - Direct transverse space charge fields $(\beta < 1)$ not accurate on structured hex-mesh
  - Space charge and resistive wall impedance solver in 2D FEM implemented
  - SIBC allows high frequency since skin depth is not meshed
- **Bench measurements of <u>broadband impedances</u>**
  - Should be cross-checked with RF simulations
  - A priori knowledge about impedance distribution required for $\underline{Z}_{\parallel}$
  - Medium frequency range
  - Twin wire method inapplicable at LF $\Rightarrow \underline{Z}_{\perp}$ by coil method down to 1 kHz
  - Quasi stationary methods at extremely LF, for both simulation and measurement

Outlook

- I will be available the whole week for any questions…

- Optimized (parallel) FEM-FD solver in 3D

  (PhD position open at TUD)

- Optimization of matching networks and conducting bench measurements (also above cut-off)

  (BSc/MSc projects open at GSI and TUD)

The End

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Thank you for your attention!

Any Questions?

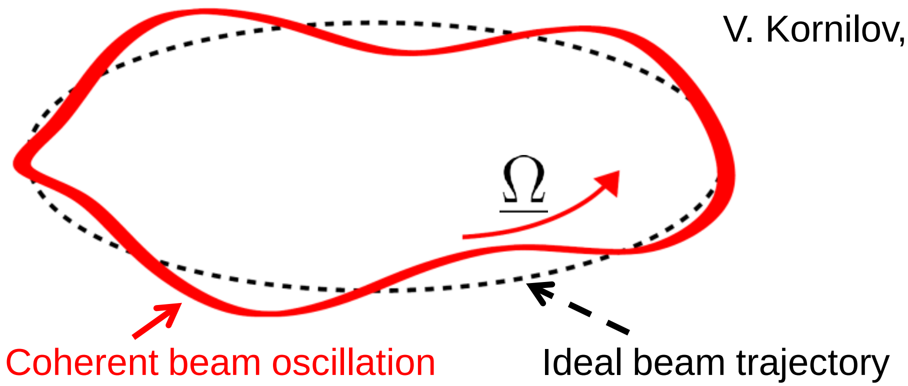
Please contact me for more references…

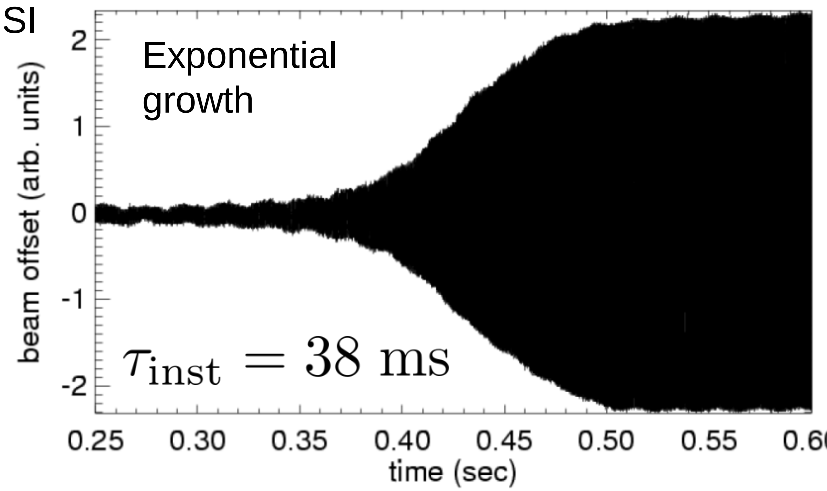
Acknowledgements:

- Elias Metral and ICE section at CERN
- Fritz Caspers, Manfred Wendt (CERN)
- Lewin Eidam, Udo Blell, Oliver Boine-Frankenheim (GSI)
- Wolfgang Ackermann, Erion Gjonaj, Ulrich Römer, Herbert De Gersem, Thomas Weiland and many more… (TEMF)

Instability Example: Transverse Coasting Beam Instability

V. Kornilov, GSI



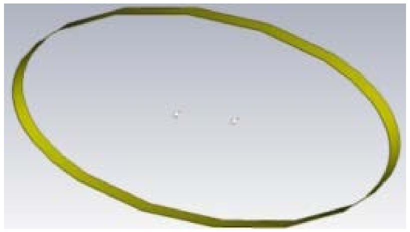


Betatron frequency

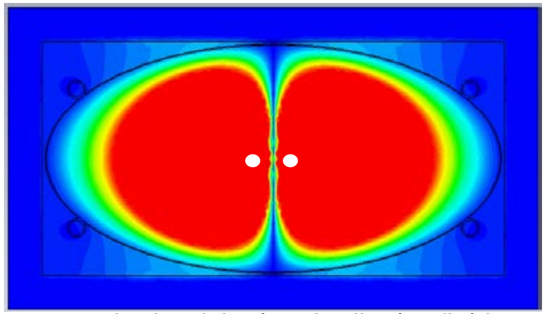
$$\ddot{y} + \omega_\beta^2 y = C \cdot N_{\text{ions}} i \underline{Z}_\perp(\underline{\Omega}) y$$

$$\frac{1}{\tau_{\text{inst}}} \propto N_{\text{ions}} \text{Re}\{\underline{Z}_\perp(\underline{\Omega})\}$$

$\underline{\Omega}$ corresponds to a very low frequency ~100 kHz



Slice of beam pipe



Magnitude of the longitudinal E-field (dipole excitation)

Details on the low frequency (LF) impedance computation:

U. Niedermayer and O. Boine-Frankenheim, "Analytical and numerical calculations of resistive wall impedances for thin beam pipe structures at low frequencies", Nucl. Instrum. Meth. *A 687, 51, 2012*

De Rham Diagram and Discretization of Sobolev Spaces

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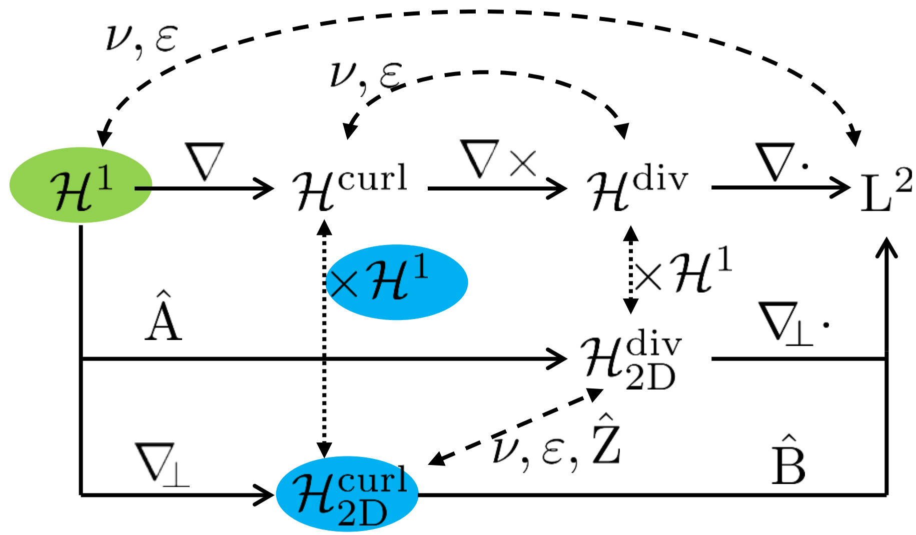

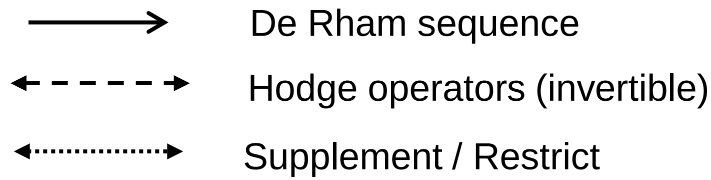

Vectorial and scalar 2D curl operators

$$\hat{\mathrm{A}} = \begin{pmatrix} \partial_y \\ -\partial_x \end{pmatrix} = -\hat{\mathrm{B}}^{\mathrm{T}}$$

- Similar De Rham diagram for discrete spaces
- Functions can be projected from continous to discrete
- If the projecton operator commutes with the exterior derivative, then the convergence of the projection implies the convergence of the FEM method!

Mathematical details see e.g.
- D. Arnold et al. ‘Finite Element Exterior Calculus, Homological Techniques, and Applications‘, Acta Numerica, 2006
- P. Monk, Finite Element Methods for Maxwell‘s Equations, Oxford University Press, 2003

2D Discretization of FCC-hh pipe
(similar to LHC pipe)

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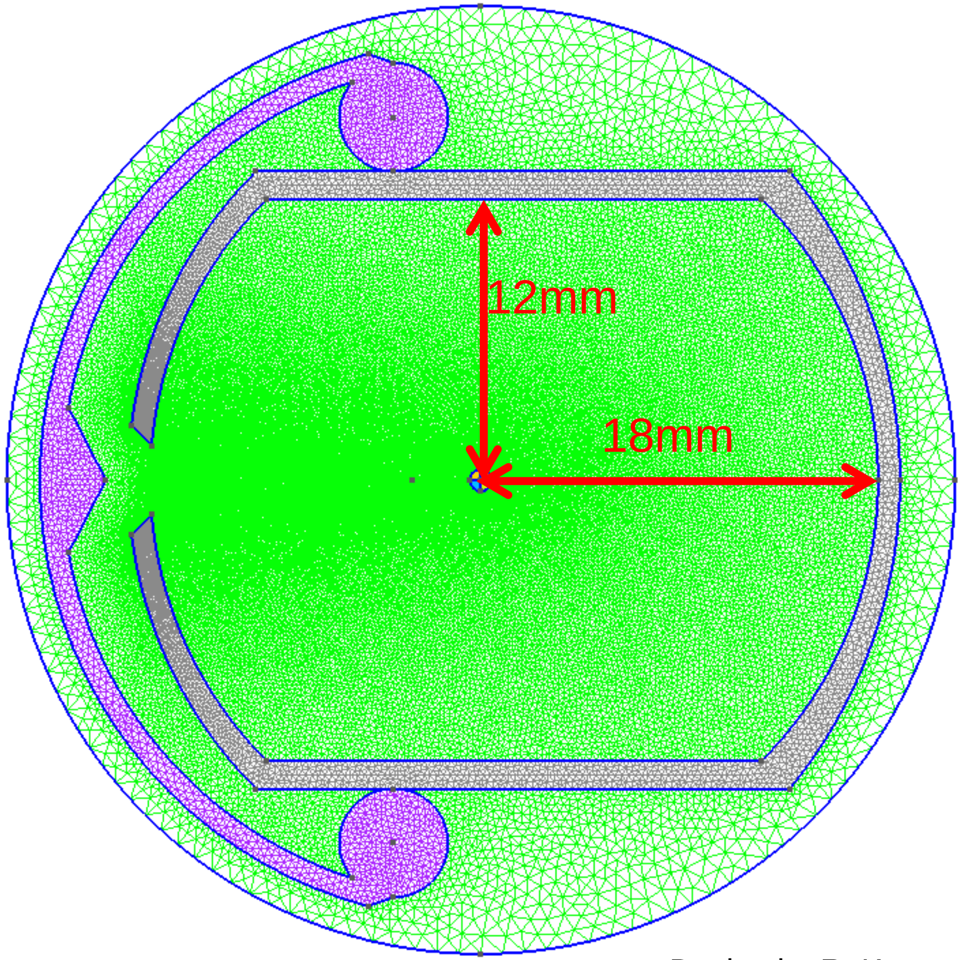


FCC-hh design study:
100TeV c.m., 100km circumf.

The first hadron collider where synchrotron radiation losses play significant role

Gmsh triangular mesh

Meshing the whole structure is required only for extremely low frequency!

Otherwise: **S**urface **I**mpedance **B**oundary **C**ondition (SIBC)

Design by R. Kersevan, CERN, mesh by T. Egenolf, TU Darmstadt

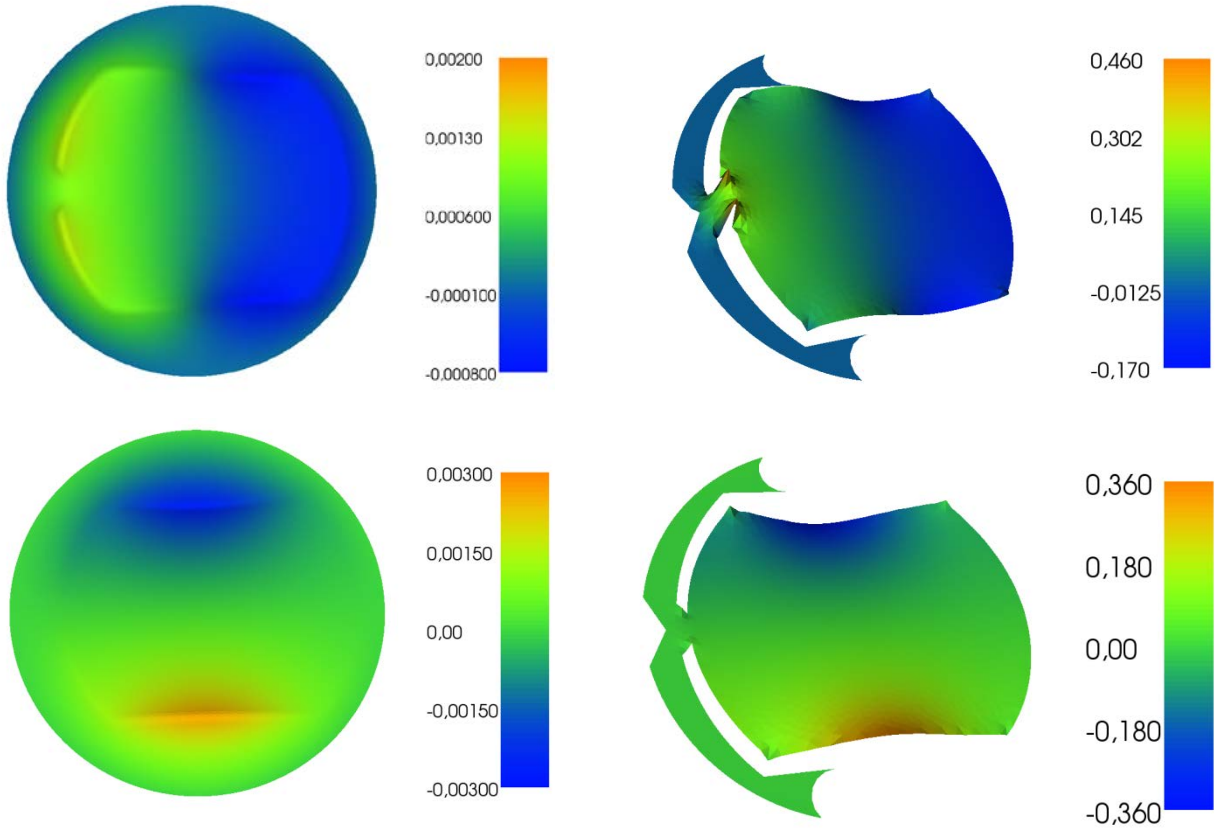
2D Simulations in the Frequency Domain

BeamImpedance2D, PYTHON code using FEniCS finite element package

$\mathrm{Re}\{E_z\}$

Horizontal

Vertical



f=100Hz f=1MHz

Penetration Depth

- Surface impedance for coated surface

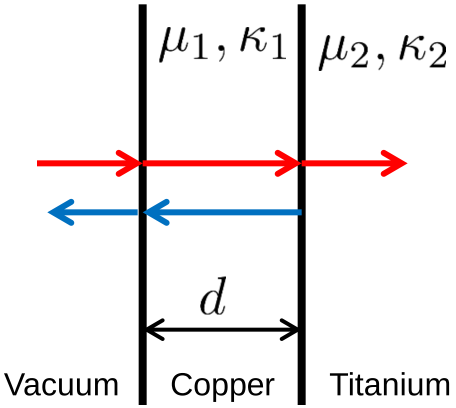
$$\delta = \sqrt{\frac{2}{\mu\kappa\omega}}$$

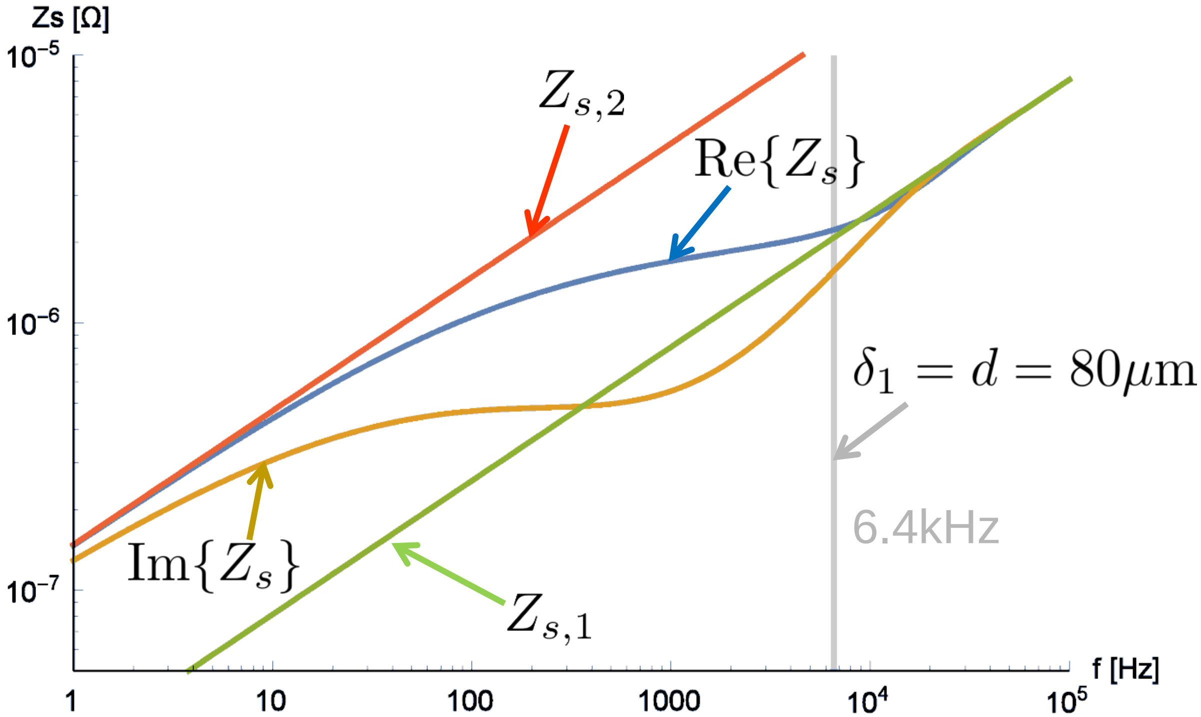
$$Z_s(\omega) = \left.\frac{E_x}{H_y}\right|_{z=0} = \frac{1+i}{\kappa_1\delta_1}\frac{Me^{ik_{z1}d}+Ne^{-ik_{z1}d}}{Me^{ik_{z1}d}-Ne^{-ik_{z1}d}}$$

$$k_z = \frac{1-i}{\delta}$$

$$M = 1 + \sqrt{\frac{\mu_1\kappa_2}{\mu_2\kappa_1}}$$

$$N = 1 - \sqrt{\frac{\mu_1\kappa_2}{\mu_2\kappa_1}}$$

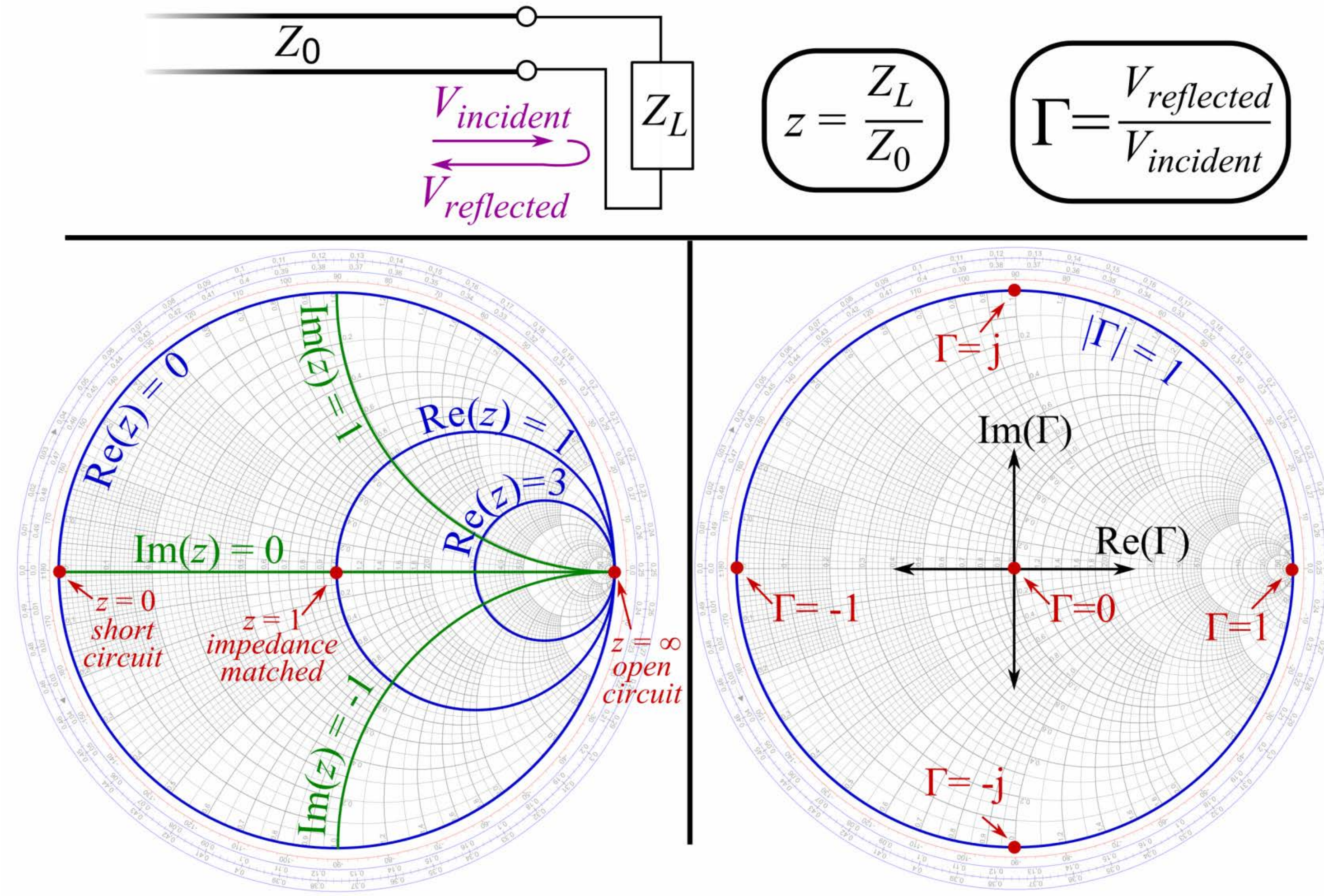




Boundary conditions

- Natural: Neumann (E-field formulation: Magnetic BC)
- Essential: Dirichlet (E-field formulation: Electric BC)
- Surface Impedance Boundary Condition (SIBC)
- Periodic
- Floquet
- Open boudary conditions
  → Mur (absorbing BC)
  → Berenger PML (perfectly matched layer)
- Waveguide ports
  → Solve 2D eigenvalue problem and connect to 3D structure
- Dedicated boundary conditions for particle beams

The Smith Chart



https://upload.wikimedia.org/wikipedia/commons/thumb/d/df/Smith_chart_explanation.svg/2666px-Smith_chart_explanation.svg.png