Observations and Diagnostics in High Intensity Beams

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Beam Observations

**Beam Instrumentation**

- Instruments and signal processing for:
  - Beam Position
  - Beam Spectra
  - Beam Profiles
  - Beam Intensity
  - Losses

**Beam Diagnostics**

- Orbit along the machine
- Beta Functions, Dispersion
- Tunes $Q_x$, $Q_y$, $Q_s$, Chromaticities, $\gamma_t$
- Momentum Spread
- Beam Distributions: 6D
- Beam Sizes: transverse, longitudinal
- Tune Shifts
- Performance, Luminosity, Losses
- Input for Feedback Systems

Both in perfect match: essential for the machine operation

Understanding of the physical basis is necessary at every stage

Beam Observations

What happens at high intensities?

Instrumentation issues (e.g. overheated wires)

- Stronger signals
- Better Signal-to-Noise ratios

But: Distorted Signals

Signals are distorted, but the physical origin did not change

Signals are different, because the physical origin has changed

- Some diagnostic methods are not possible at high intensities
- Some diagnostic methods get enriched by new opportunities
High Intensity

What happens at high intensities?

Beam parameters, machine settings, and ion optics can be changed, for example:

- the lattice can be changed;
- particle/beam oscillation frequencies are changed;
- beam loading: different rf-buckets

Some measurements should be done at low intensities

Many diagnostics can be used at high intensities, and even offer new opportunities

Horizontal beta function for SIS18 with/without space-charge, solution of the system of envelope equations for $\sigma_x(s)$, $\sigma_y(s)$, $D(s)$

M. Reiser, Theory and Design of Charged Particle Beams
M. Venturini, M. Reiser, PRL 81, 96 (1998)
Here: calculations for SIS18, A. Andreev, GSI 2014
Particle, Beam Oscillations

Incoherent oscillations

Coherent oscillations

Picture from K.Schindl, CAS2000, Greece, CERN-2005-004
Interactions Particle–Beam–Facility

\[ \Omega_{\text{inc}} \quad \text{incoherent} \quad \text{oscillations} \quad \Omega_{\text{coh}} \quad \text{coherent} \]

\[ F_{\text{inc}}^{\perp} = 0 \quad F^{\perp} = F_{\text{inc}}^{\perp} + F_{\text{coh}}^{\perp} \quad \overline{F}^{\perp} = F_{\text{coh}}^{\perp} \]

**example 1: facility components**

- **Octupole magnets**
  \[ B_x = \frac{g}{6} (-y^3 + 3x^2y) \]
  \[ B_y = \frac{g}{6} (x^3 - 3xy^2) \]
  Related to magnet axis

- Excitation by a 1-turn kicker of a mode \( \Omega_{\text{coh}} \)

  - Changes the individual particle oscillation,
  does not shift the coherent tune
  - Beam oscillation as a whole with \( \Omega_{\text{coh}} \),
  does not shift the incoherent tunes
Changes the individual tunes $Q_{\text{inc}}$, does not shift $Q_{\text{coh}}$. 

$\Omega_{\text{inc}}$ \hspace{2cm} $\Omega_{\text{coh}}$

Oscillations

Interactions

Incoherent: $F_{\perp}^{\text{inc}} = 0$

Coherent: $F_{\perp} = F_{\perp}^{\text{inc}} + F_{\perp}^{\text{coh}}$

Example 2: space charge forces

- Self-field space charge (related to beam center, moves with the beam)
- Changes the individual tunes $Q_{\text{inc}}$, does not shift $Q_{\text{coh}}$

- Round pipe, image charges
- Affects the beam as a whole, shifts $Q_{\text{coh}}$, does not shift $Q_{\text{inc}}$
Intensity Effects & Tune Shifts

Dipolar wakes: $F_{x_2} \sim \Delta x_1$
(driving)
the same for the whole trailing slice: coherent

Quadrupolar wakes: $F_{x_2} \sim \Delta x_2$
(detuning)
different for individual particles: incoherent

experiences the wake field

The facility impedances also have coherent and incoherent effects
Intensity Effects & Tune Shifts

Simplified reminder

Incoherent tune shift

\[ \Delta Q_{sc} = -g_a \frac{\lambda_0 r_p R}{4 \gamma^3 \beta^2 \varepsilon_x} \]

Coherent tune shift

\[ \Delta Q_{coh} = \frac{I_0 q_{ion}}{4 \pi \gamma mc Q_0 \omega_0} i Z_{ext} \]
\[ \text{Re}(\Delta Q_{coh}) = \kappa \text{Im}(Z_{ext}^\perp) \]
\[ \text{Im}(\Delta Q_{coh}) = \kappa \text{Re}(Z_{ext}^\perp) \]

impedance measurements

- Space-charge tune shift depends on amplitude: different for every particle
- Imaginary impedances shift the coherent frequency
- Real Impedances drive instabilities \( \langle x \rangle = x_0 \exp\{ \text{Im}(\Delta Q_{coh}) t \} \)
- Bunches: effective impedance (convolution with the bunch spectrum)
Intensity Effects & Tune Shifts

Compare the incoherent (space-charge) tune shift and the coherent (due to impedance) tune shift

\[
\Delta Q_{\text{sc}} = -\frac{\lambda_0 r_p}{\gamma Q_0} \frac{1}{\gamma \beta} \frac{Q_0 R}{4\varepsilon_{xn}}
\]

\[
\Delta Q_{\text{coh}} = \frac{\lambda_0 r_p}{\gamma Q_0} \frac{i Z^\perp}{Z_0/R}
\]

- both depend linearly on the intensity
- decrease at the ramp as \(1/\gamma\)
- space-charge: additional \(1/\gamma \beta\)

\(\varepsilon_{xn}\): normalized rms emittance

\(r_p = q^2/(4\pi \varepsilon_0 mc^2)\)

\(Z_0 = 1/(\varepsilon_0 c)\)

Special impedance: image charges

\[
Z_{IC}^\perp = -i \frac{Z_0 R \xi_{\text{geom}}}{\beta^2 \gamma^2 h^2}
\]

- decreases faster than space-charge: \(1/\gamma^2 \beta^2\)
- related to space-charge: induced fields in the pipe
- should not be confused with space-charge

Intensity Effects & Tune Shifts

Seems to be easy: by measuring $\Delta Q_{\text{coh}}$ the impedance is determined

But then, how to understand this:

![Graph showing intensity effects and tune shifts](image)

Single bunch tune measurements at the CERN SPS, J.Gareyte, EPAC2002

What has been measured?
The horizontal impedance was surely non-zero.
Intensity Effects & Tune Shifts

Laslett coefficients for coasting beams:

\[ \Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\varepsilon_1}{h^2} \]

\[ \Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{\gamma^2 h^2} \right] \]

- \( \xi_1 \): symmetries, coherent
- \( \varepsilon_1 \): unsymmetries, incoherent
- \( \frac{1}{\gamma^2} \): \( E-B \) cancellation

Elliptical pipe, \( h=b_y \) is the half-height.
Perfectly conducting pipe.
Different terms for:
- Low frequencies (ac magnetic field)
- Magnet poles
- Partial neutralization

\[ \zeta = \frac{2 r_p R^2}{\beta^2 \gamma Q_0} \]

Elliptical pipe

- \( b_y / b_x \to 0 \) parallel plates
- \( b_y / b_x \to 1 \) circular pipe

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62
Intensity Effects & Tune Shifts

Laslett coefficients for coasting beams:

\[ \Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\varepsilon_1}{h^2} \]
\[ \Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{\gamma^2 h^2} \right] \]

For bunched beams \((B=I_{av}/I_{peak})\):

\[ \Delta Q_{\text{inc}} = -\zeta \lambda_0 \frac{\varepsilon_1}{h^2} \left( \beta^2 + \frac{1}{B \gamma^2} \right) \]
\[ \Delta Q_{\text{coh}} = -\zeta \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{B \gamma^2 h^2} \right] \]

The bunching factor, the cancellation, \(\beta^2\) appear in a non-straightforward way

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62
Intensity Effects & Tune Shifts

From the first-order expansion of the forces for small perturbations, a symbolic relation:

\[ \Delta Q_{coh} = \Delta Q_{Z\perp} + \Delta Q_{inc} \]

Coherent tune shift: everything affecting \( \langle x \rangle \)
Tune shifts due to impedance: \( \langle x \rangle \) acting on individual particles. Include incoherent effects.
Facility- and lattice-related incoherent tune shifts. No space charge.

This is why there are incoherent effects in the coherent tune shift

\[ \Delta Q_{coh} = -\zeta \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2} \right] \]

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
P.Bryant, CAS1986, CERN 87-10, p.62

Intensity Effects & Tune Shifts

\[ \Delta Q_{\text{coh}} = \Delta Q_{Z}^\perp + \Delta Q_{\text{inc}} \]

\[ \Delta Q_{\text{coh}} = -\xi \lambda_0 \left[ \frac{\beta^2 \varepsilon_1}{h^2} + \frac{\xi_1}{B\gamma^2 h^2} \right] \]

Incoherent effect of the unsymmetries reduces \( \Delta Q_Z \) horizontally and enhances \( \Delta Q_Z \) vertically.

Single bunch tune measurements at the CERN SPS, J.Gareyte, EPAC2002.
Intensity Effects & Tune Shifts

What is it what you measure?
- coherent or incoherent oscillations
- frequency ranges
- impedances involved

Be aware about
- coherent or incoherent tune shifts
- self-field (space-charge) or facility tune shifts
- dipolar or quadrupolar wakes
- the difference between coherent tune shifts and impedance tune shifts
- electric or magnetic images (cancellation, bunching factor)
- penetrating or non-penetrating fields (frequencies)

Handbook of Acc. Physics and Eng. 2013, 2.4.5
K.Y.Ng, Phys. of Intensity Dep. Beam Instab., 2006
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
P.Bryant, CAS1986, CERN 87-10, p.62
Schottky Signals

W. Schottky 1918: random fluctuations due to uncorrelated arrival of charges

A beams in accelerator: signal due to discrete particles

Longitudinal (Sum) Schottky signal: \( \Sigma = T + B \)
Transverse (Delta) Schottky signal: \( \Delta = T - B \)

\[
P_{\parallel}(\Omega) \propto Z^2 f_0 N_p \sum_{m=-\infty}^{\infty} \frac{1}{|m|} \Psi \left( \frac{\Omega - m\omega_0}{\sigma_{m,\omega}} \right)
\]

Gives \( f_0 \), momentum spread, \( \eta \)

\[
\Delta \omega_0 / \omega_0 = -\eta \Delta p / p
\]

rms \( \sigma_p = \delta p / p \)

\( \Psi \) is the momentum distribution

S. Chattopadhyay, CERN 84-11 (1984)
F. Caspers, CAS Dourdan 2008, p.407

W. Schottky 1918: random fluctuations due to uncorrelated arrival of charges

A beam in an accelerator: signal due to discrete particles

Longitudinal (Sum) Schottky signal: \( \Sigma = T + B \)
Transverse (Delta) Schottky signal: \( \Delta = T - B \)

Gives the tune, chromaticity, and tune shifts

\[
\frac{2Q_f f_0}{Q_f f_0} = \frac{\delta f^-}{f_0} = \frac{\delta p}{p} \left| \eta \left( m - Q_f + \frac{\xi}{\eta} \right) \right| \\
\frac{\delta f^+}{f_0} = \frac{\delta p}{p} \left| \eta \left( m + Q_f - \frac{\xi}{\eta} \right) \right|
\]

\( Q_f \) is the fractional part of the tune
\( (Q=4.3, Q_f=0.3) \)
\( \Delta Q = \xi Q \Delta p/p \)

S. Chattopadhyay, CERN 84-11 (1984)
F. Caspers, CAS Dourdan 2008, p.407
Schottky Signals

Example:
Xe^{48+} ions in SIS18 at GSI:
m=20, \( Q_h = 4.3 \), measured \( \xi_h = -1.3 \)

![Graph showing lower and upper sidebands](image-url)
Schottky Signals

The Schottky sidebands transform at high intensities

In order to understand and to use these transformations as a new diagnostics, we need to discuss the Beam Transfer Function (BTF)

Lower sidebands for m=50, Ar^{18+} beams in SIS18 at GSI, f_0=214 kHz
S.Paret, V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 022802 (2010)
Beam Transfer Function

an excitation:

\[ x'' + \omega_{\beta i}^2 x = \hat{G} e^{-i\Omega t} \]

beam forced response:

\[ \langle x \rangle = A \cdot e^{-i\Omega t + \Delta \phi} \]

\[
R(\Omega) = 2\omega_{\beta 0} \frac{\langle x(t) \rangle}{\hat{G} e^{-i\Omega t}}
\]

kicker  
\[
\text{exciting signal} \quad \Omega, \quad \phi, \quad A^{\text{in}}
\]

network analyser

\[
\text{response} \quad \Omega, \quad \phi, \quad A^{\text{out}}
\]

beam

pick-up

BTF: amplitude(\Omega), phase(\Omega)
Beam Transfer Function

BTF is:

- Useful diagnostics; gives the tune, $\delta p$, chromaticity, beam distribution
- A fundamental function in the beam dynamics
- Necessary to describe the beam signals and Landau damping

$$R_0(\Omega) = \int \frac{f(\omega)d\omega}{\omega - \Omega}$$

$$\Delta Q = (\Omega - (m \pm Q_0)f_0)/f_0$$

$$\delta Q_\xi = |m\eta \pm (Q_f\eta - Q_0\xi)| \delta p/p$$

J.Borer, et al, PAC1979
D.Boussard, CAS 1993, CERN 95-06, p.749
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993

Handbook of Acc. Physics and Eng. 2013, 7.4.17

Beam Transfer Function

BTF cab be easily measured with a network analyser. Example here: the beam responses exactly as the theory for Gaussian beams.

Measurement in SIS18 at GSI Darmstadt, U^{73+} at 500 MeV/u, slow freq sweep (6sec), m=24, m=50. V. Kornilov, et.al., Measurements and Analysis of the Transverse Beam Transfer Function (BTF) at the SIS 18 Synchrotron, GSI-Acc-Note-2006-12-001, GSI Darmstadt (2006)
Transverse unbunched Schottky noise without collective interactions (incoherent frequencies of uncorrelated particles)

\[ P_0(\Omega) = D \Psi \left( \frac{\Omega}{n - Q_0} \right) \]

With an impedance:

\[ P = \frac{P_0}{|\varepsilon|^2}, \quad \varepsilon(\Omega) = 1 + iZ^{-1}R_0(\Omega) \]

S. Chattopadhyay, CERN 84-11 (1984)
N. Dikansky, D. Pestrikov, Physics of Intense Beams and Storage Rings (1994)

What about space charge? Just include space-charge as an impedance?

Wrong

D. Pestrikov, NIM 578, 65 (2007)
O. Boine-Frankenheim, V. Kornilov, S. Paret, PRSTAB 11, 074202 (2008)
Schottky Signals at High Intensity

\[ P(\Omega) = \frac{P_0^\Delta(\Omega)}{|\varepsilon(\Omega)|^2} \]

for an impedance

\[ P_0^\Delta(\Omega) = P_0(\Omega) \]
\[ \varepsilon = 1 + iZ^\perp R_0(\Omega) \]

for space charge

\[ P_0^\Delta(\Omega) = P_0(\Omega - \Delta \omega_{sc}) \]
\[ \varepsilon = 1 + \chi R_0(\Omega - \Delta \omega_{sc}) \]

\[ \chi = \frac{\Delta Q_{sc}}{\delta Q_{\xi}} \]
\[ \Delta Q = (\Omega - (m - Q_f) f_0) / f_0 \]
\[ \delta Q_{\xi} = |m \eta - (Q_f \eta - Q_0 \xi)| \delta p / p \]

Space-Charge: no total shift + deformation to the opposite side

Again: fundamental differences between impedance (ext coh) and space-charge (int incoh)
Schottky Signals at High Intensity

Lower sidebands for $m=50$, $\text{Ar}^{18+}$ beams in SIS18 at GSI, $f_0=214$ kHz
S.Paret, V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 022802 (2010)

From the sideband deformation and shift, the space-charge tune shift and/or the impedance can be determined

$$\Delta Q_{sc} = -0.053 \quad \delta_p = 0.78 \times 10^{-3}$$
$$\Delta Q_{sc} = -0.019 \quad \delta_p = 0.67 \times 10^{-3}$$
$$|\Delta Q_{sc}| < 0.002 \quad \delta_p = 0.28 \times 10^{-3}$$
Schottky Signals at High Intensity

Also the longitudinal (the sum) Schottky spectrum is distorted by collective interactions. Here: examples for space charge.

S. Chattopadhyay, CERN 84-11 (1984)

Transverse and longitudinal oscillations in a bunch

$Q_s$

$Q_v$

$Q_s$ is the synchrotron tune

$Q_s = f_s/f_0$

for example:

$f_0 = 200\text{kHz}, f_s = 1\text{kHz}, Q_s = 0.005$

$\Delta Q = \Delta f/f_0, \Delta f = f - (m \pm Q_f) f_0$

Sideband $((m \pm Q_f) f_0)$ in the transverse spectrum

The $k=0$ line normally dominates, but the $|k|>0$ lines can also be measured

Gives the tune, chromaticity, coherent (Z) tune shifts, $Q_s$
Transverse and longitudinal oscillations in a bunch

\[ Q_s \]

\[ Q_v \]

$Q_s$ is the synchrotron tune

$Q_s = f_s/f_0$

for example:

$f_0 = 200\text{kHz}$, $f_s = 1\text{kHz}$, $Q_s = 0.005$

\[ \Delta Q = \Delta f/f_0 \quad \Delta f = f - (m\pm Q_s f_0) f_0 \]

Transverse spectrum in LHC, 3.5TeV, $Q_s = 0.002$

E.Metral, B.Salvant, N.Monet, IPAC2011

The $k=0$ line normally dominates, but the $|k| > 0$ lines can also be measured

Gives the tune, chromaticity, coherent (Z) tune shifts, $Q_s$

Bunch spectrum is distorted by collective interaction
Space Charge in Bunches

The airbag model for arbitrary space-charge
M.Blaskiewicz, PRSTAB 1, 044201 (1998)

\[ \Delta Q_k = \frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2} \]

The space charge parameter for bunches

\[ q = -\frac{\Delta Q_{sc}}{Q_s} \]
Space Charge in Bunches

The $k=0$ line is not affected by space charge
• The line distance is not $Q_s$, more difficult to resolve
• The incoherent tune is at $(-q)$

\[ q = -\frac{\Delta Q_{sc}}{Q_s} \]
The effect of a coherent tune shift (imaginary impedance) in the airbag theory

\[
\Delta Q_k = \frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\frac{(\Delta Q_{coh} - \Delta Q_{sc})^2}{4} + k^2 Q_s^2}
\]

\(\Delta Q_{coh} = 0\)

\(\Delta Q_{coh} = 0.5 \Delta Q_{sc}\)

O. Boine-Frankenheim, V. Kornilov, PRSTAB 12, 114201 (2009)
Space Charge + Impedance in Bunches

\[ q = -\frac{\Delta Q_{\text{sc}}}{Q_s} \]

example here: \( \Delta Q_{\text{coh}} = 0.5 \Delta Q_{\text{sc}} \)

- The \( k=0 \) line is shifted by \( \Delta Q_{\text{coh}} \)
- The \( |k|>0 \) lines are shifted by both space charge and impedance
- The \( |k|>0 \) lines: it is not just everything shifted by \( \Delta Q_{\text{coh}} \)
The space-charge tune shifts of the lines in the bunch spectrum can also be measured in experiment.

Ar$^{18+}$ bunches in SIS18 at GSI Darmstadt, $Q_s=0.0032$, $q=4.5$

V. Kornilov, O. Boine-Frankenheim, PRSTAB 15, 114201 (2012)
Quadrupole Pickup

\[ \Sigma = L + T + R + B \]
\[ \Delta_h = R - L \]
\[ \Delta_v = T - B \]
\[ \kappa = T + B - R - L \]

Beam Quadrupole moment:

\[ \kappa = \sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2 \]

Beam width oscillations, lines at \((m \pm 2Q_0)\)

Also used to measure
the transverse emittances
(2 pickups needed)

First measurements were
at SLAC, R. Miller, et.al. PAC1983

Comparison of the transverse emittance measurements using the quadrupole pickup and the wire scanner in CERN PS,
A. Jansson, PRSTAB 5, 072803 (2002)
Quadrupole Pickup

Shift of the quadrupolar line due to space charge: a diagnostics which directly rely on a high-intensity effect

\[ Q_{coh,1} - 2Q_{0,x} = -\frac{1}{2}\left(3 - \frac{a_x}{a_x + a_y}\right)\Delta Q_{inc,x} \]

The quadrupole sidebands are strongly Landau damped: a dedicated quadrupole exciter is needed

W.Hardt, CERN ISR/Int 300 GS/66.2 (1966)
R.Bär, et.al., NIMA 415, 460, (1998)
R.Singh, et.al., IBIC2014
Profile Monitors

An example for a diagnostics with distorted signals, but the physical origin did not change

Ionization Profile Monitor (IPM)

The beam particles ionize the atoms of the rest gas and produce ions and electrons, which are detected on a collector.

Electrons need a transverse magnetic field (disadvantages). Ions are often used without magnetic field.

The field of beam space-charge deflect the ions and distort the resulting profiles.
Profile Monitors

Two examples for the IPM usage with beam space-charge

- **Effect of space-charge on IPM in ISIS at RAL, UK**
  - B.Pine, C.Warsop, S.Payne, EPAC2006

- **Model for IPM measurements in AGS bunched beams. R.Thern, PAC1987**

The measured/true beam size model depends on IPM design, bunch parameters, and can give accurate results.
The space-charge voltage changes the bucket potential, the line density and the particle synchrotron frequency

$$V_{sc}(z) \propto -N_p \frac{\partial \lambda}{\partial z}$$

Beam induced voltage adds to the generator voltage and changes the resulting cavity voltage

$$V_b \propto -N_p R_s$$

Reminder
high-intensity effects in the longitudinal plane:
space-charge, beam loading, impedances

Synchrotron frequencies and matched line densities in bunches with space-charge
$$\Delta Q_s = -0.5Q_{s0}, \text{ single rf.}$$

O.Boine-F., O.Chorniy, PRSTAB 10, 104202 (2007)
Longitudinal Phase Space Tomography

Phase space reconstruction for a CERN PS Booster bunch, $6.5 \times 10^{12}$ p, 100 MeV

Effect of space-charge must be taken into account for a correct reconstruction

Summary

Many things change at high intensities:
• beam parameters
• particle/beam oscillations
• lattice/rf settings
• signal propagation to the instrument
thus, the beam signals are distorted.

Some diagnostics are used similarly at low and high intensities (but: instrumental)

Some measurements should be done at the low intensities

There are diagnostic methods which do not work at high intensities

Some diagnostic methods can be used, but an additional effort is needed

There are diagnostic methods which give new opportunities at high intensities

Understanding of the physical basis is necessary