Twelve Limits in Accelerator Physics
Zwei fundamentale Erkenntnisse:

1.) Albert:  
**Energie & Masse Aequivalenz**  
\[ E = mc^2 \]
\[ E^2 = p^2c^2 + m^2c^4 \]

*Energie-Anteil aus der Bewegung*  
*Ruhe Energie*

2.) Louis de Broglie:  
**Welle-Teilchen Dualismus**

\[ \lambda = \frac{h}{p} \]

*h* = Planck’sches Wirkungsquantum  
*p* = Impuls

\[ h = 6,626 069 57(29) \cdot 10^{-34} \text{ Js} \]
\[ = 4,135 667 516(91) \cdot 10^{-15} \text{ eVs} \]

Woher wissen wir eigentlich dass Elektronen quasi punktfoermig sind ???  
\( r < 10^{-18} \)  
... HERA e/p Streuung
A Bit of History

Rutherford Scattering, 1911
Using radioactive particle sources: α-particles of some MeV energy

\[ N(\theta) = \frac{N \cdot n t Z^2 e^4}{(8 \pi \varepsilon_0)^2 r^2 K^2} \times \frac{1}{\sin^4(\theta / 2)} \]
Electrostatic Machines: 
The Cockcroft-Walton Generator

1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV

Particle source: Hydrogen discharge tube on 400 kV level
Accelerator: evacuated glas tube
Target: Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once
Electrostatic Machines: *(Tandem -) van de Graaff Accelerator (1930 ...)*

* Terminal Potential: $U \approx 12 \ldots 28 \text{ MV}$
  using high pressure gas to suppress discharge (SF$_6$)

Problems: * Particle energy limited by high voltage discharges
* high voltage *can only be applied once per particle ...*
  ... or twice?
The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H⁻) and stripping the electrons in the centre of the structure.

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg.
The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:

Energy gained after n acceleration gaps

\[ E_n = n \cdot q \cdot U_0 \cdot \sin \psi_s \]

- \( E_n \) number of gaps between the drift tubes
- \( q \) charge of the particle
- \( U_0 \) Peak voltage of the RF System
- \( \psi_s \) synchronous phase of the particle

* acceleration of the proton in the first gap
* voltage has to be „flipped“ to get the right sign in the second gap \( \Rightarrow \) RF voltage
* shield the particle in drift tubes during the negative half wave of the RF voltage
Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF

Time span of the negative half wave: \( \tau_{RF}/2 \)

Length of the Drift Tube:

\[
l_i = v_i \times \frac{\tau_{rf}}{2}
\]

Kinetic Energy of the Particles

\[
E_i = \frac{1}{2} m v^2
\]

\[
\rightarrow v_i = \sqrt{\frac{2E_i}{m}}
\]

\[
l_i = \frac{1}{v_{rf}} \times \frac{\sqrt{i \times q \times U_0 \times \sin \psi_s}}{2m}
\]

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: \( \approx 20 \text{ MeV per Nucleon} \) \( \beta \approx 0.04 \ldots 0.6 \), Particles: Protons/Ions
Accelerating structure of a Proton Linac (DESY Linac III)

\[ E_{\text{total}} = 988 \text{ MeV} \]

\[ m_0c^2 = 938 \text{ MeV} \]

\[ p = 310 \text{ MeV} / c \]

\[ E_{\text{kin}} = 50 \text{ MeV} \]

**Beam energies**

reminder of some relativistic formula

rest energy \[ m_0c^2 \]

total energy \[ E = \gamma^* E_0 = \gamma^* m_0c^2 \]

kinetic energy \[ E_{\text{kin}} = E_{\text{total}} - m_0c^2 \]

Limit III: length of the acc. structure

Energy Gain per „Gap“:

\[ W = q U_0 \sin \omega_{RF} t \]

\[ E = p^2 c^2 + m^2 c^4 \]
Largest storage ring: The Solar System

**astronomical unit:** average distance earth-sun

$1 \text{AE} \approx 150 \times 10^6 \text{ km}$

Distance Pluto-Sun $\approx 40 \text{ AE}$
1.) Introduction and Basic Ideas

"... in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force

\[ \vec{F} = q * (\vec{E} + \vec{v} \times \vec{B}) \]

typical velocity in high energy machines:

\[ v \approx c \approx 3*10^8 \text{ m/s} \]

The ideal circular orbit

condition for circular orbit:

\[ F_L = e \nu B \]

\[ F_{\text{centr}} = \frac{\gamma m_0 \nu^2}{\rho} \]

\[ \frac{p}{e} = B \rho \]

B \rho = "beam rigidity"
**Limit IV: The Magnetic Guide Field**

![Image of a storage ring dipole magnet](image)

**Circular Orbit:** dipole magnets to define the geometry

\[
\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B \rho}
\]

The angle run out in one revolution must be \(2\pi\)  
so … for a full circle

\[
\alpha = \frac{\int Bdl}{B \rho} = 2\pi \quad \rightarrow \quad \int Bdl = 2\pi \frac{p}{q}
\]

… defines the integrated dipole field

---

**LHC: 7000 GeV Proton storage ring**

- dipole magnets \(N = 1232\)
- \(l = 15\ m\)
- \(q = +1\ e\)

\[
\int B \, dl \approx N \, l \, B = 2\pi \frac{p}{e}
\]

\[
B \approx \frac{2\pi \times 7000 \times 10^9 \text{eV}}{1232 \times 15 \text{m} \times 3 \times 10^8 \frac{m}{s}} = 8.3 \text{ Tesla}
\]
**Focusing Properties – Short Excursion to Classical Mechanics**

**classical mechanics:**

**pendulum**

there is a **restoring force**, proportional to the elongation $x$:

$$m \frac{d^2 x}{dt^2} = -c \times x$$

**general solution:** free harmonic oscillation

$$x(t) = A \times \cos(\omega t + \phi)$$

**Storage Ring:** we need a **Lorentz force** that rises as a function of the distance to ....... ?

............... the design orbit

$$F(x) = q \times v \times B(x)$$
**Quadrupole Magnets:**

required: focusing forces to keep trajectories in vicinity of the ideal orbit

linear increasing Lorentz force

linear increasing magnetic field

\[ B_y = g \, x \quad B_x = g \, y \]

normalised quadrupole field:

\[ k = \frac{g}{p/e} \]

simple rule:

\[ k = 0.3 \frac{g(T/m)}{p(GeV/c)} \]

what about the vertical plane: ... Maxwell

\[ \nabla \times B = 0 \quad \Rightarrow \quad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g \]
The Equation of Motion:

\[ \frac{B(x)}{p/e} = \frac{1}{\rho} + k x + \frac{1}{2!} m x^2 + \frac{1}{3!} n x^3 + \ldots \]

Only terms linear in \( x, y \) taken into account

\( \text{dipole fields} \)
\( \text{quadrupole fields} \)

Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

Example:

heavy ion storage ring TSR
The Equation of Motion:

* Equation for the horizontal motion:

\[ x'' + x \left(\frac{1}{\rho^2} + k\right) = 0 \]

\( x = \) particle amplitude  
\( x' = \) angle of particle trajectory (wrt ideal path line)

* Equation for the vertical motion:

\[ \frac{1}{\rho^2} = 0 \quad \text{no dipoles \ldots in general \ldots} \]

\( k \leftrightarrow -k \quad \text{quadrupole field changes sign} \)

\[ y'' - k \, y = 0 \]
Solution of Trajectory Equations

Define ... hor. plane:  \[ K = \frac{1}{\rho^2} - k \]
... vert. Plane:  \[ K = k \]

Differential Equation of harmonic oscillator ... with spring constant \( K \)

Ansatz:  \[ x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s) \]

general solution: linear combination of two independent solutions

\[ x'(s) = -a_1 \omega \sin(\omega s) + a_2 \omega \cos(\omega s) \]
\[ x''(s) = -a_1 \omega^2 \cos(\omega s) - a_2 \omega^2 \sin(\omega s) = -\omega^2 x(s) \quad \rightarrow \quad \omega = \sqrt{K} \]

general solution:

\[ x(s) = a_1 \cos(\sqrt{K} s) + a_2 \sin(\sqrt{K} s) \]
Hor. Focusing Quadrupole $K > 0$:

\[
x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)
\]

\[
x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)
\]

For convenience expressed in matrix formalism:

\[
\begin{pmatrix}
x \\
x'
\end{pmatrix}_{s_1} = M_{foc} \cdot 
\begin{pmatrix}
x \\
x'
\end{pmatrix}_{s_0}
\]

Determine $a_1, a_2$ by boundary conditions:

\[
s = 0 \quad \Rightarrow \quad \begin{cases} 
  x(0) = x_0, \quad a_1 = x_0 \\
  x'(0) = x'_0, \quad a_2 = \frac{x'_0}{\sqrt{K}}
\end{cases}
\]
**hor. defocusing quadrupole:**
\[ x'' - K' x = 0 \]

*Remember from school:*
\[ f(s) = \cosh(s) \quad , \quad f'(s) = \sinh(s) \]

*Ansatz:*
\[ x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s) \]

\[
M_{\text{defoc}} = \begin{pmatrix}
\cosh \sqrt{|K|} l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|} l \\
\sqrt{|K|} \sinh \sqrt{|K|} l & \cosh \sqrt{|K|} l
\end{pmatrix}
\]

**drift space:**
\[ K = 0 \]

\[
M_{\text{drift}} = \begin{pmatrix}
1 & l \\
0 & 1
\end{pmatrix}
\]

! *with the assumptions made, the motion in the horizontal and vertical planes are independent „... the particle motion in x & y is uncoupled“*
Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

\[ M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*} \ldots \]

\[ \begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2,s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1} \]

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „

\[
\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2,s_1) \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}
\]

typical values in a strong foc. machine:
\[ x \approx \text{mm}, \ x' \leq \text{mrad} \]
5.) Orbit & Tune:

Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

LHC revolution frequency: 11.3 kHz

0.31*11.3 = 3.5 kHz

We treat the transverse movement of the particles along the accelerator as harmonic oscillations with a well defined amplitude and (Eigen-) frequency.

To avoid resonance problems

> keep the tune away from resonance conditions
Question: what will happen, if the particle performs a second turn?

... or a third one or ... $10^{10}$ turns
Astronomer Hill:

*differential equation for motions with periodic focusing properties*

„Hill‘s equation“

Example: particle motion with periodic coefficient

**equation of motion:**

\[ x''(s) - k(s)x(s) = 0 \]

restoring force ≠ const,

\[ k(s) = \text{depending on the position } s \]

\[ k(s+L) = k(s), \text{ periodic function} \]

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.
6.) The Beta Function

General solution of Hill’s equation:

\[
x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)
\]

\(\varepsilon, \Phi = \text{integration constants determined by initial conditions}\)

\(\beta(s) \text{ periodic function given by focusing properties of the lattice } \leftrightarrow \text{ quadrupoles}\)

\(\beta(s + L) = \beta(s)\)

Inserting (i) into the equation of motion ...

\[\psi(s) = \int_{0}^{s} \frac{ds}{\beta(s)}\]

\(\Psi(s) = \text{“phase advance“ of the oscillation between point “0“ and “s“ in the lattice.}\)

For one complete revolution: number of oscillations per turn “Tune“

\[Q_y = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}\]
The Beta Function

Amplitude of a particle trajectory:

\[ x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cos(\psi(s) + \varphi) \]

Maximum size of a particle amplitude

\[ \hat{x}(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \]

\( \beta \) determines the beam size (... the envelope of all particle trajectories at a given position “s” in the storage ring.

It reflects the periodicity of the magnet structure.
7.) Beam Emittance and Phase Space Ellipse

The general solution of the Hill equation

\[ \begin{align*}
\text{(1)} & \quad x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cos(\psi(s) + \phi) \\
\text{(2)} & \quad x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}} \left\{ \alpha(s) \cos(\psi(s) + \phi) + \sin(\psi(s) + \phi) \right\}
\end{align*} \]

From (1) we get

\[ \cos(\psi(s) + \phi) = \frac{x(s)}{\sqrt{\varepsilon} \sqrt{\beta(s)}} \]

Insert into (2) and solve for \( \varepsilon \)

\[ \varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) \]

* \( \varepsilon \) is a constant of the motion ... it is independent of „s“
* parametric representation of an ellipse in the \( x \ x' \) space
* shape and orientation of ellipse are given by \( \alpha, \beta, \gamma \)
Beam Emittance and Phase Space Ellipse

\[ \varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) \]

\[ A = \pi \varepsilon = \text{const} \]

\[ \varepsilon \text{ beam emittance} = \text{woozilycity of the particle ensemble, intrinsic beam parameter, cannot be changed by the foc. properties.} \]

\[ \text{Scientifiquely speaking: area covered in transverse } x, x' \text{ phase space ... and it is constant !!!} \]
Phase Space Ellipse

particle trajectory: \[ x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cos\{\psi(s) + \phi\} \]

max. Amplitude: \[ \hat{x}(s) = \sqrt{\varepsilon\beta} \rightarrow x' \text{ at that position ...?} \]

... put \( \hat{x}(s) \) into \[ \varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2 \text{ and solve for } x' \]

\[ \varepsilon = \gamma \cdot \varepsilon \beta + 2\alpha \sqrt{\varepsilon \beta} \cdot x' + \beta x'^2 \]

\[ x' = -\alpha \cdot \sqrt{\varepsilon / \beta} \]

\[ \ast \text{ A high } \beta\text{-function means a large beam size and a small beam divergence.} \]

\[ \ast \text{ et vice versa} !!! \]

\[ \ast \text{ In the middle of a quadrupole } \beta = \text{ maximum,} \]
\[ \alpha = \text{ zero} \]

\[ \{ \begin{array}{l} x' = 0 \end{array} \] \[ \text{... and the ellipse is flat} \]
**Phase Space Ellipse**

\[ \varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) \]

\[ \varepsilon = \frac{x^2}{\beta} + \frac{\alpha^2 x^2}{\beta} + 2\alpha \cdot xx' + \beta \cdot x'^2 \]

... solve for \( x' \)

\[ x'_{1,2} = \frac{-\alpha \cdot x \pm \sqrt{\varepsilon \beta - x^2}}{\beta} \]

... and determine \( \hat{x} \) via:

\[ \frac{dx'}{dx} = 0 \]

\[ \hat{x}' = \sqrt{\varepsilon \gamma} \]

\[ \hat{x} = \pm \alpha \sqrt{\gamma} \]

**shape and orientation of the phase space ellipse depend on the Twiss parameters** \( \beta \alpha \gamma \)
**Emittance of the Particle Ensemble:**

\[ x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \]

\[ \hat{x}(s) = \sqrt{\varepsilon \beta(s)} \]

**Gauß Particle Distribution:**

\[ \rho(x) = \frac{N \cdot e}{\sqrt{2\pi} \sigma_x} \cdot e^{-\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} \right)} \]

particle at distance 1 \( \sigma \) from centre

\[ \leftrightarrow 68.3\% \text{ of all beam particles} \]

*single particle trajectories, } N \approx 10^{11} \text{ per bunch}*

**LHC:**

\[ \beta = 180 \text{ m} \]

\[ \varepsilon = 5 \times 10^{-10} \text{ m rad} \]

\[ \sigma = \sqrt{\varepsilon \beta} = \sqrt{5 \times 10^{-10} \text{ m} \times 180 \text{ m}} = 0.3 \text{ mm} \]

*aperture requirements: } r_0 = 12 \times \sigma
**21.) Luminosity**

Example: Luminosity run at LHC

\[
\beta_{x,y} = 0.55 \text{ m} \\
\varepsilon_{x,y} = 5 \times 10^{-10} \text{ rad m} \\
\sigma_{x,y} = 17 \mu \text{m} \\
I_p = 584 \text{ mA}
\]

\[
f_0 = 11.245 \text{ kHz} \\
n_b = 2808
\]

\[
L = \frac{1}{4\pi e^2 f_0 n_b} \ast \frac{I_p I_{p1}}{\sigma_x \sigma_y}
\]

\[
L = 1.0 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}
\]
beam sizes in the order of my cat’s hair !!
**β-Function in a Drift:**

*let's assume we are at a symmetry point in the center of a drift.*

\[ \beta(s) = \beta_0 + \frac{s^2}{\beta_0} \]

*At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.*

*-> here we get the largest beam dimension.*

*-> keep \( l \) as small as possible*

8 individually powered quadrupole magnets are needed to match the insertion (... at least)
**Mini-β Insertions: Betafunctions**

A mini-β insertion is always a kind of special symmetric drift space.

→ greetings from Liouville

\[ \alpha^* = 0 \]

\[ \gamma^* = \frac{1 + \alpha^2}{\beta} = \frac{1}{\beta^*} \]

\[ \sigma'^* = \sqrt{\frac{\varepsilon}{\beta^*}} \]

\[ \beta^* = \frac{\sigma^*}{\sigma'^*} \]

At a symmetry point β is just the ratio of beam dimension and beam divergence.

→ At a mini-beta-insertion we have a small beam size σ

→ And a large beam divergence σ'

And both are determined by the EMITTANCE ε as quality factor of the beam
The LHC Insertions

ATLAS R1

Inner Triplet
Q1 Q2 Q3 D1 (1.38 T)
TAN* D2 Q4 (3.8 T)
Q5 Q6 Q7

Separation/Recombination
4.5 K
1.9 K

Matching Quadrupoles

Warm
1.9 K

Mini β optics
ATLAS detector in LHC for 7x7 TeV interactions

Example: Luminosity optics at LHC: $\beta^* = 55$ cm for smallest $\beta_{\text{max}}$ we have to limit the overall length and keep the distance "s" as small as possible.

Limit V: The Interaction Region of a Particle Collider
Liouville, Mini-$\beta$ & Detector Size
**Limit VI: Fixed Target Machines**

**The (Problem of the) Centre of Mass Energy**

**Fixed Target experiments**

accelerated particle beam hits a target at rest

\[ a + b \rightarrow c + d \]

lab system:

\[ p_{b_{\text{lab}}} = 0, \ E_{b_{\text{lab}}} = m_{b}c^{2} \]

centre of mass system:

\[ p_{b_{\text{cm}}} + p_{b_{\text{cm}}} = 0 \]

relativistic total energy

\[ E^{2} = p^{2}c^{2} + (mc^{2})^{2} \]

and for a single particle as well as for system of particles the overall rest energy is constant

\[ \sum E_{i}^{2} - (\sum p_{i}^{2})c^{2} = M^{2}c^{4} = \text{const} \]

\[ (E_{a_{\text{cm}}}^{cm} + E_{b_{\text{cm}}}^{cm})^{2} - (p_{a_{\text{cm}}}^{cm} + p_{b_{\text{cm}}}^{cm})^{2}c^{2} = (E_{a_{\text{lab}}}^{lab} + E_{b_{\text{lab}}}^{lab})^{2} - (p_{a_{\text{lab}}}^{lab} + p_{b_{\text{lab}}}^{lab})^{2}c^{2} \]
The (Problem of the) Centre of Mass Energy

Fixed Target experiments:

\[
(E_a^{cm} + E_b^{cm})^2 \quad - \quad (p_a^{cm} + p_b^{cm})^2 \quad c^2 = \quad (E_a^{lab} + E_b^{lab})^2 \quad - \quad (p_a^{lab} + p_b^{lab})^2 \quad c^2
\]

= 0

\[
W^2 = (E_a^{cm} + E_b^{cm})^2 \quad = \quad (E_a^{lab} + m_b c^2)^2 \quad - \quad (p_a^{lab} c)^2
\]

= 2E_a^{lab} m_b c^2 + (m_a^2 + m_b^2)c^4

for \quad E_a^{lab} >> m_a c^2, \quad m_b c^2

\Rightarrow \quad W \approx \sqrt{2E_a^{lab} m_b c^2}

Taylor/Kendall/Friedman: Discovery of the quark structure of protons and neutrons 1966-1978 ..... 1990 Nobel Price

For high energies in the centre of mass system, fixed target machines are not effective.

... \Rightarrow \text{need for colliding beams}
**Fixed target experiments:**

HARP Detector, CERN

- high event rate
- easy track identification
- asymmetric detector
- limited energy reach

**Collider experiments:**

- low event rate (luminosity)
- challenging track identification
- symmetric detector
- \( E_{lab} = E_{cm} \)

\( Z_0 \) boson discovery at the UA2 experiment (CERN). The \( Z_0 \) boson decays into an \( e^+e^- \) pair, shown as white dashed lines.
**Limit VI: Fixed Target Machines**

→ go for particle colliders

**The (Problem of the) Centre of Mass Energy**

**Colliding Beams experiments:**

\[
(E_a^{cm} + E_b^{cm})^2 - (p_a^{cm} + p_b^{cm})^2 c^2 = (E_a^{lab} + E_b^{lab})^2 - (p_a^{lab} + p_b^{lab})^2 c^2
\]

=0

\[
W^2 = (E_a^{cm} + E_b^{cm})^2
\]

=> \( W = 2E_a^{lab} \)

The full lab energy is available in the center of mass system.

*Prize to pay: we have to build colliders...
beam sizes = μm*
ATLAS event display: Higgs => two electrons & two muons
The High light of the year

production rate of events is determined by the cross section $\Sigma_{\text{react}}$ and a parameter $L$ that is given by the design of the accelerator:

... the luminosity

$1 \text{b} = 10^{-24} \text{cm}^2 = \frac{1}{\text{mio}} \times \frac{1}{\text{mio}} \times \frac{1}{\text{mio}} \times \frac{1}{100} \text{mm}^2$

The particles are “very small”

$R = L \times \Sigma_{\text{react}} \approx 10^{-12} \text{b} \cdot 25 \times \frac{1}{10^{-15} \text{b}} = \text{some} 1000 \text{H}$

The luminosity is a storage ring quality parameter and depends on beam size ($\beta$!) and stored current

$L = \frac{1}{4\pi e^2 f_0 b} \times \frac{I_1 * I_2}{\sigma_x * \sigma_y}$
Limit VIII: Data Taking Efficiency of the Detectors
“event pile up”

The LHC Performance in Run 1

Design 2012

Momentum 7 TeV/c 4 TeV/c

Luminosity (cm$^{-2}$s$^{-1}$) $10^{34}$ $7.7*10^{33}$

Protons per bunch $10^{11}$ 1.15 1.50

Number of bunches/beam 2808 1380

Nominal bunch spacing 25 ns 50 ns

rms beam size (arc) 300 μm 350 μm

rms beam size IP 17 μm 20 μm

Storage ring colliders are very efficient machines:
Bunch collision Frequency: 40 Mhz = 1/25ns
**Limit IX: Luminosity Limit due to Beam-Beam Effect**

**Beam-Beam-Effect**

the colliding bunches influence each other

=> change the focusing properties of the ring !!

for LHC a strong non-linear defoc. effect

**most simple case:**

linear beam beam tune shift

\[
\Delta Q_x = \frac{\beta_x^* \cdot r_p \cdot \gamma_p \cdot N_p}{2\pi \cdot \sigma_x + \sigma_y} \cdot \sigma_x
\]

⇒ puts a limit to \( N_p \)

Eigenfrequency of the particles is changed due to the beam beam interaction

Particles are pushed onto resonances and are lost.

courtesy. K. Schindl
**Luminosity Limits**

**Beam-Beam-Effect**

the space charge of the colliding bunches lead to a strong non-linear defoc. effect and possibly to particle loss.

\[
L = \frac{1}{4\pi} \left( f_{rev} N_p n_b \right) \left( \frac{\gamma N_p^2}{\varepsilon_n \beta^*} \right) \cdot F \cdot W
\]

**effect of beam-beam force in LHC run1**

observed particle losses when beams are brought into collision
**Limit X: RF Acceleration & Momentum Spread**

**Energy Gain per „Gap“:**

\[ W = q U_0 \sin \omega_{RF} t \]

*RF Acceleration*: multiple application of the same acceleration voltage; brillant idea to gain higher energies

**1928, Wideroe**

Drift tube structure at a proton linac (GSI Unilac)

**500 MHz cavities in an electron storage ring**
Problem: panta rhei !!!
(Heraklit: 540-480 v. Chr.)
How do we accelerate ???

Example: HERA RF:

Bunch length of Electrons $\approx 1\text{ cm}$

$\nu = 500\text{ MHz}$
$c = \lambda \nu$
$\lambda = 60 \text{ cm}$

$\sin(90^\circ) = 1$
$\sin(84^\circ) = 0.994$

$\frac{\Delta U}{U} = 6.0 \times 10^{-3}$

$\frac{\Delta p}{p} \approx 1.0 \times 10^{-3}$

typical momentum spread of an electron bunch:
The Acceleration for $\Delta p/p \neq 0$

“Phase Focusing” below transition

- Ideal particle
- Particle with $\Delta p/p > 0$ faster
- Particle with $\Delta p/p < 0$ slower

Focussing effect in the longitudinal direction
keeping the particles close together
... forming a “bunch”
... so sorry, here we need help from Albert:

\[ \gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \rightarrow \quad \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}} \]

\( v/c \)

... some when the particles do not get faster anymore

.... but heavier!
The Acceleration for $\Delta p/p \neq 0$

“Phase Focusing” above transition

- Ideal particle
- Particle with $\Delta p/p > 0$ (heavier)
- Particle with $\Delta p/p < 0$ (lighter)

Oscillation frequency:

$$f_s = f_{rev} \sqrt{\frac{h \alpha_s}{2\pi} \frac{q U_0 \cos \phi_s}{E_s}} \approx \text{some Hz}$$
Energy Gain in RF structures:
Transit Time Factor to optimise the cavities

Oscillating field at frequency $\omega$ (amplitude is assumed to be constant all along the gap)

$$E_z = E_0 \cos \omega t = \frac{V}{g} \cos \omega t$$

Consider a particle passing through the middle of the gap at time $t=0$:

$$z = vt$$

The total energy gain is:

$$\Delta W = eV \frac{g^{1/2}}{g} \int_{-g/2}^{g/2} \cos \omega \frac{z}{v} dz$$

$$T = \frac{\sin \theta/2}{\theta/2}$$ transit time factor ($0 < T < 1$)

$$\theta = \frac{\omega g}{v}$$ transit angle

ideal case: $T = \frac{\sin \theta/2}{\theta/2} \rightarrow 1 \iff \theta/2 \rightarrow 0$

el. static accelerators $\omega \rightarrow 0$

minimise acc. gap $g \rightarrow 0$
RF Cavities, Acceleration and Energy Gain

\[ dW = dE = eE_z ds \quad \Rightarrow \quad W = e \int E_z ds = eV \]

Energy Gain per turn / per passage through an acc. structure is limited by electric discharges that set a limit to the electrical field and so the achievable acceleration gradient \( \Delta E / \Delta s \)

- Typical values: sc. cavities LEP \( \Delta E / \Delta s = 5 \text{ MV/m} \)
- State of the art (ILC) \( \Delta E / \Delta s = 30 \text{ MV/m} \)

... which defines the number of resonators installed in the ring.
Limit XI: Light

c. 400,000 BC: Mankind discovers the Fire
In a circular accelerator charged particles lose energy via emission of intense light.

\[ P_s = \frac{2}{3} \alpha \hbar c^2 \gamma^4 \frac{\rho^2}{\rho^2} \]  
\[ \Delta E = \frac{4}{3} \pi \alpha \hbar c \gamma^4 \rho \]  
\[ \omega_c = \frac{3}{2} \frac{c \gamma^3}{\rho} \]

radiation power  
energy loss  
critical frequency

\[ \alpha \approx \frac{1}{137} \]  
\[ \hbar c \approx 197 \text{ MeV fm} \]

1946 observed for the first time in the General Electric Synchrotron
Synchrotron Radiation as useful tool

structure analysis with highest resolution
Ribosome molecule

Undulator to enhance the synchrotron radiation in e+/e- storage rings
Synchrotron Radiation as aggravating effect in High Energy Rings „Sawtooth Effect“ at LEP (CERN)

In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.

In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.
FCC-ee - Lepton Collider

Limit XI: Light
... the only way out: think BIG ... or think LINEAR
Planning the next generation e+ / e- Ring Colliders

Design Parameters FCC-ee

\[ E = 175 \text{ GeV / beam} \]
\[ L = 100 \text{ km} \]

\[ \Delta U_0 (\text{keV}) \approx \frac{89 \times E^4 (\text{GeV})}{\rho} \]
\[ \Delta U_0 \approx 8.62 \text{ GeV} \]

\[ \Delta P_{\text{sy}} \approx \frac{\Delta U_0}{T_0} \times N_p = \frac{10.4 \times 10^6 \text{eV} \times 1.6 \times 10^{-19} \text{Cb}}{263 \times 10^{-6} \text{s}} \times 9 \times 10^{12} \]
\[ \Delta P_{\text{sy}} \approx 47 \text{ MW} \]

Circular e+ / e- colliders are severely limited by synchrotron radiation losses and have to be replaced for higher energies by linear accelerators.
**Example: FCC-ee**

**Typical Energy of the Photons**

\[ E_{\text{crit}} = 1.2 \text{ MeV} \]

*reminder: visible light \(\approx\) some eV*

**Energy Loss per Turn**

\[ \Delta E_{\text{turn}} = 8.2 \text{ GeV} \]

**Cavity Voltage to compensate losses**

\[ U_{\Sigma \text{cav}} = 11 \text{ GV} \]

courtesy L. Rivkin
Limit XII:  Once more: the Accelerating Gradient

CLIC ... a future Linear e+/e- Accelerator

Avoid bending magnets => no synchrotron radiation losses
=> energy gain has to be obtained in ONE GO
<table>
<thead>
<tr>
<th>Description</th>
<th>500 GeV</th>
<th>3 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total (peak 1%) luminosity</td>
<td>$2.3 \times 10^{34}$</td>
<td>$5.9 \times 10^{34}$</td>
</tr>
<tr>
<td>Total site length [km]</td>
<td>13.0</td>
<td>48.4</td>
</tr>
<tr>
<td>Loaded accel. gradient [MV/m]</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Main Linac RF frequency [GHz]</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>Beam power/beam [MW]</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>Bunch charge [$10^9$ e$^+/e^-$]</td>
<td>6.8</td>
<td>3.72</td>
</tr>
<tr>
<td>Bunch separation [ns]</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Bunch length [$\mu$m]</td>
<td>72</td>
<td>44</td>
</tr>
<tr>
<td>Beam pulse duration [ns]</td>
<td>177</td>
<td>156</td>
</tr>
<tr>
<td>Repetition rate [Hz]</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Hor./vert. norm. emitt. [$10^{-6}/10^{-9}$m]</td>
<td>2.4/25</td>
<td>0.66/20</td>
</tr>
<tr>
<td>Hor./vert. IP beam size [nm]</td>
<td>202/2.3</td>
<td>40/1</td>
</tr>
</tbody>
</table>
The LHC RF system

LHC ... as a low gradient example  16 MV / 27000m

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch length (4σ)</td>
<td>1.06</td>
<td>ns</td>
</tr>
<tr>
<td>Energy spread (2σ)</td>
<td>0.22</td>
<td>10^{-3}</td>
</tr>
<tr>
<td>Synchr. rad. loss/turn</td>
<td>7</td>
<td>keV</td>
</tr>
<tr>
<td>RF frequency</td>
<td>400</td>
<td>MHz</td>
</tr>
<tr>
<td>RF voltage/beam</td>
<td>16</td>
<td>MV</td>
</tr>
<tr>
<td>Energy gain/turn</td>
<td>485</td>
<td>keV</td>
</tr>
</tbody>
</table>

4xFour-cavity cryo module 400 MHz, 16 MV/beam

For the fun of it ...

energy gain per turn = 485 keV
takes 14.4 Mio turns to get to 7 TeV
sums up to 387 Mio km

going linear we have to be much more efficient
Linear Colliders need the highest feasible Accelerating Gradient. RF break downs have to be studied and understood in detail and pushed to the limit.

as they have impact on

\[ \rightarrow \text{the accelerator performance (luminosity)} \]
\[ \rightarrow \text{beam quality} \]
\[ \rightarrow \text{and the accelerating structure itself} \]

“how far can we go and how much can we optimise such a future accelerator before we reach technical limits and how can we push these limits?”
Resume:

In order to reach higher energies and keep the machines still “compact” we need acceleration techniques that are much more efficient than the status quo.

We urgently need new and better ideas ... PWA

And we need them NOW.